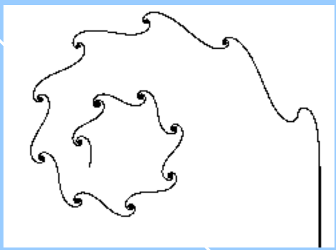
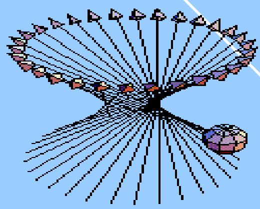
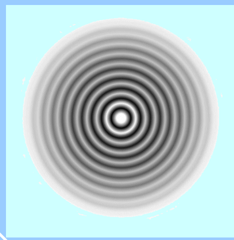
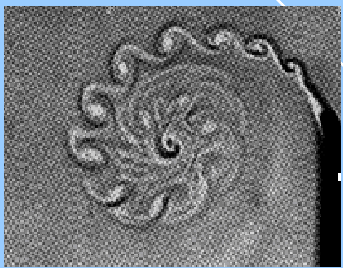
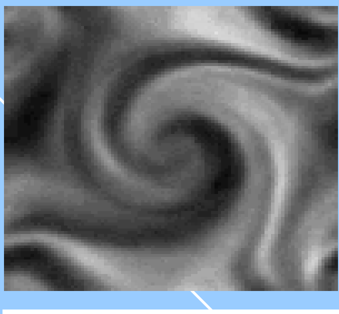
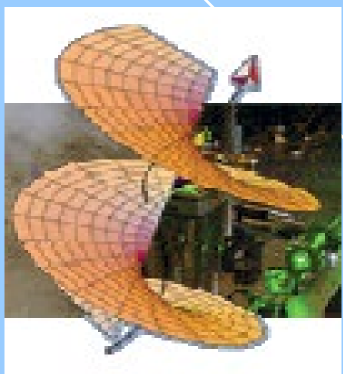












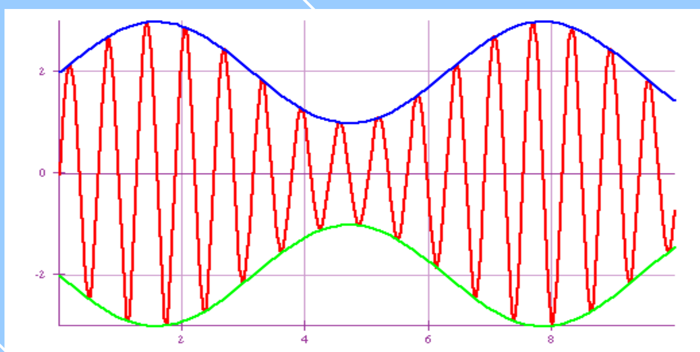
CERTAINTY of PARTICLE – WAVE UNITY

or

UNCERTAINTY of PARTICLE – WAVE DUALITY

Miodrag Prokic

1975 - 2025



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INNOVATIVE ASPECTS OF PARTICLE-WAVE DUALITY, GRAVITATION, AND ELECTROMAGNETIC THEORY ADDRESSED FROM THE POINT OF VIEW OF ELECTRIC AND MECHANICAL ANALOGIES

Author: Miodrag Prokic

Abstract:

This book introduces a novel approach to understanding Wave-Particle Duality, expanding its significance from the quantum realm of micro-world to larger cosmological scales. Through a system of multi-level physical and mathematical analogies, it offers fresh insights and hypotheses on de Broglie's matter waves, providing deeper insights into gravitation, quantum mechanics, and possible modifications to Maxwell-Faraday's Electromagnetic Theory.

The key themes include exploring gravitation as part of a unified system involving masses and fields emerging from atoms and studying the interaction between electromagnetic and mechanical linear and rotational motions. A central proposition is that gravitation, possibly with torsional or spinning field components, may be a manifestation or consequence of the electromagnetic field. Another perspective suggests that gravitational force could also arise from wave-particle duality and intrinsically associated standing matter waves, around and between masses, like phenomena of ultrasonic levitation.

Additionally, the book critiques mainstream interpretations of Orthodox Quantum Theory (QT), proposing that more appropriate mathematical models could lead to a more deterministic, and improved understanding of the quantum world. It questions the exclusive probabilistic nature of traditional QT, asserting that its self-contained mathematical formalism, largely based on assumptions, lacks a firm conceptual and causal foundation. New generalized versions of the Schrödinger and Dirac equations are smoothly and deterministically developed in this book, bypassing modern QT probabilistic assumptions.

One of major contribution in this book is the generalization of the Schrödinger equation using complex analytic signal functions (Phasors) based on the Hilbert transform. This technique establishes universally applicable relationships between a moving particle and its wave-packet, group, and phase velocity. The book also questions the inclusion of rest energy and rest mass in current QT matter-wave models, advocating for a focus on states of motional energy in de Broglie's wave-particle duality (without rest mases participation).

Moreover, the stochastic and probabilistic behavior of quantum mechanical wavefunctions is revisited. The book argues that such behavior results from the "in-average" modelling of particle-wave duality using tools from Probability, Statistics, and Signal Analysis, whenever mathematically reasonable. Orthodox QT, while conserving energy and momentum, overlooks real-time and space-dependent signal-phase, eventually leading to the "magic" effects observed (during longer time) in diffraction experiments with single particles.

The work further extends the concept of particle-wave duality to all forms of moving or temporal-spatial dependent energy flow. It proposes that any sudden variation in motional or total energy must be associated with de Broglie matter waves, producing effects that can be gravitational, mechanical, or electromagnetic forces.

Key Contributions

The book identifies several key areas where modern physics, especially Quantum Theory, could be enhanced and updated through innovative thinking:

1. Electromechanical Analogies as a Foundational Framework

By comparing and modelling physical relations using a system of generalized electromechanical analogies, it becomes possible to harmonize different domains of physics.

2. Revival of a Carrier Medium for Matter Waves

The propagation of electromagnetic waves in a vacuum requires the reestablishment of a generalized Maxwell-Faraday electromagnetic theory, potentially involving a fluid-like waves-carrier medium (what is valid for all other natural matter waves).

3. Complex Mathematical Representation of Matter-Waves

Matter-waves should be modelled using complex or hypercomplex waveforms, formulated as Analytic signal functions, or Phasors based on the Hilbert transform.

4. Reconceptualizing Quantization

Quantization should be understood as emerging from temporal-spatial periodicities, rules of signal analysis and synthesis in mater structures and motions, applicable across both the micro and macro scales of physics.

5. Reevaluation of Fundamental Forces

A reimagined framework for the fundamental forces of nature, particularly in Quantum and Relativistic physics, is proposed.

6. Limited Role for Probability and Statistics

Probability and statistics should be applied (whenever mathematically grounded and defendable) mostly in the final stages of modelling, where they help interpret results, rather than serving as foundational and ontological concepts for explaining physical phenomena.

Discussion of Key Topics

1. Mathematical and Physical Foundations

The book begins by questioning the current theoretical models and scientific principles (about matter in motion) that should form the basis for universally applicable physical and mathematical frameworks. It challenges the ontologically probabilistic models prevalent in quantum physics today, advocating for mathematical models grounded in verifiable, deterministic principles.

2. Macro-Level Physics Principles

Conservation laws, symmetries, and fundamental principles governing mechanics, fluid dynamics, thermodynamics, and electromagnetism are examined as structural and foundational framework of our universe.

The exciting and challenging works and assumptions of Nikola Tesla and Ruđer Bošković are highlighted for their indicative conceptual contributions to the broader structure of physical laws.

3. Micro-Level Structures and Approximations

The book investigates the fundamental micro-processes that evolve and govern the physical world, emphasizing the importance of connecting higher-level theories to elementary processes.

4. Electromagnetic Nature of Matter

Matter is presented as fundamentally and dominantly electromagnetic in nature, with specifically structured electromagnetic energy forming of masses and driving electro-mechanical principles. Maxwell’s electromagnetism (conveniently upgraded) serves as the foundational theory linking classical, relativistic and quantum mechanics.

5. Mathematics as the Language of Nature

Mathematics (that emerges from experimental physics) is presented as the universal language and logic of natural phenomena. An essential concept within mutually transformable space-time-frequency domains, like Parseval’s theorem, is playing a crucial role in explaining and satisfying energy conservation across physics.

6. Real and Measurable Matter Waves

Matter waves are treated as real, measurable, dimensional phenomena, emanating from classical physics to electromagnetism. Schrödinger's equation, as originally postulated in QT, is described as an “luckily correct”, but artificial construct that obscures the natural wave properties of matter.

7. Complementary Wave Coupling

All matter-waves are proposed to involve a coupling between complementary fields, such as electric and magnetic fields in electromagnetism, linear and angular motions in mechanics and gravitation, or as real and imaginary parts of an Analytic Signal. This principle is still not explored in lesser-known areas, like gravitation.

Concluding Thoughts

While presented as a brainstorming draft, the book aims to inspire creative exploration of innovative, indicative and challenging ideas addressing physics. It acknowledges its open-ended nature but offers a wealth of innovative conclusions and proposals. For traditionalists, the ideas advanced in this book may seem oversimplified and speculative, but for those open to rethinking and enriching already established theories, the book may present a stimulating and imaginative framework for significant advancement in physics.

Miodrag Prokic

2025, Le Locle, Switzerland

CONTENTS

<u>Appendix:</u>

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INTRODUCTION

Introduction to the Theory of Particle-Wave Unity and Unified Natural Fields

This book primarily focuses on developing an updated theory of particle-wave duality, or more precisely, particle-wave unity. In doing so, it opens the door to a new understanding of gravitation and provides the foundation for a unified theory of natural fields and forces. The innovative perspectives presented draw on the pioneering work of figures like Nikola Tesla and Rudjer Boskovic (see [6] and [97]).

The book has several key objectives:

1. Gravity and Particle-Wave Duality

Gravity is explored through the specific coupling of linear and angular (or rotational) motions and their associated electromagnetic and particle-matter-waves-duality complexities. These interactions originate at the atomic level of matter and extend to larger masses. The gravitational field, including its hypothetical torsional components, is proposed as an electromagnetic structure, coupled with various states of mass, energy, and momentum. This structure operates within a global standing-wave framework, resonating across both micro and macrocosmic scales. The principles of de Broglie's matter-wave theory apply to both the microscopic and macroscopic realms when proper temporal and spatial considerations are made. Synchronous conservation of stable, uniform inertial motions (both linear and angular) is satisfied and mutually coupled thanks to associated matter-wave fields (manifesting effects of gravitation).

2. Tangible Mass

Rest mass is viewed as a specific stabilized state of matter-wave energy, where atoms form ensembles of spatial-temporal oscillators or standing waves resonators. These masses are stabilized and synchronized through electromagnetic field couplings, with mass states emerging from various energy forms, quantified as $\mathbf{m^* = E_{total} / c^2}$*. Even massless particles have an energy equivalence, emphasizing that mass is a manifestation of deeper energy structures.*

3. Matter Waves

Matter waves serve as transient, intermediary states, enabling synchronization and coupling between atoms and masses in motion. These waves bridge the micro and macro worlds, acting as dual "spatial temporal" formations. Particle-wave duality arises from the transformability between translational and rotational motion symmetries and/or coupling of linear and rotational inertial states, coupled with familiar electromagnetic states, with particles formed by the superposition of matter waves.

In contemporary physics, significant progress has been made in seemingly isolated domains: Mechanics (including Gravitation Theory), Maxwell's Electromagnetic Theory (EM), Relativity (RT), and Quantum Theory (QT). However, the theory of particle-wave duality must be integrated with these frameworks to pave the way for a unified field theory. This theory posits that particle-wave duality arises from atoms, matter structure, and field couplings as associated space-time-dependent forces between involved moving objects and matter states. Without these interactions, particle-wave duality would not exist.

The book also argues that particles cannot be created without rotation and spin properties of matter, which are coupled with linear motion and torsional matter-wave components. This dynamic is akin to how solenoid-shaped magnetic fields form around moving electric charges. The term "particle-wave duality" is used temporarily, but the more accurate concept of "particle-wave unity" better explains the causal relationship between particle and wave properties.

Revisiting Established Theories

While our universe is structurally unified, current frameworks like Mechanics, RT, EM, and QT remain theoretically unsynchronized, suggesting potential foundational issues. For instance, Einstein and others attempted to unify EM and gravitation within Relativity theory (and later QT), but faced challenges due to two major conceptual gaps:

1. Special Relativity Theory (SRT)
SRT overlooks rotational, angular, vibrational, and other accelerated motions, which are intrinsic to all elementary particles and atoms. An upgraded SRT, along with a revised General Relativity (GRT), should emerge as consequences of an improved and more coherent EM theory, leading to mathematically more symmetrical formulations.

2. Electromagnetic (EM) Theory
Current Maxwell equations lack torsional field components and fail to account for longitudinal electric and magnetic fields in certain cases. This book suggests innovative and evolving, self-regenerating Maxwell's equations to include these elements, bringing a more symmetrical understanding of electric and magnetic fields, and thus, a renewed Relativity Theory (see more in Chapter 3).

To meaningfully unify RT and EM, significant upgrades of both are needed, as well as a deeper integration of QT with these frameworks. Quantum Electrodynamics (QED) has achieved experimental verification and precision; however, it does not provide a complete ontological and elemental understanding of how Quantum Theory (QT), Relativity Theory (RT), Electromagnetism (EM), and Gravitation can be naturally unified outside of axiomatic and mathematical frameworks. *This book proposes aspects of an upgraded Gravitation Theory (GT), linking gravitation more closely to specific matter-waves and electromagnetic phenomena.*

Towards a Unified Theory

Einstein's Relativity, along with modern theories like Superstrings or M-theory, represent innovative attempts to unify natural forces. However, this book argues that the foundation of any unified field theory should be rooted in the Particle-Wave Duality Concept (PWDC). Once the foundations of PWDC are solidified, future theoretical advancements, such as string theory, can be more effectively built (see more about PWDC in Chapters 4.1 and 10).

The multilevel analogies proposed in the first chapter serve as a conceptual framework to explore and unify particles and fields. These analogies help to test and refine both old and new theories, moving beyond the isolated (only locally valid) models of contemporary physics. By recognizing that all natural phenomena are interconnected, we can better unite currently separated knowledge bases in physics.

Upgrading Electromagnetic Theory and Relativity

In Chapter 3, the book demonstrates how electromagnetic (EM) theory can be upgraded to generate Lorentz-like transformations without relying on Special Relativity (SRT). Generalized definitions of electric and magnetic fields lead to an extended Maxwell-Faraday theory that includes gravitation. These improvements highlight the potential for unified field theory without the need for existing frameworks like SRT (since the new SRT will be directly generated by upgraded EM theory).

Additionally, the unity of linear and rotational motion and linear and rotational inertia is emphasized as a critical factor in understanding matter-wave phenomenology. Every particle's *motion, whether linear or rotational, is inherently coupled with complementary, "matter-waves spinning motion", suggesting that rotational dynamics must be incorporated into an upgraded understanding of Relativity and Quantum Theory.*

Reassessing Quantum Theory

This book also challenges the foundations of Orthodox Quantum Theory (QT), suggesting that its probabilistic framework, while useful, is not the only way to understand the quantum world. By applying more deterministic models and extending wave equations through Complex Analytic Signal modeling, we can achieve similar or richer results without dominantly relying on probabilistic interpretations. This rethinking opens the door to correcting misconceptions, such as those embedded in the Heisenberg Uncertainty principle.

A New Framework for Unified Physics

In summary, this book advocates for a more deterministic and unified approach to physics. By using multilevel analogies, symmetries, and upgraded electromagnetic and gravitational theories, we can create a more coherent and comprehensive understanding of the universe. The proposed Particle-Wave Duality Concept (PWDC) serves as a key element in this framework, linking established theories and offering new paths for future theoretical developments (see more about PWDC in Chapters 4.1 and 10).

The Role of Multilevel Analogies in Physics

*This book employs **multilevel analogies** as a strategic tool to explore and unify diverse areas of contemporary physics. Today, Physics is fragmented into specialized, complex theories, each developed over time by numerous researchers. Engaging deeply in any one area often requires a lifetime of specialization, making it challenging to see the bigger picture, or question established models. Additionally, unconventional ideas are met with resistance, especially when they challenge longstanding theories that already form the foundation of many scientific careers.*

Analogies: A New Approach to Physics

*The author advocates for the use of **analogies** as a neutral and accessible way to generate new ideas and hypotheses, or to correct mistakes made in the past. Analogies provide a platform for exploring gaps between different theories without inciting resistance from the mainstream scientific community. Even when predictions derived from analogies are not entirely accurate, they can yield valuable or indicative insights, bringing us closer to a more integrated understanding of physics.*

The Power of Symmetry in Physics

***Symmetry**, particularly continuous symmetry, is fundamental to understanding conservation laws and other core principles in physics. However, to fully utilize the power of symmetry, we must first establish a strong framework of relevant elements and mathematical models. This book blends multilevel analogies with a profound understanding of symmetry, supported by rigorous mathematics, to chart a course toward advancing our comprehension of the natural world (see more in the first Chapter about Analogies).*

The Evolving Role of Mathematics

***Mathematics** is indispensable in physics, aiding in modelling, prediction, interpolation, extrapolation, and bridging knowledge gaps. Yet mathematics, like physics, is continually evolving. As the best language and logic of physics, it adapts in response to our expanding* *understanding of the universe. Recognizing this evolving relationship emphasizes the importance of refining both disciplines in tandem to achieve more accurate representations of reality.*

A Critique of Modern Scientific Practices

*The author critiques certain **modern scientific practices**, particularly the over-reliance on statistical and probabilistic, ontological interpretations of the universe. These methods often lead to superficial and oversimplified explanations, such as "tunneling effects" or "dark mass and energy," which serve as placeholders for other deeper understanding. The reluctance to question such concepts often stems from the vested interests of those whose careers are tied to maintaining the status quo. This book calls for a shift toward exploring analogies and symmetries, which offer more substantial and enduring insights.*

Learning from History: The Path to Progress

*Rather than dwelling on past mistakes, the author emphasizes building on **what already works well**, using analogies and symmetries as guiding principles. By acknowledging the structural and intrinsic unity of nature, we can make stronger predictions and deepen our understanding of the universe. Historical examples, like Ptolemy's geocentric model, demonstrate that even entrenched theories can be fundamentally flawed while still sufficiently explaining many phenomena.*

A Framework for the Future

*In conclusion, this book proposes a **framework** rooted in analogies and symmetries as a foundation for future advances in physics. This approach aims to help avoid the shortcomings of current scientific practices and guide us toward a more unified and coherent understanding of the natural world.*

Challenges for Independent Thinkers

*Independent thinkers and creative amateurs face significant obstacles in engaging in established science, not just in opposition from mainstream figures but in the lack of a shared language. Formal education, while essential, does not guarantee **creativity** or innovative and original thinking. Certain gifted amateurs, although they may lack formal credentials, often bring **novel insights** through their natural intelligence and creativity.*

The Need for Independent Minds in Science

*Scientific breakthroughs often come from **independent thinkers** who challenge the status quo. As philosopher F. Nietzsche observed, extraordinary results tend to come from bold, independent personalities. Unfortunately, many established scientists are driven more by the need to publish for career advancement than by the originality of their work, producing papers that may sound compelling but rest on weak foundations.*

Critique of Established Scientific Systems

*Over time, **established scientific systems** can prioritize self-preservation over innovation. They cling to outdated or unverifiable data, perpetuating weak theories rather than replacing them with superior ones. This is evident in the increasingly complex layers of quantum theory and relativity, which are frequently modified instead of being re-examined from first principles.*

Simplicity and Openness in Physics

*The author advocates physics to be expressed in a **simple, clear, and coherent** manner, grounded in experimentally verified and mathematically described facts. **Talented outsiders** are often better positioned to see the novelties and simplicity in theories, while professionals, constrained by their education, may lose sight of some fundamental questions.*

*The complexity in modern physics often arises from its foundational assumptions. By defending these assumptions, mainstream established science resists new ideas that challenge the established framework. However, with the rise of global communication, Artificial Intelligence systens, and high-powered computing, traditional power structures are slowly being dismantled. **Mathematics, imagination, and creativity** are now becoming more prominent, enabling individuals to spread their ideas without obstruction from established authorities.*

New Channels for Creativity

*With the advent of high-power computing and global communication means, including AI (Artificial Intelligence), traditional barriers in science are breaking down. **Mathematics, imagination, and creativity** are increasingly dominant, allowing almost anyone to spread their ideas, irrespective of the opinions of conservative authorities. This tension between traditional scientific power structures and emerging channels of communication and computation is a defining characteristic of our current era, which remains in evolution and transition.*

[♣ Comments & Free-Thinking Corner ♣]

Throughout the book, you will find small sections enclosed in square brackets, such as [♣ COMMENTS & FREE-THINKING CORNER ♣]. These segments are meant as spaces for intuitive brainstorming, where quick thoughts and challenging ideas are loosely presented. The aim is to capture potentially valuable ideas and concepts that may later be developed into more refined contributions to physics.

These comments also document the real-time thought processes behind the development of ideas, offering insight into how the author was thinking while formulating new contributions. These sections can be skipped by readers focusing on the main content, but they may prove thought-provoking for those interested in the brainstorming process.

A Long Journey of Evolution

This book has been evolving for over 40 years, starting around 1975 and continuing to the present. Because of this long development period, certain ideas and concepts may appear redundant, not mutually harmonized, scattered, or disjointed, as they were written, revised, and updated at different times. While this book is an amateur effort, the author believes that the ideas presented have the potential to be innovative, motivating, indicative and significant for creating better Physics.

For Unconventional Thinkers

*This book is not aimed at those who are fully committed to the **orthodox establishment** of probabilistic quantum and relativity theories. Instead, it is intended for **unconventional thinkers, skeptics, and creative minds** who are open to exploring innovative, intuitive, hypothetical, and brainstorming approaches. Recognizing that this book is still unfinished, the author has chosen to release it as an open-ended draft, hoping it will serve as a platform for further exploration and innovation in Physics.*

Miodrag Prokic,
2025, Le Locle, Switzerland. *(Works on this book started 1975. First time published in May 2006).*

INTRODUCTION (in Serbian Language)

U analizama cesticno-talasnog dualizma materije, potrebno je pre svega imati u vidu fundamentalne situacije i pitanja, kao na primer:

1. Koji je to sadasnji kostur, metod ili struktura, raspolozivih puteva, naucnih oslonaca i teoretskih platformi odakle mozemo da delujemo, a da znamo da smo na stabilnom i plodotvornom terenu. Drugim recima, sta cemo usvojiti kao uvek vazece, univerzalno primenljive platforme u domenu matematike i fizike. Ovde se podrazumeva da ontolosko-probabilisticki koncepti, modeliranja i teorije u domenu fizike ovde ne pripadaju, ali da se veoma uspesna modeliranja procesa sa velikim brojem identicnih ili slicnih clanova mogu bazirati na teoriji verovatnoce i statistici.

2. Koja je to "makro obvojnica" onoga sto se u svetu fizike dogadja, tj. koja je to gornja granica, ili aproksimacija koju cemo imati u vidu, kad nasa partikularna saznanja i nove teorije ukalupimo u neke sire naucne i teoretske okvire (u makro svetu fizike). To ce biti zakoni odrzanja, analogije, simetrije, principi koji definisu ponasanje svih sistema sa nekim kretanjem, matematicka logika i opste istine i principi sveta kontinuuma, ili makro sveta koji nas okruzuje. Neizbezna kreativna pozadina i neki od temelja svega toga su i dela nasledjena od Nikole Tesle i Rudjera Boskovica (vidi [6] i [97]).

3. Isto tako, mozemo se pitati koji su to neosporni, unutrasnji, bottom-line ili pozadinski mikro procesi, strukture i donje aproksimacije kojima sve mora da tezi kad polazeci od nekih makro struktura nesto svodimo na najelementarnije i minimalne strukture I elemente (ali da to bude nesto opipljivo, merljivo, vidljivo i matematicki uhvatljivo…), jer visi makro sistemi i teorije moraju u nekoj svojoj nultoj, osnovnoj aproksimaciji da se glatko i kontinualno spajaju sa teorijama i situacijama koje vaze u mikro svetu fizike. Na neki nacin nasa realnost se uvek nalazi u egzistencijalnom domenu izmedju dve beskonacnosti (tj. naucno i saznajno prodiranje u mikro i makro univerzuum je najverovatnije kao prodirenje u dve suprotno orijentisane beskonacnosti).

Strategija ili odgovor koji se ovde favorizuje je da su alati koji nam trebaju: izabrana poglavlja matematike, klasicna fizika i mehanika, teorija fluida, termodinamika i u velikoj meri Maxwell-ova teorija elektromagnetizma. Tu je skoro sve kauzalno, jasno, proverljivo, uhvatljivo opstim principima i ponovljivim eksperimentima, i podleze jasnoj, prirodno povezanoj matematici (a ne bazira se na apstraktnim, postuliranim i vestacki definisanim matematickim teorijama). Sada ide pitanje zasto je to tako:

1. ***Zato sto je matematika univerzalni jezik i logika prirode i formira se na bazi eksperimentalnih, induktivnih, deduktivnih, analognih i empirijskih zapazanja i generalizacija koje imaju svoje izvore i modele u prirodi (tj. u fizici).***

2. ***Zato sto je gradja mase ili materije u svojoj dubokoj sustini elektromagnetske prirode***, gde se na specifican nacin uoblicena i upakovana elektromagnetska energija manifestuje kao masa (i moze biti u

raznim stanjima kretanja), pa posle dolazi sve ono sto se proucava u mehanici…

3. ***Zato sto prosirene, kontinualne simetrije i analogije (bazirane na « Mobility » analogijama koje su opisane u prvom poglavlju ove knjige) neosporno vaze iz vise uglova posmatranja, jer opisuju jedan isti univerzum (nas), i sve su to razne vrste prostornih i intelektualnih preseka, projekcija i korelacija iz razlicitih uglova, a zajednicko im je da ih opisuju iste ili slicne matematicke jednacine ili modeli, jer se odnose na isti i vec ujedinjeni univerzuum u kome zivimo.*** Sto mi jos uvek nismo u stanju da formulisemo univerzalnu teoriju polja, nije ni bitno, jer je ta (nasa) priroda vec ujedinjena (pre nase civilizacijske pojave) i ne mora bas nas da ceka, prati, slusa i da uvazava sve nase teorije kako bi po tim teorijama bila ujedinjena… To prirodno jedinstvo (kao sto bi ga opisivala neka nova jedinstvena teorija polja i sila) vec postoji nezavisno od nasih metoda opisivanja prirode i ostalih teorija i koncepata kojima masemo i to je izvor svih simetrija i analogija u prirodi. Mnogi se slazu da gravitacija kao poglavlje fizike nije jos uvek u svojoj zavrsnoj formi i da nije povezana sa kvantnom teorijom (a verovatno je i bespredmetno i nerealno ocekivati da moze biti povezana sa sadasnjom kvantnom teorijom). ***Prema univerzalno vazecim elektromehanickim analogijama (sto je kao neki prirodni zakon ili kostur koji se mora uvaziti), mi mozemo da vidimo da silu gravitacije ne izaziva sama masa, nego masa u kretanju ili oscilovanju, sa nekoliko pridruzenih atributa, tj. izvori gravitacije su: medjusobno spregnuti linearni i ugaoni momenti i njima pridruzeni (spoljasnji i unutrasnji) dipolni i magnetni, ili elektromagnetni momenti, tj. masi pridruzeni elektricni naboj i magnetni fluks. Mase sa takvim pridruzenim i medjusobno spregnutim atributima mogu na razne nacine da se medjusobno privalace ili odbijaju, a jedan od tih nacina privlacenja (koji ima neku svoju specificnu kombinaciju pomenutih atributa) rezira ono sto zovemo Gravitacijom. Mi isto tako znamo da se u nekoj meri gravitacija i elektromagnetizam « medjusobno osecaju » i intreaguju (jer putanje fotona bivaju savijene u jakim gravitacionim zonama), a znamo i da gravitacija i elektromegnetsko polje interaguju skoro sa svim ostalim konstituentima naseg univerzuuma.*** Ovoglediste o gravitaciji (da sama masa nije primarni izvor gravitacije) se razlikuje od gledista armije onih koji dosledno slede Njutna i Ajnstajna i kvantnu teoriju (gde vazi da samo masa predstavlja glavni gravitacioni naboj ili izvor), ali kako fizika nije politika, tu nadglasavanje i kvorum istomisljenika i njihovih poslusnih sledbenika nista ne znace. Uprosceno receno: neki dobar djak koji poseduje sva akademska priznanja ovoga sveta i koji je napisao mnogo radova o gravitaciji, nije ustvari mnogo dao i razumeo gravitaciju ako kroz svoja razmisljanja i radove pokazuje da ne postuje Mobility system elektromehanickih analogija… Tako je i Ptolomejev geocentricni sistem ostao na snazi oko 1000 godina, jer religiozni mudraci nisu zeleli da vide i uvaze neosporne astronomske cinjenice... I tada (u vreme vazenja Ptolomejevog geocentricnog koncepta) je bilo kao i danas: veoma mnogo uvazenih i visoko-pozicioniranih mainstream autoriteta su se dogovorili i rekli da njihov tj. Ptolomejev model mora da vazi po svaku cenu, a ostali nesrecno rodjeni koji su dokazivali i videli to isto drugacije su spaljivani... Sada barem

znamo da necemo biti spaljeni zbog ovakvih pisanja (ali da budemo kritikovani i ismejani zbog ovakvih misljenja, to je najmanje sto se moze realno ocekivati, sto i nije tako strasno ako znamo da se borimo za cilj kao sto je naucna istina o nasem univerzumu…).

4. Naravno, tu treba dodati apsolutno postovanje svih kontinualnih simetrija i neospornih zakona odrzanja koji su univerzalno primenljivi u fizici, mehanici i elektromagnetizmu... (bez uvodjenja probabilistickih i statistickih izuzetaka).

5. Na bazi deduktivnog pristupa mi imamo sansu da (hipoteticki i analogno) dodjemo do indikativnih elementarnih principa, pa da posle toga sve to ispitujemo i usaglasavamo induktivnim i iterativnim naucnim I matematickim metodama…

Sada dolazimo do pitanja kako startovati od takvih generalizovanih, osnovnih i okvirnih platformi i principa i sve to primeniti na svet fizike. Odgovor koji se ovde favorizuje je da je najbolje da se u najvecoj meri drzimo samo dobro selektovanih, proverenih i univerzalnih matematickih teorija, principa, zakonitosti, modela, jednacina, analogija i logike… jer ako se toga drzimo onda necemo imati sanse da mnogo pogresimo. Koji su to univerzalno primenljivi matematicki blokovi koje mozemo koristiti i koje moramo uvek postovati:

1. Parsevalova teorema koja je neka vrsta prezentacije univerzalno vazeceg zakona o odrzanju energije u vremenskom I frekventnom domenu… Na primer: Ako se toga ne drzimo, imacemo kvantno-mehanicku i ontoloski-probabilisticku talasnu funkciju i jedan apstraktni i vestacki matematicki aparat koji ce sve to podrzavati na konceptualno nedovoljno jasan nacin(sto je sadasnji slucaj sa kvantnom teorijom).

2. Talasi matterije su realni i merljivi i analiziramo ih preko univerzalnih talasnih jednacina (gde podrazumevamo da sve talasne jednacine matematicke fizike imaju korene u d'Alambert-ovoj i klasicnoj talasnoj, parcijalnoj diferencijanoj jednacini drugog reda)… Bez obzira sto je mehanika i teorija fluida izvorno proizvela te klasicne talasne jednacine, one mora na odredjeni analogan nacin da vaze i u elektromagnetizmu uz odgovarajuce dorade modela i, kako znamo, takve analogne jednacine elektromagnetskih talasa su vec izvedene u izvornoj Maxwell-ovoj teoriji… ***Cesticno-talasni dualizam matterije zapravo mora da obuhvata sve realne, merljive talasne, vibracione i oscilatorne fenomene u fizici... (a ne samo one iz domena mikro sveta koji su prirodno probabilisticki, direktno nemerljivi i jos uvek egzoticni). Ako talasnu funkciju klasicne talasne, diferencijalne jednacine drugog reda tretiramo kao funkciju koja je u formi kompleksnog analitickog signala (formiranog koristeci Hilbertovu transformaciju), mozemo direktno i lako izvesti sve talasne jednacine kvantne i mikro fizike (bez ubacivanja novih elemenata u takve jednacine, kao sto je to radjeno u kvantnoj teoriji tokom formiranja Schrödinger-ove jednacine).*** Posto postoji neka linearna i direktna veza izmedju prostornih i vremenskih dimenzija (kao sto se vidi u faznom clanu talasne funkcije (ωt - kx), odakle se izvode grupna i fazna brzina svih talasa matterije), odatle mi dobijamo sve ono sto je

Schrödinger vestacki nakalemio, pa posle toga dobijamo talasne jednacine kvantne mehanike... Dakle, Schrödinger-ov rezultat je tacan, ali metoda kako je do toga dosao je neko intuitivno, vestacko i neprirodno kalemljenje (ili dodavanje novih clanova), sto se ne sme legalizovati kao nesto univerzalno preporucljivo, dobro i naucno prihvatljivo.

3. Na raspolaganju nam stoji i analiza I sinteza signala u vremenskom i frekventnom domenu, bazirana na univerzalnom matematickom modelu svih kretanja (ukljucujuci i talasna) kao sto je analiticki signal (sto je generalizacija fazora na bazi Hilbertove transformacije).

4. Tu su i ostale vazne grane matematike kao sto je vektorska analiza itd.

Dakle, kao zakljucak, trebalo bi da se drzimo vrhunskog principa koji glasi: ***Ne pocinjimo nista drugo ili novo (prilikom modeliranja i formiranja, ili inoviranja teorija fizike) dok postojecu ili prethodnu situaciju ne razcistimo i stabilisemo na ovim, gore pomenutim, okvirima, koji treba da budu nasi osnovni i polazni nivoi, tj. temelji novih teorija…***

Normalno je da cemo u sadasnjoj fizici naci i neke kontradikcije i praznine, jer je istorijski mnogo toga zapoceto bez postojanja zajednicke osnove i formirano iz mnogo razlicitih uglova… Bez jasnih generalnih vidjenja i matematickih struktura i putokaza, mi mozemo samo da budemo izgubljeni u nizu mogucnosti sa razlicitim verovatnocama izbora, ili da budemo nesigurni i sa lose formulisanim ciljevima… Ne moze se nesto novo u fizici stvarati bez cvrstih, neospornih, zajednickih, univerzalnih temelja i bez jedinstvene matematike, logike i filozofije prirode… (sto u ranim danima formiranja Fizike i Matematike nije bio slucaj).

Kada se, na primer, uhvatimo modela Analitickog signala (koji je generalizacija FAZORA za sve sto se krece), videcemo da svaki signal ili funkcija nekog kretanja ima svoj Hilbertov, ortogonalni par, tj. drugu, komplementarnu ili pridruzenu (fazno pomerenu) funkciju kretanja… To je slicno situaciji u vezi sa elektromagnetskim talasima (i verovatno primenljivo mnogo sire), gde su elektricno i magnetno polje medjusobno povezani, tj. uvek nastaju u spregnutim parovima medjusobno komplementarnih polja… Radi se o fazno pomerenim talasnim funkcijama (za $\frac{\pi}{2}$), ali jednu od tih funkcija mi zovemo elektricnim poljem a drugu magnetnim jer im je prostorna geometrijska ili vektorska forma drugacija (jedno polje je laminarno a drugo spiralno…).

Najbolje je da uvek postujemo generalno vazece i veoma primenljive matematicke principe, modele i structure (kao sto su, least action, Euler-Lagrange equations, and Lagrangians and Hamiltonians) i da se sto manje prilagodjavamo parcijalnim, usko vazecim modeliranjima, abstraktno I vestacki definisanim teorijama i raznim fitovanjima na bazi statistike I teorije verovatnoce… Jedan od takvih generalno primenljivih koncepata na sva kretanja je i Doplerov efekat, ali je problematika strukture i kretanja matterije ukljucujuci cesticno-talasni dualizam jos sadrzajnija nego sama primena Doplerovih formula.

Najverovatnije i najprirodnije je da su ***svi talasi koji se spominju i fizici (pod raznim obicnim i neobicnim imenima kreativnih autora, ili nazvani prema***

sredini u kojoj se prostiru) zapravo deo iste univerzalne talasne, vibracione i oscilatorne stvarnosti koja je jedinstvena u fizici (tj. jedinstvena za nas univerzuum) i to su zapravo talasi matterije koje mozemo na neki nacin da merimo, vidimo, cujemo, kao na primer, zvuk, oscilacije cvrstih i fluidnih masa, svetlost, razni elektromagnetski talasi u raznim sredinama, talasi praskastih formacija, toplotni talasi, talasi subatomskih cestica i kvazicestica (tj. sve sto se da na neki nacin meriti ili detektovati kao talasna ili oscilatorna pojava). ***Zato je ista klasicna differencijalna talasna jednacina primenljiva u nizu razlicitih talasnih fenomena (a u ovom radu je pokazano da je skoro univerzalno primenljiva i u kvantnoj teoriji, tj. da se odatle dobija Schrödinger-ova jednacina i druge talasne jednacine mikro fizike, kada se talasna funkcija tretira kao kompleksni analiticki signal).***

Nema niceg drugog u ovom nasem talasajucem univerzuumu kao sto bi bili neki misticni, transformabilni, ontoloski-probabilisticki i na druge nacine neuhvatljivi talasni oblici. To sto smo mi uveli u fiziku neka nova imena i opise neobicnih ili egzoticnih talasa je deo nase nepotpune i nedovoljno tacne konceptualizacije te fenomenologije (zanemareni su neki vazni faktori u vezi sa prostornom i vremenskom prezentacijom signala, ili jos uvek nismo u stanju da sve to sto nedostaje u nasem teorijskom opisivanju empirijski i eksperimentalno merimo i sagledamo u realnom vremenu).

Razna imena koja su davana raznim talasima matterije u publikacijama iz fizike su ponekad samo matematicke i konceptualne komplikacije i mistifikacije, ili vestacke (fitting i ad-hock postulirane) tvorevine da bi se podrzala ili spasila necija hipoteza, model ili teorija, u nedostatku boljih objasnjenja i matematickih modela, tj. to su apstrakcije koje cesto ne uzimaju sve vazne elemente u obzir, ili krace receno to su losi, ili nekompletni modeli.

Dakle, de Broglievi talasi su talasi materije kojom smo okruzeni i opipljivi su, merljivi itd., ali mi smo u sadasnjoj kvantnoj teoriji nasli nacine kako da ih normalizujemo, usrednjimo i izomorfno predstavimo kao neke ontoloski-probabilisticke, bezdimenzione funkcije koje imaju formu talasa (sto je mozda korisno i upotrebljivo u mirkofizici, jer tamo imamo ogromne skupove ili ansamble slicnih, ili identicnih entiteta, pa statistika i verovatnoca tamo produktivno rade, kao sto je slucaj u termodinamici fluida, ali to ne sme da se generalizuje na sve ostale domene fizike)… Sve ostalo (sto nisu direktno merljivi talasni fenomeni) su nekompletna i delimicno pogresna sagledavanja i modeliranja talasnih pojava (gde ce se jednog dana pokazati da tu nesto nedostaje i bice evolucije u pogledu stvaranja boljih modela)... To je poruka koja se nalazi u pozadini ovakvih elaboracija. Uloga statistike i verovatnoce kao dobrih teorija za modeliranje i obradu masovnih podataka i klasifikaciju fenomena, tendencija i kretanja je veoma znacajna i mocna, ali treba da se primenjuje kao zavrsna faza veoma sadrzajnog prikazivanja rezultata koji prvo treba da se dobiju na sve druge deterministicke, empirijske, logicke, analogne i matematicke nacine. Srtatistika i verovatnoca ne mogu i ne smeju da budu implementiranje kao ontoloska, konceptualna i izvorno-modelarna baza novih teorija, vec kao neka vrsta mocne zavrsnice i donosenja dobrih generalizovanih zakljucaka.

Drugi deo gledista koje se ovde favorizuje je da ***svi ranije pomenuti realni, matterijalni talasi (koji su merljivi, vidljivi i poznati iz iskustva) mora da***

se na neki nacin manifestuju u nekakvim komplementarnim, spregnutim talasnim parovima, kao sto je sprega elektricnog i magnetskog polja koja cini elektromagnetske talase. Dakle, nesto slicno (kao kod analitickog signala) mora da postoji i kod svih ostalih, vec pomenutih talasnih i oscilatornih fenomena (u akustici, gravitaciji, vibracijama cvrstih tela i fluida...). Osnova za talasanje matterije je da dolazi do dinamickog sprezanja kineticke i potencijalne energije kada se stvara neki oscilujuci ili talasni proces, gde se ukupna energija oscilatorno preliva iz kineticke u potencijalnu i vice-versa... To oscilatorno prelivanje kineticke u potencijalnu energiju mora i moze da bude prezentirano nekom spregom dva fizicka polja ili dva fenomena koji su medjusobno komplementarni (sto je za sada poznato i primenjeno, tj. modelirano samo u elektromagnetizmu, ali je to i sastavni deo modela analitickog signala). Druga je stvar sto mi do sada jos nismo nasli kakve su to fizicke i talasne sprege u gravitaciji ili u mnogim drugim talasnim fenomenima, silama i fizickim poljima. Zapravo, u mnogim slucajevima nismo jos uvek modelirali ili koncipirali tu drugu komponentu talasnog polja (osim u slucaju elektromagnetskih talasa gde obe komponente polja poznajemo). Hipoteza koja se ovde favorizuje je da ***svi talasi matterije mora da imaju pomenutu spregu izmedju dva medjusobno komplementarna fenomena, ili polja, ili talasanja***, ali da nase matematicke konceptualizacije (i eksperimentalna praksa) jos uvek nisu dosle do tog nivoa sagledavanja (ili modeliranja) jer u nizu slucajeva nemamo, tj. neznamo tu pomenutu, komplementarnu i/ili fazno pomerenu, drugu talasnu komponentu. U ovom radu se objasnjava da je ***jedna od fundamentalnih talasnih sprega u kretanju i talasanju materije, sprega izmedju elemenata linearnog i rotacionog kretanja, ukljucujuci spinning (i oko takvog modela gravitiraju sve ostale konceptualizacije u ovoj knjizi)***... a to u nekoj daljoj konsekvenci najverovatnije evoluira do sprege elektricnog i magnetnog polja, ili do neke druge familijarne sprege dva medjusobno komplementarna fenomena (i u najvecem broju slucajeva talasa matterije i oscilacija masa, elektromagnetska sustina se najverovatnije nalazi u pozadini svih takvih talasnih fenomena, samo sto to ponekad moze da bude dobro skrivena situacija).

Dakle ***problem je u tome sto mi ustvari neznamo, ili nismo matematicki, konceptualno i empirijski uhvatili jos jednu komponentu pomenutih talasa matterije (kao sto je slucaj u elektromagnetici, gde barem znamo obe talasne komponente)... i tu dolazi do izrazaja de Broglieva filozofija, koja hipoteticki (i analogno sa fotonima) nadomesta ili uvodi tu neuhvacenu komplementarnu talasnu komponentu*** i daje tacne rezultate (najcesce tamo gde su analizirane eksperimentalne interakcije izmedju entiteta sa nekakvom elektromagnetskom prirodom). Dakle, nema ni Schrödinger-ovih, ni Heisenberg-ovih, ni de Broglie-vih talasa... Sve je to deo iste talasne stvarnosti (sve su to talasi matterije i treba da se opisuju istim matematickim modelima i odnose se na isti univerzum). Sto mi jos nismo dosli do takvog nivoa konceptualizacije (analogicnog sa elektromagnetizmom), to je druga prica… ali treba da tezimo ka tome i jednom cemo sve to sagledati kako treba. Ovaj rad o tome govori... ili barem pocinje tu pricu i otvara mali prozor u takav svet...

Kada se stvore stojeci talasi matterije koji su samo-zatvorenog tipa (zatvaraju se prostorno sami-u-sebe na nekoj zatvorenoj liniji, krugu, povrsini, zapremini...) tada nastaju talasne grupe koje pocinju da primaju osobine cestica... a grupacije takvih elementarnih cestica stvaraju vece mase... Za sada mozemo zamisliti da ovde mora da se radi najvise o talasima koji izvorno (mozda na nekin nacin dobro skriveno) imaju neku vezu sa elektromagnetizmom. Naravno, postoje i drugi talasi neelektromagnetske prirode koji mogu da formiraju grupacije stojecih talasa, ali je najverovatnije da samo elektromagnetske stojece formacije talasa stvaraju talasne grupe koje pod odredjenim uslovima postaju ono sto zovemo elementarnim cesticama. ***Da bi jedna sama-u-sebe-zatvorena, stojeca talasna grupa mogla da stvori cesticu, nije dovoljno da su njene (unutrasnje) talasne komponente jednog tipa tj. iste prirode. Mora da postoji neka interakcija ili dinamicka sprega izmedju dva tipa (medjusobno komplementarnih i spektralno konjugovanih) fizickih polja ili talasa koji se tu pojavljuju.*** Ovo tvrdjenje je bazirano na poznatim i vazecim elektromehanickim analogijama i simetrijama u fizici, kao sto su: sprega elektricnog i magnetnog polja kod elektromagnetskih talasa (ili neki oblik sprege kineticke i potencijalne energije, ili neka forma sprege linearnog i rotacionog kretanja), da bi se od takvih medjusobno komplementarnih i spregnutih stojecih talasa stvorio kompleksniji i sadrzajniji stojeci matterijalni talas i da bi se kasnije taj talas zatvorio sam u sebe (na neki nacin), a time se stvara relativno stabilna talasna grupa koja moze da vodi formiranju cestica koje ce da imaju stabilnu masu mirovanja.

U prirodi postoje i talasi koji nisu zatvoreni sami u sebe (koji nisu tipicni stojeci talasi), a to su propagirajuci, ili progresivni, ili slobodni talasi matterije... a zajednicko im je da se svi ti talasi (i slobodni i stojeci) opisuju slicnim matematickim modelima i jednacinama… ***i to se odnosi na sve talase i vibracije koje poznajemo u prirodi, jer je sve to medjusobno spregnuto i deo iste stvarnosti... pa se zbog toga i manifestuju razne analogije u opisivanju talasnih kretanja. Najverovatnije da je sve to sustinski prozeto ili komponovano (u svojoj dubokoj ontoloskoj pozadini) nekim specificnim pakovanjem elektromagnetskih polja, koja se talasaju, ili osciluju***, sto sve jos treba mnogo bolje razraditi…

Na primer, kod elektrona imamo niz direktnih indicija da su elektroni bas takve (pomenute) samo-zatvorene talasne grupacije fotona ili elektromagnetskih talasa, a protoni bi trebalo da budu nesto slicno, ali sa mnogo vise upakovane energije (jer se opet radi o naelektrisanju koje stvara elektricno polje, a kretanje svih naelektrisanih entiteta stvara magnetno polje…). Na primer, fotoni izbacuju elektrone iz atoma ili ulaze u njih i podizu energiju unutrasnjih elektrona u atomu, pa kasnije takav ekscitovani elektron opet izbaci foton. Elektron i pozitron se anihiliraju stvarajuci fotone, a postoji i inverzni proces… Komptonov i Fotoelektricni efekat govore o medjusobno slicnim interakcijama… (i nije bas slucajno da se sve to na neki nacin vezuje za fotone, ili naelektrisanja, ili za neku formu elektromagnetske energije).

Skoro da je dokazano, ali eksplicitno nepriznato u fizici, da su neutroni neki specificni parovi jednog protona i jednog elektrona i to neutron cini elektricno neutralnim, jer kad se analizira njegova struktura, nadju se jake indicije da su unutar neutrona jedan proton i jedan elektron (kao u atomu vodonika, ali drugacije upakovani ili vezani). Dakle, sada imamo sve sastojke materije: elektrone, protone, neutrone, fotone (a znamo da postoje i njihove anti-cestice). Svakako, ovde jos nedostaje i neutrino (da se tu negde smesti) i da se lepo ukomponije i objasni (jer i to je eksperimentalno uhvatljiva talasna grupa, ili neka vrsta kvazi-cestice) ... A to se nekako opet sve intuitivno svodi na pakovanje fotona ili pakovanje elektromagnetske energije (ako se udje u dublju istoriju procesa). Naravno, ovo sto je ovde izlozeno je mnogo uprosceno, ali sa svrhom da se bolje i brze razume, nije lose za pocetak diskusije i za stvaranje novih koncepata i temelja o talasima i strukturi matterije.

Sto se kretanja elektrona tice tu postoje neke znacajnije razlike u kretanju pojedinacnih, relativno izolovanih i slobodnih elektrona od kretanja mase slabo vezanih elektrona kroz metalne provodnike. Dakle, bice tesko da ih opisujemo ili koncipiramo na veoma slican nacin, jer usamljeni elektron u slobodnom prostoru predstavlja neku talasnu grupu specificno upakovanih fotona (dakle predstavlja talasnu elektromagnetsku strukturu koja ima neku svoju prostorno-vremensku lokalizovanost ili koncentraciju, ili nesto sto asocira na cesticu, ili talasnu grupu i ima svoju rezultantnu grupnu i faznu brzinu koje zavise od osobina prostora u kome se takva talasna grupa krece, kao sto i blizina okolnih objekata moze da utice na grupnu i faznu brzinu elektrona).

Po konceptima favorizovanim u ovoj knjizi, slobodni, ili izolovani elektron koji se krece linijski stvara oko sebe (oko ose svog linearnog kretanja) helikoidalno, spiralno, ili vrtlozno magnetno polje *(helikoidna linija se ovde spominje vise kao neka zamisljena vodeca, solenoidna linija sile magnetnog polja, ili kao obvojnica koju prati stvoreno, komplementarno magnetno polje, tj. ta obvojnica je vezana za svoju talasnu grupu. Ta asocijacija sa helikoidom nam pruza neku vrstu vizualizacije talasne duzine i frekvencije matterijalnog elektronskog talasa, jer je intuitivno ocigledno da pomenuta talasna duzina mora da bude jednaka rastojanju izmedju dva sukcesivna zavojka helikoidne obvojnice).* Elektron u kretanju podleze zakljuccima hipoteze matterijalnih talasa (Luis de Broglie) i talasna duzina tog pridruzenog, solenoidnog, elektromagnetnog polja ili talasa je jednaka $h/p = h/mv$ (gde je m masa elektrona, a v je njegova linearna brzina). Ta (de Broglie-va) talasna duzina, kao sto je vec pomenuto, je jednaka razmaku koji obuhvata jedan zavoj zamisljenog solenoida koji okruzuje i prati elektromagnetsko polje oko elektrona u kretanju (tj. ta razdaljina odgovara jednoj periodi pomenute helikoidalne obvojnice). Takav elektronski talas (koji ima elektricne i magnetne komponente polja) neminovno ima i svoju de Broglie-vu talasnu duzinu i frekvenciju i faznu i grupnu brzinu (a mi iz drugih analiza vec poznajemo jednacine univerzalnih veza izmedju grupne i fazne brzine, koje vaze za sve talase, pa sve to mozemo i ovde primeniti, bez ikakve potrebe da pozivamo verovatnocu i statistiku u pomoc). Grupna brzina elektrona je jednaka stvarnoj, mehanickoj ili cesticnoj, linearnoj brzini elektrona (prema modelu talasne grupe). Dakle, moguce je pridruziti elektronu koji se krece neku frekvenciju (zavisno od njegove brzine kretanja, tj. zavisno od njegove kineticke energije). Ova situacija postaje jos kompleksnija i ociglednija ako uzmemo u obzir da svi elektroni imaju svoj spin (jer je to modus postojanja elektrona, tj. bez spinskog momenta elektron nebi postojao kao cestica, sto znaci da je koncentracija i grupisanje specificno formiranog, stojeceg elektromagnetskog talasa ostvareno time sto je linearna komponenta kretanja energetski pogodnog elektromagnetskog talasa (ili fotona) harmonijski spregnuta sa pogodnom spinskom, tj. rotacionom komponentom tog istog talasnog kretanja i tako nastaje elektron).

To isto postaje komplikovanije ili teze opisati za veliki broj slabo vezanih elektrona koji se krecu unutar metalnog provodnika jer ne mozemo da na jednostavan i slican nacin izdvojimo neku usamljenu, pridruzenu solenoidalnu putanju vektora magnetnog polja i da joj odredimo talasnu duzinu ili pripadajucu frekvenciju (posto ima bezboroj elektrona koji uzajamno interaguju, interferiraju, mesaju se i nisu na istom mestu, tj. nema izolovanih helikoidalnih matterijalnih ili magnetskih linija sila). U metalnim provodnicima, elektroni se krecu ili struje, ili se talasaju na neki nacin koji pomalo moze da asocira na kretanja cestica i talasa u fluidima gde ima bezbroj malih cestica (atoma i molekula i vecih grupacija) koje su medjusobno povezane Van der Vaals-ovim i elektromagnetskim silama, pa zbog toga u neposrednoj okolini provodnika kroz koji prolazi elektricna struja necemo prepoznati izolovane helikoidalne, magnetne linije polja pojedinacnih elektrona. Tu se radi o ogromnoj masi elektrona sa slucajnim prostornim i faznim raspodelama, pa cemo imati utisak da se u okolini provodnika kroz koji tece struja nalazi kontinualno, homogeno, solenoidno magnetsko polje koje opisuje Biot-Savart-ov zakon (jer ovde u sustini imamo superponiranu masu helikoidalnih magnetnih linija sila, koje imaju svoje slucajno-raspodeljene talasne duzine, faze i frekvencije, sa zajednickom centralnom ili osnom linijom kroz koju kazemo da prolazi elektricna struja).

Miodrag Prokic

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BASIC ELECTROMECHANICAL ANALOGIES

In this book, we will explore innovative explanations of wave-particle duality, the structure of matter based on agglomerated formations of matter waves, and natural fields and forces, based on generalized definitions and analogies. These explanations will be presented as intuitively predictive models, creative arrangements, and indicative conclusions rooted in well-known electromechanical analogies. Analogies are natural indicative patterns and signs of fields and forces unity within our universe. Our goal is to introduce new concepts and theories in challenging physics domains while refining existing theories to achieve a higher level of unification, based on these analogies.

All theories in physics are grounded in mathematically established models that allow for mastering and prediction of observable phenomena. However, these models typically do not provide an exact representation of nature or an 100% accurate depiction of the actual structure of the universe. In the context of classical mechanics and geometry, which describe the macroscopic mechanical world, the models, descriptions, and visualizations tend to be more stable, coherent, and harmonious. This contrasts with the mathematical models used to describe the subatomic realm of physics, which are often provisional, evolving, and applicable only within specific contexts or timeframes. These models are valuable tools for describing and predicting events, but they are subject to continuous refinement and improvement—a process that is ongoing and likely never-ending. Clearly, there is room for optimization in how we intellectually and mathematically approach the physics of the microworld. To achieve more robust and enduring descriptions of physical systems, we must develop better frameworks for natural events mapping and referencing, akin to those used in classical mechanics. One complementary and powerful approach to establishing universally valid and applicable reference systems in physics is the use of generalized electromechanical analogies. This involves recognizing that the electromagnetic domain underpins the mechanical world, thus allowing for more effective modelling.

In this regard, analogies are crucial. They serve as tools to guide our thinking, enhance our intuition, and foster creativity in developing new theories. These analogies not only help to structure our understanding of nature, but also provide a bridge between different physical domains, aiding in the conceptualization of complex systems.

Before embarking on this multidisciplinary project, we must establish a clear strategy, algorithm, and framework for organizing and classifying the knowledge we'll examine. This "intellectual space" will be structured using conservation laws, universal principles, and established systems of electromechanical analogies in physics. Given the variety of analogy platforms developed in physics, particularly in electrical sciences, we will prioritize the most promising and broadly applicable ones. Among these, the **Mobility** type of analogies is especially useful for understanding and visualizing innovative concepts related to electromagnetism, mechanics, gravitation, matter waves, and wave-particle duality, which are the main subjects of this book.

From Fourier analysis and the works of Kotelnikov, Shannon, Whittaker, and Nyquist, we know that signals, motions, and matter states can be decomposed or synthesized using elementary harmonic functions (see more about such problematic in [57, 58, 59, 117, 118 and 119], and in Chapters 8, 9, and 10 of this book). By this reasoning, the diversity of matter in the universe can be viewed as structured combinations of elementary physical oscillators, resonators, and familiar circuits, all generating simple harmonic waves or spectrally narrow-banded wave groups. These physical oscillators, through mutual synchronization, coupling, superposition and interference, form more complex wave groups and matter states, consistent with Fourier theory and signal analysis.

This conceptual foundation explains why we begin with analogies comparing simple electrical and mechanical resonant and oscillatory circuits, as illustrated in Figure 1.1. These resonant configurations, including atomic and molecular resonant structures, as elementary building blocks of matter, are naturally coupled and synchronized, forming the various states of matter in the universe. Physics tells us that where oscillation exists, rotation or spinning phenomena are often nearby, and vice versa. Coupled resonators mutually and naturally communicate by entanglement effects and by exchanging quantized energy wave-packets.

Well-established analogies provide an initial, indicative step suggesting that seemingly different theories, concepts, or systems may share a common foundation. This implies the potential for unifying these ideas into a comprehensive theory. For example, atomic, molecular, and mechanical systems, along with various oscillators and resonant circuits, are likely composed of more fundamental micro-resonators, spatially arranged as resonant or oscillatory matter states. In other words, matter states are built from combinations of elementary physical resonators and oscillators, which, after multiple superpositions and interferences, form elementary particles, atoms, molecules, and other masses.

Moreover, the forces between these resonating matter states such as attraction, repulsion, adhesion, and cohesion are superficially electromechanical but fundamentally electromagnetic. These resonant states communicate by exchanging matter waves, photons, phonons and vibrations, which macroscopically generate mechanical properties of masses and thermodynamic phenomena. Similar resonant states are prone to create couplings, synchronizations, and entanglements. For instance, the Casimir effect illustrates such coupling between cosmic radiative energy and capacitive, metallic mirror plates, with involved photons and elementary particles entanglement effects (*see more in [103], [104] and [130]).*

Let us first establish or summarize the basic set of (Mobility type) electromechanical analogies that will be used later to formulate some of the most important ideas, concepts, and messages in this book. We can compare different configurations of electric circuits in Fig.1.1 a), b), c), d), consisting of electrical elements (resistance **R**, inductance **L**, and capacitance **C**), to different configurations of mechanical circuits, consisting of mechanical elements (mechanical resistance or damper $\mathbf{R_m}$, spring **s**, and mass **m**). From literature (addressing analogies) it is known that we can create six different electromechanical analogies (see **[1],** pages 9-18**, [2], [3]** and **[164]**). Here, we shall not present or analyze the way of formulating mentioned analogies (since this can be found in an enormous number of publications). As a general conclusion, we shall state that *all the known electromechanical analogies have been created after noticing* ***similarity of the mathematical forms of corresponding differential equations*** *(between selected, dual, or mutually similar electric and mechanical* *networks), which are describing currents, voltages, forces, and velocities in/trough/across involved electric or mechanical networks, components and involved circuit elements. We are thus establishing mathematical analogies (in a basic and simple way that presents the **first** criteria in creating analogies). The mathematical conclusion here is that all possible six analogy situations (literature **[1]** and **[2]**) are mutually equal, or equally useful.*

We shall now introduce the ***second*** analogy criteria (**which is not mathematical**) to give more weight and power to the certain set of analogies (from the existing six). Let us say that besides *mathematical* analogies we would like to have (visually, positionally, or structurally) mutually *equivalent or similar electric and mechanical circuits, meaning to have* ***the same networks, or geometric configurations*** (or topology) of their elements (if we look at how they are mutually and spatially connected in corresponding electric and mechanical circuits, or where they are physically placed). *By introducing this* ***second*** *analogy criterion, the previous set of six analogies (which is systematically analyzed in* ***[1], [2]*** *and* ***[3]****) is reduced to the set of only two dual analogy situations (as presented on Fig.1.1 a), b), c), d)). Such mathematical and* ***topographic analogy*** *or situation can be represented by "****velocity-to-voltage****", and "****force-to-current****" analogy (known in literature as the* ***Mobility*** *type of analogies).*

Fig. 1.1c

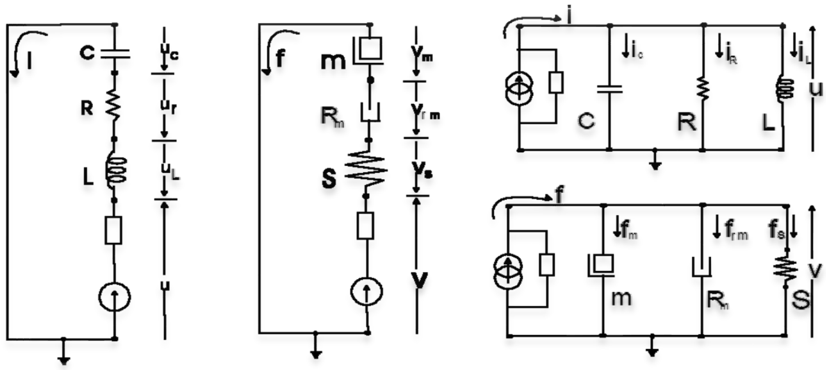

Fig.1.1a Fig. 1.1b Fig. 1.1d

Fig.1.1. Equivalent electric and mechanical circuits

The content and message that results from this *double level analogy platform* is shown in T.1.1. In this book, it will be demonstrated that the analogies from T.1.1 present the most important, most predictive, practical, and most natural (electro-mechanical) analogy platform in physics (and later, the same *Mobility type analogy platform* from T.1.1 will be widely extended; -see T.1.2 until T.1.7). Here we are also creating grounds for productive analogical and conceptual thinking, conclusions and predictions making, to understand and describe the world of Physics in a simpler, more intuitive, analogical, united, and better conceptual way.

T.1.1

Electric parameter / [unit]	Mechanical parameter / [unit]
Voltage (=) u (=) [V = volt]	Velocity (=) v (=) [m/s]
Current (=) i = dq/dt (=) [A = ampere]	Force (=) f = dp/dt (=) [N = kg m/s^2 = newton]
Resistance (=) R (=) [Ω = ohm]	Mech. Resistance (=) R_m (=) [m /N s = s/kg]
Inductance (=) L (=) [H = Henry]	Spring Stiffness (=) S (=) [m/N = s^2/kg]
Capacitance (=) C (=) [F = farad]	Mass (=) m (=) [kg]
Charge (=) q = Cu (=) [C = coulomb]	Momentum (=) p = mv (=) [kg m/s]
Magn. Flux Φ (=) [Wb = V s = Weber]	Displacement (=) x (=) [m]

To establish full (1:1) analogy and mapping and symmetry between idealized elements of electrical and mechanical circuits, as given in T.1.1 (**L ⇔ S** and **R ⇔ R_m**), it was necessary to redefine the unit of spring stiffness and unit of mechanical resistance or damping. In literature we find for spring stiffness **S (=) [N/m]**, and here we use **S (=) [m/N]**, and for mechanical resistance in today's literature it is very usual to find **R_m (=) [Ns/m]**, and here we use **R_m (=) [m/Ns]**. The consequences of such parameter redefinitions are that usual (traditional) meaning of **Mechanical Impedance = Force/Velocity** should also be redefined into **"New Mechanical Impedance" = Velocity/Force,** analog to **Voltage/Current** (presently known in Mechanics as **Mobility**), serving to fully support the **VOLTAGE-VELOCITY** and **CURRENT-FORCE** analogies. Similar redefinitions are also applied to mechanical resistance (or friction constant) and spring stiffness valid for rotation (see all equations from (1.1) until (1.9)).

Here it is recommendable to read very familiar and complementary elaboration about analogies **[164],** written by L. Stocco. "An Isolated Mass Model for Intuitive Electro-Mechanical Analogies". All other analogies (applicable in Physics) are equally useful (but only on the mathematical, formal level of analogies), and when we consider additional rules, correct use of any of these analogies will lead to the same result (but often in a more complicated, and not so visually obvious and intuitively clear way, as when using Mobility system of analogies).

Let us come back to the equivalent (and idealized) models given in Fig.1.1 and introduce angular oscillatory motion, yet another level of study for the creation of additional analogies. We now imagine two more mechanical analogies, as oscillating circuit models added to this situation, presenting the case of rotation (in series and parallel configuration of oscillatory rotating mass having a moment of inertia **J**, angular velocity **ω**, angular momentum **L** = **Jω**, torque **τ** = d**L**/dt, rotational friction **R_R**, and spring stiffness for rotational oscillations **S_R**). In such situations (analogous to cases shown in Fig.1.1 a), b), c), d)) we can establish relations between the parameters from T.1.1, adding to them similar relations concerning the rotational oscillatory motions (here are not supported with any additional figure, but easy to understand and visualize).

Generalized Ohm's Laws and Generalized Kirchoff's Laws are directly applicable to the circuit situation on Fig.1.1 (see **[2], Vol. 2**).

Kirchoff's Voltage Law states: "**The sum of all the *__across element voltage differences__* in a loop is equal to zero."** Based on the situation in Fig.1.1 a), we shall have:

$$\sum u_i = 0,\ u = u_L + u_R + u_C = L\frac{di}{dt} + Ri + \frac{1}{C}\int idt = L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C},\quad (i = i_L = i_R = i_C = \frac{dq}{dt}), \tag{1.1}$$

For mechanical oscillating circuits, the analogy to Kirchoff's Voltage Law will state, **"The sum of all the *across element velocity differences* in a loop is equal to zero."** Based on the situation in Fig.1.1 b), we shall have:

$$\sum v_i = 0,\ v = v_S + v_{Rm} + v_m = s\frac{df}{dt} + R_m f + \frac{1}{m}\int fdt = s\frac{d^2p}{dt^2} + R_m\frac{dp}{dt} + \frac{p}{m},\quad (f = f_S = f_{Rm} = f_m = \frac{dp}{dt}). \tag{1.2}$$

For a series, rotational resonant elements circuit (like Fig.1.1 b)), the analogy to Kirchoff's Voltage Law will state, **"The sum of all the angular *across element velocity differences* in a loop is equal to zero"**,

$$\sum \omega_i = 0,\ \omega = \omega_{SR} + \omega_{RR} + \omega_J = s_R\frac{d\tau}{dt} + R_R\tau + \frac{1}{J}\int \tau dt = s_R\frac{d^2L}{dt^2} + R_R\frac{dL}{dt} + \frac{L}{J},\ (\tau = \tau_{SR} = \tau_{RR} = \tau_J = \frac{dL}{dt}). \tag{1.3}$$

Kirchoff's Current Law states: **"The sum of the entire *thorough element currents* entering into a junction is equal to the sum of the *thorough element currents* out of the junction".** Given the situation in Fig.1.1 c), we shall have:

$$\sum i_{inp.} = \sum i_{outp.},\ i = i_C + i_R + i_L = C\frac{du}{dt} + \frac{u}{R} + \frac{1}{L}\int udt = C\frac{d^2\Phi}{dt^2} + \frac{1}{R}\frac{d\Phi}{dt} + \frac{\Phi}{L},\ (u = u_C = u_R = u_L = \frac{d\Phi}{dt}). \tag{1.4}$$

For similar mechanical circuits, the analogy to Kirchoff's Current Law will state, **"The sum of all the input *through element forces* is equal to the sum of the output *through element forces*"**. Looking at the situation in Fig.1.1 d), we shall have:

$$\sum f_{inp.} = \sum f_{outp.},\ f = f_m + f_{Rm} + f_S = m\frac{dv}{dt} + \frac{v}{R_m} + \frac{1}{s}\int vdt = m\frac{d^2x}{dt^2} + \frac{1}{R_m}\frac{dx}{dt} + \frac{x}{s},\ (v = v_m = v_{Rm} = v_S = \frac{dx}{dt}). \tag{1.5}$$

For an analogical rotating element being in parallel-configuration circuit (like on Fig.1.1 d)), the analogy to Kirchoff's Current Law will state, **"The sum of all the input *through element torques* is equal to the sum of the output *through element torques*"**,

$$\sum \tau_{inp.} = \sum \tau_{outp.},\ \tau = \tau_J + \tau_{RR} + \tau_{SR} = J\frac{d\omega}{dt} + \frac{\omega}{R_R} + \frac{1}{s_R}\int \omega dt = J\frac{d^2\theta}{dt^2} + \frac{1}{R_R}\frac{d\theta}{dt} + \frac{\theta}{s_R},\ (\omega = \omega_J = \omega_{RR} = \omega_{SR} = \frac{d\theta}{dt}). \tag{1.6}$$

To avoid possible (terminological) confusion in understanding the system of analogies (1.1) – (1.6), we must make a difference between the electric potential and voltage, which is a difference of potentials between the two points ($u = \varphi_2 - \varphi_1 = \Delta\varphi$). The same is valid for velocities concerning reference level of certain motion, and for the velocity «on» a certain mechanical element (measured ***across the element***), which is equal to the difference of two referential velocities. On the same way ***pass-through-element*** variables are Current, Torque, and Force, since such variables are transmitted through (or measured when passing through) certain element (see chapter 2, reference [55], Modern Control Systems, 12th Edition, Richard C. Dorf, Davis Robert H. Bishop, University of Texas at Austin, Pearson Education). A next important difference between the electric and mechanical mutually analog elements is in the fact that all electric elements from T.1.1 are scalars, and some of the analogical mechanical elements can be vectors (**v, p, f**).

On the other hand, if we now formulate the *mechanical conservation law of momentum*, we shall have:

$$\sum \mathbf{p}_{inp.} = \sum \mathbf{p}_{outp.} \Rightarrow \frac{d}{dt}\sum \mathbf{p}_{inp.} = \frac{d}{dt}\sum \mathbf{p}_{outp.} \Leftrightarrow \sum \mathbf{f}_{inp.} = \sum \mathbf{f}_{outp.} . \tag{1.7}$$

Clearly, momentum conservation (1.7) directly corresponds (or leads) to "Kirchoff's Force/Current Laws" (1.5) applicavle on linear motions. Following the same pattern of conclusion in (1.7), we can return to the electric system and develop the *total electric charge conservation law*:

$$\sum \mathbf{i}_{inp.} = \sum \mathbf{i}_{outp.} \Rightarrow \frac{d}{dt}\sum \mathbf{i}_{inp.} = \frac{d}{dt}\sum \mathbf{i}_{outp.} \Leftrightarrow \sum \mathbf{q}_{inp.} = \sum \mathbf{q}_{outp.} . \tag{1.8}$$

If we now take the mechanical conservation law of angular momentum, we shall have:

$$\sum \mathbf{L}_{inp.} = \sum \mathbf{L}_{outp.} \Rightarrow \frac{d}{dt}\sum \mathbf{L}_{inp.} = \frac{d}{dt}\sum \mathbf{L}_{outp.} \Leftrightarrow \sum \tau_{inp.} = \sum \tau_{outp.} . \tag{1.9}$$

Obviously, angular momentum conservation (1.9) also and directly corresponds (or leads) to "Kirchoff's Force/Current Laws" for angular motions (1.6), or, in other words, this is just a *torque conservation law*.

*In a previous way as elaborate with (1.1) to (1.9), step-by-step, we introduced new elements for supporting the **third** level of Mobility-type of electro-mechanical analogies, connecting them to the well-known and proven conservation laws in physics. This task must be still completed by considering all other, already known conservation laws in physics in such a way as to obtain multilevel and multidisciplinary parallelism and analogies between electric and mechanical formulas and relevant closed circuits, as before. By using such a method, we could later formulate or predict some new, up to present not explicitly and distinctively expressed, conservation laws.*

Now we can summarize all the additionally exposed analogies (comparing corresponding equations from (1.1) to (1.9)) and extend the table of Mobility-type analogies T.1.1 to T.1.2 (by adding the elements originating from "rotational" circuit situations, (see **[3]**)).

T.1.2

Electric parameter / [unit]	Mechanical parameter / [unit]
Voltage (=) u (=) [V = volt]	Velocity (=) v (=) [m/s]
	Angular Velocity (=) ω (=) [rad/s]
Current (=) i = dq/dt (=) [A = ampere]	Force (=) F = dp/dt = f (=) [N = kg m/s² = Newton]
	Torque (=) τ = dL/dt (=) [kg m²/s²]
Resistance (=) R (=) [Ω = ohm]	Mech. Resistance (=) R_m (=) [m /N s = s/kg]
	Mech. tors. Resistance (=) R_R (=) [s/kg m²]
Inductance (=) L (=) [H = henry]	Spring Stiffness (=) S (=) [m/N = s²/kg]
	Torsion. Spring Stiffness (=) S_R (=) [s²/kgm²]
Capacitance (=) C (=) [F = farad]	Mass (=) m (=) [kg]
	Moment of Inertia (=) J (=) [kg m²]
Charge (=) q = Cu (=) [q = coulomb]	Momentum (=) p = mv (=) [kg m/s]
	Angular Momentum (=) L = J ω (=) [kg m²/s]
Magn. Flux Φ (=) [Wb = V s = weber]	Displacement (=) x (=) [m]
	Angle (=) θ (=) [rad] (=) [1]

Along with T.1.2, let us **dimensionally** compare relevant formulations for linear motion in a gravitational field, the analogical situation in the electromagnetic field, and the situation related to the rotation of masses, as summarized in T.1.3, T.1.4, and T.1.5. For the common and generic names of the corresponding (internally analog) columns in the following tables, we shall use the names and symbols of relevant electromagnetic parameters (see **[2], Vol. 1**).

T.1.3	[W] = [ENERGIES]	[Q] = [CHARGES] (Integrated Through Variable)	[C] =[CAPACITANCES] (Capacitive Storage elements)
Electro-Magnetic Field	[Cu^2] (=) [qu^1] [Li^2] (=) [Φi^1]	[Cu^1] (=) [qu^0] (=) [q] [Li^1] (=) [Φi^0] (=) [Φ]	[Cu^0] (=) [qu^{-1}] (=) [C] [Li^0] (=) [L]
Gravitation	[mv^2] (=) [pv^1]	[mv^1] (=) [pv^0] (=) [p]	[mv^0] (=) [pv^{-1}] (=) [m]
Rotation	[$J\omega^2$] (=) [$L\omega^1$]	[$J\omega^1$] (=) [$L\omega^0$] (=) [L]	[$J\omega^0$] (=) [$L\omega^{-1}$] (=) [J]

T.1.4	[U] = [VOLTAGES] (Variable across Element)	[I] = [CURRENTS] = d[Q]/dt (Variable Through Element)	[Z] = [IMPEDANCES] (=[mobility] in mechanics)
Electromagnetic Field	[u] (=) [dΦ/dt]	[i] (=) [dq/dt]	[Z_e] (=) [u] / [i]
Gravitation	[v] (=) [dx/dt]	[F] (=) [dp/dt]	[Z_m] (=) [v] / [F]
Rotation	[ω] (=) [dω = dθ/dt /dt]	[τ] (=) [dL/dt]	[Z_R] (=) [ω] / [τ]

T.1.5	[L] = [INDUCTANCES] (Inductive Storage elements)	[R] = [RESISTANCES] (Energy dissipaters)	[Φ] =[DISPLACEMENTS] (Integrated Across Variable)
Electromagnetic Field	[L]	[R]	[Φ] (=) [Li]
Gravitation	[S]	[R_m]	[x] (=) [SF]
Rotation	[S_R]	[R_R]	[α] (=) [S_R τ]

Now let us create analogical and indicative conclusions, such as: The most interesting and most significant set of Mobility-type analogies in T.1.3 is related to different natural fields' charges, or fields' sources. We can see that the source of electric field is electric charge q, and that for magnetic field the most representative is magnetic flux Φ. We could also introduce an imaginative, and innovative way of *seeing mutually analogical items from T.1.3. For instance, under "[C] = [CAPACITANCES]" we find items that are on certain specific way able to accumulate electromagnetic and/or mechanical energy, or, at least, able to capture different field-charges that are analogically summarized as "[Q] = [CHARGES]". Here we also need to understand or to notice that mass m always presents a kind of energy accumulation.*

Rethinking the Origins of Gravitation and the Role of Dynamic Entities

It is well understood that the internal constituents of mass, such as atoms and molecules, consist of various field-charges and dynamic elements, including electric and magnetic charges, dipoles, multipoles, magnetic fluxes, and mechanical and electromagnetic moments. In this context, mass, specifically gravitational mass, can be seen as an organized storage or accumulator of these dynamic and spinning entities. This implies that the true origins and sources of gravitation are these dynamic entities within masses, rather than static (or rest) masses themselves. In other words, oscillating masses and atoms play a more significant role in gravitation than static masses.

As presented in T.1.3, within the fields of gravitation, rotation, and mechanics, linear momentum (p) and angular momentum (L) are of paramount importance. These are inherently coupled with electromagnetic charges, fluxes, and relevant electric and mechanical moments. Interestingly, based on here summarized analogies, there is no strong indication that static mass (m) should be the sole or primary source of gravitation. This contrasts with the expectations based on the analogy between Newton's and Coulomb's force laws. Instead, we can analogically conclude that the dynamic parameters of vibrating masses are the principal sources of gravitation or directly influence it. This conclusion is drawn from analogies between electrical and mechanical oscillatory or resonant circuits, as explored in this chapter, where oscillating masses are associated with mechanical moments.

The practical implication of these analogical insights is that the theories of gravitation proposed by Isaac Newton and Albert Einstein may eventually require some smaller updates. Additionally, this line of thought suggests that electric charges (such as electrons and protons) should always be considered dynamic, motional, or oscillatory energy states, behaving like mechanical and electromagnetic moments or equivalent matter-wave packets. This aligns with the broader understanding of mechanical and electromagnetic moments and flux entities as dynamic states that can be represented as vectors, as described in T.1.3.

Currently, physics tends to view electric charges as stable entities with fixed and static parameters (quantified in Coulombs). However, this perspective may be flawed. We can anticipate some form of electromagnetic energy exchange (currents, flux, or electromagnetic energy flow) between positive and negative electric charges, or between appropriately aligned electromagnetic dipoles, especially in the presence of oscillations. This idea lays the groundwork for an intuitive and indicative explanation of gravitation in electromagnetic terms. In electromagnetic theory, such phenomena are linked to alternating dielectric currents and electric induction.

*We also know that electric charge and magnetic flux are naturally bipolar entities, capable of forming dipoles or other polarized structures. A similar principle should apply to linear and angular momenta, which are already related to action and reaction forces, electromagnetic induction, Lorentz forces, and various inertial (electromagnetic and mechanical) effects. All of that belongs to the CPT (Charge, Parity or Position, Time) Symmetry of our Universe (but here this will be seen with an extended meaning of electromagnetic and mechanical charges, as introduced in T.1.3). In traditional CPT Symmetry "C" stands for electric charge (q) symmetry, and in **extended CPT Symmetry** "C" analogically stands for (q, Φ, p, L), adding magnetic flux, linear and angular moments as new fields charges.*

Since gravitational force is known only as an attractive force, there must be a complementary mass-energy-momentum flow that serves as a reaction force, balancing such force of gravitation. Nikola Tesla conceptualized this as "radiant energy," a form of "mass-moments" flow from all masses toward other masses. Tesla speculated that oscillating and radiating masses, essentially being most of the masses in our universe, create standing matter waves between them, as part of his Dynamic Gravity theory. Rudjer Boskovic, before Tesla, proposed a similar concept with his Universal Natural Force.

While these analogical predictions and expectations are still in their early stages and lack robust empirical support, they are highly indicative. Later in this book, we will provide a more substantial *foundation to demonstrate that electric charges, magnetic fluxes, linear and angular moments are often (or perhaps always) interconnected, complementary, and integrated into the properties of matter. These dynamic entities may constitute the most significant sources of all natural fields and forces, laying the groundwork for an innovative General Field Unification Platform. These ideas resonate with Rudjer Boskovic's "Universal Natural Force" [6] and Nikola Tesla's "Dynamic Force of Gravity [97]." Ultimately, this could lead to a fundamental revision of our understanding of the four fundamental natural forces, with far-reaching implications for modern physics.*

*Citation from PowerPedia, on Internet; -"Tesla's Dynamic Theory of Gravity: The **Dynamic Theory of Gravity** of Nikola Tesla explains the relation between gravitation and electromagnetic force as a unified field theory (a model over matter, the aether, and energy). It is a unified field theory to unify all the fundamental forces (such as the force between all masses) and particle responses into a single theoretical framework".*

[♣ COMMENTS & FREE-THINKING CORNER: *Let us make one digression towards the Special Relativity Theory. In T.1.2, we find that mass $\mathbf{m}$ is analog to the electric capacitance $\mathbf{C}$ ($\mathbf{m_0}$ - rest mass), meaning that this analogy can possibly be extended in the following way:*

$$\mathbf{m} = \frac{\mathbf{m}_0}{\sqrt{1-\frac{\mathbf{v}^2}{\mathbf{c}^2}}} \Rightarrow \mathbf{C} = \frac{\mathbf{C}_0}{\sqrt{1-\frac{\mathbf{v}^2}{\mathbf{c}^2}}} \qquad (1.10)$$

The analogy (1.10) can be strongly supported by the following example. Let us imagine the plane electrodes capacitance, where the surface of a single electrode is $\mathbf{S}$, and the distance between the electrodes is $\mathbf{d}$. In that case, electric capacitance is:

$$\mathbf{C} = \frac{\mathbf{q}}{\mathbf{u}} = \varepsilon_0 \frac{\mathbf{S}}{\mathbf{d}}. \qquad (1.11)$$

Let us suppose that the electric capacitance is moving by velocity $\mathbf{v}$ in the direction of its electrode distance (perpendicular to the electrodes surfaces). In this case, the distance between the electrodes is described by the relativistic contraction formula:

$$\mathbf{d} = \mathbf{d}_o\sqrt{1-\frac{\mathbf{v}^2}{\mathbf{c}^2}}. \qquad (1.12)$$

Introducing (1.12) in (1.11), we get the formula for capacitance (1.13) that is equal to (1.10):

$$\mathbf{C} = \frac{\mathbf{q}}{\mathbf{u}} = \frac{\varepsilon_0 \frac{\mathbf{S}}{\mathbf{d}_0}}{\sqrt{1-\frac{\mathbf{v}^2}{\mathbf{c}^2}}} = \frac{\mathbf{C}_0}{\sqrt{1-\frac{\mathbf{v}^2}{\mathbf{c}^2}}} = \frac{\frac{\mathbf{q}_0}{\mathbf{u}_0}}{\sqrt{1-\frac{\mathbf{v}^2}{\mathbf{c}^2}}}. \qquad (1.13)$$

From (1.10) and (1.13) we can get the relativistic formula for the electric charge (on its own capacitance electrodes), and mass-charge direct proportionality relations,

$$q = q_0 \frac{\frac{u}{u_0}}{\sqrt{1-\frac{v^2}{c^2}}}\ ,\ \frac{q}{m} = \frac{q_0}{m_0}\frac{u}{u_0},\ \frac{q}{q_0} = \frac{m}{m_0}\frac{u}{u_0},\ m = q\frac{m_0 u_0}{q_0 u} = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}. \qquad (1.14)$$

We know that mass is dependent on velocity. Starting from (1.14), we see that an electric charge is also velocity dependent, and that mass and its charge are mutually proportional, as follows.

$$\boxed{\begin{array}{l} q = m\cdot \text{const.} \\ (q \neq q_0) = q(v) \end{array}} \Rightarrow q = m\cdot\frac{q_0}{m_0}\frac{u}{u_0} = q_0\frac{\frac{u}{u_0}}{\sqrt{1-\frac{v^2}{c^2}}} = q(v) \Leftrightarrow \frac{q}{m} = \frac{q_0}{m_0}\frac{u}{u_0} = \text{const.}, u = u_0\frac{q}{q_0}\sqrt{1-\frac{v^2}{c^2}}. \qquad (1.15)$$

In (1.15), voltage $\mathbf{u}$ is equal to the potential difference between the capacitance electrodes,

$$u = \phi_2 - \phi_1 = \Delta\phi = u_0\frac{q}{q_0}\sqrt{1-\frac{v^2}{c^2}} = (\Delta\phi_o)\sqrt{1-\frac{v^2}{c^2}}. \qquad (1.16)$$

Also, velocities «on» analogical mechanical elements are equal to the difference of their corresponding referential velocities (if we want to treat the previous analogies correctly). Charge-to-Mass relation (or *always constant ratio) is also important or indicative from the point of view related to the direct analogy and proportionality between a mass and an electric charge (associated with the same mass). If this is valid, we could say that mass and certain associated electric charge are mutually analog (in relation to $1/r^2$, central force laws), meaning that Newton force law of universal Gravitation and Coulomb force law between equivalent electric charges are describing essentially or ontologically the same phenomenology (see more about such indicative relations in the second chapter of this book).*

Another interesting aspect of analogies can be developed if we compare the relativistic formulas for additions of the referential velocities with electrical voltages and potential (but this might be somewhat premature and complicated at this stage).

Following the same analogy and the same supporting description as in (1.10) - (1.16), it is possible to conclude that the moment of inertia (for spinning particle in rectilinear motion), most probably, could be analogically presented as (see T.1.2):

$$\mathbf{m} = \frac{\mathbf{m}_0}{\sqrt{1-\frac{\mathbf{v}^2}{\mathbf{c}^2}}} \Rightarrow \mathbf{J} = \frac{\mathbf{L}}{\omega} = \frac{\mathbf{J}_0}{\sqrt{1-\frac{\mathbf{v}^2}{\mathbf{c}^2}}} \left\{ \Rightarrow \mathbf{L} = \frac{\mathbf{J}_0\omega}{\sqrt{1-\frac{\mathbf{v}^2}{\mathbf{c}^2}}} \right\}. \quad (1.17)$$

The analogical prediction of the formula (1.17) is almost obvious as a correct result since definition of the moment of inertia is (dimensionally) the product of a certain mass and certain surface. Of course, a spinning mass, which has the moment of inertia J (described by formula (1.17)), should be moving along its axis of rotation, having a linear (axial) velocity v. In the same case, combined, non-relativistic, motional (kinetic) energy can be expressed as (v << c):

$$E_k = \frac{1}{2}mv^2 + \frac{1}{2}J\omega^2, \quad (1.18)$$

which is well applicable for observations in a center of gravity system. In relativistic motion situations (using (1.17)), when certain mass is in linear motion and makes a spinning, we can generalize (1.18) into (1.19), a little bit hypothetically, assuming the applicability of used analogies:

$$\mathbf{E}_k = (\mathbf{m} - \mathbf{m}_0)\mathbf{c}^2 + (\mathbf{J} - \mathbf{J}_0)\omega_c^2 = \frac{pv}{1+\sqrt{1-(\frac{v}{c})^2}} + \frac{L\omega}{1+\sqrt{1-(\frac{v}{c})^2}} \quad (1.19)$$

$$(\Rightarrow \mathbf{pv} + \mathbf{L}\omega = \mathbf{E}_k\left[1+\sqrt{1-(\frac{v}{c})^2}\right], \ (\frac{\omega}{\omega_c})^2 = (\frac{v}{c})^2(1-\frac{v^2}{c^2})^{-1/2}).$$

From (1.19) we could try to go back to analogous electromagnetic equivalents, using again (1.13) and T.1.2 - T.1.5, (with an objective to create corresponding and meaningful relativistic expressions, in the frame of the system of analogies presented here).

*It would be interesting to make a comparison between electric charge **q**, and momentum **p** (regarding its "Mobility-type" analog match in Mechanics). As we already know, electric charge is invariant (always stay the same, constant amount), regardless of its velocity. By analogy, the same could be valid for Linear Momentum (or quantity of motion), **p** = **m**v. If we want to make them formally analogous (regardless of this is correct or not), then such mathematical exercise would produce:*

$$\left\{ \begin{array}{l} \{q = Cu = inv.\} \Rightarrow \{p = mv^* = inv.\},\ m = m_0/\sqrt{1-v^2/c^2}, \\ C = C_0/\sqrt{1-v^2/c^2},\ \{u = u_0\sqrt{1-v^2/c^2}\} \Rightarrow \{v^* = v_0^*\sqrt{1-v^2/c^2}\} \end{array} \right\} \Rightarrow \quad (1.20)$$

$$\Rightarrow p = mv^* = (m_0/\sqrt{1-v^2/c^2}) \cdot (v_0^*\sqrt{1-v^2/c^2}) = m_0 v_0^* = inv..$$

- ***Velocity and the Spatial Matrix Framework***

The velocity v_0^* appearing in equation (1.20) could be associated with a specific spatial matrix that possesses its own inherent background velocity. This background velocity is intrinsically linked to any particle that has a non-zero rest mass. In other words, what we traditionally consider as a particle's rest mass, when viewed from a specific reference frame attached to this spatial matrix, would not be truly at rest. This perspective redefines the concept of rest mass and its relationship to motion.

A similar reasoning can be extended to an angular momentum, $L = J\omega$, which applies to rotational or spinning motion. At present, there is no definitive evidence that either linear momentum p or angular momentum L remains invariant under changes in velocity. This suggests that we still lack key components, particularly regarding the analogy between electrical and mechanical charges, that would enable us to fully validate equation (1.20).

- ***The Real Sources of Gravitation: Beyond Mass***

Newton's theory of gravitation identifies mass as the sole source of gravitational attraction. However, if we consider the analogies and theoretical developments here presented (see T1.3, T1.6 – T1.8, and discussions in Chapters 2 and 8), the real sources of gravitation may be more complex. In addition to static mass, these sources could include oscillating masses, linear and angular moments, as well as electromagnetic charges and moments arising from atomic structures.

This expanded view implies that gravitational "charges" may not only be due to static masses but could also involve the dynamic properties of particles, such as linear and angular moments, coupled with electromagnetic interactions.

- ***Addressing Incomplete Theoretical Frameworks***

In Chapters 2 and 10 of this book, we delve deeper into the concept of hidden or background velocity parameters and their role in Newtonian gravitational attraction between linear and angular moments (see equations (10.1.4) – (10.1.7)). These considerations suggest that our current understanding of gravitational interactions may be incomplete, particularly regarding how such parameters interact with known forces.

- ***Caution Regarding Relativity Theory and Premature Conclusions***

Given the ongoing debates and recent contributions surrounding the foundational aspects of Einstein's Theory of Relativity, it would be prudent to temporarily set aside premature conclusions derived solely from analogy (such as in equations (1.10) – (1.20)), which may be directly linked to relativity. A more cautious approach will ensure that these analogies are supported by empirical data and consistent theoretical foundations before being integrated into a broader framework. ♣]

An analysis of the previously established analogies, T.1.1 - T.1.5, shows that all electric and mechanical entities can be very conveniently arranged and classified into two groups, according to their definition or nature, such as: **Spatial/Geometry parameters, and Action (or dynamic) parameters**, T.1.6.

T.1.6

Spatial/Geometry parameters	Linear Motion Gravitation	Electromagnetism	Rotation	*Spatial/Geometry parameters*
Velocities and voltages	$v = dx/dt$	$u = d\Phi/dt$	$\omega = d\theta/dt$	Velocities and voltages
Displacements	$x = sF$	$\Phi = Li$	$\theta = s_R \tau$	Displacements
Reactances (+)	s	L	s_R	Reactances (+)
Resistances and Impedances	(*) $R_m = (v/F)_{Real}$ $Z_m = (v/F)_{Complex}$	(*) $R = (u/i)_{Real}$ $Z = (u/i)_{Complex}$	(*) $R_R = (\omega/\tau)_{Real}$ $Z_R = (\omega/\tau)_{Complex}$	Resistances and Impedances
Reactances (-)	m	C	J	Reactances (-)
Charges	$p = mv$	$q = Cu$	$L = J\omega$	Charges
Forces and Currents	$F = dp/dt$	$i = dq/dt$	$\tau = dL/dt$	Forces and Currents
Action Parameters	Linear Motion Gravitation	Electromagnetism	Rotation	*Action Parameters*

(*) $R = (u/i)_{Real}$ ⇔ u(t) and i(t) in phase (=) Real, Active Impedance or Resistance,
$Z = (u/i)_{Complex}$ ⇔ u(t) and i(t) out of phase (=) Complex Impedance (=) Resistance + Reactance.

As we can see, in table T.1.6 we do not have an explicit separation regarding parameters relative only to electric and only to magnetic fields. In the next chapters of this book, it will be shown that such parameters separation (by using analogy criteria) can be realized in the following way (see table T.1.7):

T.1.7

Spatial/Geometry parameters	Electric field	Magnetic field	*Spatial/Geometry parameters*
Velocities and voltages	$u = d\Phi/dt$	$i = dq/dt$	Velocities and voltages
Displacements	$\Phi = Li$	$q = Cu$	Displacements
Reactances (+)	L	C	Reactances (+)
Resistances and Impedances	(*) $R_{el.} = (u / i)_{Real}$ $Z_{el.} = (u / i)_{Complex}$	(*) $R_{mag.} = (i / u)_{Real}$ $Z_{mag.} = (i / u)_{Complex}$	Resistances and Impedances
Reactances (-)	C	L	Reactances (-)
Charges	$q = Cu$	$\Phi = Li$	Charges
Forces and Currents	$i = dq/dt$	$u = d\Phi/dt$	Forces and Currents
Action Parameters	Electric field	Magnetic field	*Action Parameters*

- **Electromechanical Analogies and Their Role in Unifying Physical Theories**

The classifications presented in Tables T.1.6 and T.1.7 provide a nearly complete and foundational overview of electromechanical analogies, structures, and the organization of key physics-related entities known today. Using these systematically arranged analogies, several hypotheses and predictions will be proposed in subsequent sections, addressing new perspectives on Gravitation, Faraday-Maxwell Electromagnetic Theory, and Quantum Mechanics (refer to sections (2.1) - (3.5), (4.1) - (4.4), and equations (5.15) and (5.16)).

- **Towards a Unified Framework for Physics**

Building on this foundation, a specific unification platform will be introduced in the later chapters, drawing on the concept of *generic Symmetries* as outlined in Table T.1.8. This platform aims to identify potential and most probable areas of unification between Gravitation, Electromagnetism, and Quantum Mechanics. The integration of these symmetries serves as a connecting thread, weaving together the analogies explored in Tables T.1.3 through T.1.8 (all of that being the part of an extended CPT Symmetry).

- **Symmetries as Indicators of Unified Physics**

The idea of *basic Symmetries* not only unifies these analogies but also conveys a deeper message: anything that exhibits analogous and symmetrical relationships could serve as an initial indicator of the broader framework for a Unified Physics or Unified Field Theory. This approach suggests that identifying these patterns of symmetry is the first step toward constructing a comprehensive theory that reconciles seemingly distinct physical phenomena under a single, cohesive model.

....

In 1905, a mathematician named **Amalie Nether** proved the following theorem (regarding universal laws of Symmetries):

-For every continuous symmetry of the laws of physics, there must exist a conservation law.
-For every conservation law, there should exist continuous symmetry.

Let us only summarize already-known conservation laws and basic symmetries in Physics, by creating table T.1.7.1 (without entering a more profound argumentation, since in later chapters of this book we will again discuss basic continuous symmetries, introducing a much wider background).

T.1.7.1 Symmetries of the Laws of Physics (mutually conjugated variables)

Original Domains $\leftrightarrow$ (spatial, geometry or static parameters)	$\leftrightarrow$ **Spectral Domains** (motional, dynamic parameters) Spectral domains
Simple symmetries of spacetime	
Time = t	**Energy** = E **(or frequency** = f**, or mass** = m**)**
Time Translational Symmetry	**Law of Energy Conservation**
Displacement = $x = S\dot{p} = SF, (F = \text{Force})$	**Momentum** = $\mathbf{p} = \mathbf{m}\dot{\mathbf{x}} = \mathbf{mv}$
Space Translational Symmetry	**Law of Conservation of Momentum**
Angle = $\theta = S_R\dot{L} = S_R\tau$	**Angular momentum** = $L = J\dot{\alpha} = J\omega$
Rotational Symmetry	**Law of Conservation of Angular Momentum**
Extended spacetime symmetry (CPT & Gauge theory symmetry)	
Electric Charge = $\mathbf{q}_{el.} = \Phi_{el.} = \mathbf{C}\dot{\mathbf{q}}_{mag.} = \mathbf{C}\,\mathbf{i}_{mag.}$	**Magn. Charge** = $\mathbf{q}_{mag.} = \Phi_{mag.} = \mathbf{L}\dot{\mathbf{q}}_{el.} = \mathbf{L}\,\mathbf{i}_{el.}$
Law of Total Electric Charge Conservation	**The Electric Charge-reversal Symmetry**
The Magnetic Charge-reversal Symmetry	**"Total Magnetic Charge" Conservation**
(Magnetic monopole or charge is not a free and self-standing physical entity)	

Mutually Conjugate Variables and Symmetries in Fourier Transformations

In Table T.1.7.1, the mutually conjugated variables represent the Fourier integral transformations between corresponding values on opposite sides of the table. These transformations can be performed in both directions, revealing deeper connections and symmetries between what we conditionally label as the *static* or *spatial* parameters of our universe (listed on the left side of the table) and their *motion-related* or *dynamically conjugated* counterparts (listed on the right side).

Table T.1.7.1 demonstrates that the conjugate pairs between the Original and Spectral domains can be systematically extended. By employing simple analogies already established in Tables T.1.3 through T.1.7.1, we can construct Table T.1.8, where these relationships are further elaborated. This extension shows a coherent pattern linking static and dynamic properties of matter in motion through well-defined mathematical relationships.

We could also make a formal, analogical transformation of T.1.2 into T.1.2.1. The new and strange terminology used on the left column side of T.1.2.1 is still conditional and only temporarily established with the intention to indicatively and analogically or hypothetically underline the possible existence and meaning of new electromagnetic parameters, if such parameters could really exist (all of that is left to be verified, more profoundly analyzed and optimized later; -see T.3.1, T.3.2,T.3.3 in Chapter 3, and 5.4.1 in Chapter 10).

T.1.2.1

Electromagnetic parameter / [unit]	Mechanical parameter / [unit]
Electric Voltage (=) u (=) [V]	Linear Velocity (=) v (=) [m/s]
Magnetic Voltage/Electric Current	Angular Velocity (=) ω (=) [rad/s]
Electric Current (=) i = dq/dt (=) [A]	Force (=) F = dp/dt = f (=) [N = kg m/s^2 = Newton]
Magnetic Current/Electric Voltage	Torque (=) τ = dL/dt (=) [kg m^2/s^2]
Electric Resistance (=) R (=) [Ω]	Mech. Resistance (=) R_m (=) [m /N s = s/kg]
Magnetic Resistance	Mech. tors. Resistance (=) R_R (=) [s/kg m^2]
Electric Inductance (=) L (=) [H]	Spring Stiffness (=) S (=) [m/N = s^2/kg]
Magnetic Inductance (=) C = [F]	Torsion. Spring Stiffness (=) S_R (=) [s^2/kgm^2]
Electric Capacitance (=) C (=) [F]	Mass (=) m (=) [kg]
Magnetic Capacitance (=) L (=) [H]	Moment of Inertia (=) J (=) [kg m^2]
Electric Charge (=) q = Cu (=) [Coulomb]	Momentum (=) p = mv (=) [kg m/s]
Magnetic Charge (=) q_{mag} (=) Φ (=) [Wb = V s]	Angular Momentum (=) L = J ω (=) [kg m^2/s]
Magnetic Flux Φ (=) [Wb = V s]	Displacement (=) x (=) [m]
Electric Flux (=) q (=) [Coulomb]	Angle (=) θ (=) [rad] (=) [1]

- **Analogies, Symmetries, and Conservation Laws**

The key insight is that these analogies and basic symmetries are fundamentally tied to some of the most important conservation laws in physics, laws that hold universally. As shown in Table T.1.8, these symmetries provide a unified framework that integrates different aspects of physical systems, making them mathematically robust and intuitively predictive.

- **Towards a Unified Predictive Framework**

Properly merging these analogies and symmetries should form the core of a powerful, unifying platform for future physics. This framework is not only theoretically compelling but also practical, as it offers predictive power across diverse physics domains. This approach is consistently applied throughout the book, illustrating its potential as a cornerstone for advancing our understanding of the physical world.

T.1.8 Generic Symmetries and Analogies of the Laws of Physics

Velocity, Voltage analogies ↓	**Original Domains ↔** (spatial, static parameters)	**↔ Spectral Domains** (motional, dynamic parameters)	Force, Current analogies↓
	Time = t	**Energy** = E	
	Time Translational Symmetry	Law of Energy Conservation	
$v = \frac{dx}{dt} = \dot{x}$ (=) Linear Velocity	**Displacement** = x	**Momentum** = p	$F = \frac{dp}{dt} = \dot{p} = m\dot{v}$ (=) Force
	Space Translational Symmetry	Law of Conservation of Momentum	
$\omega = \frac{d\theta}{dt} = \dot{\theta}$ (=) Angular velocity	**Angle** = θ	**Angular momentum** = L	$\tau = \frac{dL}{dt} = \dot{L} = J\dot{\omega}$ (=) Torque
	Rotational Symmetry	Law of Conservation of Angular Momentum	
	Electric field	**Magnetic field**	
↑ $u = \frac{d\Phi_{mag.}}{dt} = \dot{\Phi}_{mag.}$ (=) Voltage	**Electric Charge** = $q_{el.} = \Phi_{el.} = C\dot{q}_{mag.} = C\, i_{mag.}$	**Magnetic Charge** = $q_{mag.} = \Phi_{mag.} = L\dot{q}_{el.} = L\, i_{el.}$	↑ $i = \frac{d\Phi_{el.}}{dt} = \dot{\Phi}_{el.}$ (=) Electric Current
	Law of Total Electric Charge Conservation	**The Electric Charge-reversal Symmetry**	
	The Magnetic Charge-reversal Symmetry	**"Total Magnetic Charge" Conservation**	

Spatial/Geometry parameters	Linear Motion (Gravitation)	Electromagnetism	Rotation	Spatial/Geometry parameters
Displacements	$x = sF$	$\Phi = Li = q_{mag.} = \Phi_{mag.}$ $q = Cu = q_{el.} = \Phi_{el.}$	$\theta = s_R \cdot \tau$	**Displacements**
Velocities and voltages	$v = dx/dt$	$u = d\Phi_{mag.}/dt = u_{el.} = i_{mag.}$ $(i = d\Phi_{el.}/dt = u_{mag.} = i_{el.})$	$\omega = d\theta/dt$	**Velocities and voltages**
Acceleration	$a = d^2x/dt^2 = \dot{v}$	$du/dt = d^2\Phi_{mag.}/dt^2 = du_{el.}/dt = di_{mag.}/dt$ $(di/dt = d^2\Phi_{el.}/dt^2 = du_{mag.}/dt = di_{el.}/dt)$	$\dot{\omega} = d^2\theta/dt^2 = \alpha$	**Acceleration**
Reactances (+)	s	$L = q_{mag.}/i = \Phi_{mag.}/i_{el.} (=) \Phi_{el.}/i_{mag.}$	s_R	**Reactances (+)**
Resistances and Impedances	(*) $R_m = (v/F)_{Real}$ $Z_m = (v/F)_{Complex}$	(*) $R = (u/i)_{Real}$ $Z = (u/i)_{Complex}$	(*) $R_R = (\omega/\tau)_{Real}$ $Z_R = (\omega/\tau)_{Complex}$	**Resistances and Impedances**
Reactances (-)	$m = p/v$	$C = q/u = \Phi_{el.}/u_{el.} (=) \Phi_{mag.}/i_{el.}$	$J = L/\omega$	**Reactances (-)**
Charges	$p = mv$	$q = Cu = q_{el.} = \Phi_{el.}$ $\Phi = Li = q_{mag.} = \Phi_{mag.}$	$L = J\omega$	**Charges**
Forces and Currents	$F = dp/dt = ma$	$i = d\Phi_{el.}/dt = u_{mag.} = i_{el.}$ $(u = d\Phi_{mag.}/dt = u_{el.} = i_{mag.})$	$\tau = dL/dt = J\dot{\omega} = J\alpha$	**Forces and Currents**
Power	$P = Fv$	$P = iu = i_{el.}u_{el.} = u_{mag.}i_{mag.}$	$P = \tau\omega$	**Power**
Work	$Fx = P \cdot \Delta t$	$\Phi i = Li^2 = q_{mag.}i = \Phi_{mag.}i$ $qu = Cu^2 = q_{el.}u = \Phi_{el.}u$	$\tau\theta = P \cdot \Delta t$	**Work**
Kinetic or field energy	$\frac{1}{2}mv^2$	$(\Phi i = Li^2 = q_{mag.}i = \Phi_{mag.}i)$ $(qu = Cu^2 = q_{el.}u = \Phi_{el.}u)$	$\frac{1}{2}J\omega^2$	**Kinetic or field energy**
Action Parameters	**Linear Motion Gravitation**	**Electromagnetism**	**Rotation**	**Action Parameters**

(*) $R = (u/i)_{Real}$ ⇔ u(t) and i(t) in phase (=) Real, Active Impedance or Resistance,
$Z = (u/i)_{Complex}$ ⇔ u(t) and i(t) out of phase (=) Complex Impedance (=) Resistance + Reactance.
(Comment: static and permanent magnetic monopoles do not exist)

- **Extension of Symmetry Concepts in Physics**

The concept of symmetries in physics can be extended far beyond what is presented in T.1.8. However, this book focuses on utilizing unified analogies and continuous symmetries, as illustrated in T.1.8, to provide a robust predictive framework. These symmetries are deeply connected to the conservation laws in physics, particularly those related to translational and rotational symmetries.

From T.1.8, we can deduce, through analogical reasoning, that electromagnetism likely has a natural mechanical counterpart. This counterpart could involve fields that unify linear and rotational motions, manifesting as the wave-particle duality phenomena observed in physics.

Many of the symmetries, including CPT, known in physics, mathematics, and in the natural sciences are binary (or bipolar) in nature. Examples include positive and negative values of physics entities, mirror symmetries, particles and antiparticles, oppositely charged poles of particles, real and imaginary numbers, and left-right or up-down symmetries. These binary symmetries are often implicitly combined with continuous symmetries.

However, our tendency to prioritize binary symmetries may be driven by intellectual inertia, limiting our exploration of more complex symmetries. There may be more exotic symmetries involving more than two mutually opposite participants. For instance, the quantum theory concepts of quarks and neutrinos might be better understood within a framework of "ternary" or "triadic" symmetries, involving three poles. This idea could be further developed using an extended "Minkowski-Hypercomplex space wavefunctions and energy-moments entities" with three imaginary units (see Chapter 10 for more details).

- **Coupling of Linear Motion and Spinning in a New Gravitational Theory**

Analogies and symmetries suggest that linear motion and spinning should complement each other, like the way electric and magnetic fields (or currents and voltages) are coupled. This idea forms the foundation for a new theory of gravitation and wave-particle duality, which will be a central theme of this book.

This new theory is fundamentally based on translational and rotational symmetries, which correspond to the conservation laws of linear and angular momentum and energy. All stable, linear, uniform, and inertial motions are analogous, and this concept also applies to stable rotational or orbital motions, which are forms of accelerated motion. ***Wave-particle duality serves as a conceptual and phenomenological bridge between linear motions (with spatial translational symmetry) and rotational motions (with spatial rotational symmetry), ensuring the coincidental conservation of energy and linear and angular momenta.***

Furthermore, under linear motion of masses, we can draw analogies to the linear motion of electrically charged particles within an electric field. Similarly, under rotational motion of masses, we can associate rotational (spinning and helical) motion of electrically charged particles within a magnetic field. This way, analogically, we could extend and connect the mechanical meaning of inertia with electromagnetic meaning of inertia (since structure of matter and atoms internally has spinning and orbiting states of elementary particles, and electromagnetic states, moments and dipoles with constant gyromagnetic ratios).

- **Unified Fields and Forces in the Universe**

The existence of these analogies and symmetries in physics indicates that the fields, forces, and motions governing our universe are inherently interconnected and naturally unified. These connections, which are also related to wave-particle duality and the coupling of periodicities, suggest that the fundamental origin of these phenomena could be electromagnetic in nature.

1.1. Analogies, Inertia, Inertial Systems, and Inertial Motions

The indicative, analogical and challenging, still somewhat hypothetical situation in Physics is related to the project of unification of Mechanics and Electromagnetism. The working concept here is that Electromagnetism is underlaying Mechanics. The starting platform in such conceptualization is that inertial states of rest and states of uniform linear and angular motions, both in mechanics and electromagnetism, are coupled, mutually analog and symmetrical on a way as presented in the table below (T.4.2.2).

T.4.2.2. INERTIA, INERTIAL STATES, INERTIAL MOTIONS			
Mechanics		**Electromagnetism**	
Linear motions	**Angular motions, rotation, spinning**	**Motions and states in electric field**	**Motions and states in magnetic field**
Linear motion and state of rest Inertial states	**Rotating and spinning Inertial states**	**Electric field polarization related Inertia**	**Magnetic field polarization related Inertia**

- **Analogies Between Electrical and Mechanical Circuits**

At the beginning of this chapter, we established analogies between closed electrical and mechanical circuits, focusing on oscillatory or resonant systems with passive, idealized components (see Fig.1.1). Now we understand how to analogously associate electrical components with their mechanical counterparts such as charges and other related circuits components and parameters. We can compare relevant electrical and mechanical circuits using these analogies (summarized in T.1.8), and we can envision a wide range of mutually analogical configurations for electrical and mechanical circuits and networks. These analog circuits can help us develop advanced conceptual models of real electromechanical systems, as well as explaining the knowledge about forces and motions acting in our universe.

- **Key Features of Analogous Electromechanical Systems**

Among these analog electromechanical systems, particular attention should be given to self-sustaining, stable, uniform, inertial motions, both with mutually coupled mechanical and electromagnetic nature. These motions, when viewed through the lens of matter-wave duality or PWDC, intrinsically contain spatial-temporal periodicities (see Chapters 4.0, 4.1, and 10 for more on PWDC).

These analog physical circuits and networks share the following characteristics:

1. Dimensionality:
- Most current analyses focus on planar, two-dimensional circuits. We rarely explore more complex three-dimensional or multidimensional spatial cases, as the necessary technologies and devices are still in their infancy.

2. Basic Structure:
- Typically, these planar systems can be represented as four-pole networks. Each network consists of a front-end primary source or input (such as voltage, current, velocity, or force generators), a network or "black box" that processes the input, and a load impedance or output, which serves as the energy consumer.

3. Active and Passive Components:
- In addition to passive components, these two-dimensional or planar networks often include active electromagnetic and mechanical elements, such as amplifiers, transistors, current sources, different regulators, sensors, piezoelectric and magnetostrictive components, motors, and actuators. It is valuable to establish a comparative table of equivalent or analogous active elements between electromagnetic and mechanical systems.

4. Towards 3D Networks:
- We can imagine that these two-dimensional networks will naturally evolve into more complex three-dimensional, spatially distributed circuits. These would include both passive and active elements, with multiple front-end energy sources and one or more load-impedance consumers, reflecting the forces and motions present in our universe. However, we currently lack the necessary theories, practices, and technologies to fully address such three-dimensional networks.

5. Closed Systems:

- The networking concept discussed here always refers to fully connected, closed systems (or spatial matrices) of electrical, mechanical, or mixed circuits and elements. These networks might also involve matter waves, fields, forces, and radiation (as illustrated in Fig. 4.1.6 in Chapter 4.1). This reflects the structure and operation of our universe: we should not have open electrical or mechanical circuits without *front-end* sources and *last-end* consumers (loads or sinks). All physical circuits are typically closed systems, nothing exists without being fixed or connected to something, and nothing simply disappears into the unknown environment.

The natural laws governing the functionality of these circuits and networks include the conservation of momentum and energy, translational and rotational symmetry, generalized Ohm's and Kirchoff's laws, and the conservation of charges, along with Norton's and Thevenin's theorems, all of which can be analogically extended from electromagnetic phenomena to mechanics.

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From Wikipedia, the free encyclopedia:
"**Thévenin's theorem** holds, to illustrate in DC circuit theory terms, that:

1. Any linear electrical network with voltage and current sources and resistances can be replaced at terminals A-B by an equivalent voltage source V_{th} in series connection with an equivalent resistance R_{th}.
2. This equivalent voltage V_{th} is the voltage obtained at terminals A-B of the network with terminals A-B open circuited.
3. This equivalent resistance R_{th} is the resistance obtained at terminals A-B of the network with all its independent current sources open circuited and all its independent voltage sources short-circuited.
4. For AC systems, the theorem can be applied to reactive impedances as well as resistances.
5. The theorem was independently derived in 1853 by the German scientist Hermann von Helmholtz and in 1883 by Léon Charles Thévenin (1857–1926), an electrical engineer with France's national Postes et Télégraphes telecommunications organization.[1][2][3][4][5][6]
6. Thévenin's theorem and its dual, Norton's theorem, are widely used for circuit analysis simplification and to study circuit's initial-condition and steady-state response.[7][8] Thévenin's theorem can be used to convert any circuit's sources and impedances to a **Thévenin equivalent**; use of the theorem may in some cases be more convenient than use of Kirchhoff's circuit laws.[9][6] "

- **The Concept of Closed Circuits in Electronics and Physics**

In electronics, electrical engineering, and circuit theory, the concept of closed electric circuits, where there are clearly defined sources and loads at the front and back ends, involving various electrical and electromechanical components is fundamental, natural, and obligatory. Open-ended, freely hung, or "randomly floating" circuits do not have any real significance; they simply do not exist in practical physics or in our universe.

- **Extending Electrical Concepts to Mechanical Systems**

In this chapter, we explore the idea that almost everything achievable within electrical circuits and with electrical components can be analogously realized within mechanical circuits, as summarized in the analogies of T.1.8. This understanding opens the door to new ways of conceptualizing and modeling mechanical systems.

- **Issues in Contemporary Physics and Mechanics**

Turning our attention to contemporary physics, mechanics, relativity, and quantum theory, fields that deal with the motion of masses, fluids, waves, elementary particles, and light we often encounter a disconnect. In many published references, it is unclear where or how the equivalent of closed mechanical and energy-momentum circuits, with well-defined inputs and outputs (sources and sinks), are represented. Instead of these closed-circuit concepts, modern physics often relies on more complex, sometimes abstract concepts like inertia and inertial motions, reference frames, Lorentz transformations, and the conservation of "energy-momenta" through 4-vectors in the Minkowski-Einstein framework.

These approaches tie together motion and geometry with a host of associated rules, postulates, transformations, and complex mathematical constructs. While these theoretical frameworks can be intellectually stimulating and offer specific applications, they often feel disconnected, overly complex, or not entirely explainable.

- **A Simpler, Unified Approach**

A simpler, unifying approach is to recognize that all inertial (self-sustaining, stable, uniform) mechanical and electromagnetic motions are analogous to one another and linked with intrinsically associated matter-waves and the periodicities of wave-particle duality.

- **The Closed-Circuit Perspective**

In this book, we will embrace the idea that all mechanical, electrical, and matter-wave phenomena can, and should be conceptualized within fully closed circuits and networks, with real front-end and last-end elements. By adopting this closed-circuit perspective, we can revisit and update our understanding of concepts such as inertia, inertial motions, inertial forces, Lorentz transformations, wave motions, electromagnetic phenomena, relativity, and quantum theory. This also includes fundamental principles like action-reaction and induction laws in both mechanics and electromagnetism.

Taken from: http://en.wikipedia.org/wiki/Inertia: **"Inertia** is the resistance of any physical object to a change in its state of motion or rest, or the tendency of an object to resist any change in its motion (including a change in direction). The principle of inertia is one of the fundamental principles of classical physics which are used to describe the motion of matter and how it is affected by applied forces. Inertia comes from the Latin word, *iners*, meaning idle, or lazy. Isaac Newton defined inertia as his first law in his *Philosophiæ Naturalis Principia Mathematica*, which states:[1]

The *vis insita*, or innate force of matter, is a power of resisting by which everybody, as much as in it lies, endeavors to preserve its present state, whether it be of rest or of moving uniformly forward in a straight line.

In common usage the term "inertia" may refer to an object's "amount of resistance to change in velocity" (which is quantified by its mass), or sometimes to its momentum, depending on the context. The term "inertia" is more properly understood as shorthand for "the principle of inertia" as described by Newton in his First Law of Motion; that an object not subject to any net external force moves at a constant velocity. Thus an object will continue moving at its current velocity until some force causes its speed or direction to change.

Taken from: http://en.wikipedia.org/wiki/Inertial_frame_of_reference: "In physics, an **inertial frame of reference** (also **inertial reference frame** or **inertial frame** or **Galilean reference frame**) is a frame of reference that describes time and space homogeneously, isotropically, and in a time-independent manner.[1]

All inertial frames are in a state of constant, rectilinear motion with respect to one another; an accelerometer moving with any of them would detect zero acceleration. Measurements in one inertial frame can be converted to measurements in another by a simple transformation (the Galilean transformation in Newtonian physics and the Lorentz transformation in special relativity). In general relativity, in any region small enough for the curvature of space-time to be negligible, one can find a set of inertial frames that approximately describe that region.[2][3]

Physical laws take the same form in all inertial frames.[4] By contrast, in a non-inertial reference frame, the laws of physics vary depending on the acceleration of that frame with respect to an inertial frame, and the usual physical forces must be supplemented by fictitious forces.[5][6] For example, a ball dropped towards the ground does not go exactly straight down because the Earth is rotating. Someone rotating with the Earth must account for the Coriolis effect—in this case, thought of as a force—to predict the horizontal motion. Another example of such a fictitious force associated with rotating reference frames is the centrifugal effect or centrifugal force.

Taken from: http://academickids.com/encyclopedia/index.php/Inertial_frame_of_reference: When there is no force being exerted on an object then the object will move inertially. This is also called 'free motion'. For example, a space module that is not firing any thrusters. (If this space module is located in intergalactic space in a region of space where gravitational influences of surrounding galaxies cancel then that is effectively a zero-gravity environment.) A frame of reference that is defined as co-moving with that object is an **inertial frame of reference**. This definition also covers rotation. A spinning gyroscope will maintain its orientation. To change the orientation of a spinning gyroscope a torque must be applied. When this torque is applied inertia manifests itself, it is inertia that maintains the gyroscope's orientation. A gyroscope that is suspended friction-free allows an observer to maintain zero rotation with respect to the co-moving inertial frame of reference.

All reference frames that move with constant velocity and in a constant direction with respect to any inertial frame of reference are members of the group of inertial reference frames."

[♣ COMMENTS & FREE-THINKING CORNER:

- ***Inertia and Inertial Motion: A Modern Perspective***

Let us begin by considering a mass in inertial motion. According to Newtonian mechanics, such mass should have a constant velocity, meaning it remains stable or stationary in its motion (including its state of rest). Newton introduced the concept of inertia and inertial motions primarily in the context of linear motion. However, modern physics recognizes that inertial states and motions can also apply to rotational, orbital, circular, and spinning motions and their relevant rest-states. In this book, we will explore a broader and more generalized conceptualization of inertia and inertial motions, expanding beyond the limitations of the initial Newtonian definition, which is now considered outdated. Here we conceptualize Wave-Particle Duality phenomenology as a physical bridge, or tangible "energy-moments-coupling" state between linear Newtonian inertia and Rotational or Spinning inertia of motional masses, including their relevant rest states and associated electromagnetic states.

- ***Electro-mechanical Mobility System of Analogies and Planetary Motion: An Analogy***

A useful analogy in the electromechanical Mobility system equates mass and velocity with capacitance and voltage, respectively. Imagine a closed mechanical system, like any stable planetary system, where the planets move in near-inertial states of periodic motion. Each planet has a sufficiently stable orbital velocity around its local sun. Analogously, such a planetary system can be compared to a network of interconnected capacitors, where masses correspond to capacitors in the Mobility system of analogies.

In this analogy, planetary masses possess angular and tangential orbital velocities relative to a common center of mass, typically where the sun resides. Thus, the equivalent capacitors (representing these masses) relate to a common grounding point, akin to the sun. In this system, the sun acts as one electrode, while the surface of each planet serves as the second electrode for the corresponding capacitance. Both mass and capacitance represent reactive, non-dissipative impedances, idealized as systems without resistive or friction-related losses.

Since a stable planetary system behaves almost frictionlessly over exceptionally long-time intervals, we can apply the mass-capacitance analogy. This leads us to consider planetary orbital motions as nearly ideal inertial motions. The simplified idea is that inertia and/or inertial motion largely corresponds to maintaining a constant mass velocity, which in analog terms corresponds to a constant voltage across a given capacitance, with minor, negligible deviations.

- ***Extending the Analogy: Couplings and Modulations***

Furthering the analogy, just as there are interconnections between planets, there are time-evolving capacitive (or gravitational) couplings involving smaller, non-dominant capacitors. Planets rotate around their sun in elliptical orbits, leading to slight variations in orbital velocities and the associated dominant capacitances. These variations result in periodic modulations in voltage or velocity, indicating that our understanding of inertial motion must extend beyond the simplistic notion of constant-velocity motion.

We also observe that each planet in a stable planetary system has a constant and highly stable orbital momentum. This suggests that a more refined and generalized definition of inertial motion and inertial states could be based on this mechanical and associated electromagnetic stability, rather than merely on constant velocity.

- ***The Nature of Planetary Systems: DC and AC Analogies***

In this generalized conceptual framework, constant orbital velocities are analogous to constant voltages within an equivalent capacitor network. Such constant voltages can be generated by both DC and AC voltage sources. In the case of a DC source, where the voltage remains stable and constant, the corresponding planetary voltages (in analogical terms) will also be stable DC voltages. Communication within such a capacitor network would only occur during short-lived transient situations.

On the other hand, if the source voltage is AC, with a stable RMS value, the corresponding planetary voltages will also be stable AC RMS voltages. This leads to continuous, synchronized, and periodic interactions within the capacitor network. This analogy introduces a new perspective on the structural stability of planetary systems, suggesting the existence of a front-end oscillatory signal-source, akin to a rotating disk or flywheel energy storage system, which synchronizes and drives the motion of all planets.

- ***Matter-Waves and Inertial Motion***

In planetary systems, it is more probable that an oscillating matter-wave field acts as the driving force, like an oscillating voltage-velocity or force-current source. Drawing from the wave-particle duality, every particle's motion can be viewed as a matter-wave motion, defined by the de Broglie wavelength. This concept should also apply to inertial motions, despite their appearance as uniform and stable. Thus, the stable, uniform motion of particles or planets within a solar system should be seen as synchronized, stabilized matter-wave motion, forming standing wave structures, both mechanically and electromagnetically, since masses and atoms internally have electromagnetic and spinning entities mutually coupled.

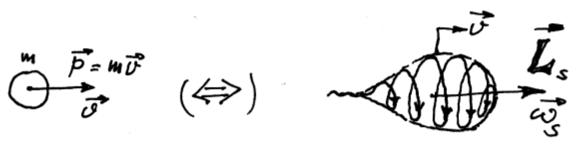
A similar concept is elaborated in Chapter 2, particularly in section 2.3.3, which discusses Macro-Cosmological Matter-Waves and Gravitation.

- ***Conclusion: Revisiting Inertia and Inertial States***

In summary, inertia and inertial states are likely linked to standing and synchronized mechanical and electromagnetic matter-wave structures, which correspond to stable, uniform motions. To fully understand and properly define inertial systems and states, we must identify the relevant, dominant, and locally superior reference systems. Without this understanding, our analyses and conclusions will remain on shaky ground, leading to arbitrary transformations and compensatory errors in our theoretical models.

While certain methods in mathematical and theoretical physics have yielded productive results over time, it is crucial to critically and creatively reexamine, redesign, and generalize the foundational concepts regarding reference and inertial frames, matter states including inertial states, and motions, particularly considering the analogies and symmetries discussed in this book.

*Later (in this book; -chapters 4.1 and 10) we will show that every particle in linear motion effectively has certain associated, angular momentum $\mathbf{L}_s$ that corresponds to certain kind of spinning, where particle's linear moment **p** and its spinning moment $\mathbf{L}_s$ are mutually collinear and coupled or coincidently present thanks to Wave Particle Duality, as indicatively illustrated on the picture below (see about PWDC in Chapter 10).*



$$E_k = \begin{Bmatrix} \frac{1}{2}mv^2 \\ \frac{1}{2}J\omega_s^2 \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} \frac{mv^2}{1+\sqrt{1-\frac{v^2}{c^2}}} \\ \frac{J\omega_s^2}{1+\sqrt{1-\frac{v^2}{c^2}}} \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} \frac{pv}{1+\sqrt{1-\frac{v^2}{c^2}}} \\ \frac{L_s\omega_s}{1+\sqrt{1-\frac{v^2}{c^2}}} \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} \frac{\vec{p}\vec{v}}{1+\sqrt{1-\frac{v^2}{c^2}}} \\ \frac{\vec{L}_s\vec{\omega}_s}{1+\sqrt{1-\frac{v^2}{c^2}}} \end{Bmatrix} \Rightarrow$$

$$pv = L_s\omega_s,\ \omega_s = 2\pi f_s \Rightarrow \vec{p} = \frac{\omega_s}{v}\vec{L}_s,\ \vec{L}_s = \frac{v}{\omega_s}\vec{p}$$

- ***Developing a New Theory in Physics: Challenges and Approaches***

Developing a specific or new theory in physics is rarely an easy task. Without a clear understanding of the theory's place within the broader framework of natural laws and its probable foundational structure, this endeavor can become arbitrary and even meaningless. The central thesis of this book is that by correctly establishing multi-level analogies and symmetries; while respecting all conservation laws, we can create a common, intuitive, and self-regulating fields and forces unification platform. This platform not only facilitates further generalizations in physics but also serves as a guide to new scientific discoveries.

Analogies, when properly constructed, can highlight inconsistencies, missing links, or weaknesses in well-established theories. They allow us to transfer successful concepts from one field to another and can inspire creative and intuitive thinking, often leading to valuable brainstorming ideas.

- ***The Historical Context of Physics Theories***

Most physical theories and philosophical frameworks have been developed independently, based on the limited data available at the time of their creation. The process was largely one of fitting the best possible facts, interpolating and extrapolating data, and constructing mathematical models that aligned with those facts. This approach has led to the development of Classical Mechanics, Relativity, Electrodynamics, Thermodynamics, Quantum Theory, and others, each addressing different aspects of the same physical universe.

To make these mathematical models practical and useful, especially when based on limited data, these theories introduce various fitting constants, axioms, theorems, postulates, and assumptions. However, there is no guarantee that these theories provide the most general and accurate description of the universe. It is possible that they offer only a locally operational and practical view, which may evolve as more data becomes available.

The most common connections between these theories are conservation laws and multi-level analogies and symmetries, since they all describe the same unified universe. By systematically applying the idea that all our knowledge about nature belongs to multilayered analogical structures, we can refine, improve, and unify our current understanding of physics. This would create a stable and comprehensive platform from which we can explore new and unknown domains of physics.

- ***The Influence of Authority and Dogma in Physics***

The author also observes that some contemporary theories in physics were established and promoted by influential and authoritative figures. These theories were later axiomatized and dogmatized by their followers, who may not have been sufficiently critical and were often professionally or existentially dependent on the theory's founders. This has created barriers to the introduction of new and different ideas.

To introduce new concepts into an environment dominated by "well-established theories," the author proposes a strategy based on the creation of a multilevel analogy and symmetries platform. This approach allows us to identify missing elements or contradictions within the broader picture of physics, facilitating the integration of new ideas.

- ***The Role of Symmetry and Analogy in Modern Physics***

Modern science has achieved significant progress, particularly in the 20th century, by recognizing that nature favors symmetry in physical theories. Symmetry is essential for constructing physical laws that are free of anomalies and divergences. The objective of this book is to demonstrate that certain symmetrical concepts should be supported and enhanced by a broad, multilevel analogy platform, making it the strongest framework for describing and mastering natural phenomena.

While some might argue that an analogy-based platform is weaker than modern topology for making relevant conclusions and predictions, this view could be misguided. We live in a unified, spatially and temporally consistent universe where all forces and fields coexist harmoniously. Nature remains united, regardless of how well we describe, accept, or understand that unity. The omnipresent signs and projections of this unity are analogies and symmetries, which we must follow creatively and with intellectual flexibility to uncover the ultimate natural laws.

- ***Towards a Unified Field Theory in Physics: The Path Forward***

It is likely that some of our current field theories are too primitive, overly simplified, or incorrectly formulated, leading to difficulties in identifying areas of unification. For instance, the perceived incompatibility between Maxwell's Theory and Gravitation, due to issues with second-rank tensors and curvature, may be temporarily and conditionally accurate. However, it also suggests that important topological components (necessary for unification) are missing from both theories.

To address these gaps, we must continue searching for more natural and accurate mathematical models and missing field components, such as incorporating rotational or torsion field components into Maxwell's Theory, Relativity, and Gravitation. The best starting point for this endeavor is the use of well-established multilevel analogies, followed by experimental and theoretical testing using modern topology, symmetries, and group theory.

Mathematics, as the language and logic of nature, is intrinsically linked to the structure of the universe. When used correctly, it can serve as an excellent tool for exploring and mastering nature, both inductively and deductively. ♣]

A Fundamental Approach to Unifying Analogies, Forces, and Fields in Modern Physics

In modern Physics, Mathematics, and Mathematical Physics, a more fundamental approach to unifying analogies, generalized forces, and fields can be identified through several key principles and frameworks:

1. Vector Analysis and Unified Application:
Vector analysis, including vector calculus, vector field analysis, and linear vector, analytical and differential geometry, represents a mathematical domain universally applicable to all vectors in physics. These vectors encompass pulses, forces, fields, and various mechanical and electromagnetic moments, applicable in Mechanics, Fluid Dynamics, Electromagnetism, and other areas. This forms a vast platform for field unification and analogy-based conclusions, initially derived from physics and later generalized by mathematics. These principles apply universally to all vectors in physics, underpinned by natural conservation laws, including the coincidental emanation of action and reaction forces and the laws of electromagnetic and electromechanical induction.

2. Energy Conservation Laws:
The law of total energy conservation is a cornerstone of physics. The simplest form states that the total energy before and after a transformation (of an isolated system) remains constant. More sophisticated approaches to energy and momentum conservation involve Probability and Statistics, and the application of Parseval's identity from Signal and Spectral Analysis. Orthodox Quantum Theory effectively exploits these without explicitly stating so. This mathematical framework is universally and analogically applicable across micro and macro physics, including Mechanics, Field Theories, Gravitation, and Wave Motion.

3. Minkowski 4-Vectors and Coupled States:
When dealing with coupled rest-energy states and vector-characterized moments, fields, and forces (whether electromagnetic or mechanical), the Minkowski 4-vectors provide a universally applicable framework. This concept unifies these diverse phenomena under a common mathematical model.

4. Universally Valid Laws and Analogies:
Laws such as "Action equals Reaction," Induction, and Inertia are universally valid and analogical across Mechanics, Gravitation, Electromagnetism, and any domain with mass-energy-moments properties. Classical wave equations, including the Schrödinger equation, always have solutions involving wavefunctions propagating in opposite directions, analogous to the action-reaction principle. These concepts manifest the unified nature of fields, with ontological origins in a common source.

5. Fourier Analysis and Decomposition:
Any physical entity with a certain shape in temporal, spatial, or spectral domains can be decomposed, recomposed, and reconstructed using Fourier and Analytic Signal Analysis. This universal principle underlies all known physics, allowing for a comprehensive understanding of matter-waves, and is closely

linked to concepts like the Shannon-Nyquist-Kotelnikov-Whittaker theorem and Denis Gabor's Analytic Signal Analysis. Uncertainty relations, a fundamental aspect of this framework, apply universally without relying solely on statistics or probability theory.

6. Wave-Particle Duality and Complex Analytic Signals:
The wave-particle duality in physics can be naturally modeled using the complex analytic signal concept established by Denis Gabor. The classical wave equation, when formulated with complex analytic signal wavefunctions, represents all wave phenomena in micro and macro physics, including the derivation of the Schrödinger equation and other quantum theory equations. Even mechanical and ultrasonic waves in solid materials can be analogically created as "ultrasonic currents," akin to electromagnetic waves and photons in open space.

7. Variational Principle of Least Action:
The principle of least or stationary action, which generates Newtonian, Lagrangian, and Hamiltonian equations of motion, is universally applicable across all domains of physics. This principle provides a consistent framework for understanding and modeling physical phenomena.

8. Resonance and Synchronization in Natural Systems:
Universally valid coupling and synchronization effects are observed in resonant systems, resonant circuits, and atomic resonators with overlapping spectral characteristics. This universal coupling indicates that all objects, masses, and atoms in the universe are at a certain level interconnected. Additionally, all natural resonators likely share ontological roots in the specific formation and packing of electromagnetic waves, offering a creative approach to understanding the Periodic Table of Elements.

9. Dimensionality and Orthogonality in the Universe:
The concept of dimensionality in the universe, extending from the familiar 4-dimensional space-time to higher multidimensional spaces, is fundamentally related to the mutual orthogonality of these dimensions. In three spatial dimensions, this concept is easily visualized using geometry and vector analysis. When extended to include time, it aligns with the orthogonality principle, placing time on an imaginary axis as in Minkowski 4-vectors. Further extension to hypercomplex numbers and quaternions allows for the creation of multidimensional Analytic Signal spaces with three spatial and three temporal dimensions, as explored in advanced signal analysis.

10. The Universality of Mathematical Frameworks:
Mathematics, based on concepts and empirical facts from physics, serves as the most universally applicable natural toolbox and logic for understanding the universe. Statistics and Probability theory, while universally applicable, excel when dealing with large numbers of identical items. By using this "natural mathematics," we achieve clear, non-contradictory explanations and analogical conclusions across all domains of physics. As mathematical logic and set theory become more widely accepted, they promise to serve as universal language for all disciplines, potentially evolving into the language of daily life, science, and philosophy (being closely connected to Artificial Intelligence programs).

- ***Caution Against Arbitrary Assumptions***

Modern science occasionally falls into the trap of making ad hoc and imaginative assumptions to explain new phenomena. For example, the "tunneling effect" in physics, though widely accepted as a name, is a superficial and oversimplified explanation. Similarly, the probabilistic nature of wavefunctions, while mathematically useful, should not be mistaken for an ontological truth. Probability and statistics are merely tools for summarizing behaviors and trends in large sets of identical participants and should not be conflated with the essence of physical phenomena.

The names or labels given to sufficiently still-unexplained forces, such as weak and strong nuclear forces, are often created too hastily, without solid or any mathematical grounding. Naming something does not equate to understanding it. Before introducing new concepts as labels, we must first consider the existing facts, laws, and frameworks that are universally valid in mathematics and physics. For instance, Parseval's identity and uncertainty relations must always be respected, regardless of the specific domain of physics being studied.

In conclusion, the path to unifying our understanding of the universe lies in grounding new theories in empirically supported mathematics and respecting the universally valid frameworks already established *in physics. This approach will ensure that we build on a solid foundation, rather than on arbitrary, artificial assumptions, or labels.*

The Role of Mathematical and Analogical Models in Physics

Mathematical analogies and other familiar indicative, conceptual frameworks serve as essential guidelines for developing new insights and concepts. These foundations provide a natural and tangible platform on which to base visionary and challenging assumptions related to newly discovered phenomena. Relying on arbitrary, ad-hoc, or "inspired labeling" in physics can be misguided approach, as is sometimes seen in modern physics. Examples of such labeling include terms like "black holes," "dark mass and energy," "neutron stars," and "nuclear forces." When theories, ideas, or concepts lack natural mathematical models or clear empirical verifications, they often reside in the realm of imaginative, artificial, ideological, or virtual constructs. While some of these constructions may be temporarily practical or operational (such as in Orthodox, Probabilistic Wave Mechanics), they risk being overly speculative or ambiguous.

- ***The Power of Normal Distribution***

Consider, for instance, the Gaussian (or Normal) statistical distribution, sometimes referred to as the Bell Curve distribution. This statistical distribution, which is central to probability theory, is widely applicable across both natural and social sciences, as well as in numerous domains within our physics and universe. When the necessary mathematical and physical conditions are met, Normal distribution effectively describes a wide range of natural phenomena.

For example, the amplitudes of signals and wavefunctions often follow a bell-curve shape in both their original and spectral domains. This leads to energy that is finite, well-localized, and limited in these domains, whether temporal, spatial or spectral. As a result, the uncertainty relations between signal durations in these Gaussian and conjugate domains tend to exhibit more "certainty" than traditionally expected. This concept should serve as a profound source of inspiration, offering analogical insights for a universal and multidisciplinary understanding of different phenomenological domains in our universe. It also holds potential for future developments in software platforms for Artificial Intelligence.

- ***The Golden Ratio as a Universal Pattern***

Another powerful example of a mathematical pattern with wide-ranging applications is the Golden Ratio. This ratio appears frequently in natural forms and structures, from plants, flowers, and animals to DNA molecules and galaxies. The Golden Ratio is closely related to the Fibonacci sequence, where the ratios of sequential Fibonacci numbers (such as 2/1, 3/2, 5/3) approach the Golden Ratio, approximately 1.618, represented by the Greek letter phi (Φ). This "divine proportion" is ubiquitous in the natural world, reflecting a unique mathematical relationship where the ratio of the sum of two numbers to the larger number equals the ratio of the larger number to the smaller.

- ***Unification in Physics Through Fundamental Constants***

Another potential platform for unifying various physical phenomena lies in the relationships between the fundamental constants of physics. Exploring these relationships could lead to a deeper understanding of the interconnectedness of different physical laws and principles.

- ***The Role of Analogies in Unified Field Theory***

Analogies serve as an intuitive and indicative foundation for exploring the concept of a unified field theory within our universe. The challenge lies in selecting the most relevant and representative situations to form analogies that are significant and potentially universally applicable. Our universe consists of both static (relatively stable) states of matter and dynamic (moving) states of matter. The latter, which involve "energy-moments," are more complex and crucial because, from these dynamic states, we can deduce or reverse-engineer the static states of matter through mathematical processes.

One strong argument supporting the focus on dynamic states is rooted in Fourier or Analytic Signal Analysis, which allows us to decompose, compose, and reconstruct any spatial-temporal function, signal, or geometry related pictures. Therefore, the most relevant system of analogies should be primarily based on the dynamic states of matter, as emphasized in this book.

We also recognize that static and dynamic states of matter are intrinsically linked, a relationship addressed in physics by the concept of wave-particle duality. The transition from dynamic to static states is associated with the formation of self-contained, standing matter waves, such as atoms and masses. This implies that static states of matter (or atomic states) are fundamentally structured by the self-stabilized agglomerations of dynamic matter entities, a concept of matter packing or formatting also related to Mendeleev's Periodic Table of Elements.

- ***Analogies as a Unification Platform***

Analogies within our universe represent the core framework for understanding natural laws and fields, providing a platform for unification in Physics. Even when considered indicatively, creatively, or as a source of scientific inspiration, analogies help us uncover the patterns and domains of natural physics unification. Many states, events, and natural fields in our universe are interconnected, multi-level synchronized, and inherently unified.

To harness this potential, we must establish order and rules within our knowledge base and adhere to proven scientific practices. This involves creating a structured, unique, and natural intellectual framework or "roadmap" for understanding, processing, writing, and reading scientific material. Such a framework can be analogically likened to formatting hard disks in IT or developing documentation for specific environments like buildings or factories.

Another crucial aspect of mastering the scientific process is respecting universally valid natural and conservation laws, along with fundamental principles and symmetries, in both their differential and integral forms. By doing so, we can develop clear algorithms for making decisions and predictions in the scientific endeavor of understanding and mastering our universe. All of this should be underpinned by the most natural and universally applicable mathematical modeling.

In conclusion, this book summarizes the importance of analogies, structured knowledge, and adherence to natural laws and mathematical modeling in the pursuit of a unified understanding of our universe.

2. Understanding Gravitation

- **Current Understanding and Challenges in Gravitational Theory**

Despite significant advancements in our understanding of gravitational phenomena, the fundamental nature of gravitation and its true sources remain elusive. While we have developed robust mathematical frameworks to describe gravity on a macroscopic scale, such as Newton's law of gravitational force and Einstein's theory of relativity; - these theories primarily address how gravity behaves rather than its essential nature. They allow us to predict and model planetary, satellite, and rocket motions, as well as the gravitational interactions between cosmic bodies, making them effective tools for macro-gravity and mechanics-related engineering. However, these theories fall short of explaining the deeper, ontological sources of gravitation, and there is no consistent coupling, compatibility and harmony between such Classical Gravitation, Electromagnetism, Quantum and Relativity theory, and Standard Model of fundamental natural forces, particle physics and matter structure.

In this book, we will attempt to go beyond the traditional Newtonian and Einsteinian perspectives, exploring gravitation from a different standpoint. Briefly summarizing, Gravitation maybe does not belong to fundamental natural forces, being only a derivative or consequence of phenomenology and laws as known in Electromagnetism and Mechanics. Also, dark-matter, dark-energy and gravitons could be some arbitrarily and analogically formulated, hypothetical assumptions, based on erroneous conceptual foundations and present mathematical modeling of gravity. Most of contemporary conceptualizations of gravity start from analogical comparison of an electric field charge, which is the source of electric field, and a mass as the charge and source of gravity, what is not completely correct. What is correct is that electric charge is analogous and comparative with mechanical and electromagnetic moments of a mass (see such analogies in the first chapter). That means, the internal mass content and related properties are sources of gravitation, meaning that gravitation is an extension of the internal superimposed atomic fields of all atoms within certain mass. In contemporary Physics we also know that electric-charge-to-mass and gyromagnetic ratios are in some way stable and too often constant values, both on the micro atomic levels and within galactic formations, meaning that sources of gravitation are both mechanical and electromagnetic, being mutually coupled and proportional. Our goal in this book is to uncover the intrinsic sources of gravity, linked to the properties of matter, mass, and atomic structures.

- **Prerequisites for Understanding Gravitation**

A comprehensive understanding of gravitation requires familiarity with several core physical theories, including mechanics, electromagnetism, matter-waves, and the wave-particle duality of quantum systems. As such, this chapter is best understood after reviewing Chapter 8, which delves into the nature of atoms. Grasping gravitation at a fundamental level necessitates understanding how atoms and masses emerge from matter-waves interactions and how these masses interact to produce the gravitational effects we observe.

- **Establishing a Conceptual Framework**

To introduce a new conceptual platform for better and deeper understanding gravitation, we must first establish a framework that synthesizes relevant ideas, concepts, and indicative facts. This framework will employ multi-level analogies, symmetries, and phenomenological parallels across different domains of physics, highlighting similarities in the manifestations of matter and energy (as elaborated in the first chapter of this book). By comparing various theories and physical phenomena, we aim to construct a broader and complementary understanding of gravitation, beyond the theories of Newton and Einstein.

- **A Broader Perspective**

Based on the preliminary discussions in the first chapter and the initial sections of this chapter, we propose several starting points for a deeper exploration of gravitation. This approach will serve as a foundation for developing a more unified theory that connects gravitational phenomena to electromagnetism and the essential properties of matter and atoms.

For those interested in further reading, notable works by researchers such as Arbab I. Arbab [63], Marçal de Oliveira Neto [64], Anthony D. Osborne and N. Vivian Pope [36], M. Prokic [3], Charles W. Lucas Jr. [89], Rudjer Boskovic [6], Nikola Tesla [97], Konstantin Meyl [99], and Jean de Climont [117] provide additional insights and perspectives on alternative gravitational theories.

Starting points for a deeper exploration of gravitation

1. Rethinking Gravitation: Exploring Potential Electromagnetic Origins

Current physics describes gravity as a fundamental force or a field surrounding masses, based on the observation that masses attract each other. Newton's model of gravity explained this attraction as an intrinsic natural force between static masses (Henry Cavendish, 1797–1798; -see Cavendish experiment | Definition & Facts | Britannica), and through a balance of centrifugal and centripetal forces in orbiting and planetary systems. Later, Einstein provided a more sophisticated and more complicated explanation with his theory of General Relativity, in which gravity arises from the curvature of spacetime caused by mass. While this model has been successful in innovative conceptual describing of gravitational interactions on a large scale (being numerically still not too much better than Newtonian gravitation), it overlooks the possibility that gravitational forces could have a more complex, underlying origin (here related to electromagnetism, wave-particle duality phenomenology, and forces and fields emanating from internal atom structure).

One hypothesis suggests that masses in accelerated or complex motion may generate weak internal electromagnetic dipoles due to the internal structure of atoms and molecules (manifesting because of the big difference between electron and proton masses). These electromagnetic dipoles, in synchronized relative motions, could create mutual attractive forces like Coulomb attraction, which acts between electrically charged micro and macroscopic masses. If this is the case, then gravity might not be an entirely independent mechanics related force but rather an emergent effect resulting from deeper electromagnetic interactions within matter.

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ **COMMENTS & FREE-THINKING CORNER…** ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

- **Analogies Between Coulomb's Law and Newton's Law of Gravitation**

Coulomb's law, which describes the force between electrically charged particles, and Newton's law of gravitation, which governs the force between neutral masses, share strikingly similar mathematical forms. Both follow an inverse-square relationship, $1/r^2$, where the force diminishes with the square of the distance, r, between the interacting bodies. While Coulomb's law applies to electrostatics and Newton's to gravitation, their structural resemblance hints at a potential underlying connection between these two forces. Unfortunately, for such simple analogical comparison between electric charge and electrically neutral mass, we know from the first chapter of this book that mass and electric charge are not mutually analog. Mass should have additional properties like mechanical and electromagnetic moments and dipoles in order to analogically behave as electric charges.

Moreover, the force between two permanent magnets can also be expressed within a similar mathematics of the Coulomb law framework, reinforcing the idea that the same fundamental principles might govern various interactions. This parallel invite the question: could gravitation and electromagnetism share a common origin?

- **Linking Gravitation to Electromagnetic Properties**

One key to understanding this potential connection lies in examining the internal properties and acting forces among atoms and atomic constituents. Electrons, protons, and neutrons each possess linear and angular mechanical momenta, along with magnetic moments, making them interact primarily through electromagnetic forces at the internal atomic level. Yet, at the macroscopic level, electromagnetically neutral masses appear to experience only gravitational attractions. This apparent discrepancy suggests that what we perceive as gravitational forces could, in fact, be the result of complex electromagnetic interactions occurring at the subatomic level, extending by superposition externally (as Rudjer Boskovic conceptualized, [6]).

- **The Role of Mass Asymmetry and Electromagnetic Polarization**

The significant mass difference between electrons and protons, combined with the continuous motion, vibrations and acceleration of all objects, points to the possibility that gravitational effects may stem from electromagnetic dipoles polarizations within matter. As objects move, their internal electromagnetic structures generate currents, potentials, fields, and forces. Thus, gravitation may not be a separate, independent, self-standing natural force at all, but rather an emergent property of these underlying electromagnetic interactions.

- **A New Perspective on Gravitation**

This perspective does not propose to unify electromagnetism and gravitation as distinct forces but rather posits that both phenomena can be understood within the framework of electromagnetic theory. For example, the universe, composed of masses in constant motion, vibrating, rotating, and spinning, may generate internally polarized electromagnetic moments and dipoles. These dipoles could produce forces that resemble both Coulomb's and Newton's laws, as well as effects like the Lorentz force, offering a unified view of natural forces.

If gravity is indeed a manifestation of electromagnetic phenomena, then the contemporary distinction between gravitational and electromagnetic forces would become more conceptual than fundamental. This would imply that our current understanding of gravitation as a separate entity needs to be re-evaluated, and new theoretical frameworks must be developed to describe gravity as an emergent property of electromagnetic interactions within matter. For further relevant details, see reference [122].

2. **Role of Accelerated Motions:** Accelerated motions of masses, such as those found in linear, angular, and oscillatory motions, are essential for mechanical, electromagnetic and matter-wave force effects. Forces also arise around energy gradients or mass-energy agglomerations, suggesting that gravitation (and other natural forces) might ontologically result from these complex internal spacetime dynamics (as elaborated in Chapter 10). All other force related theories, formulas and concepts should be consequences of mentioned primary ontological sources.

3. **Threshold Size and Gravitation:** Gravitation becomes significant above a certain spatial size of macro-masses. Below a threshold distance or size (less than 10 micrometers), gravitational effects are negligible, and electromagnetic interactions dominate. This suggests that gravitation is a macro effect of superimposed and mutually synchronized micro electromagnetic fields and interactions, emanating from atoms.

4. **Historical and Contemporary Perspectives:** Historical concepts about fields and forces from Rudjer Boskovic, Nikola Tesla, and Konstantin Meyl, along with modern interpretations from researchers like Reginald T. Cahill, [73], and Jean de Climont, [117], provide imaginative insights into gravitation as electromagnetic energy exchanges between masses.

5. **Current Practices and Future Directions:** Despite our reliance on Newton's gravitational force concepts, Einstein's equivalence of gravitational and acceleration-related forces, and curved spatial-temporal geometry in our Universe, the origins of gravitation are likely tied to electromagnetic fields and electromechanical interactions within and around atoms. Atoms, stabilized by structure of internal electromagnetic fields, superimpose, synchronize and extend these fields and forces into their environment, creating gravitational effects.

6. **Electromagnetic and Mechanical Moments:** Electromagnetic and mechanical moments within atoms are central to understanding gravitation. The gravitational force might result from the superposition of many internal electromagnetic and mechanical micro-dipole and other moments states, extending outward to create a macro effect.

7. **Analogy with motions in Electrostatic Fields:** There is an intriguing analogy between the motion of charged particles in electrostatic fields and neutral particles in gravitational fields. Although there are similarities, the comparison reveals deeper complexities, particularly in understanding gravitation.

   In an electrostatic field, a charged particle q placed in a uniform electric field E will experience an acceleration in the direction of the electric field lines. This relationship is described by the equations: $F = ma$, $F = qE = ma$, $a = qE/m$. Here, all kinematic laws and mechanics apply to the charged particle’s motion. However, the analogy between electric fields and gravitation becomes challenging when we consider that mass and electric charge are not directly analogous, as shown in the first chapter. This implies that a static mass cannot solely account for gravitation; -instead, moving, vibrating, or resonating masses with mechanical and electromagnetic moments play a crucial role.

   All masses, including those at rest, exhibit relative motion due to linear-motion inertia. This suggests that gravitation can be (at least analogically) likened to the attraction between two parallel wires carrying electric currents in the same direction. The relative motions of internally created electromagnetic dipoles produce Lorentz force effects like electric currents and magnetic fields, resulting in mutual attraction between masses.

Furthermore, the rotational inertia of internally spinning atomic constituents can create a macro gyroscopic effect when aligned and superimposed, helping stabilize the spatial position of masses against an external gravitational field. This also indicates that inertial states have both mechanical and electromagnetic nature (see more about inertia in Chapters 1, 4.2 and 10).

Electric charges (measured in coulombs), despite being dynamic and motional entities like mechanical and electromagnetic moments, are often treated as stable, constant and fixed. This implies that between electric charges or dipoles, there is a continuous flux of radiative electromagnetic energy, contributing to gravitational effects. The internal electromagnetic forces maintain the stability of masses and atoms, echoing Rudjer Boskovic's Universal Natural Force and Nikola Tesla's Dynamic Gravity concepts (see references [89], [97]).

Considering the global, accelerated, and synchronized motions of masses in our Universe, there are two possibilities: either masses are globally moving or rotating about something, or an external factor (such as an electromagnetic field, flux of elementary particles, cosmic rays, or an exotic, fluidic medium like an aether) streams around and through these masses. These motions are interconnected and tend to synchronize, suggesting that forces arise around energy gradients or mass-energy agglomerations and deagglomerations (or around atoms).

It is plausible that an external magnetic field could influence gravitational attraction between two electromagnetically neutral masses, given the internal magnetic moments of spinning and orbiting of atomic constituents.

8. Anyway, the essential and **ontological origins of gravitation** are atoms and atoms agglomerations, being mutually synchronized by matter-waves and connected to our Universe, internally and externally, with $1/r^2$ -dependent central, electromagnetic forces and entanglement effects. This is an obvious and almost trivial statement that should replace the oversimplified concept that only masses are sources of gravitation. Atoms are internally structured and stabilized thanks to specific self-closed standing matter waves, or thanks to specific structure of electromagnetic fields and forces of atoms. The extension of mentioned internal atomic field structure (of electromagnetic and mechanical moments and dipoles) towards external environment presents or creates force of Gravitation (see more in Chapter 8, under "8.3. Structure of the Field of Subatomic and Gravitation related Forces").

In summary, a deeper understanding of gravitation involves recognizing it as an effect of electromagnetic interactions within and between atoms and masses. This perspective aligns historical insights and modern theories, suggesting that gravitation is a complex, emergent phenomenon rooted in the electromagnetic and dynamic nature of matter and energy. For more details, refer to Chapters 5, 8, 9, and 10.

[♣ The parallelism between gravity and electrostatics. Taken from
http://physics.bu.edu/~duffy/PY106/Charge.html

"An electric field describes how an electric charge affects the region around it. It is a powerful concept because it allows you to determine ahead of time how a charge will be affected if it is brought into the region. Many people have trouble with the concept of a field, though, because it is something that is hard to get a real feel for. The fact is, though, that you are already familiar with a field. We have talked about gravity, and we have even used a gravitational field; we just did not call it a field.

When talking about gravity, we got into the (probably wrong) habit of calling g "the acceleration due to gravity". It's more accurate to call g the gravitational field produced by the Earth at the surface of the Earth. If you understand gravity, you can appreciate electric forces and fields because the equations that govern both have the same form.

The gravitational force between two masses (m and M) separated by a distance r is given by Newton's law of universal gravitation:

$F = -G m M / r^2$, where the constant $G = 6.67 \times 10^{-11}\ N m^2 / kg^2$

A similar equation applies to the force between two charges (q and Q) separated by a distance r:

Coulomb's Law: $F = k q Q / r^2$, where the constant $k = 8.99 \times 10^9\ N m^2 / C^2$

The force equations are identical, so the behavior of interacting masses is identical to that of interacting charges, and related analysis methods can be used. The main difference is that gravitational forces are always attractive, while electrostatic forces can be attractive or repulsive. The charge (q or Q) plays the same role in the electrostatic case that the mass (m or M) plays in the case of gravity.

A good example of a question involving two interacting masses is a projectile motion problem, where there is one mass m, the projectile, interacting with a much larger mass M, the Earth. If we throw the projectile (at some random launch angle) off a 40-meter-high cliff, the force on the projectile is given by:

F = mg.

This is the same equation as the more complicated equation above, with G, M, and the radius of the Earth, squared, incorporated into g, the gravitational field.

So, you have seen a field before, in the form of g. Electric fields operate similarly. An equivalent electrostatics problem is to launch a charge q (again, at some random angle) into a uniform electric field E, as we did for m in the Earth's gravitational field g. The force on the charge is given by **F = qE**, the same way the force on the mass m is given by **F = mg.**

We can extend the parallel between gravity and electrostatics to energy, but we will deal with that later. The bottom line is that if you can do projectile motion questions using gravity, you should be able to do them using electrostatics. In some cases, you¹ll need to apply both; in other cases, one force will be so much larger than the other that you can ignore one (generally if you can ignore one, it will be the gravitational force)". ♣**]**

9. Transformation of Linear Motion in a Magnetic Field

The linear motion of an electrically charged particle in a magnetic field can be transformed into a rotating helix-path matter-wave motion. This transformation results in a combined linear and spinning motion, akin to the motion of electrically neutral particles or planets in a gravitational field. This phenomenon can be explained, at least by analogy, using relevant mathematical forms regarding moments and energies (see T.2.1 and T.1.8).

10. Complementarity of Electric and Magnetic Fields, Gravitational Attraction and Resonant Standing Waves

Electric and magnetic fields are complementary, constantly interacting and transforming each other into. Together, they form the electromagnetic field, which manifests as electromagnetic waves. A similar coupling may exist within the realm of gravitation, although it is not yet fully understood. Concluding based on analogies between linear and rotational motion and the relationship between associated magnetic moments, gravitation might also have an electromagnetic origin. If true, this could provide a compelling explanation for the complementarity of fields and motions in gravitational phenomena, such as the interaction between linear and spinning motions and the coupling between electric and magnetic fields (also explaining the nature of wave-particle duality).

Another hypothesis suggests that gravitational attraction may arise from the interaction of masses within spatial nodal zones of stationery and standing matter waves. These waves could be specific electromagnetic standing waves, like the phenomena observed in acoustic or ultrasonic levitation (see references [150] and [151]). In this framework, gravitational attraction is explained as a force originating from the nodes in standing matter waves. This idea is supported by resonant effects seen in the nodal zones of half-wavelength standing waves in ultrasonic resonators and in acoustic levitation.

Oscillations in these standing waves might induce internal electromagnetic dipoles polarizations. This leads to the possibility that the universe could be globally oscillating in a synchronized manner across electromagnetic, mechanical, and electromechanical domains, as proposed by N. Tesla and Konstantin Meyl (see reference [99]). A particularly important resonant state occurs when temporal and spatial resonances align synchronously, forming combined temporal-spatial standing waves.

These standing waves, when self-contained, have the potential to generate stable particles with non-zero rest masses. In contrast, open-ended waves could act as progressive matter waves. The nodal and antinodal zones within these standing waves produce attractive and repulsive forces, respectively, due to the vibrational energy gradients directed toward or away from these zones.

This concept offers an alternative framework for understanding natural forces in physics, providing a fresh perspective on gravitational and other fundamental forces (see Konstantin Meyl's work in [99]). Given that the temporal and spatial domains of all motion states, along with their corresponding wave functions, are intrinsically linked (as discussed in Chapter 10), it is logical to consider both temporal and spatial periodicity and their resonant states.

The key resonant state arises when temporal and spatial resonances align, creating combined temporal-spatial standing waves. These waves, when self-contained, could form stable particles with non-zero rest masses. Alternatively, open-ended waves might serve as sources of mechanical, electromechanical, or electromagnetic matter waves. Within these standing waves, the nodal and antinodal zones would exhibit attractive and repulsive forces, respectively, due to the energy gradients in the vibrational fields.

Ultimately, gravitational attraction could be understood as a force that manifests within the nodes of standing matter waves. This aligns with the effects observed in ultrasonic resonators and acoustic levitation (see [150] and [151]). Since the temporal and spatial domains of all motion states and their wavefunctions are fundamentally linked and proportional (as explained in Chapter 10), it is reasonable to consider both temporal and spatial resonant states.

In summary, masses in the universe may behave as if immersed in a resonant matter-wave field, structured by standing waves that interact between them.

11. Omnipresence of Rotational and Oscillatory Motions in Nature

Rotational, spinning, and orbital motions, including oscillatory motions, are ubiquitous in nature. Examples include the rotation of planets around their suns, moons around planets, and solar systems around galactic centers, as well as the spinning and orbiting of atomic and subatomic particles. These rotational characteristics extend to all subatomic particles, quasiparticles, astronomical objects, and galactic formations. The origin of such motions in nature cannot be merely random; they are likely quantifiable phenomena, sometimes explicable by statistics and probability under certain conditions.

Every motion characterized by a specific spectral distribution, frequency, or wavelength should be linked to visible or hidden rotational or spinning phenomena. Linear motion is a specific case of curvilinear motion, with a straight-line path being a mathematical idealization that does not exist in the physical world. Rotational, orbital, and spinning motions at the micro and macro levels should be synchronously connected or coupled, along with oscillatory and linear movements, including inertial effects and

Download the last version here:
http://www.mastersonics.com/documents/revision_of_the_particle-wave_dualism.pdf

associated matter-wave phenomenology, considered both mechanically and electromagnetically. This coupling is a starting point for developing a new theory of gravitation. See more about inertia in Chapters 1, 4.2 and 10.

All micro and macro particles and energy states in our universe are likely to have coupled linear and angular moments, including synchronous "entanglement couplings" (see more in Chapter 10). As noted in [36], "The natural (i.e., forceless) state of motion is orbital (or angular), not rectilinear." Even if we cannot directly associate orbital and spinning moments with a particular mass or system, we can imagine a global orbiting motion about a dominant center of moving mass, indicating that matter is always in states of relative linear and angular motion. Gravitation, as a central-force phenomenon, results from this global orbiting, as supported by the concept mentioned in [36].

While cosmic events such as impacts, scattering, and explosions can produce non-rotational and pulsating motions, these events often lead to the creation of additional angular and spinning field components, ultimately contributing to the evolution of rotational phenomena (thanks to associated helicoidal matter waves).

Theoretical Considerations and Propositions:

- **Gravitational-Rotational Complementary Fields and Matter Waves:**

It is plausible that the gravitational field, associated with linear motion, is coupled with an omnipresent but still not fully understood complementary angular-motion field. While this field is not yet recognized as a distinct key concept in current physics, it may play a crucial role in gravitational interactions. The combination between gravitation and this complementary spinning field could result in a complex "gravitational-rotational" field, analogous to the relationship between electric and magnetic fields.

This proposed coupling between linear and spinning (or torsional) fields can be mathematically modeled using the Complex Analytic Signal. In this model, two phase-shifted wave functions combine to create a complex analytic signal wavefunction (for more on this, see Chapters 4.0, 4.3 and 10, and references [7], [57], and [107]). This concept mirrors the coupling of electric and magnetic field vectors in the formation of electromagnetic waves and photons (see further discussions in Chapters 4.0, 4.1, and 10).

Experimental evidence supports this theory. Japanese researchers observed that the weight of a spinning mass within Earth's gravitational field changes depending on the direction of spin. Spinning mass in the same direction as Earth's rotation weighs slightly more than when at rest, while spinning in the opposite direction results in a slight decrease in weight (see reference [36]).

A potential candidate for this complementary field might be a spinning matter wave, akin to de Broglie's matter wave concept. This idea extends beyond mechanical mass spinning and connects to a variety of physical phenomena, including inertia, gravito-magnetic induction, gyromagnetic ratios, charge-to-mass ratios, pendulum behavior, mass-spring oscillatory systems, and variety of gyroscopic motions. However, the concept remains incomplete and requires precise terminology, harmonic unification and a comprehensive theoretical framework.

Regarding the "Origins of rotation of celestial and other astronomical objects," further insights can be found in reference [51]. In essence, gravitation, along with its proposed rotational, orbiting, spinning, and torsional matter waves, may represent a specific manifestation of an intrinsic electromagnetic field.

In summary, linear gravitational motion is likely coupled with an omnipresent but poorly understood complementary angular-motion field. This field, though not fully recognized in contemporary physics, may be essential in understanding gravitational phenomena, and most probably belongs to dualistic particle-wave and matter-waves phenomenology. The interaction between gravitation and this spinning or torsional field may result in a complex "gravitational-rotational" field, like the coupling of electric and magnetic fields.

While promising, the future development of a uniting theoretical framework and more precise terminology is essential for fully understanding this potential relationship between gravitation and electromagnetism.

- **Revisiting Newton's Law of Gravitation:**

Newton's Law of Gravitation primarily describes the attractive central force between stationary or resting masses or those orbiting in stable solar systems. It does not account for velocity or momentum-dependent factors. However, since all masses are in relative motion, an updated law of gravitation should consider dynamic elements like linear and orbital moments, velocities, vibrations, and related electromagnetic dipoles, moments, and charges, relative to a dominant "center of total energy" coordinate system. This suggests that only masses with the mentioned motional and electromagnetic properties can be true sources of gravitation, rather than neutral masses at rest.

This chapter explores this conceptual framework, proposing that purely static and electromagnetically neutral masses are not the genuine and unique sources of gravitational fields. This stands in contrast to electric charges, which generate electric fields. As a result, the common analogy between Coulomb's law and Newton's law is somewhat misleading, since static masses and electric charges are not directly analogous, as discussed further in Chapter 1.

The main goal of this book is to investigate new hypothetical aspects of gravitation and electromagnetism using the extended Mobility analogies chart (see Chapter 1, T.1.1 and T.1.2). This chart highlights gaps and inconsistencies in current formulations of gravitational force, field, and energy. By filling these gaps through analogy-based reasoning, we can propose new, hypothetical and indicative relations for forces and fields (see T.2.2 for further details), just to start thinking and conceptualizing gravitation on a new way.

The book introduces new ideas primarily based on the extended Mobility-type analogies, which satisfy three levels of analogy: mathematical, topological, and the similarity of force expressions and energy-momentum conservation laws. While the accuracy of these predictions remains uncertain, the analogy-based approach highlights inconsistencies within the current system, especially concerning electric charge, mass and mechanical and magnetic moments.

These ideas were initially formulated by the author, Miodrag Prokic, in the 1975 diploma thesis "Energies Parallelism" at the University of Nis, Faculty of Electronic Engineering, Yugoslavia.

12. Understanding Natural Forces: Gravitation, Electromagnetic Phenomenology, and Analogies Between Coulomb and Newton's Laws

The foundation for understanding natural forces, including gravitation and nuclear forces, is explored in depth in Chapter 10 of this book, particularly under the section "10.02: Meaning of Natural Forces." All forces can be conceptualized as spatial and temporal gradients, essentially the first derivatives of energy or mass-density distributions and associated mechanical and electromagnetic moments. These forces are closely tied to the globally valid Uncertainty Relations (see more in Chapter 6), and omnipresent cosmic, structural, and spatial standing-matter-wave formations between masses. This comprehensive framework provides the clearest explanation for gravitation and other natural forces.

Many current approaches to natural forces rely on analogical, deductive, experimental, and hypothetical methods that are often reasonable but incomplete or not generally valid. It is a fundamental methodological error to treat mutually analogical Newton's and Coulomb's force laws as the primary foundations for understanding gravitation and electromagnetism. A more universal approach views natural forces as spatial-temporal gradients of a specific energy state, or as first temporal derivatives of energy-mass-moments states. From this perspective, the familiar forms of Newton's law of gravitation, Coulomb's law, Einstein's General Relativity, and other force equations should be deduced after certain development.

- **Revisiting Gravitational Charges and Symmetries**

To resolve intriguing aspects of "real gravitational charges", new predictions can be made based on the symmetries and analogies discussed in Chapter 1, particularly summarized in "T.1.8 Generic Symmetries and Analogies of the Laws of Physics." A direct comparison of Coulomb's and Newton's force formulas reveals a mathematical similarity, though static masses and electric charges are not directly analogous. If we treat Coulomb forces as the primary, tangible forces, serving as a model for understanding other forces, it is clear (based on analogies) that electric and magnetic charges, as well as linear and angular mechanical moments, are analogous. Therefore, gravitational attraction, as expressed by the Coulomb-Newton force law, should exist only between internal, spatially distributed linear and angular moments, dipoles, and electromagnetic charges associated with the masses in question. These forces can be described in the simplest way using the Coulomb-Newton framework, representing attraction between internal electromagnetic charges, moments, and dipoles.

- **Coupling of Linear and Angular Motion**

There should be an inherent unity and coupling between the linear and angular or spinning moments of particles, as suggested by de Broglie's concept of matter waves. Even if such motion is undetectable externally, it could manifest as internal spinning, oscillatory, electromagnetic, thermal, or micro-motion within atoms and molecules, generating spatial currents and electromagnetic charges that flow and oscillate inside and between masses, such as those centered around a galactic core.

In both the micro and macro scales of physics, magnetic and angular moments are coupled, maintaining constant gyro-magnetic ratios; -a principle valid for atoms, planetary systems, and galaxies. Understanding gravitation requires conceptualizing coupled linear and angular motions, along with their associated electromagnetic charges and dipoles, as fundamental components.

- **Superposition of Moments**

Just as the superposition of numerous micro-spinning magnetic domains creates a macroscopic magnetic moment, the total orbital or angular macro-moment of a mass can be analogized. Conversely, a macro angular moment can be broken into elementary spinning moments. This book posits that the intrinsic unity of linear and spinning (or angular) electromagnetic and mechanical motions, along with their coupled moments and charges, is the primary source of gravitation and matter waves (see more in Chapter 4.1).

Our current concept of mass needs to evolve. Mass should not be viewed only as static, electromagnetically neutral, or with fixed properties and geometry. Instead, moving mass is intrinsically linked to linear and angular moments, associated matter-wave and wave-particle-duality effects, and is electromagnetically coupled with the universe.

- **Electromagnetic Polarization and Gravitational Attraction**

Even stable rest masses exhibit a minimal level of internally non-compensated electromagnetic polarization, producing internal dipole-attracting effects. These polarized electromagnetic dipoles, generated by global and relative linear, orbital, and spinning motions (including oscillations) within the universe, create gravitational attractions, like the attraction between two wires carrying electric currents in the same direction (based on Lorentz forces). Additionally, Dr. Jovan Djuric's publications ([33], [71], and [102]) explore this phenomenology and the associated electromagnetic charges and polarizations.

We exist within a global structure of *mass-energy moments and electromagnetic charges*, where stable masses are localized in relatively stable nodal positions of surrounding mater-waves field, analogous to acoustic levitation effects (see references [150] and [151]). In our laboratory frames, we do not observe these structured holistic couplings and motions, yet gravitation serves as a reminder of their omnipresence, relative to specific dominant centers of mass or inertia.

- **Higher Dimensions and Gravitation**

If the universe consists of more than four dimensions (x, y, z, t), these holistic motions may be observable from higher-dimensional perspectives. A detailed theoretical background on gravitational central force related to orbital motion can be found in reference [36].

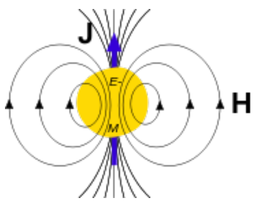
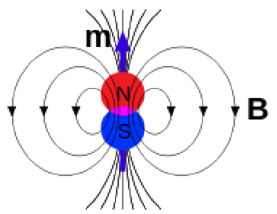
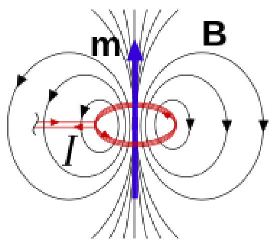
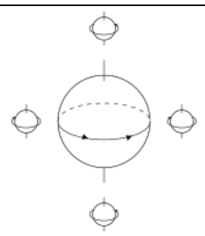
Numerous attempts have been made to create gravitational field equations analogous to Maxwell's electromagnetic equations, known as the equations of Gravitoelectromagnetism (see [65], [122], [170], and https://en.wikipedia.org/wiki/Gravitoelectromagnetism). These approaches typically and erroneously treat mass as directly analogous to electric charge, assuming mass to be the primary source of gravitation what is wrong. However, static mass alone cannot be the sole source of gravitation. Instead, mass moments (or masses in motion) and their associated electromagnetic complexity must be considered as sources of gravity, but all of that is effectively coming or radiating from internal atoms structure (see more about relevant, supporting analogies in the first chapter).

- **Reevaluating Mass and Gravitation**

Mass and electric charge are not directly analogous, but mass possesses intrinsic electromagnetic properties and internal polarizations due to omnipresent cosmic radiation and motions (see Chapter 3). The real sources of gravitation are atoms, synchronized with other masses and the universe, respecting the $1/r^2$ central force laws. This explains why mass and gravitation cannot be electromagnetically shielded or neutralized, as any shielding structure is an integral part of the global, multi-axial cosmic motions and couplings.

Ultimately, we may find that Newton's gravitational constant (G) has a hidden "cosmic angular speed" component, which could provide further insight into the analogy between mass and electric charge. Nikola Tesla's ideas about Dynamic Gravity Theory, which suggest that a "radiant energy flow" between masses creates gravitational attraction, imply that masses are not static entities (see reference [97]).

In conclusion, the most promising future gravitation theory will likely be a reformulated Gravitomagnetism theory, as an extension of Maxwell's equations. Here, gravitation-related charges or sources are subatomic and mass-spinning moments, associated with electromagnetic moments, charges, and dipoles. For more on this, see the Physical analogs between different fields [7] at https://en.wikipedia.org/wiki/Gravitoelectromagnetism.

 **Gravitomagnetism** — gravitomagnetic field **H** due to (total) angular momentum **J**.	 **Electromagnetism** — magnetic field **B** due to a dipole moment **m**...
 ... or, equivalently, current *I*, same field profile, and field generation due to the rotation.	 **Fluid mechanics** — rotational fluid drag of a solid sphere immersed in fluid, analogous directions and senses of rotation as magnetism, similar interaction to frame dragging for the gravitomagnetic interaction.

Later, in the same chapter, familiar ideas are additionally elaborated (see equations under (2.11), and Figs. 2.2 and 2.3).

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

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Every electromagnetically neutral macro-mass, or one with balanced opposing charges, was initially formed from elements that are electromagnetically charged. These elements possess specific magnetic and electric charge properties, resulting from the internal polarization and dipole or multipole behavior of their constituents. During the formation of electromagnetically neutral macro-masses, the involved electromagnetic charges spatially align to create dipoles or bipolar electromagnetic entities. This process leads to mass compacting and optimal energy packing, ultimately neutralizing the electromagnetic charges through mutual compensation. As a result, the neutral macro-mass becomes externally self-compensated in terms of electric and magnetic charges, rendering it macroscopically neutral, while remaining internally electromagnetically polarizable.

Elementary matter domains, such as atoms and various electric and magnetic dipoles (primarily composed of charged elementary particles in motion), exhibit forms of self-sustaining and auto-stabilized, internally self-contained matter waves, characterized by rotating and spinning states. These internal elementary matter and energy-momentum states function as electric dipoles and elementary magnets (or magnetic dipoles) due to their coupled motions and intrinsic spinning.

The significant mass difference between electrons and protons (with protons being 1836 times heavier) further contributes to the creation of polarized electromagnetic dipoles. If our universe possesses an intrinsic, holistic macro-rotation, the countless polarized micro-electromagnetic states could collectively contribute to the macro force of gravity. Additionally, from the mass-energy equivalence principle, it is evident that this diversity of coupled rotating and oscillating matter-wave forms has its effective masses and mechanical moments.

Since mass and gravitation are intrinsically linked (as per Newton's law), and mass represents a mode of energy packing ($E = mc^2$), this implies that gravitation could be a form or component of the electromagnetic field. Electrons, protons, and neutrons, which constitute atoms and larger masses, are inherently electromagnetic entities. Neutrons can decay into an electron and a proton, revealing the complex interactions between photons, electrons, and other elementary particles, further coupling gravitation with electromagnetic phenomena. Given that electrons and other charged particles always exhibit intrinsic magnetic, spinning, and orbital moments—mutually coupled with constant gyromagnetic and charge-to-mass ratios, mechanical rotation and electromagnetic properties are also inherently interconnected.

Moreover, many phenomena observed in one field, such as particle mechanics, can be analogically or mathematically converted to equivalent states in electromagnetism, and vice versa (refer to the literature cited in Chapter 4, T4.0, and comments surrounding equations (10.1.4) to (10.1.7) in Chapter 10, as well as Francisco Cabral and Francisco S. N. Lobo's work on gravitational waves and electrodynamics [131]).

In our universe, different centers, such as centers of mass, moments of inertia, electric and magnetic charges, and centers of gravity (where self-gravity is zero), never coincide in the same space and time spot due to the relative linear and angular motions of all masses, including possible vibratory and resonant states. This spatial displacement creates intrinsic mechanical couples or torques, as well as bipolar and multipolar tension and stress effects between these displaced centers, resulting in dipoles, multipoles, and electromagnetic polarization effects.

This situation also holds true for electric and magnetic elements and charges, meaning that it is impossible for any mass to be entirely electromagnetically neutral. There is always some degree of internal bipolar or multipolar polarization within all masses. The mutually displaced centers of these opposing couples, moments, and dipoles are also related and coupled, leading to an attractive force between the internal mechanical and electromagnetic constituents, moments, and charges of a given mass. Since the constituents of mass are atoms and molecules, the sources of gravitation are strongly linked to extended atomic forces and interactions (further discussed in Chapter 8).

This is where we should explore the electromagnetic and atomic origins of gravitational force. Experimental evidence supporting these ideas, particularly the effects of displaced centers and associated moments and torques, can be found in the works of Prof. Dr. Jovan Djuric [33], [71], and [102]. These findings also suggest that our universe possesses a certain level of inherent, intrinsic, holistic rotation, which is coupled with linear-motion parameters, creating various intrinsic resonant states that contribute to wave-particle duality and matter-wave phenomena. Similar and more detailed concepts regarding the true nature of gravitation are discussed in [89] by Charles W. Lucas Jr.

13. In 1916, Albert Einstein published his theory of General Relativity, which describes gravity as a deformation of space-time geometry around masses. Despite this groundbreaking theory, Newtonian gravity remains widely used because General Relativity rarely produces dramatically different or clearly verifiable and unique predictions. Newtonian mechanics and gravitational laws are still practical and sufficient for most applications within our solar system.

    In 1919, General Relativity was officially validated by observing the bending of light during a solar eclipse, where the sun's gravitational pull warped space-time. However, this verification (until the present many times repeated) is still not extremely precise, as it was conducted based on limited data and some assumptions. The possibility of electromagnetic interaction between light and gravitation, which could explain the phenomenon also as an electromagnetic reality, was neglected because gravity is conceptually considered as an independent natural force.

    Another success of Einstein's General Relativity was resolving an anomaly in Mercury's orbit. Mercury's perihelion, the point of its closest approach to the sun, advances with each orbit, creating a complex rosette pattern over time. Newtonian theory could not explain this anomaly. Einstein's calculations, incorporating the sun's mass and Newton's gravitational constant, correctly predicted Mercury's orbital precession. However, later studies showed that similar results could be obtained without using General Relativity, questioning its necessity for this explanation.

    For much of the 20th century, Einstein's General Relativity was believed to be in perfect agreement with astronomical observations. However, recent observations suggest otherwise. There is increasing evidence of stronger gravity in remote spinning and spiral galaxies than predicted by both Newtonian and Einstein's theories. To reconcile these theories with the observations, scientists postulated the existence of "dark matter" and "dark energy", which have never been observed, measured, or proven. These hypothetical entities were suggested to strengthen gravitational pull to match observed data (see [60]; John W. Moffat).

    Alternatively, stronger gravitational attractions in some galactic motions could be explained by the hypothetical existence of stronger electromagnetic-dipoles polarization. This suggests that both Newtonian and Einstein's gravity theories should be updated, ideally without resorting to exotic, unobservable entities. The initial search for dark matter stemmed from the assumption that mass is the sole cause of gravity. However, based on analogies from the first chapter of this book, we now understand that gravity sources include mechanical and electromagnetic moments and dipoles, coupled with masses and atoms in motion.

If the assumed dark matter is missing from our universe, there might be a surplus of mechanical and electromagnetic moments or dipoles contributing to explaining the mentioned gravitational anomalies. Consequently, Einstein's theories may soon undergo substantial reorganization and evolution (see [73]; Reginald T. Cahill).

Reevaluating Einstein's Theory of Relativity

When it comes to A. Einstein's Theory of Relativity, we should approach it with a more creatively curious, critical, and intellectually open mindset. It's essential to creatively and critically consider non-mainstream and heretical perspectives, new experimental results, and publications that challenge the established norms. Some of these perspectives suggest the following:

1. Variability in the Speed of Light (C):

There are indications that the speed of light (C), the maximum velocity of electromagnetic waves and the upper limit of speed in our universe, may not be as constant or absolute as postulated in Relativity theories. Evidence suggests that this speed can vary under certain conditions, challenging the foundational assumption of its invariability.

2. Revisiting the Concept of Ether:

We must reconsider the existence of a material texture, fluid, or specific state of matter, historically referred to as ether that could serve as the medium through which electromagnetic and maybe other matter and cosmic waves propagate. The results of the Michelson-Morley interferometer experiment, which historically dismissed the existence of aether, may have been misinterpreted or conducted with incomplete setup and assumptions. For instance, if light beams or photons oscillate only transversely, the expected flow of aether might resemble laminar or linear motion rather than the previously assumed dynamics.

As long as the dielectric permittivity and magnetic permeability of a vacuum remain stable or constant, we can accept that the maximum speed of light in a vacuum is also constant $\mathbf{C = 1/\sqrt{\varepsilon_o \mu_o}}$ *. However, given that these constants represent measurable electromagnetic properties of the vacuum (as* $\boldsymbol{\varepsilon_o}$ *,* $\boldsymbol{\mu_o}$*), it is reasonable to infer that they belong to a tangible substance with elastic and fluidic characteristics, which we refer to as aether.*

We already know that the speed of light in less exotic transparent media, such as water, varies with the motion of the medium (e.g., water flow). A similar principle could apply to the vacuum, suggesting that the electromagnetic properties of space, matter, and relative motion significantly influence the speed of light. Independent measurements have shown that the speed of light is not always constant or perfectly absolute, as Relativity theory claims. The significance of the constant $\mathbf{C = 1/\sqrt{\varepsilon\mu}}$ *in physics may require an independent interpretation related to the texture or fluidic nature of material substances and space, implying that the motion of ether (if it exists) could affect the speed of light.*

A similar argument can be made regarding Newton's gravitational constant (G), which is known to vary slightly when measured in different laboratories and spatial locations, suggesting that it may not be as constant as traditionally assumed.

3. Experimental Evidence and Relativity:

The experimental evidence supporting relativistic effects such as length contraction, time dilation, and mass distortion, central tenets of Relativity Theory is sufficiently well confirmed. While such effects related to mutual temporal-spatial coupling and forces within our galaxy do exist as predicted, they still do not fully validate all aspects of the Relativity Theory in cases of remote spiral galaxies. Of course, the bending of light beams passing about big masses is *experimentally verified many times, showing that Relativity theory in such cases is giving much better agreements (or predictions) than Newtonian theory.*

For example, the synchronization among GPS satellites and stations for related time and distance measurements on Earth almost fully proves present Relativity Theory concepts, using Lorentz transformations of time and length, while additionally applying necessary mathematical corrections and space-time compensations.

Minkowski's extension of Euclidean and Newtonian space (where time is considered stable and absolute) is creatively and correctly associated with A. Einstein's Special and General Relativity theory. However, it also exists independently and would exist even if Relativity Theory had never been developed. The most useful product of integrating Minkowski Space, Classical Mechanics, and Relativity Theory concepts is the energy-momentum 4-vector.

It is often misleadingly claimed that 4-vectors are exclusive products of Relativity Theory. A. Einstein, or more accurately, his wife Mileva Maric, who was a student of Minkowski, recognized that using Minkowski space and 4-vectors allowed for a more robust elaboration and reinforcement of his theory. This integration makes the analysis of different interactions in Mechanics elegant and mathematically efficient, particularly in predicting impact and scattering interactions in microphysics.

Without the proper merging of Minkowski's mathematical concepts with Relativity Theory, the ideas of Relativity would remain complicated and confusing. In fact, the practical usefulness and validity in experimental physics of Relativity Theory are primarily due to the applications of energy-momentum 4-vectors within Minkowski space, while respecting basic conservation laws. Other 4-vectors within Relativity Theory are also well-fitted and complementary formulas designed to demonstrate the functionality of Einstein's postulates, including Lorentz transformations.

It should be noted that there are publications and experimental results challenging some aspects of Relativity Theory. These suggest that the speed of light, c, may not always be constant and absolute, and that time and length dilation or contraction cannot be fully experimentally proven as prescribed by Lorentz transformations.

In the final chapter of this book (Chapter 10), we will show that the understanding of mass can be significantly expanded using Minkowski energy-momentum 4-vectors and corresponding phasors expressed as analytic signals (see "10.1 Hypercomplex Analytic Signal Functions and Interpretation of Energy-Momentum 4-Vectors in Relation to Matter-Waves and Particle-Wave Duality").

In conclusion, this book aims to demonstrate that the sources of gravitation are not static masses. Instead, gravitation originates from moving and oscillating masses, as well as mass-associated mechanical and electromagnetic moments and charges. Essentially, the force of gravitation stems from atoms and molecules, which are electromagnetic resonant formations and constituents of masses.

14. Here we aim to explain, update, and conceptually redesign the understanding of gravity as a field formed by dynamically stable and standing matter waves between masses, or between atoms that create interacting masses. Gravitation is viewed here as the result of specific interactions between linear and rotational motions, including oscillatory and resonant behaviors. These motions are intrinsically linked to the electromagnetic complexity of the moving masses (see later in this chapter on "2.3.3. Macro-Cosmological Matter-Waves and Gravitation"). This idea shares similarities with

the micro-world wave-particle dualism, while also aligning with the Mobility system of electromechanical analogies (discussed in Chapters 1, 8, 9, and 10).

In this book, gravitation is presented as a natural consequence of the mutual interactions within the internal states of matter, particularly the behavior of atoms and masses. These interactions involve intrinsic orbital and spin motions, which are closely connected to electromagnetic phenomena (see the conceptual framework in [36]). Traditional Newtonian gravitation, by contrast, overlooks these internal and external factors, such as linear, orbital, and spin moments, as well as the associated electromagnetic properties of interacting atoms and masses. As a result, it still relies on the effective, but simplified, Newtonian formula for gravitational force, which only considers mass.

Albert Einstein's General Relativity (GR) replaces Newton's model by describing gravity as the curvature of space-time around masses. This approach is mathematically elegant and aligns with concepts discussed in [36]. However, this book proposes a different framework, initially grounded in the Mobility System of Analogies. We argue that static masses, without dynamic parameters like different moments and oscillations, cannot generate gravitational fields or forces (see Chapter 1 for more about Analogies). Instead, it is the material fields, acting within and outside vibrating atoms, that generate central forces that depend on the inverse square of the distance ($1/r^2$), as explored in Chapter 8, Section 8.3, "Structure of the Field of Subatomic and Gravitation-Related Forces."

The real sources of gravitation, as proposed, are the mutually interacting masses and atoms, which are electromagnetically and mechanically coupled through standing matter waves. These masses, along with atoms that have internal electromagnetic charges, magnetic moments, and electric dipoles, as well as their linear and angular (spinning) mechanical moments, produce gravitational force (see Chapter 3 for more on revitalized electromagnetism). Essentially, all macroscopic masses, atoms, and astronomical objects are in motion relative to one another, interacting by creating synchronized standing matter waves and vortices of radiant electromagnetic energy, or a subtle fluid flow (potentially composed of fine particles or "ether"). These interactions generate the effects of gravitational attraction, in a manner like in Nikola Tesla's "Dynamic Gravity Theory" (discussed in Chapters 8 and 9, and referenced in [6], [97], [99], and [117]).

Each linear "mass-energy-momentum motion" is accompanied by a motional-energy component, a matter wave that behaves like a spinning or helical wave-packet motion along the path of linear motion. This can be also described as a pair of wavefunctions forming an Analytic Signal function (for a detailed discussion on Matter Waves and Analytic Signals, see Chapters 4.0, 4.1, 4.3 and 10). The concept of an Analytic Signal pair, consisting of mutually orthogonal wavefunctions, is particularly useful for modeling matter waves and understanding gravitational effects, especially in connection with the associated electromagnetic field between attracting masses.

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Understanding gravitation is inherently complex due to the interplay of several coinciding states, effects, and situations involving mechanical and electromagnetic inertial effects, matter-waves, atoms and masses. These contribute to the same gravitational force or field, and can be summarized as follows:

a) **Origin of Gravitation**: Gravitation fundamentally arises from forces acting on and around atoms, manifesting both internally and externally (see Chapter 8, section 8.3, "Structure of the Field of Subatomic and Gravitation Related Forces"). In the cosmos, a macro-scale, low-frequency standing matter-wave field exists between gravitationally attracting masses. Masses such as solar systems and galaxies are situated in spatial nodal zones of these macrocosmic waves, where we observe only attractive Newtonian forces, like in Nikola Tesla's concept of Dynamic Gravity. The field effects within these nodal, standing-wave zones generate only attractive forces. In contrast, repulsive forces, which counteract gravitation, are believed to operate also within atoms, masses, or mass agglomerations, resembling a concept akin to Rudjer Boskovic's universal Natural Force. It is important to note that the $1/r^2$ force law applies equally to Coulomb forces, Newtonian gravity, and the "external," long-distance component of Boskovic's universal natural force. Boskovic's "internal" alternating force, which balances the external force, is responsible for keeping atoms and masses stable, compact or agglomerated.

b) **Motion and Electromagnetic Effects**: All masses in the universe are in constant motion, exhibiting holistic, omnidirectional oscillations, angular and helical motions, and multiaxial rotations and spins relative to different potential-force centers. These motions generate spontaneous electromagnetic tensions, including dipole and multipole polarizations and forces within macro masses. Here, electromagnetic dipoles align and mutually attract, following Coulomb's law. Contrary to common misconceptions, electric charges (including dipoles) are not fixed or static parameters but are dynamic entities, akin to linear and angular moments in mechanics. This dynamic nature means that an electromagnetic energy flow, as speculated by Nikola Tesla, always exists between electric charges, accompanied by a Coulomb-type $1/r^2$ central force, which shares the same mathematical form as Newtonian gravity and is an "external" aspect of Boskovic's universal natural force. The primary contributors to gravitational force are the internal and external spinning (rotational and orbital) states of atoms, masses and their associated magnetic moments or domains. These moments attract opposite magnetic moments in neighboring masses. The synchronized, global cosmic motions, including linear, rotational, orbital, and spinning motions of elementary particles, electric charges, and dipoles within macro masses, effectively generate electric currents and associated magnetic field components. These aligned magnetic fields create an attractive electromagnetic force between any two masses, contributing to the Newtonian force of gravity through complex electromagnetic interactions (as presented in Chapter 3, equations 3.1–3.4). Additional insights can be found in the works of Eric Roberts Laithwaite, known as the "Father of Maglev" for his development of the linear induction motor and maglev rail system (see reference [102]).

c) **Role of Ether and Wave Equations**: For electromagnetic forces to effectively contribute to gravitation, the existence of an exceptionally fine fluid or "ether" inside and between atoms and masses must be acknowledged. This ether acts as the carrier of electromagnetic fields and waves. Gravitational, electromagnetic, quantum-mechanical, mechanical, and acoustic matter waves all adhere to the same partial differential second-order Classical Wave Equation. This equation is best exploited using the Complex Analytic Signal model (see Chapter 4.0). Under such conditions, the Schrödinger equation can be smoothly and comprehensively developed without additional assumptions or postulated mathematical attachments (see Chapters 4.3 and 10). In physics, wave equations have no significance without the presence of a suitable fluid or elastic medium, as waves can only be generated within sufficiently elastic material carriers.

Download the last version here:
http://www.mastersonics.com/documents/revision_of_the_particle-wave_dualism.pdf

2.1.1 More about Newtonian Gravitation based on analogies

Current explorations into the relationship between gravitation and electromagnetism have largely been developed through a direct analogy between Newton's gravitational force formula for two masses and Coulomb's force formula for two electric charges. This approach assumes that an electric charge is analogous to mass. By extending this assumption and applying the Lorentz force from electromagnetic theory, several publications have proposed that the gravitational field can be modelled analogously to the electromagnetic field using Maxwell's equations. Notable works in this area include A. I. Arbab's *Comprehensive Analogy Between Electromagnetism and Gravity* [63], Ling Jun Wang's *Unification of Gravitational and Electromagnetic Fields* [122], and Mohammed A. El-Lakany's *Unification of Gravity and Electromagnetism* [170].

However, the full unification of gravity and electromagnetism still remains incomplete and requires further refinement. The key challenge is that electric charge is not directly analogous to mass. A more accurate analogy is that electric charge corresponds to mechanical "energy-mass-moments," as discussed in the first chapter of this book. In this analogy, mechanical energy-mass-moments are intrinsically linked with corresponding electromagnetic moments.

To advance this work, as outlined in this book, we will focus on developing and examining indicative analogies and fundamental symmetries between gravitation and electromagnetism. As demonstrated in the first chapter (see T.1.8), this involves comparing the mathematical forms of energies and central $1/r^2$ forces across different domains of physics, including cases of mass rotation [3]. It is crucial to maintain consistency with the established analogies presented in T.1.2 and T.1.8 of the first chapter. The data required for the analogical comparisons, which form the next step in our analysis, can be found in T.2.1.

T.2.1	**Electric Field**	**Linear Motion** and Gravitation	**Magnetic Field**	**Rotation**
Energy (Motional, kinetic, fields ...)	$\frac{1}{2}Cu^2 = \frac{1}{2}qu = \frac{1}{2}\frac{q^2}{C}$	$\frac{1}{2}mv^2 = \frac{1}{2}pv = \frac{1}{2}\frac{p^2}{m} (= \frac{1}{2}Sf^2)$	$\frac{1}{2}Li^2 = \frac{1}{2}\Phi i = \frac{1}{2}\frac{\Phi^2}{L}$	$\frac{1}{2}\mathbf{J}\omega^2 = \frac{1}{2}L\omega = \frac{1}{2}\frac{L^2}{\mathbf{J}} (= \frac{1}{2}S_R\tau^2)$
Energy Density	$\frac{1}{2}\varepsilon E^2$	?	$\frac{1}{2}\mu H^2$	?
Coulomb-Newton-J. Mitchell $1/r^2$ Forces	$\frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2}$ (Coulomb)	$-G\frac{m_1 m_2}{r^2}$ (Newton)	$\mathbf{F}_{1,2} = \frac{\mu_0}{4\pi}\frac{\mathbf{q}_{m1} \cdot \mathbf{q}_{m2}}{\mathbf{r}^2}$ (John Michell)	?

Table T.2.1 contains known and newly analogically created formulas related to energy and corresponding forces in various contexts, such as electrostatic fields, linear motion related to gravitation, magnetic fields, and rotational-motion forces. Empty boxes marked with a question mark (?) are left for developing hypothetical ideas to "reconstruct" missing mathematical forms based on mobility-type analogies (from T.1.1 to T.1.8).

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

After conditionally accepting the previous analogical, indicative brainstorming formulas (see points 1 to 17 in the introductory part of this chapter), we can further and still superficially support them by filling in the missing energy and force expressions in Table T.2.1 using electromechanical analogies from T.1.2 to T.1.8. This oversimplified approach will help us create a more complete formal symmetry of indicative, mutually corresponding, and analogous energy and force expressions. While some of these forms may still be highly hypothetical or essentially incorrect, they are temporarily valuable for initiating challenging thinking and creative insights.

Explaining, updating, and redefining gravitation from the foundations established by Kepler and Newton is not an easy or straightforward task, as their work still holds strong validity. To make meaningful progress, we need to foster a creative environment that encourages new ideas and the development of new, probable, or possible directions based on analogies. Contemporary mathematical physics provides ample mathematical options and tools for this purpose.

The Newtonian gravitational force, when considered solely as the attraction between static masses, is negligible compared to other natural forces known in physics under comparable conditions. Dealing with such infinitesimal effects within our planet as a laboratory is challenging, especially if new and still unknown gravitational force components are involved.

For instance, if we succeed in extending the framework of gravitation to include interactions between "mass-energy-moments" and electromagnetic entities, rather than assuming gravitation acts exclusively between static masses, we can create richer models of motion within solar systems. This approach increases our chances of upgrading and verifying a new vision of gravitation. This strategy is promoted here, as illustrated in section 2.3.3, "Macro-Cosmological Matter-Waves and Gravitation."

In summary, while Newtonian gravitation will remain quantitatively unchanged, its internal content can be hypothetically and analogically upgraded and reorganized, by replacing T.2.1 with T.2.2, expecting that in this way new indicative assumptions and predictions could be initiated, as follows.

T.2.2	**Electric Field**	**Linear Motion** and Gravitation	**Magnetic Field**	**Rotation**
Energy	$\frac{1}{2}Cu^2 = \frac{1}{2}\frac{q^2}{C} \Downarrow$	$\frac{1}{2}mv^2 = \frac{1}{2}\frac{p^2}{m} \Downarrow$	$\frac{1}{2}Li^2 = \frac{1}{2}\frac{\Phi^2}{L} \Downarrow$	$\frac{1}{2}J\omega^2 = \frac{1}{2}\frac{L^2}{J} \Downarrow$
Energy Density	$\frac{1}{2}\varepsilon E^2 \Rightarrow$	$(\frac{1}{2}g_p G^2)^*$	$\Leftarrow \frac{1}{2}\mu H^2 \Rightarrow$	$(\frac{1}{2}g_L R^2)^*$
Coulomb Force types	$F_{ed} = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2} \Rightarrow$	$(F_{gd} = \frac{1}{4\pi g_p}\frac{p_1 p_2}{r^2})^*$	$(F_{md} = \frac{1}{4\pi\mu}\frac{\Phi_1\Phi_2}{r^2})^*$	$(F_{Rd} = \frac{1}{4\pi g_L}\frac{L_1 L_2}{r^2})^*$
Newton Force types	$(F_{es} = (?)\frac{C_1 C_2}{r^2})^*$	$F_{gs} = -G\frac{m_1 m_2}{r^2}$ $\Leftarrow$ $\Rightarrow$	$(F_{ms} = (?)\frac{L_1 L_2}{r^2})^*$	$(F_{Rs} = (?)\frac{J_1 J_2}{r^2})^*$

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

(…)* -Hypothetical formulas; $\mathbb{G}$ -Gravitation field, g_G -"Gravitational-permeability"; $\mathbb{R}$ -"Field of Rotation", g_R -"Rotational-permeability"; Indexing: ed = electro-dynamic, gd = gravito-dynamic, md = magneto-dynamic, Rd = rotational-dynamic, es = electro-static, gs = gravito-static, ms = magneto-static, Rs = rotational-static (most of them are here invented, temporary formulations for the purpose of making initial analogical expressions)

All energy and forces expressions in T.2.2 are intentionally formulated, suggested, or modified (or analogically formulated to comply with Mobility system of analogies), to have mutually similar mathematical forms regarding relevant constants (such as: $\frac{1}{4\pi\varepsilon}$, $\frac{1}{4\pi\mu}$, $\frac{1}{4\pi g_p}$, $\frac{1}{4\pi g_L}$). Of course, we already know the constant of gravitational force as, G = 6.67428 (+/- 0.00067) $\cdot 10^{-11}$ [$m^3 kg^{-1} s^{-2}$] (= $\frac{1}{4\pi g_p}$...). All of that is serving to obtain maximal formal symmetry and parallelism of mutually analog and corresponding force and energy forms, and to implicate that an analogical force member related to certain kinds of *associated rotation to linear motion* (or spinning) could also exist with its own force constant $\frac{1}{4\pi g_L}$. What could be particularly interesting in here introduced strategy is to find different, mutual and cross-platform, analogical links of relevant constants, such as the speed of light expressed as the function of electric permittivity and magnetic permeability, $c = \frac{1}{\sqrt{\mu\varepsilon}}$, or in cases of electric transmission lines as $c = \frac{1}{\sqrt{LC}}$, where **L** and **C** are inductance and capacitance per unit length. Analogically concluding, the speed of light or universal constant **c** could have another form such as, $c = \frac{1}{\sqrt{g_L g_p}} = \sqrt{\frac{4\pi G}{g_L}}\ (= \frac{1}{\sqrt{\mu\varepsilon}}) \Rightarrow g_L = \frac{4\pi G}{c^2}$ (see similar thinking in **[70],** Guido Zbiral; "*Does Gravitation Have an Influence on Electromagnetism*"). Such relations implicate the possible existence of much more profound unity of different natural fields and forces, and if we continue that way, something similar could be applicable to old and new, analogically introduced constants regarding mechanics and gravitation, as by a brainstorming exercised in T.2.2.

Citation (from the Internet): "Self-consistent gravitational constants are complete sets of fundamental constants, which are self-consistent and define various physical quantities associated with gravitation. These constants are calculated in the same way as electromagnetic constants in electrodynamics. This is possible because in the weak field equations of general relativity are simplified into equations of gravitomagnetism, similar in form to Maxwell's Equations. Similarly, in the weak field approximation equations of the covariant theory of gravitation [1] turn into equations of the Lorentz-invariant theory of gravitation (LITG). LITG equations are Maxwell-like gravitational equations, which are the same as equations of gravitomagnetism. If these equations are written with the help of self-consistent gravitational constants, there is the best similarity of equations of gravitational and electromagnetic fields". Taken from: http://en.wikiversity.org/wiki/Nonstandard_physics/Selfconsistent_gravitational_constants .

At this time, it is still better to postpone arbitrary and premature discussions about the terminology and meaning of new formulas, constants and symbols introduced in T.2.2, because it is clear (from obvious analogies) how such mathematical forms are created, and what meaning they could have (here temporarily figuring mostly for purposes of easy introduction and simplified presentation based on indicative similarities). In any case, the importance of the previous brainstorming step would be mostly in sparking possible original ideas about connections and symmetries that could exist between different central fields and forces. In addition, we will be able to rethink if we are really explaining the same known items (regarding fields and forces) in some different way, or maybe we are still not used to seeing the same facts from the analogical point of view as previously initiated. Anyway, the most general way of understanding and defining all natural forces is summarized in Chapter 10. of this book (see "10.02 MEANING OF NATURAL FORCES"). All forms (in T.2.2) with the white background, marked by an asterisk (…) *, are newly created and hypothetical. Of course, on this level, there is still no commitment as to which expression in T.2.2 has some real applicability, or a chance to be transformed towards something with Physics-related meaning. Based on the data in T.1.6, T.2.1, and T.2.2, we also see that (by analogy) magnetic flux Φ belongs to "spatial or geometry-related parameters", like rectilinear displacement and angle are. At the very least, we can also conclude that (at this time) a full analogy (in all directions) is still not satisfied with magnetic field phenomena (cf. T.1.6 and T.2.2) in the same way as it is satisfied for all other electromechanical parameters and entities. Later, we shall attempt to establish a more complete symmetry in electromagnetism; - see (3.1) to (3.5) in the third Chapter.

As we can see in T.2.1 and T.2.2, Newton and/or Coulomb or "$\mathbf{1/r^2}$" central force laws could be extended (presently only analogically and hypothetically) to rotational, orbital, and spinning motions involving some linear, angular, and orbital moments. It should also be mentioned (what it is already experimentally known) that attractive and repulsive forces between two permanent magnets (between the opposite or the same magnet poles (q_{m1}, q_{m2})) are also satisfying the same Coulomb-Newton force-law analog to electrical charges situations, $F_m = \mu \frac{q_{m1} \cdot q_{m2}}{4\pi r^2}$ (first time discovered by John Michel in 1750). Also, Ampère in 1822 introduced his first (empirically and intuitively fitted) law of electrodynamics, addressing the force between two electric current-elements, which is also analog or familiar to Newton-Coulomb "**1/r²**" force laws. Ampère's law (modernized on different ways, by number of authors) is increasingly receiving recognition (almost 200 years after its first formulation) in predicting or complying with experimental results related to forces between current elements or other electrically conductive channels (see literature references [28], [29], [30] and [36]). The significance of Ampère's law is in documenting that specific experimentally known phenomenology, which cannot be explained or predicted by contemporary Maxwell Electrodynamics, is explicable by Ampère's law. In fact, Ampère's law shows the existence of some additional (still not exploited) structural issues of modern Maxwell Electromagnetic Theory (for instance, related to Lorentz force, and to some still non-counted electromagnetic force manifestations).

There is another "**1/r²**" force law (see later (2.4-4), and [45]) *between equidistant, parallel paths moving electrical charges* (q_1, q_2), having velocities (v_1, v_2), where magnetic force between them is found to be, $F_{1,2} = \frac{\mu}{4\pi} \frac{(q_1 v_1) \cdot (q_2 v_2)}{r^2}$.

Obviously, there is certain creative and indicative potential in further elaborating and generalizations of Newton-Coulomb "$\mathbf{1/r^2}$" force laws because we find them applicable in gravitational or electromagnetically neutral masses interactions, in electric charges and magnetic poles interactions, as well as in electrodynamics' related situations between current elements and/or moving electric charges. Here, we will devote some more space to different analogical and unifying, also hypothetical aspects of "$1/r^2$" force laws. *We should strongly underline that (in this book) an analogy based only on "$1/r^2$" force laws is not at all what will be taken as the strongest, single and unique argument saying that Gravitation could have an electromagnetic nature (since we will have number of new options for such and other more relevant conclusions, later).*

[♣ COMMENTS & FREE-THINKING CORNER (brainstorming, while exploring analogies)

In electromagnetic theory, it is evident that a complete symmetry, parallelism, and mathematical analogy can be developed between electric and magnetic fields, moments, and charges (see Chapter 3 of this book and [99] from Konstantin Meyl for more details). Additionally, a significant analogy can be drawn between the Bohr atom model and solar or planetary systems (see Chapter 8 and T.2.8 for the N. Bohr hydrogen atom and planetary system analogies). This analogy intuitively suggests that gravitation is directly linked to the structure of matter and electromagnetic phenomena.

Anyway, static masses are not the true and sole sources of gravity (based on analogies from the first chapter). Instead, masses in motion, whether through vibration, linear and angular movement, or spin, along with their internal and external manifestations. Mechanical and electromagnetic moments, and other electromagnetic properties, are the actual sources of what we recognize as gravitation.

Furthermore, every linear motion, when modeled and analyzed using the Analytic Signal model, reveals an associated complementary spinning motion. Spinning, in turn, generates elements of oscillatory motion. From this understanding, we can determine the de Broglie matter wave frequency, wavelength, group and phase velocity, and relevant phase functions for moving masses. This approach allows us to connect these concepts with the wave-particle duality of matter, thereby significantly advancing our understanding of gravitation.

If we consider gravitational force as part of the standing matter-waves phenomenon—occurring between attracting masses that remain relatively stable in the nodal positions of such resonating fields—we align more closely with the concepts of gravitation proposed by Nikola Tesla and Rudjer Boskovic (see Chapters 4.0, 4.1, 8, 9, and 10, as well as [97] and [117] from Jean de Climont). These analogies and concepts are crucial as they can enhance our conceptual understanding of gravity.

For example, in real material media, high-power mechanical, ultrasonic, or acoustic energy, moments, forces, oscillations, and vibrations—such as audio signals and music—can be created and transmitted by applying various signal-modulating techniques to laser beams and dynamic plasma states. These laser and plasma states act as carriers for lower-frequency mechanical vibrations or signals (see Chapter 10, sections 10.2–2.4, and references [133] to [139] for more information).

Calculating the attractive or repulsive force between two magnets is, in a general case, an extremely complex operation, as it depends on the shape, magnetization, orientation, and separation of the magnets. The John Michell force between two magnetic poles is given by: $F_m = \mu \frac{q_{m1} \cdot q_{m2}}{4\pi r^2}$, where: F_m is force (SI unit: Newton), q_{m1} and q_{m2} are the pole strengths (SI unit: ampere-meter), μ is the permeability of the intervening medium (SI unit: Tesla, meter per ampere or Henry per meter) and r is the separation (SI unit: meter). The pole description is useful to practicing "magneticians" who design real-world magnets, but real magnets have a pole distribution more complicated than a single north and south. Therefore, implementation of the poles concept is not simple (this text is taken from the Internet: http://en.wikipedia.org/wiki/Magnet).

By analogy (from T.1.2 and T.1.6), as the electric capacitance **C** is a kind of a static reservoir or storage for the electric-charge elements, **q**, the same is (hypothetically) valid for the mass **m**, which should be a static storage of (somehow internally packed and stabilized) "momentum elements" **p and L**. Also, a

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moment of inertia $\mathbf{J}$ could be understood as a "static storage" for (somehow internally packed) "angular momentum elements" $\mathbf{L}$, what is considering mass as storage of rotating entities. The above statements and terminology are only temporarily established and conditionally suitable for the purpose of intriguing and analogical ideas generation, until some more convenient formulations are found. In fact, nothing very original and new has been claimed here since we already know that matter (or mass) elements are atoms with all their complexity of internally synchronized and well packed motional states, fields and forces. The author of this book introduced such ideas for the first time in [3].

2.1.2 More of Hypothetical Central Force Laws based on analogies

If we give freedom to our hypothetical brainstorming, again based on analogies, (see [3]), from table T.2.2 we could exercise that gravitational type of attractive force between the two moving masses (in their center-of-mass coordinate system) is not only given by the original Newton's formula, but rather by (2.1), as a superposition of two forces. One, which represents a static case of Newtonian attraction between two static masses, prefix "stat. = s", and the other, for the time being very hypothetical, which represents a dynamic interaction between corresponding moments, prefix "dyn. = d", which could be repulsive or attractive:

$$F_g = F_{g-dyn.} + F_{g-stat.} = + \frac{1}{4\pi g_p} \cdot \frac{p_1 p_2}{r^2} - G \cdot \frac{m_1 m_2}{r^2}. \qquad (2.1)$$

Similar or analogical thinking (and new "hypothetical innovation") can be applied to the situation with spinning masses (taken from T.2.2), in which case there could exist (attractive and/or repulsive or non-central) force between them. Such force could analogically be expressed as in (2.1), as static and dynamic force members' addition (with corresponding angular and spinning moments, about the same center of rotation):

$$F_R = F_{R-dyn.} + F_{R-stat.} = + \frac{1}{4\pi g_L} \cdot \frac{\mathbf{L}_1 \mathbf{L}_2}{r^2} - G_R \cdot \frac{\mathbf{J}_1 \mathbf{J}_2}{r^2}, \qquad (2.2)$$

In cases when we have the presence of combined rectilinear and spinning elements of certain complex motion of two objects, we could appropriately combine (2.1) and (2.2), to describe the Newtonian resulting force between them such as, $F_{g,R} = - G \cdot \frac{m_1(p_1, L_1, m_{01}, \mathbf{J}_1) \cdot m_2(p_2, L_2, m_{02}, \mathbf{J}_2)}{r^2}$. As we know, linear motion of certain particles could be conceptualized (or approximated) as a case of rotation, where the relevant radius is sufficiently long, practically accepting that all motions in our universe are in their nature rotational or angular motions. This would produce that (2.1) and (2.2) are mutually equivalent. *We also need to address the proper vector forms of (2.1) and (2.2) to conceptualize the 3-dimensional picture regarding such forces* (but presently this is not the primary objective here). In fact, we still do not have good conceptualization about involved (internal and external) linear and angular moments, and we also know that mentioned mechanical moments are often coupled or interacting with naturally associated electromagnetic and dipole moments and charges. Additional theoretical and conceptual grounds to the idea about Newton-type forces originating from orbital moments and spinning can be found in Chapter 10. of this book (see comments around equations (10.1.4) - (10.1.7)), as well as in [36], Anthony D. Osborne, & N. Vivian Pope, "An Angular Momentum Synthesis of 'Gravitational' and 'Electrostatic' Forces".

Of course, it is an open question whether formulas such as (2.1) and (2.2) can be experimentally proven and how (since the most realistic approach should be that all elements of linear and rotational motions are coincidently present and mutually coupled in the motion of the same mass or object. Also, we know that masses are kind of agglomerations of internally spinning, more elementary domains (since almost all subatomic entities have spin characteristics). All electrically charged particles have spin characteristics, and usually there we also find associated magnetic moments, magnetic fluxes, and magnetic dipoles. Since masses of electrons and protons are enormously different, in accelerated and non-uniform motional situations of masses, we can expect that many electric and magnetic dipoles will be created (all of them integrated and well packed within a mass). Consequently, (2.1) and (2.2) can also be united in a form that is more general. In fact, the unifying imperative produces that static-members found in (2.1) and (2.2) could be mutually identical, $G \cdot \frac{m_1 m_2}{r^2} = G_R \cdot \frac{\mathbf{J}_1 \mathbf{J}_2}{r^2} = F_{stat.} \Rightarrow$

$G \cdot m^2 = G_R \cdot J^2 \Rightarrow J = m\sqrt{\frac{G}{G_R}} \Rightarrow r = \sqrt{\frac{G}{G_R}}$, and that force elements responsible for (possible) dynamic interaction between two moving objects could have both linear and/or orbital or spin moments, $\frac{1}{4\pi g_p} \cdot \frac{p_1 p_2}{r^2} = \frac{1}{4\pi g_L} \cdot \frac{L_1 L_2}{r^2} = F_{dyn.} \Rightarrow \frac{g_L}{g_p} = \frac{L_1 L_2}{p_1 p_2} = \frac{L^2}{p^2} = r^2$, this way generalizing the force law between two moving objects as: $F_{1\text{-}2} = F_{dyn.} + F_{stat.}$. Of course, all relevant orbital and/or spin moments should be linked to the same center of rotation (or to their common center of mass), such as: $\frac{1}{2}pv = \frac{1}{2}L\omega \Leftrightarrow \frac{L}{p} = \frac{v}{\omega} = \frac{\omega r}{\omega} = r = \sqrt{\frac{G}{G_R}} = \sqrt{\frac{g_L}{g_p}}$. The situation addressed here (by applying analogical combinations) is still some kind of simple and easy introduction, and familiarization with the ideas that gravitation should not be only an attraction between static masses, and that other vibrating, dynamic, especially rotational and spinning properties of masses constituents (including electromagnetic charges, moments, and dipoles) can have much more vital importance. We will come back to the same situation several times in this chapter (see "2.3.2. Insights, "Macro-Cosmological Matter-Waves and Gravitation"), and we will see that the reality behind here introduced ideas is not as simple as expressed by (2.1) and (2.2), but also not too far from here creatively addressed insights. See later, in Chapter 4.1, around equations (4.3-0), (4.3-0)-a,b,c,d,e,f,g,h,i,j,k..., an extended conceptualization of familiar ideas (regarding spinning). See also illustrations on Fig.4.1.1 and Fig.4.1.1a. ***In chapter 10 of this book, we can find the more complete and simple explanation of the same situation regarding Newtonian attraction between important linear and angular moments (see (10.1.4) - (10.1.7)).***

Exploring the Candidates for better Understanding Gravitation and Related Forces

The essential question is whether we have clear examples that can provide a better and deeper understanding of gravitation, especially examples of repulsive and other unusual gravitation-related forces. These examples could be the foundation for establishing a new, experimentally and scientifically valid theory. Some potential answers include:

A) <u>Centrifugal Forces and Gravitational Balance</u>
One of the most probable candidates for exploring repulsive or torsional gravitational forces could involve studying centrifugal forces, gyroscope behaviors, and Coriolis forces, which are all coupled with internally structured magnetic and electric field moments and dipoles. For example, the orbiting planets in any Solar System would collapse into the Sun if not compensated with the associated, opposite direction centrifugal forces. This centrifugal force acts as a form of repulsive gravitational force, balancing gravitational attraction and maintaining stable orbits. If we consider this repulsive gravitational force as inherently related to rotation, this suggests that gravitation itself may be intrinsically connected to similar effective micro rotations and oscillations within gravitational masses. The internal constituents of ordinary mass, such as molecules, atoms, electrons, protons, and neutrons—are highly disparate in mass (e.g., the proton is nearly 2000 times heavier than the electron). Neutron, as a couple of an electron and a proton, also possess its intrinsic spin. When a mass is at relative rest, its internal components (or atoms) could be seen as self-stabilized and electromagnetically neutral. However, when the mass undergoes linear, angular and accelerated motions, internal electric and magnetic dipoles polarize due to the differing masses and mobilities of electrons and protons. These polarized dipoles may lead to electromagnetic attractions and repulsions between interacting macro masses, offering a possible explanation for the hidden electromagnetic nature of gravitation.

In classical mechanics, objects have centers of mass, gravity, and inertia related to linear motions, and similar centers can be defined for rotational and spinning movements. Extending this concept, we could establish neutrality centers of electric and magnetic charges and moments. An ideal state of electromagnetic and mechanical neutrality (for any mass or system) would occur if all centers of inertia, gravity, and field charges (regarding linear and rotational motions, including gravitational and electromagnetic fields) were linked to the same spatial spot. However, since all motions in our universe are relative and curvilinear, such an idealized stable center of mass does not exist. Consequently, a mass in motion, internally containing electrons, protons, and neutrons, will always exhibit multiple electric and magnetic dipole-polarized states. Professor J. Djuric suggested replacing the mathematically idealized center of mass reference system with a "center of zero-self-gravity" position, which accounts for real mass properties, shape, content, and density distributions, providing a more convenient framework for analyzing motion equations. Although the center of mass and center of zero gravity are typically close, their separation produces additional kinematic, linear, and torsional effects in motion equations. It is also impossible to achieve total electromagnetic neutrality or fully compensate for electromagnetic charges within a mass without spatially separating electric and magnetic charges. These non-compensated, spatially separated electromagnetic dipoles (or multipoles) likely contribute to, or even generate, the force of gravity. Significant contributions to similar ideas can be found in publications by Arbab I. Arbab [63].

B) The Influence of Rotation on Gravitational Forces
It is possible that fast-rotating or spinning objects generate additional, measurable gravitational forces, which differ significantly from the ordinary Newton-Coulomb static forces between them. For instance, Anthony D. Osborne and N. Vivian Pope's work on angular momentum synthesis of gravitational and electrostatic forces [36] and Eric Laithwaite's studies on gyroscope behavior suggest that rotation and spinning influence gravitational forces [102].

C) Coupling of Linear and Rotational Motions in Gravitation
A common factor among the hypothetical phenomena mentioned here is the combination of linear and rotational motions. In later chapters (4.xx and 10.), we will demonstrate that particle-wave duality and matter waves always result from the coupling between linear and rotational or spinning motions, where oscillatory and rotating motions are strongly interlinked.

D) Diffraction and Repulsive Gravitational Forces
It would be interesting to explore whether the diffraction of microparticles, photons, and electrically charged or neutral particles (when such beams pass through small diffraction holes) is related to the repulsive forces discussed here. Diffraction involves the divergence of a rectilinear stream of matter (with momentum-energy-mass content), which could be conceptualized as a two or multi-body interaction involving repulsive forces caused by particle rotation. The particle-wave duality of diffraction events complicates this situation, as wave concepts can explain diffraction patterns, but questions remain about associated particle repulsion. We might visualize diffraction holes as "bottlenecks," channeling particles (or matter waves) into mutually synchronized motion, creating higher density paths. Under these conditions, hypothetical repulsive gravitational forces may act, dispersing or diffracting particles after passing through the hole. This situation likely involves spinning field effects, especially in the space-time vicinity of any two-body interactions (see Chapter 10 for further explanations).

E) Electromagnetic Nature of Static and Dynamic Forces
Continuing this line of thought, we could hypothesize that static and dynamic forces between different objects have elements of an electromagnetic nature, consistent with Coulomb-Newton-J. Mitchell force laws. Furthermore, we might demonstrate that electromagnetic forces influence all other gravitation and rotation-related forces and field effects, acting as the primary source of these forces. Since masses are agglomerations of atoms with various internally spinning states, the magnetic and spin moments of elementary particles and astronomic formations are always mutually and proportionally coupled. It seems plausible that, by creating an external magnetic field, or applying strong electric fields near a structured mass, we could influence, modify, or even shield the local force of gravitation (potentially creating a repulsive gravitational-like force, as suggested by Dr. Evgeny Podkletnov [66]). The combination of electrostatic and dynamic force elements between moving, electrically charged particles was introduced long ago by Wilhelm Weber's force law [28], [29], [36]. Matter and all natural forces, fields, and conservation laws in our universe are inherently united and synchronized, regardless of our current understanding. By identifying analogies and continuous symmetries, we can discern probable connections between analogous entities, indicating that a future explanation of gravitation will likely consider additional mass properties that influence mass attraction, beyond the attraction between static rest masses (see the table of analogies T.1.8 in the first chapter, which supports these conclusions).

The challenging ideas and possibilities radiating from T.2.2 and (1.19) - (2.9) could be intuitively addressed as a unity and couplings between rectilinear and rotational motions. We usually relate gravitational field to a specific space deformation around a certain mass. However, from T.2.2, from similar expressions for energy density, we can see that gravitational and rotation-related forces could be differently introduced, in two ways:

-in static cases (prefix “stat.”), as certain spatial deformation resulting as a force proportional to the product of involved masses m_1, m_2 and/or moments of inertia J_1, J_2; and

- In a dynamic case (prefix "dyn.”), as the force proportional to products between relevant linear and/or orbital moments $\mathbf{p}$ and $\mathbf{L}$. We could also create a vector addition of both (when shows relevant), combining (2.1) and (2.2) with (2.4). Later (see (2.11.14-20), in "2.3.3-3 Standing-Waves Resonators and Gravitation”), after developing such ideas, we will notice the possibility to determine gravitational force (in the Center of Mass coordinates) as, $F_g = v\frac{(L_1 + L_2)}{r^2} = G\frac{m_1 m_2}{r^2} = G\frac{m_r m_c}{r^2}$, which could present certain progress in supporting Newton law of gravitation.

To explore possible experimental evaluations, we can examine equations (2.1) and (2.2) as alternative explanations for the diffraction of elementary micro-particles and quasi-particles. These equations may also help us understand interactions between micro-particles with spin characteristics within atomic systems (refer to [3]).

For instance, the Pauli Exclusion Principle states that "only two electrons with opposite spins can occupy the same quantum state," as conceptualized in Bohr’s planetary atom model. This principle might be related to equation (2.2). Electrons, or electron wave packets with torsional field components, generate mutually coupled magnetic and mechanical angular moments. This results in electromagnetic or electrodynamic Coulomb and Lorentz forces, as well as attraction or repulsion between their magnetic poles. These forces, which have rotational energy-mass properties, balance with other electromagnetic interactions, potentially proving the interactions between rotating objects.

It is almost intuitive that electrons in stationary atomic orbits pair up, forming pairs of elementary magnets with opposite spins. This pairing allows for attraction between opposite magnetic poles. Similarly, protons and neutrons in the atomic nucleus also have spin and obey the Pauli Principle. If a new type of hypothetical force, as described in (2.2), exists between rotating objects, it could provide insights into the nature of nuclear forces, potentially related to de Broglie matter waves (see (4.18)).

Given the strong relationship and synchronization between magnetic, orbital, and spinning moments of elementary particles (with constant gyromagnetic ratios), it is possible that what we perceive as "new and hypothetical" forces could be manifestations of well-known electromagnetic forces (see [33], [71], and [102], from Jovan Djuric; “Magnetism as Manifestation of Gravitation”).

One exciting possibility arising from equations (2.1) and (2.2) is the creation and control of a non-stationary, transient, dynamic antigravity force, like the repulsive force between like magnetic poles. This could be achieved by exploiting the vibrating, pulsing, and dynamic time-variable effects of rotating masses and angular momentum interactions within a rigidly coupled system, potentially generating transient and alternating forces that counteract local gravitational force.

We could also explore creating mixed force formulas for electric and magnetic fields by combining static and dynamic cases, using force formulas from T.2.2 (see also (3.1) - (3.5) and (4.18)). However, this would be a simplified approach compared to the more comprehensive Maxwell electromagnetic theory, which manages electromagnetic fields and forces. Additionally, recent extensions of Maxwell’s theory and alternative approaches to relativity, such as the revitalization of Wilhelm Weber’s force law, offer new insights into the connections between electromagnetism and gravitation. Chapter 3 of this book discusses these generalizations of Maxwell’s equations, supporting theories of gravitation and relativity.

These discussions aim to unify fundamental laws of classical electrodynamics, such as Gauss’s laws, Coulomb’s law, Ampere’s generalized force law, Faraday’s law, Lorentz force laws, and Lenz’s law (see literature [28], [29], [34], and [36]).

In a steady state of relative rest, mass presents a neutral electromagnetic balance between positive and negative electric constituents and mutually opposite magnetic charges or moments. However, mass in motion, including oscillations due to acceleration and dynamic states, tends to create or increase internal electrical and magnetic dipoles and associated moments, due to the significant mass difference between electrons and protons (1836 times).

The ideas presented here are initial brainstorming points for developing new force laws beyond gravitation. The goal is to initiate a creative process that considers the limitations of classical mechanics and planetary motions in our current understanding of gravitation. Future theories may offer significant upgrades by incorporating more complex electromagnetism-related situations. Examples of such creative attempts can be found in Chapter 3 (about electromagnetism), Section 8.3 (structure of the field of subatomic and gravitation-related forces), and Section 2.3.3 (macro-cosmological matter waves and gravitation).

Another, original and exciting method of generating velocity dependent, static and dynamic, force-field formulas (starting from expressions for "Newton-Coulomb" force types) is presented in [4]. Similar methods can also be applied to all force formulas from T.2.2, as well as to all possible combinations like in (2.1) and (2.2). This will additionally support here presented hypothetical thinking (since the author of [4] presented certain experimental evidence of the general validity of such force formulas). For instance, in [4], we find for all velocity-dependent "Newton-Coulomb" types of fields and forces the following expression, $E_{moving} = E_{dyn.} = E(v) = E_{stat.}(1 - \frac{v}{c}\cos\vartheta)$, *where: E(v) is the intensity of certain velocity-dependent, "Newton-Coulomb" field/force type;* $\mathbf{E_{stat.}}$ *corresponds to field/force formulas from T.2.2 and to force expressions like in (2.1) and (2.2); v is the speed of the (relevant) field charges; c is the speed of light; and* ϑ *is the angle between the direction of the charge motion (the speed v) and the direction of involved field-intensity propagation.* ♣]

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Exploring Gravitation: A Deeper Understanding of Mass and Its Interactions
In this book, we will gradually explore the nature of gravitation by delving into several key concepts:

- **Mass as a Form of Energy Packing**: We will examine mass as a specific state of energy, conceptualized as "self-closed and stabilized standing matter waves agglomerations." This includes associated and mutually coupled phenomena such as vibrations, spinning, and electromagnetic properties of atoms and masses, like electric charges, different electromagnetic and mechanical moments, and dipoles.

- **Mass and Gravitation**: We will demonstrate that only mass, when combined with its (internal and external) linear and angular moments, electromagnetic moments and dipoles, and oscillating states, can truly be considered the source of gravitation (see [36]). Even when a mass is in a state of relative rest, it remains coupled with a specific "velocity field" that belongs to the surrounding spatial and global background motion. This means that mass is always "velocity charged," possessing associated linear and angular velocity components and moments (see [73], Reginald T. Cahill). The underlying principle is that all motions in our universe, both micro and macro, tend to become oscillatory, orbital, curvilinear, or circular (i.e., angular and accelerated). These motions are mutually synchronized and coupled with phenomena such as self-spinning or helix or vortex fields, which contribute to the effects of central forces and gravitation. These effects are based on spontaneous electromagnetic dipole polarizations within masses and electric charges. Since all macroscopic masses are composed of electromagnetic entities that are interconnected and move in relative motions within a universally synchronized cosmic motion, this supports the idea that gravitation is fundamentally electromagnetic in nature.

- **Foundations of Inertial States:** We will extend the concept of inertia to include linear, rotational, spinning, and oscillatory motions, and how they contribute to self-stabilized, stationary, and standing-waves spatial formations, such as atoms and planetary systems, both mechanically and electromagnetically (see more about inertia in Chapters 1, 4.2 and 10). Different quantization phenomena known in physics are, in fact, the consequences of these self-stabilized, resonant, or standing waves, which are mutually synchronized spatial-temporal energy-momentum states.

- **Unified Gravity and Particle-Wave Duality:** We will explore how gravity and particle-wave duality can be understood through the specific couplings between linear, angular, and spinning motions with associated electromagnetic fields. This concept extends from the micro-world of subatomic structures to the macro-universe of astronomic objects. This could potentially (still hypothetically) open new channels of communication between distant orbiting and spinning states, where information might propagate much faster than the speed of light, or even instantaneously, if global conservation of angular, orbital, and spinning moments is satisfied. Such instantaneous global couplings could belong to the entanglement effects known in quantum theory. There are innovative theories suggesting that all orbital and spinning moments are universally, intrinsically, and synchronously coupled, akin to an entanglement-type of cosmic communication (see [36] and Chapter 10). There is already evidence that solar systems adhere to certain standing matter-wave spatial-temporal arrangements (see more at the end of this chapter). Additionally, there is a remarkable parallel between atomic models and planetary systems (see Chapters 8 and 10). The modeling of macrocosmic matter waves and planetary motions can be achieved using complex analytic signal functions, which could allow us to apply the same mathematical models used in microphysics to explain macrocosmic entanglements (see "2.3.3. Macro-Cosmological Matter-Waves and Gravitation" later in this chapter).

- **Reevaluating Newtonian and Einsteinian Gravitation:** Contemporary interpretations of gravitation, as proposed by Newton and Einstein, suggest that the force or field of gravity exists independently or appears as a force due to the structured (and curved) spatial-temporal geometry around masses. In Newtonian mechanics, this concept is conveniently hybridized with rules related to circular motions, and assumed centrifugal and centripetal forces, along with the empirical observation that mass acceleration creates effects like those of mass weight in a gravitational field. While this hybridization of assumptions in physics, and mathematics works well within certain limits, we have now reached the boundaries of this approach. There is a need for a more extensive and realistic foundation for a new theory of gravitation (see, for instance, [33], [71], and [102] from Jovan Djuric, "Magnetism as Manifestation of Gravitation," and the effects of orbital motions on gravity in [36]).

- **Influence of other Historical Theories**: Some of the most compelling ideas about gravitation can be traced back to Rudjer Boskovic’s Universal Natural Force theory [6] and Nikola Tesla's Dynamic Theory of Gravity [97], both of which suggest that gravitation may be a specific manifestation of universal electromagnetic force. The mathematical similarities between Coulomb's law and Newton's law of gravitation, as well as their connection to Boskovic's force law, suggest that gravitation could be seen as a residual electromagnetic force, heavily attenuated by balancing elements of Boskovic's universal force. In this context, the part of Boskovic's force curve that stabilizes atoms, and solid masses can be associated with the gravitational force. To maintain this dynamic force balance, it is necessary to have properly distributed standing waves among the structural elements of masses. Since Coulomb forces act between electrically charged particles and we consider that electric charges are dynamic properties of matter (like mechanical moments), it follows that there should be a natural and continuous flow of Tesla's radiant electromagnetic energy between masses in mutual gravitational attraction (see Chapter 9).

The author of this book contends that, in our universe, we have only two families of forces: electromagnetic forces and forces related to standing-wave structured matter waves. If all matter waves are intrinsically related to an electromagnetic nature or origin, then our universe is governed by a single family of forces, electromagnetic forces. For more on these ideas, see [117], Jean de Climont.

2.2. Ideas and proposals regarding Generalized Coulomb-Newton Force Laws

Regardless of indications that Newtonian Gravitation should be replaced by the more conceptually advanced theory (like in [36], or in General Relativity) let us explore the consequences and limits of extensions of already well operating Newtonian Gravitation. There is an easy way to upgrade Newton gravity force law in combination with relativistic understanding of mass and energy. We start from Newton attractive, gravitational force between two static masses, $\mathbf{m_{01}}$ and $\mathbf{m_{02}}$, ($m = m_0/\sqrt{1-v^2/c^2} = \gamma m_0 ,\ m_0 = const.$). Now, let us imagine that the mentioned masses are in linear motion, and let us replace the two (static or standstill) masses ($\mathbf{m_{01}}$ and $\mathbf{m_{02}}$) with their relativistic, total energy equivalents, using well-known relations from the Relativity Theory,

$$\mathbf{E_{t1} = m_1c^2,\ E_{t2} = m_2c^2,\ (E_t = E_0 + E_k} = m_0c^2/\sqrt{1-v^2/c^2} = mc^2 = \gamma m_0c^2,\ E_0 = m_0c^2,$$

$$E_k = (m-m_0)c^2 = \mathbf{pv/(1+\sqrt{1-v^2/c^2}\,)} = (\gamma-1)m_0c^2,\ p = mv = \gamma m_0 v\ ,$$

$$\mathbf{m = m_0 + \frac{E_k}{c^2} = m_0 + \frac{pv}{c^2(1+\sqrt{1-v^2/c^2})} = m(m_0, p) = m(m_0, E_k),}$$

$$\mathbf{p = mv = \gamma m_0 v = \frac{E}{c^2\sqrt{1-v^2/c^2}}v = \frac{E_0+E_k}{c^2\sqrt{1-v^2/c^2}}v = p(E_0, E_k),}$$

$$\mathbf{E_k = \frac{pv}{1+\sqrt{1-v^2/c^2}} = E_k(p)}$$

If masses (in addition) have spinning motion elements, we will need to add spinning energy amount, $\mathbf{E_{ks} = \frac{L\omega_s}{1+\sqrt{1-\frac{v^2}{c^2}}}}$ and this spinning energy is effectively increasing a total particle mass for the amount $m_s = \frac{E_{ks}}{c^2} = \frac{L_s \omega_s}{c^2 \left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)}$ producing that the new total mass will be (see later (2.11.3-1)),

$m = m_0 + \frac{E_k}{c^2} + \frac{E_s}{c^2} = m_0 + \frac{pv + L_s \omega_s}{c^2 \left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)} = m(p, L_s) = m_0 + \tilde{m}$. Practically, spinning particles, in a state of relative rest (regarding linear motion, meaning v = 0) has a mass, $m = m_0 + \frac{L_s \omega_s}{2c^2} = m_{rest}$. Here we see that resulting and effective internal spinning of mass constituents $L_{s-internal}$ (as well as external, macro spinning $L_{s-ext.}$, if exist) have a direct influence on a total mass amount (since $L_s = L_{s-internal} + L_{s-ext.}$.

We could even (still hypothetically) consider that a real, standstill or rest mass m_0 is only a kind of stabilized, superimposed agglomerate or condensate of internal spinning and mater-wave states $L_{s-internal}$, this way conceptualizing rest mas as, $m_{rest} = m_0(L_{s-internal}) + \frac{L_{s-ext.} \omega_{s-ext.}}{2c^2}$. See also familiar elaborations around equations (2.11), Fig.2.4. Tables T.2.4 and T.2.5., and equations (4.41-1) to (4.41-4) in chapter 4.3. **Eventually, we will have some reasonable grounds to say that meaning of mass, $m = m(p, L_s)$, is always closely related to associated linear and spinning moments, and the intention here is to show that real sources of gravitation can only be masses with associated linear and angular momenta, including associated mass vibrations (because spinning and rotating motions are eventually generating vibrations). On a similar way, we could also hypothetically exercise that additional mass contributors could come from associated electromagnetic energy participants, this way specifying mass as $m = m(p, L_s, q, \phi)$, where q is a certain amount of electric charge, and ϕ is specific magnetic flux. Here, we are respecting the Mobility system of analogies (from the first Chapter of this book) where we have the clear analogical prediction that sources of gravitation should only be active-charge elements such as (p, L_s, q, ϕ).**

Newton's force law between two (almost static) masses presents the traditional understanding of gravitation. Now we innovatively assume that every object or mass $m = m(p, L_s, q, \phi)$ may have (internally and externally) associated gravity-related charge attributes (p, L_s, q, ϕ). Here we should appropriately include matter waves, different acting fields, wave-particle duality facts etc. (see about PWDC in Chapters 4.1 and 10). All the mentioned energy-momentum entities can be represented as having an equivalent (relativistic) mass or total energy content based on Einstein energy-mass relation $E = mc^2$, and vice versa, and consequently, we are able to address Newton law within a much broader meaning than usual, as follows:

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

$$\left(F_g = -G\frac{m_{01}m_{02}}{r^2}\right) \to \left(F_g = -G\frac{m_1 m_2}{r^2},\ m_{1,2} = m_{0-1,2} / \sqrt{1 - v_{1,2}^2 / c^2}\right) \Leftrightarrow$$

$$F_g = -G\cdot\frac{m_1\cdot m_2}{r^2} = -\frac{G}{c^4}\cdot\frac{(m_1c^2)\cdot(m_2c^2)}{r^2} = G^*\cdot\frac{E_{t1}\cdot E_{t2}}{r^2}, (G^* = -\frac{G}{c^4} = \text{const.}) \quad (2.3)$$

$$F_g = G^*\cdot\frac{1}{r^2}(E_{k1}+E_{01})\cdot(E_{k2}+E_{02}) = G^*\cdot\frac{1}{r^2}(E_{k1}E_{k2}+E_{k1}E_{02}+E_{01}E_{k2}+E_{01}E_{02}).$$

$$\text{If}\begin{bmatrix} m_1 = m_2 = m \\ E_1 = E_2 = E_t \\ E_{k1} = E_{k2} = E_k \end{bmatrix} \Rightarrow \begin{bmatrix} F_g = G^*\frac{1}{r^2}(E_k+E_0)^2 = G^*\frac{1}{r^2}E_t^2,\ E_0 = \text{const}, dE_t = dE_k \\ dF_g = G^*\frac{2E_t}{r^2}dE_t - G^*2E_t^2\frac{dr}{r^3} = G^*\frac{2E_t}{r^2}(dE_t - E_t\frac{dr}{r}) \end{bmatrix} \quad (2.3\text{-}1)$$

The most general and very indicative expression for gravitational force, as presented in equations (2.3), (2.3-1), (2.4), (2.4-a), and (2.4-b), intuitively clarify an innovative conceptualization of gravitation. If we later express the involved energies in terms of relevant charges, moments, and forces (including electromagnetic and other matter-wave entities), this vision of gravitation becomes even more striking.

From equation (2.3-1), we can infer that gravitational force, in its differential form, has at least two components. By connecting this with the broader definitions and meanings of forces discussed in Chapter 10, under "10.02: Meaning of Natural Forces," we can derive additional, thought-provoking conclusions that may influence a revised understanding of General Relativity, Black holes, Dark mass and Dark energy.

This analogical progression, starting from equation (2.3) and extending through (2.3-1), represents a smooth transition from the classical Newtonian view of gravitation to a modified relativistic interpretation, initiated independently of Einstein's General Relativity theory. Now we know what can be involved in the most general formulation of gravitational force, such as different forms of motional energies and energies of rest states.

It is immediately clear that in the force law (2.3) there are only relations (products) between motional (kinetic or dynamic) energies, $E_{k1} = E_{1dyn.}$, $E_{k2} = E_{2dyn.}$, and state-of-rest energies, $E_{01} = E_{1stat.}$, and $E_{02} = E_{2stat.}$, of the interacting objects. Since total energy members, $E_{t1} = E_{k1} + E_{01} = E_{1dyn.} + E_{1stat.}$, $E_{t2} = E_{k2} + E_{02} = E_{2dyn.} + E_{2stat.}$, could have different origins, composed from many total-energy-contributing elements (electromagnetic, gravitational, rotation, kinetic and potential energies, etc.), we could generalize Newton force law (for instance, between two complex multi-component energy states, here marked with **"i ↔ 1"** and **"j ↔ 2")** creating the following force formula, $F_g = F_{1,2}$:

$$F_{1,2} = G^*\cdot\frac{1}{r^2}[(\sum_i E_{k1i})(\sum_j E_{k2j}) + (\sum_i E_{k1i})(\sum_j E_{02j}) +$$
$$+(\sum_i E_{01i})(\sum_j E_{k2j}) + (\sum_i E_{01i})(\sum_j E_{02j})] = \quad (2.4)$$
$$= \sum_{i,j} F_{(1-2)dyn.} + \sum_{i,j} F_{(1)dyn.(2)stat.} + \sum_{i,j} F_{(1)stat(2)dyn.} + \sum_{i,j} F_{(1-2)stat.} \ .$$

In (2.4) there is only one purely static (or constant, velocity independent) force member involving only rest masses interaction or interaction between static, stable and constant field charges (if mass in its wider meaning, as introduced here, could be characterized as a real, unique, self-standing gravitation field-charge):

$$\sum_{i,j} F_{(1-2)stat.} = G^*\cdot\frac{1}{r^2}(\sum_i E_{01i})(\sum_j E_{02j}) = -G\cdot\frac{m_{01}\cdot m_{02}}{r^2}. \quad (2.4)\text{-a}$$

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The favored concept here is that static or rest mass represents only a portion of the gravitational force picture. The other significant components of gravitational forces between two objects are related to their linear and orbital moments, intrinsic mass vibrations, and coupled electromagnetic moments, including relevant dipoles and charges. These dynamic and motional energy components—like those found in equations (2.1), (2.2), and (2.4)—may effectively represent the still mysterious entities of "dark matter, mass-energy states."

$$\sum_{i,j} \mathbf{F}_{(1-2)dyn.} = \mathbf{G}^* \cdot \frac{1}{\mathbf{r}^2}(\sum_i \mathbf{E}_{k1i})(\sum_j \mathbf{E}_{k2j}) = \mathbf{G}^* \cdot \frac{1}{\mathbf{r}^2}(\sum_i \int_{[0,r]} \mathbf{F}_{(1\text{-}2)i} d\mathbf{r}_{(1-2)i})(\sum_i \int_{[0,r]} \mathbf{F}_{(2\text{-}1)i} d\mathbf{r}_{(2-1)i}),$$
$$\sum_{i,j} \mathbf{F}_{(1)dyn.(2)stat.} = \mathbf{G}^* \cdot \frac{1}{\mathbf{r}^2}(\sum_i \mathbf{E}_{k1i})(\sum_j \mathbf{E}_{02j}) = \mathbf{G}^* \cdot \frac{1}{\mathbf{r}^2}(\sum_i \int_{[0,r]} \mathbf{F}_{(1\text{-}2)i} d\mathbf{r}_{(1-2)i})(\sum_j \mathbf{E}_{02j}), \quad (2.4)\text{-b}$$
$$\sum_{i,j} \mathbf{F}_{(1)stat(2)dyn.} = \mathbf{G}^* \cdot \frac{1}{\mathbf{r}^2}(\sum_i \mathbf{E}_{01i})(\sum_j \mathbf{E}_{k2j}) = \mathbf{G}^* \cdot \frac{1}{\mathbf{r}^2}(\sum_i \mathbf{E}_{01i})(\sum_i \int_{[0,r]} \mathbf{F}_{(2\text{-}1)i} d\mathbf{r}_{(2-1)i}) \cdot$$

We should bear in mind that the total motional energy (of both interacting objects) could have linear-motion kinetic energy components $E_k = pv/(1+\sqrt{1-v^2/c^2}) \cong \frac{1}{2}pv$ and certain amount of rotating (or spinning) motion energy $E_{kr} = L\omega/(1+\sqrt{1-v^2/c^2}) \cong \frac{1}{2}L\omega$ (in case any of interacting objects is spinning), including embedded electromagnetic and other energy forms. *The point here is that (using conclusions based on analogies) we are coming closer to understanding that Newton's law of gravitation, initially defined as an attraction of masses, is also presentable as an attraction between linear and/or angular moments (p_i and L_i) of involved mass-energy-momentum states, since,*

$$E_k = \sum_{(i)} (E_{ki} + E_{kri}) \cong \frac{1}{2} \sum_{(i)} (p_i v_i + L_i \omega_i),$$

$$\left(m = m_0 + \frac{E_k}{c^2} = m_0 + \frac{pv}{1+\sqrt{1-v^2/c^2}} \right) \left[\begin{matrix} \Leftrightarrow \\ \text{analog to} \end{matrix} \right] \left(J = J_0 + \frac{E_{kr}}{c^2} R^2 = J_0 + \frac{L\omega}{c^2(1+\sqrt{1-v^2/c^2})} R^2 \right)$$

(See the table of analogies T.1.8 from the first chapter).

For instance, the force component $\sum_{i,j} \mathbf{F}_{(1-2)dyn.} = \mathbf{G}^* \cdot \frac{1}{\mathbf{r}^2}(\sum_i \mathbf{E}_{k1i})(\sum_j \mathbf{E}_{k2j})$ from (2.4)-b can be transformed into, $\sum_{i,j} \mathbf{F}_{(1-2)dyn.} = \frac{\mathbf{G}^*}{4} \cdot \frac{1}{\mathbf{r}^2}(\sum_i \mathbf{p}_i \mathbf{v}_i)(\sum_j \mathbf{p}_j \mathbf{v}_j)$. If we now consider (of course, as a specific case) within Mobility system of analogies (from T.1.8), that electric charge analogically corresponds to linear momentum, $\mathbf{q}_i \Leftrightarrow \mathbf{p}_i$, we can create another (already known) expression that should be a force between *equidistant, parallel paths uniformly moving electrical charges* (q_1, q_2), while at the same height (see later (2.4-4), and [45]),

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$$\sum_{i,j} \mathbf{F}_{(1-2)dyn.} = \frac{\mathbf{G^*}}{\mathbf{4}} \cdot \frac{\mathbf{1}}{\mathbf{r^2}} (\sum_i \mathbf{p}_i \mathbf{v}_i)(\sum_j \mathbf{p}_j \mathbf{v}_j) \quad \begin{pmatrix} \Leftrightarrow \\ (p \Leftrightarrow q) \end{pmatrix}$$

$$\Leftrightarrow \frac{\mathbf{G^*}}{\mathbf{4}} \cdot \frac{(\sum_i \mathbf{q}_i \mathbf{v}_i)(\sum_j \mathbf{q}_j \mathbf{v}_j)}{\mathbf{r^2}} (\Leftrightarrow) \frac{\mu}{4\pi} \frac{(\mathbf{q_1 v_1}) \cdot (\mathbf{q_2 v_2})}{\mathbf{r^2}} . \tag{2.4)-c$$

$$\left[\begin{array}{l} a \cdot G^* = \mu / \pi = a \cdot G / c^4 \Leftrightarrow \mu c^4 = \pi \cdot a \cdot G = \mu / \varepsilon^2 \mu^2 = 1/\varepsilon^2 \mu \Rightarrow \pi \cdot a \cdot G\varepsilon\mu = 1/\varepsilon \Rightarrow \boxed{\pi \cdot a \cdot Gc^2 \varepsilon = 1} \;?? \\ a = \text{constant} \end{array} \right].$$

Of course, such fragmented analogical associations and involved conclusion process as in (2.4)-c is oversimplified, not well supported, and still presents an intuitive and brainstorming product, but it is on some indicative way saying that gravitational attraction could also be an attraction between some effective (or embedded) electric and/or magnetic charges, fluxes, or dipoles, as it will be exercised later (see elaborations around equations from (2.4-7) until (2.4-10)). *Here, we are also becoming familiar with the ideas that Newtonian or $1/r^2$ gravitational, central force, could have dynamic or active interacting elements like (structurally or intrinsically integrated) mechanical and electromagnetic moments, and corresponding velocities, including associated matter-waves and effects of wave-particle duality, meaning not only interactions between static masses (see more about PWDC in Chapter 10).*

Direct analogical (and hypothetical) force expression between two moving masses, based on (2.4)-c and on Mobility analogies from the first chapter, is again similar or equivalent to (2.4), as follows,

$$F_{1,2} = K' \frac{(p_1 v_1) \cdot (p_2 v_2)}{r^2} = K'' \frac{E_{k1} \cdot E_{k2}}{r^2} = K'' \frac{(E_1 - E_{01}) \cdot (E_2 - E_{02})}{r^2} =$$
$$= \frac{K''}{r^2} (E_1 E_2 - E_1 E_{02} - E_{01} E_2 + E_{01} E_{02}) = \frac{K''}{r^2} \left(\gamma_1 \gamma_2 m_1 m_2 C^4 - \gamma_1 m_1 m_2 C^4 - \gamma_2 m_1 m_2 C^4 + m_1 m_2 C^4 \right) =$$
$$= K''' \left(\gamma_1 \gamma_2 \frac{m_1 m_2}{r^2} - \gamma_1 \frac{m_1 m_2}{r^2} - \gamma_2 \frac{m_1 m_2}{r^2} + \frac{m_1 m_2}{r^2} \right) = \boxed{K''' \frac{m_1 m_2}{r^2}} \cdot (\gamma_1 \gamma_2 - \gamma_1 - \gamma_2 + 1),$$
$$(K', K'', K''') = \text{constants}$$

(2.4)-c-1

To support the existence of such an attractive force, that will be equal to Newton force of gravitation, it will be necessary that masses m_1 and m_2 are in permanent (holistic) motion, including rotation, as follows,

$$\boxed{K''' \frac{m_1 m_2}{r^2}} \cdot (\gamma_1 \gamma_2 - \gamma_1 - \gamma_2 + 1) \cong G \frac{m_1 m_2}{r^2} \Rightarrow (\gamma_1 \gamma_2 - \gamma_1 - \gamma_2 + 1) \cong \text{Const.},$$
$$G = G(\gamma_1, \gamma_2) = G(v_1, v_2) = K''' \cdot (\gamma_1 \gamma_2 - \gamma_1 - \gamma_2 + 1) \cong \text{const.}, \quad \gamma_{1,2} = (1 - v_{1,2}^2 / C^2)^{-0.5} ,$$
$$(v_1 = v_2 = v), (\gamma_1 = \gamma_2 = \gamma) \Rightarrow K''' (\gamma_1 \gamma_2 - \gamma_1 - \gamma_2 + 1) = K''' (\gamma^2 - 2\gamma + 1) = G \Rightarrow v = \underline{\text{const}}.$$

(2.4)-c-2

In our solar system and within our part of cosmic space, Newton's gravitational constant is G ≅ 6.67408 × 11^{-11} m^3 kg^{-1} s^{-2}, but in some different parts of cosmic space, it could be different, since holistic rotations, background linear velocities, and

surrounding electromagnetic fields could be different there. The logical consequence of (2.4)-c-2 is that we could imagine that our Universe is in perpetual, global, micro, and macrocosmic synchronized motion, including rotation, to support Newtonian gravitation. See similar analogical excursions later around (2.4-4), (2.4-5), (2.4-5.1), T.2.2-1 and T.2.2-1. Mentioned rotation (being an accelerated motion) is producing spatially organized dipoles, or electric and magnetic polarizations, and such electromagnetic entities are mutually experiencing effects of attractive forces, and creating rotating, spiral, elliptic, and circular, galactic, and planetary systems (as speculated later in this chapter around equations (2.4-7) and (2.4-8)).

We also know that an internal rest-mass structure has its natural constitutive elements (molecules, atoms, and subatomic entities) composed of electromagnetically or dipole-balanced, charged states. These are in perpetual motion, oscillating, spinning, and rotating in their stationary energy states, being somehow auto compensated, and stabilized like self-closed standing waves, resonant structures, or as mutually coupled elementary magnets and positive and negative charges. The characteristic property of rest masses in our universe is that all of them have specific latent (internal), rest-state energy (equal to $\mathbf{m_o c^2}$). The force between the rest-masses could still exist, even if they look (macroscopically, or externally) as fully electrically and magnetically neutral. This should be the real nature of gravitational force (because rest-masses are at the same time (or dominantly) agglomerates and ensembles of different internal spinning and oscillating states). Of course, when rest masses are in relative motion interacting with other energy states and other masses, the force of Gravitation would become only more complex, like in (2.4). There are only mutually relative motions and relative rest states among all masses and energy states in our universe and involved forces should also be mutually relative. The challenge here would be to find the essential, generalizing relations and connections between electromagnetic and gravitation forces, since mainly, all of them (when isolated) are obeying the same form of Coulomb-Newton's ($1/r^2$) force law, as shown in T.2.2.

What we are presently considering as Newton law of gravitation force is only the most static and maybe the weakest part of the generalized force expression (2.4): $\sum_{i,j} \mathbf{F}_{(1-2)stat.} = \mathbf{G}^* \cdot \frac{1}{r^2} (\sum_i \mathbf{E}_{01i})(\sum_j \mathbf{E}_{02j}) = -\mathbf{G} \cdot \frac{\mathbf{m}_{01} \cdot \mathbf{m}_{02}}{r^2}$. Obviously, that (2.4) can be modeled or mathematically transformed as in (2.1) and (2.2), or also to include linear and orbital moments, electromagnetic and other force, and energy elements. Here, we can pose the question what real, elementary, and essential sources or charges of different fields and forces in our Universe are. We can also try to model them deductively, starting from (2.4), taking the energy and different moments as the most common quantifying property for all of them (avoiding starting from specific objects and other field charges, as often practiced in Physics). Until present, official mainstream science still considers that mass is an exclusive source of gravity, as the source of an electric field is an electrical charge. Based on an extended understanding of Newton-Coulomb force law (2.4), and on respecting Mobility-type Analogies (from the first chapter), we could find or reinvent more of essential *force charges* (part of that matter being still hypothetical and based on analogies). If our present knowledge regarding mentioned analogical situations about natural forces is essentially correct, the same, already known facts regarding field sources and charges should be re-confirmable from different conceptual, symmetry-and-analogy-related platforms, becoming mutually compatible for generalizations, and being uniquely treated regarding all possible field charges, starting from (2.4), by going back to elementary relations between different field charges. As we know, this is still not the case in contemporary Physics. For instance, if we know that objects $\mathbf{m_1}$ and $\mathbf{m_2}$, from (2.3) are not only neutral masses, but also have certain (non-compensated) electrical charges and specific magnetic properties (being like permanent magnets and/or electric dipoles), we can extend the Newton-Coulomb force law (2.4) by adding two more force members. However, we need to remember that associated electromagnetic forces in many cases could be enormously stronger than any attraction caused by gravitation and that in such cases the force member belonging to static mass attraction, $-G\frac{m_{01}m_{02}}{r^2}$, could be negligible, especially in instances when masses are sufficiently small). At the same time, we still do not have any quantitative argument to say how big other (hypothetical) gravity-related force contributing members (found in expressions (2.1), (2.2) and (2.4)) could be, compared to electromagnetic forces. Most probably it will be explained that all of such "old and new force members" essentially belong to electromagnetic forces, or directly originating from them (see [33], [71] and [102], from Jovan Djuric, "Magnetism as Manifestation of Gravitation").

[♣ COMMENTS & FREE-THINKING CORNER (brainstorming)

Since both, Newton and Coulomb force laws have the same mathematical form, for developing more general (for the time being only qualitative) understanding of force expressions between two objects "charged electrically ($q_{el.} = q$), magnetically, ($q_{mag.} = \Phi$) and gravitationally, (m)"; -we could hypothetically transform (2.4 into):

$$\mathbf{F_{1,2} = F_g \Rightarrow F_{1,2}(combined,\ multiple\ charges) = F_g} + \frac{1}{4\pi\varepsilon}\frac{q_1q_2}{r^2} + \frac{1}{4\pi\mu}\frac{\Phi_1\Phi_2}{r^2} + \dots .$$

$$\begin{aligned} F_{1,2} &= \sum_{i,j} F_{(1\text{-}2)stat} + \sum_{i,j} F_{(1\text{-}2)dyn.} + \sum_{i,j} F_{(1)dyn.(2)stat} + \sum_{i,j} F_{(1)stat(2)dyn} = \\ &= \left[-G\frac{m_1m_2}{r^2} + \frac{1}{4\pi\varepsilon}\frac{q_1q_2}{r^2} + \frac{1}{4\pi\mu}\frac{\Phi_1\Phi_2}{r^2} + \dots \right] + \\ &+ \sum_{i,j} F_{(1\text{-}2)dyn.} + \sum_{i,j} F_{(1)dyn.(2)stat} + \sum_{i,j} F_{(1)stat(2)dyn} \\ &\left(F_{1,2} = F_m = \mu\frac{q_{m1}\cdot q_{m2}}{4\pi r^2} = \frac{1}{4\pi\mu}\frac{\Phi_1\Phi_2}{r^2}, \Phi_{1,2} = \mu q_{m1,2} \right), \end{aligned} \quad (2.4\text{-}1)$$

however, being modified for new motional energy contributions coming from interactions between mechanical moments, electromagnetic and all other presently known or maybe some unknown field charges (if any of such still hidden charges really exist). The reason for having such a situation is not only that Newton and Coulomb's laws have the same mathematical forms, but in the fact that every electrical or magnetic entity has its equivalent mass (or energy content). This means that the logic applied in developing (2.4-1) can be extended to any Coulomb-Newton central force situation between different field charges. At the same time, qualitatively and conceptually, (2.4-1) could present a kind of simple field unification platform, but still with some associated problems such as, that active centers of masses, electric charges and magnetic dipoles or fluxes are never completely overlapping the same area. The distances between the mentioned charged entities (between relevant charged object centers) are always different. We anyway know that motional electric and magnetic objects and states are always mutually related, electromagnetically coupled and embedded in a mass as a state of energy packing. Also, force laws (2.4) – (2.4–1) are non-linear, $1/r^2$ central force laws, and

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simple (linear) superposition and spatial homogeneity is not applicable here. Different field charges (mass, moments, electrical charge, magnetic flux ...) are only looking mutually separated and distinct to us (especially if we start comparing them quantitatively and dimensionally), but maybe we just have different "packing and energy-atomizing formats" of the same universal field (meaning that (2.4-1) presents an improbable situation). A similar concept will be additionally elaborated later; -see equations (2.4-6) – (2.4-10), just to explore how far we could advance using conclusions based on analogies.

Consequently, we could imagine that every mass internally has a combination of mass-energy elements coming from its electromagnetically neutral rest mass, and from its electromagnetically active (not compensated) elements. This way of conceptualizing, neutral rest masses $m_{1,2-n}$ will mutually attract respecting Newton law (of Gravitation), and electromagnetic elements $\frac{E_{1,2-em}}{c^2}$ will mutually interact respecting Coulomb law, as follows,

$$F_{1,2} = G\frac{m_1 m_2}{r^2} = G\frac{(m_{1-n} + \frac{E_{1-em}}{c^2})(m_{2-n} + \frac{E_{2-em}}{c^2})}{r^2} .$$

If at the same time, certain mass has spinning, rotating, torsional or angular motions (which are not electromagnetic), we could extend Newton-Coulomb law as,

$$F_{1,2} = G\frac{m_1 m_2}{r^2} = G\frac{(m_{1-n} + \frac{E_{1-spinning}}{c^2} + \frac{E_{1-em}}{c^2})(m_{2-n} + \frac{E_{2-spinning}}{c^2} + \frac{E_{2-em}}{c^2})}{r^2}$$

The well-known particle-wave **duality** (or better to say **unity**) concepts and their origins, known from Quantum Theory (first introduced by L. De Broglie, Max Planck, A. Einstein, and N. Bohr), should also be (qualitatively) recognizable in (2.4-1) for instance,

$$\begin{aligned} F_{1,2} &= \sum_{i,j} F_{(1\text{-}2)stat} + \sum_{i,j} F_{(1\text{-}2)dyn.} + \sum_{i,j} F_{(1)dyn.(2)stat} + \sum_{i,j} F_{(1)stat(2)dyn} = \\ &= F(\text{Particles-related}) + F(\text{Waves \& Fields-related}) + \\ &+ F(\text{mixed-particles-waves-related}), \\ F(\text{Particles-related}) &= \sum_{i,j} F_{(1\text{-}2)stat} = \frac{\Omega_1 \Omega_2}{r^2}, \\ F(\text{Waves \& Fields-related}) &= \sum_{i,j} F_{(1\text{-}2)dyn.}, \\ F(\text{mixed-particles-waves-related}) &= \sum_{i,j} F_{(1)dyn.(2)stat} + \sum_{i,j} F_{(1)stat(2)dyn} . \end{aligned} \quad (2.4\text{-}3)$$

$\Omega_{1,2}$ (=) here introduced symbol for generalized universal field charges.

Familiar ideas regarding unique origins of Newton-Coulomb force laws (as consequences of orbital and spin moments conservation) can also be found in [36], Anthony D. Osborne, & N. Vivian Pope, "An Angular Momentum Synthesis of 'Gravitational' and 'Electrostatic' Forces".

Gravitation explicable as a weak magnetic field's attraction

There is another $\frac{1}{r^2}$ force law *between equidistant, parallel-paths uniformly moving electrical charges* (q_1 , q_2), while at the same height, where the force is given by (see (2.4)-c and [45]),

$$F_{1,2} = \frac{\mu}{4\pi}\frac{(q_1 v_1)\cdot(q_2 v_2)}{r^2} = K\frac{(q_1 v_1)\cdot(q_2 v_2)}{r^2},\ K = \frac{\mu}{4\pi} = \text{const.} \cdot \quad (2.4\text{-}4)$$

The magnetic force between two moving charges in the more general case is,

$$\vec{F}_{1,2} = \frac{\mu}{4p}\frac{q_1 q_2}{r^2}\vec{v}_1 \times (\vec{v}_2 \times \hat{r}). \tag{2.4-4-1}$$

Here, original Coulomb's law (like in cases of static electric charges) cannot directly explain magnetic-field force between them. For the same sign on the products $(q_1 v_1)$ and $(q_2 v_2)$ the charges are drawn closer (like an attractive force between two parallel wires with currents in the same direction), and for opposite signs, the charges are brought apart (introducing repulsive force, like between two parallel wires with currents in mutually opposed directions). The most likely, still an intuitive explanation is that moving charged particles are creating helicoidally spinning electromagnetic fields around paths of motion (or at least helicoidally spinning magnetic field). Where such electromagnetic fluxes are mutually not canceling, motional electric charges will experience an attractive force, and in cases of mutually opposed fields, a repulsive force will appear. *The same idea about specific "field spinning" around any moving particle will later be shown applicable to all matter waves (see chapter 4.1), as an extended understanding of de Broglie, matter-waves hypothesis.*

Let us continue generating more intuitive, analogical, and brainstorming or hypothetical ideas regarding Gravitation, since it is evident that from the time when I. Newton created his foundations of gravitation, modern physics did not move too much (regardless A. Einstein elaborated relatively complex and seducing mathematics within General Relativity theory), and as we know, very new ideas and concepts about Gravitation are still missing. If we apply already known analogies (see tables of analogies T.1.2 until T.1.8, from the first chapter), where electrical charge q is analog to linear, $p = mv$, ($m = m_0 + \frac{E_k}{c^2} = m_0 + \frac{pv}{c^2(1+\sqrt{1-v^2/c^2})}$), or spin moment $\mathbf{L} = \mathbf{J}\omega$, or to a magnetic flux Φ, we will be able to transform (2.4-4) into another couple of interesting (and hypothetical) $\frac{1}{r^2}$ force expressions (2.4-5). Instead of different field charges we will (based on mentioned Mobility analogies) have products of corresponding motional, kinetic, or field energy members, like in (2.3), (2.4) and (2.4)-c,

$$F_{1,2} = K\frac{(q_1 v_1)\cdot(q_2 v_2)}{r^2}\begin{pmatrix}\Rightarrow \\ \text{by analogy}\end{pmatrix} F_{1,2} = \begin{Bmatrix} K_p\frac{(p_1 v_1)\cdot(p_2 v_2)}{r^2}, \text{ for neutral particles} \\ K_L\frac{(\mathbf{L}_1 v_1)\cdot(\mathbf{L}_2 v_2)}{r^2}, \text{for spinning particles} \\ K_\Phi\frac{(\Phi_1 v_1)\cdot(\Phi_2 v_2)}{r^2}, \text{ for magnetic dipoles}\end{Bmatrix}$$

$$\begin{pmatrix}\Rightarrow \\ \text{by analogy} \\ =\end{pmatrix}\begin{Bmatrix} K_p\frac{(p_1 v_1)\cdot(p_2 v_2)}{r^2} \\ K_L\frac{(\mathbf{L}_1\omega_1)\cdot(\mathbf{L}_2\omega_2)}{r^2} \\ K_\Phi\frac{(\Phi_1 u_1)\cdot(\Phi_2 u_2)}{r^2}\end{Bmatrix}\begin{pmatrix}\Rightarrow \\ \text{by analogy} \\ =\end{pmatrix}\begin{Bmatrix} K_p^{'}\frac{(E_{k1})\cdot(E_{k2})}{r^2} \\ K_L^{'}\frac{(E_{r1})\cdot(E_{r2})}{r^2} \\ K_\Phi^{'}\frac{(E_{m1})\cdot(E_{m2})}{r^2}\end{Bmatrix}, \tag{2.4-5}$$

$(K, K_p, K_L, K_\Phi, K_p^{'}, K_L^{'}, K_\Phi^{'}) = \text{Constants}, v_i (=)\text{velocity}, u_i (=)\text{voltage}, \omega_i (=)\text{angular velocity}$

Of course, (2.4-5) could have some analogical predictive power under similar conditions when (2.4-4) is applicable (between equidistant, parallel paths moving charges). ***In cases of electric charges, corresponding voltages could replace velocities (using Mobility system of analogies).***

Another, revitalized and emerging force law between electric current elements, also familiar and analogical to (2.4-4), (2.4-5), and to other Newton-Coulomb ($1/r^2$)-force laws, is based on the old Ampère's force law. Such force law is showing promising results and introduces competitive modeling compared to similar conclusions from Maxwell-Faraday and Quantum Electrodynamics (see [36], Anthony D. Osborne, & N. Vivian Pope; -Immediate Distant Action and Correlation in Modern Physics).

The real challenge in applying analogies for creating possible and brainstorming options regarding what gravitation field and force could be (in addition to what we already know) is related to the fact that only a static mass is not the entity that should be the essential, and unique source of gravitation. Of course, if we start only from the Newton and Kepler Laws, we can get a strong impression that mass is the only source of gravity. Since mobility-type analogies of mass are electric capacitance **C** and moment of inertia **J** (see more in Chapter 1.), it is difficult to imagine how such a direct analogical approach could produce Coulomb law $F_{1,2} = \frac{1}{4\pi\varepsilon}\frac{q_1 \cdot q_2}{r^2}$ from Newton law of gravitational attraction between two masses, $F_{1,2} = G\frac{m_1 \cdot m_2}{r^2}$. The reason is that mass and electric charge are not mutually analogical or equivalent (meaning not directly replaceable within the mobility-type of analogies). What analogically corresponds to electric charges (in mobility type of analogies) is a linear moment, $p = mv$, and angular moment, $L = J\omega$ (as already exercised in the first chapter and in T.2.2), but presently we do not know enough about gravitational interactions between linear and/or angular moments. Also, since mass analogically corresponds to specific capacitance, analogically created a law of (hypothetical) attraction between two capacitors $F_{1,2} = K\frac{C_1 \cdot C_2}{r^2}$ is meaningless, except when capacitors in question are electrically charged (and when we could imagine the existence of certain surrounding electric fields attraction). Obviously, to resolve this challenge and contradictions based on analogies, masses should have some hidden velocity components or attributes. On a similar way as in cases of capacitors, masses could be conveniently "velocity-charged" or activated to be like electrically charged capacitors. What makes masses mechanically and electromagnetically "charged" is certain kind of holistic, combined linear and rotational motion (including spinning and associated vibrations), meaning that masses should have linear and angular velocity components and moments to be analogically comparable to charged capacitors… As we know, Newton law still does not address any dynamic or motional parameters like velocities, but it is mathematically like Coulomb law. We could ask ourselves where missing (angular and linear) velocity components are? Most probably (and still hypothetically), something (still invisible and not properly conceptualized in physics) is permanently embedded, moving, or oscillating everywhere around and inside gravitational masses, also creating certain intrinsic, background velocity field. This way conceptualizing, we know that Newton Law is not explicitly considering mentioned ***velocity-characterized***, and (should be) omnipresent background field, but anyway, all masses (in our Universe) are on some natural way always "velocity-charged" regarding such hidden and moving, spatial texture (see more of familiar and challenging ideas in [73] and [74]). However, if we now start from the Coulomb law and familiar "$1/r^2$" (electromagnetic) force laws, such as:

T.2.2-1

$F_{1,2} = \frac{1}{4\pi\varepsilon}\frac{q_1 \cdot q_2}{r^2}$	(=) Coulomb force between electric charges
$F_{1,2} = F_m = \mu\frac{q_{m1} \cdot q_{m2}}{4\pi r^2} = \frac{1}{4\pi\mu}\frac{\Phi_1\Phi_2}{r^2}$, $\Phi_{1,2} = \mu \cdot q_{m1,2}$	(=) Coulomb-type force between two magnets (first time discovered by John Michell in 1750, see [36]).
$F_{1,2} = \frac{\mu}{4\pi}\frac{(q_1 v_1)\cdot(q_2 v_2)}{r^2}, \Phi_{1,2} = \mu \cdot q_{m1,2} = \mu \cdot q_{1,2} \cdot v_{1,2}$ $q_{m1,2} = q_{1,2} \cdot v_{1,2}$	(=) $1/r^2$ force law between equidistant, parallel-paths, uniformly moving electrical charges (q_1 , q_2); (2.4-1), (2.4-4), (2.4)-c (see [45])

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and if we consider that electric charge is (in mobility-type of analogies) replaceable with linear, orbital and/or angular moments, $p = mv$, and/or $\mathbf{L} = \mathbf{J}\omega$ (see Analogies in chapter 1.), we could apply such mobility-type analogies (on T.2.2-1) and formulate the following (hypothetical) mechanical-world (would be gravitational force) analogies,

T.2.2-2

$1/r^2$ force laws	Possible, mobility-type analogies (still hypothetical)
$F_{1,2} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 \cdot q_2}{r^2}$	$F_{1,2} = G_p \frac{p_1 \cdot p_2}{r^2} (=) G_L \frac{\mathbf{L}_1 \cdot \mathbf{L}_2}{r^2}$
$F_{1,2} = F_m = \frac{1}{4\pi\mu}\frac{\Phi_1\Phi_2}{r^2}$	$F_{1,2} = G_p \frac{p_1 \cdot p_2}{r^2} (=) G_L \frac{\mathbf{L}_1 \cdot \mathbf{L}_2}{r^2}$
$F_{1,2} = \frac{\mu}{4\pi}\frac{(q_1 v_1)\cdot(q_2 v_2)}{r^2}$	$\cdots \rightarrow F_{1,2} = G_p \frac{p_1 \cdot p_2}{r^2} (=) G_L \frac{\mathbf{L}_1 \cdot \mathbf{L}_2}{r^2}$ (?!)

How to transform $F_{1,2} = G_p \frac{p_1 \cdot p_2}{r^2} (=) G_L \frac{\mathbf{L}_1 \cdot \mathbf{L}_2}{r^2}$ into an expression of Newton gravitation force law $F_{1,2} = G\frac{m_1 \cdot m_2}{r^2}$, considering that specific "***background velocity field***" is linked to a locally surrounding spatial structure of our Universe (see [73] and [74]), is possible to imagine and analogically exercise on the following way,

$$\left\{\begin{array}{l} F_{1,2} = G_p \frac{p_1 \cdot p_2}{r^2} = G_p \frac{m_1 v_1 \cdot m_2 v_2}{r^2} = G_p v^2 \frac{m_1 m_2}{r^2} = G_{pv} \frac{m_1 m_2}{r^2} = G\frac{m_1 m_2}{r^2} \Rightarrow G_{pv} = G = G_p v^2, \\ F_{1,2} = G_L \frac{\mathbf{L}_1 \cdot \mathbf{L}_2}{r^2} = G_L \frac{\mathbf{J}_1\omega_1 \cdot \mathbf{J}_2\omega_2}{r^2} = G_L \omega^2 \frac{\mathbf{J}_1\mathbf{J}_2}{r^2} = G_{L\omega}\frac{\mathbf{J}_1\mathbf{J}_2}{r^2} = ... = G\frac{m_1 m_2}{r^2} \Rightarrow G_{L\omega} = G = G_L\omega^2, \\ \left(v = \sqrt{v_1 v_2}, \omega = \sqrt{\omega_1\omega_2}, G = G_p v^2 = G_L \omega^2 = v\omega\sqrt{G_p G_L}\right) \end{array}\right\} \Rightarrow$$

$$v/\omega = \sqrt{G_L/G_p} (=) r (=) [m] (=) \text{certain radius, or length} \Rightarrow$$

$$\text{If: } \left((G, G_p, G_L) = \text{constants}\right) \Rightarrow (v,\omega) = \text{Constants},$$

$$\text{If: } \left(G_p = (1/4\pi g_p)\ \&\ G_L = (1/4\pi g_L)\right) \Rightarrow c = (1/\sqrt{\varepsilon\mu}) = (1/\sqrt{g_p g_L}) = 4\pi\sqrt{G_p G_L} = 4\pi G/v\omega \Rightarrow$$

$$v\omega = 4\pi G/c = G/\sqrt{G_p G_L} (=) v^2/r = r\omega^2 (=) \text{central or radial acceleration } (=) \text{constant}, \sqrt{G_p G_L} = c/4\pi,$$

$$\frac{G_L}{G_p} = \frac{g_p}{g_L} = \frac{\mathbf{L}_1 \cdot \mathbf{L}_2}{p_1 \cdot p_2} (=) \frac{\mathbf{L}^2}{p^2} (=) \frac{v^2}{\omega^2} (=) r^2 (=) [m^2], \left(\frac{1}{2}pv = \frac{1}{2}\mathbf{L}\omega\right) \Leftrightarrow \left(\frac{\mathbf{L}}{p} = \frac{v}{\omega} (=) \frac{\omega r}{\omega} = r = \sqrt{\frac{G_L}{G_p}} = \sqrt{\frac{g_p}{g_L}}\right),$$

$$\left(m = m_0 + \frac{E_k}{c^2} = m_0 + \frac{pv}{c^2(1+\sqrt{1-v^2/c^2})}\right)\left[\begin{array}{c}\Leftrightarrow \\ \text{analog to}\end{array}\right]\left(\mathbf{J} = \mathbf{J}_0 + \frac{E_{kr}}{c^2} = \mathbf{J}_0 + \frac{\mathbf{L}\omega}{c^2(1+\sqrt{1-v^2/c^2})}\right), \quad (2.4\text{-}5.1)$$

where $v = \sqrt{v_1 v_2}$ and $\omega = \sqrt{\omega_1\omega_2}$ are mentioned, intrinsic and mutually coupled velocities of "*spatial matrix*", creating locally relevant "***background velocity field***" (to avoid using often criticized entity like "***ether***").

We already understand that the variety of linear, mechanical, and electromagnetic moments, dipoles, and fluxes, or relevant field charges, originates from interactions between electrons and protons within atoms and masses. These electrons and protons, along with atoms and molecules, oscillate, spin, and interact electrodynamically. Through these interactions, they synchronize, extend, or radiate their internal atomic and electromagnetic fields, charges, moments, and matter waves to other atoms and masses.

Neutrons, composed of one electron and one proton, also exhibit spinning properties. This concept suggests that all other external mechanical properties and moments of masses, including gravitation, are derived from these fundamental interactions. Additionally, it is observed that all masses in our universe exhibit slight magnetization, which implies that gravitation might be conceptualized as an attraction between mutually opposed magnets, in line with Coulomb's or John Mitchell's laws, as promoted by Dr. Jovan Djuric (see [33], [71], and [102]).

It is important to remember that the force of gravitation is a central force following the inverse-square law ($1/r^2$). This idea is also echoed in Nikola Tesla's concept of Dynamic Gravity and R. Boskovic's Universal Natural Force. For more on these related topics, see Chapters 8, 9, and 10.

This version is more concise and focused, with clearer connections between the concepts.
We already know that in the gravitational field of a big mass, if we neglect possible friction with surrounding space, two different (smaller) masses will fall with equal speed (being synchronized and arriving at the surface of a big mass at the same time), meaning that it could be $v^2 = v_1 v_2$, $v_1 = v_2 = v$. Later, we will realize that there is no linear motion without certain associated field rotation (or spinning) and vice versa, leading to a conclusion, that it could exist another, "***background velocity field***" where, by analogy, is valid, $\omega^2 = \omega_1 \omega_2$, $\omega_1 = \omega_2 = \omega$. Familiar concepts about force of gravitation, effectively equal to the product of involved moments will be exercised and extended later (in the same chapter) within equations (2.4-13), (2.4-5.1), (2.11.13-1) - (2.11.13-5), (2.11.23), (2.11.24), (2.11.14-4), and table of analogies T.2.8.

Chapter 10 of this book offers the most comprehensive explanation of the hidden or background velocity parameters, the complex nature of mass, and the Newtonian attraction between relevant linear and angular moments (refer to equations 10.1.4 through 10.1.8 for more details). In brief, our contemporary understanding of mass, as something constant or a stable building block of matter, needs to evolve significantly, becoming more closely tied to the wave-particle duality of matter.

Since interacting masses, along with their linear, orbital, or spinning moments, generate what we describe as gravitation (an interaction of relevant standing matter waves), we can also propose that any two such states effectively create a specific "half-wavelength resonant dipole." These electromechanical dipoles, in a state of standing-wave resonance, produce an attractive force in the nodal zones. Such resonant effects, where an attractive force appears within the resonant nodal zone, are well-documented in ultrasonic techniques when a half-wavelength sonotrode or resonator oscillates at its resonant frequency, as well as in cases of acoustic levitation (see references [150] and [151]).

It is highly probable that gravitation is a similar local state of attraction within a resonating universe. The key question is to identify the "divine source" of this (still unknown) oscillatory energy (see further discussions in [99] by Konstantin Meyl).

To illustrate larger spectrum of such (or familiar) ideas about *spatial, background* ***"velocity-field"***, or "***linear and orbital moments linked***" mass and electromagnetic charges and dipoles understanding, or concepts about gravitation and mutual communication (and couplings) between space, matter, and electromagnetic phenomenology, let us first read the following abstracts and citations (see next pages):
Taken from [73]; -Reginald T. Cahill, Dynamical 3-Space: Emergent Gravity:

Download the last version here:
http://www.mastersonics.com/documents/revision_of_the_particle-wave_dualism.pdf

Dynamical 3-Space
An Expanding Earth Mechanism

Extended Abstracts Book, p.5

Ettore Majorana Foundation and Centre for Scientific Culture
37th Interdisciplinary Workshop of the International School of Geophysics
Erice, Sicily, 4-9 October 2011

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Extended Abstract

In remarkably prescient work Hilgenberg in the 1930's (Scalera and Braun, 2003) proposed that the expansion of the Earth, driven by an increasing mass content, might be explained by a dynamical space that caused the generation of the new matter, and that the acceleration of gravity was nothing but the effect of an accelerated flow of that space. This was in direct conflict with the prevailing belief that space and time were integral aspects of a geometrical entity – spacetime and that gravitational acceleration was a spacetime curvature effect. This Einstein worldview had its origin in the supposedly null results from the 1887 Michelson-Morley interferometer experiment designed to detect the anisotropy of the speed of light, which would otherwise have indicated that a dynamical space, or an ether in the terminology of that era, was flowing through the detector Despite a few successes, the spacetime paradigm has faced an ever-increasing list of inexplicable failures, including the need for introducing dark matter and dark energy. However, in 2002 (Cahill and Kitto, 2003), it was discovered that the Michelson-Morley experiment was not null, and the published 1887 data, using a new calibration theory for the device, showed a space-speed to 500km/s (Fig.1). Miller's 1925-1926 experiment showed even more detailed confirming results (see also Fig.1), and recently Doppler shifts from spacecraft earth-flybys have confirmed those early results (Cahill, 2009), revealing the galactic speed of the solar system to be some 490km/s in the direction RA=4.3h, Dec=-75deg, and within 5deg of the direction that Miller had determined. As well, the flyby data also revealed an inflow of space into the Earth, confirming the expected speed of some 11km/s at the surface, as well as the sun inflow speed at 1AU of 42km/s. The Sun's surface inflow of 615km/s now follows from a new account of the deflection of starlight by the Sun (Cahill, 2009b). These developments change all of physics, and now provide a mechanism to explain the expanding earth, precisely along the lines suggested by Hilgenberg, and in accord with later developments (Carey, 1989, Scalera, 2003, Maxlow, 2005.

Dynamical 3-Space: Emergent Gravity

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Invited contribution to "Should the Laws of Gravitation be reconsidered". H´ector A. Mu´nera, ed. (Montreal: Apeiron 2011)

The laws of gravitation devised by Newton, and by Hilbert and Einstein, have failed many experimental and observational tests, namely the borehole g anomaly, flat rotation curves for spiral galaxies, supermassive blackhole mass spectrum, uniformly expanding universe, cosmic filaments, laboratory G measurements, galactic EM bending, precocious galaxy formation, … The response has been the introduction of the new epicycles: "dark matter", "dark energy", and others. To understand gravity, we must restart with the experimental discoveries by Galileo, and following a heuristic argument, we are led to a uniquely determined theory of a dynamical 3-space. That 3-space exists has been missing from the beginning of physics, although it was 1st directly detected by Michelson and Morley in 1887. Uniquely generalizing the quantum theory to include this dynamical 3-space we deduce the response of quantum matter and show that it results in a new account of gravity and explains the above anomalies and others. The dynamical theory for this 3-space involves G, which determines the dissipation rate of space by matter, and α, which experiments, and observation reveal to be the fine structure constant. For the 1st time, we have a comprehensive account of space and matter and their interaction - gravity.

Citations from [122], LING JUN WANG, UNIFICATION OF GRAVITATIONAL AND ELECTROMAGNETIC FORCES.

"Newton's law of gravitation is strikingly similar to Coulomb's law; (2) Newton's law differs from Coulomb's law, equally striking, by lacking a dynamic term dependent on the velocity of the gravitating source. As a result, Newton's law is essentially a static theory unable to describe the propagation of the interaction. This is the theoretical origin of the historical problem of action at a distance. Natural questions arise: Is the lacking of a dynamic term in Newton's law a real manifestation of the law of Nature, or a theoretical miss due to the weakness of the dynamic part of the gravitational force that escaped the detection by observational astronomers and experimental physicists? If the latter is the case, what kind of dynamic term should be added to Newton's law?

……………..

In the following sections, we will present a generalized theory of gravitation by adding a dynamic term similar to the Lorentz force. It turns out that the inclusion of a dynamic Lorentz force alone is enough to develop the whole theory of dynamic gravitation and the wave equations. It has been shown that the inverse square law is a result of Gauss' Law and Wang's Law. The newly discovered Wang's Law shows that the total linear momentum propagated into space is conserved.

……………………..

We generalize Newton's Law of gravitation to include a dynamic term like the Lorentz force in Eq. (1):

$$\mathbf{F} = k_1 \frac{qq'}{r^2}\left[\hat{r} + \frac{1}{c^2}\mathbf{v}' \times (\mathbf{v} \times \hat{r})\right], \qquad (1)$$

………………………

$$\mathbf{F} = -G \frac{mm'}{r^2}\left[\hat{r} + \frac{1}{c^2}\mathbf{v}' \times (\mathbf{v} \times \hat{r})\right], \qquad (2)$$

where v and v¢ are the velocities of the masses m and m¢, respectively".

…………………………

The background or natural texture and spatial matrix of our universe is also remarkably close to ether concepts.

The meaning and existence of ether within our universe is still not wholly addressed (solved or decided) in modern Physics. For A. Einstein Relativity theory, aether does not exist, or it is not necessary to consider it. It looks that aether is also not an essential item for electromagnetic theory (based on contemporary mainstream physics). Anyway, Maxwell created his equations very much based on analogies with real fluid-flow theory. Also, light, or electromagnetic waves speed (in different material media, including vacuum) strongly depends on measurable parameters like dielectric and magnetic permeability, since $c = 1/\sqrt{\varepsilon\mu}$, meaning that matter waves carrier media should always exist; - see familiar relations (3.7-1) and (3.7-2) from the third chapter of this book.

Modern Quantum theory is attributing, sporadically and not consistently, different matter-waves and particles properties, and vacuum energy stochastic events fluctuations, to an absolute vacuum (*whenever necessary and useful to explain something strange*, but not always, and not on the same way).

Nikola Tesla also was very much convinced, descriptive, and clear regarding ether existence (but in some way based only on his experiments, without mathematical modeling).

What is typical for different ether-understanding concepts should be that this is a fluidic state of matter, somewhat analogical to ideal gas states. Such an intrinsic, background matter state (or spatial matrix), should exist in an absolute vacuum, even without masses, particles, and waves (presenting material carrier of electromagnetic and other matter waves). Anyway, we still do not have a better and more precise explanation of what ether is, but we can read below how the same problematics is conceptualized and explained in contemporary physics, and get some challenging ideas:

Ether and gravitation [Citation from https://en.wikipedia.org/wiki/Aether_(classical_element)]

"Ether has been used in various gravitational theories as a medium to help explain gravitation and what causes it. It was used in one of Sir Isaac Newton's first published theories of gravity, *Philosophiæ Naturalis Principia Mathematica* (the *Principia*). He based the whole description of planetary motions on a theoretical law of dynamic interactions. He renounced standing attempts at accounting for this particular form of interaction between distant bodies by introducing a mechanism of propagation through an intervening medium.[24] He calls this intervening medium aether. In his aether model, Newton describes an aether as a medium that "flows" continually downward toward the Earth's surface and is partially absorbed and partially diffused. This "circulation" of aether is what he associated the force of gravity with to help explain the action of gravity in a non-mechanical fashion.[24] This theory described different aether densities, creating an aether density gradient. His theory also explains that aether was dense within objects and rare without them. As particles of denser aether interacted with the rare aether, they were attracted back to the dense aether much like cooling vapors of water are drawn back to each other to form water.[25] In the *Principia*, he attempts to explain the elasticity and movement of aether by relating aether to his static model of fluids. This elastic interaction is what caused the pull of gravity to take place, according to this early theory, and allowed an explanation for action at a distance instead of action through direct contact. Newton also explained this changing rarity and density of aether in his letter to Robert Boyle in 1679.[25] He illustrated aether and its field around objects in this letter as well and used this as a way to inform Robert Boyle about his theory.[26] Although Newton eventually changed his theory of gravitation to one involving force and the laws of motion, his starting point for the modern understanding and explanation of gravity came from his original aether model on gravitation.[27]"

The Dirac sea [Citation from https://en.wikipedia.org/wiki/Dirac_sea]
The **Dirac sea** is a theoretical model of the vacuum as an infinite sea of particles with negative energy. It was first postulated by the British physicist Paul Dirac in 1930[1] to explain the anomalous negative-energy quantum states predicted by the Dirac equation for relativistic electrons.[2] The positron, the antimatter counterpart of the electron, was initially conceived of like a hole in the Dirac sea, well before its experimental discovery in 1932.[nb 1]

Vacuum energy [Citation from https://en.wikipedia.org/wiki/Vacuum_energy]
Vacuum energy is underlying background energy that exists in space throughout the entire Universe. One contribution to the vacuum energy may be from virtual particles, which are thought to be particle pairs that blink into existence and then annihilate in a timespan too short to observe. Their behavior is codified in Heisenberg's energy-time uncertainty principle. Still, the exact effect of such fleeting bits of energy is difficult to quantify. The vacuum energy is a particular case of zero-point energy that relates to the quantum vacuum.[1]

The effects of vacuum energy can be experimentally observed in various phenomena such as spontaneous emission, the Casimir effect, and the Lamb shift, and are thought to influence the behavior of the Universe on cosmological scales. Using the upper limit of the cosmological constant, the vacuum energy of free space has been estimated to be 10^{-9} joules (10^{-2} ergs) per cubic meter.[2] However, in both quantum electrodynamics (QED) and stochastic electrodynamics (SED), consistency with the principle of Lorentz covariance and with the magnitude of the Planck constant requires it to have a much larger value of 10^{113} joules per cubic meter.[3][4] This vast discrepancy is known as the vacuum catastrophe.

Implications: Vacuum energy has many consequences. In 1948, Dutch physicists Hendrik B. G. Casimir and Dirk Polder predicted the existence of a tiny attractive force between closely placed metal plates due to resonances in the vacuum energy in the space between them. This is now known as the Casimir effect and has since been extensively experimentally verified. It is therefore believed that the vacuum energy is "real" in the same sense that more familiar conceptual objects such as electrons, magnetic fields, etc., are real. However, alternative explanations for the Casimir effect have since been proposed.[5]

Other predictions are harder to verify. Vacuum fluctuations are always created as particle-antiparticle pairs. The creation of these virtual particles near the event horizon of a black hole has been hypothesized by physicist Stephen Hawking to be a mechanism for the eventual "evaporation" of black holes.[6] If one of the pairs is pulled into the black hole before this, then the other particle becomes "real" and energy/mass is essentially radiated into space from the black hole. This loss is cumulative and could result in the black hole's disappearance over time. The time required is dependent on the mass of the black hole (the equations indicate that the smaller the black hole, the more rapidly it evaporates) but could be on the order of 10^{100} years for large solar-mass black holes.[6]

The vacuum energy also has important consequences for physical cosmology. General relativity predicts that energy is equivalent to mass, and therefore if the vacuum energy is "really there", it should exert a gravitational force. Essentially, non-zero vacuum energy is expected to contribute to the cosmological constant, which affects the expansion of the universe.[citation needed] In the special case of vacuum energy, general relativity stipulates that the gravitational field is proportional to ρ+3p (where ρ is the mass-energy density, and p is the pressure). Quantum theory of the vacuum further stipulates that the pressure of the zero-state vacuum energy is always negative and equal in magnitude to ρ. Thus, the total is ρ+3p = ρ-3ρ = -2ρ, a negative value. If indeed the vacuum ground state has non-zero energy, the calculation implies a repulsive gravitational field, giving rise to acceleration of the expansion of the universe,[citation needed]. However, the vacuum energy is mathematically infinite without renormalization, which is based on the assumption that we can only measure energy in a relative sense, which is not true if we can observe it indirectly via the cosmological constant.[citation needed]

The existence of vacuum energy is also sometimes used as a theoretical justification for the possibility of free-energy machines. It has been argued that due to the broken symmetry (in QED), free energy does not violate conservation of energy since the laws of thermodynamics only apply to equilibrium systems. However, the consensus amongst physicists is that this is unknown, as the nature of vacuum energy remains an unsolved problem.[7] In particular, the second law of thermodynamics is unaffected by the existence of vacuum energy.[citation needed] However, in Stochastic Electrodynamics, the energy density is taken to be a classical random noise wave field which consists of real electromagnetic noise waves propagating isotopically in all directions. The energy in such a wave field would seem to be accessible, e.g., with nothing more complicated than a directional coupler.[citation needed] The most obvious difficulty appears to be the spectral distribution of the energy, which compatibility with Lorentz invariance requires to take the form Kf^3, where K is a constant and f denotes frequency.[3][8] It follows that the energy and momentum flux in this wave field only becomes significant at extremely short wavelengths where directional coupler technology is currently lacking.[citation needed]

Since we know that (2.4-4) is correct force law, and since we have numerous indications that mass and surrounding velocity field are mutually coupled, we can (hypothetically) imagine that all masses in our Universe have certain (somewhat hidden), internal ***"electric-charges-weak-dipole-polarization"*** produced by global, holistic, and universal, always present orbital motions around dominant center of masses (or galactic center...). Mentioned dipole polarization is propagating or spreading from a local dominant center of mass (or galaxy center) over all surrounding and orbiting bodies and other astronomic objects, making that all the captured masses are mutually attracting (because of omnidirectional, multi-helical and properly aligned or arranged dipole polarizations of mutually opposed electromagnetic dipoles; -see more in Chapter 3. especially around equations (3.5-a), and in Chapter 8).

For instance, if $v_1 \cong v_2 \cong v = \frac{1}{\sqrt{\varepsilon\mu}} = c$, we could evaluate and exploit the validity of the following force relations between two masses that are electrically polarized like mutually opposed dipoles,

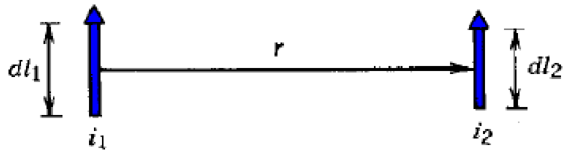
$$\text{Lim}\left(F_{1,2}\right)_{v\to c} = \text{Lim}\left[\frac{\mu}{4\pi}\frac{(q_1 v_1)(q_2 v_2)}{r^2}\right]_{v\to c} = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2} = G\frac{m_1 m_2}{r^2},$$

$$\left[\begin{array}{l} F_{1,2} = \frac{\mu}{4\pi}\frac{q_1 q_2 v^2}{r^2} = \frac{\mu}{4\pi}\frac{k^2 m_1 m_2 v^2}{r^2} = G\frac{m_1 m_2}{r^2} \Rightarrow \frac{\mu}{4\pi}k^2 v^2 = G \Rightarrow \\ v = \frac{2}{k}\sqrt{\frac{\pi G}{\mu}} = \frac{2}{k}\sqrt{\frac{\pi\varepsilon G}{\varepsilon\mu}} = c\frac{2\sqrt{\pi\varepsilon G}}{k} = \frac{1}{\sqrt{\varepsilon\mu}} = c \Rightarrow k = 2\sqrt{\pi\varepsilon G} = const., \\ q_{1,2} = km_{1,2} = m_{1,2}\sqrt{4\pi\varepsilon G} \Rightarrow q = km = m\sqrt{4\pi\varepsilon G} \Rightarrow \boxed{(qv) = m\sqrt{\frac{4\pi G}{\mu}}} \end{array}\right]. \quad (2.4\text{-}4.1)$$

Here we assume that ***one-side-electric-dipole-charge*** is directly proportional to the mass of the object in question, $q_{1,2} = km_{1,2}$, and this way we can equally use Newton or Coulomb force law for presenting the same attractive force between two masses (see more in Chapters 1, equations (1.10)-(1.16), and 3.). This is on some way (with a lot of creative imagination) insinuating or evoking memories about exotic concepts and speculations in relation to Nikola Tesla's ***Ether***, ***Radiant Energy*** flow and his ***Dynamic Gravity*** theory, since $v_1 \cong v_2 \cong v = \frac{1}{\sqrt{\varepsilon\mu}} = c$ could be the speed of some closely related electromagnetic waves (see more of similar elaborations around equations from (2.4-4) to (2.4-10)).

Another indicative $1/r^2$ force law applicable on familiar or analogically extended situations, showing that Coulomb and Newton laws are mutually equivalent, both having only an electromagnetic nature (see the picture below), here composed of two electric charges (creating two elementary current elements, i_1 and i_2) coincidently moving as mutually parallel, and in the same direction is,

$$\left|dF_{1,2}\right| = \frac{\mu}{4\pi}\frac{(i_1 dl_1)\cdot(i_2 dl_2)}{r^2} = \frac{1}{4\pi\varepsilon}\frac{dq_1\cdot dq_2}{r^2}(=) \\ = G\frac{dm_1\cdot dm_2}{r^2} \Rightarrow dm_1\cdot dm_2 = \frac{\mu}{4\pi G}i_1\cdot i_2\cdot dl_1\cdot dl_2 \quad (2.4\text{-}4.2)$$



Now, we can demonstrate the intrinsic relations between a certain mass, its electric charge, and its velocity (see more familiar elaborations in Chapters 1, equations (1.10) - (1.16), and 3.)). Again, like in (2.4-4.1), but now starting from (2.4-4.2), we will get the same result $\boxed{(qv) = m\sqrt{\frac{4\pi G}{\mu}}}$ as below (in (2.4-4.3)),

$$\left(dm_1 \cdot dm_2 = \frac{\mu}{4\pi G} i_1 \cdot i_2 \cdot dl_1 \cdot dl_2 = \frac{1}{4\pi\varepsilon} dq_1 \cdot dq_2\right) \Rightarrow$$
$$\text{for} \begin{pmatrix} m_1 = m_2 = m \\ l_1 = l_2 = l,\ i_1 = i_2 = i,\ q_1 = q_2 = q \end{pmatrix} \Rightarrow (dm)^2 = \frac{\mu}{4\pi G} i^2 \cdot (dl)^2$$
$$dm = \sqrt{\frac{\mu}{4\pi G}} idl = \sqrt{\frac{\mu}{4\pi G}} \frac{dq}{dt} dl = \sqrt{\frac{\mu}{4\pi G}} \frac{dl}{dt} dq =$$
$$= \sqrt{\frac{\mu}{4\pi G}} vdq \leq \sqrt{\frac{\mu}{4\pi G}} cdq = \frac{dq}{\sqrt{4\pi\varepsilon G}} \Rightarrow$$
$$\Rightarrow m = (qv)\sqrt{\frac{\mu}{4\pi G}} \leq \frac{q}{\sqrt{4\pi\varepsilon G}} \Rightarrow \boxed{(qv) = m\sqrt{\frac{4\pi G}{\mu}}}, G = \frac{\mu}{4\pi}\left(\frac{qv}{m}\right)^2 \leq \frac{1}{4\pi\varepsilon}\left(\frac{q}{m}\right)^2 \tag{2.4-4.3}$$
$$\Rightarrow \frac{q}{m} = \frac{1}{v}\sqrt{\frac{4\pi G}{\mu}} \geq \sqrt{4\pi\varepsilon G} = 8.616032252 \cdot 10^{-11} \left[\frac{C}{kg}\right],\ v \leq \frac{1}{\sqrt{\varepsilon\mu}} = c,$$
$$q \geq m \cdot \sqrt{4\pi\varepsilon G}.$$

Anyway, mass, and *electric charges direct proportionality* (as in (2.4-4.3), and in Chapter 8., under (8.62) and (8.63)) should also be the consequence of the total electric (or electromagnetic) charge conservation, as well as it should be (indicatively and intuitively) directly related to energy and mass conservation (of course not to understand everything literally). This is in the background regarding analogy between Newton and Coulomb force.

Citation from: https://energyeducation.ca/encyclopedia/Law_of_conservation_of_charge
Law of conservation of charge
Law of conservation of charge says that the net charge of an isolated system will always remain constant.[1] This means that any system that is not exchanging mass or energy with its surroundings will never have a different total charge at any two times. For example, if two objects in an isolated system have a net charge of zero, and one object exchanges one million electrons to the other, the object with the excess electrons will be negatively charged and the object with the reduced number of electrons will have a positive charge of the same magnitude. The total charge of the system has not and will never change.[1]
This concept is important for all nuclear reactions—alpha decay, beta decay, gamma decay, etc.— because it allows scientists to predict the composition of the final product in the reaction, shown in Figure 1.[2]
Charged particles are allowed to be created or destroyed, as long as the net charge before and after the creation/destruction stays the same. Therefore this must happen with oppositely charged pairs of matter and anti-matter.[1]

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Electromagnetic Analogy in Gravitation and Mass Interaction

As we know from electromagnetics, parallel wires carrying current in the same direction attract each other. This principle can be analogically extended to explain the attraction between two masses. Specifically, when two or more neighboring masses move in the same direction as part of a group or mechanical system, they will mutually attract if they share a common center-of-mass velocity.

Often, this center-of-mass velocity and related linear motion are not directly observed or measured because they are relative to an external, dominant inertial frame, such as the local galaxy center. As observers within the two-mass reference system, we might not perceive this external motion. However, this hidden or external motion induces internal electric and magnetic dipole polarizations, generating electrical currents in the same direction. These currents, in turn, create magnetic fields that mutually attract, as demonstrated in equations (2.4-4.1) through (2.4-4.3).

Consequently, all masses in our universe are engaged in a synchronized, holistic motion—oscillating, orbiting, and spinning around local or global centers of potential forces, cosmic masses, and galaxies.

The key conclusion here is that matter and mass in our universe are in perpetual motion, likely in a global helical, holistic rotation and spinning. Moreover, mass, electric charge, and associated spin effects or electromagnetic angular moments are always interrelated and mutually coupled. These electromagnetic and electromechanical couplings are exchangeable and reciprocally proportional, depending on the communication between matter waves, the energy involved, and various mechanical and electromagnetic moments, including dipole and multipole polarizations, as well as energy-mass (standing waves) packing and formatting (see Chapters 3 and 8 for more details).

Similar conclusions about the nature of matter will be revisited later in this chapter, particularly in the discussion surrounding equations (2.4-4) through (2.4-8), and in Chapter 5, around equations (5.2) and (5.2.2). These ideas implicitly support Nikola Tesla's visionary concepts of Ether, Radiant Energy, the Death Ray Gun, and Dynamic Gravity (see [97] and [117]). Tesla's experiments with high alternating voltages between specially assembled electrodes demonstrated the ability to evacuate gas (or the mass of gas content) from an open-ended glass tube, a process that relied on mass-electric-charge coupling like what is described in (2.4-4.3).

There is also a strong possibility that the Casimir effect represents a manifestation of Tesla's radiant energy flow. This effect could be the result of universal cosmic and atomic resonant synchronization between overlapping resonant states of matter (see [103] and [104]).

Citation [121]; -What is gravity? From Ralph Sansbury: *http://www.holoscience.com/wp/electric-gravity-in-an-electric-universe/ :*

"Gravity is due to radially oriented electrostatic dipoles inside the Earth's protons, neutrons, and electrons. [18] *The force between any two aligned electrostatic dipoles varies inversely as the fourth power of the distance between them and the combined force of similarly aligned electrostatic dipoles over a given surface is squared. The result is that the dipole-dipole force, which varies inversely as the fourth power between co-linear dipoles, becomes the familiar inverse square force of gravity for extended bodies. The gravitational and inertial response of matter can be seen to be due to the same cause. The puzzling extreme weakness of gravity (one thousand trillion trillion trillion trillion times less than the electrostatic force) is a measure of the minute distortion of subatomic particles in a gravitational field.*

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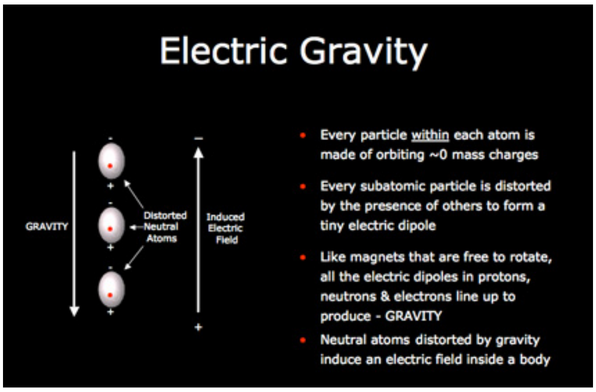

Celestial bodies are born electrically polarized from a plasma z-pinch or by core expulsion from a larger body.

The 2,000-fold difference in mass of the proton and neutron in the nucleus versus the electron means that gravity will maintain charge polarization by offsetting the core within each atom (as shown). The mass of a body is an electrical variable—just like a proton in a particle accelerator. Therefore, the so-called gravitational constant—'G' with the peculiar dimension $[L]^3/[M][T]^2$, is a variable! That is why 'G' is so difficult to pin down.

Antigravity?
Conducting metals will shield electric fields. However, the lack of movement of electrons in response to gravity explains why we cannot shield against gravity by simply standing on a metal sheet. As an electrical engineer wrote, "we [don't] have to worry about gravity affecting the electrons inside the wire leading to our coffee pot." [19] If gravity is an electric dipole force between subatomic particles, it is clear that the force "daisy chains" through matter regardless of whether it is conducting or non-conducting. Sansbury explains:

"...electrostatic dipoles within all atomic nuclei are very small, but all have a common orientation. Hence their effect on a conductive piece of metal is less to pull the free electrons in the metal to one side toward the center of the earth but to equally attract the similarly oriented electrostatic dipoles inside the nuclei and free electrons of the conductive piece of metal." [20]

This offers a clue to the reported 'gravity shielding' effects of a spinning, superconducting disk.[21] *Electrons in a superconductor exhibit a 'connectedness,' which means that their inertia is increased. Anything that interferes with the ability of the subatomic particles within the spinning disk to align their gravitationally induced dipoles with those of the earth will exhibit antigravity effects.*

Despite many experiments demonstrating antigravity effects, no one has been able to convince scientists attached to general relativity that they have been able to modify gravity. This seems to be a case of turning a blind eye to unwelcome evidence. Support for antigravity implicitly undermines Einstein's theory".[22]

Citation (below): See Ref. [121], Raymond HV Gallucci: *http://www.holoscience.com/wp/electric-gravity-in-an-electric-universe/*, "Electric Gravity in an Electric Universe." https://principia-scientific.org/electromagnetic-gravity-examination-of-the-electric-universe-theory/

One of the prime tenets of Electric Universe Theory is that electromagnetism dominates over gravity throughout the universe, given that 99% of all matter (ignoring the fictitious Dark Matter and Energy) is plasma and that electromagnetism is 39 orders of magnitude stronger than gravity.

Nonetheless, gravity is far from dismissed. In fact, Wal Thornhill, lead physicist for The Electric Universe, has developed a theory for gravity as a manifestation of an electromagnetic phenomenon that ever so slightly causes distortion within atoms such that a dipole is created that could account for gravitational force.

This paper summarizes Thornhill's theory and examines it mathematically, concluding that it is at least plausible. The Electric Universe (EU) theory postulates that gravity is just another manifestation of electromagnetism, albeit at an almost inconceivably lower force ($\sim 10^{-39}$ as strong).

This paper examines the EU conjecture about an electromagnetic basis for gravity based on simplified mathematical analysis for an idealized arrangement of three hydrogen atoms. Results suggest that the possibility of an electromagnetically induced distortion of a hydrogen atom to create an atomic dipole is at least plausible.

"Gravity is due to radially oriented electrostatic dipoles inside the Earth's protons, neutrons, and electrons. The force between any two aligned electrostatic dipoles varies inversely as the fourth power of the distance between them and the combined force of similarly aligned electrostatic dipoles over a given surface is squared. The result is that the dipole-dipole force, which varies inversely as the fourth power between co-linear dipoles, becomes the familiar inverse square force of gravity for extended bodies. The gravitational and inertial response of matter can be seen to be due to the same cause. The puzzling extreme weakness of gravity (one thousand trillion trillion, trillion times less than the electrostatic force) is a measure of the minute distortion of subatomic particles in a gravitational field. Celestial bodies are born electrically polarized from a plasma z-pinch or by core expulsion from a larger body. The 2,000-fold difference in mass of the proton and neutron in the nucleus versus the electron means that gravity will maintain charge polarization by offsetting the nucleus within each atom (as shown). The mass of a body is an electrical variable — just like a proton in a particle accelerator. Therefore, the so-called gravitational constant — 'G' with the peculiar dimension $[L]^3/[M][T]^2$, is a variable! That is why 'G' is so difficult to pin down."

This exercise attempted to interject some mathematics, greatly simplified, into the paradigm of the EU theory that gravity can be attributed to an electromagnetic effect, albeit almost inconceivably smaller, due to the distortion of atoms by their neighbors into electric dipoles. While we have not attempted to address the mathematics that would be involved in explaining the 10^{39} factor difference between the respective strengths of these forces, the possibility of an electromagnetically induced distortion to create an atomic dipole appears at least plausible.

For gravitation, rotation and mechanics related fields and forces the most important are linear moment p and angular moment L. Surprisingly, we do not find an analogical indication that a static mass m should be the primary and unique source of gravitation, but vibrating mass should produce effects of Gravitation (see more in the first chapter about analogies). The practical predictive meaning of such analogical revelations is that I. Newton and A. Einstein theories about Gravitation should be one day significantly updated. The other brainstorming insight here is that electric charges (or electrons and protons) should anyway be kind of dynamic, motional energy states (like mechanical and electromagnetic moments), since all other mutually analogical mechanical and electromagnetic moments and flux entities are also kind of moving or dynamic states (see T.1.3 in the first chapter). In Physics, presently we still wrongly consider electric charges as stable, or fixed and static parameters (as number of Coulombs). Consequently, we could expect certain continuous electromagnetic energy-exchange (or natural flow of N. Tesla Radiant energy) between any of positive and negative electric charges, or between conveniently aligned electromagnetic dipoles, this way creating necessary background for explanation of Gravitation, especially if we revitalize the existence and meaning of ether as fine particles fluid, which is the carrier of electromagnetic waves and energy.

Another perspective emphasizes the relationship between gravitation and the internal, spatially distributed magnetic moments within masses. Every macro mass, including its constituents such as molecules, atoms, elementary particles, and interatomic states, contains numerous spinning and magnetic moment states that are internally distributed throughout its volume. While electric and electrostatic charges tend to migrate to the external surface of mass, the internal magnetic entities remain confined within the mass structure.

Considering that all masses in our universe are rotating on a global scale—around galactic and other dominant mass centers—these motions are inherently accelerated. This suggests that the electromagnetic moments, or elementary magnetic dipoles within these masses, would become somewhat organized or aligned. Although this alignment may be weak on average, these magnetic elements still experience a Coulomb-like force, analogous to the forces described in electrostatics and Newtonian gravity.

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This leads to the possibility that gravitational force, observed between seemingly neutral or non-charged masses, may be predominantly linked to a very weak Coulomb force between masses with internally distributed magnetic elements. This interpretation suggests that gravitation might have been mischaracterized as a distinct natural force, when it could instead be related to these subtle electromagnetic interactions (see more in T.2.2-1, T.2.8., [33], and [36]).

Citation from the European Space Agency. "Anti-gravity Effect? Gravitational Equivalent of A Magnetic Field Measured In Lab." ScienceDaily. ScienceDaily, 26 March 2006. <www.sciencedaily.com/releases/2006/03/060325232140.htm>

Just as a moving electrical charge creates a magnetic field, a moving mass generates a gravitomagnetic field. According to Einstein's Theory of General Relativity, the effect is virtually negligible. However, Martin Tajmar, ARC Seibersdorf Research GmbH, Austria; Clovis de Matos, ESA-HQ, Paris; and colleagues have measured the effect in a laboratory.

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It demonstrates that a superconductive gyroscope can generate a powerful gravitomagnetic field, and is, therefore, the gravitational counterpart of the magnetic coil. Depending on further confirmation, this effect could form the basis for a new technological domain, which would have numerous applications in space and other high-tech sectors" says de Matos. Although just 100 millionths of the acceleration due to the Earth's gravitational field, the measured field is a surprising one hundred million trillion times larger than Einstein's General Relativity predicts. Initially, the researchers were reluctant to believe their own results. ....

Moving electric charges generate electrical currents and magnetic fields. Additionally, electric and magnetic entities with spinning and dipole (or multi-pole) moments are inherently present within the internal structure of every macro mass. This suggests that the mass of an object is directly proportional to its internal content of electromagnetic entities, including electric and magnetic micro-domains, fluxes, and charges.

From this, we can intuitively justify that the product of masses in Newton's law of gravitation is directly proportional to the product of the involved electric charges or the magnetic field fluxes of internally distributed micro-magnetic domains. However, it's important to consider that mass and electric charges are not directly analogous. Instead, mass in motion and electric charges share an analogy. Therefore, to explain gravitational force using electromagnetic concepts, we must consider the background, global, and holistic mass rotation or mass-energy flow, including the oscillations and resonance effects of standing waves.

This analogy helps clarify how electromagnetic Coulomb force can be involved in explaining gravitational force. William Hooper's conclusions, as discussed in [112], align with these ideas, highlighting the natural diversity of electric and magnetic fields related to masses in relative motion. His work effectively shows that gravitation may have an electromagnetic origin (see more in [89]).

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Citation from: (4) Quora , *Viktor T. Toth, IT pro, part-time physicist 2y*
Can you explain dark matter to a non-physicist?

“Galaxies are collections of stars that are held together by their mutual gravity.

However, galaxies spin. And they spin too fast. When we calculate how much gravity is there, given the stuff that we see, there just isn’t enough. Newton’s law of gravitation tells us that there is not enough gravitational force to keep the stars together. Galaxies should fly apart.

They don't.

Unless we got everything completely wrong, there can be two fundamental reasons for this: Either we misunderstand the law of gravity, or there is more matter in a galaxy than the eye can see.

We are very reluctant to conclude that we misunderstand the law of gravity, because it works so well everywhere else. It is more likely, then, that there is simply stuff that we do not see.

We call this stuff "dark matter" (dark, because we do not see it; but it really should be called invisible or transparent matter, as it doesn't exactly cast a shadow either, it has no effect on light whatsoever.) And we keep looking for it, and theorists keep inventing new theories as to what this dark matter might be.

So far, we have not been able to establish the existence of dark matter independently from their gravity. So, the door is still open for the alternative, modified theories of gravity. On the other hand, it so happens that presuming the existence of dark matter also improves our mathematical models of the cosmos. That is a strong point in favor of the dark matter idea.

But we won't know until and unless we have direct observational evidence or, alternatively, until we have a convincing modified theory of gravity, confirmed through testable predictions".

Citation: Dark matter. *From Wikipedia, the free encyclopedia; -* https://en.wikipedia.org/wiki/Dark_matter
*"**Dark matter** is a theorized form of matter that is thought to account for approximately 80% of the matter in the universe, and about a quarter of its total energy density. The majority of dark matter is thought to be non-baryonic in nature, possibly being composed of some as-yet-undiscovered subatomic particles.[note 1] Dark matter has not been directly observed, but its presence is implied in a variety of astrophysical observations, including gravitational effects that cannot be explained unless more matter is present than can be seen. For this reason, most experts think dark matter to be ubiquitous in the universe and to have had a strong influence on its structure and evolution. The name dark matter refers to the fact that it does not appear to interact with visible electromagnetic radiation, such as light, and is thus invisible (or 'dark') to the entire electromagnetic spectrum, making it extremely difficult to detect using normal astronomical equipment.[1] The primary evidence for dark matter is that calculations show that many galaxies would fly apart instead of rotating, or would not have formed or move as they do if they did not contain a large amount of dark matter.[2] Other lines of evidence include observations in gravitational lensing,[3] from the cosmic microwave background, from astronomical observations of the observable universe's current structure, from the formation and evolution of galaxies, from the mass location during galactic collisions,[4] and from the motion of galaxies within galaxy clusters".*

....

Here are more of imaginative speculations (just to explore different ideas): For instance, based on assumptions and concepts from (2.4-1) - (2.4-5), (2.4-4.1), (2.4-4.2) and (2.4-4.3), we could exercise that within spiral and rotating (or spinning) galaxies, rotating messes are getting additionally electrically or electromagnetically charged and polarized (internally and externally), meaning that involved central force between two masses (considered as a binary system) would have one force component being the Newton force of gravitation, and an additional force component being the Coulomb type of attraction, based on involved (free-standing, non-compensated) electric charges (or dipoles), such as,

$$\mathbf{F_{1,2} = G\frac{m_1m_2}{r^2} + \frac{1}{4\pi\varepsilon}\frac{q_1q_2}{r^2}}$$

Since, based on the system of analogies established in the first chapter of this book, electric charges are analogue to linear mechanical moments, we will have,

$$\begin{aligned}&\mathbf{F_{1,2} = G\frac{m_1m_2}{r^2} + g\frac{m_1v_1 \cdot m_2v_2}{r^2} = G\frac{m_1m_2}{r^2} + g\frac{m_1\omega_1r_1 \cdot m_2\omega_2r_2}{r^2} \cong}\\ &\mathbf{\cong G\frac{m_1m_2}{r^2} + g\frac{m_1m_2\omega^2r^2}{r^2} \cong G\frac{m_1m_2}{r^2} + g\omega^2m_1m_2 \cong m_1m_2\left(\frac{G}{r^2} + g\omega^2\right) = G\frac{m_1m_2}{r^2}(1 + \frac{g\omega^2r^2}{G}) =}\\ &\mathbf{= G\frac{m_1m_2}{r^2}(1 + \frac{gv^2}{G}),\ G,\ g\ (=)\,constants.}\end{aligned}$$

$$\left[\begin{array}{l} v=\frac{2}{k}\sqrt{\frac{\pi G}{\mu}}=\frac{2}{k}\sqrt{\frac{\pi\varepsilon G}{\varepsilon\mu}}=c\frac{2\sqrt{\pi\varepsilon G}}{k}=\frac{1}{\sqrt{\varepsilon\mu}}=c\Rightarrow k=2\sqrt{\pi\varepsilon G}=\text{const.}, \\ q_{1,2}=km_{1,2}=m_{1,2}\sqrt{4\pi\varepsilon G}\Rightarrow q=km=m\sqrt{4\pi\varepsilon G}\Rightarrow \boxed{(qv)=m\sqrt{\frac{4\pi G}{\mu}}} \\ \omega_1 r_1\cdot\omega_2 r_2\cong\omega^2 r^2=v^2,\ v=\omega r \end{array}\right].$$

This new rotation-dependent force $F_{1,2}\cong G\frac{m_1m_2}{r^2}(1+\frac{gv^2}{G})$, for small rotational velocities ω (or small tangential velocities v), will be equivalent to the Newtonian gravitational force. By conceptualizing it this way, there is no need to invoke or assume the existence of dark matter, dark mass, or dark energy. Instead of relying on such speculative concepts, it would be more productive to deepen our understanding of the electromagnetic phenomena involved (for further details, see the elaborations in Chapters 1 and 3, particularly equations (1.10) - (1.16)). This approach also considers the relevant interactions within mass-energy formation and distribution.

While the proposed ideas could be further refined and enriched, the core concept—explaining central forces (such as gravitational or Coulomb-Newton forces) through electromagnetic reasons, coupled with linear and angular momentum—is already well-established in a simplified and indicative manner.

Moreover, this concept introduces the idea that a stable solar or planetary system can be represented by an effective central mass, where all rotational, orbital energies, and rest masses of the planets $m_{planets}$ are combined into a new effective solar mass $E_{k-planets}+E_{0-planets}=\sum_{(1,n)}E_{k-i}+\sum_{(1,n)}E_{0-i}\cong m_{planets}\cdot c^2$. In this formulation, the effective solar mass $M_s^*=M_s+m_{planets}$, where M_s is the original solar mass before accounting for the planets orbiting around it.

This concept can be similarly applied to any system of particles or masses in collective motion, where the masses have linear motion components (or linear kinetic energies) as well as rotational, spinning, and angular energies. All these energy components can be effectively transformed into a central mass, which combines both the linear motion and the resulting total angular momentum or spin.

In other words, we implicitly introduce the idea that all masses in the universe are created as superpositions of rotating, spinning, and motional “mass-energy-moment” states (including electromagnetic contributions). The most elegant way to analyze such systems is by using four-vectors in Minkowski space, which also sheds light on the meaning of de Broglie matter waves in relation to resulting spinning moments.

Based on electromechanical mobility analogies, as discussed in the first chapter, we can infer that the concept of a real and active gravitational charge is not synonymous with an electromagnetically neutral rest-mass. Instead, this charge represents a mass that is dynamically charged with linear and angular momentum, electric charges, magnetic fluxes, or relevant electromagnetic dipoles $m=m(p,L_s,q,\phi)$.

The general, analogous, and more comprehensive interpretation of Newton’s law of gravitation has already been outlined in equations (2.3), (2.4), (2.4a), (2.4b), and others. Purely *electromagnetic explanations of gravitation are inadequate, as there is no known electromagnetic shielding that can stop, neutralize, or cancel gravity.*

Thus, the challenge remains to develop a practical, elegant, and easily applicable formulation of an updated law of gravitation, one that surpasses Newton's theory. It is also evident that Einstein's theory of relativity requires significant enhancement to address these concepts fully.

"https://cosmosmagazine.com/science/introducing-the-amazing-concept-of-gravito-electromagnetism/ , 30 August 2021/, Robyn Arianrhod, **Amazing gravito-electromagnetism!**

How can mathematics analogies shed light on reality?

Originally published by Cosmos as Amazing gravito-electromagnetism!

Today this so-called "gravito-electromagnetism", or GEM for short, is generally treated mathematically via the "weak field" approximation to the full GR equations – simpler versions that work well in weak fields such as that of the earth.

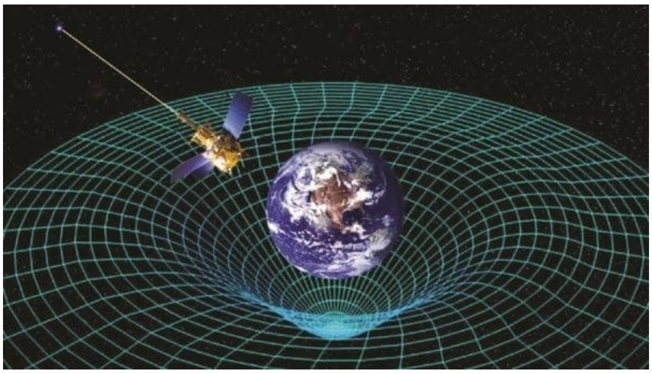
It turns out that the mathematics of weak fields includes quantities satisfying equations that look remarkably similar to Maxwell's. The "gravito-electric" part can be readily identified with the everyday Newtonian downward force that keeps us anchored to the earth. The "gravito-magnetic" part, however, is something entirely unfamiliar – a new force apparently due to the rotation of the earth (or any large mass).

It's analogous to the way a spinning electron produces a magnetic field via electromagnetic induction, except that mathematically, a massive spinning object mathematically "induces" a "dragging" of space-time itself – as if space-time were like a viscous fluid that's dragged around a rotating ball. (Einstein first identified "frame-dragging", a consequence of general relativity elaborated by Lense and Thirring.)

But how far can such mathematical analogies be pushed?

Is "gravito-magnetic induction" real? If it is, it should show up as a tiny wobble in the orbit of satellites, and – thanks also to the "geodetic" effect, the curving of space-time by matter – as a change in the direction of the axis of an orbiting gyroscope. (The latter is analogous to the way a magnetic field generated by an electric current change the orientation of a magnetic dipole.)

Finally, after a century of speculation, answers are unfolding. Independent results from several satellite missions – notably Gravity Probe B, LAGEOS, LARES, and GRACE – have confirmed the earth's geodetic and frame-dragging effects to varying degrees of precision. For frame-dragging, the best agreement with GR has been within 0.2%, with an accuracy of 5%, but astronomers expect that a new satellite (LARES 2), to be launched in 2022, will, with data from LAGEOS, give an accuracy of 0.2%.



Results from satellite missions such as Gravity Probe B have confirmed the Earth's geodetic and frame-dragging effects. Credit: Gravity Probe B team/Stanford/NASA

More accurate results will provide more stringent tests of GR, but astrophysicists have already taken gravito-magnetism on board. For instance, it suggests a mechanism to explain the mysterious jets of gas that have been observed spewing out of quasars and active galactic nuclei. Rotating supermassive black

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holes at the heart of these cosmic powerhouses would produce enormous frame-dragging and geodetic effects. A resulting gravito-magnetic field analogous to the magnetic field surrounding the two poles of a magnet would explain the alignment of the jets with the source's north-south axis of rotation." ....

Author's comments:

Electromagnetic Forces as an Alternative to Dark Matter in Explaining Galactic Structure

When considering the forces that maintain the structural integrity of rotating galaxies, the gravitational attraction between masses is often assumed to be the dominant force. This assumption leads to the postulation of dark matter, an invisible substance, to account for the hypothetically missing gravitational force needed to balance the centrifugal forces of rotation. However, an alternative explanation could be that gravitation is a manifestation of central-force electromagnetic interactions. These interactions are related to electromagnetic moments and dipole polarizations, which arise due to the global, holistic rotation of the universe.

Electromagnetic forces, such as Coulomb and Lorentz forces, are significantly stronger than gravitational forces. This strength could provide the missing central force component needed to explain the observed structural integrity of rotating spiral galaxies. In this view, the additional central attractive force is not derived from an undetectable and invisible dark energy or dark mass. Instead, it originates from conventional electromagnetic forces, specifically, Coulomb and Lorentz forces, acting within the dynamic configuration and spin of galaxies. These forces create large-scale electric circuits, generated by the galaxy's rotation and other kinematic factors, which supply the necessary central force (see more in [121], Raymond HV Gallucci, *Electromagnetic Gravity? Examination of the Electric Universe Theory*).

The core idea here is that principles governing static and moving electric charges, as well as magnetic and electric dipoles (such as Coulomb's law, Lorentz force, and induction laws), can be analogically extended to interactions between masses and their respective moments. This suggests that gravitation fundamentally has an electromagnetic origin. However, it's important to note that mass and electric charge are not directly interchangeable; instead, electric charge is comparable to linear, angular, and magnetic moments. In this context, electric charges are not static entities, but dynamic ones closely linked to the spinning, orbital, or angular moments of masses.

Thus, macro masses, or aggregations of atoms and molecules, are dynamic electromagnetic entities with magnetic moment properties, influenced by their rotational and spinning characteristics. These properties lead to electromagnetic polarizations, inductions, and currents that manifest as gravitation (see Chapters 3 and 8 for further discussion). This concept was explored by Jean de Climont in [117].

In modeling spiral galaxies, they can be approximated as rotating fluids or as spinning masses submerged within a certain galactic fluidic matter, or aether. This approach helps to explain the observed deviations in gravitational force when compared to the predictions of Kepler, Newton, and Einstein's General Relativity theory. The following pictures provide further support and analogical insights into cosmic vortex models.






Citation from [117¨]: "Fluids with spin have been proposed by M. Mathisson in 1937 then by J.P. Vigier in 1958, among others, to explain the Barnett effect. The spin density was assumed to be related to the orbital angular momentum in a rotating fluid such as a gas. As a consequence that spin was not randomly distributed. However, it is possible to envisage a fluid composed of tiny particles with an angular momentum in addition to their linear moments, both being randomly distributed in intensity as well as in direction. This kind of fluid has some very specific properties that cannot be found in gases and liquid. But if such a fluid is assumed to fill Space, the galaxy curves could be explained very simply. The orbital speeds of stars in galaxies do not conform to Kepler's laws. A first theory to solve the problem was based on the existence of an invisible mass in the halo of galaxies: the dark matter. Another proposal was the MOND theory involving a modification of Newton's law at large distances. These theories are essentially dedicated to the problem of galaxies. The curves of galaxies are only a special case of the fluid flow that explains gravitation. This very same fluid with angular moments has thus a transversal property and can be used for light and for all waves of the same celerity, as in the Cartesian approach".

…………………………………..

We should consider that cosmic masses and other matter states experience a range of interactions, including "cosmic-fluidic-friction," electromagnetic fields and forces, as well as various inductions and polarizations. These interactions also involve mutual effects such as the induction of electromagnetic currents and voltages, as well as the activity of Lorentz forces. Additionally, any spinning, orbital, rotating, or angular motion, whether circular or elliptical—is associated with intrinsic magnetic and spin moments.

These spinning and rotating motions lead to structural oscillations and resonances in the universe, creating electromagnetic and mechanical resonant states and standing waves among the involved mass-energy moments. This reinforces the idea that gravitation may have underlying connections to electromagnetism. The synchronization of universal resonant states further supports this perspective.

Moreover, in practical material media, high-power mechanical, ultrasonic, or acoustic energy can be generated and transferred through various signal-modulation techniques applied to laser beams and dynamic plasma states. Lasers and plasma states can act as carriers for lower frequency mechanical vibrations or signals. For more detailed information, refer to Chapter 10, sections 10.2-2.4, and consult the literature references from [133] to [139].

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2.2.1. WHAT THE GRAVITATION REALLY IS

In this book, we explore the concept, introduced in the first chapter through a Mobility system of analogies, that static and electromagnetically neutral masses alone are insufficient to create full scale gravitational fields and forces. Instead, gravitational effects arise from the interactions of oscillating, rotating, and electromagnetically and mechanically coupled masses. These masses, with their internal, coupled and synchronized spinning states, electromagnetic charges and dipoles, electromechanical moments, and as matter waves associated linear and angular mechanical moments, are the true sources of gravitation.

In essence, all atoms, masses, and astronomical objects in our universe interact by exchanging synchronized matter-waves and balancing their electromagnetic and mechanical moments. These interactions are mediated by fields and forces both within and extending beyond atoms. The effects of gravitational attraction manifest in spatial nodal zones within standing matter-wave formations between masses, where masses naturally stabilize on stable orbits. Linear, angular, oscillatory, spinning, and orbital motions (of involved masses or energy states) are inherently coupled, synchronized, and influenced by surrounding electric and magnetic fields, dipoles, and mechanical moments. Consequently, linear and angular forces and torques can interfere with or superimpose upon the resulting gravitational force.

This perspective allows us to alternatively explain gravitational attraction as a mass-agglomerating force around nodal zones of standing matter waves—analogous to resonant effects seen in acoustic levitation techniques (refer to [150] and [151]). Practically, masses, particles, matter-states (or self-stabilized matter-wave formations) composed of mechanical and electromagnetic "energy-moments states" are anchored to nodal zones of surrounding cosmic, standing matter waves. These standing-wave formations around and between masses create the effects of gravitational fields and forces (see Konstantin Meyl's work in [99]).

We can mathematically formalize various natural forces, including gravitation, in ways consistent with the standing wave concept, as detailed in Chapter 10, section "10.02 Meaning of Natural Forces."

Next, let's address how we can influence the gravitational force exerted by a large object. When a smaller object is within the gravitational field of a much larger object (e.g., a planet and local Sun), we can affect the gravitational force experienced by the smaller object by applying non-uniform, accelerated motions, whether linear, angular, rotational, or spinning. Such motions (of smaller masses) can positively or negatively impact the gravitational force acting from a big mass on the smaller mass or object.

Inertial, stable, uniform, and stationary motions, including stable periodic motions, are analogous to linear and angular, mechanical and electromagnetic motions. While Newtonian mechanics provides extensive insights into linear inertial motions, similar principles apply to stable, periodic, and angular motions. These stable motions do not perturb the existing gravitational force but can create new, dynamically active forces and torques when some accelerations are introduced.

For instance, consider a gyroscopic system: a stable, uniform spinning motion of a disk tends to maintain its spatial position and orientation relative to a massive object. By combining this with smaller spinning disks fixed to a larger central disk, we can achieve a stabilized spatial configuration. By modulating the rotational speeds of the smaller disks, we can produce controllable linear motions of the entire system. This concept can be applied to create moving and flying objects based on gravitational technology (though this is a simplified explanation).

The universally valid laws of "Action equals Reaction," electromagnetic induction laws, and inertial phenomena are interrelated and analogical across mechanics, gravitation, electromagnetism, and other dynamic systems involving mass-energy moments, currents, voltages, and forces (as discussed in the first chapter). There is significant potential to further develop these concepts, which offer insights into understanding, explaining, and manipulating gravitation.

Similarly, electromagnetic, electromechanical, electrostatic, and magnetic conditions associated with spinning objects affect gravitational interactions. Coulomb and Lorentz forces, which can be attractive or repulsive, combine with the existing gravitational force. Dynamic, accelerated electromagnetic conditions can also influence gravitational field (see Chapter 3 for more on induction laws).

In practical terms, vehicles, airplanes, rockets, electromotors, and flying objects work against gravitational force. Any transient driving-phase involves linear forces and torques to alter the gravitational force until achieving a stable, uniform motion. Understanding inertial motions—including rotations, orbiting, and spinning—is crucial for this analysis. See more about inertia in Chapters 1, 4.2 and 10.

If we have electromagnetically, or electromechanically created, non-uniform, startup conditions, related to the motion of the small object, this way we can create new linear force and/or torque (as vector elements) that are superimposing and interacting with the preexisting gravitational force of a big object. *See supporting background about relations between linear and rotational or spinning moments, and between linear force and torque (of the same motional object) around equations (2.4-5.1), (2,5.1-7), (2.9.1), (2.9.5-7), (2.9.5-8), (2.9.5-9), and T.2.5., from this chapter, and in Chapter 4.1, starting from (4.3), (4.3-0)-d,e,f,g,..,t, (4.3-1), T.4.2.1, Fig.4.1.2…, and later, such as,* $\mathbf{pv = L\omega}$, $\mathbf{p = (\omega / v) \cdot L}$, $\mathbf{F = dp / dt}$, $\mathbf{\tau = dL / dt}$, $\mathbf{p \cdot (dv / dt) + v \cdot F = L \cdot (d\omega / dt) + \omega \cdot \tau}$, $\mathbf{v \cdot F = \omega \cdot \tau}$, $\mathbf{pdv = Ld\omega}$. *To explain this situation better let's start with certain particle in linear motion, having kinetic energy,*

$$\mathbf{E_k = \tilde{E} = \frac{pv}{1+\sqrt{1-v^2/c^2}}\Bigg|_{v<<c} \cong \frac{pv}{2} = \frac{mv^2}{2}} . \quad (2.4\text{-}4.4)$$

If we now replace the same kinetic energy (or state of particle motion) with an equivalent spinning energy (of the same mass), we will have,

$$\mathbf{E_k = \tilde{E} = \frac{pv}{1+\sqrt{1-v^2/c^2}} = \frac{L_s\omega_s}{1+\sqrt{1-v^2/c^2}}\Bigg|_{v<<c} \cong \frac{L_s\omega_s}{2} = \frac{J\omega_s^2}{2} \Rightarrow L_s\omega_s = pv}, \quad (2.4\text{-}4.5)$$

where $\mathbf{L_s, \omega_s}$ *are particle spinning moment and angular spinning velocity, and* $\mathbf{p, v}$ *are particle linear moment and linear velocity (this way effectively visualizing certain spiral or helix-line motion around the path or line of linear particle propagation). Such spinning moment and spinning velocity could be externally visible and measurable parameters, and here we can also conceptualize that mass m internally has many atomic micro-spinning and orbiting states that are under favorable conditions mutually superimposing, orienting, and aligning (or synchronizing). Of course, spinning is introduced here mathematically, but we will upgrade such concept to the hypothetical statement saying that elements of matter-waves, radiant energy, spinning and vortices* *emanating from atoms would (on some wave-particle duality way) really appear around any motional mass or contribute to its internal mass or energy. This way conceptualizing, we get,*

$$\mathbf{pv = L_s\omega_s \Rightarrow pdv + vdp = \omega_s dL_s + L_s d\omega_s \Leftrightarrow p\frac{dv}{dt} + v\frac{dp}{dt} = L_s\frac{d\omega_s}{dt} + \omega_s\frac{dL_s}{dt} \Rightarrow}$$

$$\mathbf{\Rightarrow \begin{bmatrix} p\frac{dv}{dt} + vF = L_s\frac{d\omega_s}{dt} + \omega_s\tau_s \\ F = \frac{dp}{dt}, \tau_s = \frac{dL_s}{dt} \end{bmatrix} \Rightarrow \begin{bmatrix} p\frac{dv}{dt} = L_s\frac{d\omega_s}{dt}, \\ vF = \omega_s\tau_s = \\ = F\frac{dx}{dt} = \tau_s\frac{d\alpha_s}{dt} \end{bmatrix}.} \tag{2.4-4.6}$$

The hypothetical consequence here is that if certain linear force is applied on a moving particle, such particle can also get some angular spinning speed ω_s, *and certain spinning torque* τ_s, *aligned with the particle velocity* $\mathbf{v}$ *and with mentioned linear force* $\mathbf{F}$ (and vice – versa). *Now we can generate the following relations,*

$$\mathbf{\frac{vdp}{pdv} = \frac{\omega_s dL_s}{L_s d\omega_s} \Rightarrow \left\{ \frac{\left(\frac{dp}{p}\right)}{\left(\frac{dv}{v}\right)} = \frac{\left(\frac{dL_s}{L_s}\right)}{\left(\frac{d\omega_s}{\omega_s}\right)} \Rightarrow \frac{\ln\left|\frac{p}{p_0}\right|}{\ln\left|\frac{v}{v_0}\right|} = \frac{\ln\left|\frac{L_s}{L_{s0}}\right|}{\ln\left|\frac{\omega_s}{\omega_{s0}}\right|} \right\} \Rightarrow}$$

$$\mathbf{\Rightarrow \begin{bmatrix} \left|\frac{p}{p_0}\right| = \left|\frac{L_s}{L_{s0}}\right| \\ \left|\frac{v}{v_0}\right| = \left|\frac{\omega_s}{\omega_{s0}}\right| \end{bmatrix} \Rightarrow \begin{bmatrix} p = \left|\frac{p_0}{L_{s0}}\right| L_s \\ v = \left|\frac{v_0}{\omega_{s0}}\right| \omega_s \end{bmatrix}, [v_0, \omega_{s0}, p_0, L_{s0}] (=) \begin{bmatrix} \text{constants presenting} \\ \text{initial conditions} \end{bmatrix}.} \tag{2.4-4.7}$$

The most important, and (gravitation related) predictive relations, here are,

$$\mathbf{p = \left|\frac{p_0}{L_{s0}}\right| L_s, v = \left|\frac{v_0}{\omega_{s0}}\right| \omega_s, vF = \omega_s\tau_s, F = \left|\frac{p_0}{L_{s0}}\right| \tau_s,} \tag{2.4-4.8}$$

indicating conditions how the force of gravitation can be influenced and connected with linear and angular forces, either using a reactive rocket drive, or effects created with accelerated spinning disks. Of course, in the background of just drawn conclusions are laws of conservation of linear and angular moments, including total energy conservation (synchronously, instantly, and coincidently applicable to any motion, meaning that linear and angular motions, and force of gravitation, should also be interdependent and connected in a similar way). In a case of an ideal, uniform circular or orbiting motion, with orbiting radius R, and orbiting angular velocity $\omega = v / R$, *we can on a similar way consider the orbital moment conservation to (2.4-4.4) - (2.4-4.6), producing* $\mathbf{L_s\omega_s = pv = p\omega R = m\omega^2R^2}$. *In other cases of combined (but not ideal circular) linear and angular motions, it will be necessary to apply a total angular momentum conservation, which would be a sum of mechanical spinning and orbital moments (using methods of Lagrangian and Hamiltonian Mechanics). Cases of entanglement relations or couplings and synchronizations should be causally linked to the same problematic (as cases of couplings between linear and angular motions; -see more in [36], and in Chapter 10, under PWDC).* ***In Mechanics we know for linear and angular inertial motions (regardless of whether Newton considered only linear inertial motion). Gravitation is related to the "force-field" phenomenology existing between mentioned inertial motions that are***

mutually coupling and transforming based on effects of wave-particle duality and associated electromagnetic complexity *(see more about inertia in Chapters 1, 4.2 and 10)*.
……………………..

We can now specify conditions and properties of groups of atoms, masses, or objects that are relevant for the existence of gravitational force. Experimental facts, theoretical concepts, and imaginative ideas that contribute to our understanding of gravitation include:

1. Nikola Tesla's Contributions:
Nikola Tesla, as documented in [97], held numerous patents for devices demonstrating remote, wireless electromagnetic couplings and resonance effects between tuned electric or electromagnetic circuits. For example, Tesla's wireless energy transfer system, achieved through his "Magnifying Transformer," involved coupled resonators operating in both time and spatial domains. Many researchers have since repeated and refined Tesla's experiments, exploring electromagnetic and electromechanical couplings and the hypothetical ether as a coupling medium between masses (see [98], [99], [144]). Tesla's ideas were influenced by R. Boskovic's concept of "Universal Natural Force," which links interatomic and subatomic fields to Newtonian gravitation. Tesla's visionary but unpublished "Dynamic Gravity Theory" remains a compelling and intriguing concept, with many of his predictions eventually proving accurate. Key references on Tesla's Dynamic Gravity and Boskovic's Universal Natural Force are available online:
- [Tesla Research on Dynamic Gravity Theory](https://teslaresearch.jimdofree.com/dynamic-theory-of-gravity/ru%C4%91er-josip-bo%C5%A1kovi%C4%87-1711-1787/)
- [Tesla Research on Dynamic Gravity Theory](https://teslaresearch.jimdofree.com/dynamic-theory-of-gravity/)

2. Mechanical and Electromagnetic Resonance:
In acoustics and ultrasonic technology, resonant couplings and synchronizations are well-documented. When mechanical resonant objects (or sonotrodes) are tuned to the same frequency and placed in a liquid (like submersible ultrasonic transducers in water), even without direct mechanical connections, passive resonators placed around will begin to resonate synchronously with the active resonator (if all of them have the same natural resonant frequency). A similar principle applies to electromagnetic resonators, which require a coupling medium, often referred to as an ether. This concept suggests that fundamental natural forces might also involve ether or another coupling, fluidic medium.

3. Atomic Resonance and Energy States:
Electromagnetic and electromechanical couplings also occur within atomic energy states. Contemporary atom models, from N. Bohr's model to Quantum Wave Mechanics models, describe how stable electron states (or resonators) communicate by exchanging photons and producing matter-waves (see Chapters 8, 9, and 10 of this book). Coupled and electromagnetically polarized atoms create $1/r^2$ *dependent fields, where radiated energy density diminishes with distance.*

4. Universal Couplings and Synchronizations:
Electromagnetic, mechanical, and electromechanical interactions between atoms and masses are based on resonant couplings and synchronization effects. These effects, involving standing waves within and between different objects, contribute to gravitational phenomena (see "2.3.3. Macro-Cosmological Matter-Waves and Gravitation" later in this chapter). Static masses alone, without mechanical and electromagnetic moments, cannot generate gravitational forces. Additionally, entanglement effects and synchronous connections between directly coupled entities further support the concept of gravitational force as an electrostrictive and magnetostrictive effect (see in Chapter 10 for mathematical background).

5. Electromagnetic and Gravitational Interactions:

Macroscopic masses might appear electromagnetically neutral, yet internally, they contain spinning and vibrating entities such as atoms, electrons, protons, and neutrons. This internal distribution of electric and magnetic fields creates tension, torque, and vibrations, contributing to global rotational and linear motions in the universe. This phenomenon can be compared to the effects seen in rotating eccentric-mass disks. Future theoretical advancements may redefine concepts such as the universal center of energy or mass, considering electromagnetic charges, fluxes, and moments (see [33], [71], and [102] from Jovan Djuric). Masses in linear accelerated motion align internal spinning and magnetic moments, creating resulting macroscopic spinning moments and stable gyromagnetic relations.

6. General Relativity and Gravitational Force:

General Relativity, proposed by A. Einstein, equates gravitational force with the curvature of spacetime, reflecting what is experienced in uniformly accelerated reference frames (e.g., A. Einstein elevator). Gravitational interactions are seen as continuous motions along spatially curved surfaces surrounding masses. Although General Relativity provides a profound philosophical and theoretical framework, it is more precise, more complicated and less practical compared to Newtonian gravitation and to here promoted concept about evolving fields and forces originating from the atomic level. Here we try to align Newtonian gravity with concepts from Tesla's Dynamic Gravity Theory and R. Boskovic's ideas, suggesting a continuous flow of radiant energy or matter-waves with mechanical and electromagnetic moments emanating from atoms towards the universe (see Chapter 10, "10.02 Meaning of Natural Forces," and "2.2. Generalized Coulomb-Newton Force Laws"). There is still much to explore and integrate between Newtonian and Relativistic gravitation.

In Chapter 8, under "8.3. Structure of the Field of Subatomic and Gravitational Forces," we discuss how gravitation can be understood as matter-waves exchange effects inside and between atoms and the universe. Hypothetically, permanent, omnidirectional electromagnetic exchanges between atomic nuclei and electron clouds extend outward, connecting all atoms to the surrounding cosmos. This continuous exchange of energy and momenta aligns with Tesla's concept of "Radiant Energy" and it is detectable as the Casimir effect, which reflects mutual synchronization between resonating atomic states of two metal plates (see [103] and [104]).

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[♣ Comments and free-thinking corner.
The additional understanding of Gravitation, in relation to atoms as real sources of gravitation, can be supported by the following facts, comments, and concepts:

a) *Let us start with Newton's cannonball thought experiment, showing how we can launch and place certain small mass m in a stable or permanent, circular orbit around much bigger mass M (where r is the distance between centers of m and M, and v is the orbital or tangential velocity of the mass m). Here we limit our attention only to circular orbits since it is easier to draw important conclusions for what we need later. This is also the case when the mass m is in a stationary orbit, having its orbital velocity as,*

$$v = \sqrt{G\frac{M}{r}} \,. \qquad (2.4\text{-}4.9)$$

b) *Here we need to consider that translatory and rotational symmetry, and conservation of linear and orbital (or angular) moments are always satisfied, this way implicating that small mass m, being in a stable circular orbit around a much bigger mass M, in addition, can make some of translatory and rotating or spinning motions, while keeping constant its radius of rotation around mass M, without disturbing its stable circular orbit. Of course, here we also need to satisfy a total balance of relevant energy forms (such as kinetic, potential, spinning ...).*

c) *We also assume that rotation of the mass m around much bigger mass M will have stable, circular or stationary orbit. Since all motions are mutually relative, we could also imagine that m* *and M are not moving, especially when m is in a stationary orbit (keeping always the same distance, r, between them). Then, a surrounding spatial content, like some background matrix, or certain (still unknown) fluidic media, which belongs to the same "big picture or fluidic background" is rotating (around M, or better to say around common center of mass of the m-M system) on a way that around small mass m, circular or tangential velocity v (of mentioned fluidic background) is the same as mass m had before. Here we are exploring the meaning of mutually relative and kinematically equivalent, circular, accelerated motions, which should be mutually kinematically and mathematically replaceable.*

d) Now we can (mathematically) consider that both masses m and M are in the same center of slightly bigger mass equal to m + M, and that their spatial, joint-mass distribution covers the space which starts from the mentioned center of bigger mass M until the radius where it was the small mass m. This is also a case of rotation of a certain disk or ring with an eccentric peripheral mass m. Eccentric mass m (or m + M) will naturally produce standing matter waves around its stable circular orbit ($\lambda = \frac{H}{mv} = \frac{2\pi r}{n}$, $H = \text{const.}$, *n = 1, 2, 3, ...). Now we can summarize what will be relevant parameters of such stable, circular, and stationary, orbital motion,*

$$\frac{mv^2}{r} = G\frac{mM}{r^2} \Rightarrow \boxed{v = \sqrt{G\frac{M}{r}}}, \quad 0 < v < c \Rightarrow$$
$$\Rightarrow 0 < \sqrt{G\frac{M}{r}} < c \Rightarrow \; r = \frac{n\lambda}{2\pi} = \frac{GM}{v^2} > \boxed{r_{min} > \frac{G(M+m)}{c^2}} > 0 \,. \quad (2.4\text{-}4.10)$$

We still do not discuss what could be the nature of a surrounding fluid, which (hypothetically) rotates around the center of mass M, (or m + M), but we could deduce its properties later.

e) ***Now, we could consider that every stable small mass m in a state of rest effectively belongs to some mathematically equivalent (and hidden) rotating-or spinning background motion, which is as certain fine fluid rotation or vortex*** *(or velocity field) about a center of that static mass m. Mentioned fluidic matter rotates around mass* ***m*** *with certain (tangential) speed v. Let us take a small mass element* ***dm****, which already belongs to a mas* ***m*** *(and this could be the mass of an atom). Here (for mathematical simplicity) we also consider that mass* ***m*** *is spherical, having radius* ***r****. Speculated hidden rotation of* ***dm*** *around the center of mass* ***m*** *is in some way securing overall stability of the mass* ***m*** *(considering that involved centrifugal force, acting on* ***dm*** *is balanced with the self-gravitation or centripetal force of the mass* ***m****), as follows,*

$$\frac{dm \cdot v^2}{r} = G\frac{m \cdot dm}{r^2} \Rightarrow 0 \le \boxed{v = \sqrt{G\frac{m}{r}}} \le c \Rightarrow \boxed{0 \le \frac{m}{r} \le \frac{c^2}{G}} . \quad (2.4\text{-}4.11)$$

From here we could extrapolate Kepler-Newton vision of gravitation among planetary or solar systems. Briefly summarizing, all masses are condensed, agglomerated, superimposed rotating energy states.

*Since we know that Gravitation (as a force or field, in mathematically idealized conditions) is homogenous, isotropic, and omnidirectional, acting in the same way all around spherical mass m, meaning that every single atom inside mass m is in some way like an elementary vortex-type gravitational-sink (associating on N. Tesla vision of gravity). This supports the argument that Gravitation is directly proportional to mutually attracting masses (since the sum of all atoms is creating a total mass m). See (5.2.2) in Chapter 5 as the supporting background to intrinsic mass rotation (being in a close relation to Uncertainty relations). That means, a certain spatially complex, omnidirectional atomic-vortices-field (or force) is created as vector's superposition of all micro-vortex gyroscopes, or atomic forces and moments of internal spinning mass constituents (meaning atoms and elementary particles), eventually creating a resulting force of Gravitation (acting in all radial directions around mass m). See later in this chapter under "****2.3. How to Account for Rotation in Relation to Gravitation?****", and in*

Download the last version here:
http://www.mastersonics.com/documents/revision_of_the_particle-wave_dualism.pdf

*Chapter 8, about possible conceptualizations of such atomic forces, under "**8.3. Structure of the Field of Subatomic and Gravitation related Forces**".*

Here we can cite a (non-scientific, but indicative, motivating and amazing) summary of N. Tesla's original and imaginative visions regarding Gravitation, Natural Forces and aether vortices (just to think about). Taken from Internet Ovidiu Butucea · **December 29, 2024 at 12:06 AM** ·

"Nikola Tesla's views on vortices, toroids, and the dynamics of energy and matter were deeply tied to his concepts of "aether" and the fundamental forces of nature. While Tesla's ideas often diverged from mainstream physics, they remain a source of fascination due to their visionary nature and potential connections to modern theories.

Tesla believed in a dynamic aether, a medium that pervades all space and serves as the carrier of energy, much like a modern reinterpretation of the quantum vacuum.

He proposed that energy moves through the aether in wave-like and vortex patterns, with toroidal and vortex structures forming the basis for physical phenomena such as electricity, magnetism, and gravity.

Tesla viewed aether as highly dynamic and structured, capable of transmitting energy without loss, challenging the prevailing mechanical view of empty space.

Tesla theorized that all energy, whether in the form of light, electricity, or matter, is derived from vortical motion within the aether.

He described spiralling, vortex-like patterns of motion as fundamental to the propagation of electromagnetic waves and other forces.

Tesla suggested that gravity is not an inherent property of mass, as in Einstein's theory, but rather an effect of aether pressure gradients caused by vortex-like motion around celestial bodies.

Electromagnetic fields were also seen as manifestations of aether motion, with toroidal fields acting as the structure of magnetic phenomena.

He envisioned the Earth itself as a giant electromagnetic toroid, with energy flowing in and out in spiral patterns, much like plasma flows in the ionosphere and magnetosphere.

Tesla's unpublished Dynamic Theory of Gravity proposed that matter is formed from the condensation of energy in the aether, with vortex-like forces giving rise to mass and inertia.

He viewed the universe as a gigantic system of spinning vortices, with interconnected flows of energy forming stars, planets, and other cosmic structures.

This theory suggested that the rotation and oscillation of aether vortices could be harnessed to manipulate gravity and extract unlimited energy.

Tesla's work laid the groundwork for exploring toroidal energy systems, influencing modern concepts such as plasma physics, electromagnetic propulsion, and zero-point energy.

Modern researchers exploring toroidal physics, scalar waves, and vacuum energy often cite Tesla as a precursor to these ideas. Tesla's view of vortices and toroids placed him far ahead of his time, and they continue to inspire scientific inquiry and technological innovation."

f) ***Natural understanding of Gravitation*** *we obtain by uniting, harmonizing, and synchronizing several well-known empirical facts, effects, and laws, such as:*

1° Einstein elevator effect, or equivalency between the force of gravitation and force produced by uniform acceleration.

*2° Kepler-Newton laws applied to elliptic, orbital planetary motions, which are permanently accelerated motions, are also producing effects of gravitation (between a planet and its sun), as a balanced, circular motion state, between central (or centripetal) force of gravity and centrifugal force of such circular motion (what again has a meaning that acceleration is producing effect of gravitation). See "*Newtonian Gravitation and the Laws of Kepler (rochester.edu)".

3° Konstantin Tsiolkovsky's rocket propulsion (or kind of anti-gravity) force, also relates to mass acceleration and Newton laws, including the mass-flow or mass-propulsion as an additional cause of mass acceleration. This way we unify and generalize the force definition (see more in Chapter 10. under "10.02 MEANING OF NATURAL FORCES") to be applicable to all kind of natural forces, as follows,

$$F = \frac{\Delta E}{\Delta x} = \frac{\Delta p}{\Delta t} = \frac{dp}{dt} = \frac{d(mv)}{dt} = m\frac{dv}{dt} + v\frac{dm}{dt},$$

$$\left(p = \gamma mv, \gamma = 1/\sqrt{1 - v^2/c^2}\right) \Rightarrow F = \frac{d(\gamma mv)}{dt} = \gamma m\frac{dv}{dt} + \gamma v\frac{dm}{dt} + mv\frac{d\gamma}{dt} = \quad (2.4\text{-}4.12)$$

$$= \gamma m\left(1 + \frac{v^2/c^2}{1 - v^2/c^2}\right)\frac{dv}{dt} + \gamma v\frac{dm}{dt}\Bigg|_{v<<<c} \cong m\frac{dv}{dt} + v\frac{dm}{dt}$$

*4° Then, we need to understand that the most general cases of all motions are angular, curved, or rotational motions, where radius of rotation can be small, or in cases approaching linear motions very long (or infinite). Such conceptualization is extending and uniting applicability of Newton-Kepler laws, and uniting mathematical processing of linear and angular motions (again clarifying understanding of central forces, such as Gravitation, or Newton-Coulomb forces, since curved or angular motions have accelerations, producing radial forces and effects of Gravitation). This also supports mutual coupling of linear and angular moments (eventually and naturally leading to Wave-Particle Duality, in this book summarized under PWDC; -see more about PWDC in Chapter 10). In Mechanics we have a very rich elaborations of the "Two Body Problem" where linear motion of two mases can be effectively presented in different ways (including elements of angular motions). The same "Two Body Conceptualization" can be analogically extended to "Multiple Body Problems". Something similar can be applied to mutually related rotational or orbital motions, by connecting and mutually transforming orbital and spinning (external and internal, or macro and micro) angular motions, in relation to the fact that any mass (or agglomeration of atoms) in its structure has number of spinning entities. This way understanding, two masses, being seemingly in a mutual state of rest, still have equivalent internal elements of mutual rotations, spinning and orbiting, thanks to number of internal or atomic spinning and orbiting entities (see later in the same chapter "**2.3. How to account Rotation concerning Gravitation?** ". Here, we should also add analogy and coupling between linear motions in mechanics and phenomenology in electric fields, and analogy and coupling between rotational or angular mechanical motions and magnetic field phenomenology. In Chapter 3. of this book we can find an extended meaning of intrinsic unity and couplings between electric and magnetic fields (within upgraded Maxwell equations).*

Let us now hypothetically elaborate in a similar way, the unity and transformability among angular and linear motions, following the chain of conclusions or mathematical transformations addressing Gravitation, such as:

a) Gravitation is presently explained (measured and calculated) based on Cavendish and similar experiments measuring the force between two mutually static masses, and dynamically within Newton-Kepler laws addressing gravitational force between one static and one orbiting mass (rotating about static mass). In both cases (static and orbiting) the force of gravitation (and gravitational constant) is the same. The objective here is to conceptualize (at least intuitively and descriptively) how to make full harmony, equivalency, and mutual transformability from static mases attraction towards the case when one mass is orbiting the other (while all conservation laws are satisfied). Here we also address mutual complementarity and transformability between linear and angular motions (since static masses are cases of stable inertial and linear motions). In this book, the intermediary or

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

interface phenomenology connecting linear and angular motions is the Wave Particle Duality of matter, or PWDC (see more about PWDC in Chapters 4.1 and 10.).

b) *Let us start from the situation that electromagnetically neutral mass* m_1 *is orbiting about macro mass* m_2*, and that both masses* m_1 *and* m_2 *also have spinning moments* L_{s1} *and* L_{s2}*, being externally visible and measurable (practically mentioned macro masses are mechanically spinning about their centers). The sum of all spinning and orbiting moments, or total angular moment of this system will stay the same, conserved, or constant. Also, the total system-energy, or sum of all spinning, orbiting and rest masses energies will stay the same, or conserved. Newton gravitational force in this case (between involved masses) is* $F_{g1,2} = G\frac{m_1 m_2}{r^2}$*).*

c) *Such situation we can better present as both masses are orbiting about their common center of mass and still have spinning moments* L_{s1} *and* L_{s2} *(being externally visible and measurable). Spinning energies of masses* m_1 *and* m_2 *are* E_{s1} *and* E_{s2}*. Orbiting or kinetic energies of both masses are* $E_{orb-1} = E_{k1}$ *and* $E_{orb-2} = E_{k2}$*. Orbiting moments of* m_1 *and* m_2 *(around their common center of mass) are* L_{orb-1} *and* L_{orb-2}*. Again, here we have conserved the total angular moment and total energy of the system as before. Gravitational force will again be the same* $F_{g1,2} = G\frac{m_1 m_2}{r^2}$*.*

d) *If we present the same situation in the center of mass system, masses* m_1 *and* m_2 *will mathematically transform to* $m_c = m_1 + m_2$*, and* $m_r = m_1 \cdot m_2 / m_c$*.* L_{s1} *and* L_{s2} *will transform appropriately, but* $L_{s1} + L_{s2} = L_{sr} + L_{sc}$ *will stay constant (the same as before). Also,* L_{orb-1} *and* L_{orb-2} *could change, but the sum* $L_{orb-1} + L_{orb-2}$ *will stay constant. In fact, here is better to say that the sum of all angular moments will stay the same or constant. Gravitational force will again be the same as before,* $F_{g1,2} = G\frac{m_1 m_2}{r^2} = G\frac{m_r m_c}{r^2}$*.*

e) *Now we will formulate another equivalent transformation of externally tangible (or measurable) spinning moments* L_{s1} *and* L_{s2} *into number of internally distributed spinning entities, being on the atomic levels of* m_1 *and* m_2*. This way explaining, externally we do not see spinning of involved masses, but* L_{s1} *and* L_{s2} *still exist as the resulting vectorial sum of all interatomic, elementary spinning elements (structurally embedded inside* m_1 *and* m_2*, or in* m_c *and* m_r*). Practically, this way we mathematically "injected" spinning and orbiting energy from externally visible rotation into a total internal energy or mass of relevant particles. Now mases* m_1 *and* m_2 *will be bigger for "injected spinning energies", meaning:* m_1 *transforms to* $m_1 + E_{s1}/c^2 = m_1^*$*, and mass* m_2 *transforms to* $m_2 + E_{s2}/c^2 = m_2^*$*. What is the difference of the newly (or mathematically) created system of two masses (when compared to earlier situation): Masses are no more spinning externally (but still have internal sums of atomic spinning and orbiting equal to* L_{s1} *and* L_{s2}*). Kinetic orbital and total energy of masses* m_1^* *and* m_2^* *is now higher (for the amounts of the external spinning transformed into internal spinning). Newton force of gravitation will stay the same as before. Also, the sum of kinetic or orbiting energies will stay as before. Total system energy will be conserved.*

f) *The final step in this series of mathematically equivalent transformations involves "injecting" orbital or rotating energy into the masses involved, in a similar way how we previously "injected" spinning energy. By doing so, we achieve a state where the two masses remain at the same distance r, from each other, as in previous steps, with the same Newtonian* *gravitational force acting between them. The total angular momentum and total system energy remain constant or conserved throughout the process.*

Additionally, we should account for the direct coupling and proportionality between spinning, orbiting, and magnetic moments (or electromagnetic dipoles) at the atomic and molecular levels. This interaction, in average, generates weak magnetic fields that are conveniently polarized or aligned in a way that causes the masses to mutually attract, in accordance with Coulomb's law. For further exploration of these concepts, see such hypothetical elaborations in the publications of Dr. Jovan Djuric [33], [71], and [102].

Now we could imagine a completely reversed process, starting from two stable masses in the state of rest, mutually not moving, spinning, or orbiting externally. Going backwards we could mathematically present how a certain equivalent system of two rest masses will start mutually orbiting and spinning (but now internally present, or invisible spinning and orbiting on the atomic levels will be transformed into externally visible mechanical spinning and orbiting). Such significant mutual (and very fast, or immediate) influence, connectivity, and transformability of coupled angular moments, linear motion, and all forms of energy of certain two or multi body system is largely elaborated in [36]. This is now conceptually explaining Newtonian Gravitation in all cases of standstill masses, being also valid (in the same way) for orbiting and spinning masses (like in cases of planetary systems).

5° The next indicative phenomena in harmony with Gravitation understanding is the acoustic or ultrasonic levitation of masses, based on standing matter waves, or force-field wave formations between masses. Within such phenomenology and conceptualization, masses will take nodal zones of involved standing matter waves (where positive and negative accelerations are passing zero line), also associating on stability of solar or planetary systems. Such concept or standing waves field structure in relation to Gravitation (and probably to other natural forces) is experimentally, analogically, and intuitively, in compliance with Rudjer Boskovic "Universal natural Force" description, and Nikola Tesla's "Dynamic Gravity Theory" ideas (see more on Internet under the following web links:
https://teslaresearch.jimdofree.com/dynamic-theory-of-gravity/ru%C4%91er-josip-bo%C5%A1kovi%C4%87-1711-1787/
https://teslaresearch.jimdofree.com/dynamic-theory-of-gravity/ ♣]

Planetary, or solar systems, are analogically structured as their (mutually synchronized) atom constituents, respecting Coulomb-Newton 1/r^2 force law, and standing matter-waves resonant packing and couplings; -See more in "2.3.3. Macro-Cosmological Matter-Waves and Gravitation", "2.8. N. Bohr hydrogen atom and planetary system analogies", and "2.3.3-6, Rudjer Boskovic and Nikola Tesla's theory of Gravitation", at the end of this chapter. Such facts are directly indicating and implicating that the essential or dominant nature of Gravitation could be electromagnetic. *More familiar ideas regarding mutual coupling between mechanical and electromagnetic forces and moments can be found in Chapters 3, 8, 10, and in [117], [144].*

..

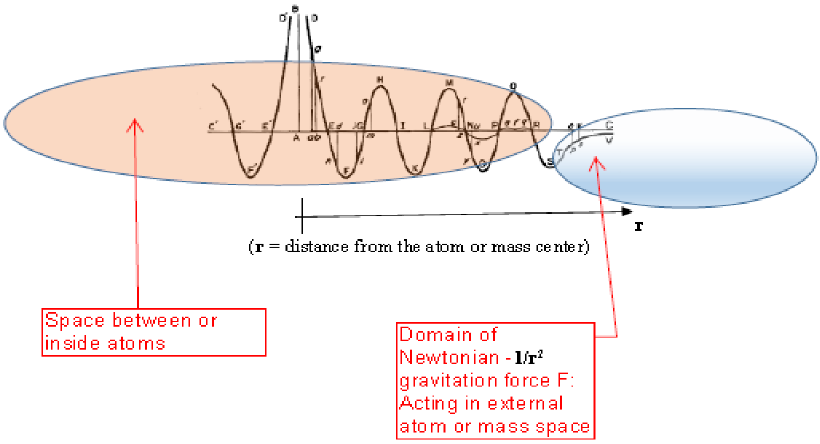
The external forces acting outside of atoms and masses are primarily electrostatic, magnetostatic, and gravitational that is naturally and essentially an electromagnetic phenomenon. Atoms communicate internally and externally by exchanging photons and absorbing cosmic rays and other forms of cosmic radiation. Practically all atoms, particles, larger objects, and even the Universe itself engage in this bi-directional (or omnidirectional) exchange, radiating electromagnetic energy and receiving the echo of gravitational attraction. This exchange may also involve Tesla's concept of cosmic radiant energy, which consists of different matter-waves. For more on these ideas, refer to Chapter 9 of this book, "Black Body Radiation, Gravitation & Photons."

The foundational ideas for this interpretation of atomic and interplanetary forces and fields can be traced back to the works of Rudjer Boskovic, particularly in his book on universal natural force in "Principles of Natural Philosophy" [6]. These concepts were later expanded in papers published in the "Herald of the Serbian Royal Academy of

Science" between 1924 and 1940 (J. Goldberg, 1924; V. Žardecki, 1940). Nikola Tesla's Dynamic Theory of Gravitation [97] also aligns closely with Boskovic's unified natural force, and with the concepts discussed in Chapter 8 of this book, particularly section "8.3. Structure of the Field of Subatomic Forces" (summarized in 8.74).

Additional insights can be found in Reginald T. Cahill's "Dynamical 3-Space: Emergent Gravity" [73] and Jean de Climont's works [117].

$$\oiiint_{\left\{ R\in[0,\infty],\ \theta\in[0,2\pi],\ \phi\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right] \right\}} \mathbf{F} d\mathbf{R} = 0 , \qquad (8.74)$$



Rudjer Boskovic's Universal Natural Force Function

If we consider gravitational force as a type of electromagnetic force, it naturally follows that both forces adhere to the Coulomb-Newton 1/r² law. This is because they are fundamentally the same force. When we examine what we refer to as gravitation, it appears to represent only the attractive portion of the R. Boskovic universal natural force curve (specifically, on the right side of the curve, extending towards infinity). Typically, natural forces are expected to have balanced positive and negative values or components. Surprisingly, however, Newtonian gravitation lacks an identifiable repulsive force counterpart.

If we accept the R. Boskovic universal natural force curve as conceptually and qualitatively relevant, it becomes clear that the repulsive, balancing forces are in the "alternating force-amplitude region", which corresponds to the masses and atomic structures situated on the left or central part of the natural force curve. More information on this can be found in Chapter 8, specifically in section 8.3, "Structure of the Field of Subatomic and Gravitation-related Forces."

We can also experimentally demonstrate the existence of both attractive and repulsive forces using "half-wavelength ultrasonic resonators" and through the effects of acoustic or ultrasonic levitation (see references [150] and [151]).

Furthermore, the standing waves mentioned earlier, which are spatially structured according to R. Boskovic's Natural Force field, should also possess radial, angular, and longitudinal components, as we are dealing with at least a three-dimensional space or field. The matter waves associated with the flow of mechanical and electromagnetic energy-momentum are always described by second-order classical partial differential wave equations. This is elaborated in Chapter 4.3 of this book (refer to equations (4.9) through (4.11) and the surrounding text). The solutions to such wave equations, including the Schrödinger equation, consistently involve two waves or can be represented by two wave functions propagating in opposite temporal and spatial directions, one inward and the other outward. These waves synchronously encapsulate the past and future aspects of an event, linking micro and macrocosmic entanglements and resonant synchronizations.

Nikola Tesla, as noted in [97], secured several patents and conducted successful experiments that demonstrate the existence of a "radiant energy fluid" or steady radiant mass flow, which possesses an electromagnetic nature. This fluid can carry both positive and negative electric charges as well as mechanical moments. Consequently, we can conclude that all atoms and masses in our universe are interconnected in a manner consistent with Rudjer Boskovic's Universal Natural Force [6], acting as frequency-tuned electromagnetic resonators. The attracting masses or agglomerated atoms can be considered to have either an electromagnetic or mechanical nature, due to their coupling through a surrounding fluidic medium, often referred to as aether. This term remains useful until a more suitable name and conceptualization are found.

According to Nikola Tesla's concepts, the properties of this ether are both electromagnetic and mechanical. It consists of extremely fine fluidic elements that serve as carriers of mechanical and electromagnetic moments and charges, and it also possesses dielectric and magnetic properties. It is important to recognize that all electrical, electromechanical, electromagnetic, acoustic, mechanical, fluidic, vibrational, and gravitational circuits, devices, and systems should form closed (moments, forces, currents, voltages and energy) circuits, networks, or systems. More about closed circuits can be found in the first chapter. These systems must have defined front-end inputs and last-end outputs, consumers, or loads. In contemporary physics and cosmology, however, we often encounter mathematical analyses based on open, or otherwise incomplete structures, formations, motions, fields, and forces. While these are mathematically acceptable simplifications, they indicate that we are still working with incomplete models, theories, and concepts.

To illustrate the broader spectrum of Ruđer Bošković's ideas about universal, united, Natural Force, let us read the following Quote:
Citation from the Internet: ***Ruđer Bošković, (English: Roger Boscovich): full biography.*** *Home* > *About Croatia* > *Culture* > *300th anniversary of the birth of Ruđer Josip Bošković* > *Ruđer Bošković: full biography. ...*"*In his work Bošković investigated various fields of science, making his most profound contribution to the understanding of the structure of matter. His theory of forces and the structure of matter is now widely accepted, making him a scientist two centuries ahead of his time. His theory was postulated on the principles of simplicity and analogy within nature, and on the principle of continuity. The practical purpose of the theory was to develop the then-topical scientific problem of collision analysis. According to Bošković, the matter is composed of points (puncta), which are simple, indivisible, non-extended, impenetrable, discrete, and homogenous, and which are sources of forces that act remotely. These points differ from mathematical points in that they possess the property*

of inertia, and in that, there is a force – Bošković's force – acting between them, which is represented by the Bošković curve (lat. curva Boscovichiana). At close distances, the force is repulsive. As distances increase, it reaches the point of neutrality, then becomes attractive, then reaches neutrality again, and finally becomes repulsive again. At farther distances, the force is attractive, in accordance with Newton's theory of gravity. B. proposed a modification in Newton's law of gravity concerning very long distances. The Bošković curve is uninterrupted, and it has two asymptotic ends (the repulsive and the attractive). It crosses the x-axis at the points of neutrality, called the points of cohesion and non-cohesion. Bošković's force is very akin to the force between atoms in a molecule or solid matter as well as to the nuclear force between nucleons (protons and neutrons). Hence, Herzfeld described it as "potential energy according to Bošković". A single law of forces existing in nature (lat. lex unica virium in natura existentium), i.e., the idea that one law can explain all of reality, constitutes Bošković's main contribution to science. The same idea has been entertained by A. Einstein, W. Heisenberg, and more contemporary scientists, but the four forces in nature (gravitational, electromagnetic, weak, and strong nuclear energy forces) have yet to be described by a unified theory. Bošković's single law is a framework for a unified theory of fields or, even more so, for a theory of everything. As a result of the unconditional assumption that the law of continuity must be observed, it followed that there can be no direct contact between particles because of the repulsive force (until then nobody had challenged the idea that there was contact between particles of matter). Modern scientists now agree with Bošković's conception of the basic elements of matter. Bošković's puncta are the most basic particles of matter and are as such comparable to quarks and leptons in modern science. Since matter consists of points, it follows that it contains a lot of empty space. This idea disproved the materialistic-corpuscular theory of matter, set foundations for a real dynamic-atomic theory, and provided a new perspective on the perception of reality. Just as the work of Copernicus resulted in the idea of the Copernican Turn, this breakthrough should be recognized as the "Boscovichan Turn" since it constitutes "the greatest triumph over the senses achieved on Earth to this time", and since Bošković and Copernicus "have been the greatest and the most victorious opponents of appearances" (F. Nietzsche, 1882., 1886).

Allowing for multiple repulsive areas in its potential, B. built the "pre-model" of "quark confinement", which is one of the central points of interest in modern elementary particle physics. Non-extended aspects of the matter are the building blocks of bigger particles, which in turn build up even bigger masses. B. speaks of them as the particles of the first, second, third, etc. order. This reflects the modern understanding of the structure of matter: quarks and antiquarks correspond with the particles of the first order, nucleons of the second, atomic nuclei of the third, atoms of the fourth, and molecules of the fifth. The properties of these particles and the distinctions between them are the result of their internal structures. B. was one of the champions of this idea, though the concept of the interconnection between the property and composition of matter was not accepted until the 19th century. (J.J. Berzelius, 1830).

The application of Bošković's law of forces to three points, two of which are placed in the foci of ellipses, is known as Bošković's "model of the atom" (1748). Long before the advent of quantum physics, this model identified the concept of "allowed" and "forbidden" orbits in nature, i.e., it quantifies the trajectory of the particle. J.J. Thomson was directly inspired by Bošković in formulating that idea (1907), which was central to the Bohr Model of the Atom (1913). »The Bohr model of the atom is a direct successor of Bošković's law of forces between microscopically removed particles"... "Where B. sowed 200 years ago, the others have reaped" (H.V. Gill, 1941). Bošković can also be considered the forerunner of thermodynamics, the kinetic theory of matter, the theory of elasticity of solid objects, and the explanation of the form of the crystal.

B. criticized Newton's conception of absolute space and time, and he construed the understanding of spatial and temporal relations as inextricable from point-like atoms and the forces between them. The extended matter is discrete rather than continual and, as such, entails a dynamic configuration of a finite number of centers of force. According to Bošković's pure dynamic atomism, the matter is not only endowed with forces (dynamic system), but it is composed of forces (dynamic system). Forces flow out of the atom and permeate empty space. This idea led to the concept of the field, much later formulated by M. Faraday (1844), who together with J.C. Maxwell introduced this idea into science. Bošković's *conception of spatially and temporally variable modes of existence (modi existendi) had ramifications, which despite all of the differences bring him into connection with Einstein's theory of relativity. Bošković can be regarded as the forerunner to the theory of relativity in three respects. First, he embraced the principle of relativity (one and a half centuries before E. Mach and A. Einstein) by proposing that direct observation and experimentation can neither distinguish between real space, relative space, time and motion, nor prove the principle of inertia. Secondly, he advocated the idea that the dimensions of an object change as its location changes. However, Bošković did not offer a quantitative measure of that change. Finally, he suggested that space might have four dimensions.*

In Philosophiae naturalis theoria… Bošković proposed the idea of an omniscient "spirit" that, based on Newton's laws and on the knowledge of all of the forces and initial positions at one moment, would have complete knowledge of the past and the future. Following essentially identical postulates, the French scientist P.S. Laplace formulated the classical determinism principle nearly half a century later (1814). That "spirit", that "intelligent entity", was termed by E. Du Bois-Reymond "Laplace's spirit" or "Laplace's demon", although it should have been named "Bošković's spirit" (S. Hondl)..."

……………………..

Citation from: [132] Dragoslav Stoiljkovic, Abstract: "In 1758 Roger Boscovich (1711-1787) published his monumental work "A Theory of natural philosophy reduced to a one unique law of forces that exist in nature". The Theory has had a major impact on Boscovich's contemporaries and resulted in many followers in the 19th and at the beginning of the 20th century. Today it is no longer present in the curricula of schools and colleges. Apart from the few individuals, our contemporaries, even highly educated people, know almost nothing about Boscovich. His life, scientific activity, and philosophical views, as well as his influence on contemporaries and followers are dealt in this monograph. His Theory is the very first quantum theory. He was the first one to draw the orbitals by which a particle moves around particles located in a center and explain that by transition from one orbital to another a particle either gains or loses a certain amount (quantum) of energy. His primary contribution was to the discovery of the structure of atoms of chemical elements. He pointed out the possibility of existence of macromolecules (i.e., polymers) and nanotubes, described the structure of these materials and their basic properties, and the structure of diamond and graphite. Following his line of thought, the ideas of neutrinos, quarks and gluons can be reached. The foundation of his theory is Boscovich's curve that describes the change in force between the particles of matter depending on the distance between them. We listed a dozen examples that confirm the validity of Boscovich's curve at several levels in the hierarchy of matter - of nucleons in atomic nucleus to the colloidal particles. The value of Boscovich's Theory is reflected in the multitude of ideas that sprout from it and that can be used to solve some of the problems of modern science. With some adaptation of Savich-Kashanin theory, we can obtain significantly more accurate calculations of solar planets densities, the volumes of matter at critical point conditions and in some other characteristic states of matter and the more correct interpretation of the mechanism and kinetics of the polymerization of ethylene and methyl methacrylate. A comparison of Boscovich's understanding of attractions and repulsions, as the essence of matter, with the understandings of Hegel and Engels are presented, too".

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♣ COMMENTS & FREE-THINKING CORNER (brainstorming):

The existence of hidden velocity-related parameters (such as associated linear and angular momentum) that are inherently embedded in Newton's force law, possibly within the universal gravitational constant, can be explored intuitively and creatively. For instance, consider the phenomenon where two parallel electric current elements (or wires) with currents flowing in the same direction attract each other.

If our universe is rotating holistically, producing electromagnetic dipoles, we might imagine that two electrically neutral masses, especially if they are close to each other or part of the same center-of-mass system, could be internally charged and slightly polarized with electric and magnetic dipoles. This polarization occurs because these masses participate in the same joint motion relative to a dominant reference system, such as a local galactic center of mass. Depending on the observer's frame of reference, there is a spatial flow or circulation of micro-electric currents within these masses, aligned with their global motion.

In this scenario, any two masses could be treated like segments of two electric wires or conductive bodies with parallel micro-currents flowing in the same direction—namely, the direction of the holistic, background angular motion (see sections 2.4-4.1). We know that such currents, or the corresponding internal electric and magnetic dipoles, create attractive magnetic fields, which locally manifest as gravitational forces.

The situation is likely more spatially and electromagnetically complex, as these currents result from the motion and oscillation of internally distributed electric and magnetic dipoles around dominant centers of mass. Nonetheless, the associated electric and magnetic field forces, including Lorentz forces, contribute significantly to what we perceive as gravitational and inertial effects.

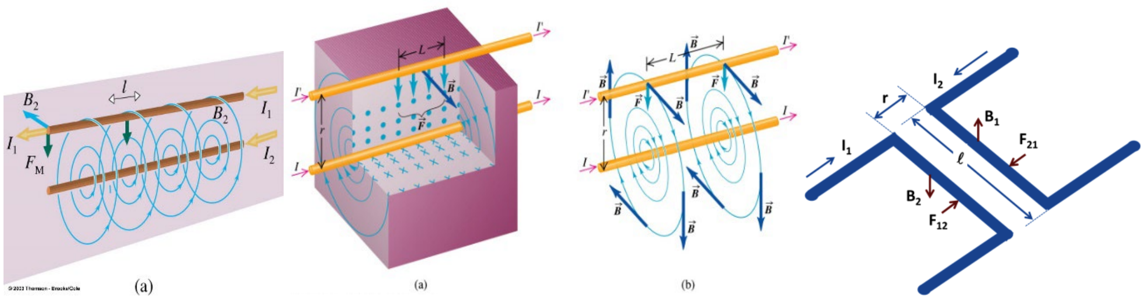
Demonstrating mathematically that these forces follow the $1/r^2$ law would require additional theoretical and mathematical work, given the influence of multiple force components—magnetic and electrostatic—on the resulting "gravitational" force. However, the intuitive and qualitative explanation provided here offers a conceptual basis for understanding why masses attract each other: they are part of a uniform, inertial, and holistic angular motion within the same mainstream.

If we consider the $1/r^2$ force law (see section 2.4-4), this scenario further supports the idea that gravitational attraction results from intrinsic, holistic motion and the internal micro-currents of electromagnetic dipoles. Ultimately, these speculations suggest that the concept of mass and "gravitational charges" has an intrinsic electromagnetic nature, which is more complex than currently conceptualized in classical mechanics, as well as in Newtonian and Einsteinian gravitation. These conclusions are in line with predictions from the Mobility system of analogies (refer to the first chapter of this book).

Theoretical reminder (taken from Internet); -Force between Two Parallel Wires:

- Two long parallel wires suspended next to each other will either attract or repel depending on the direction of the current in each wire.
- B-field produced by each wire interacts with current in the other wire.
- Produces magnetic deflecting force on other wire.
- Wires exert equal and opposite forces on each other.

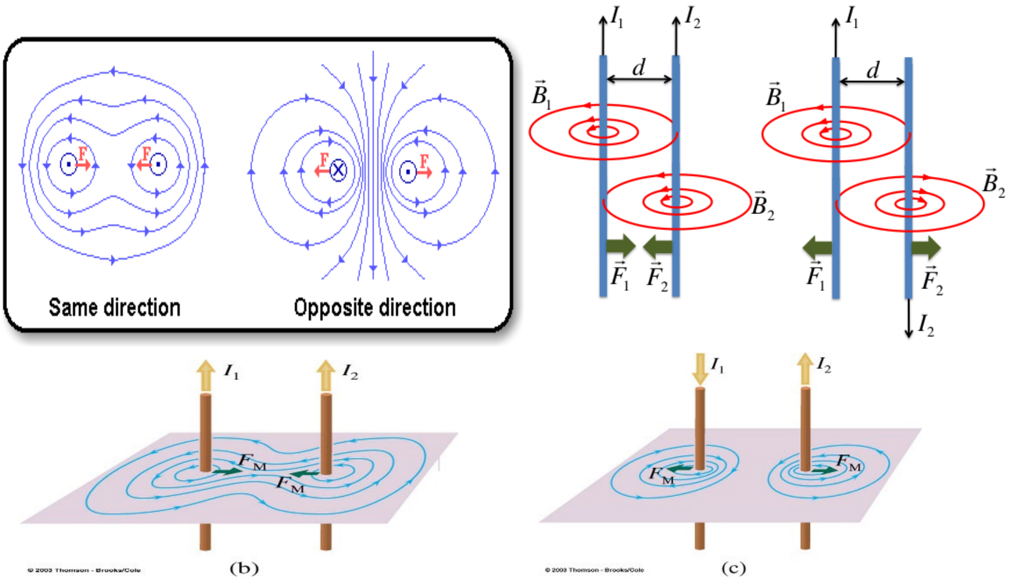
$$F_{1,2} = -F_{2,1} = \frac{\mu_o I_1 I_2}{2\pi r} \cdot L$$ (=) Force between two parallel wires ($L = l = l$ is the active length of wires)



1. Parallel wires with current flowing in the same direction attract each other.

All over this book are scattered small comments placed inside the squared brackets, such as: [♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

2. Parallel wires with current flowing in the opposite direction repel each other (*d = r*).



- Note that the force exerted on I_2 by I_1 is equal but opposite to the force exerted on I_1 by I_2.

Here is an attempt to show how force between two parallel wires with currents I_1 and I_2 could be presentable as Newtonian attraction between two effective masses $\mathbf{m_1}, \mathbf{m_2}$ of wires in question. By respecting the Mobility system of analogies (see T.2.2 in the first chapter), we know that there should be some direct correspondence or similarity between current and certain force. Of course, force here ($\mathbf{f_{1,2(axial)}} = \mathbf{m_{1,2}} \cdot \mathbf{a_{1,2}}$) should act on electrically charged particles that are creating currents I_1 and I_2, meaning that this is not a gravity force. Let us speculate how the force between two wires, $F_{1,2}$, could be equivalent to the Newtonian force of gravitational attraction, as for example,

$$F_{1,2} = -F_{2,1} = \frac{\mu_o I_1 I_2}{2\pi r} \cdot L$$

$$\left\{ \begin{array}{l} I_{1,2} \Leftrightarrow k \cdot f_{1,2(axial)} = k \cdot m_{1,2} \cdot a_{1,2} \\ f_{1,2(axial)} = m_{1,2} \cdot a_{1,2}, k = const (=) \left[\frac{A}{N}\right] \end{array} \right\} \Rightarrow$$

$$F_{1,2} = \frac{\mu_o k^2 m_1 m_2}{2\pi r} a_1 a_2 \cdot L$$

$$F_{1,2} = \frac{\mu_o k^2 m_1 m_2}{2\pi r} a_1 a_2 \cdot L = G \frac{m_1 m_2}{r^2} \Rightarrow$$

$$\Rightarrow G = \frac{\mu_o k^2 a_1 a_2 \cdot L \cdot r}{2\pi} = \frac{\mu_o k^2 a^2 \cdot L \cdot r}{2\pi} \Leftrightarrow ka = \sqrt{\frac{2\pi G}{\mu_o \cdot L \cdot r}}$$

Here we assume that axial electric charge acceleration in both wires is the same and constant, $\left(a_1 \cong a_2\right) \cong a = Const.$.

Of course, here we have a very indicative result (see (2.4-4.1) - (2.4-4.3) and later), showing that in such cases we already have Newtonian and/or Coulomb attraction of involved masses, or currents produced by motions of internal electric dipoles, supported by the following, approximated and analogical comparisons,

$$\mathbf{dm = \sqrt{\frac{\mu_0}{4\pi G}} \cdot idl \Rightarrow m = \sqrt{\frac{\mu_0}{4\pi G}} \cdot I \cdot L + m_0 = m_0 + \Delta m, \ m_0 = const. \Rightarrow}$$

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ **COMMENTS & FREE-THINKING CORNER...** ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

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$$\begin{pmatrix} F_{1,2} = \frac{\mu_0 I_1 I_2}{2\pi r} \cdot L \\ I_1 = I_2 = I \\ F_{1,2} = G\frac{m_1 m_2}{r^2} \\ m_1 = m_2 = m \end{pmatrix} \Rightarrow \begin{pmatrix} F_{1,2} = \frac{\mu_0 I^2}{2\pi r} \cdot L \\ (=) \\ F_{1,2} = G\frac{m^2}{r^2} \end{pmatrix} \Rightarrow r = \frac{1}{2} L$$

Since $r = 0.5L\ (=d)$ *and* L *are directly comparable lengths (being the same order of magnitude), the force between two parallel wires with electric currents should be roughly presentable or effectively convertible into a kind of Newton or Coulomb force.*

Gravitation conceptualized based on force between electric circuit elements

Imagine a planet with mass m, rotating around its Sun, which has mass M. The internal composition of both masses m and M consists of atoms and molecules, containing electrons and protons with negative and positive charges, as well as magnetic and spin moments. Since the electron's mass is 1,836 times smaller than that of the proton, the rotational or orbiting motion of the planet—being an accelerated motion—induces internal electric and magnetic dipoles, along with polarization and alignment of these electromagnetic entities within the atoms involved.

This concept allows us to creatively visualize the formation of internal micro-electric current elements that are synchronized in both masses m and M, following the circular path of mass m and reflecting this motion as a sort of spherical mirror image within mass M. This visualization creates a platform to model the forces between m and M as forces between two parallel electric wires, or more precisely, two coaxial and electro-conductive circles (electric circuits) behaving like parallel wires.

Taking this idea a step further, we can represent these electromagnetic forces as the centrifugal and centripetal forces acting between m and M. Eventually, this approach can be mathematically connected to Newton's explanation of gravitation (refer to sections 2.4-4.1 to 2.4-4.3). Additionally, the mutual coupling and transformation between the involved electric and magnetic field vectors can be conceptualized as elaborated in Chapter 3 of this book.

In conclusion, the force field between masses can be described as an attraction between well-coupled and synchronized electric currents and magnetic fields, which develop inside and around the masses due to the motion and polarization of internal electric and magnetic dipoles or moments. In other words, without considering certain rotating, intrinsic, global, or background motions of masses—both internally and externally—combined with cosmic resonance effects and standing waves between cosmic bodies, as well as electromagnetic field coupling and radiant energy circulation between these masses, we cannot fully explain gravitation in an integral and ontological sense.

For further insights, well-supported discussions on "New Horizons in Electric, Magnetic & Gravitational Field Theory" and the "Similarities of Motional Electric & Gravitational Forces" can be found in the works of William J. Hooper [152] and in Chapter 3 of this book.

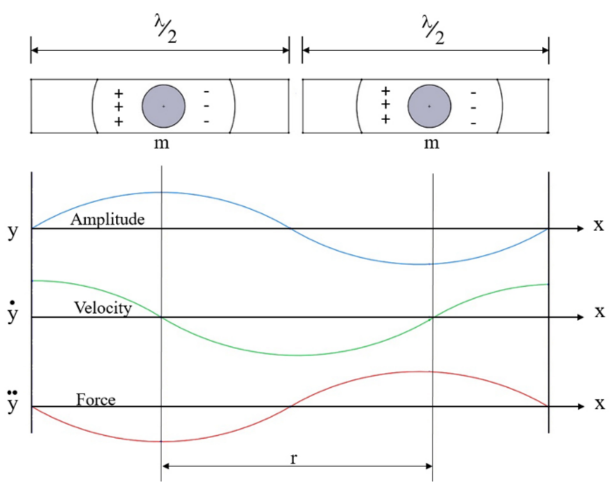
Gravitation and Natural Forces based on Standing Waves, Resonating Universe

In this section, we will explore the dominant and most probable conceptualization of gravitation, reflecting the author's preferred perspective. The discussion will show that the generalized $1/r^2$ Coulomb-Newton forces, as presented from equations (2.1) to (2.4-5.1), involve both static and dynamic force components. These forces are significantly influenced by the "energy-moments" of the interacting entities, as highlighted in equation (2.4).

Moreover, everything in our universe is in relative motion to its surroundings, and the internal structure of all particles and atoms exhibits intrinsic fields and standing-wave characteristics, with associated spinning and orbital moments. Newton-Coulomb force laws and the analogies drawn from them suggest that properties such as electric charges, masses, magnetic fluxes, orbital and spin moments, gyromagnetic ratios, and charge-to-mass ratios are interdependent and governed by the same conservation laws. Consequently, gravitation and other natural forces could be represented from several equivalent, synchronized, and convertible platforms. This leads to speculation about the existence of an immediate action at all distances, ensuring global "energy-moments" balance, synchronization, and adherence to conservation laws across both micro and macro-cosmological scales (see more in [36]).

Coulomb force laws, which govern the interactions between electric charges and magnets, are typically considered static because we often treat mass, electric charge, or permanent magnet flux as fixed entities. However, a deeper analysis reveals that mass, electric charge, and magnetism are the result of dynamic electromagnetic field formations, often structured by resonance and standing waves. Matter itself is composed of molecules, atoms, fields, and elementary particles, all of which exhibit electric charges, magnetic properties, intrinsic spin and orbital moments, and oscillatory behavior. These are the essential sources and effects that contribute to and influence gravitation.

To conceptualize gravitational attraction, consider a simplified, intuitive analogy of two identical oscillating masses, m and m, based on the induced electromagnetic forces between them within a resonating universe. This analogy likens these masses to half-wavelength mechanical (or ultrasonic) resonators. In this model, the masses m and m are part of a standing matter-wave, effectively acting as half-wave mechanical, electromechanical, and electromagnetic resonators (as illustrated in the accompanying picture, below, with rectangular bars representing these resonators). This imaginative conceptualization treats the oscillating masses as "electrically or electromagnetically charged and conveniently polarized half-wave resonators," which naturally attract each other due to their internal electric and magnetic dipole polarization.



All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

*This picture also presents a simple concept of Gravitational or Coulomb attraction based on electrostatic (also magnetic) dipoles-attraction (being in a form of standing waves) between oscillating masses m and m (what could be later upgraded as an attraction between certain Sun, with a mass M, and a planet from the same solar system with a mass m<<<M). First, the upper sinusoidal curve, $\mathbf{y}$, symbolizes oscillating amplitude of involved matter waves (like being an oscillatory displacement) between masses m and m. Middle sinusoidal curve $\dot{\mathbf{y}} = \mathbf{v}$ signifies velocity of matter waves between masses m and m. Zones where sinusoidal velocity curve has minimal (or zero) values, are zones of agglomerated (and polarized) masses and/or electromagnetic charges. The velocity curve between m and m could also be analogically compared with certain voltage or potential (respecting Mobility system of analogies, as presented in Chapter 1.). Second derivation of oscillating amplitude $\ddot{\mathbf{y}}$ corresponds to spatially distributed acceleration or force $\ddot{\mathbf{y}} \sim \mathbf{F}$ between masses m and m, <u>here having an indicative and analogical meaning of gravitational force, also on some way being analogical to the attractive force between "magnetized and/or to electric-dipoles polarized masses" m and m.</u> Maximal attractive forces are in zones where masses are agglomerated or placed, and maximal repulsive forces are where force or acceleration curve $\ddot{\mathbf{y}} \sim \mathbf{F}$ has minimal or zero values. Spatial distribution of oscillatory, **Kinetic energy** $\mathbf{E_k} = \mathbf{0.5mv^2}$ (of involved gaseous, plasma, and fields forms between masses m and m), is proportional to a squared velocity curve ($\mathbf{v^2}$) of relevant standing waves. Similar imaginative meaning of here involved (oscillatory) **Potential (gravitational, or central force), spatial energy-distribution** also corresponds to relevant acceleration, or to a force-curve $\ddot{\mathbf{y}} \sim \mathbf{F}$ between m and m, meaning that such maximal **Potential energy** is on and around mases m and m, where absolute value of the force or acceleration curve has maximal values. From thermodynamics we know that if certain gas (system of particles or body) is being heated, then its temperature is directly proportional to the average kinetic energy of involved gas particles, or to $\mathbf{0.5mv^2}$, meaning that maximal value of the square of mentioned velocity curve is the zone where temperature of involved gas and ionized particles will also be rising and getting maximal. This is the very indicative, or almost direct proof that in any solar system (where the Sun is much bigger compared to masses of all orbiting planets) temperature around it will not be maximal when measured on the Sun's surface (where standing waves velocity is almost zero). If we consider the Sun (being a part of the described two body, or two-mass, oscillating system), as a heat energy radiator, and also as a radiator of ionized and other dust and gas particles, and matter-waves, the increasing velocity curve $\mathbf{v}$ of mutually resonating masses (m and m) will start from zero (or from certain minimal value on the Sun surface), and it will rise in a direction of the second mass, this way also influencing motions and streaming of gaseous state around the Sun surface (as much as conditions around the Sun support such motions). We already know that maximal temperature about our Sun (which is between 1 to 2 million °C) is in its **Corona zone**, not very close to its surface (where velocity of resonating standing waves between involved masses is still rising or already being maximal). Where the same oscillating velocity is minimal (or close to zero, what is at the surface of the sun), our Sun is only around 6000°C hot (what is about 300 times less compared to Corona zone temperature). Of course, the Sun (as a blackbody or electromagnetic plasma radiator) in its near-to-surface zone also has certain increased density of gasses and other particles and matter-waves, which is gradually decreasing (with a distance) until arriving to a state of vacuum, meaning that temperature and kinetic energy of such evaporating and radiating gaseous and ionized state will naturally start increasing until sun's Corona zone, and then it will start decreasing, arriving in a low mass-energy-density* *conditions and high vacuum zone, toward the second mass (or towards all planets orbiting around the Sun). Solar winds or streaming of charged and ionized plasma particles (which are not dominantly hydrogen and helium atoms) will be created within the mentioned Corona zone (continuing to flow from Corona zone, towards high vacuum zone, in a direction of the other mass). Such (low density) solar wind particles are being accelerated from the sun's surface until the velocity curve maximal values, what we know from astronomic measurements (see more about solar winds, corona effects and blackbody radiation in Chapter 9.). Since the temperature on our Sun surface is about 6000°C, we can obviously conclude that the Sun could only be a molten lava (or molten stones and minerals; -not at all dominantly gaseous) state, where anyway we know that high concentration of hydrogen and helium atoms is not detected as a dominant mass population. Then, we could ask again, what really creates such high temperature of the sun in its Corona zone (and such low temperature on its surface). Corona zone should also be the consequence of the high voltage electric field or electric plasma discharges (between involved masses), conceptually meaning that what we here consider as a velocity curve, analogically also corresponds to standing-mater-waves voltage distribution between masses m and m. In other words, this about Corona zone temperature, followed by accelerated motion of plasma or solar-wind particles, directly and/or indicatively translates to the concept of* ***Resonant Universe and formations of (electromagnetic and mechanical) standing matter-waves between involved gravitational masses*** *(M and m), including possible participation of other orbiting planets and energy-mass agglomerations, and "by electromagnetic induction produced heating effects". In other words, the Sun is also, additionally, and most probably, getting internally heated, because some internal electromagnetic currents are creating induction heating effects; -see citation below). This is also in close conceptual compliance with the Electromagnetic nature of cosmic formations, Cosmic Electromagnetic Plasma phenomenology, Blackbody radiation, N. Tesla Dynamic Gravity, Radiant Energy, and Rudjer Boskovic's Universal, Natural Force Law. See more in Chapter 9. of this book, and here:*

https://teslaresearch.jimdofree.com/dynamic-theory-of-gravity/

https://teslaresearch.jimdofree.com/dynamic-theory-of-gravity/ru%C4%91er-josip-bo%C5%A1kovi%C4%87-1711-1787/ .

Citation from: https://www.space.com/17137-how-hot-is-the-sun.html :
"Temperatures in the sun's atmosphere also vary considerably between the layers. In the photosphere, temperatures reach about 10,000 degrees F (5,500 degrees C) according to the educational website The Sun Today. It is here that the sun's radiation is detected as visible light. Sunspots on the photosphere appear dark because they are cooler than the other parts of the sun's surface. The temperature of sunspots can be as low as 5,400 to 8,100 degrees F (3,000 to 4,500 degrees C) according to the University Corporation of Atmospheric Research (UCAR).
The chromosphere lies above the photosphere and temperatures range from approximately 11,000 degrees F (6,000 degrees C) nearest the photosphere to about 7,200 degrees F (4,000 degrees C) a couple of hundred miles higher up.

Now here is where things get a little bit strange. Above the chromosphere lies the corona — the outermost layer of the sun's atmosphere. The sun's corona extends thousands of miles above the visible "surface" (photosphere) of the sun. Now you might think that temperatures here must be the lowest here since we are the farthest away from the heat-generating core… but that isn't the case. At all.

The sun's corona can reach temperatures of around 1.8 million degrees F to 3.6 million degrees F (1 to 2 million degrees C), that's up to 500 times hotter than the photosphere. But how is the sun's upper atmosphere hotter than the surface? It's a great question, and one that has scientists rather stumped. ***There are some ideas about where the energy comes from that heats the corona, but a definitive conclusion is yet to be made***. If you'd like to read more about this solar mystery check out this article on "Why is the sun's atmosphere hotter than its surface?"."

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This explanation presents a simplified, visual, and analogical conceptualization of standing waves, like those observed in ultrasonic levitation experiments (see references [150] and [151]). It serves to illustrate how attracting and oscillating masses might interact through properly oriented electrostatic and magnetic moments, dipoles, and the creation of electromagnetic standing waves.

The masses, denoted as m and m, are macroscopically, electromagnetically neutral. However, internal electric dipoles, spatial distributions of electric fields, or polarizations generate electrostatic and magnetic attractions between the masses, which are equivalent to gravitational attractions. At the atomic and subatomic levels, the constituents of these masses possess magnetic and spin moments. In planetary systems, these masses are also engaged in orbital and spinning motions, generating electric currents and associated magnetic field standing waves.

Underlying the conceptualization of all natural forces, including gravitation and nuclear forces, is the idea of a Structurally Resonating Universe or Cosmos. This concept is ontologically linked to the internal, mutually coupled formations of electromagnetic, mechanical, and other matter-waves within atoms. These structural, mutually synchronized resonant states of atoms radiate or penetrate wave energy outside the atoms, creating "energy-mass-moments" agglomerations, respecting standing-waves structures. This idea aligns with Rudjer Boskovic's Universal Natural Force concept and Nikola Tesla's Dynamic Gravity theory.

To illustrate, consider a solar system. It is often oversimplified by saying that planets orbit around the Sun. However, it is more accurate to say that all planets and the Sun orbit or rotate around their common barycenter (or center of mass). This barycenter represents the combined center of mass of every planet, asteroids, the Sun, and other objects in the solar system. Its position continuously changes depending on where the planets and other objects are in their orbits. Consequently, the barycenter can range from being near the center of the Sun to outside its surface, given that the Sun is the largest mass in the planetary system. As the Sun also orbits this moving barycenter, it experiences a wobble, producing structural cosmic resonance effects and associated standing matter-waves between the planets and the Sun. These effects arise due to the mutually coupled electromagnetic fields and charges between the planets and other masses. This spatially defined, continuously variable barycenter contributes to the very high temperatures observed in the corona zone around the Sun, which are nearly a thousand times higher than the temperature at the Sun's surface. The Sun's mechanical wobbling directly influences radiant and resonance effects and "energy-moments" electromechanical couplings among all orbiting planets.

A similar conceptualization of a wobbling barycenter can be applied to the electromagnetic and electromechanical structure of atoms, creating resonant and standing matter-wave effects that are synchronized with external atoms, other masses, and the entire cosmos. This synchronization is possible because electromagnetic fields, forces, fluxes, and charges exist not only within atoms but also in the macrocosm of planets and galaxies, as conceptualized by Rudjer Boskovic and Nikola Tesla. Here, we extend the meaning of the mechanical or gravitational barycenter to encompass any barycenter of electric, magnetic, and other natural fields and charges.

The fact that wobbling mechanical and other (electromagnetic) barycenter spatial zones do not overlap add additional structural resonance complexity to our Universe. Along with the standing wave conceptualization of gravitation, we must consider that cosmic and galactic formations also host large-scale magnetic fields. We know that magnetic fields are created around electric current loops, implying that we are part of an Electric, Electromagnetic, and Electromechanical Universe. Therefore, the cosmic standing waves discussed here should be *understood as part of the electromagnetic and mechanical wave formations and vibrations between cosmic masses. These formations include plasma streaming and other electroconductive phenomena, all of which influence the creation and explanation of gravitation (see Chapter 3 of this book for a detailed discussion on symmetrical and perpetually interrelated electric and magnetic fields).*

The unipolarity of gravitation (being only an attractive force) serves as proof of the ultimate relevance of the "Standing Waves Resonating Universe" concept, where involved masses and/or energy states manifest effects like ultrasonic levitation within nodal zones. A visual indication of resonating, standing-wave fields between planets in our solar system can be observed in the rings of Saturn, Jupiter, Uranus, and Neptune. According to Wikipedia, "The most prominent and most famous planetary rings in the Solar System are those around Saturn, but the other three giant planets (Jupiter, Uranus, and Neptune) also have ring systems. There are also dust rings around the Sun at the distances of Mercury, Venus, and Earth, in mean motion resonance with these planets. Recent evidence suggests that ring systems may also be found around other types of astronomical objects, including minor planets, moons, brown dwarfs, and other stars" (source: https://en.wikipedia.org/wiki/Ring_system).

Certain planets in solar systems possess gaseous or atmospheric envelopes. These planets both spin on their axes and orbit their suns. Given that the gravitational force between gas particles and planets is extremely weak, one might wonder why these gaseous envelopes remain relatively stable and appear to be closely synchronized with their planets' rotation and orbit. The stability of these gaseous envelopes can be attributed to the interactions between ionosphere-associated electromagnetic dipoles and standing wave patterns (within relevant nodal zones, between the local planet and its ionosphere). These dominantly electrostatic interactions help maintain the gaseous envelopes in relatively stable positions relative to their planets. Additionally, the spinning and orbiting of planets interact with matter-wave effects arriving from other celestial bodies such as moons, asteroids, and the sun. Such interactions also influence tidal forces and other matter waves, winds and turbulences on the planet's liquid, gaseous and solid masses, creating also Schumann and planetary natural resonances.

Citation: "How Magnetic Fields Challenge Gravity-Centric Cosmology | Space News: ThunderboltsProject How Magnetic Fields Challenge Gravity-Centric Cosmology | Space News - YouTubeHow Magnetic Fields Challenge Gravity-Centric Cosmology | Space News - YouTube.
It has been one of the greatest surprises of the Space Age – powerful magnetic fields pervade the cosmos. Mainstream astronomers and astrophysicists do indeed acknowledge pervasive cosmic magnetism, but they did NOT predict it, and the realization has come begrudgingly. Here we explore why powerful cosmic-scale magnetic fields associated with countless celestial objects is the clearest indication that we live in an Electric Universe".

Contemporary attempts to create a Quantum field theory of Gravitation are searching for a non-existent meaning and nature of Gravitation (as traditionally and mainstream treated), just for the purpose of creating a theory that should be entirely Quantum Mechanics symmetrical, analogical, and compatible. This is unrealistic, since the field of gravitation is profoundly linked to electromagnetic phenomenology, based on structural, spatial standing-waves phenomenology of involved electromagnetic fields, currents, and electromagnetic dipoles-polarizations of involved masses, emanating from atoms. All of that (as conceptualized here) looks in some way familiar to String theory foundations, to N. Tesla Dynamic Gravity descriptions, and to Rudjer Boskovic Universal Natural Force concepts (see more in [6], [97], [117] and [144]).

Quantum nature of our Universe are manifestations of a multi-resonant assembly of different (mutually coupled) resonant states and standing matter-waves formations. All stable matter states in our Universe are structural (spatial-temporal) combinations of more elementary, vibrating, and resonant states with dualistic wave-particle properties, equally applicable to a *micro and macro world of Physics. This way we are coming closer to "**String theory" concepts**. Since the **String theory is the most promising, universal, natural fields and forces unification platform, or concept**, (see more in [144]) it would be very much beneficial to merge and unify here favored mater structure concepts based on resonant and standing matter waves (**originating from atoms**) with the contemporary String theory achievements (where the most important, or background mathematical and modelling support should comply with Kotelnikov-Shannon-Nyquist-Whittaker and Complex Analytic Signal analysis theory).*

*Citation from [144], https://www.scirp.org/journal/paperinformation.aspx?paperid=84159]: "***Abstract***, Planets are interconnected via magnetic field lines, which allow flux transfer events. In this paper, a thought experiment is performed whereby the Sun and planets are visualized as a violin with magnetic field lines analogous to strings of the violin. Frequency calculations for vibrating strings between the Earth and Sun compare favorably with measured satellite data for flux transfer events. Ideal Schumann Resonance was calculated using speed of light, diameter of the Earth and Earth's gravity for tension. The force of attraction and repulsion of magnets is analogous to tension and compression resulting from tremendously low frequency (TLF) electromechanical waves. Gravity waves are thus theorized as mechanical waves brought about by the tension of electron strings, which also act dually as electromagnetic waves. Future harmonic analysis of flux transfer wave forms for each planet would add credence to the string theory as the theory of everything".*

We know that high-power mechanical, ultrasonic, or acoustical energy—along with moments, forces, oscillations, and vibrations—can be generated and transferred by applying various signal-modulating techniques to laser beams or dynamic plasma states. These laser beams and plasma states act as carriers for lower frequency mechanical vibrations. For more information, see Chapter 10 (Sections 10.2-2.4) and references [133] to [139].

Expanding Our Understanding of Gravitation

The simplest and most descriptive explanation of gravitation stems from our existing knowledge, as outlined by Kepler, Newton and Einstein. While these theories remain useful, they offer explanations that are still superficial and incomplete. The understanding of gravitation is poised to evolve, potentially surpassing the theories established by Newton and Einstein. A deeper and more sophisticated explanation of gravitation, which may also open avenues for antigravity applications, involves the following key concepts:

1. Wave-Particle Duality: *Universal applicability of uncertainty relations to both micro and macro cosmic situations.*

2. Electromagnetism: *The effects of synchronized electromagnetic polarization can create attractive Coulomb forces that resemble gravitation. Gravitation is essentially a subtle integration of electromagnetic theory and mechanics.*

3. Atomic Interactions: *The extension of atomic fields and forces toward other masses and atoms in the universe is what generates the force of gravitation.*

4. Structural Vibrations of the Universe: *The attractive and repulsive forces associated with the nodes and anti-nodes of standing waves between masses contribute to the effects of gravitation and other fundamental natural forces, akin to concepts in String Theory (see reference [144]).*

5. Concepts from N. Tesla and R. Boskovic: *Nikola Tesla's Dynamic Gravity theory and Rudjer Boskovic's Universal Natural Force concept are fundamentally correct, although they are not yet fully formulated mathematically. For example, when a large mass attracts a smaller one, there must be a continuous energy exchange between them—what Tesla referred to as radiant energy flow. This radiant energy flow is more likely an electromagnetic phenomenon than a separate gravitational force, given the electromagnetic nature of matter.*

***6. Symmetry and Conservation Laws:** Translational and rotational symmetry, along with the conservation of linear and angular momentum, form an essential foundation for gravitation. All stable, linear, uniform, and inertial motions are relative, and similar principles apply to stable rotational or orbital motions, which involve accelerated movements.*

***7. Universal Entanglement:** Spatial-temporal-spectral synchronization also supports gravitation and the creation of standing matter waves between mutually attracting masses.*

***8. Analogies and Gravitation:** Based on universally valid analogies (as discussed in Chapter 1), the real sources of gravitation are not just static masses but also atoms in motion, which include vibrations, mechanical and electromagnetic fields, moments, and charges.*

***9. The Ether Concept:** To fully understand gravitation and electromagnetism, we may need to revive the concept of ether—an almost ideal fluid that underlies all matter in the universe.*

***10. Spatial-Temporal Unity:** The unity, proportionality, and transformability of spatial, temporal, and spectral domains are crucial for understanding gravitation and other natural forces.*

Summary and Future Directions

All these aspects (1-10) are interrelated, united, and synchronized. It's challenging to isolate simplified meanings of gravitation from the broader context of these elements. While gravitation appears to be one of the fundamental natural forces that is easily explained, it is of electromagnetic nature and highly complex.

Summary Points:

- ***Quantum Theory and Wave-Particle Duality**: The current quantum theory and wave-particle duality concepts need significant revision. The probabilistic foundation of quantum theory should be viewed as a useful mathematical tool rather than an ontological basis, applicable only in contexts where probability and statistics are mathematically justified. This book offers many ideas for revising quantum theory.*

- ***Evolution of Fundamental Forces**: The concept of four fundamental natural forces is likely to evolve, revealing current formulations as obsolete, oversimplified, and possibly meaningless despite their apparent complexity. If such changes occur, significant areas of modern physics, including the Big Bang theory, Standard Model, black holes, and dark matter/energy, may either disappear or undergo substantial transformation, requiring a more unified and complex conceptual framework.*

The author's position is that the particle-wave duality framework in modern physics should be linked to the complex nature of fields and forces associated with stationary and standing matter waves between interacting objects. Linear motion and spinning are presented as a conjugate pair of motion entities, analogous to the relationship between electric and magnetic fields in creating electromagnetic waves (see equations 4.1, 4.2, 4.3, 4.33.1, and T.4.2 and T.4.3, which support the unity of linear motion and spinning). The final picture and explanations of particle-wave duality, as proposed in this book, should become clearer, more straightforward, and more apparent than the current state of quantum theory, free from unnecessary probabilistic frameworks except where mathematically appropriate.

The author also aims to show, at a conceptual level, that the questions surrounding gravity, and the phenomena related to linear motion and rotation are not fully or clearly answered in modern physics, unlike in electromagnetic theory. While General Relativity Theory correctly describes spatial deformations caused by gravitational forces, current simple forms of Newton-Coulomb force laws apply only within a limited framework. Nevertheless, these laws suggest an electromechanical analogy and coupling between different forces and fields among all relevant participants.

This book attempts to demonstrate, through a combination of Newton-Coulomb force laws and new hypothetical proposals about a vibrating and resonating universe, that the space between two interacting objects in relative motion has a multi-component, mutually coupled electromagnetic field structure (as presented in 2.4-3 and Chapter 3). The specific fields and force components involved in particle-wave *duality are not yet fully conceptualized or understood in contemporary physics. While the relationships outlined in 2.4-3 are not yet generally applicable, they offer a brainstorming insight into the possible complexity of fields and forces between two interacting objects, pointing to new directions for exploring universal field theory.*

It is becoming clear that the most relevant set of field-charges will evolve from the current understanding, potentially being reestablished or reinvented to become more general and coherent. The conceptual platform regarding gravitation-related forces introduced in sections 2.3 to 2.4-3 and elsewhere in this book is based on interactions between the "energy-momentum" content of mutually coupled, internally resonating, and reacting matter domains, which involve elements of spinning and other angular movements. Understanding the energy-momentum content and distribution of these coupled domains should be the starting point for any new charge/field/force foundation. Consequently, the static mass currently considered the principal cause or source of gravitation will likely be replaced by interactions between mutually coupled linear and angular moments, along with other natural electromagnetic fields and charges. These interactions manifest as the spinning and electromagnetic properties of internal matter domains, including effects between oscillating or resonating masses. This suggests that gravitation could be more than a simple, conservative field or central force.

Later, in chapter 5, we will see that mass understanding could be conceptualized differently involving certain kinds of mutually coupled electric and magnetic dipole charges, and/or elements of linear motion and spinning, such as, (see (5.2) and (5.2.2)),

$$m=\frac{\Delta q_{mag.}\cdot\Delta q_{el.}}{c^2\Delta t}=\frac{\Phi\cdot q}{c^2\Delta t}=\frac{\Phi\cdot i_{el.}}{c^2}=\frac{\Delta\alpha\cdot\Delta L}{c^2\Delta t}=\frac{\Delta\alpha\cdot\tau}{c^2}=\frac{\Delta x\cdot\Delta p}{c^2\Delta t}=\frac{\Delta x\cdot F}{c^2}=\frac{\Delta E}{c^2},$$
$$\left(\begin{array}{l} i_{el.}=\frac{\Delta q}{\Delta t},\Delta q_{mag.}=\Phi=\text{magnetic flux},\ q_{el.}=q=\text{electric charge},\\ \tau=\frac{\Delta L}{\Delta t}=\frac{\Delta E}{\Delta\alpha},F=\frac{\Delta p}{\Delta t}=\frac{\Delta E}{\Delta x},\ \frac{\Delta E}{\Delta p}=\frac{\Delta x}{\Delta t},\ \Delta E=\tau\Delta\alpha=F\Delta x.\end{array}\right) \quad (2.4\text{-}6)$$

Based on (2.4-6) we should also be able to explain gravitational attraction of neighbor's masses as kind of attraction between their electrically and/or magnetically, "partially and weakly polarized dipoles", thanks to certain level of spontaneous (mutually induced and opposite) polarization of corresponding electromagnetic charges (see [121]). For instance, if such (presently hypothetical polarization) is really happening, we would have the situation that Newton masses attraction should be equal to certain Coulomb effective-charges attractions, as follows,

$$\left\{\begin{array}{l} F_{12}=G\frac{m_1m_2}{r^2}=\frac{1}{4\pi\varepsilon}\frac{q_1q_2}{r^2},p_1=\frac{m_1v_1}{\sqrt{1-\frac{v_1^2}{c^2}}}=\frac{m_2v_2}{\sqrt{1-\frac{v_2^2}{c^2}}}=p_2\\ \Rightarrow m_1m_2=\frac{1}{4\pi\varepsilon G}q_1q_2,\ m^2=\frac{1}{4\pi\varepsilon G}q^2,\\ \frac{q_1}{m_1}=\frac{q_2}{m_2}=\sqrt{4\pi\varepsilon G}=8.616032252\cdot10^{-11}\left[\frac{C}{kg}\right]=\text{const.},\\ \text{valid for vacuum.}\end{array}\right\}\Rightarrow\left\{\begin{array}{l}\frac{m_1}{m_2}=\frac{q_1}{q_2}=\frac{v_2}{v_1}\frac{\sqrt{1-\frac{v_1^2}{c^2}}}{\sqrt{1-\frac{v_2^2}{c^2}}}(\cong\frac{v_2}{v_1},v_{1,2}<<c)\\ \frac{q_1v_1}{\sqrt{1-\frac{v_1^2}{c^2}}}=\frac{q_2v_2}{\sqrt{1-\frac{v_2^2}{c^2}}}=\frac{q_1p_1}{m_1}=\frac{q_2p_2}{m_2}\end{array}\right\}. \quad (2.4\text{-}7)$$

What we see from (2.4-7) is that the "*one-sided charge-to-mass ratio"* (of slightly polarized, bat macroscopically electrically neutral masses) is velocity independent and constant ($\frac{q}{m}=\sqrt{4\pi\varepsilon G}=8.616032252\cdot10^{-11}\frac{C}{kg}$, in free space and vacuum). This could be the principal reason that Newton force between two masses (which are anyway in relative motion) is equal or proportional to Coulomb force between two corresponding effective electric or magnetic charges (of mutually opposed electric and magnetic dipoles) since Newton and Coulomb's laws have the same mathematical forms. Here, we should not forget that relevant, effective charges (in (2.4-7)) are not freestanding charges (because they belong to corresponding electrical dipoles embedded in relevant masses). Since the one-sided charge-to-mass ratio (of electrically neutral and slightly polarized masses) is a tiny number ($8.616032252 \cdot 10^{-11} \frac{C}{kg}$), this is the explanation why such effects of week spontaneous dipole (or multi-pole) polarization are still not experimentally detected. In cases of significant astronomical objects (where dipole charges in question could be significantly stronger), again there is a measurement problem since it is not possible or easy to apply direct measurements. Anyway, we know that the freestanding charge-to-mass ratio of an electron is also velocity-independent and constant (as in (2.4-7), but a much more significant number) and that it can be found experimentally, or calculated theoretically as, $\frac{e}{m_e} = 1.759 \cdot 10^{11} \frac{C}{kg}$.

Of course, the electron is not a dipole or an electrically neutral mass, and this is the reason why its charge-to-mass ratio is enormously more significant, compared to the similar ratio of electrically neutral masses. The "$q/m=e/m_e$" of an electron was successfully measured and calculated by J.J. Thomson in 1897, and more successfully by Dunnington's Method, which involves the angular momentum and deflection due to a perpendicular magnetic field.

The same way, we could calculate the gravitational attraction between two electrically neutral atoms (considering that some very small or weak electric-dipole polarization, *"(+q)↔(-q)"* would for some natural reason appear between them). In the case of two hydrogen atoms, a charge amount of such spontaneous polarization between them (here expressed as q) will be:

$$\left\{ \begin{array}{l} \frac{q}{(m_e + m_p)} = \sqrt{4\pi\varepsilon G} = 8.616032252 \cdot 10^{-11} \frac{C}{kg}, \\ \frac{e}{m_e} = 1.759 \cdot 10^{11} \frac{C}{kg},\ m_p = 1836.152701(37) \cdot m_e, \\ \frac{e}{m_e + m_p} = 957459.8775 \frac{C}{kg} \end{array} \right\} \Leftrightarrow q = 8.998844186 \cdot 10^{-15} e.$$

Calculated polarized (one side) charge is small ($q = 8.998844186 \cdot 10^{-15} e$) to be seriously considered for hydrogen atoms attraction, but in cases of astronomic objects, a similar situation could be much different, because of superposition of such elementary electromagnetic dipoles.

Analog to (2.4-7), we can extend the same chain of conclusions considering relevant dynamic "magnetic polarization charges" or (one-sided) magnetic fluxes, as for instance,

$$\left\{\begin{array}{l} F_{12} = G\frac{m_1 m_2}{r^2} = \frac{1}{4\pi\,\mu}\frac{\Phi_1\Phi_2}{r^2}, p_1 = \frac{m_1 v_1}{\sqrt{1-\frac{v_1^2}{c^2}}} = \frac{m_2 v_2}{\sqrt{1-\frac{v_2^2}{c^2}}} = p_2 \\ \Rightarrow \Phi_1\Phi_2 = 4\pi\,\mu G\cdot m_1 m_2,\ m^2 4\pi\mu G = \Phi^2, \\ \frac{\Phi_1}{m_1} = \frac{\Phi_2}{m_2} = \sqrt{4\pi\mu G} = 32.45920531\cdot 10^{-9}\left[\frac{W_b}{kg}\right], \\ \frac{q_1}{m_1} = \frac{q_2}{m_2} = \sqrt{4\pi\varepsilon G} = 8.616032252\cdot 10^{-11}\left[\frac{C}{kg}\right] \\ c = \frac{1}{\sqrt{\varepsilon\mu}} = 2.99792458\cdot 10^{8}\frac{m}{s}\ (\text{in vacuum}), \\ G = (6.67428\ \pm\ 0.00067)\cdot\ 10^{11}\left[\frac{m^3}{kg\cdot s^2}\right]. \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \frac{\Phi_1 v_1}{\sqrt{1-\frac{v_1^2}{c^2}}} = \frac{\Phi_2 v_2}{\sqrt{1-\frac{v_2^2}{c^2}}} = \frac{\Phi_1 p_1}{m_1} = \frac{\Phi_2 p_2}{m_2}, \\ \frac{m_1}{m_2} = \frac{\Phi_1}{\Phi_2} = \frac{q_1}{q_2} = \frac{v_2}{v_1}\frac{\sqrt{1-\frac{v_1^2}{c^2}}}{\sqrt{1-\frac{v_2^2}{c^2}}} (\cong \frac{v_2}{v_1}, v_{1,2} << c), \\ \frac{\Phi_1}{q_1} = \frac{\Phi_2}{q_2} = \sqrt{\frac{\mu}{\varepsilon}} (= 377\,\Omega,\ \text{for vacuum}), \\ \Phi = q\sqrt{\frac{\mu}{\varepsilon}} = m\sqrt{4\pi\mu G} = 32.45920531\cdot 10^{-9}\cdot m\,(=)\left[W_b\right], \\ q = m\sqrt{4\pi\varepsilon G} = 8.616032252\cdot 10^{-11}\cdot m\,(=)\left[C\right], \\ \frac{\Phi q}{m^2} = 4\pi G\sqrt{\varepsilon\mu} = \frac{4\pi G}{c} = 4191.690044\left[\frac{m^2}{kg\cdot s}\right] \end{array}\right\}$$

$$\Rightarrow m = \frac{\Phi}{\sqrt{4\pi\mu G}} = \frac{q}{\sqrt{4\pi\varepsilon G}} = \sqrt{\frac{c\Phi q}{4\pi G}} = \begin{pmatrix} 1.160627039\cdot 10^{10}\cdot q \\ 3.08079015\cdot 10^{7}\cdot\Phi \end{pmatrix} = 597967252.4\sqrt{\Phi q}\,(=)\left[kg\right] \qquad (2.4\text{-}8)$$

Analog to Charge-to-mass ratio we also know that Gyromagnetic ratios for many elementary particles are always constant numbers, and why not have a similar situation for any other mass including astronomic objects (like we see in (2.4-7) and (2.4-8)). Also, planets that have their own magnetic field, like our planet Earth, should have specific, quantifiable relation ($\Phi = q\sqrt{\frac{\mu}{\varepsilon}} = m\sqrt{4\pi\mu G} = 32.45920531\cdot 10^{-9}\cdot m\,(=)\left[W_b\right]$) between planet's mass and its total (one-sided) magnetic flux. In the case of planet Earth, $m\ =\ 5.976\cdot 10^{24}\ kg$, $\Phi = 32.45920531\cdot 10^{-9}\cdot 5.976\cdot 10^{24} = 193.9762109\cdot 10^{15}\ W_b$. Here we should imagine an equivalent bar-magnet and consider the total magnetic flux from one side (single pole) of the magnet. Of course, in cases of such planetary magnetic fields, we could have a superposition of some other, spinning, and internal electric currents related magnetic field components (for instance, electric dipoles rotation under favorable conditions will also produce magnetic fields). Anyway, since electric and magnetic phenomena and fields are always mutually coupled (do not exist as isolated entities), and since all matter states motions in our universe are relative linear and angular motions (including oscillatory and resonant states), it should be clear that gravitation and electromagnetism are very much connected regardless of what our contemporary theories and concepts are presenting. See more familiar elaborations in Chapters 1, equations (1.10) - (1.16), and 3.).

Again, in the case of our planet Earth, the total one-sided, dipole electric charge (2.4-8) should be $q = 8.616032252\cdot 10^{-11}\cdot m =\ 8.616032252\cdot 10^{-11}\cdot 5.976\cdot 10^{24} = 51.48940874\cdot 10^{13} C$. For comparison, the negative electron charge is $1.6021892\cdot 10^{-19} C$, producing that one "Earth-electric-dipole" side could capture $32.13690914\ \cdot 10^{32}$ electrons. Of course, such displaced electrons are distributed on a large surface. All that conceptualization (here over-simplified) looks, sooner or later, in some ways verifiable, and of course, it is mixed with familiar electromagnetic and fluid dynamics complexity (see successful, more advanced, but familiar concepts published under [63], Arbab I. Arbab). The author of this book also introduced in [3] similar ideas about effective electric masses-polarizations and equivalency or analogy between masses and electrostatic charges attraction.

*Let us imagine that **q** presents a single electron, **e =1.602176565 x 10⁻¹⁹C**. The corresponding (minimal) magnetic flux of permanent spinning, belonging to an electron charge (in a vacuum), will be* $\Phi_0 = e\sqrt{\frac{\mu_0}{\varepsilon_0}}$ ***= 6.03588480 x 10⁻¹⁷ Wb**. But, since the relation* $\Phi = q\sqrt{\frac{\mu}{\varepsilon}}$ *is formulated for electrically neutral particles that are for specific reason electrically and magnetically polarized as dipoles (2.4-8), the one-side magnetic flux value of **6.03588480 x 10⁻¹⁷ Wb** is more likely to be valid for a certain hydrogen atom state.*

Most probably, Northern and Southern Polar Lights (or Auroras) are not only related to solar winds (of electrically charged particles) that are trapped in the Earth's magnetic field. Here-hypothesized "electric-dipoles, or by **Earth and Sun mutually induced electromagnetic polarization**" should have its significant guiding or attracting contribution to such polar light and solar wind effects (or to currents and winds of electrically charged particles between the Sun and Earth, strongly influenced by standing electromagnetic and electromechanical, or gravitational matter-waves between planets and local Sun; -see more in [144]). Since solar winds are real and detectable, another reality, still under probation, would be to show that mentioned electromagnetic dipoles polarization has an essential place in explaining currents of electrically charged particles between the Sun and Earth. Of course, atmospheric thunderclouds' lightning (between electrically charged clouds and Earth) could also be in close relation to hypothesized electrostatic polarization and solar winds (what is producing cosmic electric currents and associated magnetic fields and dipoles). All of that, if shown to be correct, will give another challenging contribution to understanding electromagnetic fields and Gravitation (see later Fig.2.1 and equations (2.4-9) and (2.4-10)).

It could also be that Casimir Effect, known experimentally in Quantum Physics (see [103] and [104]), is not too far from here hypothesized attraction based on "spontaneous electromagnetic polarization" between two neighbors' masses as the effect of universal synchronization between mutually overlapping spectral domains of internal atomic states of involved matter (here, the Casimir force is an attractive force between two flat conductive surfaces in vacuum, but Casimir force can also be repulsive, depending on selected distance between conductive metal plates).

The far-reaching consequences related to here-hypothesized electromagnetic polarizations intrinsically coupled with mechanical motions and gravitational attraction of involved objects could lead to an explanation of gravitation as the specific manifestation of electromagnetic interactions (see more familiar or very indicative ideas in Chapter 3. of this book, and in [72], Dr. László Körtvélyessy; -The Electric Universe, as well as in papers from Prof. Dr. Jovan Djuric, [33], [71], [102], and in [121] and [144]). Of course, the meaning of mass (such as inertial, heavy, gravitational, rest, relativistic…) will also evolve.

[♣ COMMENTS & FREE-THINKING CORNER:

Now, after correlating and combining results and relations from (2.4-6) until (2.4-8) we could start getting another challenging insight into (2.4-4), which is an empirically verified $\frac{1}{r^2}$ force law between

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

equidistant, parallel paths moving electrical charges (q_1 , q_2). The arising message here should be that matter particles have mutually correlated, coupled and dependent intrinsic properties such as mass, relevant dipole-related electric charges, and associated magnetic moments and fluxes (as well as stable Gyromagnetic and charge-to-mass ratios). Of course, it is still too early to draw generally valid conclusions, and most probably, that lot of new conceptual and mathematical fittings and managing efforts should be implemented here, but the direction towards an innovative conceptualization of matter structure and involved forces is already paved. The following set of mutually analog relations and results, extracted from (2.4-4), (2.4-6), (2.4-7) and (2.4-8), could serve only as a brainstorming reminder about some of remaining challenging problems in connection with mentioned innovative conceptualization about Gravitation. This is a part of objectives in front of us (not to mention many other challenging issues initiated by [35], Thomas E. Phipps, Jr., Old Physics for New, and [121], Raymond HV Gallucci, Electromagnetic Gravity? Examination of the Electric Universe Theory).

$$F_{1,2} = \frac{\mu}{4\pi}\frac{(q_1v_1)\cdot(q_2v_2)}{r^2} = K\frac{(q_1v_1)\cdot(q_2v_2)}{r^2},\ K = \text{const.},$$

$$m = \frac{\Delta q_{mag.}\cdot\Delta q_{el.}}{c^2\Delta t} = \frac{\Phi\cdot q}{c^2\Delta t} = \frac{\Phi\cdot i_{el.}}{c^2} = \frac{\Delta\alpha\cdot\Delta L}{c^2\Delta t} = \frac{\Delta\alpha\cdot\tau}{c^2} = \frac{\Delta x\cdot\Delta p}{c^2\Delta t} = \frac{\Delta E}{c^2},$$

$$\frac{q_1v_1}{\sqrt{1-\frac{v_1^2}{c^2}}} = \frac{q_2v_2}{\sqrt{1-\frac{v_2^2}{c^2}}} = \frac{q_1p_1}{m_1} = \frac{q_2p_2}{m_2}\begin{pmatrix}\leftrightarrow\\ \text{analog}\end{pmatrix}\frac{\Phi_1v_1}{\sqrt{1-\frac{v_1^2}{c^2}}} = \frac{\Phi_2v_2}{\sqrt{1-\frac{v_2^2}{c^2}}} = \frac{\Phi_1p_1}{m_1} = \frac{\Phi_2p_2}{m_2},$$

$$\frac{q_1}{m_1} = \frac{q_2}{m_2} = \sqrt{4\pi\varepsilon G} = 8.616032252\cdot10^{-11}\,\frac{C}{kg}\begin{pmatrix}\leftrightarrow\\ \text{analog}\end{pmatrix}\frac{\Phi_1}{m_1} = \frac{\Phi_2}{m_2} = \sqrt{4\pi\mu G} = 32.45920531\cdot10^{-9}\,\frac{W}{m},$$

$$\frac{q_1q_2}{m_1m_2} = 4\pi\varepsilon G = 74.23601177\cdot10^{-22}\left(\frac{C}{kg}\right)^2\begin{pmatrix}\leftrightarrow\\ \text{analog}\end{pmatrix}\frac{\Phi_1\Phi_2}{m_1m_2} = 4\pi\mu G = 1053.600009\cdot10^{-18}\left(\frac{W}{m}\right)^2,$$

$$\Phi = q\sqrt{\frac{\mu}{\varepsilon}} = m\sqrt{4\pi\mu G} = 32.45920531\cdot10^{-9}\cdot m\,(=)[W_b]\begin{pmatrix}\leftrightarrow\\ \text{analog}\end{pmatrix}q = m\sqrt{4\pi\varepsilon G} = 8.616032252\cdot10^{-11}\cdot m\,(=)[C],$$

$$\frac{m_1}{m_2} = \frac{\Phi_1}{\Phi_2} = \frac{q_1}{q_2} = \frac{v_2}{v_1}\frac{\sqrt{1-\frac{v_1^2}{c^2}}}{\sqrt{1-\frac{v_2^2}{c^2}}}\left(\cong\frac{v_2}{v_1}, v_{1,2} << c\right),\ \frac{\Phi_1}{q_1} = \frac{\Phi_2}{q_2} = \sqrt{\frac{\mu}{\varepsilon}}\ (= 377\,\Omega,\ \text{for vacuum}),$$

$$\frac{\Phi q}{m^2} = 4\pi G\sqrt{\varepsilon\mu} = \frac{4\pi G}{c},\ m = \frac{\Phi}{\sqrt{4\pi\mu G}} = \frac{q}{\sqrt{4\pi\varepsilon G}} = \sqrt{\frac{c\Phi q}{4\pi G}} = \begin{pmatrix}1.160627039\cdot10^{10}\cdot q\\ 3.08079015\cdot10^{7}\cdot\Phi\end{pmatrix} = 597967252.4\sqrt{\Phi q}\,(=)[kg],$$

(2.4-4), (2.4-6), (2.4-7), (2.4-8)

$$c = \frac{1}{\sqrt{\varepsilon\mu}} = 2.99792458\cdot10^8\,\frac{m}{s}\ \text{(in vacuum)}.$$

(See more familiar elaborations in Chapters 1, equations (1.10) - (1.16), and 3.)) ♣]

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Let us now try to explore the simplified modeling of the possible partial electrical dipole polarizations between two electrically neutral masses, which are related to (2.4-7), as illustrated on Fig.2.1. Relatively isolated and mutually distant masses **m = m₁** and **M = m₂** are attracting each other by Newton gravitation force, and at the same time both masses are (hypothetically) becoming slightly electrically polarized, aligning, and orienting their electrical dipoles in a mutually opposite positions that the force between them will only be attractive (as presented on Fig.2.1). The dipole belonging to a mass **m** has electrical charges **+q*** and **–q*** and the dipole belonging to a mass **M** has electrical charges **+Q*** and –**Q*** (**q = q_1**, **Q = q_2**). The dipole length or distance between **+q*** and **–q*** is **d**, and between +**Q*** and –**Q*** is **D**, and distance which is separating centers of masses **m** and **M** is **r**.

After we represent masses attraction by equivalent and effective electrostatic dipoles attraction (using Coulomb force law, based on data and configuration from Fig.2.1, where polarized electrical charges are presented as couples of smaller circles inside of bigger circles), we will get results as shown in (2.4-9) which are equivalent to (2.4-7). What is extremely interesting

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in (2.4-9) is the fact that the force between relevant electrical dipoles (which corresponds to Newton masses attraction) has a negative sign, and here, the first time we have a clear conceptual picture and mathematical explanation why masses are experiencing only a mutually attractive force. In cases of several masses (attracting each other gravitationally), we should imagine an equivalent multi-poles electrical polarization. Of course, we still do not know why and how, at least two neighboring, electrically neutral masses could get certain electrical dipole polarizations. Since "macro-mass and atoms embedded" carriers of electrical charges (electrons and protons) have very much different masses they should be susceptible to the specific level of masses and electric dipoles separation caused by associated rotation, centrifugal forces and accelerated or curvilinear motions, producing rotation-related coupling forces between them. If this is the case, we will be able to explain Newton law and Gravitation from the platform of electromagnetic theory as in (2.4-9) or (2.4-10).

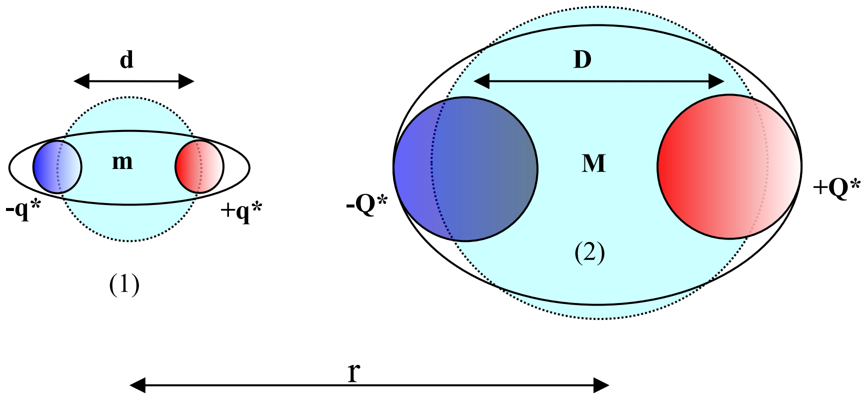

Fig.2.1. Conceptualization of electrical dipoles polarization between two masses

$$F_{12} = -G\frac{mM}{r^2} \cong \frac{1}{4\pi\varepsilon}\frac{q^*Q^*}{r^2} + \frac{1}{4\pi\varepsilon}\frac{q^*Q^*}{(r+\frac{d}{2}+\frac{D}{2})^2} - \frac{1}{4\pi\varepsilon}\frac{q^*Q^*}{(r+\frac{d}{2}-\frac{D}{2})^2} - \frac{1}{4\pi\varepsilon}\frac{q^*Q^*}{(r-\frac{d}{2}+\frac{D}{2})^2} =$$

$$= \frac{1}{4\pi\varepsilon}\frac{q^*Q^*}{r^2}\left[1 + \frac{r^2}{(r+\frac{d}{2}+\frac{D}{2})^2} - \frac{r^2}{(r+\frac{d}{2}-\frac{D}{2})^2} - \frac{r^2}{(r-\frac{d}{2}+\frac{D}{2})^2}\right] = -\frac{1}{4\pi\varepsilon}\frac{qQ}{r^2} = -G\frac{mM}{r^2}, \quad (2.4\text{-}9)$$

$$qQ = q^*Q^*\left[1 + \frac{r^2}{(r+\frac{d}{2}+\frac{D}{2})^2} - \frac{r^2}{(r+\frac{d}{2}-\frac{D}{2})^2} - \frac{r^2}{(r-\frac{d}{2}+\frac{D}{2})^2}\right] \cong q^*Q^*\left[\frac{r^2}{(r+\sqrt{d\cdot D})^2} - 1\right] < 0.$$

We can also find a couple more similar forms of attractive forces applicable to the situation from Fig.2.1, and make similar conclusions like in (2.4-9),

$$F_{12} = -G\frac{mM}{r^2} \cong \frac{1}{4\pi\varepsilon}\left[\frac{q^*Q^*}{r^2} - \frac{q^*Q^*}{(r+D)^2} - \frac{(q^*)^2}{d^2}\right] = \frac{1}{4\pi\varepsilon}\left[\frac{q^*Q^*}{r^2} - \frac{q^*Q^*}{(r+d)^2} - \frac{(Q^*)^2}{D^2}\right] =$$

$$= \frac{1}{4\pi\varepsilon}\frac{q^*Q^*}{r^2}\left[1 - \frac{r^2}{(r+D)^2} - \frac{q^*}{Q^*}(\frac{r}{D})^2\right] = \frac{1}{4\pi\varepsilon}\frac{q^*Q^*}{r^2}\left[1 - \frac{r^2}{(r+d)^2} - \frac{Q^*}{q^*}(\frac{r}{d})^2\right] = \quad (2.4\text{-}10)$$

$$= -\frac{1}{4\pi\varepsilon}\frac{qQ}{r^2} = -G\frac{mM}{r^2},\ qQ = q^*Q^*\left[1 - \frac{r^2}{(r+D)^2} - \frac{q^*}{Q^*}(\frac{r}{D})^2\right] = q^*Q^*\left[1 - \frac{r^2}{(r+d)^2} - \frac{Q^*}{q^*}(\frac{r}{d})^2\right] < 0.$$

As illustrated in equations (2.4-9) and Figure 2.1, we can conceptualize a similar framework to that in equation (2.4-8) but now focusing on the interaction between specific types of associated and equivalent "magnetic dipoles" or weakly magnetized

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

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permanent magnets that are aligned in such a way that only attractive forces between them are possible. For discrete permanent magnets, it is already established that the force between them follows a mathematical form like Coulomb's and Newton's laws. Despite being analyzed separately, electric and magnetic phenomena are interconnected; this becomes apparent when we broaden our analysis.

The simplified electrostatic model presented in Figure 2.1 can be naturally extended to include associated and mutually coupled electric and magnetic fields. The primary sources of these electromagnetic dipole (or multipole) interactions are likely due to the separation between a body's center of mass, center of inertia, center of gravitation, equivalent center of electric charge distribution, and magnetic poles. This separation, where these centers do not coincide, can be particularly significant in rotational scenarios. For instance, the substantial mass difference between positive and negative electric charges (with protons being approximately 2000 times more massive than electrons) can cause centrifugal forces and other inertial effects that result in the separation of electric and magnetic charges or the creation of dipoles.

Moreover, rotational, circular, and other curvilinear motions are likely fundamental properties of matter throughout the universe. The universe maintains a global macro-cosmological orbital momentum equilibrium, meaning that any perturbation in the orbital momentum of a small object will lead to immediate adjustments in the global equilibrium, reflecting macrocosmic entanglement effects. Since global gyromagnetic ratios and electric-charge-to-mass ratios should also be conserved universally, and since these properties are mutually coupled, we can make analogous interpretations of gravitational forces from various perspectives, as elaborated throughout this book.

Consequently, any perturbation in electromagnetic, rotational, or spinning systems within the universe will influence the overall balance of these properties, preserving global conservation laws. Circular, elliptical, pendulum, and other curvilinear motions create accelerated movements toward their centers of rotation, which we perceive as gravitational effects. Historically, gravity has been associated with mass attraction because empirical observations have given the false impression that only mass is significant in such interactions. However, rotational and oscillatory motions, including resonant states, are actual sources of gravitation.

This understanding brings us closer to why forces between electrical charges, permanent magnets, and masses follow similar mathematical forms to the Newton-Coulomb laws. These forces are all part of a globally conserved, mutually coupled system balanced across the universe, integrating with their associated orbital moments (see Chapter 10, equations (10.1.4) – (10.1.7) for a more complete explanation).

Based on equations (2.4-6) through (2.4-10), we can conclude that gravitation might effectively represent a weak electromagnetic dipole (or multipole) interaction, primarily manifested as central forces arising from intrinsic rotational motions in the universe, coupled with relevant electromagnetic charges and field complexities. Analogous and more detailed discussions on electric dipole interactions as sources of gravitation can be found in A.K.T. Assis's work [69], "Gravitation as a Fourth Order Electromagnetic Effect." Additional insights into mass-dipole-related gravitational forces are presented by Jovan Djuric in [33], [71], and [102], titled "Magnetism as Manifestation of Gravitation." Innovative and verifiable ideas that establish a rich analogy between masses, electric charges, electromagnetic fields, gravitational forces, and Coulomb forces, particularly applied to planetary systems and the N. Bohr atom model, are explored in [63] by Arbab I. Arbab and [64] by Marçal de Oliveira Neto.

*Citation from **[157]**, Robyn Arianrhod, "... Amazing gravito-electromagnetism! "An analogy that has tantalized mathematicians and physicists for a century, and which is still a hot if much-debated topic, is that between Albert Einstein's equations of gravity and James Clerk Maxwell's equations of electromagnetism. It's led to an exciting new field of research called "gravito-electromagnetism" – and to the prediction of a new force, "gravito-magnetism". The surprising idea of comparing gravity and electromagnetism began with the intriguing mathematical analogy between the equations of Newtonian gravity and Coulomb's law of electrostatics. Both sets of equations have exactly the same inverse-square form.*

In 1913, Einstein began exploring the much more complex idea of a relativistic gravitational analogue of electromagnetic induction – an idea that was developed by Josef Lense and Hans Thirring in 1918. They used Einstein's final theory of general relativity (GR), which was published in 1916.

Today this so-called "gravito-electromagnetism", or GEM for short, is generally treated mathematically via the "weak field" approximation to the full GR equations – simpler versions that work well in weak fields such as that of the earth.

It turns out that the mathematics of weak fields includes quantities satisfying equations that look remarkably similar to Maxwell's. The "gravito-electric" part can be readily identified with the everyday Newtonian downward force that keeps us anchored to the earth. The "gravito-magnetic" part, however, is something entirely unfamiliar – a new force apparently due to the rotation of the earth (or any large mass).

It's analogous to the way a spinning electron produces a magnetic field via electromagnetic induction, except that mathematically, a massive spinning object mathematically "induces" a "dragging" of space-time itself – as if space-time were like a viscous fluid that's dragged around a rotating ball. (Einstein first identified "frame-dragging", a consequence of general relativity elaborated by Lense and Thirring.) ..."

If electromagnetic forces and charges, encompassing both mechanical and electromagnetic moments are fundamental sources of gravitation arising from the atoms of involved masses, then we might expect gravitational waves to manifest as very low-frequency electromagnetic waves. These would be coupled with cosmic macro-oscillations. In stable solar and planetary systems, we should be able to observe standing and stationary macro-electromagnetic field structures between planets and a local sun (see [144] for more details).

Given that "electromagnetic gravitation" cannot be shielded or stopped using metal or solid-state plates (since such shields, being atomic in nature, will also contribute to the radiant energy flow between attracting masses), a more promising approach to shielding and attenuating gravitational effects may involve using specific cellular or spatially periodic resonant shielding envelopes. This method could potentially manipulate gravitational effects and even produce anti-gravitational effects.

Additionally, an extraordinary and complementary approach to explaining Newton-Coulomb forces is found in the holistic conservation of orbital and spin moments as discussed in [36] by Anthony D. Osborne and N. Vivian Pope, titled "An Angular Momentum Synthesis of 'Gravitational' and 'Electrostatic' Forces" (see also [121]).

There is astronomical observational and experimental evidence suggesting that orbiting and spinning bodies or systems may be strongly coupled in a manner where their orbital and spin moments communicate at speeds potentially 10 orders of magnitude greater than the speed of light (refer to [65], T. Van Flandern, J.P. Vigier). If these findings are confirmed, they could cast significant doubt on the foundational principles of Relativity Theory. Consequently, both Newtonian gravitation and Einstein's theory of relativity may need substantial revision. Gravitation appears to be a well-coupled and synchronized manifestation of electromagnetic, mechanical, electromechanical effects, and the Wave-Particle Duality (PWDC) properties of matter in our universe.

Regarding the origins of celestial and astronomical object rotation, challenging and intriguing sources can be found in the works cited under [51] by Pavle Savic, R. Kasanin, and V. Celebonovic.

For further exploration of gravitation, fundamental constants, masses, and forces in our universe, consider the following sources:

- Anatoly V. Rykov, as discussed in literature under [32], [3], [33], and [34]: For example, the Journal of Theoretics, PACS: 03.50.Kk, "Magnetism as Manifestation of Gravitation."

- Dr. Jovan Djuric, with relevant papers listed under [33], [71], and [102], and additional information available at http://www.journaloftheoretics.com/ and http://jovandjuric.tripod.com/.

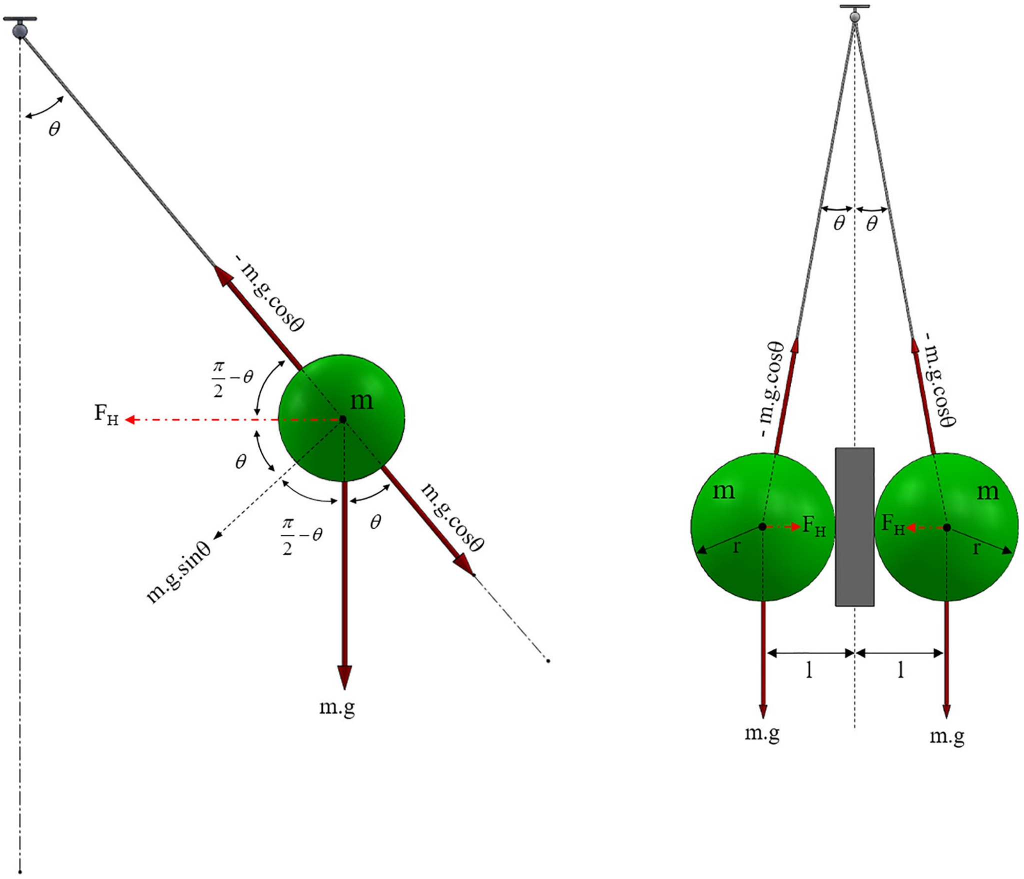
- Dipl.-Ing. Andrija S. Radović, addressing topics such as "Gravitation Field as Annulled Electromagnetic Field" and "Essence of Inertia and Gravitation" (see his website at http://www.andrijar.com/).

- Dr. László Körtvélyessy, who has contributed to discussions on the Electric Universe ([72]).

- Greg Poole's work on Cosmic String Theory: Gravity and Tension ([144]).

- Evgeny Bryndin's publication ([171]), which explores various practical, imaginative, challenging and hypothetical ideas about gravity.

[♣ COMMENTS & FREE-THINKING CORNER: How to detect hypothetical oscillations and forces related to Gravitation



Single and two masses pendulum

Let us try to explore the effects of gravitational attraction F_H between two masses, based on pendulum forces analysis (as presented in the picture above). If we consider only single mass pendulum (left side picture), horizontal force component, F_H will be,

$$F_H = mg \cdot \sin\theta \cdot \cos\theta \cdot$$

If we create a similar two-mass pendulum (as presented on the right-side picture), the attractive horizontal force between two masses, F_H, will be at least two times stronger compared to a single mass pendulum,

$$F_H = 2mg \cdot \sin\theta \cdot \cos\theta \cdot$$

However, here we also need to consider force of horizontal gravitational attraction between two masses, and this will produce,

$$F_H = 2mg \cdot \sin\theta \cdot \cos\theta + \delta F_H = 2mg \cdot \sin\theta \cdot \cos\theta + G\frac{m \cdot m}{R^2} =$$

$$= 2mg \cdot \sin\theta \cdot \cos\theta + G\frac{m^2}{R^2}, (R = 2l).$$

Another questionable assumption here is that a two-mass pendulum will intrinsically present a specific half-wave dipole or mechanical resonator. Consequently, the mentioned mechanical resonator will experience certain (probably exceptionally low amplitude) oscillatory or resonant motion, and if this could be the case, we will have, either.

$$F_H = 2mg \cdot \sin\theta \cdot \cos\theta + G\frac{m^2}{R^2}\cos\omega_H t,$$

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

or more probably,

$$F_H = 2mg \cdot \sin\theta \cdot \cos\theta + G\frac{m^2}{R^2} + \Delta F_H \cdot \cos\omega_H t \ .$$

Now, let us imagine that we place one piezoelectric element, or sensor (like an accelerometer), between two masses, as already presented on the right-side picture with a gray-color rectangular block. New experimental and analytical options, starting from here, are:

1. If the oscillatory force component ($\cos\omega_H t$) really exists, we should be able to detect specific oscillatory-voltage signals on the piezoelectric element between two masses. Moreover, we could discover or listen to some kind of "*breathing and pulsating*" of our Universe, by analyzing voltage-signal detected with a piezoelectric element.

2. Or we can measure complex electrical Impedance-Phase-Frequency curves of the two-mass system (directly on the piezoelectric element) and try to find sensitive and "*active*" impedance-frequency zones and draw some exciting conclusions,

3. Alternatively, we could externally bring specific oscillatory voltage signal on electrodes of a piezoelectric transducer and analyze the response of the two-mass pendulum system, and its interaction with our Universe and involved forces.

4. Also, we could place a permanent magnet (in different positions) in the vicinity of the two-masses-pendulum and try to detect if masses are being influenced by a magnetic field, and mutually interacting in a different way (of course, experimenting with masses that are not ferromagnetic or magnetically active).

5. A two-mass pendulum with the piezoelectric sensor could also serve as a detector of vibrations, and detector of seismic and gravitational perturbations.

The main idea (or objective) here is to verify if the oscillatory force component $\Delta F_H \cdot \cos\omega_H t$, or maybe $G\frac{m^2}{R^2}\cos\omega_H t$ (or something similar) really exists.

Later, based on geometrical relations (such as $R = 2l$, relationships for half-wavelength resonant dipoles, matter waves, etc.), we could unveil certain, still hidden, nature of gravitational forces, and show that gravitation is an attractive force only between moving or oscillating masses (or between electromechanical dipoles) with associated angular and linear mechanical moments, and most probably with some coupled electromagnetic moments and charges.

Another intention here is to show that gravitational attraction, only between neutral, not-charged, standstill (or static) masses, is not realistic. In other words, we may conclude and prove that we are living in a continuously moving (rotating and oscillating) Universe, where all motions are mutually coupled, and have linear and angular components (combined with complementary electromagnetic moments and charges).

****Summary of Gravitation as Conceptualized in This Book**: **

1. **Reconceptualizing Gravitation:**

This book introduces new perspectives on gravitation, viewing it as related to stationary and resonant standing matter-wave states of masses, or energy agglomerations within natural fields. These states involve mutually coupled and synchronized linear, orbital, and spinning motions, as well as intrinsically associated electromagnetic properties. Rather than static masses being the principal and sole source of gravitation, electromagnetically charged or polarized vibrating masses, with coupled linear and angular mechanical moments, are identified as the true sources of gravitation. This concept is supported by updated translational and rotational symmetry, along with the conservation of linear and angular moments and energy. Atoms and masses in motion, including oscillations and other angular or torsional motions, with their associated electromagnetic complexity (coupled linear, angular, and electromagnetic moments and dipoles), are genuine sources of gravitation. Consequently, atoms, or atomic fields with dynamic properties and a universal natural force structure, are primary sources of gravitation. These internal atomic field structures extend externally, synchronizing with other atoms and masses in the structurally vibrating universe, naturally creating $1/r^2$-dependent central forces (see Chapters 2, 8, and 10 for more details).

2. **Schrödinger's Wave Equations and Gravitation:**

Schrödinger-type wave equations are applicable to gravitation, particularly concerning orbital planetary motions. The Schrödinger equation is a direct product of the classical wave equation when the wavefunction model takes the form of an analytic signal. Gravitation may be a specific manifestation of mixed and mutually dependent standing electromagnetic and electromechanical wave formations. These waves reflect an essential electromagnetic force acting between motional, mutually coupled, oscillating electric and magnetic charges, dipoles, and/or moments within certain global spatial-temporal configuration (see Chapter 2 for more details). Consequently, theories related to relativity should also be redesigned and updated being as products of an appropriately revised electromagnetic theory (see Chapter 3).

3. **Gravitation and Macroscopic Masses:**

To summarize, gravitation is the force between agglomerated macroscopic masses, which are composed of atoms and molecules with intrinsic electromagnetic complexity. These masses consist of oscillating electric charges, magnetic moments, and polarized electromagnetic dipoles or multipoles, each internally structured as standing-wave resonators. Due to this internal electromagnetic complexity and the constant global linear, angular, and oscillatory motions within the universe, structurally induced effects of electromagnetic forces arise inside and around atoms and other masses, producing the observable effects of gravitation. Gravitation, as a recognizable and measurable effect, diminishes if the attracting micro-world masses or distances between them become exceedingly small (for instance, smaller than 100 micrometers). At such scales, there are not enough electrically charged and polarized electromagnetic entities to create effects of currents, voltages, induction, fluxes, and eventually produce forces that we typically interpret as gravitational effects.

4. **Shielding and Manipulating Gravitation:**

a) Current Understanding:

Experiments with various shields, such as closed cages, envelopes, or solid barriers around and between masses have shown that the weight of the masses is not reduced, suggesting that gravitation penetrates such shielding. This supports the idea that gravitation sources are extended atomic fields, which produce internal electromagnetic dipole polarization. These polarizations naturally align with the applied shield, rendering it effectively transparent to such formations. An alternative, speculative hypothesis suggests that the electromagnetic field may have additional components not covered by contemporary Maxwell theory, allowing it to penetrate electromagnetic shielding (see [152] William J. Hooper). This implies that a specific background matter state, with omnidirectional vortex and spinning formations of an invisible, fine fluid (or ether), might exist around involved masses. Zero-point quantum vacuum fluctuations and Casimir (attractive and repulsive force) effects support such ideas about ether and radiant energy. This concept aligns with the ideas of Nikola Tesla and Rudjer Boskovic concerning gravitation and radiant energy, where standing-wave formations occur between mutually attracting and polarized masses, adhering to the $1/r^2$ central Coulomb force, which penetrates applied electromagnetic shielding.

b) Speculative Possibilities:

There are speculative indications that gravitational shielding, weight reduction, and gravitation field-reflecting effects might be achievable using shielding objects with specific spatial resonant structures.

Russian professor Viktor Stepanovich Grebennikov [149] described various effects related to gravitation, levitation, and temporal-spatial matter-wave phenomena in structurally periodic geometric structures created by insects, such as beehive cells or dry honeycomb structures. Objects artificially assembled in a similar manner have been reported to create weight reduction. Additionally, in certain situations, spinning gyroscopes may influence or interfere with gravitation.

5. **Why We Only Observe Attractive Gravitational Forces:**

Our current understanding of gravitation is limited, leading us to observe only attractive gravitational forces. This limitation stems from our incomplete and restrictive conceptual models of gravitation, particularly the Newtonian and Einsteinian frameworks. Stable macroscopic masses, which are agglomerations of smaller masses and/or atoms, can be imagined as residing in the nodal zones of a structurally resonating system with standing matter waves. In these nodal zones, we naturally detect only attractive, agglomerating forces (or gravitational forces). However, standing wave structures also have anti-nodal zones, where repulsive (or anti-gravitational) forces might be observed. This suggests that our universe is structurally resonating, a concept qualitatively indicated by Nikola Tesla and Rudjer Boskovic.

6. **The Mathematical Similarity Between Newton’s and Coulomb’s Laws:**

The mathematical similarity between Newton’s law of gravitation and Coulomb’s law of electrostatic force arises because both forces fundamentally originate from electric and magnetic charges and polarizations. Masses can be conceptualized (or mathematically presented) as being directly proportional to their internal electric and magnetic dipoles and charges. Consequently, when applying Newton’s law, we are effectively or indirectly applying Coulomb’s law (see Chapter 2 for more details).

7. **Black or Dark Matter, Masses, and Energies in Relation to Gravitation:**

Mass and energy are not confined to the solid boundaries of a body or a simple “mass-energy-moments” description. Masses formations should be better understood as part of a dynamic, structurally resonating universe. Different aspects of mass and energy are interconnected within this framework.

[♣ Free thinking corner... brainstorming.

The state of macroscopic matter (whether solid, liquid, or gaseous) involves internal atomic and molecular structures. In electromagnetically neutral masses, the internal configuration of electric and magnetic items (such as electrons and protons) is mutually compensated, neutralized, and stabilized. However, when the external state of motion of a mass changes, the internal electromagnetic states of its constituent particles adjust accordingly, leading to the creation of internal, non-compensated electric and magnetic dipoles. These internal polarizations can create macroscopically measurable forces between masses, which are often mistakenly interpreted as purely mechanical or gravitational.

The existence of spontaneous dipole polarizations, if proven, would indicate that all motion in the universe is non-linear, encompassing curvilinear, torsional, angular, rotational, and spinning motions. Given the overwhelming presence of such rotational elements, we can expect the emergence of centrifugal electromagnetic and mass separations, leading to the formation of electric and magnetic dipoles. This phenomenon may result in an electromagnetic or Coulomb-J. Michel attraction between large masses, which has been traditionally characterized as gravitational attraction. This involves a unity of electromagnetic and mechanical effects, integrating linear, spinning, and rotational motions.

The external consequences of these interactions, responsible for energy-momentum coupling and exchanges, are the manifestations of matter waves associated with moving masses, which likely have a dominant electromagnetic nature. We also know that high-power mechanical, ultrasonic, or acoustic energy, as well as moments, forces, oscillations, and vibrations, can be generated and transmitted by applying various signal-modulating techniques to acoustic emitters, electromagnetic and laser beams, and dynamic plasma states, using these as carriers for lower-frequency mechanical vibrations (see Chapter 10 for further discussion). ♣]

....

It is also evident that gravitation should be causally related to the "two-body" problem, being the force between two bodies that are in a (mutually) relative motion. If bodies in question (in a Laboratory reference system) have masses m_1 and m_2, velocities v_1 and v_2, from classical mechanics we know that kinetic energy balance of such system can be presented as (2.4-11); -See Fig.4.1.5, chapter 4.1, and usual mathematical treatment of two-body problems, easily found in physics and mechanics books, is as follows,

$$\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2=\frac{1}{2}m_cv_c^2+\frac{1}{2}m_rv_r^2,$$
$$m_c=m_1+m_2,\ m_r=\frac{m_1m_2}{m_1+m_2},\ \vec{v}_r=\vec{v}_1-\vec{v}_2, \vec{v}_c=\frac{m_1\vec{v}_1+m_1\vec{v}_1}{m_1+m_2}, \qquad (2.4\text{-}11)$$

or it could also be valid (by applying a reversed analogy and most probable approximations, passing from non-relativistic motions to relativistic ones, where all velocities are related to the same laboratory system),

$$\left\{\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2=\frac{1}{2}m_cv_c^2+\frac{1}{2}m_rv_r^2\right\}\Rightarrow\left\{(\gamma_1-1)m_1c^2+(\gamma_2-1)m_2c^2=(\gamma_c-1)m_cc^2+(\gamma_r-1)m_rc^2\right\}\Leftrightarrow$$
$$\Leftrightarrow\left\{\begin{array}{l}\frac{\gamma_1m_1v_1^2}{1+\sqrt{1-v_1^2/c^2}}+\frac{\gamma_2m_2v_2^2}{1+\sqrt{1-v_2^2/c^2}}=\frac{\gamma_cm_cv_c^2}{1+\sqrt{1-v_c^2/c^2}}+\frac{\gamma_rm_rv_r^2}{1+\sqrt{1-v_r^2/c^2}},\\ \gamma_r=1/\sqrt{1-v_r^2/c^2},\ \gamma_i=1/\sqrt{1-v_i^2/c^2}\cong\left(1+\frac{1}{2}\frac{v_i^2}{c^2}\right)_{v<<c}\end{array}\right\}\Rightarrow \qquad (2.4\text{-}12)$$
$$\Rightarrow\left\{\begin{array}{l}m_1v_1dv_1+m_2v_2dv_2=m_cv_cdv_c+m_rv_rdv_r=dE_{k1}+dE_{k2}=dE_{kr}+dE_{kc}=dE_k,\\ v_1dp_1+v_2dp_2=v_cdp_c+v_rdp_r=dE_k=v_cdp,\ p_i=\gamma_im_iv_i,\vec{p}=\vec{p}_1+\vec{p}_2=\gamma_1m_1\vec{v}_1+\gamma_2m_2\vec{v}_2=\gamma_cm_c\vec{v}_c\end{array}\right\}$$

In (2.4-12) we are faced with the imaginative and challenging possibility to understand the new kinematic reality of an artificial (mathematical) object that has mass m_r and velocity v_r, but also has an equivalent zero linear momentum ($\sum\vec{p}_r=\vec{0}$), since $\vec{p}=\vec{p}_1+\vec{p}_2=\gamma_1m_1\vec{v}_1+\gamma_2m_2\vec{v}_2=\gamma_cm_c\vec{v}_c$. Apparently, such m_r should be composed of at least two objects (effectively rotating around m_c of local center of mass) with mutually canceling moments. We can additionally address the nature of such m_r by applying Newton gravitation force law between m_1 and m_2 (including Kepler laws and Binary Systems relations). We will see that the same gravitational force is acting between masses m_c and m_r, and between m_1 and m_2 when compared in the same laboratory system of coordinates,

$$F_{12}=G\frac{m_1m_2}{r_{12}^2}=G\frac{m_cm_r}{r_{12}^2}\Leftrightarrow m_1m_2=m_cm_r,$$
$$E_{kr}=\left[\left(\frac{\gamma_rm_rv_r^2}{1+\sqrt{1-v_r^2/c^2}}\right)_{v_r<<c}\cong\frac{1}{2}m_rv_r^2\right]=\int_0^{r_{12}}F_{12}dr_{12}\Rightarrow \qquad (2.4\text{-}13)$$
$$dE_{kr}=m_rv_rdv_r=F_{12}dr_{12}\Rightarrow F_{12}=m_r\frac{dv_r}{dt}=m_ra_r=m_r\frac{v_r^2}{r_{12}}=G\frac{m_1m_2}{r_{12}^2}=G\frac{m_cm_r}{r_{12}^2}\Rightarrow$$
$$\Rightarrow G=\frac{r_{12}v_r^2}{(m_1+m_2)}=\frac{r_{12}v_r^2}{m_c}\Rightarrow F_{12}=\frac{r_{12}v_r^2}{m_c}\frac{m_1m_2}{r_{12}^2}=\left(\frac{r_{12}}{m_c}\right)\frac{(m_1v_r)(m_2v_r)}{r_{12}^2}=\left(\frac{r_{12}}{m_c}\right)\frac{p_{1r}p_{2r}}{r_{12}^2}=$$
$$=\frac{r_{12}v_r^2}{m_c}\frac{m_cm_r}{r_{12}^2}=\left(\frac{r_{12}}{m_c}\right)\frac{(m_cv_r)(m_rv_r)}{r_{12}^2}=\left(\frac{r_{12}}{m_c}\right)\frac{p_{cr}p_{rr}}{r_{12}^2},\ p_{cr}=m_cv_r,p_{rr}=m_rv_r.$$

From (2.4-13) we could also conclude that force of gravitation between two moving masses is directly proportional to the product between involved moments (at least dimensionally),

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$F_{12} = \left(\frac{r_{12}}{m_c}\right)\frac{p_{1r}p_{2r}}{r_{12}^2} = \left(\frac{r_{12}}{m_c}\right)\frac{p_{cr}p_{rr}}{r_{12}^2}$, in a way that should be better analyzed and completely understood. We know that all masses in our universe are in mutual relative motions, and such intrinsic or holistic movements are bringing hidden, linear, and angular moments in the same framework, regarding gravitation (see more complete explanation about matter-wave complexity of motional messes in Chapter 10., equations (10.1.1)- (10.1.8)).

Consequently, Newtonian force presented initially as depending on the product between involved masses, could be a lucky coincidence and circumstantial result or acceptable conceptual mistake, meaning that real or effective sources of gravitation are involved as linear and angular, mechanical, and electromagnetic moments including associated, intrinsic vibration related states (not only static masses). Of course, specific intrinsic (hidden, or background) rotation (with angular and linear velocity components) should also be associated to all masses in our universe to give more practical meaning and weight to (2.4-13). Here, we are implicitly introducing the concept that any two masses in linear, mutually relative motion (in the process of approaching), are naturally being enriched (or complemented) with elements of angular motion, which in some cases could generate orbital (and spinning) motions (or planetary and satellite motions). See also similar exercises around equations (2.4-5.1). Later in the same chapter, with elaborations around equations (2.11.13-1) - (2.11.13-5), (2.11.23) and (2.11.24), we will additionally address similar analogical and challenging options. In Chapter 10 of this book, we can find the most complete explanation of the same situation regarding hidden or background velocity parameters and Newtonian attraction between relevant linear and angular moments (see (10.1.1) - (10.1.7)).

By considering validity of (2.4-13) in connection with Kepler laws, we can conclude that (from the point of view of a relevant laboratory system) reduced mass m_r tends to be “symmetrically self-balancing distributed”, and to rotate around m_c, or it should be effectively (mathematically) presentable as such rotation (later in this chapter proven by (2.11.13-1) - (2.11.13-5)). Such hidden, rotating-like situations can be exploited as the background for explaining the origins of de Broglie matter waves and planetary orbiting; -see chapter 4.1. If there were no rotation and spinning motions, including associated centrifugal forces, we would have only attractive forces between masses, with a permanent tendency of masses to agglomerate. We already know that this is not a generally valid case in our universe (at least not concerning stable planetary systems), and the same could easily and analogically be extended to orbital situations in atom or planetary models, considering Coulomb-Michel electrostatic and magnetostatic forces.

***Laboratory Coordinate System*, observes what is happening between two mutually moving bodies with masses m_1 and m_2, and it belongs to much less general and divine “kitchen”, compared to what is happening between m_r and m_c in the same system, or in a dominant local *Center of Mass System* in relation to matter-waves, orbital and spinning motions, and particles creation, or disintegration …**

Let us now try to roughly formulate (in the same Laboratory system of coordinates) the same mass-product relation $m_1m_2 = m_cm_r$ from (2.4-13), considering that masses are velocity-dependent (as in (2.4.12)). This will produce the following options,

$$\left(\begin{array}{l} m_1m_2 = m_cm_r\,,\ m_1 \rightarrow \gamma_1m_1, m_2 \rightarrow \gamma_2m_2 \\ m_c \rightarrow \gamma_cm_c = \gamma_1m_1 + \gamma_2m_2\,, m_r \rightarrow \gamma_rm_r = \dfrac{\gamma_1m_1\cdot\gamma_2m_2}{\gamma_1m_1+\gamma_2m_2} \end{array}\right) \Rightarrow \gamma_1m_1\cdot\gamma_2m_2 = \gamma_cm_c\cdot\gamma_rm_r \Leftrightarrow$$

$$\Leftrightarrow \gamma_1\cdot\gamma_2 = \gamma_c\cdot\gamma_r \Leftrightarrow \left\{\begin{array}{l} \gamma_r = \dfrac{\gamma_1\cdot\gamma_2}{\gamma_c} = \sqrt{\dfrac{1-v_c^2/c^2}{(1-v_1^2/c^2)(1-v_2^2/c^2)}} \\ v_1^2+v_2^2-\left(\dfrac{v_1v_2}{c}\right)^2 = v_r^2+v_c^2-\left(\dfrac{v_rv_c}{c}\right)^2 \end{array}\right\}. \quad (2.4\text{-}14)$$

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

Since, in (2.4-14) we are presenting mutually similar situations in the same reference system, time meaning, and/or the time-flow relevant for all associated objects should be the same. Of course, we should additionally verify if γ_r in (2.4-12) and (2.4-14) are mutually identical since the unifying, and more general case would be,

$$
\begin{aligned}
&E_{k1}+E_{k2}=E_{kc}+E_{kr}\Leftrightarrow(\gamma_1-1)m_1c^2+(\gamma_2-1)m_2c^2=(\gamma_c-1)m_cc^2+(\gamma_r-1)m_rc^2\Leftrightarrow\\
&\Leftrightarrow(\gamma_1-1)m_1c^2+(\gamma_2-1)m_2c^2=(\gamma_c-1)m_cc^2+E_{kr}\\
&\Leftrightarrow(\gamma_1m_1+\gamma_2m_2)c^2-(m_1+m_2)c^2=(\gamma_cm_c+\gamma_rm_r)c^2-(m_c+m_r)c^2\Rightarrow\\
&\Rightarrow E_{kr}=(\gamma_1-1)m_1c^2+(\gamma_2-1)m_2c^2-(\gamma_c-1)m_cc^2=\\
&=(\gamma_1-1)m_1c^2+(\gamma_2-1)m_2c^2-(\gamma_c-1)m_1c^2-(\gamma_c-1)m_2c^2=\\
&=(\gamma_1-\gamma_c)m_1c^2+(\gamma_2-\gamma_c)m_2c^2=(\gamma_r-1)m_rc^2\left(\cong\frac{1}{2}m_rv_r^2\right)_{v_r<<c}\Leftrightarrow\\
&\Rightarrow\gamma_r=\frac{1}{\sqrt{1-v_r^2/c^2}}=\left\{\begin{array}{l}1+(\gamma_1-\gamma_c)\frac{m_1+m_2}{m_2}+(\gamma_2-\gamma_c)\frac{m_1+m_2}{m_1}=1+\frac{\gamma_cm_c^2}{m_1m_2}=1+\frac{\gamma_cm_c}{m_r}\\(\text{and/or})\\\frac{\gamma_1\cdot\gamma_2}{\gamma_c}=\sqrt{\frac{1-v_c^2/c^2}{(1-v_1^2/c^2)(1-v_2^2/c^2)}}\end{array}\right\}.
\end{aligned}
\tag{2.4-15}
$$

In fact, here (with (2.4-11) – (2.4-15)) we are roughly trying to construct or elaborate conditions when a two-particle system (with particles m_1 and m_2) is evolving or transforming into another two-particle system with particles m_c and m_r. The characteristic property of both two-particle systems (observed in the same Laboratory system) is that both have the same (total) linear momentum, $\vec{p}=\vec{p}_1+\vec{p}_2=\gamma_1m_1\vec{v}_1+\gamma_2m_2\vec{v}_2=\gamma_cm_c\vec{v}_c$, the same (total) kinetic energy $E_k=(\gamma_1m_1+\gamma_2m_2)c^2-(m_1+m_2)c^2$ $=(\gamma_cm_c+\gamma_rm_r)c^2-(m_c+m_r)c^2$, and the same total energy $E=E_c=(\gamma_1m_1+\gamma_2m_2)c^2=(\gamma_cm_c+\gamma_rm_r)c^2$. This is giving a chance to formulate two 4-vectors representing such systems, $\overline{P}_4=(p,\frac{E}{c})$,$\overline{P}_{4c}=(p,\frac{E_c}{c})$,

$$
\begin{aligned}
&\left\{\begin{array}{c}\left[\begin{array}{l}\overline{P}_4^2=(p,\frac{E}{c})^2=\text{inv.},\quad \overline{P}_{4c}^2=(p,\frac{E_c}{c})^2=\text{inv.}\\ \vec{p}=\vec{p}_1+\vec{p}_2=\gamma_1m_1\vec{v}_1+\gamma_2m_2\vec{v}_2=\gamma_cm_c\vec{v}_c\end{array}\right]\Rightarrow\\
(\gamma_1m_1+\gamma_2m_2)c^2-(m_1+m_2)c^2=E_{k1}+E_{k2}=E-E_0=E_c-E_{0c}=E_{kc}+E_{kr}=(\gamma_cm_c+\gamma_rm_r)c^2-(m_c+m_r)c^2\\
E=(\gamma_1m_1+\gamma_2m_2)c^2=E_1+E_2=E_0+E_k,E_0=(m_1+m_2)c^2=m_cc^2=E_{01}+E_{02}\\
E_c=(\gamma_cm_c+\gamma_rm_r)c^2=E_{0c}+E_{kc},E_{0c}=(m_c+m_r)c^2,E_{kc}=(\gamma_c-1)m_cc^2,E_{kr}=(\gamma_r-1)m_rc^2\\
E=E_{01}+E_{k1}+E_{02}+E_{k2}=E_{0c}+E_{kc}+E_{0r}+E_{kr},E_{0r}=(m_c+m_r)c^2=E_{0c}+E_{0r}\end{array}\right\}\Rightarrow\\
&\Rightarrow\left\{\begin{array}{l}p^2-\frac{E^2}{c^2}=-\frac{E_0^2}{c^2}\Leftrightarrow p^2c^2-E^2=-E_0^2\Leftrightarrow p^2c^2+E_0^2=E^2\\ p^2-\frac{E_c^2}{c^2}=-\frac{E_{0c}^2}{c^2}\Leftrightarrow p^2c^2-E_c^2=-E_{0c}^2\Leftrightarrow p^2c^2+E_{0c}^2=E_c^2\end{array}\right\}\Rightarrow\\
&\Rightarrow\left\{\begin{array}{l}E^2-E_0^2=E_c^2-E_{0c}^2=p^2c^2=(E-E_0)(E+E_0)=(E_c-E_{0c})(E_c+E_{0c}),E-E_0=E_c-E_{0c}\\ p^2=(\gamma_1m_1+\gamma_2m_2)^2c^2-(m_1+m_2)^2c^2=(\gamma_cm_c+\gamma_rm_r)^2c^2-(m_c+m_r)^2c^2\end{array}\right\}\Rightarrow\\
&\Rightarrow\left\{\begin{array}{l}E+E_0=E_c+E_{0c}\\E-E_0=E_c-E_{0c}\end{array}\right\}\Leftrightarrow E=E_c=(\gamma_1m_1+\gamma_2m_2)c^2=(\gamma_cm_c+\gamma_rm_r)c^2\Rightarrow\\
&\Rightarrow\gamma_rm_r=\gamma_1m_1+\gamma_2m_2-\gamma_cm_c,\gamma_r=\gamma_1\frac{m_1}{m_r}+\gamma_2\frac{m_2}{m_r}-\gamma_c\frac{m_c}{m_r}.
\end{aligned}
\tag{2.4-16}
$$

Here (in (2.4-12) – (2.4-16)) we are merely speculating or hypothesizing with analogical applications and rough approximations of relativistic-like (velocity dependent) mass-energy formulations (but presently we do not take too seriously what would the same expressions be if somebody formulates them using strictly mathematical Relativity theory methods and Lorentz transformations). At least, we know for sure that kinetic energy balance (2.4-12) will generate non-relativistic kinetic energy balance (2.4-11) in cases when all relevant velocities are much smaller than the light speed c. It will be interesting to find which γ_r formulation, summarized in (2.4-17), is closest to a correct and generally valid case that will satisfy (2.4-13), (2.4-14), $m_1 m_2 = m_c m_r$, as well as to satisfy energy and momentum conservation laws ((2.4-11) – (2.4-16)) for any relative speed of mutually interacting objects, since until here we found the following options,

$$\gamma_r = \frac{1}{\sqrt{1-v_r^2/c^2}} = \begin{cases} \gamma_1 \frac{m_1}{m_r} + \gamma_2 \frac{m_2}{m_r} - \gamma_c \frac{m_c}{m_r}, & \text{or} \\ \frac{\gamma_1 \cdot \gamma_2}{\gamma_c} = \sqrt{\frac{1-v_c^2/c^2}{(1-v_1^2/c^2)(1-v_2^2/c^2)}}, & \text{or} \\ 1 - \frac{m_c}{m_r}\left[\sqrt{1+(\gamma_c \frac{v_c}{c})^2} - \gamma_c\right], & \text{or} \\ 1+(\gamma_1-\gamma_c)\frac{m_1+m_2}{m_2} + (\gamma_2-\gamma_c)\frac{m_1+m_2}{m_1} = 1+\gamma_c \frac{m_c}{m_r}. & \end{cases} \quad (2.4\text{-}17)$$

The consequences of such (little bit lose) analytical testing and speculations will contribute to showing that Lorentz contraction factor $\gamma = (1-\frac{v^2}{c^2})^{-0.5}$ and Lorentz transformations could be differently addressed or modified regarding effective particle velocity or velocity-momentum influence on Newtonian masses attraction. In addition, any mass in motion has elements of particle-wave duality, or presents a matter wave group, and can be interpreted as a complex, Analytic Signal function (see more in Chapter 10.; -equations (10.1.1) - (10.1.7)). For instance, by considering accelerated curvilinear motions, involved angular and spinning motions, gravitational potential and/or other forces and fields present during particles interactions, we may find another, more accurate understanding, or replacement for current Lorentz transformations. At the same time, we are also challenging the meaning, nature, and universality of contemporary Lorentz transformations (since we will attempt to show later (in the third chapter), that real origin of Lorentz transformations should be in Maxwell electromagnetic theory and not so much in mechanics, optics, and contemporary relativity theory based on imaginative, mental experiments). Here ((2.4-11) to (2.4-17)), the *temporal nature* and *time-flow,* for all participants are the same (since everything is presented in the same Laboratory reference frame, and the same Newtonian force is acting between mutually corresponding masses). See similar hypothetical approach around equations (2.11.24).

In this book it is shown that particle-wave duality of matter in motion is causally related to the fields and forces manifesting in two (or many) bodies interactions, or more specifically to motional energy associated with reduced mass m_r. <u>Since kinetic energy E_{kr} of reduced mass m_r cannot be treated merely as an energy of a particle in a purely linear motion, the remaining, imaginative, and most probable, or most promising case (regarding understanding formations of solar, planetary systems, atoms' structure, and origins of particle-wave duality) is that E_{kr} is a form of rotational energy (or energy which has an angular motion nature).</u> The meaning of such speculations is to conclude that reduced mass m_r has a tendency towards establishing rotational (closed line) motion around a mass m_c. All of that should also be in relation to Kepler and Coulomb laws, in case of electrically charged masses, because Newton (or Coulomb) attractive force should be balanced with a similar, centrifugal, and repulsive force. See much more about familiar situations in the same chapter under "2.3.3. Macro-Cosmological Matter-Waves and Gravitation"; -equations (2.11.13-5) - (2.11.13-5), and from (2.11.10) until (2.11.21), as well as number of supporting elements in chapter 4.1).

The approach here, regarding relativistic-like scaling, Lorentz, or gamma contraction factors γ_i, (and Lorentz transformations) is essentially non-relativistic. Briefly, what is proposed here is:

<u>First</u>: Let us forget or neglect for a moment that Relativity theory and Lorentz transformations exist in a mutual context of such somewhat hybridized, partially speculative, and probably incomplete theory (*see extremely relevant, exceptional, and unavoidable references [35], [80] and [81] for understanding such serious statements much better*). Physics anyway evolved (even before Einstein, independently from any form of modern Relativity theory) towards understanding that the total rest energy of a particle

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can be quantified as $E_0 = mc^2$. The extension of such mass-energy relation is that total particle energy in motion is increasing multiplied by "scaling, or gamma factor" $E = \gamma mc^2 = \gamma E_0$, $\gamma = (1 - v^2/c^2)^{-0.5}$, where particle kinetic energy is $E_k = (\gamma - 1)mc^2 = E - E_0$ (what is experimentally verifiable). This is additionally reinforced by the fact that for non-relativistic velocities we will get kinetic energy expression known from Classical Mechanics, $E_k = (\gamma - 1)mc^2 /_{(v<<c)} \cong \frac{1}{2}mv^2$. In other words, transitions, and connections between Classical and Relativistic Mechanics should be smoother and more natural, without conceptual, logical, and mathematical discontinuities created by postulating and inventing artificial and non-realistic missing links and challenging concepts, as sporadically practiced in Relativity theory.

*When presenting specific particle in motion, the most significant and probably only relevant links to Relativity theory (without covariant or Lorentz invariant misrepresentations), are definitions of particle "**proper time** τ", "**proper mass** m_0" and "**proper energy** E_0", such as,*

$$dt = \gamma d\tau,\ d\tau^2 = dt^2 - dr^2/c^2 = \text{invariant},\ (dr^2 = dx^2 + dy^2 + dz^2,\ v = dr/dt),$$

$$\begin{bmatrix} m = \gamma m_0 \\ E = \gamma E_0 = \gamma m_0 c^2 \\ p = \gamma p_0 = mv = \gamma m_0 v \\ \gamma = (1 - v^2/c^2)^{1/2} \end{bmatrix} \Rightarrow p^2 - \frac{E^2}{c^2} = \text{invariant} = -\frac{E_0^2}{c^2}. \quad (2.4\text{-}17.1)$$

*Proper time τ (measured with a single co-moving clock, linked to the particle in question) has the same meaning as well elaborated and clarified by Thomas E. Phipps, Jr. in [35], and proper mass and proper energy, m_0, E_0, are only analogical names introduced here to underline the analogy with a proper time ($dt = \gamma d\tau \leftrightarrow m = \gamma m_0, E = \gamma E_0$), but effectively meaning rest mass and rest energy. Everything else what we know from Einstein-Minkowski 4-vectors formalism should be explicable based on "**proper parameters**". Inertial "frame time" t is the time measured by a spatially extended set of clocks at rest in that inertial frame (and something similar is valid for inertial "frame mass" and inertial "frame energy", m and E.*

***<u>Second</u>**: Since the number of situations in relation to motions and energy and momentum conservation laws are anyway experimentally verifiable, here we merely utilize only obvious and no-doubts, well-operating energy-momentum, velocity dependent relativistic formulas, without making too many of glorifying, apologetic and generalizing links to A. Einstein Relativity theory, until we rich the next, more general level of deeper understanding. The fact is that A. Einstein Relativity theory, besides establishing challenging or imaginative (some of them maybe not easy verifiable) concepts and practices, also has few of convenient and useful building blocks, such as concepts related to "inertial frame time" and particle "proper time", and Einstein-Riemann-Minkowski 4-vectors, which are very much productive in analyzing the world of interactions in microphysics. Regardless of objections and critics of Relativity Theory, we still cannot avoid using such concepts and mathematical prescriptions, but we should not forget that much broader and more general understanding of the same problematic is still in front of us (see similar and more profound elaborations in Chapter 10.). For instance, based on (2.4-17.1) we can easily create the following table of analogies (such as introducing new complex 4-vectors and 4-scalars) that are on some way simulating Minkowski-Einstein 4-vectors formalism, being entirely correct and applicable,*

T.2.2-3

	Ref. Frame Values	Symbolic Vectors	Invariant Expressions
Time	$dt = \gamma d\tau$	$\overline{\tau} = (dt, \frac{dr}{c})$	$d\tau^2 = dt^2 - \frac{dr^2}{c^2} = inv.$
Mass	$m = \gamma m_0$	$\overline{M}_4 = (M, \frac{E/c}{c}) = = (M, \frac{p}{c})$	$\overline{M}_4^2 = M^2 - \frac{p^2}{c^2} = = M_0^2 - \frac{p_0^2}{c^2} = inv.$
Energy	$E = \gamma E_0$ $E_0 = m_0 c^2$	$\overline{\varepsilon} = (E, cp)$	$\overline{\varepsilon}^2 = E^2 - c^2 p^2 = = E_0^2 = inv.$
Momentum	$p = \gamma p_0$ $p_0 = mv$	$\overline{P}_4 = (p, \frac{E}{c})$	$\overline{P}_4^2 = p^2 - \frac{E^2}{c^2} = = -\frac{E_0^2}{c^2} = inv.$
Velocity	$v = \frac{dr}{dt} = \frac{1}{\gamma^2}\frac{d\rho}{d\tau}$	$\overline{V} = (\gamma v, \gamma c)$	$\overline{V}^2 = \gamma^2 v^2 - \gamma^2 c^2 = = -c^2 = inv.$
Distance (space interval)	$d\rho = \gamma v dt = \gamma dr$, $dr \cdot dt = d\rho \cdot d\tau$	$\overline{\tau} \cdot \overline{V}$	$\overline{\tau} \cdot \overline{V} = \gamma v dt - \gamma dr = = 0 = inv.$

We could now start establishing new 4-vectors or 4-scalars, Phasors, complex spaces, introduce new coordinate systems with imaginary units, and develop new "Super or Hyper-Complex Relativity" theory based on such exciting mathematical combinations based on T.2.2-3 (see more in Chapter 10). Of course, such creative imagination and further mathematical action steps should be guided by something experimentally verifiable, with predictive power, and be well integrated with the texture of surrounding Physics, avoiding creating artificial, non-natural, conventions and axioms-based theories.

....

[♣ The theory of relativity, as we understand it today, is the product of contributions from several brilliant minds, including Hendrik Lorentz, Henri Poincaré, Hermann Minkowski, Albert Einstein, and possibly even Einstein's first wife, Mileva Marić, who was of Serbian origin and had a strong background in mathematics and physics, may have played a significant but largely unacknowledged role in its development. The theory's evolution was shaped by a complex interplay of circumstances, leading to groundbreaking results, some of which have proven invaluable, while others may never be fully verifiable.

Had history taken a different course without Einstein, the mathematical expressions for energy-momentum 4-vectors might still have been developed by others. However, it is unlikely that the entire framework of modern relativity, with its specific assumptions, postulates, and concepts, would have emerged in the same form. This suggests that we are not compelled to accept all aspects of the theory of relativity uncritically. While the current theory provides logical and valuable formulations regarding mass, velocity, momentum, and energy, it also presents challenges. Certain elements are improvable, questionable, or unnecessarily complex, as discussed by scholars such as Thomas E. Phipps, Milutin Milanković, and Velimir Abramović.

Given this, it might be pragmatic to adopt only the proven and effective components of relativity theory while also considering insights from other scientific frameworks. This approach could guide the evolution of relativity towards more advanced and realistic concepts, potentially leading to a theory that is a direct extension of an enhanced Maxwell electromagnetic theory.

Albert Einstein, despite the complexities of his personal life, expressed a certain respect and obligation toward his first wife, Mileva Marić. He promised her the money he anticipated receiving from the future Nobel Prize, a promise he eventually fulfilled—though only after Mileva reminded him that she could write his biography to support herself. Sadly, after Mileva's death, many documents related to relativity and Einstein were removed from her apartment, leaving her potential contribution to the theory largely *unrecognized. Christopher Jon Bjerknes explores this in his work, *The Manufacture and Sale of Saint Einstein*.*

Most science historians have only sporadically acknowledged Mileva Marić's role, often downplaying her Serbian origins. Instead, they frequently link her identity to the region where she was born—Vojvodina, Serbia, then part of the Austro-Hungarian Empire, leading to implications that she might have been Croatian or Austro-Hungarian rather than Serbian.

Even into the early 21st century, the location of Mileva Marić's grave was unknown, possibly due to unpaid burial fees. Her tomb was eventually discovered in Switzerland, largely thanks to the efforts of some dedicated Serbs and local authorities. It's conceivable that her grave could one day become a notable tourist site in Switzerland. However, there is also the troubling and sarcastic possibility that Mileva Marić might be posthumously claimed as Croatian, as has happened with other prominent Serbs like Nikola Tesla and Ruđer Bošković, influenced by geopolitical and ideological motives. All such nonsenses are related to the fact that majority of Croatians are relatively recently, ideologically and forcefully preformatted Serbs.

These observations underscore the pervasive influence of mainstream inertia, manipulation, and indoctrination, even in areas that may not directly impact scientific achievements. The field of physics, from the 20th century to the present, continues to grapple with semi-false, inaccurate, and problematic interpretations, visions, and manipulated theories.

After reading publications from Thomas E. Phipps Jr. and Reginald T. Cahill on contemporary relativity theory, it becomes imaginable that the dominance of contemporary relativity theory in physics may be waning. This could also apply, albeit to a lesser extent, to current electromagnetic theory.

....

The complexity underlying equation (2.4-17) can be traced back to a nearly philosophical concept: as humans and adept mathematicians, we possess the ability to analyze and understand two-body or multi-body systems, with their internal interactions, from various perspectives. These perspectives include the laboratory (inertial) reference systems, which are more tangible and realistic to us, and the unified center-of-mass reference system, which is more abstract and mathematically derived.

From the specific viewpoint of an observer in a laboratory system, the laboratory reference frame seems more concrete. However, from the broader perspective of Nature or Physics, considered as the most neutral and significant observer, the reality associated with the center-of-mass system is far more relevant, dominant, and substantial. The complexity of particle-wave duality and the interactions within a multi-body system are intrinsically tied to their common center-of-mass system. This concept applies to systems ranging from atomic structures to planetary, solar, and galactic systems. It's important to recognize that all objects, energy states, and masses in the universe exist in states of relative motion or rest when analyzed concerning their center-of-mass systems.

For example, in the case of solar systems with many orbiting planets (refer to equations (2.11.10) to (2.11.21)), or when comparing two-body problems to N-body problems, the natural approach would be to conceptually reduce these systems to multiple instances of two-body systems.

$$\left(m_c \to \gamma_c m_c = \gamma_1 m_1 + \gamma_2 m_2\right) \Rightarrow \gamma_c m_c = \sum_{(i)} \gamma_i m_i = \gamma_c \sum_{(i)} m_i$$
$$\left(m_r \to \frac{\gamma_1 m_1 \cdot \gamma_2 m_2}{\gamma_1 m_1 + \gamma_2 m_2} = \gamma_r m_r , \gamma_r = \frac{\gamma_1 \cdot \gamma_2}{\gamma_c}\right) \Rightarrow \gamma_{r-i,j} = \frac{\gamma_i \gamma_j}{\gamma_c}. \quad (2.4\text{-}18)$$

Significant differences between Classical Mechanics, Quantum Theory, and Relativity Theory can largely be attributed to the conceptually incomplete formulations of particle-wave duality and the current, incomplete mathematical modeling in physics. Both Quantum Theory and Relativity Theory, which ideally should have evolved from an updated Electromagnetic Theory, face issues where certain binding components, such as particles and fields, are either missing or inadequately considered.

For instance, the results of the famous Michelson-Morley experiments pointed to the exceptional and universal speed of light in a vacuum. These experiments, which used a single light source split into two

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orthogonal beams, aimed to detect time delays or advances caused by the Earth's motion or a hypothetical ether wind. However, early interpretations of these results were incomplete and led to the incorrect conclusion that the ether does not exist. The Michelson-Morley interferometer was too small, and its components were too closely coupled to the Earth and the laboratory environment where the measurements took place. Consequently, the interferometer and the surrounding field medium (ether) moved synchronously with the Earth, which influenced the measurements. Furthermore, because electric and magnetic fields are strongly coupled to their sources, any movement of these sources also moves the surrounding fields, meaning the ether in such cases is not freely streaming. A larger and more sophisticated interferometer would be required to detect the influence of aether wind on light beams.

Moreover, the early conclusions drawn from the Michelson-Morley experiments have significantly influenced the foundations of Relativity Theory. These conclusions did not account for the fact that light beams and photons have specific spinning motions, and that all rotational and spinning motions in the universe are globally coupled and synchronized. Interestingly, modern GPS systems operate effectively applying Lorentz transformations and simple spatial-temporal corrections, which suggests that the complex mathematics of the present state of Relativity Theory is largely correct. Anyway, Relativity Theory should be developed further, primarily starting from Maxwell-Faraday electromagnetic theory to better address mutually dependent linear, angular, inertial, relative, and absolute motions, and such updated Electromagnetic theory should generate an optimized Relativity theory.

Some aspects of particle-wave interactions, such as diffraction, interference, and scattering phenomena, remain incompletely analyzed in modern physics. These gaps are often filled with probabilistic modeling, raising questions about the role of causality and objectivity in the natural sciences. While Quantum Theory works well within its framework, largely due to its robust mathematical foundation in signal analysis, statistics, and probability theory, it is also criticized for leading generations of scientists to believe that what we observe is merely a probability of what could be.

Quantum Theory also posits that different observers will perceive the same situation differently. While this is philosophically intriguing, it still requires a deterministic explanation. The impact of measurement methods and instruments on microphysical results does not justify the view that everything in microphysics is fundamentally stochastic. Such dogmatic positions are reminiscent of certain ideological teachings in history and are sometimes applicable to both Relativity and Quantum Theory, as well as some aspects of Maxwell Electromagnetism.

It is also important to recognize that any new theoretical approach regarding particle-wave duality cannot completely disregard the many mathematically well-established concepts and models currently used in Quantum Theory. These successful mathematical structures, found in Quantum, Relativity, and Electromagnetic Theory, could be modified, upgraded, and reused within new frameworks to formulate new models and approaches to an updated Wave-Particle Duality Theory.

However, we must remember that powerful mathematical tools do not automatically confer the same power and significance to the theories or authors using them. A good tool in the wrong hands can still result in flawed or incorrect constructions. What we currently have within Quantum and Relativity Theory may involve the substitution of complex subjects with something more trivial to support less well-established concepts. Such substitutions can be found in various ideological and political movements, where powerful art forms like music, painting, and architecture are used to support ideas that lack substantial and logical foundations.

*Our human history is full of such examples, and it seems that we have not yet fully learned from them. This irrational tendency persists to some extent within our mainstream natural sciences, often in subtle ways. Nonetheless, as suggested by J. M. T. Thompson in *Visions of the Future*, the days of unchallenged dominance of contemporary Relativity and Orthodox Quantum Theory may be coming to an end. ♣]*

2.3. How to Account for Rotation in Relation to Gravitation?

The idea, though still hypothetical and not fully developed, suggests that every mass composed of atoms and molecules inherently possesses (and it is composed of) number of micro-spinning states. We already know that almost all elementary particles, such as electrons, protons, and neutrons, exhibit spin and magnetic properties. This spin can be understood as consequence of something equivalent to spinning of internal matter-wave formations, without seeing that solid particles are externally spinning. Under favorable conditions, these internal spins and associated magnetic moments can align, polarize, and combine as vectors, leading to a macro system with a specific angular momentum and a permanent magnetic field distributed around the mass (see similar ideas from (2.4-4.9) - (2.4-4.11)). *The indicative background for supporting such internally hidden and intrinsic spinning nature of macro masses is addressed in Chapter 4.1 under "4.1.1.1. Photons and Particle-Wave Dualism."*

When a mass is in linear motion, it generates an external matter-wave field. This is due to its internal polarizable spinning elements, which are electromagnetic and electromechanical entities, with angular and magnetic moments emanating from atoms. Any mass can also oscillate (internally as thermal motions of atoms and externally) and perform angular motion relative to certain locally dominant center of mass. As a result, the linear motion of masses, especially along large circular orbits, tends to create an associated macro spinning field or spiral wave motion in the direction of motion. Simultaneously, this motion balances the attractive Newton-Coulomb forces with repulsive centrifugal forces. In stable, periodic, and orbital motions, this process leads to the formation of orbital standing matter waves.

Kepler's and Newton's Laws support these ideas on a macro-cosmological scale, applying them to solar and planetary systems. Additionally, atomic and elementary particle models often exhibit spinning, helical, oscillatory, and orbital motions with standing waves structures, coupled with associated electromagnetic complexities. There is a certain universal tendency of mass and energy formations in the universe to create self-closed orbital motions, with the radius of rotation being arbitrarily large. The local and global conservation of orbital and spin moments universally applies to these situations, involving entanglement effects. This is because all atoms and subatomic entities across the universe behave as resonators that tend to synchronize with one another, as much as their spectral characteristics overlap.

These principles hold true on both micro and macrocosmic scales, even when rotational and spinning motions are not directly observable we can assume that some motional or oscillatory phenomena exist in the background. Masses in motion effectively are specific "energy packing formats" that adhere to global energy-momentum conservation laws. This concept will be explored further in Chapter 4.3, "Mass, Particle-Wave Duality, and Real Sources of Gravitation," and in Chapter 10, "Hypercomplex Analytic Signal Functions and Interpretation of Energy-Momentum 4-Vectors in Relation to Matter-Waves and Particle-Wave Duality."

It is important to recognize that any closed-path stationary motion, whether circular, elliptical or otherwise, is an accelerated motion. This motion has a vector of orbital acceleration directed toward its center of rotation, analogous to the force of gravitation. This phenomenon is discussed later in this chapter under "2.3.3. Macro-Cosmological

Matter-Waves and Gravitation," and in Chapter 8. Masses are basically composed of atoms (with countless subatomic and elementary particles). All these exhibits rotational and oscillatory behaviors and mutually coupled orbital, linear, and magnetic moments, spin, and other motional manifestations, akin to a system of interconnected micro-gyroscopes.

We can assert that a certain level of equivalent rotational (mostly spinning) energy is permanently and randomly accumulated or stored within masses and atoms. This energy also extends as a field outside, into the space surrounding the external shapes of macro masses. The resulting effect of this "internally captured and stabilized rotation," combined with external macro-orbital, mechanical, and electromagnetic moments, serves as the source of matter waves and gravitation. The accelerated linear motion of mass also induces polarization or alignment effects on its internal spinning or gyroscopic domains, thereby creating an external macro angular momentum and torque, which in turn influences the externally measurable gravitational force. This concept is further explored as PWDC (Particle-Wave-Duality-Concept) in Chapters 4.1, 4.3 (see equations (4.41-1) to (4.45)), and in Chapter 10 (see equations (10.1.4) to (10.1.7)).

To intuitively advance these ideas, we can start by exploring the formula for centripetal force acting to a rotating particle. This can be analogous to the force of gravitation. By creating several mutually dimensionally equivalent, and analogically comparable formulas (see (2.5)), we can exercise the concept that gravitation may be an external manifestation of the cumulative angular momentum of countless mutually coupled, energy-carrying, spinning micro-masses or micro gyroscopic domains, combined with other associated orbital and electromagnetic moments. This "internally packed quantity of spinning states," when observed externally, manifests as the total mass with its mechanical, angular and electromagnetic moments, presenting the origin of gravitation.

$$F_c = \frac{mv^2}{r} (=) \left[\frac{E_k}{r}\right] (=) \left[\frac{L\omega}{r}\right] (=) \left[\frac{\tau}{r}\right] (=) \left[\frac{Lv}{r^2}\right] (=) \left[m\frac{dv}{dt}\right] (=) \left[\frac{kg \times m}{s^2}\right] (=) [\text{Force}] \ ,$$

$$\left(\begin{array}{l} [E_k](=)[mv^2](=)[J\omega^2](=)[pv](=)[L\omega] \ (=) \left[\frac{kg \times m^2}{s^2}\right] (=)[\text{Motional energy}](=)[J] \ , \\ [L](=)[J\omega](=)[mr^2\omega](=)\left[\frac{kg \times m^2}{s}\right](=)[\text{Orbital moments}] \ , \\ [\tau](=)\left[\frac{dL}{dt}\right](=)\left[\frac{kg \times m^2}{s^2}\right](=)[\text{Torque}](=)[\text{Energy}](=)[J] \ ,(\text{when multiplied by an angle in radians}), \\ [v](=)[\omega r](=)\left[\frac{m}{s}\right](=)[\text{orbital velocity}], \ [\omega](=)\left[\frac{1}{s}\right](=)[\text{angular velocity}]. \end{array}\right) \tag{2.5}$$

By making dimensional comparisons within the parameters in (2.5), such as: $m, J, p, L, \tau, v, \omega, E_k ...$), we can generate new indicative and analogies-based ideas about which physics-related parameters or values might be involved in formulas for centripetal forces, orbital moments, and their mathematical derivatives. This approach introduces an oversimplified, indicative, but also clear starting point for formulating hypothetical force laws, grounded in "Newton-Coulomb" force laws, while remaining compatible with earlier sections (2.1) through (2.4-3). If we creatively combine insights from sections (2.1) to (2.4-3), (2.5), and analogies drawn from T.1.2 and T.1.8, we might infer that a stable rest mass could be viewed as a storage or packing format for its own internal orbital and spin moments (see later sections (2.10) and (2.11)). The objective here is to propose missing force-links that are causally related to fields created by rotation or spinning, which are complementarily coupled to linear motions and externally manifest as gravitation and/or magnetic field effects. This coupling is like complementing of electric and magnetic fields in electromagnetism, or as in the attraction and repulsion of electric dipoles and permanent magnets, or as unity of real and imaginary parts of a Complex Analytic Signal (explored further in Chapter 4.0).

Atoms manifesting as sets of resonators, tend to synchronize when they share the same resonant frequencies. The intrinsic coupling between electric and magnetic fields and other material properties, as observed in electromagnetism, demonstrates that electric charges, while respecting Coulomb-Newton force laws, can perform various linear and rotational motions in electric and magnetic fields due to different forms of attractive and repulsive forces (see additional background in [144]). We also understand how to relate properties of mass and different moments to electromagnetic waves, as we find in the analyses of the Compton and Photoelectric Effects.

Since mass in motion is fundamentally linked to gravitation, one of the earliest confirmations of Relativity theory, at least qualitatively and within the limits of measurement errors was in astronomy, where it was observed that large masses bend light beams (or attract photons) in accordance with the Newtonian Law of Gravitation. This observation suggests that photons, despite being purely electromagnetic entities without rest mass, possess equivalent dynamic mass and angular or spin moments that behave similarly to other masses concerning conservation laws, mechanical moments, and gravitational forces. There are differing opinions on the gravitational attraction between photons and large masses, with some attributing it to purely electromagnetic interactions, as discussed in [89]. The challenge here lies in conceptualizing the meaning of mass. Currently, we understand gravitation as a force between nearly static macroscopic masses, but this is an oversimplification (because all masses are in relative motion). There is also a minimal mass-size and distance between two masses, below which the forces of gravity, as currently understood, lose their meaning and applicability. We also know that macroscopic masses are aggregations of atoms and molecules, and that atoms, elementary particles, and other interatomic entities usually possess electric and magnetic charges, along with associated fields and other properties, including angular, orbital, and spinning electric and mechanical moments.

In the atomic and subatomic world, spinning and orbiting are coupled with magnetic moments. Additionally, all dynamic manifestations of magnetic fields are invariably combined with complementary electric charges and electric field manifestations. Here, we establish the platform that gravitational forces are related to mechanical spinning, orbiting, and torque effects, which are mutually coupled and synchronized with similar electromagnetic manifestations in atoms and associated field charges. This natural foundation supports a direct and striking analogy between Newton's and Coulomb's force laws.

Further indicative support for "reinventing the origin of gravitation" comes from electromechanical analogies and the comparison between Coulomb's and Newton's laws, as discussed in the first chapter of this book on analogies. It becomes clear that electric charge and static mass are not analogically comparable. What corresponds to electric charges are electromagnetic and mechanical moments, including linear, angular, and vibrational states of masses, not only static masses. Consequently, the true sources of gravitation are likely to be electromagnetic and mechanical moments and charges, including relevant currents, as analogically predicted in the first chapter.

[♣ Comments & free-thinking corner: The following elaboration about gravitational sources extends based on “2.2.1. WHAT THE GRAVITATION REALLY IS”

Energy, Torque, and Gravitational Sources within Rest Mass

An intriguing observation arises when we consider the dimensional similarity between energy and torque, both measured in Joules. This similarity suggests that torque can, at least dimensionally, be viewed as an equivalent of linear motion and/or axial force.

$$\left[\frac{kg \cdot m^2}{s^2}\right] (=) [\tau] (=) [Torque] (=) [Energy] (=) [J] (=) Joule \cdot \tag{2.10}$$

For example, any torque can be presented as an equivalent linear motion of a particle along a path Δs, driven with a force $\underline{\mathbf{F}^*_{linear}}$, acting collinearly to the torque vector τ. This idea is captured in Equation (2.10.1).

$$\tau = F_{angular} \cdot \theta = F_{linear} \cdot \Delta s \ (=) \ [Nm]. \tag{2.10.1}$$

Dimensionally, torque (when multiplied by the relevant rotational angle θ, expressed in nondimensional radians) represents energy. This formal connection between torque and energy leads to an interesting hypothesis: the energy of every rest mass ($\mathbf{E = mc^2}$) could be thought of as the work done by its internal torque elements, where the full rotational work ($\theta = 2\pi$) corresponds to the mass's internal energy $\tau_{int.} \cdot 2\pi = mc^2$.

We can conceptually relate this internal mass-energy to a superposition of many micro-spinning-mass-energy domains, which are omni-directionally distributed, each possessing a small amount of torque and angular momentum. These domains are not necessarily "spinning" like microscopic gyroscopes but rather represent superposition of spin moments and orbital angular momentum from atoms within the mass.

Furthermore, when considering the total internal torque of a macro mass (which, in a state of rest, results in zero net external torque), we can infer that the individual micro torque vectors within the system are balanced. This means the total internal torque must be composed of minimum two mutually opposed torque vectors, ensuring that the resulting external torque is zero $\tau_{int.} \cdot 2\pi = \frac{1}{2} mc^2$. This concept suggests that the sources of gravitational forces in masses—both at the macroscopic and atomic scales—are likely tied to the internal spinning and torque elements within the mass, which are dynamically synchronized and anyway coupled with electromagnetic charges and moments.

This aligns with earlier discussions in Section 2.2, where a generalized force law was introduced, indicating that atoms may be the primary sources of gravitational attraction.

- **Conceptualizing the Rest Mass of a Non-Spinning Macro-Particle**

Let us now consider the rest mass m of a specific macro-particle that is externally non-spinning and electrically and magnetically neutral. This macro-particle can be thought of as a compact energy state, resulting from the mutual interactions of numerous micro-mass domains (or atoms and elementary particles). These micro-domains exist in various internal states of orbiting and spinning, and are coupled by surrounding electromagnetic forces.

On average, the vector quantities associated with these internal states cancel each other out. This neutralization leads to the overall macro spin, orbital, or angular momentum of the particle being zero from an external reference frame, such as a local laboratory system. Thus, from the outside, the macro-particle appears to be at rest and not rotating.

However, on an internal level, the macro-particle consists of a collection of micro-rotating, orbiting, and spinning energy states that are mutually coupled and synchronized. While these moments neutralize each other as vectors, the total internal angular momentum remains zero when viewed macroscopically. Externally, the particle appears as a source of gravitational force, as described by Newton's Law of Gravitation. However, we propose that the true source of gravitational attraction comes from the internal atomic spinning and orbital moments, along with the associated electromagnetic moments, charges, and dipoles (as illustrated in Figure 2.2).

- **The Role of Torque and Angular Momentum in Rest Mass**

For a particle at rest, the center of mass velocity and angular velocity should both be zero. This implies that the mass would have at least two mutually opposed vector components (of involved forces and moments) in any direction, ensuring that the external torque is zero, regardless of the observer's perspective. By considering the center of mass and angular velocity in this way, we can derive an expression that explicitly incorporates the internal orbital and spinning moments (i.e., torque or angular force components), as shown in equation (2.11).

The fundamental idea here is that a stable macro-particle can be conceptualized as a superposition of mutually synchronized micro-spinning and rotating energy states (see (2.4-4.9) - (2.4-4.11)). These internal spinning states represent a kind of (randomly, omnidirectionally distributed) motional energy, which, when appropriately combined, creates the total rest mass of the macro-particle in question, such as,

$$\left\{\begin{matrix} mc^2 = \sum\limits_{[i\in(1,N)]} \frac{1}{2} L_i \omega_i \cong \frac{N}{2}\bar{L}\bar{\omega} = \pi N \bar{L}\cdot\bar{f} = H\cdot\bar{f} \\ (N, \bar{L}, \pi, H = \text{constants}) \end{matrix}\right\} \Rightarrow \left\{\begin{matrix} mc^2 = H\cdot\bar{f} \\ c^2\cdot dm = H\cdot d\bar{f} \\ c^2\cdot\Delta m = H\cdot\Delta\bar{f} \end{matrix}\right\} \Rightarrow \left\{\begin{matrix} mc^2 = 4\pi|\tau| = H\cdot\bar{f} \\ \bar{f} = \text{internal, mean} \\ \text{spinning frequency} \end{matrix}\right\}$$

$$\vec{\omega} = \vec{\omega}^+ + \vec{\omega}^- = \frac{\sum_{(i)} J_i \vec{\omega}_i}{\sum_{(i)} J_i} = \frac{\sum_{(i)} \vec{L}_i}{\sum_{(i)} J_i} = 0 \Leftrightarrow \frac{1}{J}\iiint_{[V]} d\vec{L} = 0$$

$$\dot{\vec{\omega}} = \dot{\vec{\omega}}^+ + \dot{\vec{\omega}}^- = \frac{\sum_{(i)} J_i \dot{\vec{\omega}}_i}{\sum_{(i)} J_i} = \frac{\sum_{(i)} \dot{\vec{L}}_i}{\sum_{(i)} J_i} = \frac{\sum_{(i)} \vec{\tau}_i}{\sum_{(i)} J_i} = \frac{\vec{\tau}^+}{\sum_{(i)} J_i} + \frac{\vec{\tau}^-}{\sum_{(i)} J_i} = 0 \qquad (2.11)$$

$$\vec{\tau} = \frac{d\vec{L}}{dt},\ \dot{\vec{\omega}} \sum_{(i)} J_i = \dot{\vec{\omega}}\, J = \vec{\tau}^+ + \vec{\tau}^- = 0 \Rightarrow mc^2 = 2|\tau^+|\cdot 2\pi = 2|\tau^-|\cdot 2\pi = 2|\tau|\cdot 2\pi \Leftrightarrow$$

$$m = \frac{4\pi|\tau|}{c^2} = \frac{1}{c^2}\iiint_{[V]} \vec{\omega} d\vec{L},\ |\tau| = \frac{1}{4\pi}\iiint_{[V]} \vec{\omega} d\vec{L} = \frac{mc^2}{4\pi}.$$

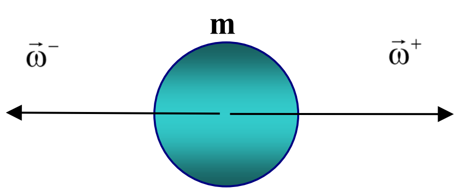


Fig. 2.2. Rest mass with internally compensated spinning and orbital moments

- **Conceptualizing the Internal Energy and Gravitational Interaction of Rest Mass**

The central idea here is that the **resulting value of internally captured spinning moments**, which produce torque τ, can also be interpreted as an **internal energy content** of the particle. This energy is proportional to the product of the torque and the rotational angle (in 2 π radians, multiplied by 2). It is nearly self-evident that any **neutral and stationary macro-particle** should possess at least two **mutually opposed torque or spinning moment components** in any given direction, such that these vector components cancel each other out. This results in the particle appearing at rest externally, with no net torque.

Building on this, we can redefine the **total mass** of the particle in terms of its internal **rotational or spinning moments**. These concepts might also serve as a foundation for rethinking inertia and gravitational interactions. For instance, Newtonian attraction could be explained by the attraction between **electric and magnetic fields** generated by the interaction of **orbital and spin moments**—being internal components of mass, such as properties of atoms, which were previously hinted at in Section 2.2.

We also recognize that any form of **orbiting, rotation, or spinning**, whether in atoms, planets, or galaxies intrinsically involves the coupling of **magnetic moments**, which often exhibit constant gyromagnetic ratios. A more complete discussion of this is presented in **Chapter 10** of this book, where the relationship between **mass, internal electromagnetic polarization**, and **linear/angular momentum** is explored in depth (see Equations (10.1.4) to (10.1.7)).

- **Coupling of Linear and Rotational Motions**

As we develop the concept of the coupling between **linear and rotational motions**, it becomes apparent that **de Broglie matter waves** and **angular forces** (Eq. 2.8) are closely related. This connection will be explored in greater detail in **Chapter 4.1**. Clearly, **Equation 2.9** offers a straightforward approach to understanding the unity of linear and angular force components, at least dimensionally. This also suggests that Newton's law of gravity may need to be **generalized** to account for rotational and spinning forces, as well as their associated **electric and magnetic polarizations**.

One speculative direction for further exploration is the potential existence of a **background rotation** or **ether-like fluid**, perhaps akin to **Nikola Tesla's radiant energy vortices**. This idea suggests that **rest mass** could be composed of, or surrounded by, **rotating or vortex-like fluid particles**, perhaps related to the concept of **aether**. While speculative, this would imply that all mass is inherently associated with some form of spatial rotational dynamics.

- **Gravitational Interaction Between Macro-Masses**

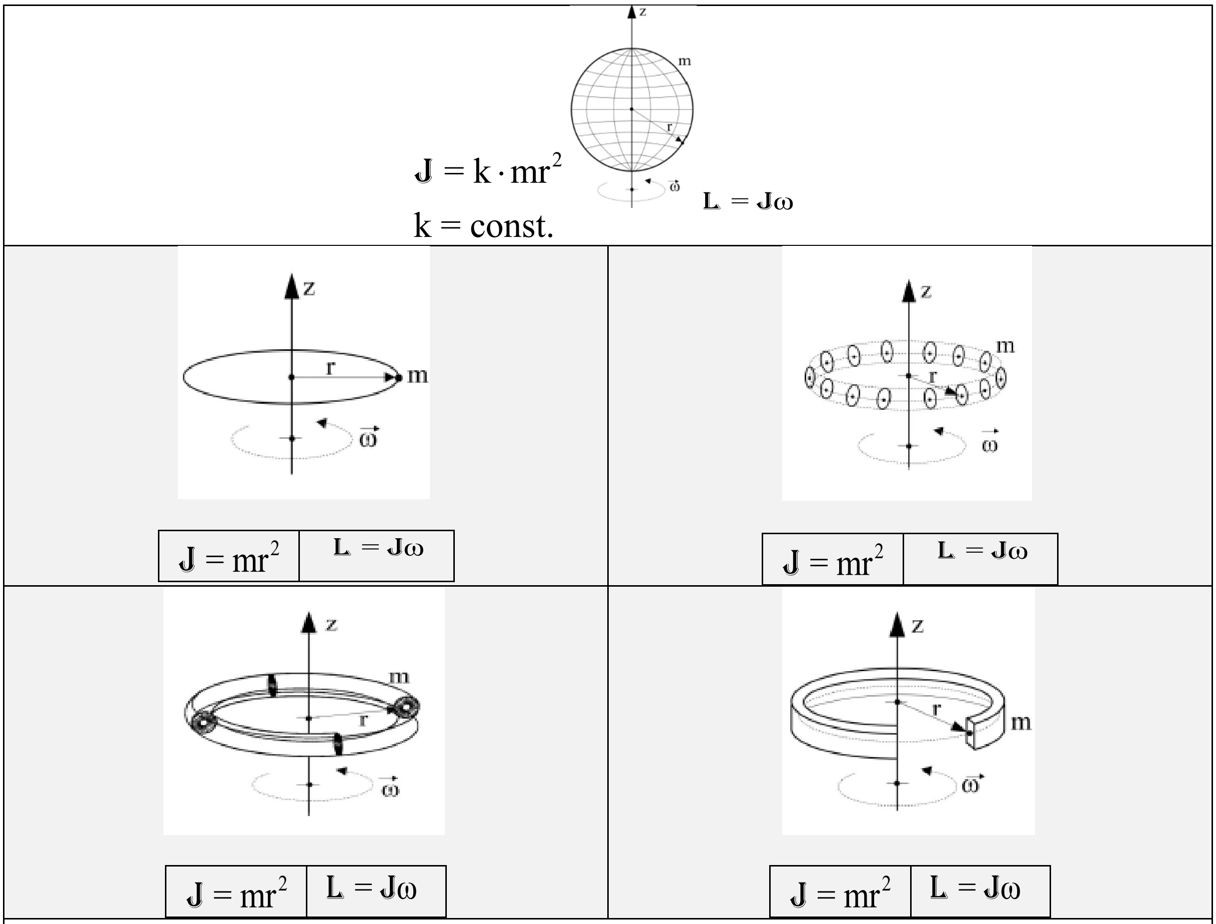
Now, let's summarize the conceptual framework regarding gravitationally attracting masses. Every **electromagnetically neutral macro-mass** (which, by definition, has non-zero rest mass, typically composed of atoms and molecules with embedded positive and negative charges) can be viewed as an **agglomerate of small, elementary micro-rotating or spinning mass formations**. These micro-domains possess **uniform, omnidirectional angular, spinning, and magnetic moments**, which, in most cases, cancel each other out as vectors.

When two or more such neutral masses come into spatial proximity, their internal **angular, spinning, and magnetic moments** begin to **reorient or polarize**, creating an **attractive force**, a force that can be described as **magneto-gravitational**. This process mirrors the interaction between two magnets or two electric charges, as outlined in the **Newton-Coulomb force laws**. This interaction is what we recognize as **gravitation**.

It is important to note that any dynamic magnetic force or field is accompanied by a complementary **electric field**, as all matter in the universe is in a state of relative motion. The **micro-spinning masses**, their associated **magnetic fields**, and **electromagnetic moments** could take many forms, each of which corresponds to different **spinning and orbiting energy states**. These states are depicted in **Figure 2.2.1** and are frequently observed in both the **micro-world of atoms** and the **macro-world of planetary systems and galaxies**.

- **The Structure of Subatomic Particles**

The structure of subatomic particles, such as **electrons** and **protons**, could also be modeled as **standing waves** on **toroidal shapes**. This concept, well-supported in the literature (e.g., [16, 17, 18, 19, 20, 22, and 120]), offers a useful analogy for understanding the rotational and orbital patterns that govern the behavior of mass at both atomic and cosmic scales.



(from: http://hyperphysics.phy-a str.gsu.edu/hbase/mi.html
https://en.wikipedia.org/wiki/Angular_momentum#Spin,_orbital,_and_total_angular_momentum
https://en.wikipedia.org/wiki/List_of_moments_of_inertia)

Moment of inertia is defined with respect to a specific rotation axis. The moment of inertia of a point mass with respect to an axis is defined as the product of the mass times the distance from the axis squared. The moment of inertia of any extended object is built up from that basic definition. The general form of the moment of inertia involves an integral.

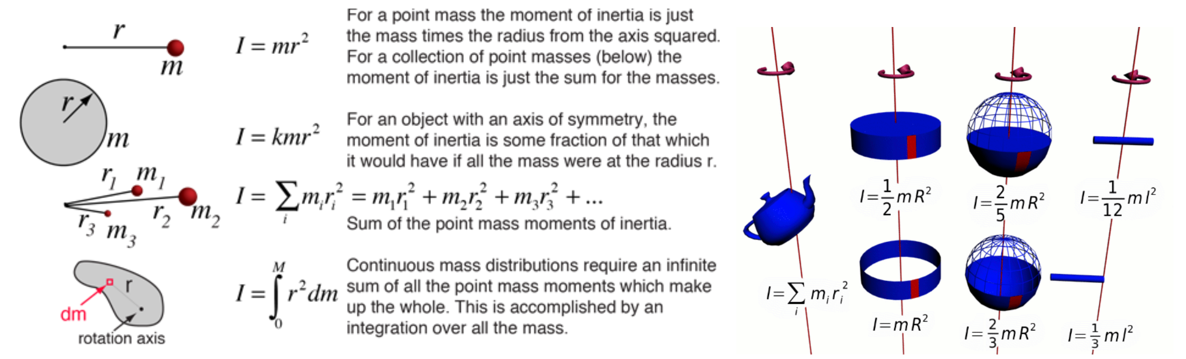


Fig. 2.2.1 Mutually isomorphic masses configurations regarding spinning

- **Superposition and Isomorphic Mapping in the Structure of Matter**

The structure of matter can be understood through multiple levels or layers of **superposition** and **isomorphic mapping** of wave-particle dual matter states (or matter-waves). Well-established mathematical theories, such as **Fourier analysis** and the concept of **Analytic Signals**, demonstrate that any time-domain function or signal can be decomposed into a series of simple harmonic (sinusoidal) components (see Sections 4.0.2 and 4.0.3 in Chapter 4). Similarly, when examining the structure of a **compact mass** or **particle**, it can be modeled as composed of numerous **internal electromechanical spinning**, **oscillatory**, and **resonant elements**. These internal elements can be viewed as combinations of **spring-mass oscillating systems** and **capacitance-inductance oscillating circuits** that are superimposed, coupled, synchronized, and unified. See more about wave-particle duality or PWDC in Chapter 10.

- **Atomic and Molecular Structure**

In addition to these internal electromechanical elements, **mass** is fundamentally composed of **atoms** and **molecules**. In this framework, we propose that **atoms** themselves represent a synchronized superposition of internal spinning states, as discussed in **Section 2.11**. This conceptual model of atomic structure extends outward to create all forms of mass, ultimately contributing to the phenomenon of **gravitational attraction**.

- **Transformation and Packing of Internal Moments**

A particularly innovative aspect of this model is the process by which **distributed internal angular moments**, including **spinning moments**, are transformed, packed, and "injected" into a given rest mass. By exploring this process, we can further test, refine, and support the theoretical framework introduced through the **generalized force-energy forms** in this chapter, particularly in **Sections 2.3 to 2.4.3** and **Sections 2.5 to 2.9**.

- **Experimental Evidence of Spinning Mass Effects**

Experimental evidence provides support for this model. For instance, Japanese researchers have observed changes in the **weight of spinning masses** within Earth's gravitational field. Specifically, they found that a spinning mass, when rotating in the same direction and plane as the Earth, measured slightly heavier than when at rest on the Earth's surface. Conversely, when the mass spun in the opposite direction, it was slightly lighter ([36]).

- **The Gyroscopic Effect and de Broglie Matter Waves**

A familiar example that illustrates the connection between spinning and linear motion is the **spinning bullet in rectilinear motion**. It is well known that the **gyroscopic effect** of the spinning bullet significantly stabilizes its linear trajectory, ensuring a straighter path over greater distances. In this context, **spinning** and **linear motion** are not independent but are mutually associated, coupled, and complementary, much like the relationship between **spinning matter** and **de Broglie matter-waves**. The mechanical spin of the bullet would, in a certain way, directly superimpose on the corresponding spinning related to **de Broglie wave's wavelength** and **frequency**, linking classical mechanics with quantum wave phenomena (see (2.4-4.9) - (2.4-4.11)).

To continue with relevant conclusions, we can go step by step, starting from the situation when a bullet is performing only rectilinear motion without self-spinning, what corresponds to,

$$E_{tot} = \gamma m_0 c^2 = E_0 + E_k = m_0 c^2 + \frac{pv}{1+\sqrt{1-v^2/c^2}}, \; p = \gamma m_0 v \tag{2.11-1}$$

If a bullet starts spinning (with an spinning energy amount E_s), we will have,

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

$$E_{tot} = \gamma\left(m_0 + \frac{E_s}{c^2}\right)c^2 = \left(E_0 + E_s\right) + E_k^* = \left(m_0 + \frac{E_s}{c^2}\right)c^2 + \frac{pv + \mathbf{L}_s\omega_s}{1+\sqrt{1-v^2/c^2}} \Rightarrow$$
$$(\gamma-1)m_0c^2 + E_s = \frac{pv + \mathbf{L}_s\omega_s}{1+\sqrt{1-v^2/c^2}}, \ (\gamma-1)m_0c^2 = \frac{pv}{1+\sqrt{1-v^2/c^2}}, E_s = \frac{\mathbf{L}_s\omega_s}{1+\sqrt{1-v^2/c^2}}. \quad (2.11\text{-}2)$$

Now, let us imagine that initially we do not have an object that has its standstill rest mass ($m_0 = 0$, v << c), or that in some way the same bullet is composed only of *matter-wave packets* as photons, or other motional energy states. Such superposition of spinning photons is hypothetically and eventually creating the resulting, total rest mass of the same bullet $m_0 > 0$. This would produce the following energy balance,

$$E_s \to m_0c^2 = \frac{\mathbf{L}_s\omega_s}{1+\sqrt{1-v^2/c^2}} \cong \frac{1}{2}\mathbf{L}_s\omega_s \Rightarrow \mathbf{L}_s\omega_s = 2m_0c^2 = \sum_{(i)}\mathbf{L}_{si}\omega_{si} \quad (2.11\text{-}3)$$

- **The Role of Spinning Matter Waves in Rest Mass and Gravitation**

The key idea presented here is that **rest mass** can be conceptualized as a superposition of **spinning matter wave states** and/or **photons**. This concept connects to a broader understanding of gravitation, particularly in relation to **spinning motions**, **orbital dynamics**, **globalized macro-cosmic rotation**, and the interplay between **linear and angular momentum**, **electromagnetic moments**, **fields**, and **matter currents** (see (2.4-4.9) - (2.4-4.11)).

This new perspective on gravitation is deeply integrated into emerging theories, research projects, and programs aimed at redefining our understanding of fundamental physical phenomena. The development of these innovative concepts has poised to significantly impact on many core areas of physics, including:

- The **Big Bang** theory
- The nature of **black holes**
- The mystery of **dark matter, mass** and **dark energy**
- The relationship between **fundamental forces of nature**
- The coexistence of **matter and antimatter**
- The interplay between **relativity** and **quantum theory**
- The foundations of **electromagnetic theory**

These advancements will gradually reshape the way we view the physical universe and challenge existing models and theories (refer to citations below for further reading).

Citation from: https://einstein.stanford.edu/SPACETIME/spacetime4.html
Gravity Probe B. W. W. Hansen Experimental, Physics Lab
Physics/Astrophysics Building, 1st Floor
452 Lomita Mall, Stanford University, MC 4085
Stanford, CA 94305-4085, support@regyro.stanford.edu
Principal Investigator, Professor Emeritus Francis Everitt
Phone: 650-725-4104, Fax: 650-725-8312, Email: francis1@stanford.edu

"The Many Faces of Spin
Many of nature's deepest mysteries come in threes. Why does space have three spatial dimensions (ones that we can see, anyway)? Why are there three fundamental dimensions in physics (mass M, length L and time T)? Why three fundamental constants in nature (Newton's gravitational constant G, the speed of light c and Planck's constant h)? Why three generations of fundamental particles in the standard model (e.g. the up/down, charm/strange and top/bottom quarks)? Why do black holes have only three properties -mass, charge, and spin? Nobody knows the answers to these questions, nor how or whether they may be connected. But some have sought clues in the last-named of these properties: spin".

Nobel laureate C.N. Yang wrote in a letter to NASA Administrator James M. Beggs in 1983 that general relativity, "though profoundly beautiful, is likely to be amended ... whatever [the] new geometrical symmetry will be, it is *likely to entangle with spin and rotation, which are related to a deep geometrical concept called torsion ... The proposed Stanford experiment [Gravity Probe B] is especially interesting since it focuses on the spin. I would not be surprised at all if it gives a result in disagreement with Einstein's theory."*

Gravito-Electromagnetism

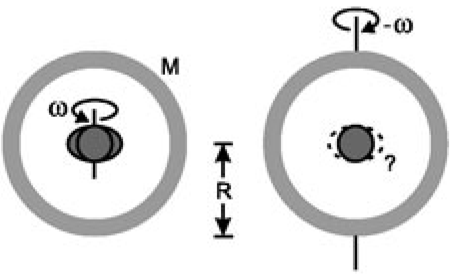
……. When gravitational fields are weak and velocities are low compared to c, then this decomposition takes on a particularly compelling physical interpretation: if we call the scalar component a "gravito-electric potential" and the vector one a "gravito-magnetic potential", then these quantities are found to obey almost exactly the same laws as their counterparts in ordinary electromagnetism! (Although little-known nowadays, the idea of parallels between gravity and electromagnetism is not a new one and goes back to Michael Faraday's experiments with "gravitational induction" beginning in 1849.) One can construct a "gravito-electric field" g and a "gravito-magnetic field H from the divergence and curl of the scalar and vector potentials, and these fields turn out to obey equations that are identical to Maxwell's equations and the Lorentz force law of ordinary electrodynamics (modulo a sign here and a factor of two there; these can be chalked up to the fact that gravity is associated with a spin-2 field rather than the spin-1 field of electromagnetism). The "field equations" of gravito-electromagnetism turn out to be of great value in interpreting the predictions of the full theory of general relativity for spinning test bodies in the field of a massive spinning body such as the earth — just as Maxwell's equations govern the behavior of electric dipoles in an external magnetic field. From symmetry considerations we can infer that the earth's gravito-electric field must be radial, and its gravito-magnetic one dipolar, as shown in the diagrams below:

Frame-dragging Effect

Frame-dragging in realistic experimental situations is not nearly that strong and the utmost ingenuity has to be exercised to detect it at all. Analysed in terms of the gravito-electromagnetic analogy, the effect arises due to the spin-spin interaction between the gyroscope and rotating central mass and is perfectly analogous to the interaction of a magnetic dipole $\boldsymbol{\mu}$ *with a magnetic field* $\mathbf{B}$ *(the basis of nuclear Magnetic Resonance Imaging or MRI). Just as a torque* $\boldsymbol{\mu}\times\mathbf{B}$ *acts in the magnetic case, so a gyroscope with spin* $\mathbf{s}$ *experiences a torque proportional to* $\mathbf{s}\times\mathbf{H}$ *in the gravitational case. For Gravity Probe B, in polar orbit 642 km above the earth, this torque causes the gyroscope spin axes to precess in the east-west direction by a mere 39 milliarcsec/yr — an angle so tiny that it is equivalent to the average angular width of the dwarf planet Pluto as seen from earth.*

…………………………

The calculations show that general-relativistic frame-dragging goes over to "perfect dragging" when the dimensions of the large mass (its size and density) become cosmological. In this limit, the distribution of matter in the universe appears sufficient to define the inertial reference frame of observers within it. For a particularly clear and simple explanation of how and why this happens, see The Unity of the Universe (1959) by Dennis Sciama. Had Mach lived 10 years longer, he could have predicted the existence of the extragalactic universe based on observations that the stars in the Milky Way rotate around a common center!



Would the earth still bulge, if it were standing still and the universe were rotating around it?

To put the cosmological significance of frame-dragging in concrete terms, imagine that the earth was standing still and that the rest of the universe was rotating around it: would its equator still bulge? Newton would have said "No". According to standard textbook physics the equatorial bulge is due to the rotation of the earth with respect to absolute space. Based on Lense and Thirring's results, however, Einstein would have had to answer "Yes"! In this respect general relativity is indeed more relativistic than its predecessors: it does not matter whether we choose to regard the earth as rotating and the heavens fixed, or the other way around: the two situations are now dynamically, as well as kinematically equivalent. ♣]

- **The Role of Wave-Particle Duality, Spinning, and Gravitation**

A strong, though still hypothetical, assumption about the relationship between **rotation, spinning**, and the explanation of **gravitation** is grounded in the concept of **Wave-Particle Duality** of matter. We know that the basic constituents of matter, **atoms, electrons, protons**, and **neutrons**, all have **spinning** and **magnetic moments** properties. This is a well-established fact, both experimentally and theoretically, supported through the **de Broglie matter wave** concept and the broader framework of **wave-particle duality** (discussed in detail in Chapters 4.0, 4.1, and 10).

- **Wave-Particle Duality and Micro-World Entities**

To briefly summarize, micro and **elementary particles** in motion, such as **atoms, electrons, protons**, and **neutrons**, can be conceptually understood as being **kinematically equivalent** to corresponding **moving wave packets**, where the dominant or mean wavelength is related to their momentum, $\lambda = h / p = h / m^* v = h / \gamma m v$. In other words, we can imagine a particle with sort of **internal spinning** and **magnetic moments** as being **encircled by a spinning, helicoidal (or spiral) wave form**, which can be modeled as a **wave-group** or **wave packet**. This wave packet naturally has **group velocity** (v) and **phase velocity** (u).

Crucially, the **kinetic energy** and **wave energy** of the particle and its corresponding wave group are equivalent. The details of this relationship will be explored further in Chapters 4.0, 4.1, and 10. (see also de Broglie's hypothesis and PWDC).

- **Spinning Particles and Angular Momentum**

A particle in linear motion, possessing linear momentum p > 0, gives rise to a specific **spinning wave-group** with its own **angular momentum** $\mathbf{L}_s$ and **angular velocity** ω_s. This is analogous to how a **mechanically spinning particle** (initially at rest, with zero linear momentum) acquires a **linear or axial thrust** when it begins accelerating. This thrust is collinear with the **spinning moment** $\mathbf{L}_s$.

These hypothetical assumptions suggest that **accelerated spinning** could potentially create a **thrust or force** that acts **against gravitation** (depending on the direction of the spin). Conversely, depending on the spin direction, the created **linear thrust** could increase the gravitational force.

- **Torque, Linear Force, and Photon-Gyroscope Analogies**

In practical situations, such as **spinning objects** combined with the effects of a **magnetic field**, **torque** and **linear force** are interdependent and transformable, similar to phenomena observed in **photons** and **gyroscopes** (see Chapter 4.1, Section T.4.0 on **Photon-Particle Analogies**, and Chapter 10, under 10.02 **The Meaning of Natural Forces**).

$$\left\{\begin{array}{c}\left[\begin{array}{l}\begin{pmatrix}\text{Linear particle motion}\\ \lambda = h/p = h/m^*v = h/\gamma mv\end{pmatrix} \Leftrightarrow \begin{pmatrix} p = m^* v \\ \tilde{E} = hf\end{pmatrix} \underline{\text{Creates matter-wave spinning } (\mathbf{L}_s = J^*\omega_s),} \\ u = \lambda f = \tilde{E}/p,\ \tilde{E} = hf = E_k = \dfrac{m^* v}{1+\sqrt{1-v^2/c^2}},\ v = u - \lambda\dfrac{du}{d\lambda} = -\lambda^2\dfrac{df}{d\lambda} = \dfrac{dE}{dp} = \dfrac{dE_k}{dp} = \dfrac{d\tilde{E}}{dp}\end{array}\right] \\ \Leftrightarrow \\ \left[\begin{array}{l}\text{Mechanically spinning particle } (\mathbf{L}_s = J^*\omega_s) \ \underline{\text{can create linear particle thrust } (p = m^* v),} \\ E_s = \dfrac{\mathbf{L}_s\omega_s}{1+\sqrt{1-v^2/c^2}} = E_k = \dfrac{pv}{1+\sqrt{1-v^2/c^2}} = \tilde{E} = hf \Rightarrow [\mathbf{L}_s\omega_s = pv]\end{array}\right]\end{array}\right\} \quad (2.11\text{-}4)$$

- **Macro-World Applicability: From Micro to Macro**

What applies to elementary particles in the micro-world also extends to **macro-astronomical masses** and systems, particularly those that are **intrinsically periodic** or repetitive, such as **solar systems**.

The key difference is that the **macro-world Planck constant (H)** would differ from the **micro-world Planck constant (h)**. This distinction will be explored further in this chapter under **Section 2.3.3**, titled **Macro-Cosmological Matter-Waves and Gravitation**.

- **The Influence of Spinning and Magnetic Properties on Gravitation**

Since **spinning** and **magnetic moments** are causally related, it is reasonable to assume that both **mass spinning** and **magnetic properties** significantly influence **gravitation**. Additionally, the existence of **magnetic properties** is intrinsically tied to **electric** and **spinning matter** properties, and vice versa. These relationships will be explored in more detail in Chapters 3, 4.0, 4.1, and 10.

- **Resonating Universe and Gravitation**

Relying solely on **electromagnetic concepts** to explain **gravitation** is not defensible, as we currently have no evidence that **electromagnetic shielding** can stop, neutralize, or cancel gravity. However, by incorporating the idea of a **Resonating Universe**—along with the **wave-particle duality** of matter in motion and combining these with **coupled electromagnetic and mechanical effects** and the natural force concepts proposed by Nikola Tesla and Rudjer Bošković, we could imagine a much more comprehensive explanation of gravity.

[♣ COMMENTS & FREE-THINKING CORNER (only a raw material for later editing)**:**

In the following brainstorming examples (labeled A through F), we will briefly hypothesize and explore ideas regarding the coupling of mechanical moments and masses. Our goal is to suggest potential consequences that could arise from creatively combining various available options. This exercise will require a high level of intellectual flexibility and tolerance to oversimplifications (see also the relevant equations in Chapter 5, from (5.4.1) to (5.4.10)).

A)

Let us begin by imagining a type of matter-wave or energy state that, by coincidence, involves both linear and rotational motion. This type of energy state could serve as a fundamental building block of mass. For example, consider a matter-wave in a specific wave-packet configuration Ψ that is in linear motion, while also incorporating elements of spinning motion (with frequency ω_s. This combination would represent a quantity of linear motion coupled with an angular (or spin) mechanical momentum. Such a configuration would correspond to a relatively stable, non-dispersive formation in both time and space.

The key idea here is that many such elementary states, which can be thought of as atomized energy domains (like wave packets), would, after certain processes of superposition and integration, combine to form what we perceive as a stable particle with rest mass. To make this concept mathematically operational, we could start by introducing the elementary matter-wave function Ψ in a formal way, as outlined in earlier chapters (see also the analogies in Chapter 1 and the equations starting with (4.9.0) in Section 4.3, which address wave functions), as follows,

$$\left[\begin{array}{c}\underline{\left\{\begin{array}{l}E_k = E_{k-lienear} + E_{k-spinning} = \frac{1}{2}mv^2 + \frac{1}{2}\mathbf{J}\omega_s^2 = \\ = \frac{1}{2}pv + \frac{1}{2}\mathbf{L}_s\omega_s = \frac{p^2}{2m} + \frac{\mathbf{L}_s^2}{2\mathbf{J}} = \frac{1}{2}(m+\Delta m)v^2 = \\ = \frac{1}{2}m^*v^2 = \frac{1}{2}p^*v,\ \Delta m = \mathbf{J}\left(\frac{\omega_s}{v}\right)^2 \\ p = mv,\ \mathbf{L}_s = \mathbf{J}\omega_s,\ p^* = (m+\Delta m)v = m^*v\end{array}\right\}} \wedge \underline{\left\{\begin{array}{l}m\\ v\\ p\\ vdp\end{array}\right\}\left(\begin{array}{c}\Leftrightarrow\\ \text{analog to}\end{array}\right)\left\{\begin{array}{l}\mathbf{J}\\ \omega\\ \mathbf{L}\\ \omega d\mathbf{L}\end{array}\right\}} \Rightarrow \\ \Rightarrow dE_k = \underline{vdp + \omega_s d\mathbf{L}_s} = \Psi^2 dt = vdp^*(=\omega d\mathbf{L}^*)\end{array}\right], \quad (2.11\text{-}5)$$

$\omega_s = 2\pi f_s$ (=) spinning frequency (around own particle axis, linked to mass center)

$\omega = 2\pi f$ (=) rotation around other externally placed point

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

or,

$$\left[\begin{array}{l}\underline{\left\{\begin{array}{l}E = \sqrt{E_0^2+p^2c^2} = E_0+E_k=\gamma mc^2,\\ E_0 = mc^2 = const.\\ p=\gamma mv,\ \gamma = (1-v^2/c^2)^{-0.5}\end{array}\right\}}\text{and}\underline{\left\{\begin{array}{l}m\\ v\\ p\\ vdp\end{array}\right\}\Leftrightarrow(\text{analog to})\left\{\begin{array}{l}J\\ \omega\\ L\\ \omega dL\end{array}\right\}}\Rightarrow\\ \Rightarrow dE = dE_k = c^2 d(\gamma m) = \underline{vdp + \omega_s dL_s} = \Psi^2 dt\end{array}\right]\Rightarrow$$

$$\Psi^2 = \frac{dE}{dt} = \underline{v\frac{dp}{dt} + \omega_s\frac{dL_s}{dt}} = \frac{dx}{dt}\frac{dp}{dt} + \frac{d\alpha}{dt}\frac{dL_s}{dt} = \underline{vF+\omega_s\tau} = Power = [W] \tag{2.5.1}$$

The concept behind equation (2.5.1) is to introduce the idea that a particle's motion—when considering its total energy content and internal structure—can be viewed as a combination of mutually coupled linear motion $\mathbf{vdp}$ and rotational (spinning) motion $\omega_s dL_s$. Both motion components play an intrinsic role in determining the total mass of the particle, with the spinning motion specifically contributing to its rest mass.

It is important to remember that any linear motion of a particle (in the absence of measurable self-spinning) can also be interpreted as rotational motion around some external point (the center of rotation), $dE = dE_k = c^2 d(\gamma m) = vdp = \omega dL$. The curvature radius (or radius of rotation) in this case could be arbitrarily large. Similarly, simple circular motion of a particle with mass m (without self-spinning) can be regarded in two equivalent ways: as linear motion along a circular path, or as rotational motion around its center of rotation. For example, the particle's motion could be expressed as:

- Linear motion along a curved path, and
- Rotational motion around an external point or axis,

$E_k = \frac{1}{2}mv^2 = \frac{1}{2}J\omega^2 = \frac{1}{2}pv = \frac{1}{2}L\omega$, $dE_k = vdp = \omega dL$, $v=\omega r, J = mr^2$, $p = mv, L = J\omega$, $\frac{L}{p} = \frac{v}{\omega} = \sqrt{\frac{J}{m}} = r$.

B)

Motional or kinetic energy of particles can be regarded as having any positive value that depends on velocity when measured externally in the space where a particle is in motion. However, when we solve the relativistic equation that connects all energy-momentum aspects of a single particle, we may encounter a solution for kinetic energy that appears as a negative value, corresponding to the particle's rest mass energy, expressed as follows:

$$\begin{array}{l}\left\{\begin{array}{l}E^2 = E_0^2+p^2c^2 = (E_0+E_k)^2 = E_0^2+2E_0E_k+E_k^2,\\ E_0 = mc^2,\ E=\gamma mc^2,\ E_k = E-E_0 = (\gamma-1)mc^2,\ \gamma=(1-v^2/c^2)^{-1/2}\end{array}\right\}\Rightarrow\\ E_k^2+2E_0E_k-p^2c^2 = 0\Rightarrow E_k = -E_0\pm\sqrt{E_0^2+p^2c^2} = -E_0\pm E\Rightarrow\\ E_k = \left\{\begin{array}{l}+E_k\\ -E_0\end{array}\right\} = \left\{\begin{array}{l}(\gamma-1)mc^2\\ -mc^2\end{array}\right\}(=)\\ (=)\left\{\begin{array}{l}\text{motional particle energy in "external space"} = (\gamma-1)mc^2\\ \text{motional particle energy internaly captured by rest mass} = -mc^2\end{array}\right\}.\end{array} \tag{2.5.1-1}$$

This result, given in equation (2.5.1-1), may seem illogical and it could be dismissed as unrealistic. The internal structure of a particle, which contributes to its rest mass, comprises various components of motion and spin that are tightly packed, self-stabilized, and internally closed.

To address this, we can consider an alternative conceptual approach: the idea that negative motional energy is associated with the ordinary motional energy that is "frozen," packed, or captured by the particle's rest mass, represented as:

$$dE = dE_k = c^2 d(\gamma m) = vdp + \omega_s d\mathbf{L}_s \Rightarrow \int_{[\omega]} \omega_s d\mathbf{L}_s = -E_0 = -mc^2 \tag{2.5.1-2}$$

The creation of stable rest mass could be viewed as part of an integration or superposition process derived from equations (2.5.1) and (2.5.1-2). In most cases, the energy component $\omega d\mathbf{L}$ remains internally blocked or by mass captured, appearing "frozen" within the particle's structure. This results in a constant and stable rest mass, which effectively becomes unobservable in differential equations since the derivation of constants yields zero.

In essence, rest mass can be understood as an accumulator or reservoir of agglomerated, self-stabilized, and well-integrated internal "spinning energy elements." While the terminology and conceptualization for this idea may still be evolving, it does not diminish the validity of the attempt to express it.

When rest mass begins to move and interacts with other objects—such as during linear motion or scattering events, the internally packed "rotational energy" or certain vortex content unfold externally. This unfolding process plays a crucial role in the creation of de Broglie matter waves and initiates interactions with nearby objects, which can be indirectly measured through the Compton effect, photoelectric effect, and diffraction of elementary particles (for further details, see chapters 4.0 and 4.1 of this book).

The most direct external indicators of these hidden rotational behaviors are the spin and orbital momentum attributes of all elementary particles. This phenomenon is not limited to subatomic particles; larger astronomical bodies such as planets, moons, and asteroids can also exhibit spinning behaviors.

C)

To be straightforward, let's consider a particle with mass m moving in its laboratory frame with a velocity v. In this scenario, the particle exhibits rectilinear motion without any measurable spinning elements (as shown in the middle column of T.2.3). Despite this apparent simplicity, we aim to demonstrate that even a mass in motion encompasses a combination of rotational and rectilinear motions. Some aspects of internal spinning are hidden within the structure of the rest mass and are not externally visible, yet they contribute to the total energy of the particle.

If the same particle in linear motion were also spinning, with an externally measurable rotation, its total motional energy would include an additional component, as indicated in the last column of T.2.3.

T.2.3 The motional particle with non-zero rest mass (in a Lab. System)	The particle which is only in rectilinear motion	Particles in rectilinear motion combined with spinning
Total particle energy $dE_t = dE = dE_k = vdp$	$E_t = E_0 + E_k =$ $= mc^2 + \int_{[v]} vdp$ $E_0 = E(L_0) = mc^2$ $E_k = (\gamma - 1)mc^2 = \int_{[v]} vdp$ $E_t^2 = E_0^2 + p^2c^2$ $\left\{ v << c \Rightarrow E_k = \frac{1}{2}mv^2 \right\}$	$E_t = E_0 + E_k =$ $= mc^2 + \int_{[v]} vdp + \int_{[\omega]} \omega dL$ $E_0 = E(L_0) = mc^2$ $E_k = \int_{[v]} vdp + \int_{[\omega]} \omega dL$ $E_t^2 = (E_0 + \int_{[\omega]} \omega dL)^2 + p^2c^2$ $\left\{ \begin{matrix} v << c, \ \omega = low \Rightarrow \\ E_k = \frac{1}{2}mv^2 + \frac{1}{2}J\omega^2 \end{matrix} \right\}$
Linear momentum	$p = \gamma mv$	$p = \gamma mv$
Orbital momentum	$\omega = 0,\ L = L_0 + J\omega = L_0$ $L_0 = const.$	$\omega \neq 0,\ \vec{L} = \vec{L}_0 + J\vec{\omega}$

....

The relations outlined in T.2.3 must align with the conservation laws of linear and orbital momentum, as well as the law of total energy conservation, regardless of the transformations experienced by the moving particle. However, before we establish proof and the conceptual framework behind these relationships, it's important to recognize that the idea of energy transformation from rotation (or spinning) to rest mass content remains largely speculative and hypothetical, as introduced in T.2.3.

This can be visualized as a conceptual model where the rest mass acts as an energy vortex or sink concerning its rotational energy states, $E_t^2 = (E_0^*)^2 + p^2c^2 = (E_0 + \int_{[\omega]} \omega dL)^2 + p^2c^2 = \gamma mc^2$. In Chapter 4.1, we will further explore this idea by conceptualizing the motion of any particle or energy state as an interaction between two bodies or two states. In this framework, the initial energy state interacts with its surroundings, which allows us to theoretically envision the motion of a corresponding center of mass, a situation akin to a "two-body" interaction.

By introducing an effective center-of-mass reference system, we can demonstrate that the energy component of this "binary system" behaves like a rotating of reduced mass around its effective center of mass. An intriguing consequence of this model is that any linear (or curvilinear) motion can be viewed as a specific form of rotational motion, where the relevant radius r is sufficiently large to render the elements of rotation imperceptible.

In essence, this implies that pure linear motion does not exist in our universe. For cases involving the differential energy balance of curvilinear motion, we will identify primary energy domains as follows:

$$dE_t = dE = dE_k = vdp = \omega dL = c^2 d(\gamma m),\ v = \omega r,\ \vec{r} \times d\vec{p} = d\vec{L}.$$

D)

Let us continue brainstorming and hypothesizing in the frames of the same idea, which is stating that linear and rotational motions are mutually complementary and united. If we start with the relativistic expression for a total moving particle energy, $\mathbf{E_t^2 = E_0^2 + p^2c^2}$, we can notice that such energy has one static or constant energy member $\mathbf{E_0}$, and another dynamic or motional energy member $\mathbf{pc}$. Because of here "postulated intrinsic unity" of linear and rotational motions, we could merely exercise that,

$$\left\{pc = \text{Dynamic or motional energy part} = E_{linear-motion} + E_{rotational-motion} = E_{lm} + E_{rm}\right\} \Rightarrow$$

$$\left\{\begin{array}{l} E_t^2 = E_0^2 + p^2c^2 = (\text{Static or constant energy part})^2 + (\text{Dynamic or motional energy part})^2 = \\ = E^2 = (\gamma mc^2)^2 = (E_0 + E_k)^2 = E_0^2 + 2E_0E_k + E_k^2, \\ E_0 = \text{Static or constant energy part} = mc^2, E_k = \text{Motional energy part} = (\gamma - 1)mc^2, \\ (pc)^2 = (E_{lm} + E_{rm})^2 = 2E_0E_k + E_k^2. \end{array}\right\}$$

$$\Rightarrow E_{rm} = pc - E_{lm} = pc - \frac{pv}{1+\sqrt{1-(\frac{v}{c})^2}} = pc(1 - \frac{\frac{v}{c}}{1+\sqrt{1-(\frac{v}{c})^2}}) = \quad (2.5.1\text{-}3)$$

$$= pc(1-\frac{u}{c}),\ u = \frac{v}{1+\sqrt{1-(\frac{v}{c})^2}}).$$

The outcome of equation (2.5.1-3) suggests that when rectilinear and rotational motions are mutually coupled, the energy associated with the rotating (or spinning) motion is constrained and analytically dependent on the parameters of its linear motion. This relationship is somewhat analogous to the behavior seen in resonant oscillatory circuits and electromagnetic waves.

A more promising and general approach to unifying linear and rotational motions is to remain within the established framework of relativity theory as much as possible. We will begin by assuming that the kinetic energy of a particle comprises both linear motion components and spinning components.

To facilitate clearer understanding, we will use different indexing for these energy components. *We will first assume that motional or kinetic energy (of a particle) would have linear motion, (* $E_{k\text{-linear}} = E_{kl}$ *) and spinning (* $E_{k\text{-rotation}} = E_{kr} = E_{spinning} = E_s$ *) components,* $E_k = E_{k\text{-linear}} + E_{k\text{-rotation}} = E_{kl} + E_{kr}$. This indexing (as k-linear, kl, k-rotation, kr, or spinning) is intended to create natural visual and intuitive connections with similar expressions and discussions found in other chapters. Given that this book has been developed and modified over an extended period, some discrepancies may arise that will need to be addressed in the future.

For now, the author prioritizes conveying the essential meanings of innovative ideas related to gravitation to open-minded, creative, and intellectually flexible readers. Later we will always have a space for improvements and optimizations.

The square of a 4-vector (in the Minkowski space) of the relevant moment $\overline{P}_4 = \overline{P}\left[p = \gamma mv,\ \frac{E}{c} = \gamma mc\right]$ should be invariant regarding referential system changes, and by continuing similar exercising as before, we will have,

$$\left\{\begin{array}{l} \overline{P}_4^2 = \overline{P}^2(\vec{p}\,,\ \frac{E}{c}) = \vec{p}^2 - \frac{E^2}{c^2} = \vec{p}'^2 - \frac{E'^2}{c^2} = \vec{p}''^2 - \frac{E''^2}{c^2} = \text{invariant}, \\ E = E_0 + E_k = E_0 + E_{k\text{-linear}} + E_{k\text{-spinning}} = E_0 + E_{kl} + E_s \end{array}\right\} \Rightarrow \quad (2.5.1\text{-}4)$$

$$\Rightarrow \vec{p}^2 - \frac{\left[E_0 + E_{kl} + E_s\right]^2}{c^2} = \vec{p}'^2 - \frac{\left[E'_0 + E'_{kl} + E'_s\right]^2}{c^2} = \text{invariant}.$$

From (2.5.1-4) we can elaborate the following possibilities regarding particle states in a particular system of reference (kind of the center of a mass system), where the particle would have a specific state of rest,

$$\vec{p}^2 - \frac{E^2}{c^2} = \vec{p}'^2 - \frac{[E'_0 + E'_{kl} + E'_s]^2}{c^2} = \begin{cases} -\frac{1}{c^2}(mc^2)^2 = -m^2c^2 \ \text{ if } \vec{p}' = \vec{0},\ E'_s = 0 \\ (\textit{no linear motion, no spinning}), \\ ----------------- \\ -\frac{1}{c^2}(mc^2 + E'_s)^2 \quad \text{ if } \vec{p}' = \vec{0},\ E'_s \neq 0 \\ (\textit{no linear motion, only spinning}). \end{cases} \tag{2.5.1-5}$$

From (2.5.1-5) it is almost evident that spinning energy states could effectively be considered entering rest mass or rest energy states ($M = m + \frac{E'_s}{c^2}$) since we can generalize (2.5.1-5) as,

$$\begin{aligned} &\vec{p}^2 - \frac{E^2}{c^2} = \vec{p}'^2 - \frac{[E'_0 + E'_{kl} + E'_s]^2}{c^2} = -\frac{1}{c^2}(Mc^2)^2 = -M^2c^2, \\ &E = E_0 + E_{kl} + E_s = \gamma Mc^2 = \gamma(m + \frac{E_s}{c^2})c^2,\ E_0 = Mc^2, M = m + \frac{E_s}{c^2}, E_s = E'_s, \\ &E_{kl} = (\gamma - 1)Mc^2 = (\gamma - 1)(m + \frac{E_s}{c^2})c^2,\ p = \gamma Mv = \gamma(m + \frac{E_s}{c^2})v. \end{aligned} \tag{2.5.1-6}$$

For further elaboration on this concept, please refer to equations (2.11.4), (2.11.5), and (2.11.13-5) as well as similar discussions in Chapter 4.1.

Notably, in equations (2.5.1-4) to (2.5.1-6), we implicitly introduce a "parameterized" state of rest, where a certain rest mass can exhibit zero linear velocity $v = 0, E_{kl} = 0, p = 0$ while either spinning around an axis through its center of gravity $E_{kr} = E_s \neq 0$, or remaining entirely at rest without any externally measurable linear motion or spinning, $v = 0, \vec{p} = \vec{0}, E_s = 0$. The energy associated with self-spinning particles effectively contributes to their rest mass, $M = m + \frac{E_s}{c^2}$. Consequently, we can neglect (or account for) this intrinsic particle rotation and consider the particle as performing only linear motion externally.

It is important to recognize that other "parameterized rest-mass" scenarios can also be envisioned, involving additional energy components. The main intention here is to demonstrate that certain masses represent well-integrated or compact spinning states and to relate this concept to the field of gravitation.

Now, let us continue exploring new ideas. The expression in equation (2.5.1-6) effectively describes the momentum-energy states of a specific particle that appears to be in linear motion, with spinning "attached" to the rest mass state. We can also assert that every linear motion is, in fact, a specific case of curvilinear motion, where the radius of rotation r is arbitrarily large (and where the tangential velocity of the particle is $v = \omega r$. In the case of idealized rectilinear motion, the radius of rotation approaches infinity ($r \to \infty$), while in all other scenarios, the radius of rotation is finite ($0 \leq r < \infty$).

This perspective allows us to introduce an equivalent rotation into the relevant four-vectors of relativity theory, based on a straightforward analogical formulation, such as for instance,

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

$$\left\{\bar{P}_4^2 = \bar{P}^2(\vec{p}\,,\,\frac{E}{c}) = \vec{p}^2 - \frac{E^2}{c^2} = \vec{p}'^2 - \frac{E'^2}{c^2} = \vec{p}''^2 - \frac{E''^2}{c^2} = \text{invariant} = -\frac{E_0^2}{c^2} = -M^2c^2\,,\right\} \Rightarrow$$

$$\Rightarrow \left\{\bar{L}_4^2 = \bar{L}^2(\vec{L}\,,\,\frac{E}{\omega_c}) = \vec{L}^2 - \frac{E^2}{\omega_c^2} = \vec{L}'^2 - \frac{E'^2}{\omega_c^2} = \vec{L}''^2 - \frac{E''^2}{\omega_c^2} = \text{invariant} = -\frac{E_0^2}{\omega_c^2} = -J^2\omega_c^2\right\},$$

$$E^2 = E_0^2 + \vec{p}^2c^2 = E_0^2 + \vec{L}^2\omega_c^2,\ E = \gamma Mc^2 = \frac{c^2}{rv}L\,,\ M = m + \frac{E_s}{c^2},$$

$$E_0 = mc^2 + E_s = Mc^2 = J\omega_c^2 = \frac{c^2}{r\gamma v}L = E - E_k\,,$$

$$\vec{p} = \gamma M\vec{v},\ \vec{L} = \vec{L} + \vec{S} = \gamma J\vec{\omega} = \vec{r}\times\vec{p}\,,$$

$$\vec{p}^2c^2 - (E_k + E_0)^2 = -E_0^2 \Leftrightarrow \vec{p}^2c^2 = 2E_kE_0 = 2E_k(mc^2 + E_s) \Rightarrow$$

$$E_k = E - E_0 = (\gamma-1)Mc^2 = \frac{pv}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{\vec{p}^2c^2}{2(mc^2 + E_s)} =$$

$$= (\gamma-1)J\omega_c^2 = \frac{L\omega}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{\left|\vec{L}+\vec{S}\right|\omega}{1+\sqrt{1-\frac{v^2}{c^2}}}, \qquad (2.5.1\text{-}7)$$

$$v = \omega r\,,\ pv = L\omega\,,\ pc = L\omega_c\,,\ \frac{c}{\omega_c} = \frac{v}{\omega} = \sqrt{\frac{J}{M}}\,,\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{\omega^2}{\omega_c^2}}},$$

$$dE = dE_k = vdp = \omega dL\,,\ pdv = Ld\omega\,,\ vdp + pdv = \omega dL + Ld\omega\,.$$

It is evident that here we are testing the concept that internal, effective spinning is an intrinsic property of a rest mass (even in cases when externally we do not see or detect spinning). Of course, other "*parameterized" total-energy* situations are also imaginable (as having additional energy members).

E)

If we accept that the internal structure of elementary particles is a complex, self-contained, and rotating waves formation, we cannot simultaneously claim that there is another finite and initial rest mass within it. In other words, the rest mass we observe, and measure externally should be viewed as the result of specific internal wave-energy packing, which gives the impression of a stable and solid particle.

Consequently, none of the variables in the differential relations presented in (2.5.1) can be treated as constant:

$$d(\gamma\, m) = md\gamma + \gamma dm,\ d(\gamma\, mv) = mvd\gamma + \gamma vdm + \gamma mdv = \frac{1}{c^2}dE\,, \qquad (2.5.2)$$

However, after integration, we can obtain a constant rest mass as a resulting product.

F)

Another message that could be extracted starting from (2.5.1) and later, is that resulting force acting on any mass in motion (not only between elementary particles) should have one linear, $F = F_{linear}$, and one angular, $\tau = F_{angular}$ or spinning component. Moreover, the **original Newton force definition, as the time derivation of linear momentum, $F = \frac{dp}{dt}$, should be (analogically) generalized to have both, angular, $\tau = \frac{dL}{dt}$ and linear force component/s**, as for instance,

$$\frac{dE}{dt} = v\frac{dp}{dt} + \omega\frac{dL}{dt} = vF + \omega\tau = \frac{1}{dt}\cdot\left[dx\cdot\frac{dp}{dt}\right] + \frac{1}{dt}\cdot\left[d\alpha\cdot\frac{dL}{dt}\right] =$$
$$= \frac{1}{dt}\cdot\left[\begin{pmatrix}\text{Energy realized}\\ \text{by linear}\\ \text{force}\\ \text{component}\end{pmatrix} = vFdt\right] + \frac{1}{dt}\cdot\left[\begin{pmatrix}\text{Energy realized}\\ \text{by angular}\\ \text{force}\\ \text{component}\end{pmatrix} = \omega\tau dt\right] = c^2\frac{d(\gamma m)}{dt} = \Psi^2 \quad (2.5.3)$$

The concept of forces, as highlighted here, belongs to classical mechanics and deserves further elaboration. However, the principal message is already clear: rotation and linear motion are always intertwined, and their respective force components (F and τ) interact according to the rules of vector algebra—provided that the necessary dimensional arrangements are correctly applied.

For a moment, let's set aside questions regarding the specific fields and forces involved in producing motion and start from a general premise: if an object (such as a particle or wave) possesses a specific energy and effective mass, then all matter in motion should have one linear and one angular force component. The first component is the linear force acting along its axial path (displacement Δx), and the second sweeps an angle along its circular path (angle segment $\Delta\alpha$), as indicated in (2.5.3), while also considering a certain external, static, or relative rest energy level $\mathbf{E_0}$:

$$E_{tot.} = F_{linear}\cdot\Delta x + F_{angular}\cdot\Delta\alpha + E_0 = E_{k-linear} + E_{spinning} + E_0 = E_t = \gamma Mc^2 \quad (2.6)$$

Next, let's mathematically rearrange the expression for the definition of linear force, focusing on the force acting on a particle in linear motion based on Newton's definition of force, $\mathbf{F} = \mathbf{F_{linear}} = \mathbf{dp/dt}$. We will replace mass with its relativistic energy equivalent, following the approach previously used in developing the force expressions (2.4) to (2.4-3).

$$F = \frac{dp}{dt} = \frac{d(\gamma mv)}{dt} = \frac{d(\frac{E_t}{c^2}v)}{dt} = \frac{d(\frac{E_0+E_k}{c^2}v)}{dt} = \frac{E_t}{c^2}\frac{dv}{dt} + \frac{v}{c^2}\frac{dE_t}{dt} =$$
$$= \frac{E_0}{c^2}\frac{dv}{dt} + \frac{E_k}{c^2}\frac{dv}{dt} + \frac{v}{c^2}\frac{dE_0}{dt} + \frac{v}{c^2}\frac{dE_k}{dt} = \underline{\frac{E_0}{c^2}\frac{dv}{dt} + \frac{E_k}{c^2}\frac{dv}{dt} + \frac{v}{c^2}\frac{dE_k}{dt}} = F_{linear}(=)\left[\frac{kg\times m}{s^2}\right], \quad (2.7)$$
$$E_k = E_{k\text{-linear}} = (\gamma - 1)mc^2.$$

By analogy with (2.7), we could hypothetically exercise the expression for the missing angular (or rotational) force definition, known as torque,

$$\tau = \frac{dL}{dt} = \frac{E_t}{\omega_c^2}\frac{d\omega}{dt} + \frac{\omega}{\omega_c^2}\frac{dE_t}{dt} = \underline{\frac{E_0}{\omega_c^2}\frac{d\omega}{dt} + \frac{E_s}{\omega_c^2}\frac{d\omega}{dt} + \frac{\omega}{\omega_c^2}\frac{dE_s}{dt}} = F_{angular}(=)\left[\frac{kg\cdot m^2}{s^2}\right], \quad (2.8)$$
$$E_k(=)E_{k-spinning} = E_s,\ \omega_c(=)[c/r](=)[1/s].$$

Since the total energy, (2.6), and an entire resulting force, should have both, linear and rotational (or spinning) elements (here mentioned as angular force elements), we can merely combine (2.7) and (2.8),

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

$$\left.\begin{array}{l} E_{tot.} = E_t = \iiint_{[x,\alpha]} (v dp + \omega dL) = F_{linear} \cdot \Delta x \ + \ F_{angular} \cdot \Delta\alpha \ + \ E_0 = \\ = (\frac{E_0}{c^2}\frac{dv}{dt} + \frac{E_k}{c^2}\frac{dv}{dt} + \frac{v}{c^2}\frac{dE_k}{dt})_{linear} \cdot \Delta x + \\ +(\frac{E_0}{\omega_c^2}\frac{d\omega}{dt} + \frac{E_k}{\omega_c^2}\frac{d\omega}{dt} + \frac{\omega}{\omega_c^2}\frac{dE_k}{dt})_{angular} \cdot \Delta\alpha \ + \ E_0 = \sqrt{E_0^2 + p^2c^2} \\ E_k = E_{k-linear} + E_{k-angular} = F_{linear} \cdot \Delta x \ + \ F_{angular} \cdot \Delta\alpha = \int_{[\Delta t]} \Psi^2(t) dt. \end{array}\right] \Rightarrow$$

$$\Rightarrow \left[\begin{array}{l} \overline{E}_t = E_0 \pm I \cdot pc = E_t \cdot e^{\pm I\theta} = \sqrt{E_0^2 + p^2c^2} \cdot e^{\pm I \cdot \mathrm{arctg}\frac{pc}{E_0}} = \gamma M c^2 \cdot e^{\pm I\theta} = \gamma \overline{M} c^2, \\ \overline{M} = M \cdot e^{\pm I\theta} = M \cdot \cos\theta \pm I \cdot M \cdot \sin\theta = M_r \pm I \cdot M_i, \ I^2 = -1, \\ \gamma M c^2 = \sqrt{E_0^2 + p^2c^2} \Leftrightarrow (\gamma M)^2 = (\frac{E_0}{c^2})^2 + (\frac{p}{c})^2, \gamma = 1/\sqrt{1-(\frac{v}{c})^2}, \\ E_0 = mc^2 + E_s = Mc^2, \vec{p} = \gamma M \vec{v}, \ \overline{P} = \gamma \overline{M} v = p \cdot e^{\pm I\theta}, \ E_s = E_{k-spinning}, \\ \theta = \mathrm{arctg}\frac{pc}{E_0} = \mathrm{arctg}(\gamma \frac{v}{c}) = \mathrm{arctg}\frac{\frac{v}{c}}{\sqrt{1-(\frac{v}{c})^2}}, \\ (\Rightarrow \overline{\Psi}^2 = \frac{\partial \overline{E}_t}{\partial t} = \pm Ic \frac{\partial p}{\partial t}). \end{array}\right. \qquad (2.9)$$

It appears that linear momentum p, when considering equations (2.7) to (2.9), represents a mixed entity that combines elements of linear motion with associated rotation or spinning. This concept aligns with Louis de Broglie's theory of matter waves ($\vec{p} = \frac{\omega}{v}\vec{L} = \gamma m\vec{v}, \vec{L} = \frac{v}{\omega}\vec{p} = \frac{v}{\omega}\gamma m\vec{v}$; -see section 4.3-0 and Chapter 4.1). A more comprehensive explanation of the relationships between linear and angular momentum can be found in Chapter 10 of this book (see equations (10.1.4) to (10.1.7)).

Next, consider the typical situation where the group velocity and phase velocity of a specific energy-momentum state in motion are not equal. This discrepancy introduces additional complexity into the concepts of gravitational and inertial mass, which is related to particle-wave duality. Such an understanding necessitates a reformulation of Newton's laws.

Gravitation, mass spinning, and energy

Newton law expresses gravitational attraction between two masses,

$$F_{12} = -F_{21} = G\frac{m_1 m_2}{r^2} \Leftrightarrow F_{12} + F_{21} = 0 \ .$$

If we imagine that two isolated masses that are mutually attracting with their own gravitational force would experience virtual and infinitesimal displacement along the line which is connecting their centers, an effective consequence will be that mutual mass exchange could happen (such as $dm_1 = -dm_2$), but the total mass of such system is conserved ($m_1 + m_2 = const.$), as follows,

$$F_{12}dr + F_{21}dr = 0 \Leftrightarrow c^2 dm_1 + c^2 dm_2 = 0 \Leftrightarrow dm_1 + dm_2 = 0, dm_1 = -dm_2 \Rightarrow m_1 + m_2 = const.$$

Obviously, gravitational attraction is presenting or implicitly complying with a mass exchange between mutually attracting masses ($dm_1 = -dm_2$). Here we are considering only attraction between purely gravitational masses (but something similar should be extendable to all kinds of energy states).

Let us explore rare motional and inertial matter states where relevant group and phase velocity are mutually equal. Of course, group and phase velocity (v, u) should be measured related to a coordinate system linked to the inertial state in question,

$$\left\{\begin{array}{l} v = u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda}, \\ v = u \end{array}\right\} \Rightarrow \lambda \frac{du}{d\lambda} = 0 \Rightarrow u = \lambda f = const. = C.$$

Based on Relativity theory assumptions we know that group and phase velocity could be mutually equal only in two cases:

$$\{v = u\} \Rightarrow \left\{\begin{array}{l} v = u = 0 \\ v = u = C\ (=\ \text{universal constant equal to speed of light}) \end{array}\right. .$$

For both limiting cases, we could say that group and phase velocity are not only mutually equal but also constant (meaning that there is no acceleration involved). With the help of some imagination (and a little bit of a creative brainstorming guesswork), we could ask ourselves if such two cases ($v = u = C \vee 0$) are in some ways (under certain conditions) describing the same motional state. Such "motional states" (if realistic) could be something like freely propagating photons, or mass in a state of rest (as a "frozen matter-waves condensate"). Anyway, we have arguments to say that motional states in our universe are states of relative motions (including states of rest), where all related velocities are relative to a certain system of reference. Only speed of light (based mostly on somewhat questionable assumptions, within Relativity theory) has a specific privileged or exceptional character, which, in here introduced guesswork, could be applicable to a certain state of rest ($(v = u = 0) \Leftrightarrow (v = u = C)$).

In other cases, when group (or phase) speed is very small, we will have

$$\left\{ v <<< C \Rightarrow v \cong 2u \cong u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda} \right\} \Rightarrow v = -\lambda \frac{dv}{d\lambda} \Rightarrow \lambda v = 2\lambda u = 2\lambda^2 f = \lambda_0 v_0 = const.$$

Inertial states are generally related to uniform motions, where there is no force involved (force equals zero). *Here, we should be very careful with the understanding of universal constant C. Such constant exist, it has velocity dimension, and specific definite value, but there are measurements and mathematical explorations showing situations when the speed of light is not constant (when compared to constant C), as follows,*

$$\left\{\begin{array}{l} F = \frac{dp}{dt} = \frac{d(\gamma mv)}{dt} = \frac{d(m^* v)}{dt} = m^* \frac{dv}{dt} + v \frac{dm^*}{dt} = 0 \\ m^* = \gamma m,\ \gamma = (1 - v^2/c^2)^{1/2} = \frac{m^*}{m} \end{array}\right\} \Rightarrow \left\{ dp = m^* dv + v dm^* = 0 \Leftrightarrow \frac{dm^*}{m^*} = -\frac{dv}{v} \right\} \Rightarrow$$

$$\Rightarrow \left| \frac{m^*}{m_0} \frac{v}{v_0} \right| = 1,\ m^* v = m_0 v_0 = p = \gamma mv =\ \text{Const.},\ (m_0, v_0 = \text{constants})$$

Obviously, for uniform, inertial, and linear motions, linear momentum is conserved and constant ($p = \gamma mv = m^* v = m_0 v_0$), but this also gives the impression that only group-speed of such motion is uniform and constant. Also, we know that one of the universally valid conservation laws is linear momentum conservation (which is in some way directly interfering with the definition of linear, uniform, and inertial motions). Here, to give a more complete explanation of linear inertial motions, we are exploring different relations between group and phase velocity, because group and phase velocity belong to a more selective and more prosperous concept which can help in establishing bridges between worlds of particles and waves (since Newton laws are not sensitive to such conceptualization).

We also know that many uniform and inertial motions in our universe are rotational or orbital (such as planetary motions, motions associated to atoms internal structure…). By analogy (with linear uniform motions), in cases of uniform angular, rotational, or orbital motions, orbital momentum should also be conserved ($\vec{L} = \vec{r}\times\vec{p} = \gamma m\cdot\vec{r}\times\vec{v} = m^*\cdot\vec{r}\times\vec{v} = m_0\cdot\vec{r}\times\vec{v}_0$) and constant, meaning that both of universally valid *moments conservation laws* are mutually related. More advanced concepts applicable to stable orbital motions are implicating that qualifications like *states of "uniform and inertial"* will be extended and upgraded by addressing associated mechanical and electromagnetic phenomenology of *stationary, standing waves and quantized, periodical motional states* (what will be shown later).

……………………..

Consequently, it would not be surprising (see literature under [23] – [26]) if one day we conclude that natural forces and matter charges, we are presently considering as mutually distinctive, or very much different and specific, have deeper origins and differences only within electromagnetic charges, and their "packing formats". This is also valid for interactions linked to conservation laws related to coupled, orbital, spin, and linear moments ..... ♣]

2.3.1. Extended Understanding of Inertial States

Free style compilation and citations about Inertia, from different sources:

"Linear and Angular Inertia

Inertia equals resistance or tendency to keep a certain motional state. Linear and angular Inertia are the resistance to changes in linear and angular motions. Just as mass m resists changes in linear motion, the moment of inertia J resists changes in rotational motion.

Linear inertia *(linear momentum, p = mv), deals with the resistance of mass to changes in speed along a straight path.*

Angular inertia (angular momentum, L = Jω) and moment of inertia J*, refers to an object's resistance to changes in its rotational motion or rotational speed.*

Linear Inertia depends only on mass. The more massive an object, the greater its inertia, making it harder to move, slow down, or stop. A good way to measure linear inertia is through ***momentum (or we can also say linear momentum)****. Momentum is defined as p = mv, where m is mass and v is velocity. If no net external force acts on the object, it will continue moving at constant velocity. This follows directly from Newton's First Law. To overcome an object's inertia and change its momentum, a net external force is required. Momentum is the product of an object's mass and velocity, and it changes when there is an external force. If dp=d(mv)=m dv, represents a change in momentum (overcoming inertia), then the rate of change of momentum is: dp/dt=d(mv)/dt=mdv/dt=ma, where a is acceleration, or the rate of change of velocity. Thus, the net force required to change the velocity of a mass is given by Newton's Second Law, f=ma. In summary, Linear* ***inertia*** *refers to an object's resistance to changes in its motion, while* ***momentum*** *involves the amount of motion an object has, depending on both its mass and velocity. The key difference is that mass is conserved, while the moment of inertia may not be. "*

Following the same patterns of thinking, here we are arriving closer to the possibility and necessity to **reformulate the universal law and concepts of inertia regarding free, natural, uniform, stable, stationary, and relative mechanical and electromagnetic motions (considering as a generally valid case, the existence of specific intrinsic coupling of linear and rotational (or orbital) motions, including spinning, characterized by $\left\{[m, J], [p, \dot{p}], [L, \dot{L}]\right\}$). Of course, linear and angular motions are also coupled with associated electromagnetic reality. The mentioned specific coupling of linear and circular motions is deeply related to the Particle-Wave Duality and Matter-Waves concepts elaborated in this book (equally valid for micro and macro world motions). Thoughts of inertial**

movements presented in this book agree with innovative inertia concepts elaborated in [36], by Anthony D. Osborne, and N. Vivian Pope, but this should be extended or united with elements of Matter-Waves and Particle-Wave duality concepts (see more about inertia in Chapters 1, 4.2 and 10)**.**

New understanding of inertial motions could be initially formulated as: "All naturally free and inertial, stable and stationary, uniform (mechanical and electromagnetic) motions of objects, particles and other energy states in our universe have intrinsic elements of mutually coupled linear and rotational, or orbital motions (see (2.5.1-7)), tending to keep unchanged the combination of their initial (linear and angular) moments-related states, unless acted upon by an unbalanced (external) force.

For changing an inertial-motion state of an object, at least one of external (unbalanced) linear ($F = \frac{dp}{dt} = \dot{p} = F_{linear}$ ***) or orbital (or torque*** $\tau = \frac{dL}{dt} = \dot{L} = F_{angular}$ ***) force components should be involved."***

Saying the same differently we should consider that every stable, non-dispersive, motional mass formation (particle or its mass-energy equivalent E/c²), in relation to certain system-of-reference, being in some of its inertial states, should present certain stationary and stable (coordinates invariant) orbital state of the following mutually correlated attributes: $\{[m, J], [p, \dot{p}], [L, \dot{L}]\}$ ***, naturally following the path of least resistance, or least action. Specifically coupled linear and orbital elements of certain inertial motion*** $\{[m, J], [p, \dot{p}], [L, \dot{L}]\}$ ***have a natural tendency to host or populate self-closed standing matter-waves. Here, we should not forget that mentioned mechanical attributes are also intrinsically coupled with associated electromagnetic properties or charges.*** *See later* (2.9.4) as the step forward in expressing here extended universal law of Inertia.

Let us try to clarify additionally what an inertial state is. *Since inertial motions present a tendency to maintain the same state of orbital motion (ideally endlessly), we could say that this is a kind of uniform, steady, stationary, often resonant, or standing matter-waves state, which is conserving, or keeping stable important parameters, such as its linear and orbital mechanical moments (*$p = const. \Rightarrow F = dp/dt = 0$*, and/or* $L = const. \Rightarrow \tau = dL/dt = 0$*).* When an object, body or energy-state exists in such steady motional conditions, we could consider it as an inertial state. Of course, we know that many stable motional states also belong to uniform rotational or spinning motions, and we should extend the meaning of Newtonian inertial states to such steady rotational motions, and to their combinations, like orbital and spinning motions of planets in stable solar systems and spinning of subatomic micro particles. ***In fact, (since every linear motion could be presented as an asymptotic case of certain orbital motion with radius of rotation increasing to infinity) linear and angular momentum of certain motion are not separately or independently conserved values (as formulated in Classical Mechanics). Consequently, every motion of a certain entity has at the same time both linear and angular moments (being***

mutually coupled and convertible, thanks to associated matter-waves, what is very much analogical to couplings between an electric and magnetic field in cases of electromagnetic waves, or electromagnetic oscillators and resonant circuits). Later in this chapter, we will demonstrate that inertial planetary, orbital, and periodical motions are also hosting standing matter waves (see 2.3.3. Macro-Cosmological Matter-Waves and Gravitation).

In the first chapter of this book (**1.1 Inertia, Inertial Systems, and Inertial Motions**) we already introduced the concept of inertia and inertial systems, based on "mass – capacitor" analogies, as summarized in T.1.8. It started to be clear that such an idea is only locally applicable "in-average" to relative (or dominant) stability within specific periodical motion conditions, where synchronized and standing matter-waves are involved. In fact, every stable and uniform relative motion, or inertial-state motion is in direct relation with periodic movements and associated matter-waves phenomenology. When matter-waves are becoming uniform, stationary, stable, periodical, and like self-closed standing waves, we can define or describe such states as being inertial states (see more about inertia in Chapters 1, 4.2 and 10). This is now becoming a more complicated situation than simple and isolated inertial states of linear motions, as initially specified by I. Newton.

Since any linear (or rectilinear) motion is only an idealization and boundary case of certain rotating or orbital motion (with sufficiently large radius of rotation $\mathbf{r}$, where actual velocity in question is $v = \omega_r r$), we could say that any linear inertial motion, and inertial system, also belong to a specific uniform (or stabilized) rotational movement. We can conclude from Kepler laws that there is a natural tendency of moving masses towards creating circular or elliptic, closed orbits, when Newton-Coulomb attractive forces are balanced by a repulsive centrifugal force (since in opposite case, we would have only a global tendency towards masses agglomerations). Another natural property of microcosmic entities is to have and maintain stable angular and spin moments. Uniform, stabilized, steady, stationary, and essentially periodical, rotational motions are also presentable as standing-waves resonant structures, or known as possible hosting zones for standing-waves formations. ***In the world of Physics, we are considering universally valid and coincidently applicable laws of conservation of linear and orbital moments (including total energy conservation).*** Let us follow the first and most traditional description of an inertial state, when its linear moment $\mathbf{p}$ is constant (without specifying the state of its orbital moment $\mathbf{L}$), and combine it with de Broglie, matter waves conceptualization. Also, we will bear in mind that linear motion is just a particular case of specific rotating motion with sufficiently large radius $\mathbf{r}$. This will produce the following chain of conclusions,

$$\underline{p = \gamma mv = const., \Rightarrow \lambda = \frac{h}{p} = \frac{2\pi r}{n} = Const., 2\pi r = n\lambda = \overline{const.}, \; r =} \frac{\overline{const.}}{2\pi} \Rightarrow$$

$$\Rightarrow \left\{ \begin{array}{l} \left(\overline{P}_4\right)^2 = (p, \frac{E}{c})^2 = inv. \Leftrightarrow p^2 - (\frac{E}{c})^2 = -\left(\frac{mc^2}{c}\right)^2, E = mc^2 + E_k = mc^2 + \tilde{E} = \gamma mc^2 \\ \tilde{E} = hf = E_k = \frac{\gamma mv^2}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{\gamma J\omega_r^2}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{pv}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{L\omega_r}{1+\sqrt{1-\frac{v^2}{c^2}}} = \\ = \frac{2\pi Lf_r}{1+\sqrt{1-\frac{(2\pi f_r r)^2}{c^2}}} = \frac{Lc^2}{2\pi r^2}\left(1-\sqrt{1-\frac{v^2}{c^2}}\right)\frac{1}{f_r} = \frac{Lc^2}{vr}\left(1-\sqrt{1-\frac{v^2}{c^2}}\right), \\ \gamma mv^2 = \gamma J\omega_r^2 = pv = L\omega_r, v = \omega_r r, \; \omega_r = 2\pi f_r, u = \lambda f = \frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}} \end{array} \right\} \Leftrightarrow$$

$$\Leftrightarrow \underline{p = \gamma mv = \frac{h}{\lambda} = \frac{nh}{2\pi r} = \frac{\gamma Jv}{r^2} = \frac{L}{r} = const. \Leftrightarrow \left(r = \underline{const.}, \; L = CONST.\right)} \qquad (2.9.1)$$

The fact that linear moment **p** *is constant, is producing that at the same time, relevant orbital moment* L *(of the same inertial motion: see equations* (4.3-0) - a,b,c,d,e,f,g,h,i,j,k... in chapter 4.1*) is also constant.* <u>*This is valid only under the assumption that such uniform motion is ideally circular and hosting specific stable, standing-waves formation (*$2\pi r = n\lambda = n\frac{h}{p} = \overline{const.}$*).*</u> For any other closed or elliptic orbit, we can analogically apply Wilson-Sommerfeld-Bohr integrals and keep the same meaning of standing waves formations, leading to the constant orbital moment L .

$$\left\{ \oint_{C_n} p_r dr \; = \; n_r h \; \Rightarrow \oint_{C_n} L d\alpha \; = \; n_\alpha h \right\} \Leftrightarrow L = n_\alpha \frac{h}{2\pi} = \text{Constant}, \qquad (2.9.2)$$
$$0 \le \alpha \le 2\pi, \; n_r = 1,2,3..., \;\; n_\alpha = 1,2,3...,n \;.$$

Now we have a much better picture of what inertial states are. ***To satisfy coincidently both linear and angular moments conservation (what we are anyway considering as universally valid), it is essentially necessary to have a standing matter-waves structure around, and this is equivalent to an inertial state*** *(here related to closed, periodical, circular motions).*

Later, we will show that the framework of (2.9.1) and (2.9.2) is equally applicable to microcosmic and astronomical (orbital) motions, considering that instead of micro-world Planck constant **h**, we should apply new "*Planck gravitational constant*" $H >>> h$ (see later "2.3.3. Macro-Cosmological Matter-Waves and Gravitation"). Similar elaboration regarding new constants which are analog to Planck-constant (but most probably generally applicable to any self-closed macro motion which creates standing waves) can be found in chapter 4.1, around equations (4.3-0), (4.3-0)-a,b,c,d,e,f,g,h,i,j,k... See also illustrations on Fig.4.1.1 and Fig.4.1.1a.

Here we are not facing something very new, and too original. It is the fact that everything in our universe, from galaxies to elementary particles, is in states or relative motions to its environment, either rotating or spinning, or manifesting like being effectively related to certain kind of matter-waves rotation, or having linear and angular motional elements (of course, also being naturally connected to coupled oscillatory motions). Here mentioned, natural and mutually coupled linear and rotational motional elements are like the sort of coupling between electric and magnetic fields when an electromagnetic wave is being created. The familiar type of coupling is also described in chapter 4.0 with Analytic Signal relations between an original (real) wave function and its (imaginary) Hilbert-transform couple. Later, we will discover that in cases of seemingly linear particle motions (where rotation is not externally detectable), there is still a presence of matter-waves coupling between rotation and linear motion, where rotating energy elements are captured by the internal rest-mass structure (see more complete explanation in chapter 10, equations (10.1.4) - (10.1.7)).

Initially, Newton's First Law, or Law of Inertia is only saying that **an object at rest tends to stay at rest, and an object in motion tends to stay in motion with the same speed and in the same direction unless acted upon by an unbalanced force**. Such formulation is primarily addressing steady linear inertial motions of masses. We will see that, to extend and upgrade our understanding about Gravitation, we need to understand and accept that orbital, circular, rotational, and spinning motions (of masses including associated matter-waves) can also be inertial motions (because of natural and direct matter-waves coupling between linear and rotational motions). We could even say that all movements in our universe are cases of rotational or orbital motions, where a relevant radius of rotation could be arbitrary long. This way, we will transform Newton First Law (of linear inertial motions) towards the law of Inertia addressing only rotational and orbital motions (as the most general case), where linear motions are only specific (extreme and asymptotic) cases of orbital motions. What is underlined in this book is that masses in motions should be surrounded or guided with a (Hilbert) couple of gravitation-related fields (manifesting as coupled linear and rotational motional states, including surrounding matter-waves), which are in a way analog to the concept of coupling between electric and magnetic fields known in Electromagnetic theory (see much more about electromagnetic and mechanical effects coupling in Chapter 3.). Such extended Newton Law of Inertia should also be a familiar form to phenomenology known in electromagnetism under the Faraday and Lenz's laws. Of course, this is becoming evident when proper analogical comparisons and conclusions are implemented, like in the first chapter; -see "T.1.8 Generic Symmetries and Analogies of the Laws of Physics", and in equations under (4.2), in chapter 4.1. For instance, Faraday's law of electromagnetic induction states that $u = -\frac{d\Phi}{dt}$, where u is the electromotive force (emf) in volts, and Φ is the magnetic flux in Webbers. Further, Lenz's law gives the direction of the induced emf, thus: The emf induced in an electric circuit always acts in such a direction that the current it drives around the circuit opposes the change in magnetic flux that produces the emf. In other words, electric current or flow of electrical charges is like a mass flow, by their common tendency to resist initial motion sudden changes (and something similar applies to inertial spinning motions). Inertia could also be understandable in the conceptual framework of electromagnetic dipoles polarization, which could be the real source of gravitation (see (2.4-8) - (2.4-10)), because most of the particles and other masses in our universe are composed of atoms, and atom constituents are electrons, protons, and neutrons (having electric charges, spinning states and magnetic moments). Of course, neutron is effectively presenting certain "exotic coupling" between a proton and an electron. Since the mass difference between a proton and an electron is enormous (1836 times), every non-linear, transient, and accelerated motion of some mass will result in specific (internal) electric dipoles polarization. The presence of electric dipoles will be linked to the applicability of Coulomb force, Faraday, and Lenz's laws, giving another background for understanding and conceptualizing Inertia and gravitation). See more about inertia in Chapters 1, 4.2 and 10.

If we consider that coordinate-systems invariance of the square of a 4-vector energy-momentum (in Minkowski space) is something that should be respected, we will be able to give much more general framework regarding forces related to linear and rotational motions, as for example,

$$\overline{P}_4^2 = (\vec{p}, \frac{E}{c})^2 = inv. \Rightarrow \left[\int_{[\Delta t]} \frac{d}{dt}(\vec{p}, \frac{E}{c})dt\right]^2 = inv. \Rightarrow \left[\int_{[\Delta t]} \frac{d\vec{p}}{dt}dt, \frac{1}{c}\int_{[\Delta t]} \frac{dE}{dt}dt\right]^2 = inv. \Rightarrow$$

$$\Rightarrow \left[(\int_{[\Delta t]} F dt, \frac{1}{c}\int_{[\Delta t]} \frac{v dp}{dt} dt)\right]^2 = inv. \Rightarrow (\int_{[\Delta t]} F dt, \frac{1}{c}\int_{[\Delta t]} vF dt)^2 = inv. \Rightarrow (\int_{[\Delta t]} F dt, \frac{1}{c}\int_{[\Delta t]} F dr)^2 = inv. \quad (2.9.3)$$

Now we could develop different scenarios regarding forces involved in linear motions, where one of them is,

$$\left(\int_{[\Delta t]} F dt\right)^2_{(1)} - \left(\frac{1}{c}\int_{[\Delta t]} F dr\right)^2_{(1)} = -\left(\frac{1}{c}\int_{[\Delta t]} F dr\right)^2_{(2)} . \quad (2.9.4)$$

If we consider any linear motion as relatively good approximation for specific rotational or orbital movement (with the corresponding radius of rotation $\mathbf{r}$), we could transform 4-vector of linear momentum into a 4-vector of belonging orbital momentum and get invariance expression that is combining orbital (or torque $\vec{\tau} = \frac{dL}{dt}$, $\vec{L} = \sum_{(i)}(\vec{L}_i + \vec{S}_i)$, $\vec{S}_i$ = spin) and linear forces ($F = \frac{dp}{dt}$), as for instance,

$$\left\{(\vec{p}, \frac{E}{c})^2 = inv. \Rightarrow p^2 - (\frac{E}{c})^2 = -(\frac{E_0}{c})^2 \Leftrightarrow p^2c^2 - E^2 = -E_0^2 \Leftrightarrow p^2c^2 - (\gamma mc^2)^2 = -(mc^2)^2\right\} \Rightarrow$$

$$\Rightarrow \left[\frac{\vec{r}}{r} \times (\vec{p}, \frac{E}{c})\right]^2 = \left[\frac{1}{r}(\vec{L}, \frac{E}{c}\vec{r})\right]^2 = inv., \Leftrightarrow (\frac{\vec{L}}{r}, \frac{E}{c})^2 = inv. \Rightarrow$$

$$\Rightarrow \left[\int_{[\Delta t]} \frac{d}{dt}(\frac{\vec{L}}{r}, \frac{E}{c})dt\right]^2 = inv. \Rightarrow \left[\int_{[\Delta t]} \left(\frac{d}{dt}(\frac{\vec{L}}{r}), \frac{1}{c}\frac{dE}{dt}\right)dt\right]^2 = inv. \Rightarrow$$

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

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$$\Rightarrow\left[\int_{[\Delta t]}\left(\frac{r\frac{d\vec{L}}{dt}-\vec{L}\frac{dr}{dt}}{r^2},\frac{v}{c}\frac{dp}{dt}\right)dt\right]^2=\text{inv.}\Rightarrow\left\{\int_{[\Delta t]}\left[\left(\frac{1}{r}\frac{d\vec{L}}{dt}-\frac{v}{r^2}\vec{L}\right),\frac{v}{c}\mathbf{F}\right]dt\right\}^2=\text{inv.}\Rightarrow$$

$$\Rightarrow\left\{\int_{[\Delta t]}\left[\left(\frac{1}{r}\vec{\tau}-\frac{v}{r^2}\vec{L}\right),\frac{v}{c}\mathbf{F}\right]dt\right\}^2=\text{inv.}\Leftrightarrow\left[\int_{[\Delta t]}\left(\frac{1}{r}\vec{\tau}-\frac{v}{r^2}\vec{L}\right)dt,\int_{[\Delta t]}\frac{v}{c}\mathbf{F}dt\right]^2=\text{inv.}\Leftrightarrow \qquad (2.9.5)$$

$$\Leftrightarrow\left[\int_{[\Delta t]}\left(\frac{1}{r}\vec{\tau}-\frac{v}{r^2}\vec{L}\right)dt,\frac{1}{c}\int_{[\Delta t]}\mathbf{F}dr\right]^2=\text{inv.}\Leftrightarrow\left[\int_{[\Delta t]}\left(\frac{1}{r}\vec{\tau}-\frac{v}{r^2}\vec{L}\right)dt\right]^2-\left[\frac{1}{c}\int_{[\Delta t]}\mathbf{F}dr\right]^2=-(\frac{E_0}{c})^2$$

There would be many exciting options resulting from an extended, creative and intellectually flexible development of (2.9.4) and (2.9.5) because (2.9.5) is combining different force and orbital momentum components and expressing or supporting *here formulated Law of Inertia*. For instance, an (extended) inertial state should conserve its linear and orbital moments as shown in (2.9.1). This will transform (2.9.4) and (2.9.5) into another controversial and challenging (to think about) result,

$$\left(\vec{p}=\underline{\text{const.}},\vec{L}=\overline{\text{const.}}\right)\Rightarrow\left(\vec{F}=0,\vec{\tau}=0\right)\Rightarrow$$

$$\left[\int_{[\Delta t]}\left(\frac{1}{r}\vec{\tau}-\frac{v}{r^2}\vec{L}\right)dt,\frac{1}{c}\int_{[\Delta t]}\mathbf{F}dr\right]^2=\text{inv.}\Leftrightarrow\left\{\begin{array}{l}\left[\int_{[\Delta t]}\left(\frac{1}{r}\vec{\tau}-\frac{v}{r^2}\vec{L}\right)dt\right]^2_{(1)}-\left[\frac{1}{c}\int_{[\Delta t]}\mathbf{F}dr\right]^2_{(1)}=-(\frac{E_0}{c})^2\\ \left(\int_{[\Delta t]}\mathbf{F}dt\right)^2_{(1)}-\left(\frac{1}{c}\int_{[\Delta t]}\mathbf{F}dr\right)^2_{(1)}=-\left(\frac{1}{c}\int_{[\Delta t]}\mathbf{F}dr\right)^2_{(2)}\end{array}\right\}\Rightarrow$$

$$\Rightarrow\left\{\begin{array}{l}\left(-\mathbf{L}\int_{[\Delta t]}\frac{vdt}{r^2}\right)^2=-(\frac{E_0}{c})^2\Leftrightarrow\left(\int_{[\Delta t]}\frac{vdt}{r^2}\right)^2=\left(\int_{[\Delta t]}\frac{dr}{r^2}\right)^2=-(\frac{E_0}{c\mathbf{L}})^2,\\ \left(\int_{[\Delta t]}\mathbf{F}dt\right)^2=\left[\int_{[\Delta t]}\left(\frac{1}{r}\vec{\tau}-\frac{v}{r^2}\vec{L}\right)dt\right]^2,-\left(\frac{1}{c}\int_{[\Delta t]}\mathbf{F}dr\right)^2=-(\frac{E_0}{c})^2\end{array}\right\}.$$

(2.9.6)

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2.3.2. Rotation and stable rest-mass creation

Let us imagine that certain particles (like a thin disk, or thin walls toroidal object) is rotating around the fixed-point **O** (see the picture Fig.2.4 below; -case A), with a radius of rotation r. The same particle is then passing through an "energy-mass-moments" transformation, becoming only the state of spinning of a compact and small particle ($r \to 0$), and now performing only rotation around its own axis around the same fixed-point **O** (see the picture Fig.2.4 below, case B). All energy and momentum conservation laws should be satisfied between all phases, or motional-states transformation between A and B. Eventually, (let us hypothetically imagine), that the same particle (state B) is finally transformed into an equivalent "standstill" mass (see the picture Fig.2.4 below, case C), of course appropriately interacting with its environment (mechanically or electromechanically). Here we are trying to speculate how motional energy of any kind (especially rotating and spinning) could be "transformed, packed or4 injected" into a standstill rest mass, and that we still satisfy energy and angular momentum conservation. To describe such (still hypothetical) process, we will imaginatively speculate with an analogical formulation of energy forms between linear and rotational motion (see T.2.4 and T.2.5), presently without entering reasons if, when, how, and why such analogical comparisons are realistic, applicable, possible, or valid. ***In fact, here we (hypothetically) speculate that linear and angular moments of certain inertial motion (or a steady state) are not always independently conserved (as formulated in Classical Mechanics), being mutually coupled and convertible (as electric and magnetic fields are always coupled in cases of electromagnetic phenomenology). We also need to answer where and how orbital and spinning moments will disappear, or be in some way hidden in the transformation process A→ B→ C.***

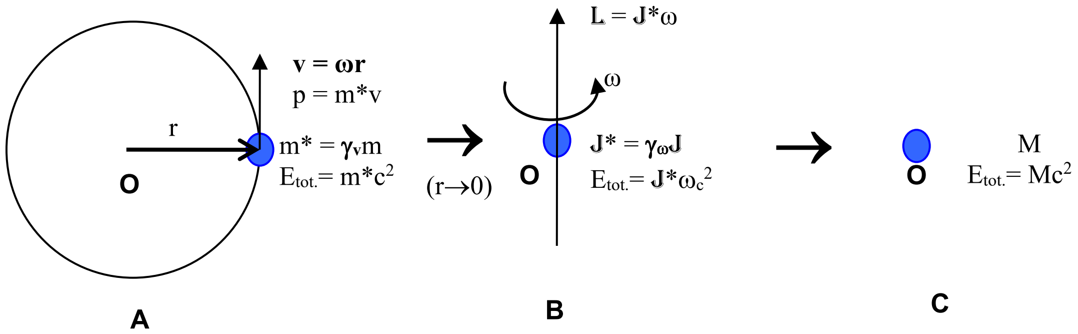

Fig.2.4. Hypothetical evolution of a rotating mass towards standstill mass

Case A: Particle $\mathbf{m}$ is only rotating around some externally fixed point (not spinning around its own axis).
Case B: Particle $\mathbf{m}$ is only spinning around its own axis and not making any other motion.
Case C: Particle $\mathbf{m}$ "energy-wise" transformed into a standstill or rest mass $\mathbf{M}$ (where revolving and spinning energy is also included).

In all cases, here-involved particles could also have the form of a thin disk, or thin walls, hollow toroidal object.

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T.2.4. Motional or kinetic energy expressions formulated based on analogies

Case → Value ↓	**A** **Linear motion**	**B** **Spinning**	**C** **Standstill**
Linear speed: v	$v = v_A$	$\leq c$	0
Angular speed: ω	$\omega = \omega_A$	$\omega = \omega_B$	0
Initial Rest Mass: m	$m = m_A$	m_B	m_C
Motional mass: m*	$m_A^* = m^* = \gamma_v m_A$	$m_B^* = m^*$	0
Total mass: M	$M = m_A^*$	$M = m_B^*$	$M = m_A^* = m_B^* = m^*$
Linear Moment: p	$p = p_A = mv, p^* = p_A^* = m^* v$	Not applicable, $(p = 0)$	Not applicable, $(p = 0)$
Static Moment of Inertia: I	$\mathbf{J}_A$	$\mathbf{J}_B$	$\mathbf{J}_C$
"Motional" Moment of Inertia: I*	$\mathbf{J}_A^*$	$\mathbf{J}_B^*$	0
Angular moment: L	$\mathbf{L}_A = \mathbf{J}_A^* \cdot \omega_A = \mathbf{L}$	$\mathbf{L}_B = \mathbf{J}_B^* \cdot \omega_B = \mathbf{L}_A = \mathbf{L}$	$= 0$, internally $= \mathbf{L}$, externally
Total Energy: $E_{tot.}$	$E_{tot.A} = m_A^* \cdot c^2 = \mathbf{J}_A^* \cdot \omega_{cA}^2 = \sqrt{(mc^2)^2 + p^2c^2}$	$E_{tot.B} = m_B^* \cdot c^2 = \mathbf{J}_B^* \cdot \omega_{cB}^2$	$E_{tot.A} = E_{tot.B} = E_{tot.C} = M \cdot c^2$
Motional Energy: E_m , E_k	$E_m = E_k = (m_A^* - m) \cdot c^2 = (\gamma_v - 1)mc^2 = (\mathbf{J}_A^* - \mathbf{J}_A) \cdot \omega_{cA}^2 = (\gamma_\omega - 1)\mathbf{J}_A \omega_{cA}^2 = \frac{pv}{1 + \sqrt{1 - v^2/c^2}}$	$E_m = (m_B^* - m) \cdot c^2 = (\gamma_v - 1)mc^2 = (\mathbf{J}_B^* - \mathbf{J}_B) \cdot \omega_{cB}^2 = (\gamma_\omega - 1)\mathbf{J}_B \omega_{cB}^2 = \frac{\mathbf{L}\omega}{1 + \sqrt{1 - \omega^2/\omega_c^2}}$	$= 0$, externally $= M \cdot c^2$, internally
Rest Energy: E_0	$m_A \cdot c^2 = \mathbf{J}_A \omega_{cA}^2$	$m_B \cdot c^2 = \mathbf{J}_B \omega_{cB}^2$	$M \cdot c^2$
Lorentz factor for linear motion: γ_v	$\gamma_v = (1 - v^2/c^2)^{-0.5} = \gamma$	Not applicable	Not applicable
Analogically formulated "Lorentz factor" for the rotational motion: γ_ω	$\gamma_\omega = (1 - \omega_A^2/\omega_{cA}^2)^{-0.5}$	$\gamma_\omega = (1 - \omega_B^2/\omega_{cB}^2)^{-0.5}$	Not applicable
	$\gamma_v = \gamma_\omega = \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - \omega^2/\omega_c^2}}$ $v = \omega r \Leftrightarrow c = \omega_c r, \frac{\omega}{\omega_c} = \frac{v}{c}, \omega_c = c\sqrt{\frac{m}{\mathbf{J}}} = c\sqrt{\frac{m^*}{\mathbf{J}^*}} = \omega \frac{c}{v}$.		

Let us additionally elaborate the same (hypothetical) situation from Fig.2.4 regarding total motional energy transformation towards a rest mass (with internally captured and packed, rotational, motional energy of the process: A → B → C), by extending kinetic energy expressions from the table T.2.4 into equivalent energy expressions as given in T.2.5. In fact, here we are searching how to transform an orbiting mass (with a non-zero radius of rotation) into an ***only-spinning***, or ***only-standstill*** mass (when orbiting radius reduces to zero). The more realistic example from Physics, addressing such situations, could be to analyze differences between orbiting electrons (on stationary orbits), and spinning electrons (see chapters 8 and 10).

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T.2.5. Motional or kinetic energy expressions formulated based on analogies

<table>
<tr><th></th><th>Linear motion: Case A</th><th>Spinning: Case B</th></tr>
<tr><td rowspan="3">Kinetic Energy</td><td>$$E_k = (m^* - m)\cdot c^2 = (\gamma_v - 1)mc^2 = \\ = \frac{\gamma_v mv^2}{1+1/\gamma_v} = \frac{\gamma_v (mr^2)\omega^2}{1+1/\gamma_v} = \frac{\alpha\cdot \mathbf{J}\cdot\omega^2}{1+1/\gamma_v} \\ = \frac{\mathbf{J}^*\cdot\omega^2}{1+1/\gamma_v} = \frac{p\cdot v}{1+1/\gamma_v} = \frac{\gamma_v (mr^2\omega)\omega}{1+1/\gamma_v} \\ = \frac{\mathbf{L}\cdot\omega}{1+1/\gamma_v} = \frac{\mathbf{L}\cdot\omega}{1+\sqrt{1-v^2/c^2}}, \\ dE_k = vdp = \omega d\mathbf{L} = c^2 dm^* = dE_{tot.} \\ \mathbf{J}^* = \gamma_v (mr^2) = \alpha\cdot\mathbf{J} = m^* r^2 = \\ = \frac{p\cdot v}{\omega^2} = \frac{p\cdot r}{\omega} = \frac{\mathbf{L}}{\omega}, \\ m^* = \gamma_v m,\ v = \omega r,\ p = \gamma_v mv = m^* v, \\ \mathbf{L} = \mathbf{J}^*\omega = \alpha\cdot\mathbf{J}\cdot\omega = p\cdot r, \alpha > 0, \\ \alpha = \frac{\gamma_v mv^2}{\mathbf{J}\cdot\omega^2} = \frac{p\cdot v}{\mathbf{J}\cdot\omega^2} = \frac{\mathbf{J}^*}{\mathbf{J}} = \frac{p\cdot r}{\mathbf{J}\cdot\omega} = \frac{\mathbf{L}}{\mathbf{J}\cdot\omega}$$</td><td>$$E_k = \frac{\alpha\cdot\mathbf{J}\cdot\omega^2}{1+1/\gamma_v} = \frac{\mathbf{J}^*\cdot\omega^2}{1+1/\gamma_v} = \\ = \frac{\mathbf{L}\cdot\omega}{1+1/\gamma_v} = \frac{\mathbf{L}\cdot\omega}{1+\sqrt{1-v^2/c^2}} \\ dE_k = \omega d\mathbf{L} = c^2 dm^* = dE_{tot.}$$</td></tr>
<tr><td colspan="2">**Case C:** Here, the externally visible rotation should somehow be transformed into “internally captured” rotation, $r\to 0$, and mass is macroscopically becoming standstill, but internally, as states of atoms and molecules, we still have number of small spinning and orbiting including vibrating cases.</td></tr>
<tr><td colspan="2">$$Lim(\frac{\mathbf{J}^*\cdot\omega^2}{1+1/\gamma_v})_{r\to 0} = E_{tot.} = E_k = \frac{\mathbf{J}^*\cdot\omega^2}{2} = \frac{\mathbf{L}\cdot\omega}{2} = m^* c^2 = Mc^2, \\ (r\to 0) \Rightarrow v = \omega r \to 0, \gamma_v \to 1.$$</td></tr>
</table>

In all other situations of combined motions where certain mass is performing linear motion and spinning around its own axis (for instance, connecting case A and case B), motional energy will be,

$$dE_k = vdp + \omega d\mathbf{L} = c^2 dm^* = dE_{tot.}. \quad (2.11.1)$$

In case if “rotating particle” is a photon, its motional energy will be, based on data from T.2.5:

$$\tilde{E} = E_k = Lim(\frac{p\cdot v}{1+1/\gamma_v})_{v\to c} = Lim(\frac{\mathbf{L}\cdot\omega}{1+1/\gamma_v})_{v\to c} = p\cdot c = \mathbf{L}\cdot\omega \quad (2.11.2)$$

Apparently, if we would like to get Planck’s expression for photon energy $\tilde{E} = hf$, from (2.11.2), $E_k = \mathbf{L}\cdot\omega$, the following conditions should be satisfied:

$$\begin{aligned} & E_k = \tilde{E} = p\cdot c = \mathbf{L}\cdot\omega = hf \\ & \omega = 2\pi f,\ \mathbf{L} = h/2\pi,\ p = hf/c = mc,\ m = hf/c^2 \\ & pr = (hf/c)r = \mathbf{L} = h/2\pi \Rightarrow \\ & \Rightarrow r = c/2\pi f,\ 2\pi r = c/f = \lambda = h/p = h/mc. \end{aligned} \quad (2.11.3)$$

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We already know (from analyses of Compton and Photoelectric effects) that photon energy is equal to $\tilde{E} = hf$. From Maxwell electromagnetic theory we know that coupled electric and magnetic field vectors (of a photon) are rotating (or spinning) along its path of propagation. Consequently, our picture about ***the photon, as an energy-packet, can get some additional conceptual grounds related to modeling it with a kind of equivalent spinning disk, ring, or torus*** *as in (2.11.3).* Such a chain of conclusions is indeed oversimplified, but still good to make some indicative (and hypothetical) findings based on analogies. From large experimental and theoretical knowledge base accumulated in Quantum Theory, we also know that other narrow-band elementary energy packets in the world of microphysics, which are not necessarily photons (and maybe not forms of electromagnetic energy), also have energies that can be expressed by $\tilde{E} = hf$. By analogy, a kind of intrinsic (matter waves) rotation is also involved there as in the case of photons. Furthermore, experimentally it is known how sufficiently high-energy-photon can be fully transformed into an electron-positron couple, or how contact of an electron and positron (or some other particle and its anti-particle) is annihilating initial participants and producing, for instance, two high-energy photons. A bit later (see: (2.11.9-1) - (2.11.9-4)), it will be shown how an electron can be analogically modeled as a rotating ring, torus, or disk, and in chapter 4.1 (*see T.4.3 and (4.5-1) - (4.5-4)*), similar ideas about explaining matter waves will also be elaborated.

The intention here is to exercise the idea that elementary "seeds and grains" of all macro-objects, or everything that has a rest mass, are particularly packed and coupled, spinning, and orbiting, self-closed and standing electromagnetic matter-waves, composed of some simpler (sinusoidal) matter waveforms. Eventually, (and most probably) we would find only photons (or some vibrating strings) as real, most-elementary matter constituents or waveforms (in a broad background of everything else created based on electromagnetic energy structural formatting; - see also T.4.0, chapter 4.1). Specific energy packing of different matter waves, causally related to rotation, and spinning, is creating particles that are self-standing, and have non-zero rest masses, and here is the meaning of the concept that every mass should be an energy-packing format. Presently (or also historically), we consider mass as being the source or direct cause of gravitation. We are gradually starting to get familiar with the concept that internal mass content are rotating matter waves (captured as atoms and other elementary particles in different forms of stationery and standing waves with spin properties, including associated electromagnetic attributes). In addition, we know that all masses in our universe are in mutually relative motions (having linear and angular moments). All of that is pointing to the conclusion that Newton law of gravitation should be upgraded with some dynamic, velocities and orbital moments dependent members, and that real sources of gravity are linked to certain "vortex-radiant-energy waves", and mater-waves fields between motional masses, radiating from atoms. In Chapter 10. of this book, we can find the most complete explanation of the same situation regarding still hypothetical *background-velocity* parameters, and Newtonian attraction between important linear and angular moments (see (10.1.4) - (10.1.7)).

Relativistic mass (as defined in Relativity theory) is a mass that is velocity-dependent, and it is effectively increasing when its velocity is increasing. We could ask here, what is effectively increasing. Can we see, measure, touch or visualize such effects of mass increase. The short answer is that an effective matter-wave energy equal to a mass motion (or to its kinetic) energy is creating and increasing mentioned (velocity dependent) matter-wave mass $\tilde{m}$. This way, we are introducing the notion of the matter-wave mass $\tilde{m}$, and we can present that this motional matter-wave mass is something what is effectively coming from a particle kinetic or wave energy, as for example,

$$m = m_0 + \tilde{m} = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma m_0 \Rightarrow \tilde{m} = m - m_0 = m_0 \frac{1-\sqrt{1-\frac{v^2}{c^2}}}{\sqrt{1-\frac{v^2}{c^2}}} = m\left(1-\sqrt{1-\frac{v^2}{c^2}}\right),$$

$$E_k = (m-m_0)c^2 = m_0c^2 \frac{1-\sqrt{1-\frac{v^2}{c^2}}}{\sqrt{1-\frac{v^2}{c^2}}} = \tilde{m}c^2 = \tilde{E} = pu = mvu = mv\lambda f = hf \Rightarrow \qquad (2.11.3\text{-}1)$$

$$\Rightarrow \lambda = \frac{h}{mv} = \frac{h}{p},\ u = \lambda f\ (=)\ \text{phase velocity},\ p = mv = (m_0 + \tilde{m})v = m_0 v + \tilde{m}v.$$

What will really materialize or happen regarding this *matter-wave mass* $\tilde{m}$ depends on number of factors, such as influence of other interaction participants, and on reaching necessary energy-level thresholds, while satisfying involved conservation laws and boundary conditions (related to a moving mass in question). For instance, we expect to generate new particles, photons, electrons etc.

Here, *within (2.10), (2.11), T.2.4., T.2.5. until (2.11.3-1), we already have all significant elements for the foundation of de Broglie matter-waves hypothesis (that should obviously be applicable not only to microworld Physics). In fact, here we are arriving at the situation that our present conceptualization of a total energy and moments conservation of combined or coupled linear and rotational motions and rest matter states, should be upgraded. For instance, as being familiar with updated Minkowski 4-vectors combined with involved angular and electromagnetic moments. Both linear and angular mechanical moments of certain isolated systems should be coincidently conserved, and this creates matter waves and wave-particle duality effects.*

Presently we conceptualize Gravitation both as static masses attraction (Newton-Rudjer Boskovic-Cavendish), and as mutually orbiting/rotating/spinning, dynamic mases attraction involving centrifugal and centripetal forces (in cases like planetary systems). Such dual nature of Gravitation is possible only in the frames of combined, coupled and mutually transformable "mass, energy, moments" relations, when angular and electromagnetic moments could be coincidently and synchronously conserved externally (between macro masses), and internally as mass-involved atom properties.

The process of "fusion between the rest mass and its matter-wave", spinning or rotating energy-mass equivalent ($E_{spinning}/c^2$) can be conceptualized even simpler. Let us imagine that certain standstill (rest) mass m is passing the process (of its energy transformation) from level **A**, through level **B**, and ending with the level **C**, where relevant energy levels are defined by the following tables (T.2.6.1. and T.2.6.2.):

T.2.6.1. A process of rotational energy "injection" into a rest mass

Levels/States of mass transformation	A Initial Standstill, Rest Mass	B Only Spinning Mass Mass from **A** starts spinning	C Final, new, Equivalent Rest Mass after the spinning mass is being transformed into a standstill mass
Rest Mass	$m = m_0$	$m = m_0$	$M = m + E_{spinning} / c^2$
Total Mass	$m = m_0$	$M = m + E_{spinning} / c^2$	$M = m + E_{spinning} / c^2$
Rest Energy	mc^2	mc^2	Mc^2
Total Energy	mc^2	Mc^2	Mc^2
Motional Energy	0	$E_{speening} = E_s$	0

Energy evolution should be analogically presentable as stated in T.2.6.2.

T.2.6.2. Energy Analogies between Linear and Spinning motions

	Only linear motion	Only spinning	Combined
Motional or kinetic energy	$E_k = \frac{1}{2}mv^2 = \frac{1}{2}pv$	$E_s = \frac{1}{2}J\omega_s^2 = \frac{1}{2}L\omega_s$	$E_k + E_s = \frac{1}{2}mv^2 + \frac{1}{2}J\omega^2 =$ $= \frac{1}{2}(m + \Delta m)v^2 = \frac{1}{2}Mv^2$
Motional or kinetic energy (relativistic, analogical formulation)	$E_k = (m - m_0)c^2 =$ $= (\gamma_L - 1)m_0c^2$ $\gamma_L = (1 - v^2 / c^2)^{-1/2}$ $m = \gamma_L m_0$	$E_s = (J - J_0)\omega_c^2 =$ $= (\gamma_s - 1)J_0\omega_c^2 = c^2\Delta m$ $\approx \frac{1}{2}J\omega_s^2 = \frac{1}{2}L\omega_s$ (for $\omega_s << \omega_c$), $\gamma_s = (1 - \omega_s^2 / \omega_c^2)^{-1/2}$ (Analogically formulated)	$(m - m_0)c^2 + (J - J_0)\omega_c^2 =$ $= (\gamma_L - 1)m_0c^2 + (\gamma_s - 1)J_0\omega_c^2 =$ $= E_k + E_s =$ $= [(m + \Delta m) - m_0]c^2 =$ $= (\gamma_L - 1)(m_0 + \Delta m)c^2 =$ $= (M - m_o)c^2$
Total rest energy	$m_0c^2 = (\sum_{(i)} m_{0i})c^2$	$J_0\omega_c^2 == (\sum_{(i)} J_{0i})\omega_c^2$ (Analogically formulated)	$(m_0 + \Delta m)c^2 = (J_0 + \Delta J)\omega_c^2$ (Analogically formulated)
Total motional energy	$\gamma_L m_0c^2$	$\gamma_s J_0\omega_c^2$	$\gamma_L(m_0 + \Delta m)c^2 = \gamma_s(J_0 + \Delta J)\omega_c^2$
	$dE = dE_k = vdp$	$dE = dE_k = \omega_s dL$	$dE = dE_k = vdp + \omega_s dL$

Of course, here (in T.2.6.1 and T.2.6.2.) we are not saying how, when, and under which circumstances, a "rest mass carrier" would capture its spinning energy. What is implicitly stated here (regardless of terminology that is still conditional and evolving) is that *under certain conditions spinning energy can be transformed, caught or packed in the form of a stable rest mass, and that every rest mass is an equivalent "packing" form of a certain amount of "frozen rotating or spinning energy"*.

The generalized case of every particle in motion (which has non-zero rest mass m), based on situations from (2.5.1-4) - (2.5.1-6), T.2.4, T.2.5 and T.2.6, is that total particle energy should always have a combination of static (rest), rotating, and liner motion energy members, as for instance,

$$\begin{aligned}
&E_{tot} = (m + E_{spinning} / c^2)c^2 + \frac{pv}{1+\sqrt{1-v^2/c^2}} = Mc^2 + E_{k-lin} = \sqrt{(Mc^2)^2 + p^2c^2} = \gamma Mc^2 = E, \\
&M = m + E_{spinning} / c^2 \text{ , } p = \gamma Mv, \; E_{k-lin} = E_k = \frac{pv}{1+\sqrt{1-v^2/c^2}} \text{ ,} \\
&dE = dE_{tot} = dE_k = vdp = \omega d\mathbf{L} .
\end{aligned} \tag{2.11.4}$$

Even the rest mass m, from (2.11.4), is also presenting a "frozen rotating energy state" which was created or stabilized in some earlier phase of initial particle creation (what could implicate a preexistence of succession of such states). The same ideas will be more elaborated later in this chapter, regarding innovative modeling of an electron (see: (2.11.9-1) - (2.11.9-4)), also around equations (2.11.13-5) - (2.11.13-5), and in chapter 4.1 (see T.4.3 and (4.5-1) - (4.5-4)). Imaginative, structural "in-depth" particle modeling, which is also considering particle past, during its formation, is presented in Chapter 6., (Different possibilities for mathematical foundations of multidimensional universe; -See comments around equations from (6.18) to (6.23)).

From (2.11.4), we can also extract the roots of particle-wave duality, *if we accept that matter-wave energy (at least in cases of micro-world of atoms and its constituents) is equal to a relevant particle kinetic or motional energy*, $\tilde{E} = E_k = E_{tot} - Mc^2 = hf$, *and that it can also be analogically presented as the wave-packet or photon energy in (2.11.3)*, as for instance,

$$\begin{aligned}
&p = \frac{1}{c}\sqrt{E_{tot}^2 - (Mc^2)^2} = \frac{1+\sqrt{1-v^2/c^2}}{v}(E_{tot} - Mc^2), \\
&\frac{v}{1+\sqrt{1-v^2/c^2}} = c\sqrt{\frac{E_{tot} - Mc^2}{E_{tot} + Mc^2}} = u = \lambda f = v_{phase} , \\
&\lambda = \frac{c}{f}\sqrt{\frac{E_{tot} - Mc^2}{E_{tot} + Mc^2}} = \frac{\left[\frac{(E_{tot} - Mc^2)}{f}\right]}{p} = \frac{\tilde{E}/f}{p} = \begin{cases} \frac{h}{p} \text{ (for micro world)} \\ \frac{H}{p} \text{ (for macro world)} \end{cases} , \; (H, h) = \text{constants} \\
&\tilde{E} = E_k = E_{tot} - Mc^2 = E_{tot} - mc^2 - E_{spinning} = \begin{cases} hf \text{ (for micro world)} \\ Hf \text{ (for macro world)} \end{cases}, \; H >>> h .
\end{aligned} \tag{2.11.5}$$

Nothing in (2.11.5) is strictly saying that h should only be the Planck constant ($h \approx 6{,}6260755 \times 10^{-34}$ J.s), when we deal with macro-world objects. Later, (in "2.3.2. Macro-Cosmological Matter-Waves and Gravitation"), we will find that matter-waves relations, as in (2.11.5), are equally applicable to gravitational orbital motions of planetary systems, except that Planck constant $\mathbf{h}$ will be replaced with another constant H with similar meaning $\Rightarrow \lambda = \frac{H}{p}, \; E_k = \tilde{E} = Hf, \; H >>> h$. It can be demonstrated that such kind of quantization and matter waves concept ($\lambda = H/p, \; \tilde{E} = Hf$) is even applicable in cases of "Vortex shedding flowmeter" or oscillatory flowmeter (based on detecting the vibrations of the downstream vortices caused by the barrier placed in a moving stream). The vibrating frequency of vortex shedding can be related to the velocity of

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the liquid flow; see more in chapter 4.1, around equations (4.3-0) and (4.3-0)-a,b,c,d,e,f,g,h,i…

The philosophical and conceptual consequence of results from (2.11.5) is that particle-wave duality and matter waves' nature is related to the internal spinning energy content of a particle in motion. For instance, photon modeling as a micro-gyroscope, vortex form, rotating disk, ring, or torus (2.11.2) - (2.11.3), and similar innovative modeling of electron structure (2.11.9-1) - (2.11.9-4) are indicative examples for such concepts. Consequently, matter waves should be an "*external unfolding manifestation*" of already existing "*internally folded*" and rotating matter waves. This will be elaborated later in the same chapter, and much more in chapters 4.1, 4.2 and 4.3. In other words, masses in motion are creating a space-time web or matrix, capturing, and sensing each other by creating angular mater-waves motions (supported by Kepler, Newton, and Coulomb laws), and at the same time manifesting forces, and fields or Gravitation. Also, rotating solar system (a spinning sun with many planets rotating and spinning around) could be effectively conceptualized as a (big) single spinning mass in the process of agglomeration (or mass integration), thanks to gravitation as a natural "vortex-sink-like" tendency of mutually approaching masses. *In other words, rotating and spinning energy effectively belongs (or migrates) to a relevant rest mass (or it is eventually creating a rest mass).* Here it is good to mention that the total mass of all planets in our solar system (or in any other solar system) presents only a few percent of the mass of our Sun (just to visualize how ***small all planets are*** compared to a local Sun). Planets in a certain planetary system are mutually interacting, rotating, and spinning around their sun, having specific planetary orbits, this way following or respecting the structure of gravitational and other (mutually coupled) electromagnetic fields involved in such situations, as well as respecting basic energy-momentum conservation laws. We are still considering gravitation as the central force between two masses, acting on the line connecting their centers (Newton law), and such simplified concept will be significantly upgraded, eventually (see later in the same chapter an extension of the same concept around equations (2.11.10) – (2.11.20); -"2.3.2. Macro-Cosmological Matter-Waves and Gravitation").

The micro-world of sufficiently isolated atoms, subatomic particles, and photons is dominantly respecting Planck-Einstein-de-Broglie's, energy-wavelength formulations, $\tilde{E} = hf, \lambda = h/p, \quad u = \lambda f = \omega/k, v = d\omega/dk$ *. The idea here is to show that for macro-objects, like planets, specific and characteristic wavelength also exists, analog to de Broglie matter-waves wavelength, but no longer proportional to Planck's constant h. The problem concerning de Broglie type of wavelength for macro-objects is that such wavelength (*$\lambda = h/p$*) is extremely small and meaningless, if we are using Planck's constant* $\boldsymbol{h}$*. Since macro-objects in motion should also create associated matter-waves, like any micro-world object, the macro-wavelength in question should be much bigger. It will be shown later (by analyzing planetary motions) that presently known micro-world Planck constant* $\boldsymbol{h}$ *should be replaced by another analogous (much bigger) macro-world constant* $\boldsymbol{H}$ *(see (2.11.10) – (2.11.20)), especially valid in cases of self-closed, standing matter waves (such as solar systems are). Other macro matter-waves respecting (analogical) concept are waves on a water surface formed by some moving object (a boat), where the water surface is visualizing matter waves of a moving object. Similar matter waves understanding could also be associated to pendulum* *motion, if we observe the pendulum from another inertial reference system that is in relative movement to the pendulum system of reference.*

To show more directly that rotation (or spinning) is an essential (ontological) source responsible for particles creation, we can again start from the relativistic particle expression that is connecting particle's total energy $E = E_{tot} = \gamma mc^2$, its linear-motion momentum $p = \gamma mv = \gamma_v mv$, and its rest mass $m = E_0 / c^2$,

$$\left[E_{tot}^2 = E_0^2 + p^2c^2\right]/c^2 \Leftrightarrow \left(\frac{E_{tot}}{c}\right)^2 = \left(\frac{E_0}{c}\right)^2 + p^2. \quad (2.11.6)$$

If the particle is at the same time performing kind of circular (rotational) motion around a specified fixed point (without spinning around its own axis), where r_0 is the distance between that fixed point and moving particle, we can consider that the particle also has certain orbital momentum $\vec{L} = \vec{r}_0 \, x \, \vec{p}$, $L = r_0 \cdot p$, $dE = \omega \cdot dL = v \cdot dp = dE_k = dE_{tot}$. Now, we can multiply both sides of (2.11.6) with the radius of rotation r_0 and get kind of relativistic relation where relevant orbital moments are involved,

$$\begin{array}{lcccc} \left(\frac{E_{tot}}{c}\right)^2 & = & \left(\frac{E_0}{c}\right)^2 & + & p^2 \\ \left(r_0 \cdot \frac{E_{tot}}{c}\right)^2 & = & \left(r_0 \cdot \frac{E_0}{c}\right)^2 & + & (r_0 \cdot p)^2 \\ L_{tot}^2 & = & L_0^2 & + & L^2 \end{array} \quad (2.11.7)$$

From the analogical point of view, to facilitate comparison between (2.11.6) and (2.11.7), it is reasonable to create (2.11.8), for exposing analogies of linear and orbital moments, such as,

$$\left\{\begin{array}{l} \left(\frac{E_{tot}}{c}\right)^2 = \left(\frac{E_0}{c}\right)^2 + p^2 \\ p_{tot}^2 = p_0^2 + p^2 \\ p_{tot} = \frac{E_{tot}}{c} = \gamma_v mc \\ p_0 = \frac{E_0}{c} = mc \\ p = \gamma_v mv \\ \\ \gamma_v = (1 - v^2/c^2)^{-0.5} \\ 0 \leq v \leq c \end{array}\right\} \underset{\rightleftarrows}{\text{analog or equivalent to}} \left\{\begin{array}{l} \left(\frac{L_{tot}}{r_0}\right)^2 = \left(\frac{L_0}{r_0}\right)^2 + \left(\frac{L}{r_0}\right)^2 \\ L_{tot}^2 = L_0^2 + L^2 \\ L_{tot} = r_0 \cdot p_{tot} = r_0 \cdot \frac{E_{tot}}{c} = r_0 \cdot \gamma_v mc = \gamma_\omega J \omega_c \\ L_0 = r_0 \cdot p_0 = r_0 \cdot \frac{E_0}{c} = r_0 \cdot mc = J \omega_c \\ L = r_0 \cdot p = r_0 \cdot \gamma_v mv = \gamma_\omega J \omega \\ \\ \gamma_\omega = (1 - \omega^2/\omega_c^2)^{-0.5} \\ 0 \leq \omega \leq \omega_c \end{array}\right\}. \quad (2.11.8)$$

Concluding based on (2.11.8), (2.5.1-6), (2.5.1-7), T.2.4, T.2.5, and T.2.6 is becoming evident that the rest mass $m = E_0 / c^2$ should have an origin causally related to certain

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

spatially localized and stabilized spinning of its internal constituents (or to $\mathbf{L}, \mathbf{L}_0, \mathbf{J}$, ω, ω_c) because,

$$\left\{\left(\frac{E_{tot}}{c}\right)^2 = \left(\frac{E_0}{c}\right)^2 + p^2\right\} \equiv \left\{\left(\frac{\mathbf{L}_{tot}}{r_0}\right)^2 = \left(\frac{\mathbf{L}_0}{r_0}\right)^2 + \left(\frac{\mathbf{L}}{r_0}\right)^2\right\} \Rightarrow$$

$$\Rightarrow \begin{Bmatrix} \frac{E_{tot}}{c} = \frac{\mathbf{L}_{tot}}{r_0} = p_{tot} = \gamma_v mc = \frac{\gamma_\omega \mathbf{J}\,\omega_c}{r_0}, \\ \frac{E_0}{c} = \frac{\mathbf{L}_0}{r_0} = p_0 = mc = \frac{\mathbf{J}\,\omega_c}{r_0}, \\ p = \frac{\mathbf{L}}{r_0} = \gamma_v mv = \frac{\gamma_\omega \mathbf{J}\,\omega}{r_0}, \\ dE = \omega \cdot d\mathbf{L} = v \cdot dp = dE_k = dE_{tot} \end{Bmatrix} \Rightarrow$$

$$\Rightarrow \begin{Bmatrix} m = \frac{\mathbf{L}_{tot}}{r_0\gamma_v c} = \frac{\mathbf{L}}{r_0\gamma_v v} = \frac{\gamma_\omega \mathbf{J}\,\omega_c}{r_0\gamma_v c} = \frac{\gamma_\omega \mathbf{J}\,\omega}{r_0\gamma_v v} = \frac{\mathbf{L}_0}{r_0 c} = \frac{\mathbf{J}\,\omega_c}{r_0 c} = \\ = \frac{E_{tot}}{\gamma_v c^2} = \frac{p_{tot}}{\gamma_v c} = \frac{p}{\gamma_v v} = \frac{p_0}{c} = \frac{E_0}{c^2} = \frac{1}{c^2}\iiint_{[\omega]} \vec{\omega}\, d\vec{\mathbf{L}} = \frac{\mathbf{J}\,\omega_c^2}{c^2}, \\ \frac{\omega}{\omega_c} = \frac{v}{c} = \frac{\mathbf{L}}{\mathbf{L}_{tot}} = \frac{cp}{E_{tot}} = \frac{p}{\gamma_v p_0} \end{Bmatrix}. \qquad (2.11.9)$$

In addition to such conceptualization, we can say that all kinds of motions in our universe are curvilinear, or combinations of linear and rotational (see later T.4.3 and (4.5-1) - (4.5-4) in chapter 4.1). If this were not the case, we would not have stable particles with non-zero rest masses. Also, based on (2.11.1) to (2.11.5) and (2.11.7), we know that internal particle structure (its rest mass) has natural elements of rotation, which are directly coupled to all kinds of externally rotating motions (including oscillatory and resonant states). Such kind of coupling of internal and external aspects of rotation should be part of explanation of Gravitation.

The unity, or complementary nature of linear and rotational motions (including universal law of Inertia), should be naturally integrated into laws of energy and moments conservation, as well as being the part of 4-vector of Energy-Momentum in the Minkowski space (as shown in (2.9.5), and later). Let us summarize different (mutually analogical and somewhat speculative) situations when a specific particle has different kinds of motional components, for example,

Particle is performing only rectilinear motion:

$$\begin{aligned} &(p, \frac{E}{c})^2 = inv., p = \gamma mv, E = \gamma mc^2, E_0 = mc^2 \Rightarrow \\ &p^2c^2 - E^2 = -E_0^2,\ E = E(p),\ \frac{\partial}{\partial p}\left(p^2c^2 - E^2 = -E_0^2\right) = 0 \Leftrightarrow \\ &2c^2p - 2E\frac{\partial E}{\partial p} = 0 \Rightarrow \frac{\partial E}{\partial p} = c^2\frac{p}{E} = c^2\frac{\gamma mv}{mc^2} = v,\ dE = vdp \end{aligned} \qquad (2.9.5\text{-}1)$$

Particle is performing only circular motion:

Depending on how we define a dominant inertial system here, we could have at least two different situations, for example,

a)
$$\left\{\begin{pmatrix} p=\gamma mv, E=\gamma mc^2, v=\omega r, \vec{v}=\vec{\omega}\times\vec{r}, \vec{p}\times\vec{r}=\vec{L}, \\ \frac{1}{2}mv^2=\frac{1}{2}J\omega^2 \Leftrightarrow mv^2=J\omega^2=pv=L\omega, p=\frac{L}{r}, \\ v=\omega r \Rightarrow c=\omega_c r,\ \omega_c=\frac{c}{r} \end{pmatrix}, (p,\frac{E}{c})^2=inv.\right\} \Rightarrow \tag{2.9.5-2}$$

$$p^2c^2-E^2=-E_0^2 \Leftrightarrow (\frac{L}{r})^2c^2-E^2=-E_0^2 \Leftrightarrow (\frac{\vec{L}}{r},\frac{E}{c})^2=inv.$$

$$E=E(L) \Rightarrow \frac{\partial}{\partial L}\left[(\frac{L}{r})^2c^2-E^2=-E_0^2\right]=0 \Leftrightarrow 2\frac{c^2}{r^2}L-2E\frac{\partial E}{\partial L}=0 \Rightarrow$$

$$\frac{\partial E}{\partial L}=\frac{c^2}{r^2}\cdot\frac{L}{E}=\frac{c^2}{r^2}\cdot\frac{J\omega}{J\omega_c^2}=(\frac{c}{\omega_c r})^2\omega=\omega,\ dE=\omega dL\ .$$

Another imaginable situation for making relation between two inertial systems addressing the same circular motion is,

b)
$$\left[(\vec{p},\frac{E}{c})^2=inv. \Rightarrow p^2-(\frac{E}{c})^2=-(\frac{E_0}{c})^2 \Leftrightarrow p^2c^2-E^2=-E_0^2 \Leftrightarrow p^2c^2-(\gamma mc^2)^2=-(mc^2)^2\right] \Rightarrow$$

$$\Rightarrow \left\{\begin{matrix} \left[\begin{matrix} \left(\frac{\vec{r}}{r}\times(\vec{p},\frac{E}{c})\right)^2=\left(\frac{1}{r}(\vec{L},\frac{E}{c}\vec{r})\right)^2=inv., \\ \gamma=\gamma_1,\gamma_2=1,\ \vec{L}=\overline{const.} \end{matrix}\right] \Rightarrow \frac{1}{r_1^2}\left[\vec{L}_1^2-(\frac{E}{c})^2r_1^2\right]=\frac{1}{r_2^2}\left[\vec{L}_2^2-(\frac{E_0}{c})^2r_2^2\right] \Rightarrow \\ \Rightarrow L=L_1=L_2=\frac{r_1r_2}{c}\sqrt{\frac{E^2-E_0^2}{r_2^2-r_1^2}}=mcr_1r_2\sqrt{\frac{\gamma_1^2-1}{r_2^2-r_1^2}}=const., \Rightarrow r_1r_2\sqrt{\frac{\gamma_1^2-1}{r_2^2-r_1^2}}=const. \end{matrix}\right\} \tag{2.9.5-3}$$

$$L=\frac{r_1r_2}{c}\sqrt{\frac{E^2-E_0^2}{r_2^2-r_1^2}}, E^2=E_0^2+\frac{L^2(r_2^2-r_1^2)c^2}{(r_1r_2)^2}, 2E\frac{\partial E}{\partial L}=2L\frac{(r_2^2-r_1^2)c^2}{(r_1r_2)^2},$$

$$\frac{\partial E}{\partial L}=\frac{L}{E}\frac{(r_2^2-r_1^2)c^2}{(r_1r_2)^2}=\frac{L}{E}\frac{c^2}{r_1^2}-\frac{L}{E}\frac{c^2}{r_2^2}=(\omega_1-\omega_2)=\omega,\ E=\frac{L}{\omega}\frac{(r_2^2-r_1^2)c^2}{(r_1r_2)^2}=J\frac{(r_2^2-r_1^2)c^2}{(r_1r_2)^2}=\gamma mc^2 \Rightarrow$$

$$J=\gamma m\frac{(r_1r_2)^2}{(r_2^2-r_1^2)}.$$

Particles are performing only spinning:

If a particle is in a kind of state of rest (by the fact of not being in linear motion), but spinning (around certain of its axes), it is evident that the total particle energy should have two components, one of them mc^2 and another equal to the energy of spinning $E_s \neq 0$,

$$\left.\begin{cases}\left(p=0, E=E_0+E_s=mc^2+E_s, E_s\neq 0, v=0, \vec{\mathbf{L}}=\vec{\mathbf{S}}\right), \\ (p, \frac{E}{c})^2=\text{inv.} \Leftrightarrow (0, \frac{E}{c})^2=\text{inv.} \Leftrightarrow (0, \frac{E_0+E_s}{c})^2=\text{inv.}\end{cases}\right\}\Rightarrow$$

$$\frac{E^2}{c^2}=\frac{E_0^2+2E_0E_s+E_0^2}{c^2} \Leftrightarrow E^2=E_0^2+2E_0E_s+E_0^2 \tag{2.9.5-4}$$

$$E=E(\mathbf{S}) \Rightarrow \frac{\partial}{\partial \mathbf{S}}\left[E^2=E_0^2+2E_0E_s+E_s^2\right] \Leftrightarrow 2E\frac{\partial E}{\partial \mathbf{S}}=2E_0\frac{\partial E_s}{\partial \mathbf{S}}+2E_s\frac{\partial E_s}{\partial \mathbf{S}} \Rightarrow$$

$$\frac{\partial E}{\partial \mathbf{S}}=\frac{E_0}{E}\frac{\partial E_s}{\partial \mathbf{S}}+\frac{E_s}{E}\frac{\partial E_s}{\partial \mathbf{S}}=\frac{1}{E}(E_0+E_s)\frac{\partial E_s}{\partial \mathbf{S}}=\frac{\partial E_s}{\partial \mathbf{S}}=\omega_s, dE=dE_s=\omega_s d\mathbf{S}$$

A particle is performing circular motion and spinning:

$$\left[\begin{pmatrix} p=\gamma mv, E=\gamma mc^2, v=\omega r, \vec{v}=\vec{\omega}\times\vec{r}, \vec{p}\times\vec{r}=\vec{\mathbf{L}}, \\ \frac{1}{2}mv^2=\frac{1}{2}\mathbf{J}\omega^2 \Leftrightarrow mv^2=\mathbf{J}\omega^2=pv=\mathbf{L}\omega, p=\frac{\mathbf{L}}{r}, \\ v=\omega r \Rightarrow c=\omega_c r,\ \omega_c=\frac{c}{r}, \\ (\vec{p},\frac{E}{c})^2=\text{inv.} \Rightarrow p^2-(\frac{E}{c})^2=-(\frac{E_0}{c})^2 \Leftrightarrow \\ p^2c^2-E^2=-E_0^2 \Leftrightarrow p^2c^2-(\gamma mc^2)^2=-(mc^2)^2 \end{pmatrix}, \begin{pmatrix} \vec{\mathbf{L}}\to\vec{\mathbf{L}}+\vec{\mathbf{S}}=\overrightarrow{\text{const.}}\ ,\ E\to E+E_s \\ \left(\frac{\vec{r}}{r}\times(\vec{p},\frac{E}{c})\right)^2=\left(\frac{1}{r}(\vec{\mathbf{L}},\frac{E}{c}\vec{r})\right)^2=\text{inv.}\end{pmatrix}\right]\Rightarrow$$

$$\Rightarrow \frac{1}{r^2}\left[(\vec{\mathbf{L}})^2-(\frac{E}{c})^2r^2\right]=\frac{1}{r^2}\left[-(\frac{E_0+E_s}{c})^2r^2\right]\Rightarrow\frac{(\vec{\mathbf{L}})^2}{r^2}-(\frac{E}{c})^2=-(\frac{E_0+E_s}{c})^2,$$

$$E^2=\frac{(\vec{\mathbf{L}})^2}{r^2}c^2+(E_0+E_s)^2, E=E(\mathbf{L})\Rightarrow\frac{\partial}{\partial\mathbf{L}}\left[(\frac{\mathbf{L}}{r})^2c^2-E^2=-(E_0+E_s)^2\right]\Leftrightarrow \tag{2.9.5-5}$$

$$2\frac{c^2}{r^2}\mathbf{L}-2E\frac{\partial E}{\partial\mathbf{L}}=-2E_s\frac{\partial E_s}{\partial\mathbf{L}}\Rightarrow 2E\frac{\partial E}{\partial\mathbf{L}}=2\mathbf{L}\frac{c^2}{r^2}+2E_s\frac{\partial E_s}{\partial\mathbf{L}}\Leftrightarrow$$

$$\frac{\partial E}{\partial\mathbf{L}}=\frac{\mathbf{L}}{E}\cdot\frac{c^2}{r^2}+\frac{E_s}{E}\frac{\partial E_s}{\partial\mathbf{L}}=\omega+\frac{E_s}{E}\omega_s\,.$$

Matter Waves and Particle-Wave Duality Concept:

Let us take into consideration only the energy-momentum state of certain motional mass that is in linear and inertial motion. We will be able to find (at least) two additional, mutually equivalent ways to present the same mass as performing rotation and spinning. The common for all such equivalent motional states (of the same mass) would be equivalence of their kinetic energy between linear and rotational states of the same motional energy. Of course, all other conservation laws should be satisfied. It is not difficult to explain such (mathematical) strategy. Intuitively clear is that every case of linear mass motion is only a case of an orbital rotation around certain center (if we consider that radius of such rotation can be arbitrarily long). In frames of such concept, all motional masses are cases of certain rotational or curvilinear movements.

The kinetic energy of the motional particle (in inertial, linear motion) is presentable in the following way,

$$E_k = (\gamma - 1)mc^2 = \frac{pv}{1 + \sqrt{1 - v^2/c^2}}$$

$$\begin{bmatrix} \overline{P}_4 = (p, \frac{E}{c}) \Rightarrow p^2 - \frac{E^2}{c^2} = -m^2c^2 \\ E^2 = m^2c^4 + p^2c^2 = E_0^2 + p^2c^2 \\ p = m\gamma v, \gamma = (1 - v^2 / c^2)^{-0.5} \\ dE_k = vdp = mc^2 d\gamma = dE_{tot} = d\tilde{E} \\ E = E_{tot} = \gamma mc^2, \ E_k = (\gamma - 1)mc^2 \end{bmatrix} \quad (2.9.5\text{-}6)$$

If we now consider the same mass rotate around the specific center (with a large radius of rotation $\vec{R}$, having angular revolving frequency $\omega_c = 2\pi f_c$), we can present such motion as,

$$E_k = \frac{L_c\omega_c}{1 + \sqrt{1 - v^2/c^2}} = \frac{pv}{1 + \sqrt{1 - v^2/c^2}} = (\gamma - 1)mc^2, \vec{v} = \vec{\omega}_c \times \vec{R}, \ L_c\omega_c = pv. \quad (2.9.5\text{-}7)$$

The same mass on the same linear path could also make helicoidal spinning (around its propagation path), with spinning angular frequency $\omega_s = 2\pi f_s$ and spinning moment L_s. This is another type of rotation (we could say a mathematical equivalent to the same motion), which will have the same kinetic or spinning energy (as before). Mass in question will effectively perform helical motion when observed from the common center of rotation.

$$E_k = \frac{L_s\omega_s}{1 + \sqrt{1 - v^2/c^2}} = \frac{L_c\omega_c}{1 + \sqrt{1 - v^2/c^2}} = \frac{pv}{1 + \sqrt{1 - v^2/c^2}} = (\gamma - 1)mc^2, \quad (2.9.5\text{-}8)$$
$$\vec{v} = \vec{\omega}_c \times \vec{R}, \ L_s\omega_s = L_c\omega_c = pv, \ (\vec{L}_s, \vec{\omega}_s, \vec{p}, \vec{v} \text{ mutually colinear})$$

It looks that the spinning (which is creating a helix line around the path of mass propagation) is only an artificial mathematical example to show how motional energy of such spinning could be equal to the kinetic energy of linear and rotational motion initially introduced. However, we could also try to give another meaning to such spinning. For instance, to consider it as being a specific de Broglie matter waves generator. Matter waves generated by such helicoidal motion will have the wavelength λ_s, frequency f_s, group velocity v, and phase velocity u, as follows,

$$\lambda = \lambda_s = \frac{H}{p}, k = \frac{2\pi}{\lambda_s} = \frac{2\pi}{H} p,\ H = \text{Const.}$$
$$u = \lambda_s f_s = \frac{\lambda_s \omega_s}{2\pi} = \frac{\mathbf{L}_s \omega_s / p}{1+\sqrt{1-v^2/c^2}} = \frac{v}{1+\sqrt{1-v^2/c^2}}$$
$$E_k = \frac{\mathbf{L}_s \omega_s}{1+\sqrt{1-v^2/c^2}} = \frac{pv}{1+\sqrt{1-v^2/c^2}} = pu = \tilde{E} = Hf_s$$
$$\left\{ \begin{array}{l} dE_k = vdp = \omega_s d\mathbf{L}_s = mc^2 d\gamma = dE_{tot} = d\tilde{E} = pdu + udp \\ v = u - \lambda \dfrac{du}{d\lambda} \end{array} \right\} \Rightarrow \qquad (2.9.5\text{-}9)$$
$$\Rightarrow \frac{du}{v-u} = \frac{dp}{p} \Rightarrow \frac{d\lambda}{\lambda} = -\frac{dp}{p} \Rightarrow$$
$$\ln\left|\frac{\lambda}{\lambda_0}\right| = -\ln\left|\frac{p}{p_0}\right| \Leftrightarrow \left|\frac{\lambda p}{\lambda_0 p_0}\right| = 1 \Leftrightarrow \lambda p = \lambda_0 p_0 = \text{Const.} = H$$
$$\vec{v} = \vec{\omega}_c \times \vec{R},\ \mathbf{L}_s\omega_s = \mathbf{L}_c\omega_c = pv,\ (\vec{\mathbf{L}}_s, \vec{\omega}_s, \vec{p}, \vec{v} \ \text{ mutually colinear}).$$

Now (in (2.9.5-9)) we have a perfect match of mutually non-contradicting parameters of matter waves integrated with motional mass properties. Imaginatively introduced helix matter wave could also be interpreted as an Analytic Signal wavefunction couple, where $\Psi(t)$ is the original, linear-motion and power-related function, and $\hat{\Psi}(t)$ is its phase-shifted Hilbert couple, both creating $\overline{\Psi}(t) = \Psi(t) + j\hat{\Psi}(t) = (1 + jH)\,\Psi\,(t)$. Now, from the relevant Analytic Signal model we will be able to determine matter waves frequency, phase, wavelength, and other results as presented in (2.9.5-9). See much more about Analytic Signals in chapters 4.0, 4.1 and 10. This gives an improved conceptual picture of de Broglie matter waves compared to the contemporary view of matter waves (see much more in chapter 4.1 around equations (4.3-0) and (4.3-0)-a,b,c,d,e,f,g,h,I,k...). Since Planck constant **h** and its macro-cosmos equivalent **H** are mutually much different ($H >>> h$), the number of questions related to unity and harmony between Quantum and Relativistic world theories will be initiated. What is common for micro and macro world constants **h** and **H** is that systems, objects, or entities where such constants are naturally significant are different, self-closed, standing matter-waves structures.

[♣ COMMENTS & FREE-THINKING CORNER:

As an illustration of the rest mass origins in rotation, let us exercise (or hypothesize) by considering an electron (in its state of rest) as being space-limited or localized, self-stabilized and internally captured electromagnetic wave that is creating a standing-waves structure on a closed circular line or on a toroidal form (the same also applicable to a proton, positron etc.). When following the same idea, it is becoming evident that the amount of electromagnetic energy (or specific group of photons), which is on that way creating an electron did not have any initial rest mass. Only after the act of standing-wave creation, we can associate the rest mass with an electron. The origin of electron rest mass is related to the fact that electromagnetic standing waves are being created on a self-closed circular path, while the internal content of such rest mass presents the matter wave or motional energy. Here, at the same time, we are also establishing a specific association between an electron and state of electromagnetic energy that is rotating on a closed circular line. Such internally rotating electromagnetic waves formation, or kind of group of photons (or rotating distributed mass), should have certain wave energy amount equal to $\tilde{E}_c = hf_c$, or maybe to $\tilde{E}_c = n \cdot hf_c,\ n = 1, 2, 3 \ldots$. Consequently, since we know (experimentally) the amount of electron rest mass, we could say that its total (rest) energy is, $E_e = m_e c^2 = \tilde{E}_c = hf_c \Rightarrow$ $f_c = \frac{m_e c^2}{h}$ or maybe $f_c = \frac{m_e c^2}{n \cdot h}$, $n = 1, 2, 3, ...$, where f_c is relevant, mean, standing-electromagnetic-waves frequency. By analogy, we could now consider that electron in some ways presents an equivalent rotating and distributed mass, which has a form of a spinning ring, disk, or torus. Now we will adopt another strategy, based on analogies, and address the rotation of such distributed mass as, $dE_e = c^2 dm_e = v_e dp_e = \omega_e d\mathbf{L}_e$, where ω_e is the effective angular speed of such rotating and distributed mass. For the sake of generality, we will assume that $\omega_e = 2\pi f_e \neq 2\pi f_c$. Since a standstill electron should present stable, fixed, stationary structure, it is clear that $\omega_e = 2\pi f_e$ should be specific constant angular frequency (or sort of equivalent, mechanical, rotating, angular speed), and that such spinning ring, disk, or torus should have steady angular momentum. Let us now make the integration,

$$dE_e = c^2 dm_e = \omega_e d\mathbf{L}_e = hdf_c \Rightarrow \int dE_e = \int c^2 dm_e = \int \omega_e d\mathbf{L}_e = \int hdf_c \Rightarrow$$
$$E_e = m_e c^2 = \omega_e \mathbf{L}_e = \omega_e^2 \mathbf{J}_e = hf_c \Rightarrow \mathbf{J}_e = \frac{m_e c^2}{\omega_e^2} = \frac{hf_c}{\omega_e^2}. \qquad (2.11.9\text{-}1)$$

If we limit our elaborations only to spinning rings or toroidal structures (see (2.11.3)), where dimensional conditions (relations between relevant diameters) enable us to assume that moment of inertia is $\mathbf{J}_e = m_e r_e^2$, we will be able to find the effective radius of an electron as,

$$\mathbf{J}_e = \frac{m_e c^2}{\omega_e^2} = \frac{hf_c}{\omega_e^2} = m_e r_e^2 = m_e \left(\frac{v_e}{\omega_e}\right)^2, \; r_e = \frac{v_e}{\omega_e}, v_e = c \Rightarrow r_e = \frac{c}{\omega_e} = \frac{1}{\omega_e}\sqrt{\frac{hf_c}{m_e}}. \qquad (2.11.9\text{-}2)$$

Of course, the solution that is more general would be $\mathbf{J}_e = \frac{n \cdot hf_c}{\omega_e^2}$, $r_e = \frac{1}{\omega_e}\sqrt{\frac{n \cdot hf_c}{m_e}}$.

Here we are underlining, as a general case, the difference between the angular speed of equivalent mass spinning and its internal, comparable electromagnetic energy angular frequency $\omega_e = 2\pi f_e \neq 2\pi f_c$. The reason is that once we are treating electron almost mechanically, as an equivalent spinning ring or spinning torus, and in parallel, we are developing the idea that internal content of such spinning object effectively presents electromagnetic standing waves (see later, in chapter 4.1, more about the spinning nature of matter waves; -equations (4.3-1) – (4.3-3)).

In the very beginning of this exercise, we assumed that electron rest mass is created by self-closing of an electromagnetic standing-waves structure, on a circular line, and this concept is giving the opportunity to describe elements of such standing waves as,

$$\begin{bmatrix} 2\pi r_e = k \cdot \lambda_c \Leftrightarrow \lambda_c = \frac{2\pi r_e}{k}, \; k = 1,2,3... \\ m_e c^2 = hf_c \Leftrightarrow f_c = \frac{m_e c^2}{h} = \frac{\mathbf{J}\omega_e^2}{h} = \frac{m_e r_e^2 \omega_e^2}{h} \end{bmatrix} \Rightarrow \lambda_c f_c = u_c = c = \frac{2\pi r_e}{k} \cdot \frac{m_e c^2}{h} \Rightarrow$$
$$k \cdot h \cdot c = 2\pi r_e m_e c^2 \Rightarrow r_e = \frac{k}{2\pi} \cdot \frac{h}{m_e c} = \frac{c}{\omega_e} \Leftrightarrow \omega_e = \frac{m_e c^2}{k \cdot \frac{h}{2\pi}} = 2\pi f_e = \frac{1}{r_e}\sqrt{\frac{hf_c}{m_e}}, \text{ or} \qquad (2.11.9\text{-}3)$$
$$\omega_e = \frac{m_e c^2}{k \cdot \frac{h}{2\pi}} = 2\pi f_e = \frac{1}{r_e}\sqrt{\frac{n \cdot hf_c}{m_e}}, \; k, n = 1,2,3...$$

Let us now analyze the same situation, seeing the particle (electron, for instance) from its "*external space*" (now we do not care what is happening inside such particle, and we only see that this is a compact, stable particle, which has the rest mass m). If the same elementary particle is in a process of motion (relative to a Laboratory system: see (2.11.5)), de Broglie matter wave, which is associated to such particle (like a moving electron, positron, proton…), can be specified as,

$$\lambda = \frac{h}{p} = \frac{h}{\gamma mv} = \frac{h}{mc}\cdot\frac{c}{\gamma v} = \lambda_c \cdot \left(\frac{c}{\gamma v}\right), \lambda_c = \frac{h}{mc} = \text{const.}, \gamma = 1/\sqrt{1-\frac{v^2}{c^2}},$$

$$f = \frac{1}{\Delta t} = \frac{E_k}{h} = \frac{mc^2}{h}(\gamma - 1) = f_c \cdot (\gamma - 1) = \frac{1}{\Delta t_c}\cdot(\gamma - 1),\ f_c = \frac{mc^2}{h} = \text{Const.} \Rightarrow \Delta t = \frac{\Delta t_c}{(\gamma - 1)},$$

$$u = \lambda f = \lambda_c f_c \cdot \frac{\frac{\gamma v}{c}}{1+\gamma} = \frac{\gamma}{1+\gamma}\cdot v = \frac{\omega}{k} = \frac{\tilde{E}}{p} = \frac{hf}{p} = \frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{E_k}{p},\ \lambda_c f_c = u_c = c, \tag{2.11.9-4}$$

$$v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} = u + p\frac{du}{dp} = \frac{d\omega}{dk} = \frac{d\tilde{E}}{dp} = h\frac{df}{dp} = \frac{2u}{1+\frac{uv}{c^2}} = (1+\frac{1}{\gamma})u\ ,$$

$$\Rightarrow 0 \le u < 2u \le \sqrt{uv} \le v \le c.$$

By comparing here obtained results with similar results available in other literature, we will certainly notice differences. The significance of this example is related to how internal electron structure is conceptualized and how it is connected to "*external*" understanding of matter waves (or how internal matter waves are related and connected to familiar, external matter waves). We are also paving the way of understanding how particles with stable rest masses are being created, and how field of Gravitation is appearing externally. The vision of the particle-wave dualism of all matter in our universe, underlined here, is that "*external*" matter waves are an extension, or unfolding of the internal wave structure of matter (in the process of motion), in addition to the complexity of 2-body or n-body interactions. If we now start analyzing two-body problems (or binary interactions), considering internal and external wave nature of moving bodies, as well as their rest and velocity-dependent masses, we will be able to address number of known particle-wave manifestations on a much more tangible and conceptually clearer way than we find presently in physics. This will certainly initiate the process of brainstorming contributions that will gradually upgrade and optimize the same (here elaborated) concept, eventually leading to a much more complete modeling of elementary particles and atoms, and to the conclusion that electron cannot be only a point particle with "magic" attributes (valid also for other elementary particles). See similar and in many aspects more profound modeling of elementary particles in literature from [16] to [20]; Authors: Bergman, David L., and Lucas, Jr., Charles W, http://www.commonsensescience.org/papers.html.

To mention that authors Bergman, David L., and Lucas, Jr., Charles W, besides ingenious and original modeling of elementary particles, are, in the same package, softly and sporadically (for some reason) promoting their religious convictions. The author of this book is not recommending such religious background attachments to be considered as a here-relevant references (because there is no need and scientifically defendable place for such inclusions). ♣]

2.3.3. Macro-Cosmological Matter-Waves and Gravitation

Rethinking Gravitation: From Newton to Einstein and Beyond

Gravitation, as described by General Relativity, largely aligns with the results predicted by Newton's classical mechanics. Within the context of our planet and solar system, the gravitational frameworks developed by Newton and Kepler remain highly effective. It is interesting to acknowledge that the foundations of Kepler's laws were originally established during a period of flourishing Arab science and culture, long before Kepler revitalized these concepts.

However, when we consider larger cosmic scales, such as the vast spaces containing galaxies, astronomical observations suggest that our current understanding of gravitation requires revision. Although Einstein's General Relativity introduces a more complex mathematical framework, it does not significantly deviate from Newton's predictions and shares some of the same limitations. Einstein proposed that the presence of mass curves space-time, and this curved geometry must be considered to describe motion within these deformations. Consequently, gravitation is equated with uniform accelerated motion.

In simpler terms, masses can be understood as concentrations of stored and stabilized energy, typically in the form of atoms. Classical Mechanics describes forces as gradients of local spatial energy-mass or potential energy concentrations. Einstein replaced this universal force definition with the concept that curved space-time geometry around masses naturally produces components of accelerating motion, or gravity-related forces. In practical terms, the motions of planets and satellites, central to our current mechanical and cosmic engineering, still yield results comparable to those predicted by Newton's theory, even though the mathematics in Einstein's framework is far more complex.

While General Relativity performs well within our solar system, its predictions for vast cosmic spaces, such as those containing spiral galaxies, are sometimes questionable or incorrect. These inconsistencies have led to the introduction of hypothetical entities like dark matter and dark energy to account for these anomalies.

Contemporary scientific authorities often attempt to extend and unify Relativity and Quantum theories by introducing these hypothetical constructs. This approach has gained widespread recognition and funding, but it risks prematurely accepting these theories as final and unchangeable truths. As a result, alternative scientific theories and concepts are often ideologically subordinated to the foundations of Relativity and Quantum theory. When inconsistencies arise, new entities, such as dark matter, dark energy, or other insufficiently clear phenomena, are invented and labeled as real, despite their speculative nature. These labels often serve merely to fill gaps in existing theories without substantial meaning.

However, as suggested in this book, if we consider that mass is not the sole source of gravity, and that the true sources of gravitation are different electromagnetic and mechanical moments and dipoles that are spatially polarized and/or aligned, then the need for concepts like missing mass or dark energy diminishes.

Download the last version here:
http://www.mastersonics.com/documents/revision_of_the_particle-wave_dualism.pdf

Mathematical models based on Newtonian gravitation and Relativity work well in controlled environments, such as laboratories on Earth or within stable solar systems. However, when these theories are applied to larger astronomical distances or to the extremely small scales of subatomic environments, their limitations become apparent. They are neither complete nor universally applicable, underscoring the need for a new and improved theory of gravitation. Instead of relying on hypothetical entities like dark matter and energy, we should focus on refining Relativity and Quantum theory, an evolution akin to the progression from Real Numbers to Complex Numbers, and later to Hypercomplex Numbers and Analytic Signal functions.

Currently, Classical Mechanics, Newton's theory of gravitation, and the theory of Relativity are well-defined and operate effectively within a smooth, continuous, and deterministic space-time framework. Conversely, contemporary Quantum theory models the motions and energy states of matter domains statistically and probabilistically, often using averaged values, functions, and distributions. This approach, coupled with a generalized, though not always mathematically precise, discretization and quantization of energy states, motions, and interactions, introduces a certain level of conceptual ambiguity.

These artificial or postulated mathematical frameworks in modern Relativity and Quantum theory cannot be easily integrated without significant conceptual, theoretical, and innovative redesign. To achieve a more unified understanding, it will be necessary to apply more general mathematical concepts than those currently in use, such as the "Analytic Signal" and joint time-frequency analysis based on the Hilbert Transform, as established by Denis Gabor, as well as the "Kotelnikov-Shannon, Whittaker-Nyquist Sampling and Signal Recovery Theory" (discussed further in Chapters 4.0 and 10).

A fascinating trend in the evolving understanding of gravitation is the successful conceptualization, modeling, and quantization of planetary systems, orbits, and motions, in a manner analogous to Niels Bohr's atom model. This approach, discussed in this chapter and elaborated further in Chapter 8, yields accurate and verifiable results based on astronomical observations and measurements across various planetary and solar systems.

Interestingly, concepts from the micro-world, such as wave-particle duality and matter waves, which are well-integrated into atomic and subatomic modeling, can be extended analogically to macro systems like planetary systems, without relying heavily on stochastic or probabilistic methods. In this book, the concept of particle-wave duality and de Broglie matter waves are expanded to include ideas about the complementarity of linear and rotational (or spinning) motions. This extended framework becomes universally applicable to both the micro and macro worlds of physics, encompassing both subatomic and astronomical phenomena. This is analogous to the complementarity of electric and magnetic fields.

In summary, the concept of gravitation presented in this book is linked to the phenomenon of spatially complex, stationary, and standing cosmic matter waves. These waves manifest within a structurally and multi-dimensionally resonating universe, where orbital, linear, and spinning inertial motions are complementarily and structurally united (both mechanically and electromagnetically).

Mass formations in this stabilized, structurally resonating (and rotating) universe occupy nodal zones of the highest energy-mass densities and oscillatory accelerations, where oscillation amplitudes are minimal. This is like the behavior observed in half-wave resonators in High Power Ultrasonics technology and in acoustic or ultrasonic levitation, where attractive forces act toward these nodal zones.

These spatially resonating, stationary, and standing waves take the form of atomic, solar, stellar, and galactic systems, adhering to the laws of energy-momentum conservation. The involved linear and rotational (or spinning) motions are specifically united and mutually complementary, much like the relationship between electromagnetic fields and the oscillatory behavior of mass-spring or inductance-capacitance circuits. The matter waves and particle-wave duality manifestations, along with the associated and strongly coupled mechanical and electromagnetic complexities, enable "energy-momentum communications" within these dynamic, self-stabilizing, standing-matter waves and resonant spatial formations.

However, the fundamental questions of what the principal macro or global "external, divine source" of these vibrations are, and how such universal and pervasive structural oscillations, spinning, and rotations are initially created, synchronized, and maintained within our universe, remain unanswered.

A set of updated and generalized second-order differential wave equations, evolving from the Classical Wave Equation and a redesigned Schrödinger equation, and formulated using the Analytic Signal, Complex, and Hypercomplex functions based on Hilbert transformation, currently offers the best mathematical framework to address structural oscillations within our micro and macro-universe. Extending this framework, we find that Nikola Tesla's ideas about Dynamic Gravity theory are familiar to these thoughts and are compatible with, or complementary to, Rudjer Boskovic's descriptions of universal natural forces.

The understanding of gravitation can be significantly simplified as follows: Since we know that gravitational masses are composed of atoms, and atoms contain internal constituents that perform orbital and spinning motions, creating magnetic moments, the most probable source of gravitational attraction should be these interacting internal magnetic moments. These magnetic moments, distributed spatially and specifically polarized, act like elementary magnets within the involved masses. There is always a certain number of uncompensated internal "magnets" within macro masses. Due to the existence of global (and accelerated) macro-motions within our universe, these internal magnetic elements mutually polarize, leading to forces between neighboring masses that adhere to attractive forces as described by Coulomb or Newton's laws (similarly applicable to forces between magnets). Additionally, the significantly different masses of spinning electrons and protons facilitate electric dipole polarization, supporting attractive Coulomb forces.

These accelerated, motion-induced polarizations, standing wave formations, and attractive forces are what contemporary physics has recognized as gravitation—incorrectly attributed solely to the mysterious attraction of masses. Instead, the motions of masses, including their oscillations, produce matter waves that form part of the surrounding stationary and standing matter waves and fields, which are primarily electromagnetic in nature. Yet, we continue to misinterpret these mixed and dynamic electromagnetic effects as an independent and self-standing force of gravitation, based solely on mutual and inexplicable static mass attraction.

In the first chapter of this book, it is demonstrated that static and electromagnetically neutral masses cannot be sources of gravitation. However, by analogy, it can be concluded that oscillating electric charges, properly oriented dipoles, and magnetic moments or fluxes, along with the associated linear and angular mechanical moments, forces, and torques emanating from atoms and agglomerated masses, are the real sources of gravitation. Consequently, most contemporary theories based on outdated concepts of Relativity, Gravitation, and other natural forces need significant evolution or should be completely replaced with better and new concepts.

In the case of micro-universe of atoms and elementary particles, de Broglie matter waves are manageable using the following relations (see more in chapter 4.1 and Chapter 10., concerning PWDC):

Wave or motional energy (=) $\tilde{E} = hf = E_k$, *(for a narrow band wave group)*
Matter waves wavelength (=) $\lambda = h/p$,
Phase velocity (=) $u = \lambda f = \omega/k = E_k/p$,
Group or particle velocity (=) $v = d\omega/dk = dE_k/dp$.

Let us now try to construct or exercise what could be the ***macro-universe equivalent*** to de Broglie matter-waves concept. The idea here is to show that solar systems, planets, satellites and similar macro-objects (in orbital and spinning motions) are also analogically respecting certain periodicity and "standing macro matter-waves packing rules", like de Broglie matter waves in a micro-universe (analogically like in N. Bohr atom model; -see more in Chapter 8.), but instead of Planck constant **h**, new macro-world constant **H** >>> **h** is becoming relevant in a similar way as **h** is in a micro world of Physics. See an indicative introduction to such concept given by equations (2.11.5) - (2.11.9), (2.11.9-1) - (2.11.9-4) and (2.9.5-1) - (2.9.5-5).

The best way for exercising the mentioned brainstorming is to start from Kepler's third law (of planetary orbital motions), which is also applicable to all satellite and lunar, inertial motions around a specific planet or big mass. Let us temporarily focus our attention only on idealized circular motions, where the radius of rotation is R , to be able to use simpler mathematical expressions (approximating different orbital planetary motions, as being circular, where **R** is a planet semi-major orbital radius). Kepler's third law is showing that the period T of a planet (or satellite) with mass **m**, orbiting around a big mass $M >>> m$ (or its sun), is given by (2.11.10),

$$T^2 = \left(\frac{4\pi^2}{G(M+m)}\right)R^3 \cong \left(\frac{4\pi^2}{GM}\right)R^3 \quad . \qquad (2.11.10)$$

We will later also need to extract from Newton-Kepler theory the expression for a maximal-orbital or escape velocity v_e , and escape kinetic energy E_e (when planet, rocket or satellite would escape its stationary, circular orbit), which can be found as,

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

$$\left(E_e = \frac{1}{2}mv_e^2 = \frac{GmM}{R}\right) \Leftrightarrow v_e = \sqrt{\frac{2GM}{R}} \Rightarrow v_e^2 R = 2GM = \text{constant}. \qquad (2.11.11)$$

Escape velocity v_e is a "*flexibility-parameter of boundary orbital-stability limit*" of all motions within and around specific planet or sun (where relevant planet or sun could be approximated as a local center of mass for such mutually related motions). Later, we will see that similar relation "$v^2R = v_n^2R_n = \text{constant}$" is also valid for all planets of solar systems. For instance, for all planets of our solar system, we always have the same constant, $v^2R = v_n^2R_n = \mathbf{1.3256E+20}$ (see this result later in the same chapter inside T.2.3.3). For other planetary and satellite systems, such constants will be different. We will then see that the relations mentioned are consequences of periodicity and standing, macro-matter-waves structures within stable planetary and satellite systems.

Kepler laws are also showing the intrinsic tendency of (mutually approaching) motional masses, planets, and satellites, to eventually stabilize in some form of elliptical, rotational, orbital and inertial motions around certain big mass (local star or sun), which is at the same time very close to a local center of mass (concerning relevant planetary or solar system). For having an additional and supporting background, see introductory elaborations in this chapter, around equations (2.4-11) – (2.4-17)), where we can find that for the stability of a specific orbital motion (planetary system), the main request is that its total orbital and spinning momentum is conserved (meaning constant). If this (about orbital motions) were not the global tendency of mutually approaching masses, our universe would collapse in the process of permanent masses attraction and agglomeration. We also know that the conservation of orbital and spin moments is equally valid and important on a micro-world scale (analogically as we find in N. Bohr atom model). The dominant tendency of macro masses, also valid for micro and elementary particles, is to create stable (standing matter-waves), periodic and orbital motions, based on balancing between attractive forces with repulsive centrifugal forces (see new trends in modeling atoms and elementary particles in literature references from [16] to [22], Bergman, Lucas, Kanarev and others). Natural, non-forced uniform and stable, orbital planetary motions are at the same time inertial (uniform, continuous, self-closed and self-standing), periodical motions, which are coincidently conserving their linear and orbital moments, and potentially hosting standing matter-waves formations, as shown in (2.9.1). Also, consequences of stable self-closed standing matter-wave's orbital formations (that are causally related to periodical motions) are various energy, spin and orbital momentum relations and quantizing situations, being very much analogical to N. Bohr atom model (see more in Chapters 8., 10., and in literature references under **[63]**).

*Citation from [63], under **25) - Spin - orbit coupling in gravitational systems:***
"We employ in this work the analogy existing between electromagnetism and gravitation [1]. We extend this analogy to include all phenomena occurring at atomic level and assume that they also do occur at the gravitation level and are governed by analogous rules (equations). The spin-orbit interaction that exists in hydrogen atom, due to the magnetic field if we introduce the concept of gravitomagnetic field that is analogous to the ordinary magnetic field. We have seen that the spin-orbit interaction is the same interaction that Einstein attributed to the curvature of the space [2]. And since all planets do have spin, the spin-orbit interaction is intrinsically prevailing in all star-planet systems. Bear in mind that some atoms can have zero total spin angular momentum. Note that the spin of planets remained a kinematical quantity in Newton and Kepler formulation of planetary motion. But we will show here that the spin is

a dynamical quantity without which the planets would not remain stable in their orbits. Moreover, without spin there is no orbital motion. How much a planet should spin will depend on how much it is needed to conform with the orbital one. Equating the gravitational energy of a star-planet system to the spin-orbit interaction yields a formula that relates the primary star-planet system parameters to each other. Moreover, we found that such a system exists only if the spin and orbital angular momenta are proportional to the planet mass to the star mass ratio. This condition represents a dynamical balance between the two angular momenta. We call the resulting equation the Kepler's fourth law which represents the missing equation (law) to determine a star-planet system completely.
…………
As in hydrogen atom, which is analogous to the solar system, there is an interaction between the internal magnetic field arising from the electron orbital momentum, and its spin angular momentum. This is normally known as the spin-orbit interaction. The gravitomagnetic field is analogous to the magnetic field arises from the motion of the electron around the nucleus".

Let us now attempt to show that (like in case of de Broglie matter waves applied on Bohr's hydrogen, or planetary atom model) a circular planetary (or satellite) orbit, or its perimeter, is susceptible to hosting some gravitational (also electromagnetic and inertial), orbital, standing matter-wave. *Such macro matter-wave should have an orbital frequency* $f_o = f_{on}$, *wavelength* $\lambda_o = \frac{2\pi R}{n} = \lambda_{on}$, $n = \text{Integer}$, *group or orbital speed v (equal to planet semi-major orbital radius velocity), and associated phase speed* $u = \lambda_o f_o = u_n$. *Integer n should serve as a principal quantum number, being mainly related to the number of days in one year of certain planet orbiting its sun. The same quantum number could be related to presence of satellites, moons, and to involved angular and spinning moments, since relevant (and associated) standing-waves structure will be affected by all involved items (what introduces additional quantum numbers for arranging structural and spatial, standing waves packing and synchronization). Familiar matter-waves related conceptualization, and results are initially addressed in chapters 4.1, 10, and in this chapter among equations (2.9.1), (2.9.2), (2.11.5), (2.11.14)-h, in tables T.2.3.3-a, T.2.3.3-1, and such conceptualization is widely applicable, both in a micro and macro world of our Universe. One good example showing coupling of linear and spinning motions are cases of vortex-shedding flowmeter, presented around equations (4.3-0) and (4.3-0)-a,b,c,d,e,f,g,h,I, from Chapter 4.1. Many ideas showing or constructing rich and well-operating (astronomic observations verifiable) analogy between planetary systems and N. Bohr atom model, can be found in [63], Arbab I. Arbab, [64], Marçal de Oliveira Neto, and in [67] Johan Hansson.*

In Chapter 8. of this book, under "8.3. Structure of the Field of Subatomic Forces" (see equations from (8.64) until (8.74)), we can find proposals how to conceptualize ***<u>spatial standing-waves-structured forces</u>***, *emanating from internal atoms field structure, where both orbital and radial quantizing rules are applicable and mutually synchronized. It will be challenging to apply similar modeling to stable planetary systems. One of the consequences of such modeling could be that gravitational forces between agglomerated atoms and masses are the result of Lorentz-forces and Coulomb attractions between half-wave resonating (and rotating) mass-dipoles (or better to say attractions between electromagnetically polarized, charged and mutually oscillating dipoles).*

Effectively, here we attempt to present the motional energy of an orbiting planet as an equivalent macro-matter-wave packet or wave group (which is the concept often and successfully applied in micro-world physics). Specific planetary rotation around certain sun (see below (2.11.12)) has a period $T = T_m$ (or this is its one planetary year duration) and frequency of such (mechanical) rotation is $f_m = 1/T = 1/T_m$. f_m is not necessarily the frequency f_o of the associated, orbital, standing and macro matter-wave. For understanding the difference between mechanical (mass or particle) revolving frequency f_m and orbital macro matter-wave frequency f_o, we will first assume (and prove later) that $f_m \neq f_o$. Since the framework of this exercise implicitly accepts that relevant planetary or satellite (orbital) velocities are much lower compared to the light speed ($v << c$), we could safely say that any planetary or group velocity (its orbital velocity) should be two times higher than its phase velocity, $v = 2u = 2\lambda_o f_o$. See better explanation why and when $v = 2u$ in Chapters 4.0, 4.1, and 10., with equations (4.0.78) – (4.0.81) ... In other words, if the analogy with de Broglie matter-waves hypothesis also applies to planetary orbital motions, then the kinetic energy of specific planet should be equal to its equivalent matter-wave energy (or its matter-wave packet), $E_k = \tilde{E} = \frac{1}{2}mv^2 = Hf_0$, where H is a kind of gravitational, Planck's-analog, constant (all of that being very much analogical to matter waves and PWDC, as presented in Chapter 10.; -see "10.00 DEEPER MEANING OF PWDC").

Now we can find mentioned orbital frequency, wavelength, group, and phase speed of such (hypothetical), planetary standing matter-wave as (2.11.12),

$$\left\{\begin{array}{l} 2\pi r = n\lambda_o,\ T = \frac{1}{f_m} = \frac{2\pi}{\omega_m} = T_m,\ v = \frac{2\pi R}{T} = 2\pi R f_m = \omega_m R \cong 2u,\ n = \text{Integer} \\ u = \lambda_o f_o = u_n \cong \frac{1}{2} v = \frac{\pi R}{T} = \pi R f_m,\ T^2 = \left(\frac{4\pi^2}{GM}\right) R^3 = \frac{1}{f_m^2}, v_e = \sqrt{\frac{2GM}{R}}, \\ v = u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda} = v_n\ , u = \frac{v}{1+\sqrt{1-v^2/c^2}} = u_n \end{array}\right\} \Rightarrow$$

$$\lambda_o = \frac{2\pi R}{n} = \lambda_{on},\ f_o = \frac{u}{\lambda_o} = f_{on} = n\frac{u}{2\pi R} = n\frac{f_m}{2} = \frac{n}{2T} = \frac{n\sqrt{GM}}{4\pi R^{3/2}}, f_m = \frac{2f_o}{n} = \frac{1}{T} = \frac{\sqrt{GM}}{2\pi R^{3/2}}, \qquad (2.11.12)$$

$$u = \frac{v}{2} = \frac{1}{2}\sqrt{\frac{GM}{R}} = \frac{1}{2\sqrt{2}} v_e,\ v = 2u = \sqrt{\frac{GM}{R}} = \frac{1}{\sqrt{2}} v_e << c,\ m << M, \forall n = \text{Integer}\ .$$

Based on the group or planet's orbital speed, $v = \frac{2\pi R}{T} = 2u = 2\lambda_0 f_0 = \left(2\frac{H}{p} f_0\right)$ from (2.11.12), the wave energy or kinetic energy and gravitational Planck constant H, of an orbiting planet, which has mass $m << M$, and naturally keeps its angular momentum **L** = constant, can be mutually supporting and connected as:

$$\tilde{E} = E_k = \frac{1}{2}mv^2 = mvu = pu = 2mu^2 = \frac{1}{4}mv_e^2 = \frac{GmM}{2R} = \frac{E_e}{2} = \frac{1}{2}\cdot\left(\frac{GmM}{R^2}\right)\cdot R = \frac{1}{2}\cdot F_{m-M}\cdot R =$$
$$= \frac{m}{2}\left(\frac{2\pi R}{T}\right)^2 = \frac{8m\pi^2R^2}{n^2}f_o^2 = 2m(\pi Rf_m)^2 = (2\pi mR^2f_m)\cdot(\pi f_m) = L\pi f_m = (\frac{2\pi}{n}L)\cdot f_o = Hf_o, \tag{2.11.13}$$

$$\left\{\begin{array}{l} L = n\frac{H}{2\pi} = 2\pi mR^2f_m,\ p = mv = \frac{\tilde{E}}{u} = \frac{Hf_o}{u} = \frac{H}{\lambda_o} = n\frac{H}{2\pi R} = m\sqrt{\frac{GM}{R}},\ G = 6.67\cdot10^{-11}Nm^2kg^{-2}, \\ F_{m-M} = F_g = \frac{GmM}{R^2} = \frac{mv^2}{R},\ \lambda_o = \frac{H}{p} = \frac{2\pi r}{n},\ f_o = \frac{nf_m}{2} = \frac{n}{2T} = \frac{n\sqrt{GM}}{4\pi R^{3/2}}, H = \frac{2\pi}{n}L = const.. \end{array}\right\}.$$

$$\Rightarrow \left\{\begin{array}{l} E_k = \tilde{E} = \frac{1}{2}mv^2 = \frac{1}{2}m\left(2\frac{H}{p}f_0\right)^2 = 2\frac{H^2}{mv^2}f_0^2 = Hf_0 \\ 2\pi R = n\lambda_0 = n\frac{H}{mv},\ H = Const.\ n = 1,2,3,... \end{array}\right\} \Rightarrow H = \frac{2\pi mvR}{n} = \frac{2\pi m_iv_iR_i}{n_i} \Rightarrow$$
$$\Rightarrow \frac{m_iv_iR_i}{n_i} = \frac{m_jv_jR_j}{n_j} = \frac{H}{2\pi}.$$

If we formulate an Analytic Signal, power-related wave, or Phasor function, that represents matter-wave of a specific orbiting planet (since we know its kinetic or wave energy), the same results, (2.11.12) and (2.11.13), should be associable to such complex matter-wave (see more about Analytic Signal in chapters 4.0, 4.1 and 10.).

[♣ COMMENTS & FREE-THINKING CORNER:

To get more tangible feeling what should be measurable effects of mentioned "standing gravitational waves" (or gravitational field phenomena that has phase velocity $u = \lambda_o f_o \cong \frac{1}{2}v = \frac{\pi R}{T} = \pi Rf_m$ *), we could analyze tidal waves on our planet Earth in relation to the Earth spinning and Moon rotation around the Earth, as well as Earth-Moon rotation around our local Sun. This way we should be able to establish predictable and measurable correlations (which should comply with (2.11.12) and (2.11.13)). Of course, it will also be necessary to consider proper values for* $u, \lambda_o, f_o, v, R, T, f_m$*, valid for Moon's rotation around Earth.*

We could also consider the orbital, matter wave frequency f_o *as the* ***time-frequency-train reference*** *for measuring our real-time flow. We are effectively using such time-reference on different ways, since it has remarkably high stability, like quartz crystal or atomic clock oscillators, and it is the most significant for measuring our time flow (what will become more evident later; -see (2.11.14)-g)).*

Let us address ***gravitational field intensity and potential****. The* ***gravitational intensity or gravitational field*** $I = E_g$ *is traditionally defined as the gravitational force experienced by a unit mass* m *when placed in the gravitational field of another mass. From Newton's gravitational force we will get,*

$F_{m-M} = F_g = G\frac{mM}{R^2}$*, we can get,* $I = \frac{F_g}{m} = G\frac{M}{R^2}$.

In this book (see the first chapter about Analogies), we know, based on analyzed electromechanical analogies, that ***mass itself should not be the real and only source of gravitation. Only an oscillatory mass or mass in motion, which has linear and orbital***

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

moments, is (analogically and still hypothetically) better fitted to present gravitational charge *(see later (2.11.13-6) - (2.11.13-8), and more complete explanations in chapter 10, equations (10.1.4) - (10.1.7)). Effectively similar or familiar conclusions (what real sources of gravity are) can be drawn from the number of works of Dr. Jovan Djuric, [33], [71] and [102]. Consequently, Newton gravitational force should have specific hidden or intrinsic, linear, and angular velocity parameters, embedded inside gravitational constant* ***G*** *(here, this unknown linear velocity parameter is* $v_0 = \text{const.}$*).*

We could now formulate another equivalent expression for Newton gravitational force, as being dependent of the product of relevant moments of attracting masses,

$$F_{m-M} = F_g = \frac{GmM}{R^2} = \frac{G}{v_0^2}\frac{(mv_0)\cdot(Mv_0)}{R^2} = \frac{G}{v_0^2}\frac{(p_m)\cdot(p_M)}{R^2} = K_G\frac{p_m \cdot p_M}{R^2},\ K_G = \frac{G}{v_0^2} = \text{const.}.$$

Based on such modified formulation of Newton force, and newly introduced gravitational charges p_m *and* p_M*, we can redefine an adjusted* ***<u>gravitational intensity (or gravitational field</u>*** *I* = E*g**)** as,*

$$I^* = \frac{F_g}{p_m} = \frac{K_G p_M}{R^2} = \frac{G}{v_0^2}\frac{p_M}{R^2} = \frac{G}{v_0^2}\frac{Mv_0}{R^2} = \frac{G}{v_0}\frac{M}{R^2} = \frac{I}{v_0}.$$

We see, after we compare the ordinary (traditional) definition of gravitational intensity I, and modified gravitational intensity I, that qualitatively nothing significant changed (since* $v_0 = \text{const.}$*), because we are the part of the same global rotation (or motion), but we also see that kind of hidden, and maybe universal or background velocity parameter has its influence on modified Gravitational Intensity.*

Consequently, if we respect analogic predictions (from the first chapter about analogies in Physics), we need to admit (still hypothetically) that real sources of gravitation are relevant and mutually coupled, linear and angular momenta. In such cases, Gravitational field intensity will be given by expressions (2.11.13-6) - (2.11.13-8).

The ***<u>gravitational potential</u>*** ***V*** *at a certain point in the gravitational field is traditionally defined as the work done in taking a unit mass* ***m*** *from that point to infinity against the force of relevant gravitational attraction. From such definition, we have,*

$$V(R) = \frac{\text{Energy}}{\text{unit mass}} = -\frac{1}{m}\int_r^\infty F dR = -\frac{1}{m}\int_r^\infty \frac{GMm}{R^2} dR = -\int_r^\infty \frac{GM}{R^2} dR = -\int_r^\infty I\, dR = -\frac{GM}{R} (=) \left[\frac{m^2}{s^2}\right].$$

If we now consider that real charges (or sources) of gravitation are masses with linear and/or orbital moments, as in (2.11.13-6) - (2.11.13-8), we can introduce ***<u>modified gravitational potential</u>*** *as,*

$$V^*(R) = \frac{\text{Energy}}{\text{unit moment}} = -\frac{1}{mv_0}\int_r^\infty F dR = -\frac{1}{mv_0}\int_r^\infty \frac{GMm}{R^2} dR = -\frac{1}{v_0}\int_r^\infty \frac{GM}{R^2} dR = -\int_r^\infty I^* dR = -\frac{G}{v_0}\frac{M}{R} (=) \left[\frac{m}{s}\right],$$

Practically, here we are recognizing that every (relatively small) mass in gravitation field or on orbit around a certain planet (or a big mass), including the situation when such smaller mass is touching a big mass, still has certain linear, orbit, or rotating speed (or intrinsically associated linear moment), because a big mass (or planet) is orbiting about its sun.

$$V^*(R) = \frac{E_{km}}{mv} = -G\frac{M}{2Rv} = -\frac{v}{2} = -\frac{R}{2}\omega_m = -\frac{G}{v_0}\frac{M}{R}, v_0 = 2\frac{G}{v}\frac{M}{R} = 2G\frac{M}{R^2\omega_m} = 2v$$

$$\left\{\begin{array}{l} E_{km} = \frac{R}{2}\left|F_g\right| = G\frac{mM}{2R} = G\frac{m_r m_c}{2R} = \frac{mv^2}{2} = \frac{J_m\omega_m^2}{2} \\ \left|F_g\right| = G\frac{mM}{R^2} = G\frac{m_r m_c}{R^2} = \frac{mv^2}{R} = \frac{J_m\omega_m^2}{R} = \frac{2E_{km}}{R} \end{array}\right\},$$

alternatively, if we consider that only angular moments are dominantly relevant, we will get,

$$V^*(R) = \frac{E_{km}}{J_m\omega_m} = \frac{1}{2}\omega_m, \left(E_{km} = \frac{J_m\omega_m^2}{2}\right)..$$

We can again see, when we compare ordinary (traditional) definition of gravitational potential, and modified gravitational potential, that nothing significantly changes, and that a relevant angular velocity (or angular moment) should be the most significant for hypothetically and analogically innovated expressions of gravitational field intensity and potential (see later the same resume in (2.11.13-6) - (2.11.13-8)).

*Now we can redefine **gravitational potential energy**. The work obtained in bringing a body from infinity to a point in the gravitational field is called the gravitational potential energy of the body at that point. Potential energy **U = E$_p$** is usually presented mathematically as,*

$$U = E_p = -F_g \cdot R = G\frac{mM}{R} = K_G\frac{p_m \cdot p_M}{R}.$$

As we can see, the traditional definition of gravitational potential energy is identical to the modified gravitational potential energy (and it is evident that introducing the new meaning of gravitational charges should not be a problem). The gravitational potential energy at infinity is assumed (to be) zero.

Here we also see that we similarly redefine gravitational intensity and potential as we do with electric charges and fields. This could be an intuitive or indicative argument in relation to relevant electromechanical analogies, in a direction that gravitation could be a specific hidden manifestation of electromagnetic forces, since rotating and spinning motions are producing electric charges separation (or electric dipoles) and spinning matter states are related to elementary magnets that are getting properly arranged during mentioned rotation.

*As the support to (2.11.13), it is convenient to mention that total mechanical energy (without rest-mass energy), $E_m = E_k + E_p$, for an object with mass **m**, in a closed circular orbit with radius **R**, in a central gravitational field around a body with mass **M,** such as a planet orbiting about local sun, is equal to the sum of its kinetic energy $E_k = \frac{1}{2}mv^2$, and its potential, or "positional" energy $E_p = -G\frac{mM}{R}$. The gravitational potential energy is defined as a negative value, equal to the kinetic energy that the object would gain by falling from an infinite distance to its current position. At considerable distances from the Sun, the object would have zero potential energy (since it would not have picked up any speed by falling). Objects close to the Sun have considerable (although negative) potential energies, corresponding to the speed they would gain by dropping a long way.*

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

$$E_m = E_k + E_p = \frac{1}{2}mv^2 - \left(G\frac{mM}{R^2}\right)\cdot R,\ F_c = F_g = m\frac{v^2}{R} = G\frac{mM}{R^2} \Rightarrow 2E_k = mv^2 = G\frac{mM}{R} \Rightarrow$$
$$\Rightarrow E_m = E_p + E_k = \frac{1}{2}G\frac{mM}{R} - G\frac{mM}{R} = -G\frac{mM}{2R} = \frac{1}{2}E_p,\ E_k = G\frac{mM}{2R} = -E_m = -\frac{1}{2}E_p\ .$$

Remarkably similar, or better to say identical result for E_m also holds for an elliptical orbit, after we generalize it by replacing the radius of orbit R by the relevant orbital semi-major axis, as usually applied for Newton's derivation of Kepler's third law. Now, such total mechanical energy is constant and has similar forms for circular and elliptical orbits ($E_m = -G\frac{mM}{2R} = \frac{1}{2}E_p$, where R is semi-major axis). In ideal circular orbits, since there, speed v is constant, kinetic, and potential energies are constant. In elliptical orbits, the kinetic and potential energy is not constant, but somewhat variable on the way that one is large when the other is small and vice versa. For elliptic orbits, where $0 < e < 1$ is the eccentricity of the elliptical orbit, the following equations can be derived:

T.2.7.

Energy type	**R = R$_{min.}$**	**R = R$_{max.}$**
Potential, E_p	$2E_m/(1-e)$	$2E_m/(1+e)$
Kinetic, E_k	$-E_m(1+e)/(1-e)$	$-E_m(1-e)/(1+e)$

We see that when orbit eccentricity $e = 0$, all latest results for elliptical orbits again correspond to a circular orbit result. The larger the eccentricity e, the larger the variation of the potential and kinetic energies during each period of orbital motion.

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*What is the meaning of the gravitation force and associated matter waves energy between two masses **m** and **M** can be briefly explained based on simplified two-body problem analysis? Here, we will use the same symbolic and meanings for associated parameters, as in (2.11.10) – (2.11.13). If we have two isolated, static (or standstill) masses, **m**, and **M**, in the same inertial, reference frame, where a distance between them is equal to R, we can say that the total energy, $E_{tot.}$, of such system and Newton force of gravitation F_g, between them, are,*

$$E_{tot.} = mc^2 + Mc^2 = (m+M)c^2 = m_c c^2,\ F_g = -G\frac{mM}{R^2}, m_c = m + M\,. \tag{2.11.13-1}$$

*If such masses are in the same reference frame and have specific mutually relative motion (where **m** has velocity $\vec{v}_1$ and **M** has velocity $\vec{v}_2$), the total energy of such two-body system and force of gravitation between them are,*

$$\begin{aligned} &E_{tot.} = mc^2 + \frac{1}{2}mv_1^2 + Mc^2 + \frac{1}{2}Mv_2^2 = \\ &= (m+M)c^2 + \frac{1}{2}(m+M)v_c^2 + \frac{1}{2}m_r v_r^2 = m_c c^2 + \frac{1}{2}m_c v_c^2 + \frac{1}{2}m_r v_r^2, \\ &F_g = -G\frac{mM}{R^2} = -G\frac{m_r m_c}{R^2},\ m_c = m + M,\ m_r = \frac{mM}{m+M},\ \vec{v}_r = \vec{v}_1 - \vec{v}_2, \vec{v}_c = \frac{m\vec{v}_1 + M\vec{v}_2}{m+M}. \end{aligned} \tag{2.11.13-2}$$

If, also, each of masses is self-spinning (has its spin moment), the same situation with the total energy (like elaborations around equations (2.5.1-4) - (2.5.1-7) from the same chapter) and force of gravitation will be,

$$\left\{\begin{array}{l} E_{tot.} = mc^2 + \frac{1}{2}mv_1^2 + \frac{1}{2}J_{s1}\omega_{s1}^2 + Mc^2 + \frac{1}{2}Mv_2^2 + \frac{1}{2}J_{s2}\omega_{s2}^2 = \\ = (m + \Delta m_s)c^2 + \frac{1}{2}(m + \Delta m_s)v_1^2 + (M + \Delta M_s)c^2 + \frac{1}{2}(M + \Delta M_s)v_2^2, \\ \Delta m_s c^2 = \frac{1}{2}J_{s1}\omega_{s1}^2, \ \Delta M_s c^2 = \frac{1}{2}J_{s2}\omega_{s2}^2 \end{array}\right\} \Rightarrow \quad (2.11.13\text{-}3)$$

$$\Rightarrow \left\{\begin{array}{l} E_{tot.} = m^*c^2 + \frac{1}{2}m^*v_1^2 + M^*c^2 + \frac{1}{2}M^*v_2^2 = m_c^*c^2 + \frac{1}{2}m_c^*v_c^2 + \frac{1}{2}m_r^*v_r^2 \\ m^* = m + \Delta m_s, \ M^* = M + \Delta M_s, \\ F_g = -G\frac{m^*M^*}{R^2} = -G\frac{m_r^*m_c^*}{R^2}, m_c^* = m^* + M^*, \ m_r^* = \frac{m^*M^*}{m^*+M^*}, \ \vec{v}_r = \vec{v}_1 - \vec{v}_2, \vec{v}_c = \frac{m^*\vec{v}_1 + M^*\vec{v}_2}{m^*+M^*} \end{array}\right\}.$$

Within one or the other option (for masses without, or with self-spinning), we are coming to the possibility to express reduced-mass kinetic energy $\frac{1}{2}m_r^*v_r^2 = E_r$ *as specific orbital, rotation energy* $\frac{1}{2}J_r^*\omega_r^2$ *of relative mass* m_r^* *about its center of mass* m_c^**, and this energy corresponds to the associated matter wave energy* $\tilde{E}$ *of relative mas in its (would or could be) orbital-like motion (as in (2.11.13)),*

$$E_{tot.} = m_c^*c^2 + \frac{1}{2}m_c^*v_c^2 + \frac{1}{2}m_r^*v_r^2 = m_c^*c^2 + \frac{1}{2}m_c^*v_c^2 + \frac{1}{2}J_r^*\omega_r^2, \ E_r = \frac{1}{2}m_r^*v_r^2 = \frac{1}{2}J_r^*\omega_r^2 = \tilde{E}. \quad (2.11.13\text{-}4)$$

Now, we will introduce the decisive, essential, questionable, and innovative assumption regarding gravitation. ***If we assume that rotating (orbital-like) motional energy*** $\frac{1}{2}J_r^*\omega_r^2$ ***, of the relative mass*** $m_r^* = \frac{m^*M^*}{m^*+M^*} = \mu$ ***, about*** $m_c^* = m^* + M^*$***, is equal to one half of the work (or energy) of Newton gravitational force*** $F_g \cdot R$ ***(between masses*** m^* ***and*** M^****, along the distance*** R***, necessary to unite masses*** m^* ***and*** M^****, as already seen in (2.11.13)), we will be able to develop, or reformulate the same (already known) form of the Kepler Third Law*** *(2.11.10), as follows,*

$$\left[\begin{array}{l} 2\tilde{E} = 2E_r = 2\cdot\frac{1}{2}J_r^*\omega_r^2 = 2\cdot\frac{1}{2}m_r^*v_r^2 = F_g\cdot R = G\frac{m^*M^*}{R^2}\cdot R = G\frac{m_r^*m_c^*}{R} = -E_p = -2E_m, \\ E_m = E_p + E_k = -G\frac{m^*M^*}{2R} = \frac{1}{2}E_p = -E_r, \ E_k = G\frac{m^*M^*}{2R} = -E_m = -\frac{1}{2}E_p = E_r \end{array}\right] \Rightarrow$$

$$\Rightarrow \left[\begin{array}{l} 2\cdot\frac{1}{2}v_r^2 = G\frac{m_c^*}{R} = 2\cdot\frac{1}{2}(\omega_r R)^2 = 2\cdot\frac{1}{2}(2\pi f_r R)^2 = 4\pi^2 f_r^2 R^2 = \frac{4\pi^2 R^2}{T_r^2}, \\ v_r = \omega_r R = 2\pi f_r R = \frac{2\pi R}{T_r} = \frac{2\pi R}{T}, \ f_r = f_m = \frac{1}{T} = \frac{1}{T_r} \end{array}\right] \Rightarrow \quad (2.11.13\text{-}5)$$

$$\Rightarrow T_r^2 = \frac{4\pi^2}{Gm_c^*}\cdot R^3 \Leftrightarrow T^2 = \frac{4\pi^2}{G(M+m)}\cdot R^3 \cong \frac{4\pi^2}{GM}R^3.$$

$$\tau = 2\pi\sqrt{\frac{m}{k}}\,a^{3/2}.$$

For the family of planets orbiting the sun, $k/m = GM$ where M is the mass of the sun. Thus, the period of a planet is proportional to the $3/2$ power of the semi-major axis of the planet's orbit; it does not depend on the mass of the planet or the eccentricity of the orbit. This is **Kepler's third law** of planetary motion.[4] See Fig. 1.11 and note that the slope of the log-log plot is $3/2$.

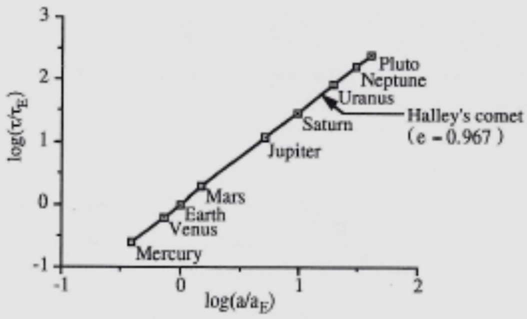

Fig. 1.11. Kepler's third law

Kepler's second law, plus some geometry, can also give the way the particle moves around the orbit as a function of time.[5] However, for modern readers more familiar with calculus than with geometry, it is probably easier to obtain this by integrating the equation for radial motion

The picture was taken from Lagrangian and Hamiltonian Mechanics, M. G. Calkin
ISBN: 978-981-02-2672-5

With (2.11.13-5), (2.4-13), (2.4-5.1) and with many familiar elaborations in Chapter 4.1, we are supporting and defending the concept of the real existence of planetary, macro matter waves, as introduced in (2.11.12) and (2.11.13). ***We also see that natural inertial, meaning orbital motions and self-closed standing matter-waves structures are appearing coincidently.*** *Different quantizing (or integer dependent) formulas are also the consequence of standing waves formations. Later, we will collect more arguments in the direction that gravitation-related matter waves (and gravitation) are most probably the consequences and effects of fundamentally electromagnetic, electromechanical, electro and magnetostrictive background.*

From widely elaborated electromechanical analogies, (see chapter 1), and from common definitions for the electric (and magnetic) field, we can analogically speculate about a new meaning for the field of gravitation, where real sources of gravity will be involved angular and linear moments (and associated electromagnetic charges and fluxes), instead of masses. *For instance, the gravitational force between masses m and M, where m is orbiting about M, can be formulated as,*

$$\left|F_g\right| = G\frac{mM}{R^2} = G\frac{m_r m_c}{R^2} = \frac{mv^2}{R} = \frac{J_m\omega_m^2}{R} = \frac{2E_{km}}{R}, \left(v = v_m = \omega_m R\right),$$
$$v_m = \sqrt{G\frac{M}{R}} = \omega_m R,\ \omega_m = \sqrt{G\frac{M}{R^3}}, \qquad (2.11.13\text{-}6)$$
$$v_r = \sqrt{G\frac{m_c}{R}} = \omega_{mr} R,\ \omega_{mr} = \sqrt{G\frac{m_c}{R^3}}, \frac{v_m}{v_r} = \frac{\omega_m}{\omega_{mr}}.$$

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

Using the analogical field-intensity definition, where a field is developed from its force divided by its source-charge, and if the relevant gravitational charge is either linear or angular momentum, (not a mass, like in traditional Newtonian gravitation), we will have,

$$E_g = \left\{ \begin{array}{l} \dfrac{F_g}{mv} \\ \text{or} \\ \dfrac{F_g}{J_m \omega_m} \end{array} \right\} = \left\{ \begin{array}{l} \dfrac{mv^2}{mvR} \\ \text{or} \\ \dfrac{J_m \omega_m^2}{J_m \omega_m R} \end{array} \right\} = \left\{ \begin{array}{l} \dfrac{v}{R} = \omega_m \\ \text{or} \\ \dfrac{1}{R}\omega_m \end{array} \right\}. \qquad (2.11.13\text{-}7)$$

*In both cases of (*2.11.13-7*), here-modified field of gravitation E_g is directly proportional to the proper angular (or orbital) velocity ω_m (as we see in (2.11.13-7)).*

*The standard and traditional definition of the gravitational field (where only a mass is the source of gravitation) is much different from (*2.11.13-7), for example,

$$E_g = \frac{F_g}{m} = G\frac{M}{R^2} = \frac{v^2}{R} = \omega_m^2 R. \qquad (2.11.13\text{-}8)$$

Consequently, here we are on some intuitive and brainstorming (or hypothetical) way generating conclusions that the most relevant gravitation-related sources should be angular, orbital, and spinning moments and relevant derivatives as linear forces and torques as angular forces (see more in chapter 4.1). This could be additionally supported if we try to specify what is common, for both the micro-world of atoms and subatomic entities, and the macro-world of planetary systems and galaxies. The typical common items are micro and macro systems, and events with rotations, spinning states, and orbital motions, all of them characterized by angular moments and dimensionally or directly proportional to certain relevant angular velocity, very much similar as we see in the new definition of the gravitational field (2.11.13-7). In Chapter 1. of this book (about analogies), we also find that relevant sources of gravitation (based on analogical conclusions) are not masses, but linear and orbital moments, electric charges and magnetic moments and fluxes (related to rotation and spinning). This is similar or at least sufficiently common theoretical platform about the dominant place of rotation and spinning in our universe, as we can find in publications under [36], Anthony D. Osborne & N. Vivian Pope.

Practically and briefly summarizing, we could say that:

1° All-natural motions in our universe are curved, and
2° All stable, stationary, periodical, and inertial motions should be orbital motions.
3° Also, our Universe is, globally, intrinsically, and holistically rotating (oscillating and resonating) on all micro and macro structural levels.

Consequently, only angular, or orbital and spinning moments should be the most relevant regarding phenomenology we understand and describe under gravitation. Since spinning, rotation and orbiting are very often coupled with associated magnetic fields and moments, here is the place for understanding electromagnetic background nature of gravity.

Until here, we mostly analyzed somewhat static and simple-motion situations between two (electromagnetically neutral, and internally balanced) masses, but similar conclusions can be drawn from the general two-body problem. For instance, the same energy balance and

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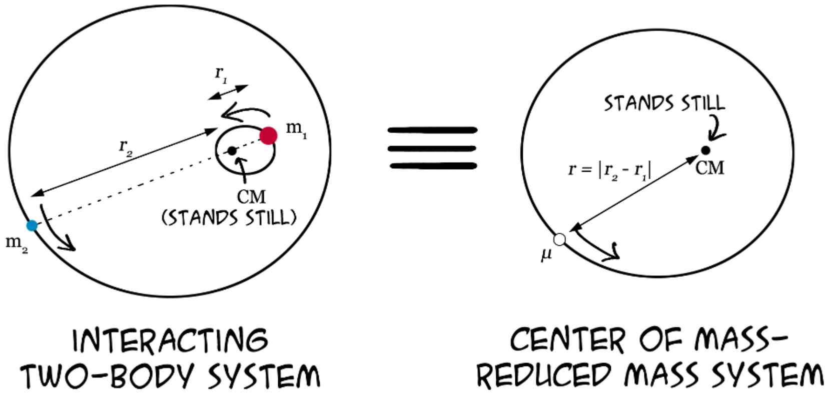
gravitational attraction between two bodies, as given with (2.11.13-2) and (2.11.13-6), is also applicable in cases of motional masses, when we transform such a two-body system to an equivalent orbiting motion of the reduced mass around its central mass. ***The more appropriate conceptualization here will be to treat the two-body problem as an impact situation and to search for evolving angular and rotating elements in such mutually interacting motions.*** *In other words, each two-body motional system is generating elements of angular, or somewhat circular (spiral and orbital) accelerated movements, what we can see when we play with different, mutually related reference frames, as presented with (2.11.13-9), and on the picture, below.*



Taken from: *https://quantumredpill.files.wordpress.com/2013/01/two-body-cm-systems.png*

Mentioned two or multibody systems with evolving elements of orbital and circular motions are naturally creating matter-waves. If involved masses are also electromagnetically charged, and have spinning moments, mentioned revolving, circular, and resulting spinning motions will be intensified. In addition, if electromagnetically neutral or non-charged masses are getting closer, because of specified self-generated and evolving elements of angular and rotational movements, we can naturally expect to get internal electromagnetic dipoles polarizations (inside of masses, because masses are composed of molecules, atoms, electrons, protons, and neutrons, all of them are on some way oscillating, rotating, and spinning). This way, certain kinds of electric and magnetic dipoles-related internal currents and magnetic fields will be created (since masses m and M are also performing linear and circular motions, depending on the point of view, what is influencing and stimulating spontaneous, but organized electromagnetic dipoles polarization). Mentioned internal electromagnetic dipoles-related currents and fluxes are effectively creating a spatial situation like parallel wires with electric currents passing in the same direction, this way magnetically attracting each other, what we detect as gravitation. At the same time, because of periodical, spatial and temporal motions of involved masses and because of associated elements of angular, rotational, spinning and helical movements, matter waves will be naturally created.

Two-body energy balance and involved gravitational force with elements of linear and circular motions can be summarized as,

$$E_{tot} \cong mc^2 + \frac{1}{2}mv_m^2 + Mc^2 + \frac{1}{2}Mv_M^2 = m_c c^2 + \frac{1}{2}m_c v_c^2 + \frac{1}{2}m_r v_r^2 =$$

$$= \boxed{m_c c^2 + \frac{1}{2}J_m\omega_m^2 + \frac{1}{2}J_M\omega_M^2 = m_c c^2 + \frac{1}{2}J_c\omega_c^2 + \frac{1}{2}J_r\omega_r^2},$$

$$\left|F_g\right| = G\frac{mM}{R^2} = G\frac{m_r m_c}{R^2} = \frac{mv^2}{R}\left(= \frac{m_r v_r^2}{R}\right) = \frac{2E_{km}}{R} = \boxed{\frac{J_m\omega_m^2}{R}},$$

$$\left(\begin{array}{l} E_{km} = \frac{1}{2}mv_m^2 = \frac{1}{2}J_m\omega_m^2 = \frac{R\left|F_g\right|}{2} = G\frac{mM}{2R} = G\frac{m_r m_c}{2R},\ v = v_m = \omega_m \cdot R \cong 2u, \\ \Psi_m^2(t)dt = d\tilde{E}_m = dE_{km} = v_m dp_m = d\left(\frac{1}{2}mv_m^2\right) = mv_m dv_m = p_m dv_m = \\ = \omega_m d\left(J_m\omega_m\right) = \left(J_m\omega_m\right)d\omega_m,\ \Delta\Psi_m(x,t) - \frac{1}{u_m^2}\frac{\partial^2\Psi_m(x,t)}{\partial t^2} = 0,\ u_m = \lambda_m f_m = v_m/2 \end{array} \right), \quad (2.11.13\text{-}9)$$

$$\omega_m = 2\pi f_m,\ n\lambda_m = 2\pi R,\ \vec{v}_c = \frac{m\vec{v}_m + M\vec{v}_M}{m+M} = K\frac{J_m\vec{\omega}_m + J_M\vec{\omega}_M}{J_m + J_M} = K\vec{\omega}_c,\ K = Const.$$

See much more in chapter 4.1, where similar problems of helical matter waves are additionally elaborated. In addition, if mutually approaching and interacting particles or masses already have spinning and electromagnetic moments and charges, and if specific spontaneous electromagnetic dipoles polarization is produced, beside gravitational forces and effects, we will need to account presence of Coulomb force interactions, indicating that all mentioned forces and fields essentially have an electromagnetic origin or background.

Now is the right place to mention an analogy between here-introduced concepts (of couplings and equivalency between linear and angular motions) as already presented in this chapter and later in chapter 4.1 (on illustrations on Fig.4.1, Fig.4.1.2, Fig.4.1.3, Fig.4.1.4, Fig.4.1.5, and within equations under (4.3) and later).

Citation took from the Internet: *http://www.school-for-champions.com/science/gravitation_orbit_center_of_mass_derivation.htm#.V6SSvKL4i6E*

Derivation of Circular Orbits Around Center of Mass

by Ron Kurtus (revised 14 May 2011)

Circular orbits of two objects around the center of mass (CM) between them require tangential velocities that equalize the gravitational attraction between the objects.

Tangential velocities tend to keep the objects traveling in a straight line, according to the Law of Inertia. If gravitation cases an inward deviation from straight-line travel, the result is an outward centrifugal force. By setting the gravitational force equal to the centrifugal forces, you can derive the required tangential velocities for circular orbits.

The orbit equations can be in simplified forms when the masses of the two objects are the same and when the mass of one object is much greater than that of the other.

Questions you may have included:

- *What are the factors involved in derivation?*
- *What are the equations for the velocities of objects?*
- *What happens when one object is much larger than the other is?*

This lesson will answer those questions. Useful tool: Units Conversion

Factors in determining orbital velocities

The linear tangential velocities required for two objects to be in circular orbits around the center of mass (CM) between them is found by comparing their gravitational force of attraction with the outward centrifugal force for each object.

***Note**: A linear tangential velocity is a straight-line velocity is perpendicular to the axis between the two objects. It is tangent to the curved path and is different from rotational speed.*

(See Center of Mass and Tangential Gravitational Motion for more information.)

Assume no initial radial velocities

When two objects in space are traveling toward the general vicinity of each other, they both have radial and tangential velocities concerning the center of mass (CM) between them. However, to simplify the derivation of circular orbits, we will only look at the case where there are no inward or outward radial velocities and be concerned about the tangential velocities.

This is like the case of Newton's cannonball going into orbit or sending a satellite into orbit around the Earth.

(See Gravity and Newton's Cannon for more information.)

Since there is no radial motion, a separation between the objects remains constant, which is a requirement for circular orbits.

The gravitational force of attraction

The gravitational force of attraction between two objects is:

$$F = GMm/R^2$$

where

- ***F** is the force of attraction between two objects in newtons (N)*
- ***G** is the Universal Gravitational Constant = 6.674×10^{-17} N-km^2/kg^2*
- ***M** and **m** are the masses of the two objects in kilograms (kg)*
- ***R** is the separation in kilometers (km) between the objects, as measured from their centers of mass*

***Note**: Since force is usually stated in newtons, but motion between astronomical bodies is usually stated in km/s, an adjusted value for **G** is used, with N-km^2/kg^2 as the unit instead of N-m^2/kg^2. **G** is also sometimes stated as 6.674×10^{-20} km^3/kg-s^2.*

(See Universal Gravitation Equation for more information.)

Separation of objects

*As the objects orbit the CM, their total separation, **R**, remains constant. The individual separations between the objects and CM are also constant and determined by **R** and their masses:*

$R = R_M + R_m$

where

- ***R_M*** *is the separation between the center of object **M** and the CM in km*
- ***R_m*** *is the separation between the center of object **m** and the CM in km*

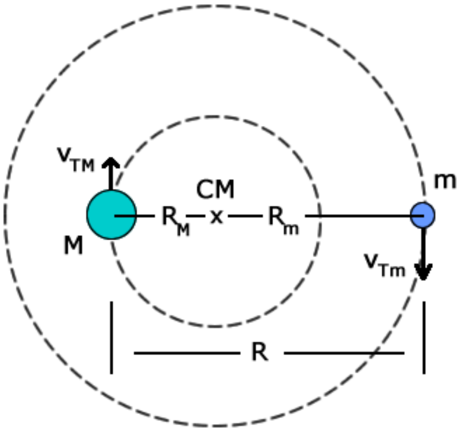
*The values of **R_M** and **R_m** are according to the equations:*

$R_M = mR/(M + m)$

$R_m = MR/(M + m)$

(See Center of Mass Definitions for more information.)

The factors involved can be seen in the illustration below:



Factors in objects orbiting CM

***Note**: Although the Earth orbits the Sun in a counterclockwise direction, we usually indicate motion in a clockwise direction.*

(See Direction Convention for Gravitational Motion for more information.)

Centrifugal force

The centrifugal inertial force on each object relates to its circle of travel:

$F_M = Mv_{TM}^2/R_M$,

$F_m = mv_{Tm}^2/R_m$,

where,

- ***F_M*** *is the centrifugal inertial force on mass **M***
- ***v_{TM}*** *is the tangential velocity of mass **M***
- ***F_m*** *is the centrifugal inertial force on mass **m***
- ***v_{Tm}*** *is the tangential velocity of mass **m***

***Note**: Centrifugal force is caused by inertia and is not considered a "true" force. It is sometimes called a pseudo- or virtual force.*

Substituting $R_M = mR/(M + m)$ *and* $R_m = MR/(M + m)$ *in the above equations gives you:*

$F_M = Mv_{TM}^2(M + m)/mR$

$F_m = mv_{Tm}^2(M + m)/MR$

Solve for individual velocities

Since the centrifugal force equals the gravitational force for a circular orbit, you can solve for the velocity.

The object with mass m

*In the case of the object with mass **m**:*

$F_m = F$

Substitute equations:

$mv_{Tm}^2(M + m)/MR = GMm/R^2$

*Multiply both sides by **MR** and divide by **m**:*

$v_{Tm}^2(M + m) = GM^2/R$

*Divide both sides by **(M + m)**:*

$v_{Tm}^2 = GM^2/R(M + m)$

Take the square root:

$v_{Tm} = \pm\sqrt{[GM^2/R(M + m)]}$

This means the velocity can be in either direction for a circular orbit. Since direction is not relevant here:

$v_{Tm} = \sqrt{[GM^2/R(M + m)]}$ *km/s*

The object with mass M

*Likewise, for the object of mass **M**:*

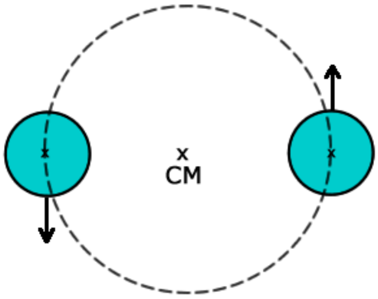
$v_{TM} = \sqrt{[Gm^2/R(M + m)]}$ *km/s*

Sizes of objects

The equations for the tangential orbital velocities can be simplified when both objects are the same size, as well as when one object has a much greater mass than the other does.

Objects have the same mass

There are situations in space where two stars have close to the same mass and orbit the CM between them. Astronomers call them double stars.



Double stars follow the same orbit around CM

*If the objects are the same mass, then **M = m** and the velocity equation for each becomes:*

$v_{TM} = \sqrt{[Gm^2/R(m + m)]}$ km/s

The equation reduces to,

$v_{TM} = \sqrt{[Gm/2R]}$ km/s

Since both objects or stars have the same orbital velocity and the same separation from the CM, they follow the same orbit around the CM.

One object is much more massive than the other

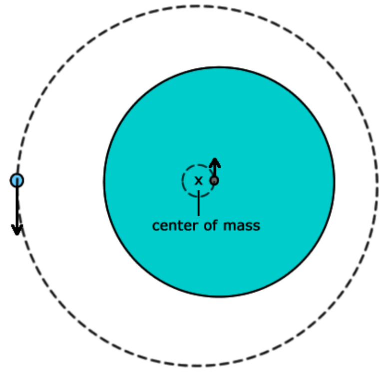
Another situation often seen in space is when one object is much larger than the other is. In this case, the CM between them is almost at the more massive object's geometric center. This results in simplifying the equation for orbital velocity. The small object then seems to orbit the more massive object.

For example, the CM between a satellite orbiting the Earth is near the geometric center of the Earth. Likewise, the CM between the Earth and the Sun is near the center of the Sun.

*Suppose **M >> m** (**M** is much greater than **m**). Then:*

$M + m \approx M$

where ≈ means "approximately equal to".



Orbits, when one object is much larger than other, is

*Substitute **M + m ≈ M** into the equation for the velocity of the smaller object:*

$v_{Tm} = \sqrt{[GM^2/R(M + m)]}$

$v_{Tm} = \sqrt{(GM^2/RM)}$

Reducing the equation results in:

$v_{Tm} = \sqrt{(GM/R)}$ *km/s*

This is the same as the standard equation for the orbital velocity of one object around another.

(See Orbital Motion Relative to Other Object for more information.)

Summary

When two objects are moving at the correct tangential velocities, they will go in circular orbits around their CM. The velocity equations are determined by setting the gravitational force equal to the outward centrifugal forces caused by their tangential velocities.

The velocity equations are:

$v_{Tm} = \sqrt{[GM^2/R(M + m)]}$ *km/s*

$v_{TM} = \sqrt{[Gm^2/R(M + m)]}$ *km/s*

When the mass of each object is the same, the velocity equation is simplified. The same is true when the mass of one object is much greater than that of the other.

To illustrate (or approximate on a highly speculative way) the meaning of global, universal, cosmic, or holistic rotation (which could be hidden or invisible for us, but we suppose that it effectively exists and produces gravitational force), let us imagine that small mass m is sitting on a big mass M, being attracted by mutual gravitational force. For instance, M could be certain planet and m will be a small spherical object, where the following approximations are applicable: M >>> m, R >>> r, d <<< R_0. Here, d is the distance between centers of M and m, and R_0 is the distance from certain distant and common, dominant, or significant center C of

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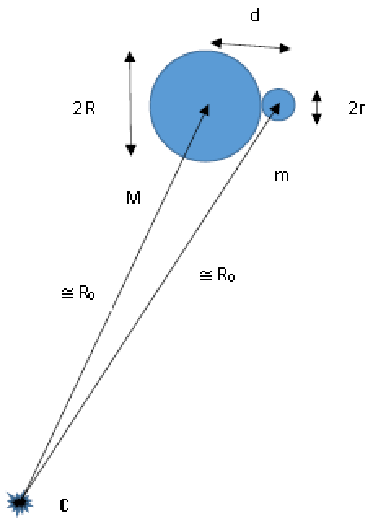
global or holistic (universal) rotation (see the picture below). We do not know where exactly the center of universal cosmic rotation is to place our reference system there, but mathematically we could operate with such an imaginative (and still hypothetical) center of rotation.



Both masses m and M, are effectively rotating synchronously (or coincidently) around their global and universal cosmic center of rotation, here marked with C. Both masses also have the same angular or revolving frequency around center C. Such (in some way) hidden, and mathematically effective rotation could also be specific complicated angular motion locally presentable as rotation (since center C should not always be center of mass of the relevant local, planetary, or solar system). Between masses, M, and m, we can safely say that it exists an attractive gravitational force, as follows,

$$\left|F_g\right| = G\frac{mM}{d^2} = \frac{mv_0^2}{R_0} = \frac{2E_{km}}{R_0} = \frac{J_m\omega_m^2}{R_0} = \frac{J_m\omega_0^2}{R_0} = m\,\omega_0^2\,R_0,\ \omega_M = \omega_m = \omega_0 = \frac{v_0}{R_0}.$$

If we now assume that gravitational force between m and M is the consequence of global, universal rotation (about center C), where relevant orbital moments are dominant factors (instead of involved masses), we will have,

$$G\frac{mM}{d^2} = G_1\frac{J_m\omega_m \cdot J_M\omega_M}{d^2} = G_1\frac{J_mJ_M\omega_0^2}{d^2} = G_1\frac{mR_0^2 \cdot MR_0^2}{d^2} = G_1\frac{mMR_0^4}{d^2}$$
$$= \frac{mv_0^2}{R_0} = \frac{J_m\omega_m^2}{R_0} = \frac{J_m\omega_0^2}{R_0} = m\,\omega_0^2\,R_0,\ \omega_M = \omega_m = \omega_0 = \frac{v_0}{R_0} \Rightarrow$$
$$\Rightarrow R_0 = \frac{GM}{d^2\omega_0^2}, G_1 = \frac{G}{R_0^4}...$$

We could also speculate that universal cosmic rotation (probably combined with linear, spinning, and helical motion) is the primary cause of internal electrostatic and magnetic polarizations of involved masses, this way giving grounds to explain gravitational attraction as an electromagnetic dipoles' attraction. This example is just a brainstorming draft of a future, more elaborate modeling. ♣]

The magnitude of the angular momentum $\mathbb{L}$ from (2.11.13), of a periodically orbiting planet or satellite, its relevant, orbital mean-radius $\mathbf{R}$ (or semi-major axis), and

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associated characteristic speeds are quantified. Such orbital quantization is based on a planet-associated resonant and standing matter-waves (having a particular group and phase velocity, like in any periodical wave motion), respecting (in average) simple geometrical fittings (such as $2\pi R = n\lambda_o$), and can be summarized as results shown in (2.11.14), below.

Similar concepts and results are presented in [43], M. Pitkänen; [38], [39], F. Florentin Smarandache and Vic Christianto; [40], D. Da Rocha and Laurent Nottale; [64], Marçal de Oliveira Neto; and in **[125],** Markus J. Aschwanden; (see in (2.11.14)).

Here is a place to underline that nobody of the authors mentioned is interpreting such results in a direct and robust relation to group and phase velocity of associated matter-waves groups, or wave-packets obtained by superposition of significant number of mutually agglomerated harmonic, elementary waves, like in the micro-world physics (what is an innovative contribution in this book).

$$
\left[L = pR = mvR = mR^2\omega_m = mR^2\frac{2\pi}{T} = 2\pi mR^2 f_m = m\sqrt{GMR} = n\frac{H}{2\pi} = n\hbar_{gr.} = \text{const},\ m \cong \mu = \frac{mM}{m+M}\right] \Rightarrow
$$

$$
\Rightarrow \left\{ \begin{array}{l}
\left[\begin{array}{l}
R = R_n = n^2\dfrac{H^2}{4\pi^2 GMm^2} = n^2\dfrac{GM}{v_0^2} = \dfrac{GM}{v_n^2} = n\dfrac{\lambda_o}{2\pi} = n\dfrac{\hbar_{gr.}}{mv},\ \lambda_o = \dfrac{H}{mv} = \dfrac{H}{p} = \dfrac{2\pi R}{n} = \lambda_{on}, \\
T = \dfrac{2\pi R}{v} = \dfrac{1}{f_m} = \dfrac{nT_0}{2} = \dfrac{n}{2f_o} = \dfrac{(nH)^3}{4\pi^2 G^2 M^2 m^3} = T_m,\ v = u - \lambda_o\dfrac{du}{d\lambda_o} = -\lambda_0^2\dfrac{df_0}{d\lambda_0} \cong 2u = 2u_n, \\
f_o = n\dfrac{f_m}{2} = n\dfrac{1}{2T} = n\dfrac{\sqrt{GM}}{4\pi R^{3/2}} = f_{on},\ T_o = \dfrac{1}{f_o} = \dfrac{2T}{n} = \dfrac{4\pi R}{nv},\ E_k = \tilde{E} = \tilde{E}_n = Hf_o = n\dfrac{Hf_m}{2}, \\
v = v_n = \dfrac{v_0}{n} = \dfrac{2\pi}{nH}GMm = 2u = \dfrac{v_e}{\sqrt{2}} = \sqrt{\dfrac{GM}{R_n}},\ \ u = u_n = \lambda_o f_o = \dfrac{1}{2}\sqrt{\dfrac{GM}{R_n}} = \dfrac{v_0}{2n} \cong \dfrac{v}{2}, \\
v_0 = nv_n = \dfrac{2\pi}{H}GMm = \dfrac{GMm}{\hbar_{gr.}} = 2un = \dfrac{nv_e}{\sqrt{2}} = n\sqrt{\dfrac{GM}{R_n}},\ \ n, n_i = \text{Integers}],\ v << c.
\end{array} \right] \Rightarrow \\
\Rightarrow \left[\begin{array}{l}
\left\{ \begin{array}{l}
\text{For two planets on orbits 1 and 2: } H = \text{const.} \Rightarrow \\
\dfrac{R_1}{R_2} = \dfrac{n_1^2}{n_2^2}\dfrac{m_2^2}{m_1^2} = \left(\dfrac{n_1 m_2}{n_2 m_1}\right)^2, \dfrac{T_1}{T_2} = \left(\dfrac{n_1 m_2}{n_2 m_1}\right)^3 = \left(\dfrac{R_1}{R_2}\right)^{3/2} = \dfrac{v_2}{v_1}\cdot\dfrac{R_1}{R_2} = \dfrac{T_{1m}}{T_{2m}} = \dfrac{n_1}{n_2}\cdot\dfrac{f_{o2}}{f_{o1}} = \dfrac{n_1}{n_2}\cdot\dfrac{T_{o1}}{T_{o2}} \\
\dfrac{v_1}{v_2} = \dfrac{u_1}{u_2} = \dfrac{\lambda_{o1}f_{o1}}{\lambda_{o2}f_{o2}} = \dfrac{n_2}{n_1}\cdot\dfrac{m_1}{m_2} = \sqrt{\dfrac{R_2}{R_1}} \Leftrightarrow \underline{R_i v_i^2 = R_j v_j^2},\ \dfrac{\lambda_{o1}}{\lambda_{o2}} = \dfrac{n_2}{n_1}\cdot\dfrac{R_1}{R_2}
\end{array} \right\} \\
-------------------- \\
\left\{ \begin{array}{l}
\text{For the same planet passing between two orbits: } H = \text{const.} \Rightarrow \\
(m_1 \cong m_2), \dfrac{R_1}{R_2} = \left(\dfrac{n_1}{n_2}\right)^2, \dfrac{T_1}{T_2} = \left(\dfrac{n_1}{n_2}\right)^3
\end{array} \right\}
\end{array} \right]
\end{array} \right\} \quad (2.11.14)
$$

Effectively, sitting on results and assumptions from (2.11.14), Titus – Bede's law (related to quantization of planetary orbits) is significantly rectified and optimized by Markus J. Aschwanden; **[125]** - "Self-organizing systems in planetary physics: Harmonic resonances of planet and moon orbits". The mentioned reference, **[125]**, is strongly supporting and reinforcing here-elaborated model of planetary, standing matter-waves, and additionally giving more legitimacy to the corrected and upgraded Titus-Bode law.

[♣ COMMENTS & FREE-THINKING CORNER:

The same or equivalent, quantizing-like results, and conclusions (as in (2.11.14)) can be formulated almost directly, analogically, and much faster if we consider that in certain solar system, the Sun analogically presents a proton, and planets are like electrons orbiting around. Quantizing is applicable if the system can be treated as a *2-body problem* (see more about 2-body situations and direct proportionality between certain mass and its electromagnetic charges in Chapter 4.1). If we exploit the mathematical identity between the electrostatic Coulomb force in the hydrogen atom, and Newton's "static" gravitational force, and systematically substitute $\frac{Ze^2}{4\pi\varepsilon_0} \to GmM$ (or $Ze^2 \leftrightarrow mM$, $\frac{1}{4\pi\varepsilon_0} \leftrightarrow G$) in all relevant results known from hydrogen atom analyzes, where M is mass of the sun, m is mass of certain planet, H is macrocosmic planetary constant analog to Planck constant h, ($H >>> h$) and G is gravitational constant. See much more of such background in [63] Arbab I. Arbab, and in [67], including other familiar publications from Johan Hansson [77], Newtonian Quantum Gravity. "*Gravito-static versus electrostatic analogy*" should not be only a mathematical curiosity, coincidence, and academic discussion option, after we consider as realistic the possibility that solar system elements are mutually electrically (and magnetically) polarized like mutually attracting electric (and magnetic) dipoles and multi-poles (since there are electromagnetic fields and forces around them). Such electromagnetic polarization option is already presented in this chapter (see ***2.2. Generalized Coulomb-Newton Force Laws***; -equations from (2.3) until (2.4-10)). In addition, Chapter 8. of this book (Bohr Model) develops and presents most of analogical, quantized results (see results from (8.23) to (8.33)), as found in (2.11.14), where mutual correspondence and full analogy of such results can be established by applying "*Gravito-static versus electrostatic analogy*". Of course, here analogy means more than mathematical similarity or identity, since in the case of gravitation within planetary systems relevant results are also correct, verified by astronomic measurements, and other theoretical and experimental observations. Consequently, here we deal with accurate empirical, natural, and scientific facts and the main consequence should be that gravitation and associated electromagnetic complexity are coincidently present and mutually coupled, at least in cases of solar or planetary systems (see also elaborations around equations (2.11.20) - (2.11.22) and Fig.2.6.). *See also supporting arguments in Chapter 4.1., under "4.1.2. Matter Waves Unity and Complementarity of Linear, Angular and Fluid Motions". The direct proportionality (and some kind of formal analogy) between electric charges and their relevant mases is also elaborated in Chapter 1., regarding Analogies, under (1.14) – (1.16), in Chapter 2., regarding Gravitation, within equations (2.4-4.1) – (2.4-4.3), (2.4-4) – (2.4-8), and the table "T.2.8. N. Bohr hydrogen atom and planetary system analogies", and in Chapter 3. under (3.5-a). All that gives us chances to analogically treat two-body mechanical systems, as two-body electric, or electromagnetic systems (when electromagnetic charges are significantly involved), profiting from already well-developed analyses of two-body mechanical systems.*

Let us directly apply analogical substitutions of quantized expressions relevant for orbiting electrons, and their associated standing matter waves (as shown in [77], Johan Hansson, Newtonian Quantum Gravity, in results from the Chapter 8., regarding Bohr atom model, and in (2.4-8)), and to results (8.4), and (8.26) - (8.30). This way, we will create analogical and quantized, standing-waves formulations as summarized in T.2.8., comparable to results from (2.11.14), valid and correct for orbiting planets (including planetary macro matter waves),

T.2.8. N. Bohr hydrogen atom and planetary system analogies

We will apply the following analogies and formal replacements:

$$\left\{\frac{1}{4\pi\varepsilon_0}\frac{q\cdot Q}{r^2}=\frac{1}{4\pi\varepsilon_0}\frac{(ze)\cdot(Ze)}{r^2}=\frac{1}{4\pi\varepsilon_0}\frac{zZ\cdot e^2}{r^2}\Leftrightarrow G\frac{m\cdot M}{r^2}\right\}\Rightarrow$$

$$\left[\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2},\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{r^2}\right]\Leftrightarrow\left[G\frac{m^2}{r^2},G\frac{M^2}{r^2}\right], q=ze\Leftrightarrow m\sqrt{4\pi\varepsilon_0 G},\ Q\Leftrightarrow Ze=M\sqrt{4\pi\varepsilon_0 G},$$

$$zZe^2\Leftrightarrow mM4\pi\varepsilon_0 G,\ e^2\Leftrightarrow\frac{mM}{zZ}4\pi\varepsilon_0 G,\ e\sqrt{zZ}\Leftrightarrow\sqrt{mM}\sqrt{4\pi\varepsilon_0 G},\frac{1}{4\pi\varepsilon_0}\leftrightarrow G,\ h\leftrightarrow H,\quad\Rightarrow$$

$$\left[\frac{e}{\sqrt{4\pi\varepsilon_0 G}}\sqrt{\frac{zZ}{m+M}}\Leftrightarrow\sqrt{\frac{mM}{m+M}}=\sqrt{\mu}\right]\Rightarrow\frac{e}{\sqrt{4\pi\varepsilon_0}}\sqrt{\frac{zZ}{m+M}}\Leftrightarrow\sqrt{\frac{mM}{m+M}}\cdot\sqrt{G}=\sqrt{\mu}\cdot\sqrt{G}$$

$$\frac{ze}{\sqrt{4\pi\varepsilon_0}}\Leftrightarrow m\cdot\sqrt{G},\ \frac{Ze}{\sqrt{4\pi\varepsilon_0}}\Leftrightarrow M\cdot\sqrt{G},\ \mu=\frac{mM}{m+M}$$

$$\Rightarrow\left[\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2}\Leftrightarrow G\frac{m\cdot M}{r^2}\right]\Rightarrow\left[\frac{Ze^2}{4\pi\varepsilon_0}=\frac{e}{\sqrt{4\pi\varepsilon_0}}\cdot\frac{Ze}{\sqrt{4\pi\varepsilon_0}}\Leftrightarrow m\sqrt{G}\cdot M\sqrt{G}\right]\Rightarrow$$

$$\left[\frac{e}{\sqrt{4\pi\varepsilon_0}}\Leftrightarrow m\sqrt{G},\ \frac{Ze}{\sqrt{4\pi\varepsilon_0}}\Leftrightarrow M\sqrt{G}\right]\Rightarrow Z\Leftrightarrow\frac{M}{m};\ \frac{1}{4\pi\varepsilon_0}\leftrightarrow G,\ h\leftrightarrow H,\ \frac{Ze^2}{4\pi\varepsilon_0}\Leftrightarrow Gm\cdot M$$

Effectively, here we assume that every mass or agglomerated atoms have the corresponding amount of polarizable electric dipoles or electric charges in a way that Coulomb and Newton's laws are mutually equivalent or replaceable (see (2.4-4.1) - (2.4-4.3) developed earlier in this second Chapter). This way, masses attraction can be treated as adequately oriented electric dipoles attraction (or by analogically extending the same conceptualization to associated magnetic dipoles attraction, since in such dynamically stable and self-closed systems electric and magnetic performances are mutually balanced like in capacitance-inductance or mass-spring resonant circuits). Here we also assume the existence of some omnipresent, holistic angular cosmic motion (including oscillatory and resonant states), being the ultimate cause or source of mentioned intrinsic and volumetric masses, electromagnetic polarization. In case if our Universe has more dimensions than 4 [more than (x, y, z, t)], we could imagine that mentioned holistic motion would be recognizable from, for us still not detectible, higher dimensional spaces. In some distant parts of our universe (for instance concerning spiral galactic formations...), such universal angular motion, and associated electromagnetic polarization could produce stronger Coulomb or Newtonian attractions (than in our part of Cosmos), and we maybe wrongly associate improvable "dark mass and dark energy" mystifications to such very much clear, natural, and explicable phenomenology (see [121], Raymond HV Gallucci).

Analogically created relations between N. Bohr atom model and planetary systems are, as follows:

the phase velocity of an electron wave	the phase velocity of a planetary wave
$u_s\approx\frac{1}{2}v=\frac{Ze^2}{4nh\varepsilon_0}=\lambda_s f_s,\ n=1,2,3...$ (8.26)	$u_n\approx\frac{1}{2}v=\frac{\pi GmM}{nH}=\lambda_n f_n, n=1,2,3,...$ ($u_n=\lambda_{on}f_{on}=\frac{1}{2}\sqrt{\frac{GM}{R_n}}=\frac{v_0}{2n}\cong\frac{v}{2}$, (2.11.14))

All over this book are scattered small comments placed inside the squared brackets, such as: [♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

Download the last version here:
http://www.mastersonics.com/documents/revision_of_the_particle-wave_dualism.pdf

the group velocity of an electron wave	the group velocity of a planetary wave
$v_s \approx 2u_s = \frac{Ze^2}{2nh\varepsilon_0}$, (8.27)	$v_n \approx 2u_n = \frac{2\pi GmM}{nH}$, ($v_n = \frac{v_0}{n} = 2u = \frac{v_e}{\sqrt{2}} = \sqrt{\frac{GM}{R_n}} = \frac{2\pi}{nH}GmM$, (2.11.14))

a frequency of an orbital electron wave	a frequency of an orbital planetary wave
$f_s \approx \frac{mZ^2e^4}{8n^2h^3\varepsilon_0^2}$ (8.28)	$f_n \approx \frac{2\pi^2G^2m^3M^2}{n^2H^3}$, ($f_n = f_{on} = n\frac{f_m}{2} = n\frac{1}{2T_n} = n\frac{\sqrt{GM}}{4\pi R^{3/2}} = \frac{\tilde{E}_n}{H}$, (2.11.14))

a wavelength of an orbital electron wave	a wavelength of an orbital planetary wave
$\lambda_s = \frac{h}{p} \approx \frac{2nh^2\varepsilon_0}{mZe^2}$ (8.29)	$\lambda_n = \frac{H}{p} \approx \frac{nH^2}{Gm^2M}$, ($\lambda_n = \lambda_{on} = \frac{H}{mv} = \frac{H}{p} = \frac{2\pi R}{n}$, (2.11.14))

the energy of a stationary electron wave	the energy of a stationary planetary wave
From N. Bohr atom model: $\varepsilon_s = \varepsilon_n = hf_s = \frac{1}{2}\mu v^2 = \frac{\mu Z^2e^4}{8n^2h^2\varepsilon_0^2}$ (8.30)	$\varepsilon_n = \tilde{E}_n = Hf_n \approx \frac{1}{2}\mu v^2 = \frac{2\pi^2G^2m^3M^2}{n^2H^2}$, ($E_k = \tilde{E}_n = Hf_{on} = n\frac{Hf_m}{2}$, (2.11.14))
--	--
From the solutions of the Schrödinger equation in spherical coordinates: $\varepsilon_s = \varepsilon_n = \frac{(\frac{h}{2\pi})^2}{2\mu a_0^2 n^2}$,	Analogically formulated, hypothetical: $\varepsilon_n = \frac{(\frac{H}{2\pi})^2}{2\mu a_{0g}^2 n^2}$, (hypothetical; -analogically created)
$a_0 = \frac{(\frac{h}{2\pi})^2}{\mu e^2} = \text{Bohr radius}$, $\frac{2\mu e^2}{(\frac{h}{2\pi})^2\delta} = \frac{2j+k+1}{2} = j+\ell+1 = n$, $\ell(\ell+1) = \frac{k^2-1}{4}, \left(\frac{\delta}{2}\right)^2 = -\frac{2\mu\varepsilon_s}{(\frac{h}{2\pi})^2}, k = 2\ell+1$, $\delta = \frac{2\mu e^2}{(\frac{h}{2\pi})^2 n} = \frac{2}{a_0 n}$, $(k, j, \ell, n) = \text{integers}$.	$a_{0g} = \frac{(\frac{H}{2\pi})^2}{G\mu^2 M} = \text{Gravitational, Bohr radius}$, $\frac{2\mu^2 M}{(\frac{H}{2\pi})^2\delta_g} = \frac{2j+k+1}{2} = j+\ell+1 = n$, $\ell(\ell+1) = \frac{k^2-1}{4}, \left(\frac{\delta_g}{2}\right)^2 = -\frac{2\mu\varepsilon_s}{(\frac{H}{2\pi})^2}, k = 2\ell+1$, $\delta_g = \frac{2\mu^2 M}{(\frac{H}{2\pi})^2 n} = \frac{2}{a_{0g} n}$, $(k, j, \ell, n) = \text{integers}$.

<table>
<tr><th>a radius of an electron orbit</th><th></th><th>a radius of a planetary orbit</th></tr>
<tr><td>$r_n = \frac{n^2 h^2 \varepsilon_0}{\pi \mu e^2 Z}$ (8.4)

$\langle r_{n,\ell} \rangle = \frac{a_0}{2}\left[3n^2 - \ell(\ell+1)\right]$,
$\langle r^2_{n,\ell} \rangle = \frac{a_0^2 n^2}{2}\left[5n^2 + 1 - 3\ell(\ell+1)\right]$,
$\left\langle \frac{1}{r_{n,\ell}} \right\rangle = \frac{1}{a_0 n^2}, \left\langle \frac{1}{r^2_{n,\ell}} \right\rangle = \frac{1}{a_0^2 n^3 (\ell + 1/2)}$,
$n = k + \ell = 1, 2, 3...$
$\ell = 0, 1, 2, ..., n-1$
$m = -\ell, -\ell+1, ..., \ell-1, \ell$</td><td></td><td>$R_n = \frac{n^2 H^2}{4\pi^2 G m^2 M}$,
$(R_n = n^2 \frac{GM}{v_0^2} = \frac{GM}{v_n^2} = n\frac{\lambda_{on}}{2\pi} = \frac{n^2 H^2}{4\pi^2 G m^2 M}$,
(2.11.14))

(hypothetical; -analogically created)
$\langle R_{n,\ell} \rangle = \frac{a_{0g}}{2}\left[3n^2 - \ell(\ell+1)\right]$,
$\langle R^2_{n,\ell} \rangle = \frac{a_{0g}^2 n^2}{2}\left[5n^2 + 1 - 3\ell(\ell+1)\right]$,
$\left\langle \frac{1}{R_{n,\ell}} \right\rangle = \frac{1}{a_{0g} n^2}, \left\langle \frac{1}{R^2_{n,\ell}} \right\rangle = \frac{1}{a_{0g}^2 n^3 (\ell + 1/2)}$,
$n = k + \ell = 1, 2, 3...$
$\ell = 0, 1, 2, ..., n-1$
$m = -\ell, -\ell+1, ..., \ell-1, \ell$</td></tr>
</table>

Also, for the specific planet, we can analogically determine gravitational fine-structure constant,

Fine-structure constant	Gravitational fine-structure constant
$\alpha = \frac{e^2}{4\pi\varepsilon_0 (\frac{h}{2\pi})c} \cong \frac{1}{137}$	$\alpha_g = \frac{Gm^{*2}}{(\frac{H}{2\pi})c}$

Of course, in T.2.8., reduced masses $\mu = \frac{m_1 m_2}{m_1 + m_2}$ are different in the case of Bohr atom model and for planets of a specific solar system (because different masses m_1 and m_2 are involved).

The results discussed in T.2.8 are largely supported by existing astronomical observations, publications, and theoretical analyses, confirming their accuracy. Notably, there is a striking analogical and quantitative proportionality between the masses involved and their analogical electric charges. This raises an intriguing question: how can this be, given that gravitational attraction also exists between electromagnetically neutral or internally balanced masses, where not macroscopically measurable magnetic or electric fields are present?

One logical explanation is that the masses, composed of agglomerations of atoms, are slightly internally polarized, electrically and magnetically—almost beyond our experimental detection. This subtle polarization may result in uniformly organized, very weak electric and magnetic dipoles. When spatially aligned, these dipoles could exert Coulomb-type attractions, analogous to the Newtonian gravitational force law, where only masses are considered. We assume that a certain mass is directly proportional to the number of its internal constituents—atoms, or the number of electrostatic dipoles and/or elementary magnets within those atoms.

From the first chapter, we know that direct analogies between electric and magnetic charges (or fluxes) and masses involve linear and spinning or orbital moments. To complete this analogy in the context of Coulomb and Newton's laws, we hypothesize the existence of a constant, intrinsic linear and/or angular speed within our Universe. This global background velocity could represent a cosmic motion where linear and angular movements, rotation, and spinning are combined.

This rotational motion, and the associated centrifugal force, could influence weakly polarized electromagnetic dipoles, producing Coulomb-type electromagnetic forces that are mistakenly *interpreted as Newtonian gravitational interactions between masses. Electrons, protons, and almost all subatomic entities possess spinning moments and magnetic properties. In macroscopically neutral masses, these internal electromagnetic states cancel out spatially. However, if there is a global background rotation of the Universe, a small electrostatic dipole polarization could appear on the surfaces of these masses, and internal mass domains may experience some alignment of spinning or magnetic moments. Both electric dipoles and internal magnetic moments respect Coulomb's law, and in some cases, these effects might be misinterpreted as gravitational phenomena.*

In the case of rotating galactic masses, the electromagnetic dipoles are stronger and more favorably aligned, producing stronger Coulomb-like attractions. This is often mistakenly attributed to the presence of additional dark matter or energy, used to explain observed gravitational effects and to uphold the validity of Newtonian and Einsteinian theories of gravity. However, this approach might merely delay the search for a new theory that better explains the true nature of gravitation.

Oscillating masses, atoms, and their associated force fields are the true sources of gravitation, rather than static masses. These structures are mutually coupled and synchronized, as seen in cosmic systems where atomic models apply (see Chapter 8 for more details).

Planetary systems, with their mechanical and periodic motions, can be thought of as creating or adhering to a stationary and standing wave structure, much like mechanical resonators. This analogy extends to the N. Bohr atomic model and electromagnetic resonators, where stationary waves exist, and electrostatic forces are dominant. Consequently, we should expect significant electromechanical couplings between electric and magnetic dipoles or multipoles within these resonant systems, like the piezoelectric effect. Every stable planetary system should function as a single, synchronized resonant system, with harmonized radial, orbital, and standing wave resonant frequencies, whether observed as mechanical or electromagnetic resonators. Any perturbation in these coupled resonators will produce synchronous electromagnetic or electromechanical disturbances, as seen in piezoelectric devices.

This electromechanical coupling may explain the electromagnetic nature of gravitation. Within standing matter-waves, nodal zones create attractive forces, leading to mass agglomerations like Newtonian attractions. To unify the radial, axial, orbital, and transverse resonant behaviors of planetary systems, electromagnetic waves should have both transverse and longitudinal components. This necessitates an update to Maxwell's equations to support longitudinal waves, a concept introduced in Chapter 3 (see relations (3.7-1) and (3.7-2)).

Nikola Tesla measured these structural and stationary planetary resonant waves, both mechanical and electromagnetic, forming the basis for his ideas on a new Dynamic Theory of Gravitation (though, unfortunately, he never published or finalized it). See references [97], [98], [99], and [117] for more details.

....

Common properties between planetary systems and atoms include:

1. Planetary or solar systems are large and complex agglomerations of atoms.
2. Both are characterized by structural periodicities or periodic motions of their internal constituents.
3. Both can be represented as self-contained, circular, stationary, and standing matter-wave structures.
4. Both internally host mutually coupled and synchronized constituents in entanglement relations.
5. Both exhibit a variety of analogous, quantified relationships.
6. In both cases, the complexity of the involved electromagnetic fields is a dominant property, suggesting that gravitation is a derivative of the intrinsic electromagnetic nature.

Ling Jun Wang: -Citation (see [122]), … "presented a theory of unification of gravitational force and the electromagnetic force based on the generalization of Newton's law of gravitation to include a dynamic term inferred from the Lorentz force of electromagnetic interaction. The inclusion of this dynamic term alone in the gravitational force is enough to develop the entire dynamic theory of gravitation parallel to that of electrodynamics".

Familiar ideas about the extension of the Lorentz force are elaborated in the third chapter of this book. If we connect electric and magnetic (dipoles and multipoles) polarizations, and global (holistic) motions *and rotations within our Universe, with Lorentz force effects, the picture of unity between gravitation and electromagnetism will be much clearer and more indicative.*

The bottom-line simplified explanation about masses coupling is related to the fact that masses are composed of atoms. Atoms internally have many discrete, stationary, and standing waves energy states, meaning resonant states, or we could also say physical resonators. Resonators with mutually overlapping spectral characteristics are naturally synchronized (in zones where they have the same resonant frequencies). This way, compact and united macro mass starts behaving like a big, united atom with number of internal, discrete (atomic and molecular) energy states. Since all macro masses are such kind of complex resonant states, it is natural to expect that mentioned (mutually overlapping) resonant states from any of two separate masses will again mutually synchronize and on some way energetically communicate (by creating standing electromagnetic waves between them), producing the effects of Gravitation (see more in Chapter 8.). Familiar innovative concept about Gravitation, where mases of planets are in states of permanent mutual electromagnetic energy exchanges and coupling, being at the same time transmitters and receivers of electromagnetic energy, and where relevant solar system structure is creating standing waves fields between the sun and planets, including many of additional imaginative and challenging excursions towards other domains of modern Physics, can be found in [144], Poole, G. (2018) Cosmic Wireless Power Transfer System and the Equation for Everything.

Citation from [144] "**Abstract**: By representing the Earth as a rotating spherical antenna several historic and scientific breakthroughs are achieved. Visualizing the Sun as a transmitter and the planets as receivers the solar system can be represented as a long wave radio system operating at Tremendously Low Frequency (TLF). Results again confirm that the "near-field" is Tesla's "dynamic gravity", better known to engineers as dynamic braking or to physicists as centripetal acceleration, or simply (g). …
A new law of cosmic efficiency is also proposed that equates vibratory force and pressure with volume acceleration of the solar system. Lorentz force is broken down into centripetal and gravitational waves. …
Spherical antenna patterns for planets are presented and flux transfer frequency is calculated using distance to planets as wavelengths. The galactic grid operates at a Schumann Resonance of 7.83 Hz, …
The Sun and the planets are tuned to transmit and receive electrical power like resonating Tesla coils".

As we know, the original N. Bohr's Planetary Atom Model is upgraded, and successfully exploited, by applying Schrödinger's equation, and ideas of particle-wave duality. Consequently, we should be able, because of the validity of mentioned "Gravito-static versus electrostatic analogies" (based on $\boxed{\frac{Ze^2}{4\pi\varepsilon_0} \to GmM}$), to analogically apply relevant wave functions and Schrödinger equation to familiar planetary, and other astronomic situations (with periodic and circular, orbital, and inertial motions). Elaborations and analysis of planetary systems, starting from (2.11.12) until (2.11.14), are anyway clearly indicating that planetary systems, presented as macro-cosmological matter-waves, behave very much analogically as is known in microphysics. Consequently, here we have enough grounds to apply the Schrödinger equation (see such attempts, later, around equations (2.11.20) - (2.11.22), and Fig.2.6.). The Analogy in question is not at all establishing different, non-doubted, definitive grounds that *probabilistic methodology* of quantum theory is relevant here (opposite to what certain authors are implicitly forging as the fact). Schrödinger equation is applicable here mostly because stable, self-closed orbits, hosting periodical and standing waves, create necessary conditions to formulate and apply such equation (and there is nothing to connect it with probabilities and statistics). Much more striking, challenging, and significant here is the fact that gravitation and electromagnetic field are on some way (more than only analogically) connected, and that sources of gravity are most probably of electromagnetic nature (see much more of similar ideas in [72], Dr. László Körtvélyessy. The Electric Universe).

Citation taken from **[63]**, under **24)**: Arbab Ibrahim Arbab. The Generalized Newton's Law of Gravitation versus the General Theory of Relativity. **Journal of Modern Physics.**

"We have shown that gravitomagnetism and the general theory of relativity are two theories of the same phenomenon. This entitles us to accept the analogy existing between electromagnetism and gravity fully. Hence, electromagnetism and gravity are unified phenomena. The precession of the perihelion of planets and binary pulsars may be interpreted as a spin-orbit interaction of gravitating objects. The spin of a planet is directly proportional to its orbital angular momentum and mass, weighted by the Sun's

mass. Alternatively, the spin is directly proportional to the square of the orbiting planet's mass and inversely proportional to its velocity".

Since in T.2.8., we have the analogical expression for gravitational, fine structure constant $\alpha_g = \frac{Gm^{*2}}{(\frac{H}{2\pi})c}$, and since ordinary (atomic and electromagnetic) fine structure constant is known as extraordinarily stable, α = 7.2973525698(24)x10^{-3}, α^{-1}=137.035999074(44), we could search for (analogic) conditions when gravitational fine structure constant will be equal to atomic, or electromagnetic fine structure constant, meaning,

$$\alpha_g = \frac{Gm^{*2}}{(\frac{H}{2\pi})c} \cong \alpha = 7.2973525698(24)\cdot 10^{-3} \cdot \quad (2.11.14\text{-}1)$$

From (2.11.14-1) we can determine the hypothetical (analogically founded) value for macro-cosmological or gravitational Planck constant H, as,

$$H = \frac{2\pi G}{\alpha c} m^{*2} \cong 137.035999074 \frac{2\pi G}{c} m^{*2} . \quad (2.11.14\text{-}2)$$

On a similar way, as we are converging gravitational fine-structure constant to the atomic or electromagnetic fine structure constant $\alpha_g \to \alpha$, $m^* \to m^*_{minimal}$, gravitational or macro cosmological Planck constant H, from (2.11.14-2), would be at the same time (analogically) converted into microworld Planck constant h, meaning $H \to h$. The minimal value of the mass $m^* = m^*_{minimal}$, which has specific gravitational mass, when is still meaningful or possible to detect and measure effects of gravitation in the Newtonian framework, could be one among masses of the proton, neutron, or electron, but here we will find that this is not the case.

$$\left(H \to h = \frac{2\pi G}{\alpha c}(m^*_{minimal})^2 \cong 137.035999074 \frac{2\pi G}{c}(m^*_{minimal})^2 \right) \Rightarrow \\ m^*_{minimal} \cong \sqrt{\frac{hc}{137.035999074 \cdot 2\pi G}} = 1.44308 \cdot 10^{-19} \text{ [kg]} . \quad (2.11.14\text{-}3)$$

We can calculate from (2.11.14-3) that the minimal mass, which has specific gravitational meaning (under here introduced analogical framework) $m^* \cong 1.44308 \cdot 10^{-19}$ [kg], is $8.62766 \cdot 10^7$ times bigger than the mass of the proton, or $8.61592 \cdot 10^7$ times more significant than the mass of the neutron, and $1.58417 \cdot 10^{11}$ times bigger than the mass of an electron.

Anyway, it is experimentally known that the validity of Newton gravitational force law (between two masses) is testable and provable until the lower distance limits of 300 micrometers (approximately). One of the conclusions here could be that gravitation has a meaning only for a relatively large group of electromagnetically polarizable atoms (for instance, for a minimum of $8.62766 \cdot 10^7$ *hydrogen atoms), above certain threshold mass amount (*$m^* \cong 1.44308 \cdot 10^{-19}$ [kg]*), and for distances between two masses higher than 300 micrometers. All of that indicates that gravitation could be a manifestation of electromagnetic forces between masses with specific electric dipoles polarization (as speculated at the beginning of this chapter, around equations from (2.4-7) to (2.4-10)). If electromagnetic forces and charges are essential sources of gravitation, consequently, what we expect to detect as gravitational waves should be some very low-frequency electromagnetic waves. In cases of stable solar or planetary systems, we should be able to find such standing and stationary, macro electromagnetic field structures between planets and the local sun. It is still too early to draw definite conclusions, but at least, here we got specific indicative numbers (regarding validity of gravitation), under certain sufficiently well-defined and challenging conditions.*

In T.2.8. we explored formal analogies based on a comparison between the Bohr planetary atom model and a real planetary system such as,

$$\left\{\frac{Ze^2}{4\pi\varepsilon_0 r^2} \Leftrightarrow \frac{GmM}{r^2},\ Ze \Leftrightarrow M, e \Leftrightarrow m \cong \frac{mM}{m+M} = \mu, Z \Leftrightarrow \frac{M}{m}, \frac{1}{4\pi\varepsilon_0} \Leftrightarrow G, h \Leftrightarrow H\right\} \Rightarrow \left\{\frac{Ze^2}{4\pi\varepsilon_0} \to GmM, Ze^2 \to mM \cong \mu M\right\}. \tag{2.11.14-4}$$

From the first chapter of this book (see T.1.2 until T.1.8) we know that when respecting Mobility system of electromechanical analogies, electric charges are analog to linear and orbital moments, meaning that Newton law of Gravitation should present specific force-field between linear and orbital moments, like already exercised and summarized in T.2.2, T.2.2-2 and (2.4-5.1). If we consider that in the Newton law of gravitational force, instead of mutually attracting masses, we should have attraction of corresponding linear and orbital moments (which are on some way implicitly present, but still hidden, and somewhat hypothetical), we can reformulate mentioned initial analogies (2.11.14-4) on the following way,

$$(G = gv_c^2 = Const) \Rightarrow$$
$$\left\{\frac{Ze^2}{4\pi\varepsilon_0 r^2} \Leftrightarrow g\frac{(mv_c)(Mv_c)}{r^2} = g\frac{p_c \cdot P_c}{r^2}, Ze \Leftrightarrow Mv_c = P_c, e \Leftrightarrow mv_c = p_c \cong \frac{mM}{m+M}v_c = \mu v_c, \frac{1}{4\pi\varepsilon_0} \Leftrightarrow g, h \Leftrightarrow H\right\} \Rightarrow \tag{2.11.14-5}$$
$$\Rightarrow \left\{\frac{Ze^2}{4\pi\varepsilon_0} \to g\cdot(p_c \cdot P_c) \Rightarrow Ze^2 \Leftrightarrow p_c \cdot P_c = mMv_c^2 \cong \mu M v_c^2\right\}$$

Practically, in (2.11.14-5) we see a way to extract (or expose) missing or hidden velocities of interacting masses from Newton gravitational constant, $G = gv_c^2 = Const.$, where speeds of both masses **m** and **M** are the same and equal to $v_c \cong const.$. This way, instead of static masses product, mM we created the outcome of associated linear moments $p_c \cdot P_c = (mv_c)(Mv_c)$, satisfying analogy that electric charge corresponds to linear momentum (like in T.1.8), and consequently transforming Newton law of gravitation to be the force between involved mechanical (linear and/or orbital) moments. In other words, both masses, **m** and **M,** are globally moving concerning specific reference system with a certain velocity, and making certain, linear and/or oscillatory motion (like oscillating dipoles). It is also evident that such a speculative and intuitive situation (regarding hidden, or background velocity parameters) should be better elaborated and explained. ***Of course, later we also need to find a way to involve angular and spin moments of interacting masses in Newton law, but what is important here is to show that Newton force of gravitation could evolve towards richer conceptualization***. In chapter 10 of this book, we can find the complete explanation of the same situation regarding unknown or background velocity parameters and Newtonian attraction between important linear and angular moments (see (10.1.4) - (10.1.7)).

The analogies between the Bohr atom model and planetary systems are both striking and suggestive (see T.2.8 and Chapter 8). Since the extended atom model also involves magnetic, orbital, and spin moments, we can analogically and hypothetically expect that planetary systems operate within a similar framework of orbital moments, spins, surrounding electric and magnetic fields, and standing matter waves, resulting from stable periodic motions. This concept is explored throughout this book and in several publications, including works by Anthony D. Osborne, N. Vivian Pope, Arbab I. Arbab, and Jovan Djuric, particularly in "Magnetism as Manifestation of Gravitation" [36], [63], [33], [71], and [102].

If we had the necessary technical and observational tools, we could potentially visualize the relevant electromagnetic matter waves and the standing wave structures related to mass distributions in planetary and galactic systems. A further hypothesis emerging from T.2.8 is that this relationship may not just be a system of analogies between electromechanical, gravitational, and electromagnetic entities. Instead, it might represent a unified phenomenology, described through combined mechanical and electromagnetic concepts.

In other words, the phenomena we observe could be mutually analogous and equivalent, involving mechanically, electromagnetically, and electromechanically coupled entities of the same fundamental force. This force has been independently conceptualized in physics—either as Newtonian mechanics and gravitation or as electromagnetism related to Coulomb's law—depending on the historical context.

Here, we propose a possible unification of electromechanical phenomenology, as previously discussed in equations (2.4-7) through (2.4-10) earlier in this chapter. Different motions of masses may create internal, spatially distributed electromagnetic entities, such as dipoles, moments, and charges. Consequently, planetary or mechanical motions could be described using either a predominantly mechanical framework or an electromagnetic one, depending on the context. ♣]

It can also be roughly (numerically) verified that quantum number n, which appears in $2\pi R = n\lambda_o$, (2.11.12) - (2.11.14), is taking the same order of magnitude as number of days in a year of relevant planet, indicating where we should search for the meaning of planetary, standing matter waves quantizing (see (2.11.14)-g, h).

Analog to vortex shedding phenomenology, known in fluid motions as "Karman Vortex Street", we could say that certain kind of "Planetary Karman Street" is on some way following planets and astronomic size objects, like a helix oscillatory tail. As shown in chapter 4.1, around equations (4.3-0) and (4.3-0)-a,b,c,d,e,f,g,h,i such vortex shedding is in agreement with matter waves quantization $\lambda = H/p, \tilde{E} = Hf$, or on a similar way equivalent to (2.11.12), (2.11.13) and (2.11.14). ***For instance, the frequency of vortex shedding is directly proportional to relevant fluid or particle velocity, meaning that in (2.11.14) we may have*** $f_o = f_{on} = const \cdot v = C(n) \cdot v$***, what is explaining that*** v_0 ***for the specific planetary system could be constant, as follows,***

$$f_o = const \cdot v = C(n) \cdot v = C(n) \cdot v_n \Rightarrow$$

$$\left\{ \begin{array}{l} R_n = n^2 \frac{GM}{v_0^2} = \frac{GM}{v_n^2} = n\frac{\lambda_{on}}{2\pi} = \frac{n^2 H^2}{4\pi^2 G m^2 M}, \ v_n = \frac{v_0}{n} = 2u = \frac{v_e}{\sqrt{2}} = \sqrt{\frac{GM}{R_n}} = \frac{2\pi}{nH} GmM \\ R_i v_i^2 = R_j v_j^2 \Rightarrow R_n v_n^2 = R_n \frac{v_0^2}{n^2} = GM \Leftrightarrow R_n v_n^2 = Rv^2 = constant, \end{array} \right\} \Rightarrow \quad (2.11.14)\text{-a}$$

$$v_0 = \frac{nv_e}{\sqrt{2}} = nv_n = \frac{2\pi}{H} GMm = \frac{GMm}{\hbar_{gr.}} = 2un = n\sqrt{\frac{GM}{R_n}}, \ n = 1,2,3...$$

$$f_o = n\frac{f_m}{2} = n\frac{1}{2T} = n\frac{\sqrt{GM}}{4\pi R^{3/2}} = \frac{n}{4\pi R_n}\sqrt{\frac{GM}{R_n}} = \left(\frac{n}{4\pi R_n}\right) v_n = \left(\frac{v_0^2}{4\pi n GM}\right) v_n = \left(\frac{1}{4\pi R_n}\right) v_0 = C(n) \cdot v_n .$$

[♣ COMMENTS & FREE-THINKING CORNER:

There is increasing evidence from astronomical measurements (spectral, Doppler redshifts of the electromagnetic radiation passing about galactic centers; - [37] Tifft, [40] Nottale, [41] Rubćić, A., & J. Rubćić, [43] M. Pitkänen) that v_0 (appearing in (2.11.14) and (2.11.14)-a) is a characteristic velocity parameter applicable for many planetary systems (like other universal or fundamental constants known in Physics) having the value $v_0 = n\sqrt{\frac{GM}{R_n}} = 144.7 \pm 0.7$ km/s. ***Nottale is shown in [40] that such fundamental velocity constant is observed from the planetary scales to the extragalactic scales (see the diagram below). His theoretical predictions, based on "Scale Relativity Theory" agree very well with the observed values of the actual planetary orbital parameters, including those of the asteroid belts.*** *Mentioned observations support the legitimacy of all other quantized parameters (from (2.11.14)) like orbital radius, phase, and group velocity* R_n, u_n, v_n*, etc.*

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

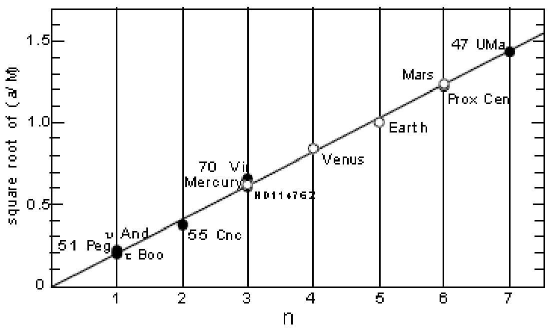


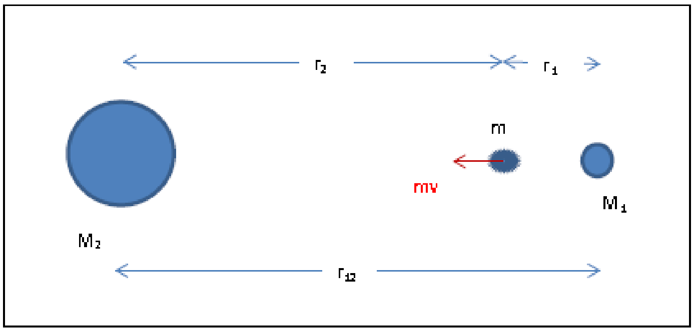
Figure 2. Square root of the ratio (a/M) (a in AU and M in $M_{\odot}$), where a is the semi-major axis of a planet and M the mass of its star, versus n integer, for inner solar system planets and extra-solar companions. The line corresponds to the law $\sqrt{a/GM} = n/w_0$, with $w_0 = 144$ km/s.

This picture is taken from [40]; -the "Letter to the Editor; Scale-relativity and quantization of extra-solar planetary systems. L. Nottale", DAEC, CNRS et Université Paris VII, Observatoire de Paris-Meudon, F-92195 Meudon Cedex, France, Received 9 May 1996 / Accepted 5 September 1996.

Some gravitational or inertial standing waves field structure, which also has specific electromagnetic nature (whatever that means), really exist around and behind planets in stationary orbital motions. See more of supporting remarks later, related to measured red shifts, around equations (2.11.15) to (2.11.19), and [63], Arbab I. Arbab, [67], Johan Hansson and [68], Charles W. Lucas, Jr. … All of that is giving chances that some "Planetary Karman Street" should exist behind every planet in orbital motion. Since certain electromagnetic nature is intrinsically incorporated (see [63]) into such planetary and gravitational formations (and quantization of planetary systems is the fact), the imprints and traces of "Planetary Karman Streets" should exist and be measurable as electromagnetic and Doppler-shifts spectral signatures (as Tifft measured), and very probably, as charged particles currents and plasma-related manifestations.

Let us observe two astronomic objects with masses M_1 and M_2. One of them $M_1 << M_2$ could be small planet or satellite, and the other M_2 could be a bigger planet or local sun, or we could have two independent and "self-standing" cosmic masses M_1 and M_2. The distance between M_1 and M_2 is $r_{12} = r_1 + r_2$, as presented in the following picture.

Let us now imagine that specific small mass $m <<< M_1$ is projected (like a gun bullet) from M_1 towards M_2. We could assume that initial mass, speed, and linear moment of the bullet-mass m are constant and known ($(r_1 = \text{minimum}) \Rightarrow m = m_0 = \text{const.}\ v = v_0 = \text{Const.},\ p = p_0 = m_0 v_0$). We could also consider that M_1 and M_2 are relatively stable and static masses.



There is also an attractive field of gravitation between all involved masses m, M_1 and M_2, making that bullet mass m will have an increasing speed and linear moment. This time, we will neglect the possible presence and influence of electromagnetic fields and forces. To be more general, we could imagine that all the involved masses should have certain linear and angular moments (what will be correct in cases of planetary systems), but for analyzing this example, we will assume that masses M_1 and M_2 are *sufficiently static (or approximately standstill) and stable. The objective here will be to find evolving effective mass, velocity, energy, and momentum of a small gun bullet m.*

The first step in such analyzing is to apply energy and momentum conservation laws (this time neglecting possible involvement of angular moments and electromagnetic fields and forces),

$$\begin{aligned}
&\vec{P}_{total} = \vec{p}_m + \vec{p}_1 + \vec{p}_2 = const. \\
&\vec{p}_m = m\vec{v}, \vec{p}_1 = M_1\vec{v}_1, \vec{p}_2 = M_2\vec{v}_2 \Leftrightarrow (\vec{p}_m = \gamma m_0\vec{v}, \vec{p}_1 = \gamma_1 M_{10}\vec{v}_1, \vec{p}_2 = \gamma_2 M_{20}\vec{v}_2) \\
&(\vec{p}_{m0} = m_0\vec{v}_0) \Rightarrow \vec{p}_{10} = M_{10}\vec{v}_{10}, \vec{p}_{20} = M_{20}\vec{v}_{20}, \\
&E_{total} = E_m + E_1 + E_2 = (m_0c^2 + E_{km}) + (M_{01}c^2 + E_{kM1}) + (M_{02}c^2 + E_{kM2}) = const. \\
&(\vec{P}_{total}, \frac{E_{total}}{c}) = invariant. \Rightarrow \vec{P}^2_{total} - \frac{E^2_{total}}{c^2} = -\frac{E^2_0}{c^2}, \ E_0 = m_0c^2 + M_{01}c^2 + M_{02}c^2, \\
&E_{km} = (\gamma - 1)m_0c^2 \cong \frac{1}{2}mv^2, E_{kM1} = (\gamma_1 - 1)M_{01}c^2 \cong \frac{1}{2}M_1v_1^2, E_{kM2} = (\gamma_2 - 1)M_{02}c^2 \cong \frac{1}{2}M_2v_2^2
\end{aligned} \tag{2.11.14)-a-1}$$

It is evident that bullet mass m, its velocity, and momentum will be dependent on speeds and moments states of M_1 and M_2. The small mass m is in the field of attractive gravitational forces of masses M_1 and M_2 (acting in mutually opposite directions and being distance-dependent, based on Newton law), meaning that all the involved masses will have evolved and mutually dependent moments.

It is also possible to present motional mass m as a matter-wave packet or photon, where we could start exploiting associated group and phase speed, wave energy, matter-wave wavelength, and matter-wave frequency. For instance, in cases of microparticles like electrons, protons, positrons, etc. it will be, $\tilde{E}_m = hf = E_{km} = (\gamma - 1)m_0c^2 \cong \frac{1}{2}mv^2, \ p = \gamma m_0 v$ *and* $v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} = \frac{d\tilde{E}}{dp}, \ u = \lambda f = \frac{\tilde{E}}{p}$. *We will find that useful (and analogical) matter-wave characteristics of the bullet mass m, or an equivalent photon (like wavelength and frequency) will also evolve, being distance, velocity, and all initial masses dependent (basically getting certain observer-dependent Doppler, red and blue frequency shifts). Here is a place to underline that Planck's constant* h *is applicable only for cases involving microparticles and photons. Typical examples where we can verify the existence of such analogical particle-wave parallelism situations are innovative analyses of Compton, and photoelectric effects, including the continuous spectrum of x-rays (or photons), caused by impacts of electrons accelerated in an electrical field between two electrodes (see such analyzes in chapter 4.2). For other macroparticles, certain new, and analogical* H *constant will be more appropriate instead of Planck constant* h *(especially in cases when self-closed, matter-waves structures are being involved or created).*

The next step can be to imagine that the kinetic energy of a small bullet-mass m (as in analogical cases of elementary microparticles and photons) will be replaced or cinematically represented by an equivalent matter-wave or photon energy $\tilde{E}_m$ *(see equations under (4.2) and T.4.0 from chapter 4.1),*

$$\begin{bmatrix}
(0 \le 2u \le \sqrt{uv} \le v \le c, m = \gamma m_0) \Rightarrow E_{km} = \tilde{E}_m = hf = (m - m_0)c^2 = \dfrac{pv}{1 + 1/\gamma}, \gamma = 1/\sqrt{1 - \dfrac{v^2}{c^2}}, \\
(0 \le v \cong 2u << c) \Rightarrow E_{km} = \tilde{E}_m = hf \cong \dfrac{1}{2}mv^2 = \dfrac{1}{2}pv
\end{bmatrix}, \tag{2.11.14)-a-2}$$

$$\begin{bmatrix}
(v \cong u \cong c) \Rightarrow \left(\dfrac{\tilde{E}_m}{p} \cong \dfrac{d\tilde{E}_m}{dp}\right) \Leftrightarrow \dfrac{dp}{p} \cong \dfrac{d\tilde{E}_m}{\tilde{E}_m} \Rightarrow \tilde{E}_m = hf \cong \dfrac{E_0}{p_0}p \cong cp, \dfrac{E_0}{p_0} = \dfrac{E_a}{p_a} = \dfrac{E_b}{p_b} = ... = \dfrac{E_m}{p} \\
(E_0 = m_0c^2, p_0 = m_0 c) = constants \Rightarrow v = \dfrac{d\tilde{E}_m}{dp} \cong \dfrac{E_0}{p_0} \cong c, \ u = \lambda f = \dfrac{\tilde{E}_m}{p} \cong c
\end{bmatrix}.$$

From results in (2.11.14)-a-2 we see that relevant matter wave model (in cases of microparticles and photons) could have its initial particle-like part (like a non-zero rest mass, $(E_0, p_0) = constants$), and waving-tail part concerning its phase velocity. For macro masses or macroparticles, we should be able to construct similar mathematical modeling with new H constant (like already exercised for planetary or solar systems, in this chapter). Of course, this mathematically challenging situation could be much better elaborated, and we should not forget that until here we neglected angular moments and associated electromagnetic complexity.

Interactions between Photons and Gravitation

By detecting and analyzing photons emitted from distant sources, such as stars or galaxies, we can infer the evolving parameters of these photons as they are influenced by the gravitational forces of masses between the source and the receiver. Light waves reaching our astronomical observatories carry imprints, modulations, or signatures from the planetary and galactic systems that lie between the distant source of light and our observatory.

Tifft [37] conducted numerous spectral analyses of light from remote sources and discovered that these light waves often exhibit certain quantized or discrete frequency shifts, commonly referred to as "redshifts." These shifts are likely gravitational or electromagnetic imprints left by astronomical objects that the light waves encounter along their journey. It is also possible to detect "blue shifts" within the same context.

This evidence strongly suggests the existence of quantized gravitational and electromagnetic structures, as well as macro-cosmological matter waves (discussed in this book), that interact with and influence the photons as they propagate. If the gravitational intensity, related velocities, and orbital diameters of planetary systems along the path of light are naturally quantized, the light waves received from such distant sources will also exhibit similar quantization, manifesting as red or blue Doppler shifts.

Effectively, here we assume gravitational force-interaction between photons and big gravitational masses around. ***Initial conditions****, relations, assumptions, and necessary mathematical relations (see chapter 4.1) applicable to a photon propagating from a very distant source (towards its observer or receiver) are:*

$$\left[\tilde{E} = hf = \tilde{m}c^2, V = V(r) = V_0 = \mathbf{0}\right] \Leftrightarrow \left[u = v = u_0 = v_0 = c\right] \Rightarrow \tilde{E}_0 = hf_0 = \tilde{m}_0 c^2 \text{ (=) initial photon},$$

$$V = V(r) = \frac{GM}{r} = \text{gravitational potential (=)} \left[\left(\frac{m}{s}\right)^2, \text{ velocity squared}\right], G = \text{gravitational const.}$$

$$\tilde{E}_0 = \text{photon energy on its distant source where } V = V_0 = \text{Lim}\left(V\right)_{r\to\infty} = \mathbf{0},\ V = V(r) = \frac{GM}{r},$$

Index "0" means that certain value relates to its distant radiation source, where $V = V_0 = \mathbf{0}$,

Photon source or emitter (in this case) has negligible mass and zero gravitational intensity.

$$\tilde{E} = hf = \tilde{m}c^2 = \text{wave energy of a photon},$$

$$\tilde{m} = \text{mass-equivalent of a photon} = \tilde{E}/c^2,$$

$$d\tilde{E} = hdf = c^2 d\tilde{m} = vd\tilde{p} = Fdr = -V\tilde{m}\frac{dr}{r},$$

$$F = \text{Force acting on a photon} = \frac{d\tilde{p}}{dt} = -G\frac{M\tilde{m}}{r^2} = -V\frac{\tilde{m}}{r},$$

$$v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} = \frac{d\tilde{E}}{d\tilde{p}} = \text{group speed},$$

$$u = \lambda f = \frac{\tilde{E}}{\tilde{p}} = \text{phase speed},$$

$$\mathbf{0} \leq \mathbf{2}u \leq \sqrt{uv} \leq v \leq c,$$

$$\lambda = \frac{h}{\tilde{p}} = \text{photon wavelength}$$

$$F = \frac{d\tilde{E}}{dr} = h\frac{df}{dr} = c^2\frac{d\tilde{m}}{dr} = v\frac{d\tilde{p}}{dr} = -G\frac{M\tilde{m}}{r^2}$$

*Possible accurate approximations (regarding propagating photon) that can be quickly developed from just stated **initial conditions** are:*

$$\tilde{m} = \tilde{m}_0 \cdot e^{\frac{V}{c^2}} \cong \tilde{m}_0 \cdot (1 + \frac{V}{c^2}) \cong \frac{\tilde{m}_0}{\sqrt{1 - \frac{2V}{c^2}}} \geq \tilde{m}_0,\ V << c^2$$

$$f = f_0 \cdot e^{-\frac{V}{c^2}} \cong f_0 \cdot (1 - \frac{V}{c^2}) \cong \frac{f_0}{\sqrt{1 + \frac{2V}{c^2}}} \leq f_0,$$

$$\lambda = \frac{h}{\tilde{p}} = \frac{\lambda_0}{u_0}\frac{\tilde{E}}{\tilde{p}} \cdot e^{\frac{V}{c^2}} = \lambda_0 \frac{\tilde{E}}{c\tilde{p}} \cdot e^{\frac{V}{c^2}} \cong \lambda_0 \frac{\tilde{E}}{c\tilde{p}} \cdot (1 + \frac{V}{c^2}) \cong \frac{\lambda_0 \frac{\tilde{E}}{c\tilde{p}}}{\sqrt{1 - \frac{2V}{c^2}}} \geq \lambda_0 \frac{\tilde{E}}{c\tilde{p}},$$

$$\lambda_0 = \frac{h}{\tilde{p}_0} = \frac{c}{f} \cdot e^{-\frac{V}{c^2}} \cong \frac{c}{f} \cdot (1 - \frac{V}{c^2}) \cong \frac{\frac{c}{f}}{\sqrt{1 + \frac{2V}{c^2}}} \leq \frac{c}{f}$$

$$\tilde{E} = hf_0 \cdot e^{-\frac{V}{c^2}} = \tilde{E}_0 \cdot e^{-\frac{V}{c^2}} = hf = \tilde{m}c^2 \cong \tilde{E}_0 \cdot (1 - \frac{V}{c^2}) \cong \frac{\tilde{E}_0}{\sqrt{1 + \frac{2V}{c^2}}} \geq \tilde{E}_0 = hf_0,$$

$$u = \frac{\tilde{E}}{\tilde{p}} = \lambda f = \lambda f_0 \cdot e^{-\frac{V}{c^2}} = \frac{\lambda}{\lambda_0} u_0 \cdot e^{-\frac{V}{c^2}} = c\frac{\lambda}{\lambda_0} \cdot e^{-\frac{V}{c^2}} \cong c\frac{\lambda}{\lambda_0} \cdot (1 - \frac{V}{c^2}) \cong \frac{c\frac{\lambda}{\lambda_0}}{\sqrt{1 + \frac{2V}{c^2}}} \geq c\frac{\lambda}{\lambda_0},$$

$$u_0 = \lambda_0 f_0 = \frac{\tilde{E}_0}{\tilde{p}_0} = c = v_0,$$

$$\left\{\begin{array}{l} \tilde{p} = \tilde{p}_0 + \frac{\tilde{m}_0 V}{v_0} = \tilde{p}_0 + \frac{\tilde{m}_0 V}{c} = \frac{hf}{c} \cdot e^{\frac{V}{c^2}} + \frac{\tilde{m}_0 V}{c} = \frac{hf}{c} \cdot e^{\frac{V}{c^2}} + \frac{hf_0 V}{c^3} = \tilde{m}c \cdot e^{\frac{V}{c^2}} + \tilde{m}_0 c\frac{V}{c^2} \\ \tilde{p} = \tilde{m}_0 c \cdot e^{2\frac{V}{c^2}} + \tilde{m}_0 c\frac{V}{c^2} = \tilde{m}_0 c \cdot \left(e^{2\frac{V}{c^2}} + \frac{V}{c^2}\right) \cong \tilde{p}_0 \cdot \left(1 + 3\frac{V}{c^2}\right) \cong \frac{\tilde{p}_0}{1 + \frac{V}{c^2}} \cong \tilde{p}_0\sqrt{1 - 2\frac{V}{c^2}}, \\ \tilde{p}_0 = \tilde{m}_0 c = \frac{hf_0}{c} = \frac{hf}{c} \cdot e^{\frac{V}{c^2}} \cong \frac{hf}{c}\left(1 + 3\frac{V}{c^2}\right) = \tilde{m}c \cdot \left(1 + 3\frac{V}{c^2}\right) \cong \frac{\tilde{m}c}{1 + \frac{V}{c^2}} \cong \tilde{m}c\sqrt{1 - 2\frac{V}{c^2}}, \\ \tilde{m} = \tilde{m}_0 \cdot e^{\frac{V}{c^2}} \cong \tilde{m}_0 \cdot (1 + \frac{V}{c^2}) \cong \frac{\tilde{m}_0}{\sqrt{1 - 2\frac{V}{c^2}}} \cong \frac{\tilde{m}_0}{1 + 3\frac{V}{c^2}}. \end{array}\right\} \Rightarrow$$

$$\Rightarrow \tilde{p} = \frac{h}{\lambda} \cong \tilde{m}c \cdot \left(1 + 3\frac{V}{c^2}\right)^2 = \tilde{p}_0 \cdot \left(1 + 3\frac{V}{c^2}\right) \cong \tilde{m}c \cdot \left(1 - 2\frac{V}{c^2}\right) \cong \tilde{p}_0 \cdot (1 + \frac{V}{c^2}) \cdot \left(1 + 3\frac{V}{c^2}\right)^2 \cong$$

$$\cong \frac{\tilde{p}_0}{\sqrt{1 - 2\frac{V}{c^2}}} \cdot \left(1 + 3\frac{V}{c^2}\right)^2 \cong \tilde{m}c\sqrt{1 - 2\frac{V}{c^2}} \cong \frac{\tilde{m}c}{1 + \frac{V}{c^2}} \cong \frac{\tilde{p}_0}{\sqrt{1 - 2\frac{V}{c^2}}} \cong \tilde{m}c.$$

– –

$$v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} = \frac{d\tilde{E}}{d\tilde{p}} = \frac{\tilde{E}_0}{c^2}\frac{dV}{d\tilde{p}}\cdot e^{-\frac{V}{c^2}} = \tilde{m}_0\frac{dV}{d\tilde{p}}\cdot e^{-\frac{V}{c^2}} \cong \tilde{m}_0\frac{dV}{d\tilde{p}}\cdot(\mathbf{1}-\frac{V}{c^2}),$$

$$v_0 = \tilde{m}_0\left(\frac{dV}{d\tilde{p}}\right)_{V\to 0} \Rightarrow v_0 d\tilde{p} = \tilde{m}_0 dV \Rightarrow v_0(\tilde{p}-\tilde{p}_0) = \tilde{m}_0 V \Rightarrow v_0 = \frac{\tilde{m}_0 V}{\tilde{p}-\tilde{p}_0} = c,$$

$$v = \tilde{m}_0\frac{dV}{d\tilde{p}}\cdot e^{-\frac{V}{c^2}} = c\cdot e^{-\frac{V}{c^2}} \cong \tilde{m}_0\frac{dV}{d\tilde{p}}\cdot(\mathbf{1}-\frac{V}{c^2}) = c(\mathbf{1}-\frac{V}{c^2}) \cong \frac{c}{\sqrt{\mathbf{1}+\mathbf{2}\frac{V}{c^2}}},$$

*Several of the here-obtained results are similar or identical to published results about **Gravitational Redshift and Gravitational Effects on Light Propagation** concerning General relativity Theory. Familiar **Blueshift** phenomenology measurements are also on the way to being theoretically (or naturally) integrated here. What is significant here is that we confirm that big cosmic, gravitational systems are an integral part of universal **P**article-**W**ave **D**uality **C**oncept (also confirmable in different ways; see more familiar elaborations in Chapter 10.). ♣]*

As the direct support to quantizing concepts, assumptions and results found in (2.11.14), and (2.11.14)-a, ($v_n^2 R_n = GM = \underline{const.}$), we can verify (based on very long time known, and many times published measurements) that product between Semi-major Axis of planet revolution **R**, and square of a mean Semi-major orbital (or group) velocity **v**, for each of members of certain stable planetary, or satellite system, is a constant number, $v^2 R = v_n^2 R_n = \text{constant}$ (see the table T.2.3.3, below).

T.2.3.3

Planets	m, Planet mass [kg]	R, Semi-major Axis of revolution around the Sun (mean radius of rotation) [m]	Mean Semi-major Orbital (or group) velocity, v (=) [m/s]	v^2R [m³/s²]	m/v [kg s/m]	nH = C₁(m/v)
Mercury	3.3022.E+23	5.80E+10	4.7828E+04	1.3256E+20	6.9043238270469.E+18	5.7506047172213.E+39
Venus	4.8690.E+24	1.08E+11	3.5017E+04	1.3256E+20	1.3904674872205.E+20	1.1581190412636.E+41
Earth	5.9742.E+24	1.50E+11	2.9771E+04	1.3257E+20	2.0067179469954.E+20	1.6715195460789.E+41
Mars	6.4191.E+23	2.28E+11	2.4121E+04	1.3256E+20	2.6612080759504.E+19	2.2165176631950.E+40
Jupiter	1.8988.E+27	7.78E+11	1.3052E+04	1.3256E+20	1.4547961998161.E+23	1.2116983645068.E+44
Saturn	5.6850.E+26	1.43E+12	9.6383E+03	1.3256E+20	5.8983430687984.E+22	4.9127243050721.E+43
Uranus	8.6625.E+25	2.87E+12	6.7951E+03	1.3253E+20	1.2748156760018.E+22	1.0615524611320.E+43
Neptune	1.0278.E+26	4.50E+12	5.4276E+03	1.3256E+20	1.8936546539907.E+22	1.5772231515805.E+43
Pluto	1.5000.E+22	5.91E+12	4.7365E+03	1.3257E+20	3.1668953868891.E+18	2.6379031231061.E+39
AVERAGE	2.9650.E+26	1.78087E+12	1.9599E+04	1.3256E+20	2.6280461756991.E+22	2.1888788009444.E+43

The relation $v^2 R = v_n^2 R_n = \text{constant}$ is originally discovered only mathematically, by finding strong numerical relationships between involved factors (based on measured data), but here it is theoretically and conceptually founded (as the consequence of standing matter waves formations), getting much higher significance and generalized weight. It can be additionally confirmed on many similar examples and looks as generally applicable to all stable solar and satellite systems, and it is substantially related to the satellite escape velocity (2.11.11). There is a big chance that such a relationship could already be considered as the law of contemporary Physics if it were properly understood and respected before the establishment of Kepler and Newton laws (at least, it is not inferior compared to Kepler and Newton laws). It is possible to show that Newton gravitational force between two masses (one of them, $\mathbf{m} = \mathbf{m_1}$, rotating on a stable circular orbit around bigger mass $\mathbf{M} = \mathbf{m_2}$) can be postulated, invented, or analogically formulated from mentioned relation $v^2R = v_n^2R_n = \text{constant}$. Based on analogies from the first chapter, summarized in T.1.8., and Coulomb-Newton force laws, as given in T.2.2, T.2.2-2, (2.1), (2.2), (2.4-5.1), (2.11.14-5), we can see that what should be analog to Coulomb electrostatic force between two electric charges q_1 and q_2 is similar relationship between two linear moments p_1 and p_2,

$$\left\{\begin{array}{l} F_e = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{R^2}\left(\begin{array}{c}\leftrightarrow\\ \text{analogical}\end{array}\right)F_g = \frac{1}{4\pi g_p}\cdot\frac{p_1p_2}{R^2} = \frac{1}{4\pi g_p}\cdot\frac{m_1v_1\cdot m_2v_2}{R^2} \\ q_{1,2}\leftrightarrow p_{1,2} = m_{1,2}v_{1,2}\end{array}\right\}\Rightarrow$$

$$F_g = \frac{v_1v_2}{4\pi g_p}\cdot\frac{m_1m_2}{R^2} = G\frac{m_1m_2}{R^2} = G\frac{mM}{R^2}, \left(G = \frac{v_1v_2}{4\pi g_p} = \text{const.}\leftrightarrow\frac{1}{4\pi\varepsilon_0}\right)\Leftrightarrow$$

$$\Leftrightarrow\left\{\begin{array}{l} v^2R = \text{constant}\Leftrightarrow v^2 = \frac{\text{constant}}{R}\Leftrightarrow mv^2 = m\cdot\frac{\text{constant}}{R}\Rightarrow \\ F_c = \frac{mv^2}{R} = m\cdot\frac{\text{constant}}{R^2}\left(\begin{array}{c}\leftrightarrow\\ \text{analogical}\end{array}\right)F_e = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{R^2}\Rightarrow \\ F_e = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{R^2}\left(\begin{array}{c}\leftrightarrow\\ \text{analogical}\end{array}\right)F_g = \frac{1}{4\pi g_p}\cdot\frac{p_1p_2}{R^2} = G\frac{mM}{R^2} = F_c(R) \\ G = \frac{v_1v_2}{4\pi g_p} = \frac{v_c^2}{4\pi g_p} = \frac{c^2}{4\pi g} = \text{const.}\Rightarrow v_1v_2 = v_c^2 = \text{ constant}\end{array}\right\}. \quad (2.11.14)\text{-b}$$

The way the expression for Newton gravitational force developed here (in (2.11.14)-b) is implicating that anyway, in any case of the gravitational attraction of masses, we should have elements of stationary rotations (on stable and inertial-motion orbits with standing matter-wave structures) to be able to apply such force law. If in some instances we do not see such elements of stable orbiting (between attracting masses), this is most probably because we are the part of specific complex or more substantial scale rotation (concerning a larger or more general reference frame). In other words, the gravitational force is not and should not only be a central force $F_c(R)$ between static masses. Dynamic parameters like linear and/or angular moments should also be in some essential way involved here, in the broader reference frame (as it is very well supported in [36]).

We still do not have solid arguments to be undoubtedly and generally sure in $(v^2R = \text{constant}) = GM$, but this looks very convincing, based on astronomic observations (see T.2.3.3), and as such is intuitively (and analogically) invented or postulated by Newton (see [61], Mark McCutcheon).

From published literature is known that Gravitational force and Coulomb force are two familiar examples with $F_c(R)$ being proportional to $1/R^2$. Both, neutral and electrically charged masses in such force field with negative $F_c(R)$ (presenting an attractive force) obey Kepler's laws of planetary motion.

-Also, the force-field of a spatial harmonic oscillator is central, with $F_c(R)$ proportional to **R**, and negative.

-Bertrand's theorem formulates more significant support to Kepler-Newton Laws, when saying, $F_c(R) = -k/R^2$, and $F(R) = -kR$, are the only possible central force fields with stable and closed orbits.

We could explore other consequences of "$v^2R = v_n^2R_n = \text{constant}$"concerning orbital quantization, to estimate numerical value of gravitational Planck constant H as follows (see T.2.3.3, (2.11.13) and (2.11.14)),

$$v^2R = v_n^2R_n = \text{constant} \Leftrightarrow 2\pi Rv^2 = 2\pi\cdot\text{constant} = C_1 = 2\pi GM \Rightarrow \quad \text{(2.11.14)-c}$$

$$\Rightarrow \begin{Bmatrix} 2\pi R = n\lambda_0 \\ \lambda_0 = H/p \\ n\lambda_0 v^2 = C_1 \end{Bmatrix} \Rightarrow \begin{Bmatrix} H = \frac{2\pi}{n}\cdot(v^2R)\cdot(\frac{m}{v}) = \frac{C_1}{n}(\frac{m}{v}) = \frac{2\pi m\sqrt{GMR}}{n} \\ nH = C_1(\frac{m}{v}) = 2\pi GM(\frac{m}{v}) = 2\pi m\sqrt{GMR}, n = \text{Integer} \end{Bmatrix},$$

$$\Rightarrow \begin{Bmatrix} H = \frac{C_1}{n}(\frac{m}{v}) = \text{Const.} \\ \tilde{E} = E_k = \frac{mv^2}{2} = Hf_0 \end{Bmatrix} \Rightarrow \frac{m}{nv} = \text{const.} \Leftrightarrow \frac{m_i}{m_j} = \frac{n_i}{n_j}\cdot\frac{v_i}{v_j}, \frac{m_i v_i^2}{m_j v_j^2} = \frac{n_i v_i^3}{n_j v_j^3} = \frac{f_{0i}}{f_{0j}}.$$

From data in T.2.3.3 it is possible to find that an "average gravitational Planck constant" **H** applicable in case of our planetary system could be somewhere inside the following estimations:

$$\begin{Bmatrix} H = \frac{2\pi m\sqrt{GMR}}{n}, n = \text{Integer}, \\ \langle H\rangle \in \left(\frac{9.33\cdot E{+}42}{n}, \frac{2.1888788009444\cdot E{+}43}{n}\right) \end{Bmatrix} \Rightarrow \left(2\pi m\sqrt{GMR} \leq 2.1888788009444\cdot E{+}43 \Rightarrow R \leq \frac{0.121362271}{m^2GM}\cdot E{+}86\right).$$

(2.11.14)-d

It is almost obvious from (2.11.14)-a,b,c,d that in $v^2R = v_n^2R_n = \text{constant} = GM$ something could be wrong with **G**, since estimated **H** is too far from being the universal constant, meaning that Newton law of gravitation should have certain weak sides. What remains is that we could creatively exploit relations:

$$\left(\frac{m}{nv} = \text{const.}\right) \Leftrightarrow \left(\frac{m_i}{m_j} = \frac{n_i}{n_j}\cdot\frac{v_i}{v_j}\right) \Leftrightarrow (H = \text{Const.}) \Rightarrow \left(\frac{m_i v_i^2}{m_j v_j^2} = \frac{n_i v_i^3}{n_j v_j^3} = \frac{f_{0i}}{f_{0j}}\right), \quad \text{(2.11.14)-d1}$$

and draw new conclusions and consequences regarding relations between gravitation, **H**-constant, and planetary masses.

We can also exploit $H = \frac{2\pi mvR}{n} = \frac{2\pi m_i v_i R_i}{n_i}$ from (2.11.13), and calculate the set of possible (or approximate) values for the gravitational Planck-like constant H, as,

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

T.2.3.3-a

Planets	m, Planet Mass,	R, Semi-major Axis of revolution around the Sun (mean radius of rotation)	v, Mean Semi-major Orbital (or group) velocity	n, number of days in one planetary year	$\frac{m_i v_i R_i}{n_i}$	H (=) Gravitational Planck constant
	[kg]	[m]	v (=) [m/s]	[1]	[H/2p] (=) [kg m²/s]	[H] (=) [kg m²/s]
Mercury	3.3022E+23	5.8000E+10	4.7828E+04	1.5000E+00	6.1069E+38	3.8351E+39
Venus	4.8690E+24	1.0800E+11	3.5017E+04	9.2500E-01	1.9907E+40	1.2501E+41
Earth	5.9742E+24	1.5000E+11	2.9771E+04	3.6600E+02	7.2893E+37	4.5777E+38
Mars	6.4191E+23	2.2800E+11	2.4121E+04	6.7000E+02	5.2690E+36	3.3089E+37
Jupiter	1.8988E+27	7.7800E+11	1.3052E+04	1.0500E+04	1.8363E+39	1.1532E+40
Saturn	5.6850E+26	1.4300E+12	9.6383E+03	2.4200E+04	3.2378E+38	2.0333E+39
Uranus	8.6625E+25	2.8700E+12	6.7951E+03	4.2700E+04	3.9563E+37	2.4846E+38
Neptune	1.0278E+26	4.5000E+12	5.4276E+03	8.9700E+04	2.7986E+37	1.7575E+38
Pluto	1.5000E+22	5.9100E+12	4.7365E+03			
Average	*2.9650E+26*	*1.7813E+12*	*1.9599E+04*	*2.1017E+04*	*2.8529E+39*	*1.7916E+40*

Obviously that n, as the principal quantum number (from T.2.3.3-a, temporarily specified as the number of days during one planetary year), because of existence of moons and satellites, should be combined or composed of different quantum numbers in relation to planets' orbital and spinning moments, what analogically also exist in the N. Bohr atom model (see T.2.8. N. Bohr hydrogen atom and planetary system analogies).

Another aspect of $v^2R = v_n^2R_n = \text{constant}$ is that this is also the way to determine the planetary (or satellite) escape speed, based on (2.11.11). For instance, for planets of our Solar system, we have $v^2R \cong 1.3256E{+}20$, (see T.2.3.3), and similar escape speed for every planet can be found as $v_{en} \cong 1.41 \cdot v_n$ (meaning that specific planet can be removed from its stable orbit if its orbital velocity will suddenly increase 1.41 times):

$$\left(E_e = \frac{1}{2}mv_e^2 = \frac{GmM}{R}\right) \Leftrightarrow v_e = \sqrt{\frac{2GM}{R}} \Rightarrow v_e^2R = 1.3256E{+}20 \;=\; 2GM \Rightarrow$$
$$\Rightarrow v_{en} = \sqrt{\frac{2GM}{R_n}} = \sqrt{\frac{1.3256E{+}20}{R_n}} = v_n\sqrt{2} = \frac{v_0}{n}\sqrt{2} = u_n 2\sqrt{2},$$
$$\frac{v_{en}}{v_n} = \frac{1}{v_n}\sqrt{\frac{2GM}{R_n}} = \frac{1}{v_n}\sqrt{\frac{1.3256E{+}20}{R_n}} = \frac{v_0}{nv_n}\sqrt{2} = \frac{u_n 2\sqrt{2}}{v_n} = \sqrt{2},\; v_{en} \cong 1.41 \cdot v_n \;. \qquad \text{(2.11.14)-d2}$$

[♣ COMMENTS & FREE-THINKING CORNER:

If matter-wave Earth's wavelength is related only to the period of one Earth Day, *such Earth wavelength will be:*

$$\lambda_{1-day} = vT_{(1-day)} = \frac{H}{p} = \frac{H}{mv} = 2.56617E+9\,[m] \Rightarrow H = mv^2T_{(1-day)} \cong 4.56E+38,$$

$T_{(1\text{-}day)}$ = 8.62E+04 [s],

v = 29771 [m/s] (=) Earth Mean Semi-Major Orbital (or group) velocity,

m = 5.9742 E+24 [kg] (=) Earth mass,

R = 1.4959826 x 10^{11} m (=) Earth Mean, or Semi-major Orbital radius.

Since one Earth year has 365.26 Earth days, it should also have 365.26 single wavelengths, and we can easily verify that, $365.26 \cdot \lambda_{1-day} \cong 2\pi R \cong 9.399E+11\,[m]$ *, meaning that n = 365.26, and associated, relevant macrocosmic Planck constant (in this case) could be* $H \cong 4.56E+38$ *. If we check on a similar way, the same situation regarding other planets in our solar system, we will get indicative* *and encouraging, not extremely large divergences, or dispersion of results related to orbital perimeters, relevant matter wavelengths, and this way calculated macrocosmic Planck constant. Naturally, after certain process-refinement, we could generalize such concepts and results (as elaborated in the Appendix, under Chapter 10). See later T.2.3.3-1, where the same idea is applied to all planets of our solar system. ♣]*

It is almost evident that the more complete picture about quantization in stable planetary systems should also take care about additional angular and spinning quantum numbers (of involved planets, moons, asteroids, meteorites, and satellites). In [64], Marçal de Oliveira Neto, we can find (effectively based on (2.11.14)) very precisely and convincingly presented, fitted, and calculated, quantizing results, applied to our planetary system.

The integer "**n**", or some kind of quantum number (appearing in all expressions from (2.11.12) until (2.11.14)-a,b,c,d) could be an arbitrarily high number, and this is presenting a difficulty regarding understanding and using precise and meaningful quantization of planetary systems. It would be much easier if we could say, for instance considering our Solar system, that Mercury is the first and closest planet orbiting our Sun, and it should be characterized as orbit number 1 (one). The same way, Venus is on the second planetary orbit around the Sun, and it should be characterized as the orbit number 2. Earth and Mars will have orbits 3 and 4, etc. Such orbital numbers can be considered as principal, significant quantum numbers. Here, we will use symbol "i" for mentioned numbers ($\mathbf{i = 1, 2, 3, \ldots}$). Obviously, such orbital numbers are not at all equal to integer "**n**" appearing in (2.11.12) - (2.11.14)-d. We can try to present the integer **n** in relation to orbital quantum number i, as $n = iN$, where **N** is a certain constant number (also integer), being the same (and valid) for all planets of certain planetary system. ***We will consider that every planetary system has its own characteristic number N, and its own unique constant H, while*** $v_n^2 R_n = \text{constant}$ ***.*** Now, relations developed under (2.11.14) will evolve too,

$$\Rightarrow \left[\begin{array}{l} \left\{\begin{array}{l} \text{For two planets on orbits 1 and 2:} \\ H = \text{const.} \Rightarrow \dfrac{R_1}{R_2} = \dfrac{n_1^2}{n_2^2}\dfrac{m_2^2}{m_1^2} = \left(\dfrac{n_1 m_2}{n_2 m_1}\right)^2, \dfrac{T_1}{T_2} = \left(\dfrac{n_1 m_2}{n_2 m_1}\right)^3 \end{array}\right\} \\ ------------------- \\ \left\{\begin{array}{l} \text{For the same planet passing between two orbits:} \\ (m_1 \cong m_2), H = \text{const.} \Rightarrow \dfrac{R_1}{R_2} = \left(\dfrac{n_1}{n_2}\right)^2, \dfrac{T_1}{T_2} = \left(\dfrac{n_1}{n_2}\right)^3 \end{array}\right\} \end{array}\right] \Rightarrow$$

$$\Rightarrow \left[\left\{\begin{array}{l} \text{For two planets on orbits i and j: } H = \text{const.} \Rightarrow \\ \dfrac{R_i}{R_j} = \left(\dfrac{n_i m_j}{n_j m_i}\right)^2 = \left(\dfrac{i \cdot N \cdot m_j}{j \cdot N \cdot m_i}\right)^2 = \left(\dfrac{i \cdot m_j}{j \cdot m_i}\right)^2, \\ \dfrac{T_i}{T_j} = \left(\dfrac{n_i m_j}{n_j m_i}\right)^3 = \left(\dfrac{i \cdot N \cdot m_j}{j \cdot N \cdot m_i}\right)^3 = \left(\dfrac{i \cdot m_j}{j \cdot m_i}\right)^3 \\ i, j \in [1, 2, 3, \ldots],\ n_i = i \cdot N, n_j = j \cdot N, n_i = \dfrac{i}{i-1} n_{i-1} \end{array}\right\}, \left\{\begin{array}{l} \text{For the same planet passing between two orbits:} \\ (m_i \cong m_j), H = \text{const.} \Rightarrow \\ \dfrac{R_i}{R_j} = \left(\dfrac{n_i}{n_j}\right)^2 = \left(\dfrac{i}{j}\right)^2, \dfrac{T_i}{T_j} = \left(\dfrac{n_1}{n_2}\right)^3 = \left(\dfrac{i}{j}\right)^3 \\ i, j \in [1, 2, 3, \ldots],\ n_i = i \cdot N, n_j = j \cdot N, n_i = \dfrac{i}{i-1} n_{i-1} \end{array}\right\}\right]. \quad (2.11.14)\text{-e}$$

Relations (2.11.14)-e are also identical, analogical, or equivalent to links found in [64], Marçal de Oliveira Neto. Here is the chance to exploit and extend results presented in [64]. As we can see in [64], mentioned relations are creatively and "ad hock" fitted (by Marçal de Oliveira Neto), to satisfy relevant astronomic observations, to the following forms,

$$\left(\frac{R_i}{R_j} = \left(\frac{i}{j}\right)^2, \frac{T_i}{T_j} = \left(\frac{i}{j}\right)^3\right) \Leftrightarrow \left(R_i = R_j \cdot \left(\frac{i}{j}\right)^2, T_i = T_j \cdot \left(\frac{i}{j}\right)^3\right) \Rightarrow$$

$$\Rightarrow \text{(as in [64], Marçal de Oliveira Neto)} \Rightarrow \qquad (2.11.14)\text{-f}$$

$$\Rightarrow R_{i+1} = R_i \cdot \left(\frac{i+1}{i}\right)^2, T_{i+1} = T_i \cdot \left(\frac{i+1}{i}\right)^3 \Rightarrow$$

$$\Rightarrow R_{i,j} = R_{j,j} \cdot \left(\frac{i^2 + j^2}{j^2 + j^2}\right), T_{i,j} = T_{j,j} \cdot \left(\frac{i^2 + j^2}{j^2 + j^2}\right)^{3/2}$$

$$i, j \in [1, 2, 3, ...],\ n_i = i \cdot N, n_j = j \cdot N, n_1 = 1 \cdot N, n_2 = 2 \cdot N, n_3 = 3 \cdot N, ...$$

Citation from [64]:

*"The application of Eq. (2.11.14)-f can be illustrated by considering the mean planetary radii and orbital periods, in astronomical units, within the solar system. The semi-major axis, which is the same as the mean planetary distance to the Sun, is expressed in terms of the mean distance from the Earth to the Sun (designated as one astronomic unit or AU). In astronomical units, the orbital period of the Earth (one year) defines the unit of time. Mercury is the closest planet to the Sun (hence its orbit corresponds to n = 1), and its observed mean radius (R_1) and orbital period (T_1) are 0.387 AU and 0.241 years, respectively. Based on these values, the other mean planetary radii and orbital periods can be calculated from Eq. (2.11.14)-f by setting **content i** in the range between 1 and 13 (Table 1).*

Regarding planetary orbits where i = 2 and i = 3, the sum of the squares of i , and a second ad-hoc integer j, taking values from 0 to i, must be considered using expressions analogous to those of Eq. (2.11.14)-f. This procedure may be illustrated by reference to the series of orbits with i = 2. Starting from the state i = 2, j = 2, which is associated with the orbit of Mars, the state i = 2, j = 1 corresponds to Earth's orbit and i = 2, j = 0 corresponds to that of Venus. The respective mean planetary radii and orbital periods are given by the following calculations:

$$R_{2,1} = R_{2,2}\left(\frac{2^2 + 1^2}{2^2 + 2^2}\right) = 0.9677,\ R_{2,0} = R_{2,2}\left(\frac{2^2 + 0^2}{2^2 + 2^2}\right) = 0.7742,$$

$$T_{2,1} = T_{2,2}\left(\frac{5}{8}\right)^{3/2} = 0.953,\ T_{2,0} = T_{2,2}\left(\frac{4}{8}\right)^{3/2} = 0.682.$$

in which the values R2,2 = 1.548 and T2,2 = 1.928 correspond to the parameters for Mars. It is worth noting that the splitting of states associated with i equal to 2 or 3 is analogous to the spectral series derived from modern atomic theory. Moreover, this splitting of states occurs in the region corresponding to the distance of Jupiter from the Sun (i = 4) and may be linked with the unusual characteristic of this planet that, together with its many rings and satellites, almost constitutes a mini-solar system in its own right.

Regarding the terrestrial and the gas giant planets, as well as the dwarf planets Pluto, Makemake and Eris, the theoretical mean radii and orbital periods predicted by this model are in reasonable agreement with the observed values (Johnston's Archive; Space and Astronomy, 2010).

Furthermore, there is a significant agreement between the theoretical and observed results (Table 1) regarding the positions of some asteroids found in the solar system. The model predicts, for example, the orbits of the inner (i = 3; j = 0; HIL) and outer (i = 3; j = 1; HOL) limits of the Hungaria asteroids at mean observed radii between 1.780 and 2.000 AU. The asteroids Vesta (i = 3; j = 2) and Camilla (i = 3; j = 3) are correctly located in the inner (2.361 AU) and outer (3.477 AU) rings of the main asteroid belt, which lies between the orbits of Mars and Jupiter and contains approximately 2000 objects orbiting the Sun. The asteroid Chiron, a Centaur object, is positioned between the orbits of Saturn and Uranus at an observed mean radius of 13.698 AU. Moreover, the calculated mean radius of 24.768 AU is associated with the recently discovered asteroids, also Centaur bodies, named Nessus and Hylonome, whose mean distances are 24.617 and 25.031 AU from the Sun, respectively. Additionally, this model predicts the orbit of trans-Neptunian objects in the region of space where the Plutoids are found, including that of the asteroid 1999 DE9 (i = 12) at an observed mean radius of 55.455 AU, a value that accords very well with the theoretical result of 55.728 AU (Johnston's Archive; Space and Astronomy, 2010)."

Table 1 from [64]

Orbital position		Planet / Asteroid	R (=) Mean radius (AU),		T (=) Orbital period (years)	
i	j		Calculated	Observed	Calculated	Observed
1	1	Mercury	0.387	0.387	0.241	0.241
2	0	Venus	0.774	0.723	0.682	0.615
2	1	Earth	0.968	1.000	0.953	1.000
2	2	Mars	1.548	1.523	1.928	1.881
3	0	HIL	1.742	1.780	2.300	2.375
3	1	HOL	1.935	2.000	2.694	2.828
3	2	Vesta	2.515	2.361	3.994	3.630
3	3	Camilla	3.483	3.478	6.507	6.487
4		Jupiter	6.192	5.203	15.424	11.864
5		Saturn	9.675	9.537	30.125	29.433
6		Chiron	13.932	13.698	52.056	50.760
7		Uranus	18.963	19.191	82.663	83.530
8		Nessus	24.768	24.617	123.392	122.420
9		Neptune	31.347	30.069	175.689	163.786
10		Pluto	38.700	39.808	241.000	251.160
11		Makemake	46.827	45.346	320.771	309.880
12		1999 DE9	55.728	55.455	416.448	412.960
13		Eris	65.403	68.049	529.477	558.070

At least, with (2.11.14)-e and (2.11.14)-f, and results from [64], Table 1, we have specific, indicative and intuitive, justification of the relation that connects newly introduced, orbital quantum number $i = 1, 2, 3, ...$, and initial quantum number **n**, as $n_i = i \cdot N$, where **n** is figuring in T.2.3.3, and in equations (2.11.12) - (2.11.14)-a,b,c,d. Here, **N** is specific constant (and integer), valid for all planets of its solar system. This way, we also have an insight regarding a deeper understanding of quantization in stable planetary and asteroid systems, with the overwhelming analogy with Bohr planetary atom model (see also [63], Arbab I. Arbab, and [67] Johan Hansson). From now on we have orbital quantum numbers (taken from [64]) associated with planets of our Solar system, $i \in [1, 10], n_i = i \cdot N$, it will be possible to make additional numerical speculations about gravitational, macrocosmic, Planck-like constant **H**, by introducing specific values of $i \in [1, 10]$ into last, right column of T.2.3.3.

Since constant N can be almost arbitrary big integer (and **n** is like number of days in a year for relevant planet; -see T.2.3.3-1), we can also conclude from (2.11.14), that for specific stable planetary system, exist the common, sufficiently high-frequency time-train (or frequency carrier), which is universally applicable for time-flow counting, for all planets belonging to the same solar system, as follows.

$$\left\{\begin{array}{l} f_o = n\frac{f_m}{2} = n\frac{1}{2T} = n\frac{\sqrt{GM}}{4\pi R^{3/2}} = f_{on}\,,\ T_o = \frac{1}{f_o} = \frac{2T}{n} = \frac{4\pi R}{nv} \\ H = \frac{2\pi m\sqrt{GMR}}{n},\quad n = i\cdot N,\ i = 1,2,3...\ N = \text{integer} \end{array}\right\} \Rightarrow$$

$$f_o = \frac{1}{T_o} = i\cdot N\cdot\frac{f_m}{2} = i\cdot N\cdot\frac{1}{2T} = i\cdot N\cdot\frac{\sqrt{GM}}{4\pi R^{3/2}} = f_{on}\,(=),\ f_m = \frac{2f_0}{n},$$

$$T_o = \frac{1}{f_o} = \frac{2T}{i\cdot N} = \frac{4\pi R}{i\cdot N\cdot v}\,,\ T = T_m = T_y = \frac{1}{f_m} = \frac{2\pi R}{v} = \frac{nT_0}{2} = \frac{n}{2f_o} = \frac{(nH)^3}{4\pi^2G^2M^2m^3}(=)\,\text{one year}\,,$$

$$\tilde{E} = Hf_o = \tilde{E}_n = n\frac{Hf_m}{2} = E_k = \frac{1}{2}mv^2 = mvu = pu = 2mu^2 = G\frac{mM}{2R}. \qquad (2.11.14)\text{-g}$$

This looks like establishing a precise mathematical way for understanding planetary systems synchronization, discretization, gearing, and digitalization, enriching our understanding of stability and integration of planets inside their solar systems (still without any need for using probability and statistics as in modern quantum theory).

As an example, let us creatively apply (2.11.14)-a,b,c, (2.11.14)-g, and standing matter waves concept from Chapter 10, to (all planets of) our Solar system, as found in T.2.3.3-1, which is created (as the spreadsheet, MS Excel table, with 22 columns), using known astronomic data and observations (mostly from very recent NASA publications). We will just start with the obvious fact that the number of days in a year N_{dy} (for every planet), multiplied with one-day time-duration $T_{(1-day)} = \frac{T_y}{N_{dy}} = \frac{T_y}{n} = \frac{T}{n} = \frac{T_0}{2} = \frac{1}{2f_0}$ is equal to the whole year time-duration $T_y = T = T_m$ (of a relevant planet). Consequently, the number of days in a year $N_{dy} = \frac{T_y}{T_{(1-day)}} = n = i\cdot N$ (for every planet), multiplied with one planetary matter-wave wavelength $\lambda_{(1-day)}$ (found for every planet as, $\lambda = \lambda_{(one\ day)} = \frac{H}{p} = V\cdot T_{(one\ day)}$) is equal to the orbital circumference of a relevant planet $2\pi R = n\lambda = N_{dy}\lambda$, calculated using the following relations,

$$\left\{\begin{array}{l} 2\pi r = n\lambda_0 = N_{dy}\lambda_{(1-day)}\,,\lambda_0 = H/p,\ v_n^2R_n = v^2R\ = GM = \underline{const.},\ v \cong 2u = 2\lambda_{(1-day)}f_{(1-day)} \\ H = \frac{2\pi mvR}{n} = \frac{2\pi m\sqrt{GMR}}{n} = \frac{2\pi mGM}{nv} = \lambda_0 p = \lambda_{(1-day)}mv,\ f_0 = f_{(1-day)} = \frac{1}{2T_{(1-day)}} \\ \lambda = \lambda_0 = \lambda_{(1-day)} = vT_{(1-day)} = 2uT_{(1-day)} = \frac{H}{p} = \frac{H}{mv} \Rightarrow H = mv^2T_{(1-day)}\,, N_{dy} = \frac{T_y}{T_{(1-day)}} = n \\ T_{(1-day)} = \frac{\lambda_{(1-day)}}{v} = \frac{\lambda_{(1-day)}}{2u}, u = \lambda_{(1-day)}f_{(1-day)} = \frac{\lambda_{(1-day)}}{2T_{(1-day)}}, v_0 = nv_n = nv = N_{dy}v = n\sqrt{\frac{GM}{R_n}} \end{array}\right\} \Rightarrow$$

$$\Rightarrow H = \frac{2\pi mvR}{n} = \frac{2\pi m\sqrt{GMR}}{N_{dy}} = \frac{2\pi mGM}{nv} = mv^2T_{(1-day)} = \frac{mv^2T_y}{N_{dy}} = \qquad (2.11.14)\text{-h}$$

$$= 2E_kT_{(1-day)} = \frac{E_k}{f_{(1-day)}} = \lambda p = \lambda mv.$$

Relations (2.11.14)-h are almost in full agreement with the helically spinning matter waves concept (associated with moving masses), as elaborated mainly in chapter 4.1. Table T.2.3.3-1 is created using relations from (2.11.14)-h, by applying relevant planetary data (taken from NASA publications). There, we can see that specific data initially known only from astronomic measurements (and from other observations) are getting completely verified, calculable, and confirmable from the here-established conceptual framework of planetary standing matter-waves (see Appendix, Chapter 10, where standing matter-waves concept is additionally summarized). We can also see that only planet Saturn is still an exception (for an order of magnitude) related to predictions from (2.11.14)-h and results from the table

T.2.3.3-1 (see columns 19 and 22). If we would like to make Saturn behave like other planets in relation to T.2.3.3-1, its mean orbital radius should be about 10 times larger, compared to what we presently know regarding Saturn (but the final answer related to the planet Saturn will be more complicated than such simple solution).

T.2.3.3-1 Gravitational Planck Constant & Standing Matter Waves of our Solar System (see columns from 1 to 22)

1	2	3	4	5	6	7	8	9	10
Planets	**Mean Radius of rotation / Semi-major orbital radius around the Sun, R (=) [m]**	**Orbit Circumference = 2πR (=) [m]**	**Planet mass, m (=) [kg]**	**Sun mass, M (=) kg**	**G**	**π**	**Average, Orbital (group) velocity, v (=) [m/s]**	**Average, Orbital phase velocity, $u = \lambda_{(1\text{-day})} f_{(1\text{-day})} = v/2$ (=) [m/s]**	**Linear, orbital moment, p = mv (=) [kg·m /s]**
Mercury	5.79E+10	3.60E+11	3.30E+23	1.99E+30	6.67E-11	3.14	4.74E+04	2.37E+04	1.56E+28
Venus	1.08E+11	6.80E+11	4.87E+24	1.99E+30	6.67E-11	3.14	3.50E+04	1.75E+04	1.70E+29
Earth	1.50E+11	9.40E+11	5.97E+24	1.99E+30	6.67E-11	3.14	2.98E+04	1.49E+04	1.78E+29
Mars	2.28E+11	1.43E+12	6.42E+23	1.99E+30	6.67E-11	3.14	2.41E+04	1.20E+04	1.54E+28
Jupiter	7.78E+11	4.89E+12	1.90E+27	1.99E+30	6.67E-11	3.14	1.31E+04	6.53E+03	2.48E+31
Saturn	1.43E+12	8.96E+12	5.68E+26	1.99E+30	6.67E-11	3.14	9.64E+04	4.82E+04	5.48E+31
Uranus	2.87E+12	1.80E+13	8.68E+25	1.99E+30	6.67E-11	3.14	6.80E+03	3.40E+03	5.90E+29
	4.50E+12	2.83E+13	1.02E+26	1.99E+30	6.67E-11	3.14	5.43E+03	2.72E+03	5.57E+29
AVERAGE	**1.24E+12**	**7.80E+12**	**3.33E+26**	**1.99E+30**	**6.67E-11**	**3.14**	**3.22E+04**	**1.61E+04**	**1.01E+31**

11	12	13	14	15	16	17
Sidereal Orbit period/Period of full rotation around the Sun/Length of Year, T_y (=) [Earth days]	**Sidereal Orbit period/Period of full rotation around the Sun/Length of Year, T_y (=) [s]**	**Sidereal Rotation Period / One self-revolution period / Length of Day (=) Rotation period, $T_{(1\text{-day})}$ (=) [Earth days]**	**Sidereal Rotation Period / One self-revolution period / Length of Day (=) Rotation period, $T_{(1\text{-day})}$ (=) [s]**	**$\lambda_{(1\text{-day})} = vT_{(1\text{-day})} = \lambda = H/mv$ (=) [m]**	**$H = mv^2 T_{(1\text{-day})} = 2 E_k \cdot T_{(1\text{-day})} = E_k/f_{(1\text{-day})}$ (=) [kg·m²/s]**	**Sidereal Number of days in a year (=) [Columns 11,12,13, 14] (=) $T_y/T_{(1\text{-day})} = N_{dy}$**
87.969	7.58E+06	58.646	5.05E+06	2.39E+11	3.74E+39	1.50E+00
224.700	1.94E+07	243.018	2.09E+07	7.33E+11	1.25E+41	9.25E-01
365.260	3.15E+07	0.997	8.59E+04	2.56E+09	4.55E+38	3.66E+02
686.980	5.92E+07	1.026	8.84E+04	2.13E+09	3.29E+37	6.70E+02
4332.820	3.73E+08	0.414	3.56E+04	4.65E+08	1.15E+40	1.05E+04
10755.700	9.27E+08	0.444	3.83E+04	3.69E+09	2.02E+41	2.42E+04
30687.150	2.64E+09	0.718	6.19E+04	4.21E+08	2.48E+38	4.27E+04
60190.030	5.19E+09	0.671	5.78E+04	3.14E+08	1.75E+38	8.97E+04
1.34E+04	**1.16E+09**	**3.82E+01**	**3.29E+06**	**1.40E+11**	**2.02E+40**	**2.10E+04**

18	19	20	21	22
$C = N_{dy}\lambda_{(1\text{-day})} = N_{dy}\, vT_{(1\text{-day})}$ (=) (Column-17) * (Column-15) (=) Orbit Circumference (=) [m]	Orbit Circumference/Orbit Circumference (=) (Column-3)/(Column-18) (=) 2πR/C	Orbital, Kinetic energy, $E_k = mv^2/2 = H \cdot f_{(1\text{-day})}$ (=) $[kg \cdot m^2/s^2]$	$H = \frac{2\pi m\sqrt{GMr}}{N_{dy}}$ (=) $[kg \cdot m^2/s]$	H-constant (Column-21)/ (H-constant Column-16) (=) H/H
3.59E+11	1.002825	3.70E+32	3.83E+39	1.024627
6.78E+11	1.002785	2.98E+33	1.25E+41	1.002949
9.37E+11	1.002751	2.65E+33	4.57E+38	1.003012
1.43E+12	1.002693	1.86E+32	3.31E+37	1.007272
4.87E+12	1.002842	1.62E+35	1.16E+40	1.003635
8.93E+13	0.100281	2.64E+36	2.03E+39	0.010042
1.80E+13	1.002928	2.01E+33	2.49E+38	1.003490
2.82E+13	1.002624	1.51E+33	1.75E+38	1.002312
7.78E+12	**1.002779**	**2.45E+34**	**2.02E+40**	**1.006757**

In the table T.2.3.3-1, column 17, and earlier in T.2.3.3-a, we can find that calculated number of days in a year, $n = N_{dy}$ (in relation to H constant) is not an integer (as under ideal and mathematically preferable conditions should be), since here we are operating with mean or average values of related orbital parameters (and still neglecting involved spin characteristics). This is also linked to the reference platform from where our astronomic measurements are valid, and to the fact that solar or planetary systems are dynamically stable, space and time-evolving motions.

It is evident that in T.2.3.3-1, we are getting significant results (see Columns 19 and 22) implicitly because all planets of our solar system (including the Sun) have orbital and spinning moments. In column 19, we find that values of planetary orbits circumferences calculated in two different ways (compared to known astronomic measurements, as in column 3, and to standing macro matter-waves concept, as in column 18) are producing almost identical values. In column 22, we can also find that macrocosmic **H** constant values, calculated in two different ways (one based on known astronomic data, and the other based on standing macro-matter-waves concept), are mutually almost identical. This way, we are building the legitimacy of standing macro matter-waves concept in relation to planetary systems. Such a situation should be much better exploited to enrich our understanding of periodicity and standing matter waves quantization within stable solar systems. For instance, planet Earth's Moon is (helically) rotating around planet Earth, and its mean orbit circumference is 2.41E+09 m. In the column 15 of T.2.3.3-1, we can find that "1-day" Earth wavelength, $\lambda_{(1\text{-day})} = vT_{(1\text{-day})} = \lambda = H/mv$ is 2.56E+09 m, not very much different from 2.41E+09 m, meaning that Earth's Moon should be on some way captured or channeled by helical macro-matter-wave field associated to planet Earth's orbital motion. ***Since calculated H constants (columns 16 and 21) are still too much mutually different, this indicates that additional, new quantizing, or new standing waves parameters should be considered, meaning that presented modeling is still oversimplified. The most promising strategy here would be to consider specific electromagnetic background involved in the structuring of planetary systems.***

[♣ COMMENTS & FREE-THINKING CORNER:

Solar and/or planetary systems adhere to the principles of standing waves and spatial arrangements, a concept that also extends to galaxies, albeit in a more complex manner. In these systems, standing waves manifest as radial and angular or circular formations. Each solar system, including its planets and the local sun, operates as an integrated, synchronized structure of spatial, macrocosmic standing waves. These standing waves can be temporarily considered as waves of the gravitational field;

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

however, their fundamental nature is more likely rooted in electromagnetic phenomena, which are naturally coupled with acoustic or mechanical oscillations.

Standing waves are characterized by attractive forces concentrated at nodal points or zones. These forces can be demonstrated and measured through experiments with ultrasonic resonators that produce standing waves, as well as through acoustic levitation effects observed in air or other fluids (see references [150] and [151]). In these scenarios, particles or masses aggregate and stabilize at nodal points of the standing waves. This analogy may also apply to planetary systems, where each planet or mass could be viewed as forming part of an agglomerated spatial-temporal standing wave structure. Acoustic standing wave levitation effects may extend to similar electromagnetic standing waves, given that masses consist of electromagnetically polarizable atoms (refer to Konstantin Meyl's work in [99]).

The presence of standing waves necessitates an external source of vibrations. It is hypothesized that such a source exists within the cosmic background of our universe, driving resonant oscillations and creating a complex, multidimensional resonating environment. For large-scale systems, resonant frequencies could be extremely low, even below 1 Hz. The existence of cosmic matter waves and their quantization, as discussed in this chapter, aligns with observable astronomical measurements.

When standing waves involve matter or masses, electromagnetic dipoles (or multipoles) are also organized within the same standing wave structure. Atoms, composed of electric charges and possess spin and magnetic properties, contribute to this organization. Electric charges within neutral atoms can become polarized, creating spatially oriented electric dipoles due to accelerated motions, as electrons are significantly lighter than protons. Additionally, the internal spinning of atomic particles creates small magnets or magnetic moments, which align within the macroscopic standing wave structure. This results in macroscopic 3D formations of gravitational and mass-related standing matter waves, which coincide with standing electromagnetic waves.

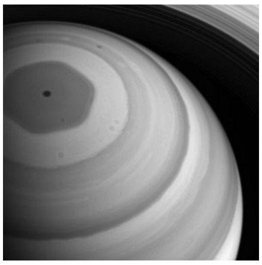
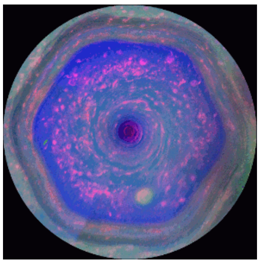
Assuming an ideal vacuum contains a fine, fluidic medium (referred to as "ether") that behaves like an ideal gas, this medium would possess measurable electromagnetic properties, such as magnetic and electric permeability. This ether would be a weak carrier of electromagnetic fields and waves, suggesting it has a material nature that allows it to support electromagnetic oscillations and forces. The ether's properties imply that gravitation, while weaker compared to electromagnetic forces, operates within the spatial-temporal matrix of this ether fluid.

Professor Jovan Djuric has demonstrated that small non-magnetic masses, including various metals and organic materials, can self-orient in alignment with local geomagnetic lines, reflecting the influence of the local magnetic field on these masses. Although these effects are subtle and difficult to observe, Djuric's experimental findings and mathematical models provide a basis for understanding how organic and inorganic masses respond to external magnetic fields, contributing to what we currently conceptualize as gravitational forces. In macrocosmic environments, such as planetary or solar systems, standing wave formations are expected, and within these standing matter waves, we should detect synchronized magnetic and electric fields, analogous to acoustic levitation effects.

This framework closes the loop on understanding gravitational attraction as resulting from the nodal spots of standing waves and their associated forces. This perspective aligns with Nikola Tesla's and Rudjer Boscovich's ideas about dynamic gravitation and universal natural forces. Tesla speculated about a streaming or flow of "radiant energy" between mutually attracting masses, analogous to ether streaming, while Boscovich qualitatively described a universal force acting between and within all masses. This conceptualization can be integrated with ether-flow effects and the circulation of kinetic and potential energy in standing waves. **♣]**

Saturn rings should present a perfect (observational) case of specific *standing-waves-like* mass density distribution, where we could search for "signatures" and effects of associated orbital standing waves (or gravity related matter waves). Since Saturn also has a strong magnetic field, and its rings are rotating (becoming somewhat electrically charged and polarized), we could conceptualize specific electromagnetic explanation of the structure of Saturn rings, analogical to N. Bohr model. We could also speculate that involved gravitational nature and attracting force effects (about Saturn and its rings) are direct consequences of a primarily electromagnetic phenomenology, since isolated and static magnetic and electric field components cannot exist as mutually separated in dynamic (motional) situations. This way we will imaginatively enter the space of Nikola Tesla Dynamic Gravity speculations.

Citation from https://solarsystem.nasa.gov/news/531/saturns-famous-hexagon-may-tower-above-the-clouds/ "A new long-term study using data from NASA's Cassini spacecraft has revealed a surprising feature emerging at Saturn's northern pole as it nears summertime: warming, a high-altitude vortex with a hexagonal shape, akin to the famous hexagon seen deeper down in Saturn's clouds.
The finding, published Sept. 3 in Nature Communications, is intriguing because it suggests that the lower-altitude hexagon may influence what happens above and that it could be a towering structure hundreds of miles in height.


"The edges of this newly found vortex appear to be hexagonal, precisely matching a famous and bizarre hexagonal cloud pattern we see deeper down in Saturn's atmosphere," said Leigh Fletcher of the University of Leicester, lead author of the new study.
Saturn's cloud levels host most of the planet's weather, including the pre-existing north polar hexagon. This feature was discovered by NASA's Voyager spacecraft in the 1980s and has been studied for decades; a long-lasting wave potentially tied to Saturn's rotation, it is a type of phenomenon also seen on Earth, as in the Polar Jet Stream.

For more on the new study, visit the European Space Agency's story here: http://sci.esa.int/cassini-huygens/60589-saturn-s-famous-hexagon-may-tower-above-the-clouds/"

[♣ Anyway, Newton-Kepler foundations of gravitation can be presently understood mostly as the best intuitive guess about planetary orbits fitting, based on observations, while several structural and theoretical miss-concepts and autocorrecting steps are approximately and creatively implemented, producing still sufficiently useful mathematical models. This will have a significant impact on our future and improved understanding of Gravitation, orbital motions, and micro-world modeling of motions within atoms. See the following citation from [127].

Non-Conservativeness of Natural Orbital Systems

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The Newtonian mechanic and contemporary physics model the non-circular orbital systems on all scales as essentially conservative, closed path zero-work systems and circumvent the obvious contradictions (rotor-free 'field' of 'force', in spite of its inverse proportionality to squared time-varying distance) by exploiting both energy and momentum conservation, along specific initial conditions, to be arriving at technically more or less satisfactory solutions, but leaving many of unexplained puzzles. In sharp difference to it, in recently developed thermo-gravitational oscillator approach movement of a body in planetary orbital systems is modeled in such a way that it results as consequence of two counteracting mechanisms represented by respective central forces, that is gravitational and anti-gravitational accelerations, in that the actual orbital trajectory comes out through direct application of the Least Action Principle taken as minimization of work (to be) done or, equivalently, a closed-path integral of increments (or time-rate of change) of kinetic energy. Based on the insights

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[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

gained, a critique of the conventional methodology and practices reveals shortcomings that can be the cause of the numerous difficulties the modern physics has been facing: anomalies (as gravitational and Pioneer 10/11), three or more bodies problem, postulations in modern cosmology of dark matter and dark energy, the quite problematic foundation of quantum mechanics, etc. Furthermore, for their overcoming, indispensability of the Aether as an energy-substrate for all physical phenomena is gaining a very strong support and based on recent developments in Aetherodynamics the Descartes' Vortex Physics may become largely reaffirmed in the near future.

1. Introduction

Following the Newton's fitting of elliptical planetary orbits to the single central force inversely proportional to the square of its distance to the Sun, all natural systems
- from atomic to galactic scales - have been treated as non-conservative (work over closed loop in the field of potential force equaling to zero). The exclusive reliance on gravitation as the only central force does not allow for the formally exact prediction of the planet's trajectories in accordance with the Kepler's First law [1], and furthermore orbit fitting to an elliptical shape is contingent on the initial conditions [2]. The basic shortcoming of Newton's theory of orbital motion is the presumed absence of the tangential acceleration component, quite contrary to well established observational results, which is deduced either from the 'naive' interpretation of the Kepler's Third law, which actually is related to the aver- age values of the orbital radius and elapsed time, or from the improper interpretation of Kepler's Second law as angular momentum, its presumed constancy implying only the circular motion.

For theoretical foundations and practical calculations, the factual time-dependence of the force (thus non-zero rotor field) is neglected, and one proceeds from the constancy of the sum of kinetic and potential energies, on one side, and the constancy of the angular momentum, on the other, although in actuality neither of the two is the case.

Only recently, within explorations of biological molecular systems, as well as in certain domains of particle physics, the need starts arising for looking at such systems as non-conservative, the so-called "open systems", which within the classical formalisms turn out to become the "non-integrable" orbital systems (inability to be reduced to "circular coordinates" by even applying the time-varying transformations of the coordinate systems). This has led to modifications and specializations of the formalisms of the classical axiomatic mechanics having been developed by Euler, Lagrange, Hamilton, Noether and others for essentially conservative systems to be applicable to the non-conservative ones. However, a critical analysis of the matters suggests that all the natural orbital systems are open, that is non-conservative (including the planetary, atomic and galactic ones), and that neither the energy nor the (angular) impulse is constant over the time, so that the very basic foundations turn out to be erroneous.

………………..

Another resurfacing of the work not intended for publication is Feynman's scrutinizing and attempting to over- come the noticed week point in Newton's geometrical fit- ting of elliptical orbits to the central force inversely proportional to the squared distance is the above first cited [1], where Feynman had attempted to correct the inconsistency of Newton's geometrical fitting of the elliptic path to the squared distance inverse central force. It is deplorable indeed, that Feynman did not persevere and was not able to apply his favorite Least Action Principle to that problem, instead of stepping into the further support the otherwise unsoundingly set-up quantum mechanics by calculation of the (notably non-zero!?) works on all possible paths of an electron and assigning their reciprocal values to the probabilities, and further going into quite controversial development of the "Quantum Gravity".

…………………

2. Critique of the conventional approach in solving the Kepler's/Newton's problems

When it comes to determining the intrinsic feature of an orbital system, that is whether is it conservative or non-conservative, by all means of prime importance is the topic of a system energy balancing — evaluation of difference between the de-facto performed work and the (knowingly) available applied energy (re)sources.

……………………….

If the former exceeds the latter, or if the traditionally conceived and established law of sum of kinetic and potential energy conservation does not 'hold', we must be missing the awareness of the true nature mechanisms and the availability of the unaccounted for 'environmental' effective energy input(s).

As the historically firstly considered, the Sun's planetary orbital system should indeed be the right one for these considerations, in particular that the established theory and its further developments have detrimentally affected all other physics' and in general science do- mains — form the atom-to galactic-levels, and from chemistry to biology. In direct relation to the orbital energy balancing stands the concept of energy conservation with the related work over a closed path being equaled to zero, as intrinsic feature of the so-called potential fields (the 'central' force vector field having form of gradient of a scalar potential field). …………..♣]

As an example, how to consider mutually coupled (and mutually interacting) orbital and spinning moments (of solar systems) as vectors, it is sufficiently illustrative to see familiar conceptualization in chapter 4.1, presented by the table "T.4.2.1, Analogies Between **n**-Body Coupled Inertial Motions in a Laboratory System". Such approach should result in the more precise numerical estimation of macrocosmic Planck constant **H**, since here, for every planet, we have different **H** constant because we are neglecting orbital and spinning moments as mutually coupled vectors (see (2.11.14)-h and T.2.3.3-1, columns 16 and 21). All orbital and spinning moments of specific solar system are so well mutually integrated and coupled, that effective, planetary moments should be established somewhat similar as in two-body problem, where we will create central and reduced moment of inertia, as well as center of inertia angular velocity, and relative angular and spinning speeds for every planet. This will be like reduced and center of mass terms in the two-body problem but now using terms of rotational and spinning motions.

For instance, the individual solar or planetary system can be characterized by the following set of parameters:

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$$
\begin{aligned}
& m_i (=) \text{mass of certain planet, } i \in [1, 2, 3,...] \\
& M_s (=) \text{mass of the local sun} \\
& \vec{v}_i (=) \text{ planet orbital velocity} (=) \ \vec{\omega}_i R_i \\
& \vec{v}_s (=) \text{sun orbital velocity (relative to local galaxy center)} \\
& \vec{\omega}_i (=) \text{planet angular velocity} \\
& R_i (=) \text{planet mean orbital radius} \\
& I_i (=) \text{planet moment of inertia} (\cong) \, m_i R_i^2 \\
& I_{sun} (=) \text{sun moment of inertia} \\
& \vec{L}_i (=) I_i \vec{\omega}_i \cong m_i R_i^2 \vec{\omega}_i = m_i R_i \vec{v}_i \, (=) \text{planet orbital moment} \\
& \vec{L}_{si} (=) \text{planet spin moment} (=) \, I_s \vec{\omega}_s \\
& \vec{L}_{sun} (=) \text{total angular moment of local sun} (=) \, I_{sun} \vec{\omega}_{sun}
\end{aligned}
$$

Since solar systems are (sufficiently and longtime) stable, we can consider that some of the orbital and spin moments of all planets, and the local sun is conserved (or constant), and this way we will be able to determine the value of local macrocosmic Planck constant $\mathbf{H}$, as,

$$
\begin{aligned}
& \vec{L}_{total} = \vec{L}_{sun} + \sum_{(i)} (\vec{L}_i + \vec{L}_{si}) = I_c \vec{\omega}_c = \overrightarrow{const.} \Rightarrow \left| \vec{L}_{total} \right| = \frac{H}{2\pi} \Rightarrow \\
& \Rightarrow \left| \vec{\omega}_c \right| = \frac{\left| \vec{L}_{total} \right|}{I_c} = \frac{H}{2\pi I_c} = 2\pi f_c = \frac{2\pi}{T_c} \Leftrightarrow H = 4\pi^2 I_c f_c = \frac{4\pi^2 I_c}{T_c}.
\end{aligned} \tag{2.11.14)-i}
$$

Apparently, in a larger picture, if we attempt to determine unique value of macrocosmic $\mathbf{H}$ constant, we should not neglect the contribution of all (involved) orbital and spin moments, as well as participation of associated, mutually coupled electromagnetic and other fields and forces within solar systems (as roughly conceptualized in (2.11.14)-i). If such $\mathbf{H}$ is a stable and constant value, we could speculate around "entanglement ideas" that all mutually coupled orbital and spin moments within the specific stable solar system are communicating at enormously high speed (or instantaneously).

With (2.11.14)-i we are effectively presenting (or replacing) a whole solar or planetary system with a single, central, spinning solar mass M_c, which also has its center of mass velocity $\vec{v}_c$ (relative to local galaxy center), as follows,

$$
M_c = M_s + \sum_{(i)} m_i \ , \ \vec{v}_c = \frac{M_s \vec{v}_s + \sum_{(i)} m_i \vec{v}_i}{M_c}. \tag{2.11.14)-j}
$$

If we consider our local galaxy center as a new reference frame, our solar system M_c will make orbital motion around galaxy center (having orbital velocity $\vec{v}_c$), and planets will create progressive helical movements around mass M_c and direction of $\vec{v}_c$ (as in (2.11.14)-j). Orbital motions of planets about the local sun are elliptical or close to circular (in the reference system linked to the local sun), but this is because we neglect that complete solar system is also orbiting about its local galaxy center (observed from the reference system linked to a galaxy center in question). This way, we are again coming to clear helical motions concepts that are in any case associated with linear movements, like elaborated in chapter 4.1 (valid both for micro and macro physics world). The much simpler example for visualizing such helical motion is rotating (or orbiting) movement of the specific moon about its local planet (from the reference system linked to the local sun or local planetary system center of mass). Here is also the more in-depth background or nature of matter waves and particle-wave duality (and not at all in a probability or possibility of "*chances that something could happen*"). Modern micro particles accelerators and colliders also generate streams of energy-momentum products that respect similar unity of linear and spinning motions, particle-waves duality, and helical motions framework.

On specific planetary systems, we can attempt to create and apply the most common, universally valid, space-time measuring referential frame linked to the local galaxy center, on the following way. Mass M_c *from (2.11.14)-j will be placed in the center of the (newly created), orthogonal axes* (*X-Y*) *plane, and* *Z*-*axis will be perpendicular to such* (*X-Y*) *plane and collinear (or coaxial) with the center of mass velocity* $\vec{v}_c$. *Total spinning moment of mass* M_c *will have its value* $\vec{L}_{total}$ *relative to such* (*X, Y, Z*) *coordinate system (using similar mathematics as in (2.11.14)-i). At the same time, we can place another* ***(x, y, z)*** *reference system (linked to the center of a local solar system) where* M_c *is again in the center of orthogonal axes* ***(x-y)*** *plane, and perpendicular* ***z****-axis has the direction of* $\vec{L}_{total}$. *Of course,* ***(X-Y)*** *and* ***(x-y)*** *planes in general case are not overlapping, except having common (0,0) and (0, 0, 0) center, and* $\vec{L}_{total}$ *will be different in* (*X-Y-Z*) *and* (*x-y-z*) *coordinates. We will always be able to use coordinates rotation and make connections between orthogonal* ***(x, y, z)*** *and* ***(X, Y, Z)*** *reference frames. The proposed concept is effectively describing an equivalent (and big, remarkably high mechanical quality factor) gyroscope replacing the complete solar system and could be more complicated than here simplified example. Such an equivalent gyroscope is spinning, making precessions around different axes, and at the same time, creating a large-scale rotational motion or orbiting in the* ***(X, Y, Z)*** *coordinate system (linked to galaxy center). Such solar-system gyroscope has its center of mass velocity* $\vec{v}_c$, *its mass* M_c, *and its linear, orbital, and spinning moments, including electromagnetic moments, and associated electromagnetic properties and charges.*

Now, we can introduce the hybrid, four-dimensional, space-time coordinate system ***(x, y, z, t)*** *where the time flow or time direction (or time axis) will be linked to the center of mass velocity* $\vec{v}_c$ *(from (2.11.14)-j), similar as in Minkowski space of Relativity theory, using the planetary coordinates basis* $(x, y, z, Iv_c t)$. *Here, "****I****" presents Hypercomplex imaginary unit (*$I^2 = -1$*), composed from three more elementary imaginary units, as introduced in chapter 4.0. This way, we will make the most natural space-time frame for describing and visualizing helical planetary motions in relation to local galaxy center (see the picture below, taken from [57], showing different aspects of an Analytic Signal of specific attenuated oscillatory process), or for visualizing helical motions of moons around certain planet.*

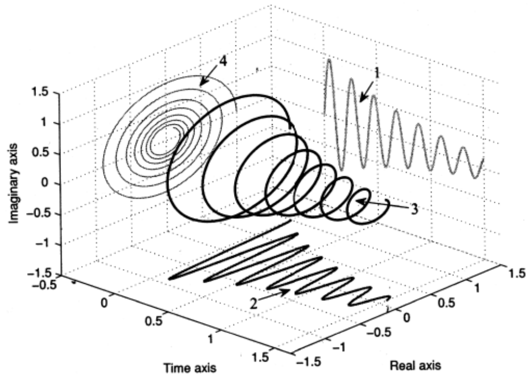
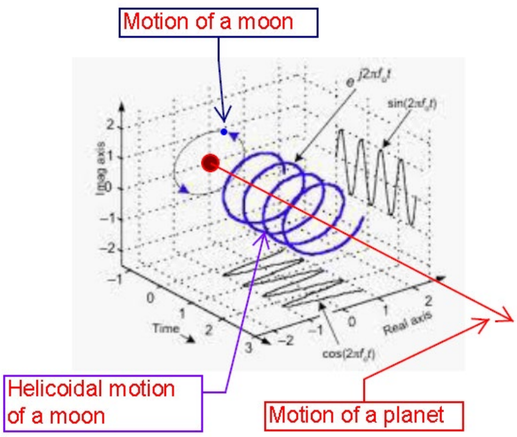


Figure 2.3 The HT projection (1), the real signal (2), the analytic signal (3), and the phasor in complex plain (4) (Feldman, ©2011 by Elsevier)



The ultimate evolution of such conceptualization is to arrive to unite and extended Minkowski, 4-vectors Hypercomplex space (x, y, z, Ict) *and Hypercomplex Analytic Signal functions (based on Hilbert transform, HT), to characterize different motions and associated energy-momentum relations (as elaborated in chapters 4.0., 4.3. and 10.). If we intellectually, creatively, and philosophically extrapolate this situation, we will understand that similar conceptualization applies to the whole micro and macro universe.*

What is relatively new and original in the discussion surrounding equations (2.11.12) - (2.11.14)-a,b,c,…) is the introduction of a framework that integrates group and phase velocities with orbital and planetary motions. This framework aligns with universally valid wave-motion concepts, including Lorentz transformations and the relativistic four-vectors of energy-momentum relations. This approach implicitly introduces the concept of a wave group or wave packet associated with orbital and planetary matter waves, analogous to the quantum theory's wave packets and wave functions. This opens the possibility for applying wave functions and wave equations in modeling planetary systems, like how Schrödinger and Heisenberg applied these concepts in quantum theory.

Further development of this modeling could incorporate ideas like the Bohr-Sommerfeld quantization conditions, as discussed in the Appendix, Chapter 10: "PARTICLES AND SELF-CLOSED STANDING MATTER WAVES." The early quantum mechanics model proposed by N. Bohr and his followers is more naturally applicable to the matter waves of planetary systems than to atomic models. There is clear and substantial analogy and connection between micro and macro world matter waves, as shown in references [63], [64], [67], and [68].

Using equations and relations from (2.11.14)-a,b,c,…, we can predict and verify surprisingly accurate quantization of celestial orbits within specific solar systems, as documented in [38], [39], and [64]. This supports the validity of the planetary standing wave field structure discussed in this book. Although the orbital periods of planets are extremely long and their associated frequencies are small, this is a matter of perception and measurement reference systems. Given that gravitation is weaker compared to electric and magnetic forces, and that Newton's and Coulomb's force laws are mathematically identical, gravitation might be an indirect manifestation of electromagnetic forces.

As discussed earlier in this book, small electric and magnetic dipoles or multipoles create unbalanced electromagnetic distributions due to the significant mass difference between electrons and protons, which are constantly rotating and spinning. These effects can be interpreted as manifestations of gravitational forces (see equations (2.4-6) - (2.4-10)). The quantization effects observed in planetary systems, which imitate quantization in electromagnetic fields like atomic systems, suggest that planetary systems may have a more significant electromagnetic nature than currently recognized (see [67]). This implies that magnetic fields, electric currents, and charges are crucial in maintaining the dynamic and stable structure of planetary systems, applicable across the universe on both micro and macro scales.

The key point here is that the same mathematical concepts can be applied to both micro and macro scales, provided we maintain intellectual flexibility and avoid taking concepts too literally. This applies to planetary systems with gravitational forces as well as Coulomb and other electromagnetic interactions between charged microparticles within atoms. Once this foundation is established, it becomes feasible to apply Schrödinger's wave quantum mechanics to planetary systems, as will be explored in equations (2.11.20) – (2.11.23). However, there is a significant difference between Schrödinger's wave mechanics in quantum theory and its application to planetary systems. The probabilistic and stochastic nature of quantum wave functions may not be necessary or appropriate for addressing all matter waves in both micro and macro contexts, except where mathematical conditions support such modeling.

Nevertheless, the mathematics of Quantum Theory remains a valuable and complementary tool for modeling matter waves, providing useful results even if its foundational concepts are not entirely realistic. It is remarkable how the founders of Quantum Theory created a sophisticated and abstract mathematical structure that, despite its non-realistic nature, produces beneficial results. The widespread admiration and acceptance of Quantum Theory, often to the exclusion of alternative views, highlights the influence of its creators.

The accuracy of predictions based on equations (2.11.14)-a,b,c,…, and their potential, is related to several approximations and omissions in contemporary mechanics regarding stable planetary systems:

a) We assume that planetary masses are small, homogeneous, isotropic solid spheres compared to the central sun, which is considered at rest without rotation. Planetary systems, including their suns, rotate around a common center of mass, and the sun does not align perfectly with this center. Planets vary in composition and structure, with some having liquid cores and differing moments of inertia. More accurately, the Solar System moves through space, orbiting the center of its galaxy, with planets following spiraling or helical paths when viewed from a galactic reference frame.

b) We often approximate that all planets and the sun orbit in a nearly flat plane, which is not always the case. Orbital and rotational moments should be considered as vectors in calculations of macrocosmic constants.

c) Interplanetary, orbital, and electromagnetic interactions are frequently neglected, with dominant forces and fields between each planet and its sun being assumed.

d) The effects of planetary spin on angular momentum conservation are not always precisely accounted for.

e) The influence of a solar system's motion relative to its local galactic center, which produces helical planetary paths, is not thoroughly considered. Solid bodies, like planets and satellites, can be viewed as conductive masses, leading to the idea that gravitational forces might be analogous to electromagnetic forces between current-carrying wires, as suggested by Nikola Tesla.

f) If planetary orbit quantization based on standing waves is valid, additional angular quantum numbers related to orbital and spin moments should be introduced.

....

Let us go back to planetary systems, where our reference frame is linked to the common center of inertia (or center of mass) of such system (not considering motions relative to galactic center…). It will become evident that expression for planetary macro-wave (motional or kinetic) energy, $E_k = Hf_o$ from (2.11.13), is directly analog to Planck's narrow-band, wave-packet energy of a photon, $\tilde{E} = hf$, as well as macro equivalent for a wavelength $\lambda = \frac{H}{p}$ is also analog to a micro-world de Broglie matter wavelength $\lambda = \frac{h}{p}$, where new "macro-world Planck-like constant" H is,

$$\begin{aligned} & H = H(m,R,n) = \frac{2\pi}{n}L = \frac{2\pi}{n}\frac{GMm}{v} = \frac{2\pi m\sqrt{GMR_n}}{n} = \frac{2\pi GMm}{v_0} = \\ & = \frac{2\pi R_n vm}{n} = \frac{\tilde{E}}{f_o} = \tilde{E}T = \text{const.} >> h, \ (n \in [1,2,3...], v << c, \ m << M) \\ & \Rightarrow \left(\hbar = \frac{h}{2\pi}\right) \text{analog to} \left(\hbar_{gr.} = \frac{H}{2\pi} = \frac{L}{n} = \frac{m\sqrt{GMR_n}}{n} = \frac{GMm}{v_0}\right). \end{aligned} \tag{2.11.15}$$

The difference between Planck's constant h and analog constant of planetary macro waves H is that h is already known as universally valid constant (for world of atoms elementary particles and photons), and H could be different for every planetary and satellite system… Of course, for specific planetary system, and for a sufficiently high integer $n = n_{max.}$ in (2.11.15), we should be able to find when H will be equal to h, as for instance,

$$H = h = \frac{2\pi m\sqrt{GMR}}{n_{max.}} = \frac{2\pi GMm}{v_0} = 6.626\ 0693(11)\cdot 10^{-34}\, J\cdot s,$$
$$n_{max.} = v_0\sqrt{\frac{R}{GM}} = 0.948252278\cdot 10^{34}\cdot m\sqrt{GMR} = 0.77443828\cdot 10^{29} m\sqrt{MR},$$
$$(v_0 = 0.77443828\cdot 10^{29}\sqrt{G}\cdot m\cdot M),$$

but such high quantum numbers are obviously unrealistic for characterizing macrocosmic objects and planetary systems.

In fact, for the specific planetary system (where each of planets concerning a common Sun could be approximated as a Binary System), we should be able to find H that will be the same constant for each planet. For instance (see (2.11.14) and (2.11.14-20)), if the ratio between any two of H constants (2.11.16), apply for planets (with circular orbits) from the same solar system, should be equal to one, then we can say that H is at least locally applicable constant, as for instance,

$$H = H_1 = H_2 \Rightarrow \frac{H_1}{H_2} = \frac{H(m_1, R_1, n_1)}{H(m_2, R_2, n_2)} = \frac{n_2}{n_1}\frac{m_1}{m_2}\sqrt{\frac{R_1}{R_2}} = \frac{n_2}{n_1}\frac{m_1}{m_2}\frac{v_2}{v_1} = 1,\ (n_1, n_2)\in[1,2,3...),$$
$$\Rightarrow \left\{n_2\cdot m_1\cdot\sqrt{R_1} = n_1\cdot m_2\cdot\sqrt{R_2} \Leftrightarrow n_2\cdot m_1\cdot v_2 = n_1\cdot m_2\cdot v_1\right\}\Rightarrow$$
$$\frac{L_2}{L_1} = \frac{m_2}{m_1}\frac{v_1}{v_2} = \frac{n_2}{n_1},\ \frac{L_2}{n_2}\frac{n_1}{L_1} = \frac{n_1}{n_2}\frac{m_2}{m_1}\frac{v_1}{v_2} = 1 \Rightarrow \qquad (2.11.16)$$
$$\frac{H}{2\pi} = \frac{L_1}{n_1} = \frac{L_2}{n_2} \Rightarrow \frac{\left|\vec{L}_1 + \vec{L}_2\right|}{n_1 + n_2} = \frac{\left|\vec{L}_i\right|}{n_i} = \frac{GMm}{v_0} \Rightarrow \frac{H}{2\pi} = \frac{\sum_{(i)}\left|\vec{L}_i\right|}{\sum_{(i)} n_i} = \frac{GMm}{v_0}.$$

The limitations of **H** constant expressions related to (2.11.15) and (2.11.16) are that we still approximate all orbits with circles (which are in the same plane) and we do not consider any planetary "self-spinning" momentum. Another limitation involved here is that orbital and spin moments conservation is entirely valid only if we bring in consideration (as a vector) a resulting total moment (including spin moments) of all planets, moons, and satellites of a solar system in question (see [36], Anthony D. Osborne, & N. Vivian Pope).

Consequently, after implementing more elaborated analyses, we should be able to find more general and more precise expressions for **H** (see (2.11.14)-i,j). Present comments regarding planetary-world **H** constant are still indicative brainstorming directions serving to establish the grounds for defending the utility of such constants. Another exciting situation here is how to explain that micro-world **h**-constant (or Planck constant) is unique and universally valid for all atomic and subatomic entities (or we just consider it as universally valid). Do we have (in our Universe) a succession of **H**-constants, starting from certain big **H**-numbers (for galactic formations) which are gradually descending towards smaller numbers with unique and constant **h**-value at the opposite subatomic end, could be a question to answer? *One common fact is almost apparent: **h** or **H** constants are products of stable, periodical, circular, or closed domain (standing waves) motions where orbital moments are conserved (meaning constant).*

[♣ COMMENTS & FREE-THINKING CORNER (still in preparation and brainstorming phase)**:**

Here we are combining dynamics of orbital motions with certain kinds of stable space packing expressed by the necessity of standing waves formation, which is in close relation to proper angular (and spin) moments conservation, also applicable to inclinations of planetary orbits. In addition, since certain wave-like spatial-periodicity and stable packing in periodical planetary motions exist, it could be presentable as integer multiple "n_α" of angular segments $\alpha = \frac{2\pi}{n_\alpha}$, capturing the angle of a full circle that is in average equal $n_\alpha \alpha = 2\pi = n\lambda_o / R,\ n_\alpha = 1,2,3...$ (of course, indicative for an idealized and oversimplified situation, just to give a direction of thinking about angular quantizing),

$$\begin{aligned} & L = n\frac{H}{2\pi} = \frac{n}{n_\alpha}\cdot\frac{H}{\alpha} = \frac{H}{\lambda_o}R = \frac{2\pi\sqrt{GM}}{n\lambda_o}\cdot mR^{3/2} = m\sqrt{GMR}, \\ & \alpha = \frac{2\pi}{n_\alpha} = \frac{n}{n_\alpha}\cdot\frac{H}{L} = \frac{n}{n_\alpha}\cdot\frac{\lambda_o}{r} = \frac{2\pi\sqrt{GM}}{n_\alpha L}\cdot mR^{1/2},\ (n, n_\alpha) \in [1,2,3,...). \end{aligned} \tag{2.11.17}$$

For instance, the spherical coordinate system (which should naturally be most applicable here) has one radial coordinate and two angular coordinates, and we should use a minimum of three different quantum numbers for describing such spatial standing waves packing. The more general approach regarding inclinations of planetary orbits, instead (2.11.17) should be an angular or spatial quantizing of relevant orbital matter waves, like in A. Sommerfeld quantization and semi-classical quantization of angular momentum (see [40], D. Da Roacha and L. Nottale). See later more of supporting background under "Wavelength analogies in different frameworks", T.4.2, as well as extended matter-waves conceptualization with equations 4.3-1, 4.3-2, 4.3-3 and Fig.4.1.2, Fig.4.1.3 and Fig.4.1.4, all from Chapter 4.1.

The ideas, modeling and documented astronomic observations about solar systems quantization of orbital radius and relevant velocities (like results in (2.11.14)) are already known from the publications of William Tifft, Rubćić, A., & J. Rubćić, V. Christianto, Nottale, L. and their followers (see literature under [37], [38], [39], [40], [41], and [42]). The possibility, suggested by the observation of velocity quantization (72 km/s, Tifft, [37]) in the redshifts of galaxies, that wave-particle duality with a much larger value of Planck's constant may apply at galactic distances is also examined. For instance, in (2.11.14), we have the phase velocity found as, $u = u_n = \frac{1}{2}\sqrt{\frac{GM}{R}} = \frac{v_0}{2n} \cong \frac{v}{2}$. The galactic (phase) velocity redshifts measured by Tifft are often found to be around 72 km/s (see [37]), and the best-known estimate for specific (galactic) velocity v_0 is, $v_0 = 144.7 \pm 0.7\left[\frac{km}{s}\right] = 2\,x\,72.35 \pm 0.35\left[\frac{km}{s}\right]$ (see (2.11.14) and (2.11.14)-a). Of course, for higher values of quantum numbers $n = 2, 3$, we should be able to detect other galactic (phase-velocity related) redshifts such as, $u = \frac{v_0}{2n} \cong \frac{144}{2n} = \frac{72}{n}\left[\frac{km}{s}\right],\ n = 1,2,3,... \Rightarrow$ $u \in (\frac{72}{2} = 36, \frac{72}{3} = 24, \frac{72}{4} = 18...)\left[km/s\right]$. It is also found that orbital velocities (see (2.11.14)) of planets and satellites belonging to our Solar System, $v_n = \sqrt{\frac{GM}{R_n}}$ multiplied by $\mathbf{n}$ ($\mathbf{n} = \mathbf{1, 2, 3...}$) are equal to the multiple of a fundamental velocity, which is close to 24$\left[km/s\right]$. Also, increments of the intrinsic galactic redshifts are found to be $\cong 24\left[km/s\right]$ (see [40] Nottale; [41] Rubćić, A., & J. Rubćić; [43] M. Pitkänen), very much like the predictable situation regarding quantized orbital, planetary and satellite velocities from (2.11.14). Surprisingly, in here mentioned literature regarding redshifts, nobody related such case to phase velocity on the way as conceptualized here (related to orbital, macro-cosmological matter waves). Since measured spectral redshifts are really affected by orbital phase velocity $u = u_n$, it is almost evident that here hypothesized standing-waves field structure should exist. ♣]

We can conceptualize the same problematic (related to redshifts and, why not to "blueshifts") concerning matter-wave duality in the following way:

-An observer on our planet (or on a satellite orbiting our planet) is analyzing spectral content of a light coming from a specific distant galaxy.
-Distant galaxy is composed of many stars, solar systems, asteroids, meteorites, etc.
-Majority of such galaxy entities are solid bodies in mutually relative motions (and in a motion in relation to the distant observer), and many of them have specific magnetic field and emitting light or photons (including direct and secondary emissions of light).
-Since galaxy entities are motional bodies, we could associate matter-waves fields and wave properties to such motions (as we did all over this book).
-Let us consider that specific and dominant mass $\mathbf{M}$ of the galaxy in question has certain center-of-mass or group velocity $\mathbf{v}$. At the same time, such group velocity is presentable as,

$$v = u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda},\ \lambda = \frac{H}{Mv}, u = \lambda f\ .$$

-If the mass $\mathbf{M}$ is emitting photons, and if our observer detects and analyzes spectral signature of such photons, we should naturally have an interference and superposition between light waves or photons and matter waves of motional galactic mass $\mathbf{M}$.
-Any light waves should also be presentable as having active group and phase velocity,

$$v_p = u_p - \lambda_p \frac{du_p}{d\lambda_p} = -\lambda_p^2 \frac{df_p}{d\lambda_p}, u_p = \lambda_p f_p\ .$$

-In such a situation, group, and phase speeds of the resulting light emission, being detected by the distant observer, are naturally modulated by matter-waves of the motional galactic body $\mathbf{M}$. Such modulation should produce red and blue (Doppler), spectral-shifts, and our distant observer should be able to detect such frequency alterations. Let us consider (for mathematical simplicity, to avoid using vectors) only extreme cases when we would have just additions or only subtractions of corresponding (group and phase) velocities. Resulting, effective (modulated) light received by the distant observer will have new, modified group and phase velocity ($v^* \text{and } u^*$),

$$v^* = v \pm v_p - \lambda \frac{du}{d\lambda} \mp \lambda_p \frac{du_p}{d\lambda_p} = -\lambda^2 \frac{df}{d\lambda} \mp \lambda_p^2 \frac{df_p}{d\lambda_p} = u^* - \lambda^* \frac{du^*}{d\lambda^*} = -\lambda^{*2} \frac{df^*}{d\lambda^*} \Rightarrow$$

$$\Rightarrow \begin{Bmatrix} v^* = v \pm v_p, u^* = u \pm u_p, \\ \lambda^* \frac{du^*}{d\lambda^*} = \lambda \frac{du}{d\lambda} \pm \lambda_p \frac{du_p}{d\lambda_p}, \\ \lambda^{*2} \frac{df^*}{d\lambda^*} = \lambda^2 \frac{df}{d\lambda} \pm \lambda_p^2 \frac{df_p}{d\lambda_p}, \\ u = \lambda f, u_p = \lambda_p f_p, u^* = \lambda^* f^* \end{Bmatrix}.$$

This section explores how concepts from Relativity Theory and electromagnetic interactions can explain observed phenomena such as redshifts. According to Relativity Theory, regardless of what we do with photons or light waves, the group speed of modulated photons remains constant at the speed of light $v^* = c = \text{constant}$. This supports the idea that mass-velocity related spectral shifts, such as those measured by William Tifft, are intrinsic to the nature of these interactions. Although this concept needs further development, particularly regarding velocity additions, it provides a qualitative foundation for understanding redshifts.

A similar approach could clarify the results of the Michelson-Morley experiment. This experiment, which initially failed to detect the influence of Earth's motion on the speed of light, might benefit from a revised setup that eliminates immediate entanglement and synchronization between light beams. The confusion in the original Michelson-Morley results could also stem from the fact that light beams or photons oscillate transversally, suggesting that any ether-flow involved should be laminar or linear.

The concept of redshift implies that electromagnetic dipole polarization among rotating and spinning astronomical objects could influence the associated electromagnetic field structures. For instance, William Tifft's observations of redshifts could be analogously related to the hypothetical "Planetary Vortex Shedding" phenomenon, akin to the "Kármán Vortex Street" seen in fluid dynamics. If such a "Planetary Kármán Street" follows planets and large astronomical objects, light passing through these regions would experience velocity modulation, leading to observed redshifts and possibly blue shifts. Further details on vortex shedding are discussed in Chapter 4.1, particularly around equations (4.3-0) and (4.3-0)-a,b,c,d,e,f,g,h,i.

Additionally, the frequency shifts of electromagnetic waves from distant sources passing through planetary systems—potentially applicable to other cosmic rays, x-rays, and neutrinos—show quantized and predictable patterns. These quantizations can be related to the group and phase velocities of planetary systems, as explored in equations (2.11.12) – (2.11.14). This connection could validate innovative concepts regarding gravitation and "temporal-spatial" motions, as discussed in Chapter 10.

Other signal modulations, such as amplitude and phase changes of light waves passing through complex electromagnetic and gravitational structures, should also be considered. The low frequencies of these phenomena might make them less apparent but potentially significant. For example, such effects could be more detectable in planets with high mass, high orbital speed, and smaller perimeters, like Mercury (notably observed in the precession of its perihelion).

Rotation and spinning motions are often associated with magnetic fields. Gyromagnetic ratios maintain constant value across different scales, adding complexity but also supporting the stability of these phenomena. Many celestial bodies exhibit stable magnetic fields, suggesting that rotational or spinning motions are a fundamental background effect.

All elements supporting the extended theory of gravitation are available; we need to learn how to recognize, understand, and apply these facts. Chapter 10 of this book provides a comprehensive explanation of the background velocity parameters and Newtonian-attraction related to important linear and angular moments (see equations (10.1.4) - (10.1.7)).

An insightful publication by Charles W. Lucas, Jr. titled "The Symmetry and Beauty of the Universe" (see [54]) discusses spiraling quantized orbits of planets and moons, linked to a universal electrodynamics force law and an updated force of gravitation. This symmetry, observable across all scales from elementary particles to cosmic structures, could explain the quantization of planetary orbits, Tifft redshifts, and the "Planetary Vortex Shedding" phenomenon, providing a unified perspective on these phenomena.

Anyway, our macro-universe is known to behave like a big and very precise astronomic clock, where periodical motions are its intrinsic property. It will be just a matter of finding or fitting proper integers $(n, n_{\alpha}) \in [1, 2, 3, ...)$ into above given (or similar) macro matter-waves relations, to support here presented concept. Of course, the situation analyzed here is presently addressing only purely circular planetary orbits (for having mathematical simplicity and faster introduction), and in later analyses, we would need to consider elliptic and other self-closed planetary orbits (and, most probably, we will generate additional quantum numbers or integers like n, n_{α}). Quantizing of planetary orbital motions presented here is realized using extremely simple, geometrical concepts analog to N. Bohr atom model, such as $n\lambda = 2\pi r_n, \lambda = H/p = H/mv_n$. The natural development of such quantized model of planetary systems will be in some ways like the evolution of Bohr's planetary atom model towards Sommerfeld's atom model (related to the period before the wave and probabilistic quantum mechanics and Schrödinger equations started to be dominant theoretical approach). For micro-world, we are merely implementing or associate universal quantization by default, since this is well-known practice tested in many cases (Planck, L. de Broglie, Einstein, Sommerfeld…). We should not forget that analogical planetary, orbital quantizing is also valid because of global periodical motions, and macro-universe conservation of important orbital and spinning moments (see (2.9.1) and (2.9.1)).

Also, as a significant theoretical background and support to the innovative concept of Macro-Cosmological stability and gravitation (presented here) the following reference should be considered: [36], Anthony D. Osborne, & N. Vivian Pope, "An Angular Momentum Synthesis of 'Gravitational' and 'Electrostatic' Forces".

[♣ COMMENTS & FREE-THINKING CORNER:

Recent empirical evidence and advances in satellite technology provide growing support for the unity and coupling of linear, circular, and spinning motions, particularly in the context of Liapunov (or Lyapunov) stability as applied to spinning satellites. According to a summary from Wikipedia:

“A spin-stabilized spacecraft is a satellite that maintains its orientation by spinning around an axis, utilizing the gyroscopic effect. The attitude of a satellite—or any rigid body—refers to its orientation in space. If a body starts with a fixed orientation relative to inertial space, it will begin to rotate due to small torques. The most natural way to stabilize such a body is to give it an initial spin around an axis of minimum or maximum moment of inertia. Rotation about the axis of minimum moment of inertia corresponds to a maximum energy state for a given angular momentum, while rotation about the axis of maximum moment of inertia corresponds to a minimum energy state. In the presence of energy loss, as in satellite dynamics, the spin axis will tend to drift towards the maximum moment of inertia. For long-term stabilization, spin stabilization about the axis of maximum moment of inertia is preferred, although short-term stabilization can use the axis of minimum moment of inertia.”

Interestingly, similar principles can be observed in the motion of bullets. Rifling—the process of creating helical grooves in a gun's barrel—imparts a spin to the projectile along its longitudinal axis. This spinning stabilizes the bullet gyroscopically, enhancing its aerodynamic stability and accuracy. Without spin, a bullet would tumble in flight, but the gyroscopic effect ensures it travels in a straight line until affected by external forces like gravity, wind, or impact. This stability is essential for achieving high accuracy in modern firearms.

To relate this to matter waves, consider the analogy between a spinning bullet and the matter waves associated with it. Linear motions, whether inertial or non-inertial, are relative. The same applies to rotational and spinning motions. For example, spinning and rotation are relative angular motions from an observer’s perspective, depending on the axis and frame of reference. See more about inertia in Chapters 1, 4.2 and 10.

If an observer’s reference frame is fixed to the longitudinal axis of a spinning bullet, the bullet may appear stationary in terms of rotational motion. The stability and accuracy of the bullet, due to the gyroscopic effect, suggests that similar principles might apply to particles in linear motion. A particle in stable, inertial motion could have a helical, spinning matter wave around its propagation axis, reflecting the mutual relative spinning motion.

Astronomical objects like planets, moons, and asteroids also exhibit natural spinning. This supports the idea of “Macro Cosmological Matter Waves,” suggesting that linear motions in both micro and macro systems are inherently coupled with spinning motions. This concept is further explored in Chapter 4.1, particularly in equations (4.3-0) and (4.3-0)-a,b,c,d,e,f,g,h,i,j,k. Additionally, in this book, we consider rest masses as “condensed, frozen, or stabilized spinning motions” (see Chapter 2.3.2: “Rotation and Stable Rest-Mass Creation”).

Similar couplings between linear and spinning motions can be observed in phenomena like laser beams or electron beams interacting with matter. Electrons, viewed as specific energy-momentum states of high-energy photons, exhibit spin attributes alongside photons. Both photons and electrons are examples of dualistic wave-particle entities, as demonstrated by effects such as the Compton Effect, the Photoelectric Effect, and various interference and diffraction phenomena.

Given that gravitational force is relatively weak compared to other forces (like electromagnetic and nuclear forces), observations on Earth alone may not suffice. Instead, we should consider the macro-universe as a laboratory for studying gravitation and testing new theories. For instance, light interacts with gravitational fields, electric and magnetic fields, and various matter states. It is logical to hypothesize that light might also interact with hypothetical aspects of gravitation, such as spinning helix-like matter waves or vortex shedding phenomena.

To test these hypotheses, we can use astronomical observations. First, measure the spectrum of a stable, distant light source with no intervening matter. Next, measure the spectrum again as the light passes through regions with known gravitational or electromagnetic fields. Differences in spectral measurements could indicate modifications or modulations due to interactions with these fields. This approach could be applied to other cosmic rays or light sources as well.

Another approach involves using two satellites in stationary orbits around Earth. By sending laser beams between them, we can establish reference spectral signatures. By comparing these signatures with those obtained when the beams pass through planetary or galactic zones, we can detect potential effects of “Planetary Karman Streets” or other gravitational phenomena.

Finally, Rainer W. Kühne’s theory in [52], “Gauge Theory of Gravity Requires Massive Torsion Field,” supports the idea of coupling linear, torsion, and spinning motions. Kühne suggests that a quantum field theory of gravity should include torsion and an associated spin field, aligning with the need to explore these complex interactions in gravitational theory.

For stable and planar solar systems, we already know quantization rules (2.11.14) applicable for an orbit radius and relevant planetary (or tangential) velocity. Most stable solar systems are not necessarily planar, and we should consider the existence of similar quantization (or spatial orbits packing) regarding angular orbits positions (or orbit inclination towards specific reference orbital plane).

To generalize the same concept (already elaborated with equations from (2.11.13) to (2.11.17)) for any closed planetary orbit, we can apply Wilson-Bohr-Sommerfeld action integrals (used in supporting N. Bohr’s Planetary Atom Model). Wilson-Bohr-Sommerfeld action integrals (see [9]), related to any periodical motion on a self-closed stationary orbit C_n, applied over one period of the movement, presenting the kind of general quantifying rule (for all self-closed standing-waves, which are energy carrying structures, having constant angular momentum). Sommerfeld (see chapter 5; equations (5.4.1)) extended Bohr atom model to cover elliptic (and circular) electron (or planetary) orbits, where the semi-major axis is **“a”** and semi-minor axis is **“b”**. We can (just to initiate brainstorming in that direction) analogically (also still hypothetically, and highly speculatively) apply the same strategy on a planet which has mass $\mathbf{m}$ and which is rotating around its sun, which has mass $\mathbf{M} >> \mathbf{m}$, in the following way,

$$\left.\begin{matrix} \left\{\oint_{C_n} L\,d\alpha = n_\alpha H,\ L = \text{Constant},\ 0 \le \alpha \le 2\pi\right\} \Rightarrow L = n_\alpha \frac{H}{2\pi},\ n_\alpha = 1,2,3...,n \\ \left\{\oint_{C_n} p_r\,dr = n_r H\right\} \Rightarrow L(\frac{a}{b}-1) = n_r \frac{H}{2\pi},\ n_r = 0,1,2,3... \end{matrix}\right\} \Rightarrow \qquad (2.11.18)$$

$$\Rightarrow a = \frac{m+M}{(mM)^2}\frac{(\frac{H}{2\pi})^2}{G} n^2 \cong \frac{(\frac{H}{2\pi})^2}{GMm^2} n^2 = a_0 n^2,\ b = a\frac{n_\alpha}{n} = a_0 n_\alpha n,\ n \equiv n_\alpha + n_r = 1,2,3,4...$$

In addition to (2.11.18), for a certain stable (planar) planetary system with several planets (or even for our universe) it should also be valid that its total angular momentum is constant (including spinning moments of planets, moons, and asteroids),

$$\left.\left\{\begin{matrix} \left[\begin{matrix} \vec{\omega}_c = \frac{\sum_{(i)} J_i \vec{\omega}_i}{\sum_{(i)} J_i} = \frac{\sum_{(i)} \vec{L}_i}{\sum_{(i)} J_i} \\ \oint_{C_n} L\,d\alpha = n_\alpha H,\ \oint_{C_n} p\,dr = n_r H \\ n_\alpha, n_r \in [1,2,3,...] \end{matrix}\right] \Rightarrow \left[\begin{matrix} \vec{L} = \sum_{(i)} \vec{L}_i = \sum_{(i)} J_i \vec{\omega}_i = \vec{\omega}_c \sum_{(i)} J_i = \text{const.}(= n\frac{H}{2\pi}) \\ \sum_{(i)} \oint_{C_n} L_i\,d\alpha = \sum_{(i)} n_i H_i = nH = \text{const.} \end{matrix}\right] \\ \& \\ \left[(\vec{p}, \frac{E}{c})^2 = \text{inv.} \Rightarrow p^2 - (\frac{E}{c})^2 = -(\frac{E_0}{c})^2 \Leftrightarrow p^2c^2 - E^2 = -E_0^2 \Leftrightarrow p^2c^2 - (\gamma mc^2)^2 = -(mc^2)^2\right] \end{matrix}\right.\right\} \Rightarrow$$

$$\Rightarrow \left\{ \begin{array}{l} \left[\begin{array}{l} \left(\frac{\vec{r}}{r} \times (\vec{p}, \frac{E}{c}) \right)^2 = \left(\frac{1}{r} (\vec{L}, \frac{E}{c} \vec{r}) \right)^2 = \text{inv.}, \\ \gamma = \gamma_1, \gamma_2 = 1, \ \vec{L} = \overrightarrow{\text{const.}} \end{array} \right] \Rightarrow \frac{1}{r_1^2} \left[\vec{L}_1^2 - (\frac{E}{c})^2 r_1^2 \right] = \frac{1}{r_2^2} \left[\vec{L}_2^2 - (\frac{E_0}{c})^2 r_2^2 \right] \Rightarrow \\ \Rightarrow L = L_1 = L_2 = \frac{r_1 r_2}{c} \sqrt{\frac{E^2 - E_0^2}{r_2^2 - r_1^2}} = mcr_1 r_2 \sqrt{\frac{\gamma_1^2 - 1}{r_2^2 - r_1^2}} = \text{const.}, \Rightarrow r_1 r_2 \sqrt{\frac{\gamma_1^2 - 1}{r_2^2 - r_1^2}} = \text{const.} \end{array} \right\} \quad (2.11.19)$$

Of course, if we have combinations of orbital (**L**) and spin moments (**S**), we will need to replace **L** with **L**+**S**. As we can see in [36], Anthony D. Osborne, & N. Vivian Pope effectively and analogically (could be unintentional, too) made a significant extension of Sommerfeld concept to the macro-world of planets, stars, and galaxies, and it is evident that this way the new chapter of Cosmology and Astronomy is being initiated. ♣]

Elements of Stable Space-Time Structures and Periodic Motions

Planetary systems, characterized by periodic motions, are stabilized by fields and forces that integrate them into coherent macro systems. A key aspect of this stability is the creation of standing waves within the fields involved. The positions and trajectories of planets in these periodic systems are governed by stable or stationary energy-momentum conditions, which minimize energy dissipation and maximize the mechanical quality factor. These conditions are often derived from solving the Euler-Lagrange-Hamilton equations.

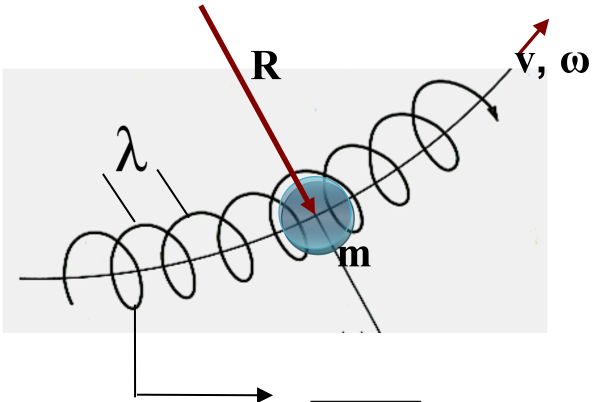
To understand how this quantum-like conceptualization of gravitation could evolve, refer to the Appendix of this book, which innovatively explores "Bohr's Model of the Hydrogen Atom and Particle-Wave Dualism." While further elaboration and verification of concepts discussed in sections (2.11.10) to (2.11.21) are needed, significant progress has been made.

Our perspective should move beyond the traditional view of time-stable and spatially isolated linear and circular planetary orbits. Instead, consider that planetary masses in orbital motion are embedded in energy-momentum fields that create standing waves. These waves represent material and mass-energy connections within planetary systems. What we observe as planetary masses and orbits, as described by Kepler and Newton, are effective centers or channels of these broader energy-momentum structures, which are structured as standing waves in a larger space-time framework.

Based on the planetary macro-waves conceptualization presented in sections (2.11.12) to (2.11.19), we can formulate an equation analogous to Schrödinger's equation for quantized mass or energy-momentum distributions, particularly in planetary systems. Here, mass should be considered in its extended form—as relativistic, velocity-dependent, spatially distributed, and coupled with surrounding energy-momentum states and fields. This is like the conceptualization discussed in "2.2. Generalized Coulomb-Newton Force Laws", particularly equations (2.3) to (2.4-3).

In this context, the wave function Ψ is related to the motional energy of a planet or satellite on its orbit, analogous to standing waves on a circular string (see [144]). Geometrically and analogically, this model involves closed spatial structures where periodic motional elements become stationary and stable. Such periodic structures have an integer number of specific elementary wavelengths or other relevant domains, qualifying them as quantized states. This concept aligns with the principles of Quantum Theory, highlighting the resonance and stability inherent in these structures.

For creating a better idea about such "standing waves *packing",* it is useful to see equations under (2.9.5-9), *Chapter 4.1, T.4.2., Wavelength analogies in different frameworks,* and *Chapter 5, T.5.3., Analogical Parallelism between Different Aspects of Matter Waves*. If we insist on creating some clear, preliminary, and conceptual visualization of planetary orbital motions with associated gravitational matter-waves structure (as spatially distributed energy-momentum states, enclosed in ***toroidal*** forms), this could be intuitively linked to the illustration on Fig. 2.6. and strongly related to future creative modeling and innovative solutions resulting from equations (2.11.20) to (2.11.23).



$$\mathbf{u} = \mathbf{v}/(1+\sqrt{1-v^2/c^2}) = \lambda f$$

Fig.2.6. Gravitational matter waves and orbital planetary motion

The natural way of modeling such geometrical forms (standing waves) is causally related to Analytic Signal concept (see chapter 4.0. and [57]) and Schrödinger equation, which has its conceptual, historical, and analogical origins in generalization of d'Alembert, Classical Wave Equation, which is related to standing waves oscillations of an ordinary (self-closed) string; -see more in [144]. In addition, the same, Classical wave-equation (in specific analogical form of d'Alembert equation) has been known in different fields of Classical Mechanics, Fluid Mechanics, Acoustics and Maxwell Electromagnetic Theory long before being "renamed, modified and analogically applied" to waves phenomenology in micro-Physics by Schrödinger and other Quantum Theory founders. Another striking analogy (showing that a mass in motion should have some kind of associated helix-spinning field or oscillating matter wave tail) is related to fluid flow vortices and vortex flow-meters, as speculated in ***chapter 4.0 and 4.1; -see equations (4.3-0), and (4.3-0)-a,b,c,d,e,f,g,h,l,j,k...***

This concept of associated helicoidally spinning field (around a path of linear motion) is causally related to Analytic Signal modeling (as presented in chapter 4.0). Let us consider a specific linear movement of a particle, or an equivalent wave group (in the same state of linear motion). If we present this motion (meaning its power or force function, or relevant field function) with specific

wave function $\Psi(t)$, using the Analytic Signal model, we can create another, associated wave function $\hat{\Psi}(t)$, where a couple of such functions ($\Psi(t)$ and $\hat{\Psi}(t)$) are generating the Complex Analytic Signal $\overline{\Psi}(t) = \Psi(t) + j\hat{\Psi}(t) = (1 + jH)\,\Psi\,(t)$ (see much more in chapter 4.0). Now, based on such Analytic Signal modeling, we can determine de Broglie or matter-wave frequency, wavelength, amplitude, and phase functions. The healthy, and fundamental mathematics, well connected to the stable and naturally stable body of Physics (without artificial and postulated theoretical concepts), is always producing good and realistic mathematical predictions, meaning that both $\Psi(t)$ and $\hat{\Psi}(t)$ must be realistic, measurable wave functions of something that exists in our Physics and Universe. One of the examples for such coupled wave functions is the electromagnetic field that is combining electric and magnetic field functions in a similar way as realized in the Analytic Signal model. Here, as a case causally related to gravitation, we have the situation that any linear and spinning or helix wave motion should be the same coupled (creating an Analytic Signal).

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Anyway, in many cases, we can conclude that linear and helix or spinning and rotating motions (of masses) are mutually complementary and united (concerning matter-waves, or PWDM elaborated in this book). Such concepts could be, (imaginatively, creatively, and analogically) extrapolated from atoms to planetary systems and galactic formations. On some way, our universe is globally rotating and spinning, following helix-like paths of associated matter-waves. What we see as red or blue, Doppler shifts (of electromagnetic radiation) coming from a remote deep space, could be effects of such globally present, macro-rotating effects. As we know, the tangential velocity of the certain rotating mass, v_t is equal to the product of relevant orbital velocity, ω, and relevant radius R, $v_t = \omega R$. Hubble's law is maybe saying something similar, such as, $v = (v_t) = H_0 R$, where H_0 is Hubble constant, which could be certain metagalaxy, orbital (or helicoidally spinning associated matter-waves), tangential velocity. ♣]

Citation took from the Internet; -Wikipedia, the free encyclopedia:

*"**Hubble's law** or **Lemaître's law** is the name for the astronomical observation in physical cosmology that: (1) all objects observed in deep space (intergalactic space) are found to have a Doppler shift observable relative velocity to Earth, and to each other; and (2) that this Doppler-shift-measured velocity, of various galaxies receding from the Earth, is proportional to their distance from the Earth and all other interstellar bodies. In effect, the space-time volume of the observable universe is expanding, and Hubble's law is the direct physical observation of this process.[1] It is considered the first observational basis for the expanding space paradigm and today serves as one of the pieces of evidence most often cited in support of the Big Bang model. Although widely attributed to Edwin Hubble, the law was first derived from the General Relativity equations by Georges Lemaître in a 1927 article where he proposed that the Universe is expanding and suggested an estimated value of the rate of expansion, now called the **Hubble constant**.[2][3][4][5][6] Two years later Edwin Hubble confirmed the existence of that law and determined a more accurate value for the constant that now bears his name.[7] The recession velocity of the objects was inferred from their redshifts, many measured earlier by Vesto Slipher (1917) and related to velocity by him.[8]*

*The law is often expressed by the equation $v = H_0D$, with H_0 the constant of proportionality (the **Hubble constant**) between the "proper distance" D to a galaxy (which can change over time, unlike the*

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http://www.mastersonics.com/documents/revision_of_the_particle-wave_dualism.pdf

comoving distance) and its velocity v (i.e. the derivative of proper length with respect to cosmological time coordinate; see Uses of the appropriate distance for some discussion of the subtleties of this definition of 'velocity'). The SI unit of H_0 is s^{-1}, but it is most frequently quoted in (km/s)/Mpc, thus giving the speed in km/s of a galaxy 1 megaparsec (3.09×10^{19} km) away. The reciprocal of H_0 is the Hubble time.

As of 3rd Oct 2012, the Hubble constant, as measured by NASA's Spitzer Telescope and reported in Science Daily, is 74.3 ± 2.1 (km/s)/Mpc"

The formulation of the Schrödinger equation is well-known and, in this book, additionally elaborated and generalized (later, in chapter 4.3). Anyway, from different publications we already have definite confirmation that Schrödinger equation is well applicable to solar systems quantizing (see [63], Arbab I. Arbab, and [67], Johan Hansson), since results of Schrödinger equations related to N. Bohr hydrogen atom are directly generating all results of planetary orbit parameters quantizing, as in (2.11.14), when we apply analogical replacement $\boxed{\frac{Ze^2}{4\pi\varepsilon_0} \to GmM}$. Since orbiting planets are respecting certain periodicity and "*macro matter-waves packing rules*", like standing, de Broglie matter-waves in a micro-universe (see explanations around equations (2.9.5-9), (2.11.5) - (2.11.9), (2.11.9-1) - (2.11.9-4), (2.9.5-1) - (2.9.5-5) and (2.11.12) - (2.11.14)), we are in the position to apply relevant, generalized Schrödinger-like equations, as equations (4.10), to planetary orbital motions. Of course, it is essential to address accurately all relevant parameters and analogical replacements.

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To understand the relationship between Schrödinger's equation and various aspects of modern physics, let us explore the following logical steps and facts:

1. Schrödinger's Equation in Microphysics: Schrödinger's equation is fundamental to microphysics and modern Quantum Theory. Its early success, particularly in spherical coordinates, was exemplified by N. Bohr's hydrogen atom model. This model, derived from Schrödinger's equation, accurately predicted experimental results and supported the old quantum theory. Although the development of Schrödinger's equation involved mathematical trial and error, its current form has proven exceptionally useful and reliable. Despite its effectiveness, the historical process of its formulation lacked systematic and logical rigor.

2. Alternative Mathematical Approach: In this book (see Chapter 4.3), an alternative approach is presented that begins with the classical, universally valid wave equation. This approach yields results that are consistent with, and often more logically clear than, those produced by Schrödinger's equation. By using Analytic Signal and Hilbert Transform modeling, this method avoids the complications of probability and statistical assumptions, offering a more straightforward and intuitive framework.

3. Consistency and Significance: Both the traditional and innovative forms of Schrödinger's equation produce accurate results, such as those related to N. Bohr's hydrogen atom model. This consistency suggests that there is something fundamentally important about Schrödinger's equations that merits further exploration in physics and our understanding of the universe.

4. Analogy with Planetary Systems: Several authors, including the author of this book, have noted a striking analogy between N. Bohr's hydrogen atom model (which involves quantized electron energy, velocity, and radius) and similar quantized properties in planetary and solar systems (refer to equations (2.11.12) – (2.11.14) and Table T.2.8). Although this analogy has not been definitively proven, the similarities observed are highly indicative. This suggests a profound and simple underlying connection between microcosmic and macrocosmic systems.

5. Exploring Gravitational Quantization: Given the significant role of Schrödinger's equation in describing atomic models and its analogy with planetary systems, it is logical to apply a similar approach to quantizing stable solar systems. Innovated forms of Schrödinger's equations, incorporating Analytic Signal Wave-functions and formulated in spherical coordinates, could provide new insights into gravitation. This approach promises to enhance our understanding of gravitational phenomena beyond the frameworks of Newtonian mechanics and General Relativity. ♣]

It is essential to underline that Schrödinger-like, analogically formulated wave equation, applicable to gravitational fields, and macro-mechanical motions within planetary systems has almost nothing to do with stochastic and probability concepts applied in contemporary Quantum Theory.

Let us briefly specify a mathematical chain of initial and final forms and conclusions in the process of analog formulation of such wave equations. We can start by complying with generalized Schrödinger equation (4.10) from the Chapter 4.3, which will address deterministic (certainly non-stochastic) planetary and satellites orbital motions, including associated (deterministic and dimensional) wave functions in a field of gravitation, as for instance,

$$\left\{\begin{array}{l}\left[\begin{array}{l}\frac{\hbar^2}{\tilde{m}}(\frac{u}{v})\Delta\overline{\Psi}+(\tilde{E}+E_0-U_p)\overline{\Psi}=0,\\ \frac{\hbar^2}{\tilde{m}}(\frac{u}{v})\Delta\overline{\Psi}-U_p\overline{\Psi}=-(\tilde{E}+E_0)\overline{\Psi}=-j\hbar\frac{\partial\overline{\Psi}}{\partial t}-U_p\overline{\Psi}=\frac{\hbar^2}{\tilde{E}+E_0-U_p}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}-U_p\overline{\Psi}=\\ =j\hbar u\nabla\overline{\Psi}-U_p\overline{\Psi},\ (\frac{E_{total}-U_p}{\hbar})^2\cdot\overline{\Psi}+\frac{\partial^2\overline{\Psi}}{\partial t^2}=0,\frac{\partial\overline{\Psi}}{\partial t}+u\nabla\overline{\Psi}=0,\\ \nabla=i\frac{\partial}{\partial x}+j\frac{\partial}{\partial y}+k\frac{\partial}{\partial z}=grad,\ \Delta=\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2},\ \overline{\Psi}=\Psi+j\hat{\Psi},\ j^2=-1\end{array}\right](4.10-1),\text{chapter 4.3}\\ \&\\ \left[\begin{array}{l}v=\omega_m R\cong 2u=2\lambda_0 f_0=\frac{2\pi r}{T}=2\pi R f_m=\sqrt{\frac{GM}{R}}<<c,\ (R=R_0\frac{1+e}{1+e\cdot\cos\theta}),\\ \omega_m=2\pi f_m=\frac{4\pi f_0}{n}=\frac{2\pi}{T}=\frac{\sqrt{GM}}{R^{3/2}}=\frac{v}{R},\ p=mv=\frac{H}{\lambda_0}=\frac{nH}{2\pi R},\ n=1,2,3,...\\ L=n\frac{H}{2\pi}=pR=mvR=mR^2\omega_m=mR^2\frac{2\pi}{T}=2\pi mR^2 f_m=\sqrt{GMR}\cdot m,\\ \tilde{E}=E_k=\frac{1}{2}mv^2=Hf_0=\frac{GMm}{2R},\ U_p=-\frac{GMm}{R},\ E_k-U_p=\frac{3}{2}\frac{GMm}{R}\end{array}\right]\begin{array}{l}(2.11.13)\\(2.11.14),\\(2.11.15)\end{array}\\ \&\\ \left[\begin{array}{l}h\leftrightarrow H=\frac{2\pi\sqrt{GMR}}{n}\cdot m,\ \hbar=\frac{h}{2\pi}\leftrightarrow\hbar_{gr.}=\frac{H}{2\pi}=\frac{\sqrt{GMR}}{n}\cdot m=\frac{GMm}{v_0},\\ m\leftrightarrow\frac{mM}{m+M}=\mu\cong m<<M,\ \frac{Ze^2}{4\pi\varepsilon_0}\to GmM.\end{array}\right]\end{array}\right\}\Rightarrow$$

$$\left[\begin{array}{l}\frac{(\frac{H}{2\pi})^2}{2m}\Delta\overline{\Psi}+(E_k+E_0-U_p)\overline{\Psi}=0,\ U_p=-\frac{GMm}{R},\ E_k=\frac{GMm}{2R},\ E_0=mc^2,\\ \frac{(\frac{H}{2\pi})^2}{2m}\Delta\overline{\Psi}-U_p\overline{\Psi}=-(E_k+E_0)\overline{\Psi}=-j\hbar\frac{\partial\overline{\Psi}}{\partial t}-U_p\overline{\Psi}=\frac{\hbar^2}{E_k+E_0-U_p}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}-U_p\overline{\Psi}=\\ =j\hbar u\nabla\overline{\Psi}-U_p\overline{\Psi},\ (\frac{E_{total}-U_p}{\hbar})^2\cdot\overline{\Psi}+\frac{\partial^2\overline{\Psi}}{\partial t^2}=0,\frac{\partial\overline{\Psi}}{\partial t}+u\nabla\overline{\Psi}=0,\ u=v/2.\end{array}\right] \quad (2.11.20)$$

The solutions of (2.11.20) will show that planetary and satellite orbits (here populated by a wave function $\overline{\Psi}$) are closed circular or elliptic lines, but only approximately. Certain harmonic, exceptionally low frequency standing waves, having helical paths, or amplitude-modulation of relevant radius, with toroidal and helix, field-envelope should be measurable, when planets and satellites' orbits are very precisely monitored (because of rotation, spinning and mutual interactions among participants).

The closest, extraordinary, and unique publications about existence and grounds of such (helix and toroidal) planetary motions are coming from Lucas Jr. Charles when he is explaining Chiral Symmetry of Spiraling Planetary Orbits (on Surface of a Toroid) about the Sun (see [54]). Unfortunately, the publications and ideas of Mr. Lucas are not enough addressed in the mainstream of officially supported science, probably because he is too original and sometimes gravitating around arbitrary, religiously flavored environments, this way maybe creating some minor doubts regarding his scientific objectivity and ideological neutrality. Anyway, regardless of the personal ideological preferences of Mr. Lucas, his conceptualization of spiraling planetary orbits on the surface of the toroid is amazingly seducing and *significant contribution to understanding Gravitation (and familiar to ideas and concepts elaborated in this book).*

It is evident that the future development of the here-introduced macro-cosmological matter-waves concept will significantly enrich our understanding of Gravitation.

To get generally valid and entirely natural solutions $\overline{\Psi} = \overline{\Psi}(\mathbf{R}, \theta, \varphi)$ of (2.11.20), involved operators ($\nabla, \nabla^2 = \Delta$) should be applied in spherical, polar coordinates ($\mathbf{R}, \theta, \varphi$). In addition, elliptic planetary orbits should be considered (of course, after we upgrade all equations from (2.11.12) until (2.11.18), which are valid for ideal circular orbits, into new and equivalent expressions applicable for elliptic planetary orbits, and consequently, all of that will modify differential equations found in (2.11.20)). The same gravitational, Schrödinger equation, (2.11.20), in its natural, spherical coordinates will eventually look like the equation applicable on N. Bohr, the hydrogen atom model,

$$\frac{(\frac{\mathbf{H}}{2\pi})^2}{2\mathbf{m}}\left[\frac{1}{\mathbf{R}^2}\frac{\partial}{\partial \mathbf{R}}\left(\mathbf{R}^2\frac{\partial}{\partial \mathbf{R}}\right)+\frac{1}{\mathbf{R}^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{\mathbf{R}^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]\cdot\overline{\Psi}+\mathbf{U}_p(\mathbf{R})=\mathbf{E}_k\cdot\overline{\Psi}, \tag{2.11.21}$$

$$\overline{\Psi} = \overline{\Psi}(\mathbf{R}, \theta, \phi) = \mathbf{X}(\mathbf{R}) \cdot \mathbf{Y}(\theta, \phi),$$

where $\mathbf{X}(\mathbf{R})$ is expressible in terms of associated Laguerre functions, and $\mathbf{Y}(\theta, \phi)$ are the spherical harmonic functions.

To find solutions of (2.11.21) will not be easy, but to rely on analogies between a specific planetary system and N. Bohr, hydrogen atom model, and directly make results conversions, like in T.2.8., will be much more comfortable, since the validity of analogical replacements, $\left\{\frac{Ze^2}{4\pi\varepsilon_0 r^2} \Leftrightarrow \frac{GmM}{r^2}, Ze \Leftrightarrow M, e \Leftrightarrow m \cong \frac{mM}{m+M} = \mu, Z \Leftrightarrow \frac{M}{m}, \frac{1}{4\pi\varepsilon_0} \Leftrightarrow G, h \Leftrightarrow H\right\} \Rightarrow \left\{\frac{Ze^2}{4\pi\varepsilon_0} \to GmM\right\}$ is shown as working very well. For instance, to get an (analogical) idea about possible spatial shapes of gravitational, matter-wave function from (2.11.21), we can see the picture given below, which is addressing hydrogen wave function (taken from Quantum Theory, standard literature):

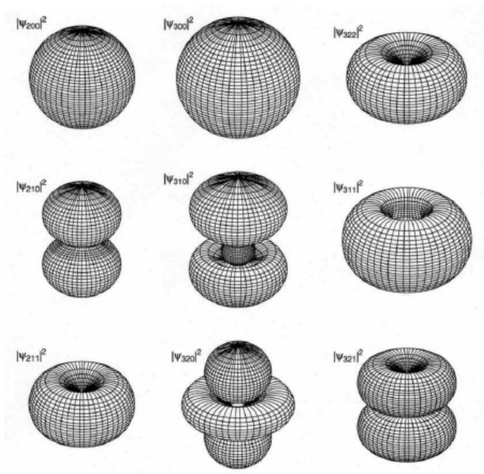

Fig.2.6. Surfaces of the constant $|\Psi|^2$ for the first few hydrogen wavefunctions

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

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In fact, a more correct explanation of the ideas found in (2.4) - (2.11.21) could be complicated compared to here-presented, but for the purpose of introducing new concepts about Wave and Quantum Gravitation, Particle-Wave Duality, force-field charges, and unification between linear and rotational elements of every motion, hire initiated, conceptual platform is already sufficiently clear. To understand the broader meaning of wave functions it is useful to see the chapter "4.3 Wave Function and Generalized Schrödinger Equation"; -equations: (4.33-1), (4.41-1) to (4.45), T.4.2 and T.4.3, as well as matter-waves conceptualization around equations (4.3) and (4.3-1) in the chapter 4.1.

Also, based on Parseval's identity (that is universally valid, and connecting time and frequency domains of any wave function), we can establish a very realistic and deterministic meaning of the matter-waves, wave function. For more information, see chapter 4.0, and equations (4.0.4).

The wave function $\overline{\Psi} = \overline{\Psi}(\mathbf{R}, \theta, \varphi, \mathbf{t})$, instead of being certain probability, any oscillating amplitude, or specific displacement, or harmonically modulated orbital radius (like in (2.11.20) and (2.11.21)), could analogically get an extended meaning of spatially distributed, matter-wave power (see (2.11.22)). In such situations, (instead of Probability and Statistics concepts and postulations), the number of mutually linked, universally valid conditions and relations would be naturally satisfied, as for instance,

$$\Psi^2 = \frac{dE}{dt} = \frac{dE_k}{dt} = \frac{d\tilde{E}}{dt} = v\frac{dp}{dt} = \omega_m \frac{dL}{dt} = vF = \omega_m \tau = \text{Power} = P,$$

$$\left\{\begin{aligned} & E_k = \tilde{E} = \frac{1}{2}mv^2 = mvu = pu = 2mu^2 = \frac{1}{4}mv_e^2 = \frac{GmM}{2R} = \frac{1}{2}\cdot\left(\frac{GmM}{R^2}\right)\cdot R = \frac{1}{2}\cdot F_{m-M}\cdot R = \\ & = \frac{m}{2}\left(\frac{2\pi R}{T}\right)^2 = \frac{8m\pi^2 R^2}{n^2} f_o^2 = 2m(\pi R f_m)^2 = (2\pi m R^2 f_m)\cdot(\pi f_m) = L\pi f_m = (\frac{2L\pi}{n})\cdot f_o = Hf_o = \\ & = \int_{-\infty}^{+\infty}\Psi^2(t)dt = \int_{-\infty}^{+\infty}\hat{\Psi}^2(t)dt = \frac{1}{2}\int_{-\infty}^{+\infty}\left|\overline{\Psi}(t)\right|^2 dt = \frac{1}{2}\int_{-\infty}^{+\infty}a^2(t)dt = \int_{-\infty}^{+\infty}\left[\frac{a(t)}{\sqrt{2}}\right]^2 dt = \\ & = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\left|\overline{U}(\omega)\right|^2 d\omega = \int_{-\infty}^{+\infty}\left|\frac{\overline{U}(\omega)}{\sqrt{2\pi}}\right|^2 d\omega = \frac{1}{\pi}\int_0^{\infty}\left[A(\omega)\right]^2 d\omega = \int_0^{\infty}\left[\frac{A(\omega)}{\sqrt{\pi}}\right]^2 d\omega = \\ & = \int_{-\infty}^{+\infty}P(t)dt\ (=)\ [J] \end{aligned}\right\},$$

$$\left\{\begin{aligned} & v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} = u + p\frac{du}{dp} = \frac{d\omega}{dk} = \frac{d\tilde{E}}{dp} = H\frac{df}{dp} = \frac{df}{df_s} = \frac{2u}{1+\frac{uv}{c^2}}, \\ & u = \lambda f = \frac{\omega}{k} = \frac{\tilde{E}}{p} = \frac{Hf}{p} = \frac{f}{f_s} = \frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{E_k}{p},\ f_s = k/2\pi, \lambda = \frac{H}{p}, \\ & \Rightarrow 0 \le 2u \le \sqrt{uv} \le v \le c \end{aligned}\right\}, \qquad (2.11.22)$$

because, regarding orbital planetary motions we have harmonic and periodical wave-functions (and motions), which create stable, self-closed, spatial standing-waves, and resonant-like field states. The nature of fields and forces involved here is related to mechanical motions and gravitation. Such movements are also mixed with associated electromagnetic fields (at least because mutually identical mathematical forms of Newton and Coulomb's force laws are applicable). We should not exclude the possibility that Gravitation has its primary and essential origin in Electromagnetism (see such electromagnetic forces conceptualizations around equations from (2.4-6) to (2.4-10), in the same chapter; -see also [54]). Deterministic, *square of the power-related macrocosmic wave-function* from (2.11.21) and (2.11.22) is a product between two of relevant, mutually linked, dynamic and motional-energy related values, such as relevant current and voltage, or force and velocity, etc. (see (4.0.82) from chapter 4.0, with more of supporting background), as follows,

$$\Psi^2(t,R) = P(t,R) = \frac{d\tilde{E}}{dt} (=) \text{Active Power } (=)$$

$$(=)\begin{Bmatrix} i(t)\cdot u(t) & (=) & [\text{Current}\cdot\text{Voltage}], \text{ or} \\ f(t)\cdot v(t) & (=) & [\text{Force}\cdot\text{Velocity}], \text{ or} \\ \tau(t)\cdot\omega(t) & (=) & [\text{Orb.}-\text{moment}\cdot\text{Angular velocity}], \text{or} \\ (\vec{E}\times\vec{H})\cdot\vec{S} & (=) & \left[\overrightarrow{\text{Pointyng Vector}}\right]\cdot\overrightarrow{\text{Surface}} \\ ----- & (=) & --------- \\ s_1(t)\cdot s_2(t) & (=) & [(\text{signal}-1)\cdot(\text{signal}-2)] \end{Bmatrix}. \quad (4.0.82)$$

It is crucial to emphasize that future progress in merging wave-function environments (as discussed in equations (2.11.22) and (4.0.82)) with wave equations (such as (2.11.21)) will require a new, innovative, and intellectually flexible approach. The current work represents preliminary brainstorming and initial intuitive steps towards developing a new theory of wave and quantum gravity.

Regarding gravitational waves, the author of this book posits that they are manifestations of cosmic, electrostrictive, and/or magnetostrictive oscillations. These waves would be akin to the effects observed in piezoelectric and magnetostrictive transducers and would include associated electromagnetic waves, fields, and forces affecting particle distributions and motions, like wave phenomena in fluids.

In our astronomical environment, we observe various electromagnetic waves, photons, streams of electrically charged and neutral particles, and surrounding electric and magnetic fields. Additionally, different mass-energy-momentum flows, particles, waves, and fluids are present in the cosmos. Primarily, electromagnetic effects, as the original sources of vibrations, produce secondary electromechanical effects that contemporary physics describes as gravitational forces and waves.

Gravitational phenomena are not solely the result of static masses but rather arise from the motion and oscillation of masses with linear and orbital moments. These masses are specific configurations of structured and polarized electromagnetic entities, such as electrons, protons, neutrons, positrons, and photons. Despite the absence of a comprehensive global unification theory, the Universe is inherently stable and unified. Therefore, a universal force should govern both the micro and macro realms of physics. The most logical candidate for such a universal force is the phenomenology related to electromagnetic fields (see Chapter 3 for further details). According to R. Boskovic's theory of universal natural force [6], this force should be singular and unique, rather than a combination of gravitational, electromagnetic, weak nuclear, and strong nuclear forces.

2.3.3-5 Uncertainty and Entanglement in Gravitation

Until the present, certain defendable legitimacy regarding planetary and gravitational wave functions and wave equations has been established. Consequently, whenever we have wave functions, we can analyze associated couples of mutually conjugate, space, time, and spectral domains. Uncertainty Relations, which are generally applicable to such situations (in mathematics), are mutually relating durations of mutually conjugate, original, and spectral domains, of involved wave functions. To better understand such, generally valid uncertainty relations, it is recommendable to read chapter 5. of this book (around relations (5.5)),

$$\begin{aligned}&\mathrm{TF} > \frac{1}{2},\ \mathrm{T}\Omega > \pi,\ \Omega = 2\pi\mathrm{F}\\&\mathrm{T}-\text{absolute time duration of the } \Psi \text{ function}\\&\mathrm{F}-\text{absolute frequency duration of the } \Psi \text{ function.}\end{aligned} \quad (5.5)$$

Obviously, in cases of planetary systems, Planck constant **h** has not its natural place there and new and analogous $\mathbf{H} >> \mathbf{h}$ constant is becoming much more relevant. The number of uncertainty relations can be now formulated for gravitational and planetary wave-functions, in a similar way as practiced in Quantum Theory concerning constant **H** (for covering much more extensive background about wave functions and Uncertainty Relations; -see chapters 4.0 and 5). As shown in chapter 5, we will conclude that typically mechanical and motional entities, as practiced in present interpretations of gravitation, should be enriched with naturally associated electromagnetic set of relevant parameters (see equations from (5.2) until (5 .4.1)), if we like to have more complete picture about Gravitation.

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Uncertainty Relations in Micro and Macro Physics

Uncertainty Relations are universally applicable, spanning both the microscopic and macroscopic realms of physics. This applicability stems from the fact that the simple geometric or spatial dimensions, durations, and sizes associated with rest mass are significantly smaller than the mass-energy-momentum matter-wave packets linked to the same mass, including all related electromagnetic components. When applying Uncertainty Relations, we must consider the durations of matter waves in both their original and spectral domains.

High-power mechanical, ultrasonic, or acoustic energies, along with moments, forces, oscillations, and vibrations can be created and transferred using various signal-modulating techniques applied to laser beams and dynamic plasma states. Lasers and plasma states can act as carriers for lower-frequency mechanical vibrations or signals. For more details, see Chapter 10, sections (10.2-2.4), and the references from [133] to [139].

Gravitational Entanglement and Planetary Wave Functions

An intriguing and speculative concept in gravitational physics is "Gravitational Entanglement." This idea, discussed in Chapter 4.3 (equations (4.10-12)) and Chapter

10, involves exploring how planetary wave functions might exhibit entanglement. In astronomy and gravitation, this could manifest as globally coupled orbital and spinning moments of planetary and galactic formations. In such systems, rotating and spinning states of planetary members would communicate, balance, and compensate for perturbations at an infinite speed. This entanglement could occur both locally within a specific planetary system and globally with the larger astronomical environment.

For further reading on concepts like Gravitational Entanglement, see the work of Anthony D. Osborne and N. Vivian Pope in reference [36].

[♣ COMMENTS & FREE-THINKING CORNER:

2.3.3-6 Rudjer Boskovic and Nikola Tesla's theory of Gravitation

****Exploring Gravitation Through Historical and Innovative Frameworks****

The most profound conceptualization of gravitation presented in this book builds upon the framework established by Rudjer Boskovic, who described it as the "Universal Natural Force" [6]. This idea is further enriched by Nikola Tesla's intuitive concepts of the "Dynamic Theory of Gravity" [97]. By creatively integrating these frameworks, we can see that the forces and fields discussed by Boskovic and Tesla are already present in the forces that stabilize atoms, much like Bohr's atomic model.

The results presented in this book reveal significant analogies between solar and planetary systems and Bohr's model of the atom. For instance, Chapter "2.3.3. Macro-Cosmological Matter-Waves and Gravitation" (equations (2.11.10) to (2.11.22)) and "T.2.8. N. Bohr's Hydrogen Atom and Planetary System Analogies" demonstrate these parallels. An innovative extension of Bohr's model, discussed in "8. BOHR'S MODEL OF HYDROGEN ATOM AND PARTICLE-WAVE DUALISM" (see equations (8.64) to (8.74) and "8.3. Structure of the Field of Subatomic Forces"), suggests that structures such as hydrogen atoms and planetary systems are surrounded by a complex force field $\mathbf{F}(r, \theta, \phi, t)$. This field could be modeled as Boskovic's Universal Natural Force and be conceptually aligned with Tesla's Dynamic Force of Gravity. Tesla's ideas were, in fact, influenced by Boskovic's work.

In addition, we should revisit the foundational ideas about gravitation discussed in "2.2.1. WHAT IS GRAVITATION REALLY?" at the beginning of this chapter. This section speculates that all atoms and masses in the universe communicate electromagnetically, mechanically, and electromechanically in a continuous and synchronous manner.

Electric charge and magnetic flux are naturally bipolar entities that can create dipole structures. Similarly, linear and angular moments are known to be related to action-reaction forces, electromagnetic induction, and various inertial effects. Since gravitational force is traditionally viewed as purely attractive, it is reasonable to hypothesize that there is a corresponding mass-energy-momentum flow serving as a reaction-force complement to gravitation. Tesla conceptualized this as "radiant energy" and the flow of mass from one body to another [97].

We can speculate that electric charges, magnetic fluxes, and linear and angular moments are always mutually coupled and packed, forming a fundamental source of natural fields and forces. This idea aligns with Boskovic's "Universal Natural Force" [6] and Tesla's "Dynamic Force of Gravity" [97], presenting a unified framework for understanding gravitation.

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*Citation from PowerPedia, on Internet; -"Tesla's Dynamic Theory of Gravity: The **Dynamic Theory of Gravity** of Nikola Tesla explains the relation between gravitation and electromagnetic force as a unified field theory (a model over matter, the aether, and energy). It is a unified field theory to unify all the fundamental forces (such as the force between all masses) and particle responses into a single theoretical framework".*

*****Evolution of Fields and Wave Phenomena in Physics*****

In physics, all fields and wave phenomena can be viewed as the natural evolution of elementary microparticles and fluid states into a diverse range of macro momentum-energy or mass states. These phenomena act as a "communicating, coupling, and gluing medium" within the space between particles. Waves, in essence, are oscillations within a certain medium, fluid, or elastic matter, where energy fluctuates between kinetic and potential forms.

In the context of an absolute vacuum, where electromagnetic waves, neutrinos, and various forms of cosmic radiation propagate, there may still be a form of fluidic matter—a concept not yet fully conceptualized in contemporary physics. Nikola Tesla frequently alluded to this idea [97].

Similarly, particles can be considered specifically condensed or solidified states of energy, derived from self-sustaining, internally folded forms of rotating matter waves. The stabilization of these particles is causally related to the formation of internal standing waves. For more on these ideas, refer to Jean de Climont's work [117].

R. Boskovic's and N. Tesla universal and dynamic force $\mathbf{F}(r, \theta, \phi, t)$, to comply with Bohr's planetary atom model, and to analogical solar systems modeling (as elaborated earlier in this chapter), should have, at least two force components $\mathbf{F_1}(r, \theta, \phi, t) + \mathbf{F_2}(r, \theta, \phi, t)$, (one being potential, and the other solenoidal vector field, as already exercised in the chapter 8.), as for instance:

$$\mathbf{F}(r, \theta, \phi, t) = \mathbf{F_1}(r, \theta, \phi, t) + \mathbf{F_2}(r, \theta, \phi, t),\ \nabla\times\mathbf{F} \neq 0,\ \nabla\mathbf{F} \neq 0 \Rightarrow$$
$$\Rightarrow \begin{cases} \nabla\times\mathbf{F}_1 = 0, \nabla\mathbf{F}_1 \neq 0 \\ \nabla\times\mathbf{F}_2 \neq 0, \nabla\mathbf{F}_2 = 0 \end{cases} . \qquad (2.11.23)$$

****Exploring Gravitational Forces through Atomic Analogies****

Forces and fields within an atom are fundamentally electromagnetic. Given the striking analogy between atomic and planetary systems, we can model gravitational forces similarly by treating them as an extension of interatomic forces. This approach is detailed in the Extended Bohr Atom Model, as discussed in Chapter 8. We can conceptualize gravitational forces as a superposition of solenoidal and potential vector fields, aligning with the ideas of Rudjer Boskovic and Nikola Tesla.

Gravitational attraction suggests that there is an underlying energy fluctuation and gravitational potential associated with the interaction of masses (see Chapter 2, equations 2.4-11 to 2.4-17). These fluctuations and communications, related to standing waves, could impact the interpretation of particle velocity within a gravitational field that comprises both solenoidal and potential components.

Chapter 8 of this book, particularly Section 8.3 ("Structure of the Field of Subatomic Forces"), discusses gravitation as a result of matter-wave exchanges between atoms and the universe. The balance of potential energy from attractive and repulsive forces within an atom, as described in equation 8.74, may be influenced by continuous electromagnetic exchanges between the nucleus and the electron shell. This interchange, occurring bidirectionally, suggests that gravitational forces might extend beyond atoms, affecting cosmic structures and connecting all atoms in the universe.

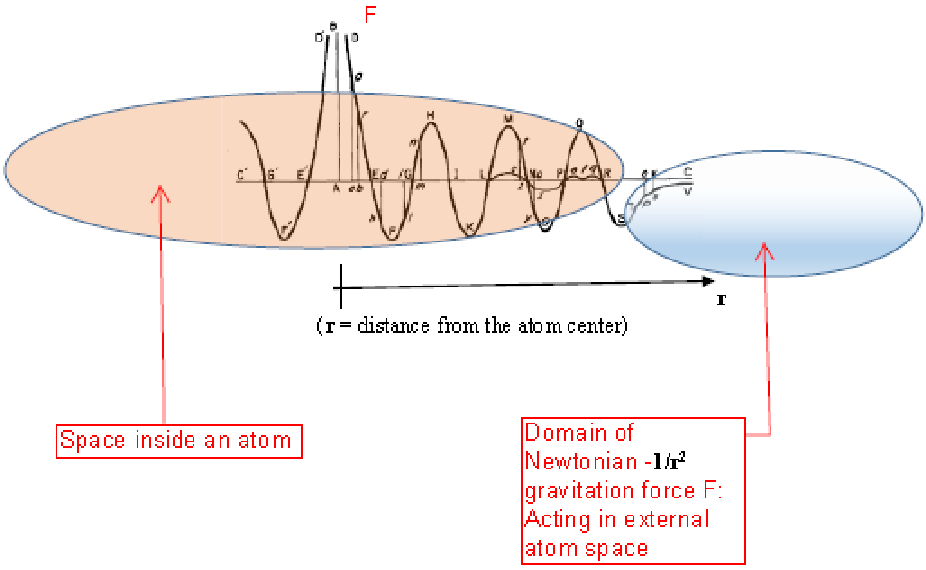
Nikola Tesla's concept of "radiant energy" or radiant fluid flow, which moves from atoms to the universe and vice versa, aligns with this view. Gravitational attraction outside of neutral atoms can be seen as an effect of these electromagnetic interactions. Every mechanical force can be considered a time derivative of momentum (F = dp/dt). Given the mutual compensation of action and reaction forces, we can imagine a steady flow of "radiant" fluid with linear momentum, offering a way to conceptualize gravitational force.

Planetary and solar systems share structural analogies with atomic systems, characterized by standing matter-wave resonances and couplings (see Sections 2.3.3 and 2.8 for details on the analogies between hydrogen atoms and planetary systems).

Gravitational forces, manifesting attractive effects outside atoms and masses, are fundamentally electromagnetic. Atoms, particles, and cosmic objects communicate bi-directionally through electromagnetic radiation, reflecting "Electromagnetic-Gravitational" forces.

This interpretation of alternating force-fields, resembling standing wave structures, has roots in the work of Rudjer Boskovic on universal natural force ([6], Principles of the Natural Philosophy) and in papers published in the *Herald of the Serbian Royal Academy of Science* between 1924 and 1940 (e.g., J. Goldberg, 1924; V. Žardecki, 1940). Nikola Tesla's Dynamic Theory of Gravitation also aligns with Boskovic's unified natural force, and the extended Bohr Atom Model discussed in Chapter 8. Further ideas on this topic can be found in Reginald T. Cahill's "Dynamical 3-Space: Emergent Gravity" [73]. Boskovic's work on repulsive (anti-gravitation) force elements explains the dual nature of natural forces and charges, akin to attractive and repulsive forces detectable with ultrasonic resonators, where nodal zones exhibit attractive forces, and anti-nodal zones exhibit repulsion.

$$\oiiint_{\left\{ r\in[0,\infty],\ \theta\in[0,2\pi],\ \phi\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right] \right\}} \mathbf{F}d\mathbf{R} = 0 \quad , \qquad (8.74)$$



Rudjer Boskovic's Universal Natural Force Function

Nikola Tesla, in his patents and successful experiments, demonstrated the existence of what he termed "radiant fluid." This radiant fluid is characterized by its electromagnetic nature, capable of carrying both positive and negative electric charges as well as mechanical moments [97]. Tesla's work supports the idea that all atoms and masses in the universe are in continuous communication. This concept aligns with Rudjer Boskovic's theory of the Universal Natural Force [6], which posits that atoms and masses function as mutually coupled and tuned resonators. This interaction is mediated by surrounding and coupling medium, historically referred to as aether (or a similar term until a more precise conceptualization is developed).

According to Tesla's ideas, this ether exhibits both electromagnetic and mechanical properties. It contains extremely fine carriers of mechanical and electromagnetic moments and charges and possesses characteristics of both dielectric and magnetic materials.

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

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2.4. How to unite Gravitation, Rotation, and Electromagnetism?

It should be increasingly clear that all forces and fields in our Universe are inherently unified, even if we have yet to formulate a comprehensive Unified Field Theory. In this context, we can explore promising avenues for developing a new theoretical framework that integrates rectilinear and rotational motions, drawing inspiration from the Faraday-Maxwell electromagnetic field definitions.

For instance, the Lorentz and Laplace forces explicitly link the rectilinear motion of electrical charges (currents) with magnetic fields. These forces can be adapted into analogous forms to describe interactions involving both rectilinear and rotational movements. The Ampère-Maxwell, Biot-Savart, and Faraday's induction laws could be reformulated to provide a more precise description of "linear-rotational" fields by translating these laws into corresponding analog expressions. For a broader context on electromagnetic and mechanical complexity, and the fundamental origins of gravitation, refer to Chapter 3.

Modern science has made numerous attempts to develop a Maxwell-like theory of gravitation and to explain the origins of inertia. Traditional gravitation theories treat mass as the primary source of gravity. However, this book proposes that more crucial sources of gravity-related phenomena are found in interactions between moving objects, particularly their linear and angular (or spinning) moments, which are coupled with electric and magnetic moments or dipoles, as well as their kinetic and rest energies (see Chapter 4.1 for more on de Broglie matter waves). Updating Wilhelm Weber's force law to encompass both electromagnetic and gravitational interactions, incorporating both linear and rotational motion elements, could provide a solid foundation for a new Maxwell-like theory of gravitation (see literature references [28] and [29]).

Additionally, many force and field manifestations, such as Coriolis, centrifugal, centripetal, gyroscopic effects, pendulum oscillations, inertial forces, and gravitomagnetic induction from General Relativity, could potentially be integrated and interpreted within the proposed gravitational framework. The connections between rotation, linear motion, and electromagnetism are conceptually outlined in this book (see Chapter 10, equations (10.1.4) to (10.1.7)), and further elaborated in equations (4.18), (4.22) to (4.29), and (5.15) and (5.16).

By establishing a theoretical foundation for understanding the complementary nature of "linear-rotational" fields and motions (see Chapters 4.0 to 4.3), we can develop a complete set of "Gravity-Rotation" field equations. Initially, these equations could be analogous to Maxwell's equations for electromagnetic fields. They would later be refined and expanded to accurately reflect the nature of different natural fields and forces (see the development of equations (4.22) to (4.29)). In Chapter 3, we will explore how Maxwell's Theory might be slightly revised to align with the updated theory of gravitation (refer to literature [23] to [26]).

To make a significant and novel contribution to our understanding of gravitation, we must introduce unique and original concepts that go beyond merely presenting redundant or analogous variations of existing field theories. Let us begin with one such innovative idea.

2.5. New Platforms for Understanding Gravitation

Gravitation can be conceptualized using analogies from mechanical or acoustic resonators. Imagine our macro-universe as a fluid-like medium filled with various particles or mass agglomerations, such as cosmic objects, planets, stars, galaxies, dust, atoms, and plasma states. Picture this cosmic fluid being vibrated at a constant frequency by an external, currently unknown source of mechanical vibrations.

To visualize this concept, consider an experiment with a vessel filled with liquid mixed with solid particles. When the vessel is vibrated, a three-dimensional standing wave pattern forms. The submerged and suspended particles create higher mass densities at the nodal areas of these standing waves. This effect, known as acoustic or ultrasonic levitation, demonstrates that particles in nodal zones experience minimal oscillating velocities and maximal mass density.

In this setup, if we introduce a small test particle near a nodal zone with high mass density, it will be attracted to the nearest nodal area or particle. This phenomenon simplifies our initial brainstorming by excluding electromagnetic forces. Similar attractive forces can be observed in resonant, standing-wave oscillations of solid resonators used in ultrasonic technology. If external vibrations driving these resonators are turned off, the attractive forces toward the nodal areas disappear.

Now, imagine our universe as a mechanical fluid-like system that is in a constant state of low-frequency resonant and standing wave oscillations (see time-frequency relations (5.14-1) in Chapter 5, Uncertainty). These standing waves would force astronomical objects to occupy stable nodal positions or orbits, forming a kind of spatial matrix. Test masses around these objects would experience forces analogous to gravitational attraction.

This initial concept can be expanded to include linear and rotational motions of cosmic objects, which would comply with agglomeration rules around global standing-wave nodal areas, consistent with Euler-Lagrange-Hamilton mechanics (see similar concepts in [99] by Konstantin Meyl). Here, mass can be conceptualized as a form of matter-wave energy packing or "frozen, rotating energy states."

While our universe contains significant "empty space" or vacuum states, which poses challenges for mechanical vibrations, these challenges are mitigated by the fact that acoustic, mechanical, electromechanical, and electromagnetic vibrations are mutually coupled. To understand the specific nature of this fluid-like substance that carries mechanical vibrations, we need to explore new and innovative approaches.

Contemporary physics has dismissed the idea of ether-type fluids due to lack of experimental confirmation. However, in mechanics and acoustics, we know that a vacuum cannot carry mechanical vibrations. Thus, our concept of gravitation requires a mechanism for resonant and standing wave states in the universe. Electromagnetic, magnetostrictive, or electrostrictive coupling might play a role in allowing mechanical vibrations to penetrate vacuum states, acknowledging that vacuum is not truly empty.

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This conceptualization of gravitation offers a richer and different perspective compared to Newtonian, Keplerian, and Einsteinian frameworks, which do not explain why gravitation manifests solely as an attractive force. Standing-wave mechanical resonators demonstrate attractive forces in their nodal areas, creating acoustic levitation. By analogy, this could offer insights into the nature of gravitation (see more in [150] and [151]).

The key question remains: What is the source of these cosmic standing waves or vibrations? Since all constituents of our universe are interconnected and in constant relative motion, understanding matter-waves associated with mass motions is crucial to answering this question.

Einstein's General Relativity Theory explains gravitation through space and field geometry modifications, but it does not speculate about resonant standing wave formations as a possible cause. Any new theory of gravitation should consider the coupling of electromagnetic and other forces with gravitation. Although gravitational forces are significantly weak compared to electromagnetic and nuclear forces, a comprehensive theory should integrate these interactions.

If the proposed concept has solid practical and theoretical support, it could represent a breakthrough in understanding gravitation and potentially other forces. The Superstrings or M-theory, which explores multi-dimensional space with vibrating strings and membranes, offers a familiar approach to these concepts but remains an evolving field. Any new theory of gravitation should be simple, elegant, and well-integrated with established physics, with further modeling attempts to follow.

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Let us review original abstracts from different publications, showing new, emerging aspects of still evolving, and future theory of gravitation:

A) **Dynamical 3-Space. Emergent Gravity**; Reginald T. Cahill, School of Chemical and Physical Sciences, Flinders University, Adelaide 5001, Australia, E-mail: Reg.Cahill@inders.edu.au . Invited contribution to: **Should the Laws of Gravitation be reconsidered**? Héctor A. Munera, ed. (Montreal: Apeiron 2011), **[73].**

The laws of gravitation devised by Newton, and by Hilbert and Einstein, have failed many experimental and observational tests, namely the borehole g anomaly, at rotation curves for spiral galaxies, supermassive black hole mass spectrum, uniformly expanding universe, cosmic filaments, laboratory G measurements, galactic EM bending, precocious galaxy formation, … The response has been the introduction of the new epicycles: "dark matter", "dark energy", and others. To understand gravity, we must restart with the experimental discoveries by Galileo, and following a heuristic argument, we are led to a uniquely determined theory of a dynamical 3-space. That 3-space exists has been missing from the beginning of physics, although it was first directly detected by Michelson and Morley in 1887. Uniquely generalizing the quantum theory to include this dynamical 3-space, we deduce the response of quantum matter and show that it results in a new account of gravity and explains the above anomalies and others. The dynamical theory for this 3-space involves G, which determines the dissipation rate of space by matter, and α, which experiments, and observation reveal to be the fine structure constant. For the first time, we have a comprehensive account of space and matter and their interaction - gravity.

B) **The Nature of Space and Gravitation; Jacob Schaff.** Instituto de Fsica, Universidade Federal do Rio Grande do Sul (UFRGS), Porto Alegre, Brazil.

Email: schaf@if.ufrgs.br. Received May 12, 2012; revised June 8, 2012; accepted July 1, 2012. doi: 10.4236/jmp.2012.38097. Published Online August 2012 (http://www.SciRP.org/journal/jmp), **[74].**

Many recent highly precise and unmistakable observational facts achieved thanks to the tightly synchronized clocks of the GPS provide consistent evidence that the gravitational fields are created by velocity fields of real space itself, a vigorous and very stable quantum fluid like spatial medium, the same space that rules the propagation of light and the inertial motion of matter. It is shown that motion of this real space in the ordinary, three dimensions around the Earth, round the Sun and round the galactic centers throughout the universe, according to velocity fields strictly consistent with the local main astronomical motions, correctly induces the gravitational dynamics observed within these gravitational fields. In these space-dynamics, the celestial bodies all firmly rest with respect to real space, which, forth-rightly leads to the observed null results of the Michelson light anisotropy experiments, as well as to the absence of effects of the solar and galactic gravitational fields, on the rate of clocks moving with Earth, as recently discovered with the help of the GPS clocks. This space dynamic exempts us from explaining the circular orbital motions of the planets around the Sun; likewise, the rotation of Earth exempted people from disclosing the diurnal transit of the heavens in the days of Copernicus and Galileo because it is space itself that so moves. This space dynamic also eliminates the need for dark matter and dark energy to explain the galactic gravitational dynamics and the accelerated expansion of the universe, respectively. It also straightforwardly accounts regarding well-known and genuine physical effects for all the other observed effects, caused by the gravitational fields on the velocity of light and the rate of clocks, including all the new effects recently discovered with the help of the GPS. It moreover simulates the non-Euclidean metric underlying Einstein's space-time curvature. This space dynamic is the crucial innovation in the current world conception that definitively resolves all at once the troubles afflicting the current theories of space and gravitation.

C) **Deriving gravitation from electromagnetism**. Can. J. Phys. 70, 330- 340 (1992). A. K. T. ASSIS[1]. *Department of Cosmic Rays and Chronology, Institute of Physics, State University of Campinas,* C. *P. 6165, 13081 Campinas, Sao Paulo, Brazil.* Received November I, 1991. Can. J. Phys. 70.330 (1992), **[75].**

We present a generalized Weber force law for electromagnetism including terms of fourth and higher order in v/c. We show that these additional terms yield an attractive force between two neural dipoles in which the negative charges oscillate around the positions of equilibrium. This attractive force can be interpreted as the usual Newtonian gravitational force as it is of the correct order of magnitude, is along the line joining the dipoles, follows Newton's action and reaction law, and falls off as the inverse square of the distance.

D) **The Electrodynamic Origin of the Force of Gravity, Part 1; ($F = Gm_1m_2/r^2$). Charles W. Lucas, Jr.** 29045 Livingston Drive, Mechanicsville, MD 20659-3271, bill@commonsensescience.org, **[76].**

The force of gravity is shown to be a small average residual force due to the fourth order terms in v/c of the derived universal electrodynamic contact force between vibrating neutral electric dipoles consisting of atomic electrons vibrating concerning protons in the nucleus of atoms. The derived gravitational force has the familiar radial term plus a new non-radial term. From the radial term, the gravitational mass can be defined in terms of electrodynamic parameters. The non-radial term causes the orbits of the planets about the sun to spiral about a circular orbit giving the appearance of an elliptical orbit tilted concerning the equatorial plane of the sun and the quantization of the orbits as roughly described by Bode's law. The vibrational mechanism that causes the gravitational force is shown to decay over time, giving rise to numerous phenomena, including the expansion of the planets (including the earth) and moons in our solar system, the cosmic background radiation, Hubble's redshifts versus distance (due primarily to gravitational redshifting), Tifft's quantized redshifts (Bode's law on a universal scale), Tifft's measured rapid decay of the magnitude of redshifts over time, the Tulley-Fisher relationship for luminosity of spiral galaxies, the unexpected high velocities of the outer stars of spiral galaxies, and Roscoe's observed quantization of the luminosity and size (Bode's law) of 900 spiral galaxies. Arguments are given that this

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derived law of gravity is superior to Newton's Universal Law of Gravitation ($F = Gm_1m_2/r^2$) and Einstein's General Relativity Theory ($G_{\mu\nu} = -8\pi G/c^2 T_{\mu\nu}$).

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The next citation (or copy) from ***[83], David L. Bergman, editor: Selected Correspondence on Common Sense Science #1. November 2013, Volume 16, number 4. Common Sense Science, P.O. Box 767306, Roswell, GA 30076-7306, E-mail: bergmandavid@comcast.net***

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Gravity Dilemma. I seem to remember that gravity cannot be shielded. An internet search agrees. Yet I believe electromagnetism can be shielded. If so, how can gravity be of electrodynamic origin?

Russel Moe,
Wildwood, Florida

Reply by Dave Bergman. More than once I have asked myself the same question. Eventually I resolved the dilemma in this way. First, recall that matter is composed of elementary particles—mostly electrons and protons.

Second, every electron and every proton has self-generated electromagnetic fields surrounding the charged particle and spreading outward into an increasing volume of space with field intensities that decrease in accordance with the inverse square law that applies also to gravity.

Third, these electromagnetic fields have an *oscillating component* and a *non-oscillating component*, corresponding respectively to radiation of energy (including **light** energy) and **gravity**.

Fourth, actual measurements show that shielding of an oscillating electromagnetic field ranges from *no shielding* to *partial shielding* to *high shielding* in correspondence with the **wavelength** of the oscillation [D. G. Fink, Editor-in-Chief, **Standard Handbook for Electrical Engineers, Tenth Edition,** McGraw-Hill Book Company, p. 29-23, fig. 29-40 (1957).]

♦♦♦♦♦♦♦♦♦♦♦

Reply by Dr. Lucas. Good question. The answer is that the experiments of William J. Hooper as described in his book **New Horizons in Electric, Magnetic and Gravitational Field Theory** identify that all three types of electric and magnetic fields have different empirical properties. For instance in Chapter 1 Hooper lists 14 empirical properties of E fields. In Table 1 he gives what these properties are for electrostatic E fields, E fields dependent on dA/dt, and E fields dependent on motion $V \times B$. In particular he notes in property 6 that the motionally caused E fields cannot be shielded.

There is an interesting story regarding this E field that cannot be shielded. When Hooper discovered this effect, he invented and patented a speedometer for airplanes. When a plane was flying with respect to the surface of the earth, Hooper's speedometer measured the $V \times B$ term due to the velocity of the plane with respect to magnetic field lines of the earth being crossed.

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

He arranged a demonstration with the military. They flew Hooper's speedometer and compared it readings with respect to other methods. The military testers confirmed that Hooper's speedometer worked very well. However, when Hooper applied for a government contract to supply the military with these speedometers, his application was rejected by the scientific reviewers, because they said that the metal hull of the military aircraft would shield the effect. According to Maxwell's theory of electrodynamics, all three forms of the electric field have the same identical properties. The scientific reviewers ignored the fact that the military testers had tested Hooper's speedometer inside a metal-hulled military airplane, and it worked inside that shielded environment.

In our derivation of an improved and more general version of the electrodynamic force law, we treat each of the three types or sources of electric fields separately as distinctly different types of E fields.

It is the term proportional to $R(R \times V)$ in the universal electrodynamic force law that gives rise to the force of gravity. It is equivalent to Hooper's motional E field.

…………………………………..

specifically to the helical charged particles. Thus, inertial mass was not fundamental, but a specific, calculable property of the unit charged particles due to their electro-magnetic structure.

Your approach differs in considering a net neutral, but finite size, structured combination of particles, interacting with all the other such dipole particles in the universe. Thus, the amount of inertia demonstrated by a particle's resistance to motion is not even a specific amount, calculable from the charge, speed of light c and physical dimensions, but depends on the amount and distribution of all the other particles in the universe.

…………………………………………………

There is a hierarchy of interactions in electrodynamics in order of decreasing strength as follows:

1. Charge to charge – Coulomb force
2. Charge to neutral electric dipole – inertial force
3. Neutral electric dipole to neutral electric dipole – gravitational force
4. Charge to neutral electric quadruple – dust or plasma aggregation & rotation
5. Neutral electric dipole to neutral electric quadruple – dust aggregation & rotation
6. Neutral electric quadruple to neutral electric quadruple – dust aggregation & rotation etc.
7. The charge to vibrating neutral electric dipole force is second order in the hierarchy.

The sentence "In the next section the interaction force that gives rise to the force of gravity will be considered." should have been "On the grand scale the interaction force that gives rise to the force of gravity is defined in terms of inertial mass as…."

> "Thus the interaction force that Einstein referred to above that gives rise to the force of inertia of any specific dipole pair is due to the vibration of that pair, and all the other vibrating, neutral, electric dipoles in the rest of the universe."

would perhaps be better stated

> "Thus the interaction force that Einstein referred to above that gives rise to the force of inertia of any specific dipole pair is due to the interaction of that dipole with all the other charges in the universe."

In a similar manner one could say that

> "The interaction force that gives rise to the force of gravity on any specific electric dipole pair is due to the interaction of that electric dipole with all the other electric dipoles in the universe."

The notion of mass is somewhat poorly defined in science. In the Standard Model of elementary particles and Einstein's theory of relativity, mass is an inherent property of a point-elementary-particle. Thus the specifics of the interactions within the structure of the particle are not taken into account. However, if one measures the *mass* of an atom, it is not the sum of the masses of the electrons, protons, and neutrons, but something less indicating that if the mass of a particle depends on internal structure, it changes within the environment of the atom. If one measures the total *charge* of an atom, it is exactly the sum of the charges of the electrons, protons, and neutrons in the atom. *Thus mass is not an inherent fixed quantity like charge!*

From the definition of the force of inertia $F = ma$, mass m is a measure of resistance to motion in some environment. In fluid dynamics the resistance to motion of a particle depends on the density of the fluid around it. Thus mass depends more on the environment than on fixed inherent local properties of the particle of mass. The total resistance to motion is not inherent to the particle, but depends on its environment.

In the work of Barnes and Bergman there is a feedback effect due to Lenz's Law on a charged elementary particle causing it to have a mass of inertia. From Mach's Principle we can see that there are two parts to this mass. The first is the local asymmetry term, and the second is the contribution on the grand scale from the rest of the universe. The first term will depend intimately on the internal structure of the particle. The second term will depend heavily on the symmetry of the universe.

There can be multiple types of contributions to inertial mass. In my work so far I have concentrated on the hierarchy of electrodynamic interactions, i.e.

1. charge to charge - Coulomb force
2. charge to electric dipole - force of inertia
3. electric dipole to electric dipole - force of gravity, etc.

Besides these supposedly primary contributions, there can be secondary and tertiary contributions just as in the case of the atom where there are fine structure and hyper-fine structure contributions. My electrodynamic force derivation of the force of inertia is able to explain the origin of the cosmic microwave background radiation and its spectrum, the unusual gyroscope experiments of Eric Laithwaite in defiance of Newton's force of inertia, the constant high velocity of the outer arms of spiral galaxies in defiance of General Relativity requiring the invention of dark matter, and the general expansion of the universe in defiance of General Relativity requiring the invention of dark energy.

……………………………………………….

Reply by Charles Wm. (Bill) Lucas, Jr. See the URLs below for the "politically correct view" that Russell Humphreys relies upon.

http://en.wikipedia.org/wiki/Hafele%E2%80%93Keating_experiment

http://en.wikipedia.org/wiki/Time_dilation_of_moving_particles

http://en.wikipedia.org/wiki/Pound%E2%80%93Rebka_experiment

For time dilation of moving particles, Special and General Relativity theories are point-particle theories. No real elementary particles such as protons, neutron, muons, etc. are point-particles. All have both finite size and internal structure. Since in my work these elementary particles consist of multiple charge current loops, the elementary particles experience an electromagnetic feedback effect when they move that compresses the particle and increases its binding energy and effective mass. The increased binding energy causes the half-life of the muon to increase with velocity. Special Relativity theory mathematically predicts the same result, but for the wrong reasons due to its use of many idealizations such as the point-particle idealization and that space is homogeneous and isotropic that does not correspond to reality.

The gravitational redshift—a tenet extrapolated from Einstein's theory of general relativity—claims that clock rates change with gravitational potential, as a result of space-time being bent by objects of large mass. Ed Dowdye has shown that for starlight passing near the sun there is no bending of the path of the light due to general relativity theory. The only bending of starlight that occurs is when the light passes through the electrical plasma rim of the sun due to electromagnetic effects. At larger distances the predicted bendings of light due to general relativity theory are *not* observed.

With regard to the Pound-Rebka experiment at Harvard in 1959 to detect the redshift and blue-shift in light moving in a gravitational field due to clocks running at different rates at different places in a gravitational field, note that the Mossbauer effect was used. In 1958 Mossbauer had reported that all the atoms in a solid lattice absorb the recoil energy when a single atom in the lattice emits a gamma ray. The test is based on the following principle: When an electron in an atom transits from an excited state to a ground state, it emits a photon with a specific frequency and energy. When an electron in an atom of the same species in its ground state encounters a photon with that same frequency and energy, it will absorb that photon and transit to the excited state. If the photon's frequency and energy is different by even a little, the atom cannot absorb it (this is the basis of quantum theory). When the photon travels through a gravitational field, its frequency and therefore its energy will change due to the gravitational redshift. As a result, the receiving atom cannot absorb it. But if the emitting atom moves with just the right speed relative to the receiving atom the resulting Doppler shift cancels out the gravitational shift and the receiving atom can absorb the photon.

http://en.wikipedia.org/wiki/Doppler_shift

The "right" relative speed of the atoms is therefore a measure of the gravitational shift. The frequency of the photon "falling" towards the bottom of the tower is blue-shifted. Pound and Rebka countered the gravitational blue-shift by moving the emitter away from the receiver, thus generating a relativistic Doppler redshift. The energy associated with gravitational redshift over a distance of 22.5 meters is very small. The fractional change in energy is given by $\partial E/E = gh/c^2 = 2.5 \times 10^{-15}$. Therefore short wavelength high energy photons are required to detect such minute differences.

http://en.wikipedia.org/wiki/Electromagnetic_spectrum

When it transitions to its base state, the 14 keV gamma rays emitted by iron-57 proved to be sufficient for this experiment.

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The Doppler shift required to compensate for this recoil effect would be much larger (about 5 orders of magnitude) than the Doppler shift required to offset the gravitational redshift. But in 1958, Mössbauer reported that all atoms in a solid lattice absorb the recoil energy when a single atom in the lattice emits a gamma ray.

http://en.wikipedia.org/wiki/Rudolf_M%C3%B6%C3%9Fbauer

http://en.wikipedia.org/wiki/Lattice_model_(physics)

http://en.wikipedia.org/wiki/M%C3%B6ssbauer_effect

<http://en.wikipedia.org/wiki/Pound%E2%80%93Rebka_experiment#>

Therefore the emitting atom will move very little. However, the notion of a photon being emitted or absorbed on a single electron of an atom is not supported by the Mossbauer effect. The wave nature of light is supported by the Mossbauer effect where the crystal lattice acts as an antenna for emission and absorption of light.

In Dr. Lucas's electrodynamic theory of gravity, vibrating neutral electric dipoles are the source of the gravitational force. The movement of the vibrating neutral electric dipoles changes the strength of the gravitational field to match the red and blue shifts. Thus there is no role for Einstein's general relativity theory. This result is also consistent with the work of Ed Dowdye showing that there is no gravitational bending of starlight due to general relativity. Thus the so-called gravitational redshift of light supports only the electrodynamic theory of gravity, *not* General Relativity theory.

Charles W. Lucas, Jr.
Mechanicsville, Maryland

2.6. A short resume of possibilities for direct experimental and theoretical verification of innovated theory of Gravitation concerning Particle-Wave Duality and Matter Waves:

1. Vortex flow meter, Karman Vortex Streets, vortices-frequency, and Strouhal-Reynolds number, in linear and robust relationship to fluid flow velocity, can be explained using here-elaborated coupling between linear and spinning motions, including associated matter-waves and particle-wave duality concepts. A similar concept can be analogically extended to the motions of planets within planetary systems. Modern engineering is using vortex flowmeters for an exceptionally long time, without real and fundamental explanation and insight why fluid flow velocity is directly and linearly proportional to vortices frequency (see Chapter 4.1, where vortex flow meter equation is developed and explained as the consequence of liner and spinning motions coupling).

2. *The work of matter-waves and associated gravitation related forces we can find in analyzing a **spin-stabilized satellite.** This is a satellite, which has the motion of one axis held (relatively) fixed by spinning the spacecraft around that axis, using the gyroscopic effect. The attitude of a satellite or any rigid body is its orientation in space. If such a body initially has a fixed orientation relative to inertial space, it will start to rotate, because it will always be subject to small torques. The most natural form of attitude stabilization is to give the rigid body an initial spin around an axis of minimum or maximum moment of inertia, meaning in the same direction where helical matter waves tend to be naturally created. The body will then have a stable rotation in inertial space. Rotation about the axis of minimum moment of inertia is at an energy maximum for a given angular momentum, whereas rotation about the axis of maximum moment of inertia is at a minimum energy level for a given angular momentum. In the presence of energy loss, as is the case in satellite dynamics, the spin axis will always drift towards the "axis of maximal moment of inertia". For short-term stabilization, for example, during satellite insertion, it is also possible to spin-stabilize the satellite about the axis of minimum moment of inertia. However, for long-term stabilization of a spacecraft, spin stabilization about its axis of maximal moment of inertia must be used (read about Lyapunov stability concept, and see more about inertia in Chapters 1, 4.2 and 10). Here is the place to explain, harmonize and generalize mentioned items with **PWD**, matter waves conceptualization, and convenient mathematics (as vastly elaborated in this book).*

We can also find supporting background to helical matter waves associated with modern-guns bullets motion. Rifling is the process of making helical grooves in the barrel of a gun or firearm, which imparts a spin to a projectile around its long axis. This spin serves to stabilize a projectile gyroscopically, improving its aerodynamic stability and accuracy. Bullet stability depends primarily on gyroscopic forces, the spin around the longitudinal axis of the bullet imparted by the twist of the rifling. Once the spinning bullet is pointed in the direction the shooter wants, it tends to travel in a straight line, until it is influenced by outside forces, such as gravity, wind, and impact with the target. Without spinning, the bullet would tumble in flight. Modern rifles are only capable of such fantastic accuracy because the bullet is stable in flight (thanks to the gyroscopic effect). Even spherical projectiles must have a spin to achieve any acceptable accuracy. We could analogically reverse the cause and effect in the same situation (of spinning bullets propagation), and say that particles in inertial, stationary, and stable motion should have helical, spinning matter wave around its long axis of propagation, because such matter wave and moving particle are in mutually relative spinning motion (either one or the other is spinning). Such factual situation (of enormously increased bullets accuracy and stability) should be verifiable by mathematical relations that are used in matter waves' conceptualization.

3. In addition, since linear and helix or spinning and rotating motions (of masses) are mutually complementary and united (in relation to matter-waves, or **PWDC** elaborated in this book, Chapter 4.1), we could imaginatively, creatively, and analogically extrapolate such concepts from atoms to planetary systems and galactic formations and try to differently address Hubble's law. On some way, our universe is globally rotating and spinning, following helix-like paths of associated matter waves. What we observe as red or blue, Doppler shifts (of electromagnetic radiation) coming from remote, deep space, could be consequences of such globally present, macro-rotating effects of distant masses. As we know, the tangential velocity of the certain rotating mass, $\mathbf{v_t}$ is equal to the product of relevant orbital velocity ω, and relevant radius $\mathbf{R}$, $v_t = \omega R$. Hubble's law is maybe saying something similar, such as, $v = (v_t) = H_0 R$, where H_0 is Hubble constant, which could be certain metagalaxy, orbital (or helicoidally spinning associated matter waves) velocity. If an expansion of our Universe is in some ways partially mistaken and masked by, or related to such macro-rotation, this will open a window into amazing new Cosmology research areas.

4. Another similar phenomenon is related to helical liquid funneling (spiral spinning) when liquid is in a vessel that has an open hole, or a sink at the bottom. Outgoing liquid flow speed should be directly proportional to the spiral, funneling frequency of the liquid in the same vessel (this is the proposal for an experimental verification). The explanation should take into consideration that every linear motion is intrinsically linked to spinning in the same direction of movement (as elaborated in this book).

5. Macro matter-waves related situation (that is analogical to micro-world, de Broglie matter-waves) are also waves on a quiet water surface created by some moving object (a boat), where the water surface is visualizing matter-waves associated to a moving object. An average wavelength of such surface water-waves should be roughly equal to $\lambda = H/p = H/mv$ (or inversely proportional to the motional object speed, where (m, H) = constants). Of course, here we should be able to show the applicability of other matter-waves relations elaborated in this book, such as: $E_k = \tilde{E} = \frac{mv^2}{2} = \frac{pv}{2} = Hf, \;\; \Rightarrow u = \lambda f = \frac{v}{2}$. For instance, we can easily measure if the phase speed of surface water-waves, u, behind moving particle (or boat), is two times smaller than particle speed v.

6. Turbulences (including vortex turbulences) are also direct manifestations, or imprints, streaming and reverberations of matter-waves associated with masses motions in and around fluid environments. Fluids (in relative motion to the specific particle) should present sensitive sensor bodies (or spatial antennas and displays) for detecting and visualizing matter waves, vortices, spinning, and turbulences, including the possibility to detect astronomic macro matter-waves. Of course, cosmic macro matter waves could be detected when observing low-frequency acoustic fields' complexity inside vast lakes and ocean spaces. Present conceptualization related to Fluid dynamics and Navier-Stokes equations should also be enriched and optimized by considering here-elaborated matter-waves manifestations, as helically rotating fields' perturbations around and behind motional particles (see chapter 4.1), and the same should apply to enriching matter-waves concepts with elements of Fluid dynamics and Navier-Stokes equations. Also, we could extend familiar analogical associations to orbital planetary

motions in solar systems (see Chapter 2. Gravitation; 2.3.3. Macro-Cosmological Matter-Waves and Gravitation).

7. Many publications show (and experimentally documenting) weight reduction when certain spinning discs, spinning magnets, and gyroscopes, combined with other rotating and oscillating motions are specifically coupled (see much more in [36]). Such, seemingly anti-gravity effects are in fact consequences of natural, linear, and rotational motion couplings (including associated electromagnetic dipoles separation and polarization), when involved spinning discs and oscillatory systems are interacting with balanced natural motions, producing unbalanced effects of weight reduction (if we maintain such movements). Here, we should not forget that besides Newton linear motion forces, we also have coupled effects of rotational or torque forces. A certain specific combination of implemented torque components (by spinning disks or magnets) can be the forces acting against gravitation.

8. Universally valid and known effects of diffraction of light rays, particle beams, fluid beams, jets, and similar phenomena, could also be explicable if we consider that certain repulsive force (in relation to associated matter-waves and spinning) is developing between effective mass (or matter-waves) packets of "parallel flow elements". Such repulsive force should be causally related to the resonant half-wavelength of involved matter-waves, being the consequence of unity and couplings between linear and spinning motions. The force law that addresses diffraction (or beams repulsion) should respect Newton-Coulomb $1/r^2$ force-law (see [3]). If centers of active mass packets in described parallel motion are mutually separated by one half-wavelength, the repulsive force (or measured diffraction angle) should be maximal.

9. Theories related to "standing-waves" quantizing of planetary systems (as one elaborated in this book, in chapter 2.) that are directly analog to quantizing in Bohr atom model, are practically showing the triumph of here-elaborated matter waves and linear-rotational motions coupling concepts. In such modeling it is possible to integrate electromagnetic effects, like in early atom models (see [63] and [67]), showing existence of more complete unity between mechanical motions, gravitation, and electromagnetic fields, since such rich quantization in planetary systems (like in atoms) cannot exist without the dominant presence of electromagnetic forces and fields. In such quantized systems of synchronized and periodic (planetary) motions with standing-waves structure, it is possible to associate masses presence only to stable stationary orbits that are equivalent to spatial nodal zones (where orbital acceleration and density are maximal, and oscillating amplitudes minimal, like in resonant mechanical systems and standing waves related to acoustic and/or ultrasonic levitation). See more familiar concepts in [150], and [151] and in [99] from Konstantin Meyl.

10. Our sun and a countless number of stars can be regarded as a variety of blackbody objects (concerning blackbody radiation). Planck's blackbody radiation formula is mathematically fitted to experimentally measured situations, but the real, essential explanation of what is happening inside of a blackbody cavity is still missing. We can try to estimate what happens inside a blackbody cavity where we have complex, random motion of hot gas particles, random light emissions, absorptions, photons, and electrically charged particles collisions and scattering (including participation of particles with magnetic moments). We only know from Planck's formula the resulting (fitted and averaged) spectral distribution of outgoing light emission, in the case when we make a small hole on the surface of a blackbody, and let photons be radiated and measured in the external, free space of a blackbody. This external light radiation is characterized by free photons where each photon has the same phase and group

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http://www.mastersonics.com/documents/revision_of_the_particle-wave_dualism.pdf

velocity $v = u = c = \text{constant}$. This is not the case inside the blackbody cavity, since there are many mechanical and fields interactions between photons, gas particles, matter waves, and cavity walls, and there we have broad distributions of group and phase velocities of different energy-momentum entities, $0 \leq 2u \leq \sqrt{uv} \leq v \leq c$. A significant number of wave packets (de Broglie matter wave groups with mutually united or coupled mechanical and electromagnetic properties, with linear and spinning motion components), inside a blackbody cavity, permanently interact (among themselves, as well as with the cavity and gas particles), and we cannot consider them being freely propagating wave groups, or stable and synchronized standing waves formations. It is logical (as the starting point in an analysis of such case) to imagine that the mean particle or group velocity of such wave groups is (in average) directly proportional to the blackbody temperature, and when gas temperature (inside a black body radiator) is relatively low, then we should dominantly have motions with non-relativistic particle velocities ($v << c \Rightarrow v \cong 2u$). When a temperature is sufficiently (or remarkably) high, we should dominantly have the case of relativistic particle motions with high speeds ($v \approx c \Leftrightarrow v \approx u \approx c$). There is a big difference between free wave groups, like free photons in open space, and mutually interacting (de Broglie) matter-waves (inside of a limited space of a black body cavity). On the contrary, in most analyses of similar situations in modern Quantum Mechanics, we do not find that such differentiation is explicitly underlined and adequately treated (mostly we see that de Broglie matter waves are treated similarly to free photons or to other free wave groups, or as virtual and artificial probability waves). Also, in mathematical development of Planck's blackbody radiation law, we can essentially find specific particularly suitable (oversimplified, mainly poor, and unrealistic) modeling and curve-fitting situations, where phase velocities of a blackbody photons are always treated as the velocity of free (externally radiated) photons, or as $u = \lambda f = v = c = \text{Constant}$. This book offers new elements to help us to understand and develop blackbody radiation formula in a more natural way (see such elaborations in chapter 4.1 and chapter 9).

11. Integration of Mechanical and Electromagnetic Circuit Concepts

A largely overlooked but potentially insightful approach in understanding mechanical systems is to draw analogies with electric circuits and electromagnetic theory. In electrical engineering, circuits are typically analyzed as closed networks comprising various components, including a source (input generator) and a load (output). Internal components are treated as closed current-flow circuits, whether they handle DC, AC, or mixed currents.

Similarly, mechanical systems should be viewed through this lens. Mechanical systems, including those involving acoustics, should also have distinct input sources and output loads. In these systems, mechanical "currents" are analogous to forces and angular moments, representing the flow of linear and angular momentum.

Contemporary physics often analyzes mechanical systems as open-ended, either lacking input or output, or both. This approach limits our understanding of the complete nature of mechanical systems and their interactions with other systems. By considering these systems as closed circuits, we can better understand their energy and momentum exchanges, and their connections with different states of matter and matter waves.

Mechanical currents can exhibit both DC and AC characteristics, paralleling electric currents (see the first chapter of this book for detailed analogies). The innovative proposals presented in this book suggest that electric currents and mechanical orbital moments within particle and mass motion could be coupled and synchronized, potentially resulting in gravitational effects. Understanding these concepts in the context of closed mechanical systems could provide deeper insights into the workings of our universe. Nikola Tesla's ideas on electromagnetic phenomena and "Dynamic Gravity Theory" offer creative illustrations of these concepts, depicting closed electromechanical circuits and matter-wave interactions (refer to literature [97]–[101], the first chapter on Analogies, Chapter 4.1, Fig. 4.1.6, and Chapters 8 and 9).

Connections Between Mechanical and Electromagnetic Concepts

This section also explores the connections between closed current circuits, couplings, and the analogies between linear and angular motions, electric currents, and magnetic field effects. These connections are visually represented in Chapter 4.1, including Figures 4.1, 4.1.2 through 4.1.5, and are further detailed in equations (4.3) and subsequent sections.

Additionally, Professor Eric Laithwaite's experiments with closed circuits of energy-momentum flow demonstrate unusual couplings between linear and rotational motions of spinning gyroscopes, revealing magnetic field effects that influence gravitational forces (see [102]).

The chapter further elaborates on the existence of matter wave connections between masses, particularly focusing on planetary macro matter waves, as detailed in equations from (2.11.14) to (2.11.14)-h.

2.7 Dark Matter, Dark Energy, and Related Concepts

Dark Matter, Dark Energy, and other hypothetical entities can be qualitatively and mathematically explained through the concept of mass, moments, and energy expressed as "4-vectors of Energy-Momentum" in Relativity Theory (Minkowski space formalism). These can be represented as complex or hyper-complex mathematical functions, encompassing Real, Imaginary, and Apparent parts. For more details, see Chapter 10, particularly Section 10.1 on Hypercomplex Analytic Signal Functions and their interpretation concerning energy-momentum 4-vectors, matter-waves, and particle-wave duality. This framework supports the concept of resonant synchronization, entanglement, and the existence of Dark Matter and Energy, presenting these "dark" entities as components of a dynamic, resonant universe.

2.8 Revisiting Gravity: Questions and Challenges

To understand gravitation, we start with Newton's and Einstein's theories, which are well-established mathematically and effective in spatial engineering. However, there remain fundamental questions about the origins and nature of universal gravitation:

1. Shielding and Manipulating Gravitation:
   - Current experiments show that gravitational effects are not reduced by shielding, implying that gravitational sources extend beyond the atomic fields and interact through electromagnetic polarization. This concept aligns with Nikola Tesla's and Rudjer Boskovic's theories about gravitational and radiant energy, suggesting a background vortex or rotation in an invisible fluid surrounding mass.
   - There are speculative and highly hypothetical indications that specific spatial resonant structures could influence gravitational effects, such as weight reduction observed in certain geometric structures created by natural or artificial means (see Viktor Stepanovich Grebennikov's work [149]).

2. Attractive vs. Repulsive Gravitation:
   - Current models only account for attractive forces because they are based on incomplete conceptualizations. Stable masses are often seen as aggregates of smaller masses positioned in nodal zones of resonating systems, where only attractive forces are detected. However,

anti-nodal zones might exhibit repulsive forces, suggesting a need for a broader understanding of gravitational forces.

3. Mathematical Similarity Between Newton's and Coulomb's Laws:
 - Both laws share similar mathematical forms because they stem from fundamental electric and magnetic interactions. Masses can be conceptualized in terms of internal electric and magnetic dipoles, making Newton's law analogous to Coulomb's law.

4. Understanding Dark Matter and Energies:
 - Mass and energy are not confined to solid boundaries but are part of a dynamic, resonant universe. The anomalies observed in remote spiral galaxies, where Newtonian and Einsteinian gravitation theories fall short, may involve contributions from free-standing electric and magnetic charges.

In conclusion, addressing these questions conceptually and innovatively offers new insights into gravitation, though practical and mathematically validated theories and devices are still under development. The goal is not to discard Newtonian and Einsteinian theories but to expand our understanding and open new engineering possibilities.

[♣ COMMENTS & FREE-THINKING CORNER (still in preparation and brainstorming phase)**:**

2.3.3-1 Binary Systems, Kepler and Newton Laws and Matter Waves Hosting

The essential fact in the background of (2.11.10) - (2.11.14) is that gravitation is the central force. Its direction is always along a radius, either towards or away from a point we are using as an origin or force center. The magnitude of such central force depends solely upon the distance from its origin, **r**. We can present such forces as, $F_{m-M} = \frac{GmM}{r^2} = F(r), \vec{F}(r) = F(r)\cdot\frac{\vec{r}}{r}$. Central forces are interesting because we find them very often in physics. The gravitational and electrostatic forces are central forces (as well as forces between permanent magnets). Much of classical mechanics or physics can be placed in the framework of elegant applications of Newton Laws. Let us start with the Second Newton Law, $\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt}$, and express the associated torque and angular momentum as, $\vec{\tau} = \vec{r}\times\vec{F} = \vec{r}\times m\vec{a}$, $\vec{L} = \vec{r}\times\vec{p}$. Since torque is the time derivative of angular momentum, let us find the torque for central forces (where $\vec{F}(r)$ is parallel with $\vec{r}$) as, $\left(\vec{\tau} = \frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r}\times\vec{p}) = \vec{r}\times m\vec{a} = \vec{r}\times\vec{F} = 0\right) \Rightarrow \vec{L} = \text{const.}$ Consequently, an orbital and stable planetary motion has constant angular momentum because gravitational force is the central force. A little bit later we will see that this is the real origin of quantization in physics (equally applicable to Coulomb Law for related analogical situations), as well as to micro-world of atoms and elementary particles where building blocks have constant angular moments or spin characteristics. Or saying the same differently, every self-closed, circular, elliptic, periodical, and stable motion is potentially hosting standing waves formations, leading directly to quantizing in Physics.

Let us now analyze the simplified case of gravitational attraction between two masses $m_1 = m$ and $m_2 = M$ from the point of view of Binary Systems relations in their center of mass coordinate system. The total separation (or distance) between centers of the two masses is $\vec{r} = \vec{r}_1 + \vec{r}_2$. We may define the center of a mass (being between two objects) through the equations,

$$m_1 r_1 = m_2 r_2 \text{, } r = r_1 + r_2 \text{, } r_1 = \frac{m_2}{m_1 + m_2} r \text{, } r_2 = \frac{m_1}{m_1 + m_2} r \text{ .} \quad (2.11.14\text{-}1)$$

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

Gravitational attraction between m_1 and m_2 will not change if we imagine that m_1 and m_2 may be in a uniform rotational motion around their common center of mass since masses will at the same time experience mutually repulsive, balancing centrifugal force (possible spinning is still not considered). Let us imagine that m_1 and m_2 are rotating (around their common center of mass) with certain angular speed $\omega = \frac{2\pi}{T}$, what can be described with another set of equations,

$$\omega = \frac{v_1}{r_1} = \frac{v_2}{r_2} = \frac{v}{r}, \frac{v_1}{v_2} = \frac{r_1}{r_2}, p_1 = m_1 v_1 = m_2 v_2 = p_2 = p = p_r = m_r v_r, \vec{p}_1 + \vec{p}_2 = 0,$$
$$\vec{v}_i = \frac{d\vec{r}_i}{dt}, \vec{v} = \frac{d\vec{r}}{dt} = \vec{v}_r = \vec{v}_1 + \vec{v}_2, v_1 v_2 = \omega^2 r_1 r_2, v_i = \omega r_i, \frac{v_1 v_2}{v^2} = \frac{r_1 r_2}{r^2}. \quad (2.11.14\text{-}2)$$

Here v_1 and v_2 are tangential velocities of m_1 and m_2. In cases of such circular, rotational motions, every mass is experiencing certain centrifugal (mutually opposed) force with a tendency to separate them, for example,

$$F_c = \frac{m_1 v_1^2}{r_1} = \frac{m_2 v_2^2}{r_2} = \frac{dp}{dt} = \frac{dp_r}{dt} = \frac{m_r v_r^2}{r} = \frac{p_r v_r}{r} \Leftrightarrow m_1 v_1 = m_2 v_2 (= p = m_r v_r = p_r) \Rightarrow$$
$$\left\{ \begin{array}{c} \Rightarrow \text{after integration} \Rightarrow \\ v_r = \frac{dr}{dt} = \frac{p_0}{m_r} \cdot \frac{r}{r_0}, p_r = m_r v_r = p_0 \cdot \frac{r}{r_0}, F_r = F_c = \frac{dp_r}{dt} = \frac{p_0}{r_0} v_r = p_0 \omega \cdot \frac{r}{r_0}, [p_0, r_0] = \text{constants} \end{array} \right\}. \quad (2.11.14\text{-}3)$$

If the distance between two masses m_1 and m_2 is remaining unchanged (stable orbital motions), mutually opposed (or repulsive) centrifugal forces should be balanced with similar (central) attractive force between them, which is Newton force of gravitation F_g. Conceptualizing a given case of a stable Binary System this way, we are developing and formulating Kepler's third law, as follows.

$$\left\{ \begin{array}{l} F_g = G\frac{m_1 m_2}{r^2} = F_c = \frac{m_1 v_1^2}{r_1} = \frac{m_2 v_2^2}{r_2} = m_1 v_1 \omega = m_2 v_2 \omega = m_1 r_1 \omega^2 = m_2 r_2 \omega^2 = \frac{m_1 m_2}{m_1 + m_2} v_r \omega = \frac{m_1 m_2}{m_1 + m_2} r\omega^2 = m_r r\omega^2 = \frac{m_r v_r^2}{r} \\ m_r = \frac{m_1 m_2}{m_1 + m_2}, v_r = \omega r = \frac{dr}{dt}, v_1 = \frac{m_2}{m_1 + m_2} v_r = \omega r_1, v_2 = \frac{m_1}{m_1 + m_2} v_r = \omega r_2 \end{array} \right\} \Rightarrow$$

$$\Rightarrow \omega = \sqrt{G\frac{m_1 + m_2}{r^3}} = \frac{2\pi}{T}, \Leftrightarrow \left(\frac{T}{2\pi}\right)^2 = \frac{r^3}{G(m_1 + m_2)}. \quad G\frac{m_1 m_2}{r^2} = G\frac{m_r m_c}{r^2} = F_g, \ m_c = m_1 + m_2. \quad (2.11.14\text{-}4)$$

Another conclusion radiating from here is that natural tendency of masses (regarding stable Binary Systems, or multi-mass systems) is to create uniform or stationary rotational motions (around their common center of mass), this way balancing attractive Newton force with associated centrifugal force. If such rotation is not a visible case, at least mathematically and by respecting relevant conservation laws every Binary System could be equally presentable as a case of mutually coupled rotating bodies (including rotating disks, toroids…). The coupling force in cases of electromagnetically neutral bodies m_1, m_2 is the gravitation.

We could also say that boundary or asymptotic tendency (or just mathematically equivalent state in the same center of mass coordinates) of Binary Systems is that initial masses m_1 and m_2 can be effectively replaced by one bigger central mass which is equal $m_c = m_1 + m_2$ and placed in their common center of mass position (being there in a state of rest). In addition to such central mass m_c, there is another, (mathematically generated) reduced mass $m_r = \frac{m_1 m_2}{m_1 + m_2}$, which is rotating around the central mass m_c. Such reduced mass m_r will have the total kinetic energy and orbital moment of masses m_1 and m_2. The distance between m_r and m_c (or relevant circle radius) is again the same as before $r = r_1 + r_2$, $r_1 = \frac{m_2}{m_1+m_2} r$, $r_2 = \frac{m_1}{m_1+m_2} r$, $m_1 r_1 = m_2 r_2 \Rightarrow p = m_1 v_1 = m_2 v_2$. Angular (mechanical rotating) velocity ω of the new Binary System m_r and m_c will stay the same as found previously for Binary System of masses m_1 and m_2 ($\omega = v_1 / r_1 = v_2 / r_2 = v_r / r = 2\pi f_m$). The attractive gravitational force between m_1 and m_2 will be the same as the attractive force between m_r and m_c, for instance:

$$F_g = G\frac{m_1 m_2}{r^2} = G\frac{m_r m_c}{r^2} = \frac{m_r v_r^2}{r} = \frac{m_1 v_1^2}{r_1} = \frac{m_2 v_2^2}{r_2} = \frac{m_1 v_1^2 + m_2 v_2^2}{r_1 + r_2} = m_r r\omega^2 = m_1 r_1 \omega^2 = m_2 r_2 \omega^2 . \tag{2.11.14-5}$$

We can also find involved orbital moments of rotating masses m_1, m_2 and m_r, taking into account that the total orbital moment of a Binary System is conserved (constant).

$$\left\{ \begin{bmatrix} \mathbf{L}_1 = p_1 r_1 = m_1 v_1 r_1 = m_1 r_1^2 \omega = \frac{m_1 v_1^2}{\omega} = \mathbf{J}_1 \omega, \\ \mathbf{L}_2 = p_2 r_2 = m_2 v_2 r_2 = m_2 r_2^2 \omega = \frac{m_2 v_2^2}{\omega} = \mathbf{J}_2 \omega, \end{bmatrix} \& \begin{bmatrix} \frac{\mathbf{L}_1}{\mathbf{L}_2} = \frac{\mathbf{J}_1}{\mathbf{J}_2} = \frac{r_1}{r_2} = \frac{v_1}{v_2}, \ p_1 = p_2 = p = p_r \\ \mathbf{L}_r = p_r r = m_r v_r r = m_r r^2 \omega = \frac{m_r v_r^2}{\omega} = \mathbf{J}_r \omega \end{bmatrix} \right\} \Rightarrow$$

$$\begin{aligned} &\vec{\mathbf{L}}_i = \vec{r}_i \times \vec{p}_i \Rightarrow \mathbf{L}_i = m_i v_i r_i = m_i r_i^2 \omega = \frac{m_i v_i^2}{\omega} = \mathbf{J}_i \omega = \frac{2}{\omega} E_{ki}, \\ &\mathbf{L} = \mathbf{L}_1 + \mathbf{L}_2 = (\mathbf{J}_1 + \mathbf{J}_2)\omega = \frac{2}{\omega}\left(\frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2}\right) = \frac{2}{\omega}(E_{k1} + E_{k2}) = \frac{2}{\omega} E_{orbital} = \frac{m_r v_r^2}{\omega} = \frac{\mathbf{J}_r \omega^2}{\omega} = \\ &= \mathbf{J}_r \omega = \mathbf{L}_r = (\mathbf{J}_1 + \mathbf{J}_2)\sqrt{G\frac{m_1 + m_2}{r^3}} = \text{const.}, \ \mathbf{J}_r = \mathbf{J}_1 + \mathbf{J}_2, \ v_r = v_1 + v_2 = \omega r, \\ &E_{ki} = \frac{1}{2} m_i v_i^2 = \frac{1}{2} p_i v_i = \frac{1}{2} m_r v_r^2 \frac{r_i}{r} = \frac{1}{2} \mathbf{J}_i \omega^2 = \frac{1}{2} \mathbf{L}_i \omega, \ E_{kr} = E_{ki} \frac{v_r}{v_i} = E_{k1} + E_{k2}, \\ &\frac{E_{k1}}{v_1} = \frac{E_{k2}}{v_2} = \frac{E_{kr}}{v_r} = \frac{E_{ki}}{v_i} = \frac{m_i v_i}{2} = \frac{m_r v_r}{2} = \frac{1}{2} p, v_i = \omega r_i . \end{aligned} \tag{2.11.14-6}$$

Now we will be able to show that for (isolated) Binary Systems that are conserving total orbital moment, specific orbital (or kinetic, or motional) energy is in some way quantized, or given by similar expression like Planck's energy of a photon (except that new Planck-like H-constant will be much bigger compared to Planck constant of micro-world).

$$\begin{aligned} &E_{orbital} = E_{k1} + E_{k2} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} \mathbf{J}_1 \omega^2 + \frac{1}{2} \mathbf{J}_2 \omega^2 = \frac{1}{2} (\mathbf{J}_1 + \mathbf{J}_2) \omega^2 = \frac{1}{2} (\mathbf{L}_1 + \mathbf{L}_2) \omega = \\ &= \frac{1}{2} m_r v_r^2 \frac{r_1}{r} + \frac{1}{2} m_r v_r^2 \frac{r_2}{r} = \frac{1}{2} m_r v_r^2 \left(\frac{r_1}{r} + \frac{r_2}{r}\right) = \frac{1}{2} m_r v_r^2 = \frac{1}{2} \mathbf{J}_r \omega^2 = E_r = \frac{1}{2}\left[(\mathbf{J}_1 + \mathbf{J}_2)\omega\right] \cdot \omega = \\ &= \frac{1}{2}\left[\text{const}\right] \cdot \omega = \text{Const} \cdot \omega = H \cdot f = H \cdot f_0, \ \omega = 2\pi f_m = \frac{2\pi}{T} = \frac{H}{\text{Const}} \cdot f_0, \ f = f_0 \neq f_m . \end{aligned} \tag{2.11.14-7}$$

The next significant remark here (relevant for Binary Systems) is that ***to experience an attractive gravitational force, rotating bodies should rotate in the same direction (both having mutually collinear angular speed and angular moments vectors).*** If the rotation is not externally (or macroscopically) detectable, it should be internally (intrinsically) present in Binary Systems relations in some way. Simply, gravitation without rotation cannot be explained. We can also find expressions for such inherently associated angular velocity and angular momentum of Binary Systems as,

$$\begin{aligned}
&F_g = G\frac{m_1m_2}{r^2} = G\frac{m_rm_c}{r^2} = F_c = \frac{m_rv_r^2}{r} = \frac{m_1v_1^2}{r_1} = \frac{m_2v_2^2}{r_2} = m_1v_1r_1\frac{v_1}{r_1^2} = m_2v_2r_2\frac{v_2}{r_2^2} = J_1\omega\frac{v_1}{r_1^2} = J_2\omega\frac{v_2}{r_2^2} = L_1\frac{v_1}{r_1^2} = L_2\frac{v_2}{r_2^2} = J_r\omega\frac{v_r}{r^2} = L_r\frac{v_r}{r^2},\\
&F_gr = G\frac{m_1m_2}{r^2}r = G\frac{m_1m_2}{r} = G\frac{m_rm_c}{r} = F_cr = \frac{m_rv_r^2}{r}r = m_rv_r^2 = m_rr^2\omega^2 = J_r\omega^2 = L_r\omega = 2E_r = 2E_{orbital} = 2Hf,\\
&\left(m_1r_1 = m_2r_2,\ m_1v_1 = m_2v_2 = m_rv_r = p\right) \Rightarrow F_t = \frac{dp}{dt} = F_\tau,\ \left(\vec{\tau} = \vec{r}\times\vec{F}_t = \frac{d\vec{L}}{dt},\ \vec{F}_\tau = \frac{\vec{\tau}}{r} = \frac{\vec{r}\times\vec{F}_t}{r} = \frac{1}{r}\frac{d\vec{L}}{dt}\right),\\
&L_rv_r = Gm_1m_2 = \frac{L_1L_2}{L_1+L_2}v_r \Leftrightarrow G = \frac{L_1L_2}{L_1+L_2}\frac{v_r}{m_1m_2} = L_r\frac{\omega r}{m_1m_2} = \frac{J_1J_2}{J_1+J_2}\frac{\omega^2r}{m_1m_2} = J_r\frac{\omega^2r}{m_1m_2},\\
&J_r = \frac{J_1J_2}{J_1+J_2} = J_1 + J_2 = \sqrt{J_1\cdot J_2} = \frac{m_1m_2}{m_1+m_2}r^2 = m_rr^2,\\
&L_r = J_r\omega = L = \frac{L_1L_2}{L_1+L_2} = L_1 + L_2 = \sqrt{L_1\cdot L_2} = \sqrt{G\frac{m_1m_2J_r}{r}} \Rightarrow \omega = \sqrt{G\frac{m_1m_2}{J_rr}} = \sqrt{G\frac{m_rm_c}{J_rr}}\ .
\end{aligned} \tag{2.11.14-8}$$

Another conclusion to draw is that gravitational constant **G** is the measure of here elaborated intrinsic rotation or angular (mechanical revolving) speed of Binary Systems, leading to another alternative form of Kepler's third law as,

$$\frac{1}{\omega^2} = \frac{1}{(2\pi f_m)^2} = \left(\frac{T}{2\pi}\right)^2 = \frac{J_rr}{Gm_1m_2} = \frac{r^3}{G(m_1+m_2)}, \tag{2.11.14-9}$$

f_m (=) Mechanical (planet or satellite) revolving or orbiting frequency.

Shall we have a repulsive gravitational force in cases when masses in Binary Systems are not rotating in the same direction (when important angular moments' vectors are mutually opposed or maybe not collinear) is one of the logical questions to ask here? Let us exercise what could be the answer on a similar question if mentioned masses are also self-spinning (having finite spin moments $\vec{L}_{s1}$ and $\vec{L}_{s2}$, $\vec{L}_i \to (\vec{L}_i + \vec{L}_{si})$), and how such spinning moments would influence the attractive force/s between them?

$$\begin{aligned}
&F_g = F_c = G\frac{m_1m_2}{r^2} = G\frac{m_rm_c}{r^2} = (\frac{\pi G}{c^4})\frac{m_1c^2m_2c^2}{\pi r^2} = (\frac{G}{c^4})\frac{m_rc^2m_cc^2}{r^2} = (\frac{\pi G}{c^4})\frac{E_{t1}E_{t2}}{\pi r^2} = (\frac{\pi G}{c^4})\frac{E_{tr}E_{tc}}{\pi r^2},\\
&\left\{\begin{array}{l}(v_i << c) \Rightarrow E_{ti} = m_ic^2 = \gamma_im_ic^2 \Rightarrow E_{ti} \cong m_ic^2 + \frac{1}{2}m_iv_i^2 = m_ic^2 + \frac{1}{2}J_i\omega^2 = m_ic^2 + \frac{1}{2}\vec{L}_i\vec{\omega}\\ \left(\vec{L}_i \to (\vec{L}_i + \vec{L}_{si}), \vec{L}_i = J_i\vec{\omega},\ \vec{L}_{si} = J_i\vec{\omega}_{si}\right) \Rightarrow E_{ti} \cong m_ic^2 + \frac{1}{2}(\vec{L}_i + \vec{L}_{si})\vec{\omega}\end{array}\right\} \Rightarrow\\
&F_g \cong (\frac{G}{c^4})\frac{\left[m_1c^2 + \frac{1}{2}(\vec{L}_1 + \vec{L}_{s1})\vec{\omega}\right]\left[m_2c^2 + \frac{1}{2}(\vec{L}_2 + \vec{L}_{s2})\vec{\omega}\right]}{r^2} = (\frac{G}{c^4})\frac{\left[m_rc^2 + \frac{1}{2}(\vec{L}_r + \vec{L}_{sr})\vec{\omega}\right]\left[m_cc^2 + \frac{1}{2}(\vec{L}_c + \vec{L}_{sc})\vec{\omega}\right]}{r^2}.
\end{aligned} \tag{2.11.14-10}$$

If there is a stable ground in here hypothesized exercise about gravitational force, the presence of spin and orbital moments (of participants) could increase or decrease the total gravitational force between two bodies in a Binary System (depending on relative mutual positions of important orbital and spin moments). Most probably, such contributive spin-related members are too small compared to other involved energy-related members (in cases of planetary or solar systems), and it has been not easy to notice such possibility for addressing modifications of the old Newton Law. ***Here we should enrich the same situation by paying more attention to matter-waves nature of such binary interactions by additional elaborations around equations (2.4-11) to (2.4-17) from the same chapter.***

Apparently, in the absence of repulsive centrifugal forces, planets (or orbits) of specific Solar System would collapse and unite masses with their Sun if there are no orbital rotations. Since the repulsive (centrifugal) gravitational force (as formulated here) is something exclusively related to rotation, most probably that the hidden nature of Gravitation itself is on a similarly effective way intrinsically and inherently also associated with specific (equivalent) rotation inside of matter substance of gravitational masses.

Additional exercising and hypothesizing of the same situation (regarding essence of Gravitation in Binary Systems relations) is to notice connections between different aspects of (involved) energy components and work of "matter vortices" characterized by orbital and spin moments which should have certain torque. There is a tiny imaginative step here to start thinking about how to conceptualize rest masses as some "frozen or self-stabilized matter vortices' states" (since dimensionally torque is measured by the same units as energy).

Until here we did not address any of relativistic aspects of motional masses, since by the nature of astronomic, gravitational Binary Systems (or planetary systems), we can consider that in majority of relevant cases relevant orbital velocities are much smaller compared to the speed of light, and in such cases, it is clearly valid,

$$(v_{1,2} << c) \Rightarrow F_g = G\frac{m_1m_2}{r^2} = G\frac{m_rm_c}{r^2} \Leftrightarrow m_1m_2 = m_rm_c, m_r = \frac{m_1m_2}{m_1+m_2}, m_c = m_1+m_2 . \quad (2.11.14\text{-}11)$$

Let us now imagine that some Binary Systems (not necessarily of exclusively gravitational nature) could be orbital speed sensitive and let us analyze the consequences (again about the important center of mass).

$$\left\{ m_i \to \gamma_i m_i = \frac{m_i}{\sqrt{1-\frac{v_i^2}{c^2}}} = m_i^* \right\} \Rightarrow \left\{ \begin{array}{c} \left(m_r = \frac{m_1m_2}{m_1+m_2}\right) \to \frac{\gamma_1m_1\gamma_2m_2}{\gamma_1m_1+\gamma_2m_2} = \frac{m_1^*m_2^*}{m_1^*+m_2^*} = m_r^* = \gamma_r m_r \\ (m_c = m_1+m_2) \to \gamma_1m_1+\gamma_2m_2 = m_1^*+m_2^* = m_c^* \\ (m_1m_2 = m_rm_c) \to \gamma_1m_1\gamma_2m_2 = m_1^*m_2^* \end{array} \right\} \Rightarrow \quad (2.11.14\text{-}12)$$

$$\Rightarrow \left\{ \begin{array}{c} F_g = G\frac{m_1m_2}{r^2} = G\frac{m_rm_c}{r^2} = G\frac{m_1^*m_2^*}{r^2} = G\frac{m_r^*m_c^*}{r^2} \\ \gamma_r = \frac{\gamma_1\gamma_2}{\gamma_1\frac{m_1}{m_1+m_2}+\gamma_2\frac{m_2}{m_1+m_2}} \end{array} \right\}.$$

To make a simple validity test, of here elaborated relativistic relations between masses, it would be very indicative and almost sufficient to present the case when one of masses is enormously more significant compared to other,

$$(m_1 = m << m_2 = M) \Rightarrow \left\{ \begin{array}{c} m_r = \frac{m_1m_2}{m_1+m_2} \cong m = m_1 \Rightarrow \gamma_r = \frac{\gamma_1\gamma_2}{\gamma_1\frac{m_1}{m_1+m_2}+\gamma_2\frac{m_2}{m_1+m_2}} \cong \gamma_1 \\ E_{k1}+E_{k2} = E_{kr} = E_{ki}\frac{v_r}{v_i} = (\gamma_r-1)m_rc^2 \cong (\gamma_1-1)m_1c^2 = E_{k1} \end{array} \right\} \quad (2.11.14\text{-}13)$$

what already looks like the correct result (see also equations (2.4-11) - (2.4-18)).

Many possible consequences are starting from here. For instance, any stable planetary system (with one big solar mass M_s) and number of orbiting planets with masses $\{m_i\}_{i=1}^{n}$ can be *decomposed and analyzed as an ensemble of simple Binary Systems* with masses m_i orbiting around $(M_c - m_i)$, for example,

$$m_i \cdot (M_c - m_i) = m_{r-i} \cdot M_c, \; M_c = M_s + \sum_{i=1}^{n} m_i, \; m_{r-i} = \frac{m_i \cdot [M_c - m_i]}{M_c}, \; \forall i \in (1,n), \quad (2.11.14\text{-}14)$$

where m_i and M_s are only an approximation of a Binary System masses when $M_c \cong M_s >> \sum_{i=1}^{n} m_i$.

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

Let us now extrapolate two-body problem analysis to an equivalent (or analogical) n-body situation. For instance, imagine that n astronomic objects (like planets, including one massive star) are mutually approaching, entering specific n-body interaction, and becoming a stable planetary or solar system (this time we will analyze such situation without considering impacts). Kinetic energy balance in such case will be:

$$\left\{\begin{array}{l} 2-\text{body situation} \\ M_c = m_1 + m_2,\ m_r = \dfrac{m_1 \cdot m_2}{m_1 + m_2} = \dfrac{m_1 \cdot m_2}{M_c} \\ E_{k1} + E_{k2} = E_{kc} + E_{kr} \Leftrightarrow \\ \dfrac{1}{2} m_1 v_1^2 + \dfrac{1}{2} m_2 v_2^2 = \dfrac{1}{2} M_c v_c^2 + \dfrac{1}{2} m_r v_r^2 \end{array}\right\} \Rightarrow \text{by analogy}$$

$$\left\{\begin{array}{l} n-\text{body situation} \\ M_c = \sum_{(i)} m_i \ \ (\text{including solar mass}) \\ \sum_{(i)} E_{ki} = E_{kc} + \sum_{(i)} E_{kr-i} \Leftrightarrow \\ \dfrac{1}{2}\sum_{(i)} m_i v_i^2 = \dfrac{1}{2} M_c v_c^2 + \dfrac{1}{2}\sum_{(i)} m_{r-i} v_{r-i}^2 = \dfrac{1}{2} M_c v_c^2 + \dfrac{1}{2} M_r v_r^2 \end{array}\right\} \Rightarrow \quad (2.11.14\text{-}14\text{-a})$$

$$\sum_{(i)} E_{ri} = \sum_{(i)} E_{ki} - E_{kc} = \frac{1}{2}\sum_{(i)} m_i (v_i^2 - v_c^2) = \frac{1}{2}\sum_{(i)} m_{r-i} v_{r-i}^2 .$$

From (2.11.14-14-a) we could easily establish the following, natural understanding of planetary systems with many planets orbiting around one big central mass. We can say that each planet has its reduced mass that is exactly equal to its ordinary mass $m_{r-i} = m_i$ (not modified) and that only relative velocities of masses are modified,

$$\begin{array}{l} \dfrac{1}{2}\sum_{(i)} m_i (v_i^2 - v_c^2) = \dfrac{1}{2}\sum_{(i)} m_{r-i} v_{r-i}^2 \Rightarrow \\ \sum_{(i)} m_{r-i} v_{r-i}^2 = \sum_{(i)} m_i v_{r-i}^2 = \sum_{(i)} m_i (v_i^2 - v_c^2) = M_r v_r^2 \Rightarrow \\ m_{r-i} = m_i,\ v_{r-i}^2 = v_i^2 - v_c^2 \Leftrightarrow v_i^2 = v_c^2 + v_{r-i}^2 \end{array} \quad (2.11.14\text{-}14\text{-b})$$

From (2.11.14-14-b), helix rotation of planets (observed from specific Laboratory System) is a natural conclusion (since $v_i^2 = v_c^2 + v_{r-i}^2$), but in the important center of mass system, we will only have orbiting (or rotation) of planets around the central mass.

We could on a similar way exercise the situation of *an ensemble of Binary Systems* with masses m_i and $(M_c - m_i)$, for example,

$$\begin{array}{l} m_{r-i} = \dfrac{m_i \cdot [M_c - m_i]}{M_c} \\ \dfrac{1}{2}\sum_{(i)} m_i (v_i^2 - v_c^2) = \dfrac{1}{2}\sum_{(i)} m_{r-i} v_{r-i}^2 = \dfrac{1}{2}\sum_{(i)} \dfrac{m_i \cdot [M_c - m_i]}{M_c} v_{r-i}^2 = \dfrac{1}{2} M_r v_r^2 \Rightarrow v_i^2 - v_c^2 = \dfrac{M_c - m_i}{M_c} v_{r-i}^2 \end{array} \quad (2.11.14\text{-}14\text{-c})$$

Of course, similar elaborations can additionally be extended to other n-body problems. In the familiar mainstream of thinking, we can imagine that initial participants of n-body interaction have orbital and spinning moments and implement laws of linear and orbital moments' conservation to establish a much more powerful analyzing framework that will consider mutual interactions within many-body systems. In cases when participants also have free and/or dipole-types of electromagnetic charges, the same situation is becoming richer for similar analyses. This will give us a chance to explore other non-Newtonian gravitation-related interactions between masses with spin and orbital moment's attributes, and electromagnetic charges (not to forget matter waves spinning associated with motions of masses).

It is also a good conclusion that in such situations (as analyzed here) the Nature or Physics dealing with Natural Forces is dominantly respecting "center of mass system", and we, as observers, mathematicians and scientists naturally belong to our local laboratory system, what should be appropriately (mathematically) considered. The deeper message of the author here is that any electric or magnetic field situation, seemingly isolated, is connected within certain, united, and mutually complementary electromagnetic field (when conveniently analyzed from a proper reference or coordinates system). Analogically, this should also be valid for unity and complementarity of linear and angular or spinning, mechanical motions.

2.3.3-2 Quantizing and Matter Waves Hosting

Circular orbits of stable Binary Systems (including most stable solar or planetary systems), as conceptualized here, are presenting uniform, stationary, periodical, and inertial motions. For inertial motions, we have seen in (2.9.1) and (2.9.2) that coincident-validity and applicability of relevant linear and orbital momentum conservation is causally linked to standing matter waves formations. Consequently, stable Binary and Planetary Systems' Orbits as inertial motions, besides hosting orbiting masses could also host certain mutually synchronized standing matter waves formations, where synchronizing (or waves packing, or quantizing) criteria concerning the relevant center of mass coordinates system should be,

$$\begin{bmatrix} 2\pi r_i = n_i\lambda_i,\ \mathbf{L}_i = p_i r_i = m_i v_i r_i = m_i r_i^2\omega = \frac{m_i v_i^2}{\omega} = \mathbf{J}_i\omega = \frac{H}{\lambda_i} r_i,\ \lambda_i = \frac{2\pi r_i}{n_i} = \frac{H}{p_i},\ H = \text{const.}, \\ \frac{\mathbf{L}_1}{\mathbf{L}_2} = \frac{\mathbf{J}_1}{\mathbf{J}_2} = \frac{r_1}{r_2} = \frac{v_1}{v_2} = \frac{n_1}{n_2} = \frac{E_{k1}}{E_{k2}},\ \frac{n_1}{r_1} = \frac{n_2}{r_2} = \frac{n_r}{r} = \frac{n_i}{r_i} = \frac{2\pi}{\lambda_i},\ n_i \in [1,2,3,...], \\ \omega = \frac{v_1}{r_1} = \frac{v_2}{r_2} = \frac{v_i}{r_i} = \frac{v_1+v_2}{r_1+r_2} = \frac{v}{r},\ p_1 = m_1v_1 = m_2v_2 = p_2 = p = p_r,\ v_1v_2 = \omega^2 r_1 r_2 \end{bmatrix} \Rightarrow$$

$$\frac{H}{2\pi} = \mathbf{L}_1\frac{\lambda_1}{2\pi r_1} = \mathbf{L}_2\frac{\lambda_2}{2\pi r_2} = \mathbf{L}_r\frac{\lambda}{2\pi r} = (\mathbf{L}_1+\mathbf{L}_2)\frac{\lambda_1+\lambda_2}{2\pi(r_1+r_2)} =$$

$$= \frac{\mathbf{L}_1}{n_1} = \frac{\mathbf{L}_2}{n_2} = \frac{\mathbf{L}_1+\mathbf{L}_2}{n_1+n_2} = \frac{\mathbf{L}_r}{n_r} = \frac{\mathbf{L}_i}{n_i} = \frac{\mathbf{J}_i}{n_i}\omega = \frac{2}{n_i\omega}\frac{\mathbf{J}_i\omega^2}{2} = \frac{2}{n_i\omega}E_{ki} = \hbar_{gr.},$$

$$(v_i << c) \Rightarrow E_{ki} = \frac{n_i\omega}{2}\frac{H}{2\pi} = \frac{n_i v_i}{2r_i}\frac{H}{2\pi} \cong \frac{n_i 2u_i}{2r_i}\frac{H}{2\pi} = \frac{n_i u_i}{r_i}\frac{H}{2\pi} = H\frac{n_i\lambda_i f_i}{2\pi r_i} = Hf_i, \quad (2.11.14\text{-}15)$$

$$\omega r_i = v_i \cong 2u_i = 2\lambda_i f_i,\ \omega = \frac{2\pi}{T} = 2\pi f_m \cong 2\lambda_i\frac{f_i}{r_i} = 2\lambda\frac{f}{r} = \frac{4\pi}{n}f,\ f_m \cong \frac{\lambda}{\pi r}f = \frac{2}{n}f = \frac{2}{n_i}f_i,\ f = f_0,$$

$$E_{orbital} = E_{k1} + E_{k2} = H(f_1+f_2) = Hf = \frac{1}{2}F_g r = \frac{1}{2}G\frac{m_1m_2}{r^2}r = \frac{1}{2}G\frac{m_1m_2}{r} = \frac{1}{2}G\frac{m_r m_c}{r} = \frac{1}{2}F_c r = \frac{1}{2}\frac{m_r v_r^2}{r}r =$$

$$= \frac{1}{2}m_r v_r^2 = \frac{1}{2}m_r r^2\omega^2 = \frac{1}{2}\mathbf{J}_r\omega^2 = \frac{1}{2}\mathbf{L}_r\omega = E_r = \frac{H}{4\pi}(n_1+n_2)\omega = \frac{H}{2}(n_1+n_2)f_m = \frac{H}{2}nf_m,\ n = n_1+n_2 = n_r.$$

We could again attempt to characterize and quantify unity of orbital moments of specific stable Solar System ($\mathbf{L}_s, n_s$) with many orbiting planets ($\mathbf{L}_i, n_i$), considering the Sun as enormously more significant mass compared to any of related planets, $m_s >> m_i$) on a similar way, for instance,

$$\left[\frac{H}{2\pi} = \frac{\mathbf{L}_1}{n_1} = \frac{\mathbf{L}_2}{n_2} = \frac{\mathbf{L}_1+\mathbf{L}_2}{n_1+n_2} = \frac{\mathbf{L}_r}{n_r} = \frac{\mathbf{L}_i}{n_i}\right] \Rightarrow \frac{H}{2\pi} = \frac{\mathbf{L}_s}{n_s} = \frac{\mathbf{L}_i}{n_i} = \frac{\sum_{(i)}\mathbf{L}_i}{\sum_{(i)} n_i} = \frac{\mathbf{L}_s + \sum_{(i)}\mathbf{L}_i}{n_s + \sum_{(i)} n_i} = \hbar_{gr.}, \quad (2.11.14\text{-}16)$$

where L_s and n_s are characteristic parameters of the common Sun, L_i and n_i are related to each planet. This way the same Solar System can be decomposed on many simple binary systems (where each planet and the sun are presenting one elementary Binary System). Of course, such a strategy should be additionally elaborated and united with masses decomposition criteria from (2.11.14-14).

There is still certain confusion and ambiguity in physics literature regarding relations between mechanical revolving (or orbital, rotating) frequency $f_m = \omega / 2\pi$ and associated, specific orbital, matter wave frequency $f = \omega_0 / 2\pi = f_0$, and resolutions of such discrepancies are being explained by *postulating correspondence principles* (what is not a real and very scientific explanation). The background of the discrepancies mentioned is closely related to the nature of wave motions, to particle-wave duality and to specific relations between a group and phase velocity of the matter wave packet (which represents an energy-momentum wave model of a moving particle). For instance, the relation between group and phase velocity (where group velocity is at the same time real, measurable particle velocity) can be found as (see chapters 4.0 and 4.1),

$$v = v_g = u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda},\ u = \lambda f,\ u = \frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow \begin{bmatrix} (v << c) \Rightarrow \omega r_i = v_i \cong 2u_i = 2\lambda_i f_i \\ (v \approx c) \Rightarrow \omega r_i = v_i \cong u_i = \lambda_i f_i \end{bmatrix} \Rightarrow$$

$$\Rightarrow \begin{bmatrix} \omega = \frac{2\pi}{T} = 2\pi f_m = \frac{v_i}{r_i} \cong \frac{2u_i}{r_i} \cong 2\lambda_i \frac{f_i}{r_i} = 2\lambda \frac{f}{r} = \frac{4\pi}{n} f,\ f_m \cong \frac{\lambda}{\pi r} f = \frac{2}{n} f = 2\frac{f_i}{n_i}, f_i = \frac{n_i}{n} f, (v << c) \\ \omega = \frac{2\pi}{T} = 2\pi f_m = \omega_m = \frac{v_i}{r_i} \cong \frac{u_i}{r_i} \cong \lambda_i \frac{f_i}{r_i} = \lambda \frac{f}{r} = \frac{2\pi}{n} f,\ f_m \cong \frac{\lambda}{2\pi r} f = \frac{1}{n} f = \frac{f_i}{n_i}, f_i = \frac{n_i}{n} f, (v \approx c) \end{bmatrix} \Rightarrow$$

$$\Rightarrow E_{ki} = \frac{n_i \omega}{2} \frac{H}{2\pi} = \frac{n_i v_i}{2r_i} \frac{H}{2\pi} = \frac{2n_i u_i}{2r_i} \frac{H}{2\pi} = H \frac{n_i \lambda_i f_i}{2\pi r_i} = Hf_i \Rightarrow \qquad (2.11.14\text{-}17)$$

$$\Rightarrow E_{orbital} = E_{k1} + E_{k2} = H(f_1 + f_2) = Hf,\ f_1 + f_2 = f,\ n_1 + n_2 = n,\ n_i \lambda_i = 2\pi r_i,$$

where,

$$\begin{bmatrix} (v << c) \Rightarrow v \cong 2u = 2\lambda f = u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda} \Leftrightarrow u \cong -\lambda \frac{du}{d\lambda} \Leftrightarrow 2\frac{d\lambda}{\lambda} = -\frac{df}{f} \Rightarrow \ln\left|\frac{\lambda}{\lambda_0}\right|^2 \left|\frac{f}{f_0}\right| = 0 \Rightarrow \left|\frac{\lambda}{\lambda_0}\right|^2 \left|\frac{f}{f_0}\right| = 1 \\ \Leftrightarrow \lambda^2 f = \lambda_0^2 f_0, u\lambda = u_0 \lambda_0,\ \lambda = \lambda_0 \sqrt{\frac{f_0}{f}} = \frac{H}{p},\ u = u_0 \frac{\lambda_0}{\lambda}, (f_0, \lambda_0) = const., p = \frac{Hf}{c}\sqrt{\frac{f_0}{f}} = \frac{nHf_m}{2c}\sqrt{\frac{f_0}{f}} \end{bmatrix}, \qquad (2.11.14\text{-}18)$$

$$\begin{bmatrix} (v \approx c) \Rightarrow v \cong u = \lambda f = u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda} \Leftrightarrow \frac{du}{d\lambda} = \frac{f d\lambda + \lambda df}{d\lambda} \cong 0 \Leftrightarrow \frac{d\lambda}{\lambda} = -\frac{df}{f} \Rightarrow \ln\left|\frac{\lambda}{\lambda_0}\right|\left|\frac{f}{f_0}\right| = 0 \Rightarrow \\ \left|\frac{\lambda}{\lambda_0}\right|\left|\frac{f}{f_0}\right| = 1 \Leftrightarrow u = \lambda f = \lambda_0 f_0 = u_0 = c = const., (f_0, \lambda_0) = const.,\ \lambda = \lambda_0 \frac{f_0}{f} = \frac{c}{f} = \frac{H}{p}, p = \frac{Hf}{c} = \frac{nHf_m}{c} \end{bmatrix}.$$

Until here we analyzed a Binary System composed of two rotating bodies (m_1 and m_2) around their common center of mass (where the total kinetic energy of both rotating participants is $E_{orbital} = E_{k1} + E_{k2} = H(f_1 + f_2) = Hf = E_r$). Alternatively, we can present the same situation as another (artificial and equivalent) Binary System where one of involved masses $m_c = m_1 + m_2$ is much bigger than other ($m_1 = m << m_2 = M \cong m_c$) and staying at rest in their common center of mass (having zero orbital, kinetic energy), and second (much smaller) mass $m_r = \frac{m_1 m_2}{m_1 + m_2} \cong m << m_c \cong M$ is rotating around much bigger mass m_c, having again the same total orbital energy as before ($E_{orbital} = E_{k1} + E_{k2} = E_{k1} = Hf_1 = Hf = Hf_0$).

If we imagine that the last phase of a Binary System evolution is its collapse towards the creation of a single spinning mass $m_c = m_1 + m_2$ (where mechanical spinning of m_c is characterized by ω_c), we will have (still in the center of the mass coordinate system),

$$E_{orbital} = E_{k1} + E_{k2} = \frac{1}{2} m_r v_r^2 = H(f_1 + f_2) = Hf = E_r = \frac{1}{2} \mathbf{J}_r \omega^2 = \frac{1}{2} (\mathbf{J}_1 + \mathbf{J}_2) \omega^2 = \frac{1}{2} \mathbf{J}_c \omega_c^2 \Rightarrow$$

$$(\mathbf{J}_1 + \mathbf{J}_2)\omega^2 = \mathbf{J}_c \omega_c^2, \ \omega_c = 2\pi f_c = \omega \sqrt{\frac{\mathbf{J}_1 + \mathbf{J}_2}{\mathbf{J}_c}} = \frac{2\pi}{T} \sqrt{\frac{\mathbf{J}_1 + \mathbf{J}_2}{\mathbf{J}_c}} = 2\pi f_m \sqrt{\frac{\mathbf{J}_1 + \mathbf{J}_2}{\mathbf{J}_c}} = \begin{Bmatrix} \frac{4\pi f_0}{n} \sqrt{\frac{\mathbf{J}_1 + \mathbf{J}_2}{\mathbf{J}_c}}, v_r << c \\ \frac{2\pi f_0}{n} \sqrt{\frac{\mathbf{J}_1 + \mathbf{J}_2}{\mathbf{J}_c}}, v_r \approx c \end{Bmatrix}. \quad (2.11.14\text{-}19)$$

Binary Systems (as conceptualized here) are planar motional systems, meaning that involved circular motions are in the same fixed plane, and this is the reason why quantizing or synchronizing, or standing waves packing criteria is related only to one orbital quantum number. Here, we should not forget that involved mechanical rotations and spinning have much different angular velocities ω_m, ω_c, compared to associated (surrounding) matter waves angular velocities $\omega_0 = 2\pi f_0 = 2\pi f$. Of course, all of that is an idealization or approximation, since more real are multi-body systems, like planetary or solar systems (including micro-world and subatomic systems), where orbital single-plane circular motions are becoming multi-planar elliptical motions (having quantized inclinations for relevant planetary orbits). Consequently, new quantizing or wave synchronizing rules are getting additional angular quantum numbers, like in semi-classical quantization of angular momentum (see [40], D. Da Rocha and L. Nottale). In mentioned Multi-component Systems (including Binary Systems), very appropriate quantizing and generalizing approach will be to apply, *creatively and with intellectual flexibility*, Wilson-Bohr-Sommerfeld Action Integrals, combined with familiar theoretical concepts published by Anthony D. Osborne, & N. Vivian Pope (see [36]). Ironically, the early days of Classical Quantum Physics related to N. Bohr Hydrogen Atom Model is much more a Quantum approach to macrocosmic, real planetary orbital motions, than anything that explains or conceptualizes the real nature of hydrogen atom. Here (in relation with Binary Solar Systems) we are still not specifying what kind of matter waves we are talking about, but a solid candidate (besides others related to inertial effects, rotation, and gravitation) that cannot be excluded are electromagnetic fields and waves.

Quantization in Physics is merely a consequence of the existence of stable Binary and Multi-component Systems and energy-momentum communications between them (but we should not forget that other, transient, and non-stable systems have a place in our universe). This is also the area where modern-day Quantum Theory started being complex and fuzzy, since for managing such situations (in the absence of real, clear, natural, and obvious conceptualization), it was necessary to establish new, primarily mathematically operating theories and postulates, which have been deductively generating "second-hand", luckily useful results.

2.3.3-3 Standing-Waves Resonators and Gravitation

Another aspect of imaginable, stable standing-waves field structures in relation to gravitation is the fact that every two masses (of specific Binary System, including static masses that are mutually touching) can be presented as a kind of half-wave ($\lambda/2$) resonator, or a gravitation-dipole, where the distance between two of such masses is equal to $r = \lambda/2 = c_{gr}/2f_{gr}$. Here c_{gr} is the radial (central) gravitational-waves velocity acting along the distance r connecting centers of masses in question and f_{gr} is the relevant, resonant frequency of the associated standing wave (see (2.11.14-15) and (2.11.14-16)). This can mathematically be described as,

$$\left\{\begin{array}{l} r=\frac{\lambda}{2}=\frac{c_{gr}}{2f_{gr}}=r_1+r_2,\ r_1=\frac{m_2}{m_1+m_2}r,\ r_2=\frac{m_1}{m_1+m_2}r,\ m_1r_1=m_2r_2, \\ E_{orbital}=E_{k1}+E_{k2}=H(f_1+f_2)=Hf=\frac{1}{2}F_g r=\frac{1}{2}G\frac{m_1m_2}{r^2}r=\frac{1}{2}G\frac{m_1m_2}{r}=\frac{1}{2}G\frac{m_r m_c}{r}= \\ =\frac{1}{2}F_c r=\frac{1}{2}\frac{m_r v_r^2}{r}r=\frac{1}{2}m_r v_r^2=\frac{1}{2}m_r r^2\omega^2=\frac{1}{2}J_r\omega^2=\frac{1}{2}L_r\omega=E_r=\frac{H}{4\pi}(n_1+n_2)\omega= \\ =\frac{H}{2}(n_1+n_2)f_m=\frac{H}{2}nf_m,\ n=n_1+n_2=n_r,\ nf_m=2f,\ L_1\to(L_1+L_{s1}),\ L_2\to(L_2+L_{s2}),\ n\to(n+n_s) \\ \frac{H}{2\pi}=\frac{L_1}{n_1}=\frac{L_2}{n_2}=\frac{L_1+L_2}{n_1+n_2}=\frac{L_r}{n_r}=\frac{L_i}{n_i}=\frac{L_1+L_2}{n}=\frac{(L_1+L_{s1})+(L_2+L_{s2})}{n+n_s} \end{array}\right\}\Rightarrow$$

$$E_{orbital}=E_{k1}+E_{k2}=H(f_1+f_2)=Hf=\frac{H}{2}(n_1+n_2)f_m=\frac{H}{2}nf_m=\frac{1}{2}F_g r=\frac{G}{c_{gr}}m_1m_2f_{gr}=\frac{G}{c_{gr}}m_r m_c f_{gr}\Rightarrow$$

$$\Rightarrow H=H\cdot\frac{nf_m}{2f}=\frac{G}{c_{gr}}\frac{m_1m_2f_{gr}}{f}=\frac{G}{c_{gr}}\frac{m_r m_c f_{gr}}{f}=2\frac{G}{c_{gr}}\frac{m_1m_2f_{gr}}{nf_m}=2\frac{G}{c_{gr}}\frac{m_r m_c f_{gr}}{nf_m}=$$

$$=2\frac{F_g}{c_{gr}}\frac{f_{gr}}{nf_m}r^2=\frac{F_g}{c_{gr}}\frac{f_{gr}}{f}r^2=2\pi\frac{L_1+L_2}{n}=\text{constant},\ \frac{nf_m}{2f}=\frac{G}{c_{gr}H}\frac{m_1m_2f_{gr}}{f}=\frac{G}{c_{gr}H}\frac{m_r m_c f_{gr}}{f}=1\Rightarrow$$

$$\Rightarrow F_g=\frac{\pi c_{gr}nf_m}{nf_{gr}}\frac{(L_1+L_2)}{r^2}=\frac{2\pi c_{gr}f}{nf_{gr}}\frac{(L_1+L_2)}{r^2}=\frac{4\pi f}{n}\frac{(L_1+L_2)}{r}=2\pi f_m\frac{(L_1+L_2)}{r}=$$

$$=\omega_m\frac{(L_1+L_2)}{r}=v\frac{(L_1+L_2)}{r^2}=G\frac{m_1m_2}{r^2}=G\frac{m_r m_c}{r^2},$$

$$\omega_m=\omega=\frac{2\pi}{T}=2\pi f_m=\frac{v}{r}=\frac{v_1}{r_1}=\frac{v_2}{r_2}=\frac{v_r}{r},\ \frac{v_1}{v_2}=\frac{r_1}{r_2},\ v(L_1+L_2)=G\,m_1m_2,$$

$$p_1=m_1v_1=m_2v_2=p_2=p=p_r=m_r v_r,\ \vec{p}_1+\vec{p}_2=\vec{0},\ \vec{r}=\vec{r}_1+\vec{r}_2,$$

$$\vec{v}_i=\frac{d\vec{r}_i}{dt},\ \vec{v}=\frac{d\vec{r}}{dt}=\vec{v}_r=\vec{v}_1+\vec{v}_2,\ v_1v_2=\omega^2 r_1r_2,\ v_i=\omega r_i,\ \frac{v_1v_2}{v^2}=\frac{r_1r_2}{r^2}. \qquad (2.11.14\text{-}20)$$

There are many challenging (still hypothetical) options regarding understanding Gravitation starting from results found in (2.11.14-20). One of such possibilities, *offering the replacement for Newton Law ($F_g=v\frac{(L_1+L_2)}{r^2}=G\frac{m_1m_2}{r^2}$), is that gravitational force is directly dependent on the total resulting vector of angular and spin moments of Binary System participants.* Such angular moments ($\vec{L}=\vec{L}_1+\vec{L}_2$) are externally visible (and measurable), and some of their components could be states related to spinning, or to another kind of hidden rotation of belonging subatomic entities (see also (2.2), (2.4-5), (2.5) and (2.11)). What is significant here is that all three vectors $\vec{r},\ \vec{v},\vec{L}$ are mutually orthogonal, meaning that relevant vectors' product will produce a vector of gravitational force $\vec{F}_g$ collinear with $\vec{r}$. Consequently, now we can be sure that the origin of gravitation is in an interaction between angular, orbital and/or spin moments of mutually attracting masses.

Half-wave resonator, as an intuitive concept for explaining gravitational attraction between two pulsating or oscillating masses (elaborated in (2.11.14-20)) can also be approximated and modeled as the situation when specific springs mutually connect two masses in question (Binary System). Such springs (obviously having non-linear spring coefficients k_1 and k_2), are effectively realizing Newton gravitational force, between masses in question and can be supported by the following (at least dimensionally correct and still hypothetical) relations,

$$\left\{\begin{array}{l} F_g = k_1 r_1 = k_2 r_2 = G\frac{m_1 m_2}{r^2},\ m_1 r_1 = m_2 r_2 = m_r r,\ \ m_r = \frac{m_1 m_2}{m_1+m_2}, \\ f_{gr} = \frac{1}{2\pi}\sqrt{\frac{k_1}{m_1}} = \frac{1}{2\pi}\sqrt{\frac{k_2}{m_2}}\ (=)[Hz], \\ r_1 = \frac{m_2}{m_1+m_2} r = \frac{\lambda_1}{4},\ r_2 = \frac{m_1}{m_1+m_2} r = \frac{\lambda_2}{4}, \lambda_i = \frac{c_{gr-i}}{f_{gr}} = \frac{H}{p_{gr-i}}, \\ r = r_1 + r_2 = \frac{\lambda_1}{4} + \frac{\lambda_2}{4} = \frac{\lambda}{2} = \frac{c_{gr1}}{4f_{gr}} + \frac{c_{gr2}}{4f_{gr}} = \frac{c_{gr1}+c_{gr2}}{4f_{gr}} = \frac{c_{gr}}{2f_{gr}}, \\ F_g r = G\frac{m_1 m_2}{r} = G\frac{m_r m_c}{r} = k_1 r_1^2 + k_2 r_2^2 = 2Hf = nHf_m = \frac{2Gm_1 m_2}{c_{gr}} f_{gr}. \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \frac{k_1}{k_2} = \frac{m_1}{m_2} = \frac{r_2}{r_1} = \frac{c_{gr2}}{c_{gr1}} = \frac{\lambda_2}{\lambda_1} \\ c_{gr-i} = \lambda_i f_{gr} = 4r_i f_{gr} = \frac{2r_i}{\pi}\sqrt{\frac{k_i}{m_i}} \end{array}\right\}$$

$$\Rightarrow \left\{\begin{array}{l} \lambda_1 = \frac{4m_2}{m_1+m_2} r = \frac{c_{gr-1}}{f_{gr}} = \frac{H}{p_{gr-1}},\ \lambda_2 = \frac{4m_1}{m_1+m_2} r = \frac{c_{gr-2}}{f_{gr}} = \frac{H}{p_{gr-1}}, \\ H = \lambda_1 p_{gr-1} = \lambda_2 p_{gr-2} = \frac{4m_2}{m_1+m_2} p_{gr-1} r = \frac{4m_1}{m_1+m_2} p_{gr-2} r = \frac{c_{gr-1}}{f_{gr}} p_{gr-1} = \frac{c_{gr-2}}{f_{gr}} p_{gr-2} = \\ = H\cdot\frac{nf_m}{2f} = \frac{G}{c_{gr}}\frac{m_1 m_2 f_{gr}}{f} = \frac{G}{c_{gr}}\frac{m_r m_c f_{gr}}{f} = 2\frac{G}{c_{gr}}\frac{m_1 m_2 f_{gr}}{nf_m} = 2\frac{G}{c_{gr}}\frac{m_r m_c f_{gr}}{nf_m} = \\ = 2\frac{F_g}{c_{gr}}\frac{f_{gr}}{nf_m} r^2 = \frac{F_g}{c_{gr}}\frac{f_{gr}}{f} r^2 = 2\pi\frac{L_1 + L_2}{n} = \text{constant} \end{array}\right\}$$

$$\Rightarrow \left\{\begin{array}{l} p_{gr-1} = m_1\cdot\frac{G(m_1+m_2)}{4c_{gr} r}\frac{f_{gr}}{f} = m_1\cdot\frac{G(m_1+m_2)}{2c_{gr} r}\frac{f_{gr}}{nf_m} = m_1\cdot v_1^*, \\ p_{gr-2} = m_2\cdot\frac{G(m_1+m_2)}{4c_{gr} r}\frac{f_{gr}}{f} = m_2\cdot\frac{G(m_1+m_2)}{2c_{gr} r}\frac{f_{gr}}{nf_m} = m_2\cdot v_2^*, \\ v_1^* = v_2^* = \frac{G(m_1+m_2)}{4c_{gr} r}\frac{f_{gr}}{f} = \frac{G(m_1+m_2)}{2c_{gr} r}\frac{f_{gr}}{nf_m} = v^* \end{array}\right\} \quad (2.11.14\text{-}21)$$

What is interesting in (2.11.14-21) is that Binary Systems relations are conclusively showing that two masses, mutually exercising the Newton force of gravitation (as a Binary System), can be analyzed in a certain approximate way as two weakly coupled mass-spring oscillators (linked to their common center of mass), having the same resonant frequency on both sides. To achieve a global forces balance (like in cases of stable planetary systems, where attractive gravitational force is balanced by repulsive centrifugal force), attractive forces of such non-linear springs (between masses) should be compensated by equal repulsive forces of the other two springs (connected in line with two masses in question in the mutually opposing directions). This way we can represent gravitational attraction between each of masses and the rest of the universe. This way (see Fig.2.5), we will be able to analyze (almost) independently, each of two mass-spring systems as an equivalent, macro $\lambda/4$ resonator, as already practiced in (2.11.14-21).

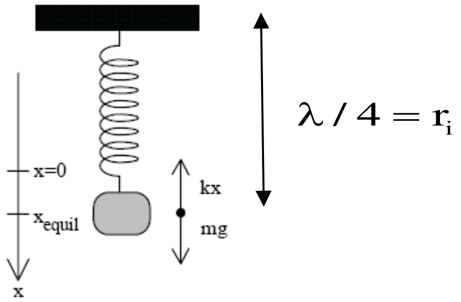


Fig.2.5. Simple Mass-Spring oscillator

The mass-spring oscillations (where mass m_i is oscillating with a certain amplitude Δr_i, Fig.2.5) can be mathematically presented by simple harmonic function $x = (\Delta r_i)\cos(\omega t + \varphi)$. In reality, we could imagine that (valid for both masses) distance r_i between a mass m_i and common center of both masses is pulsating (or harmonically oscillating) between two values: $r_i + \Delta r_i$ and $r_i - \Delta r_i$. This will (after applying few mathematical steps valid for mass-spring systems, and applicable to particle-wave duality situations) extend the relation of proportionality between relevant elements of a Binary System in question (found in (2.11.14-21)) to,

$$\frac{k_1}{k_2} = \frac{m_1}{m_2} = \frac{r_2}{r_1} = \frac{c_{gr2}}{c_{gr1}} = \frac{\lambda_2}{\lambda_1} = \frac{(\Delta r_2)^2}{(\Delta r_1)^2},$$
$$\left(r = r_1 + r_2 = \frac{\lambda_1}{4} + \frac{\lambda_2}{4} = \frac{c_{gr1}}{4f_{gr}} + \frac{c_{gr2}}{4f_{gr}} = \frac{\langle c_{gr} \rangle}{2f_{gr}} = \frac{c_{gr}}{2f_{gr}} \right). \tag{2.11.14-22}$$

If such (standing waves and resonant) oscillations exist between two astronomic objects, we should be able to detect them in some way. For instance, if one of masses is our Sun and the other of masses is our planet Earth, the light beam coming from the Sun and detected on the Earth (by certain prism) should be *wavelength-modulated* producing that every specific color should have its bandwidth, directly proportional to the oscillatory speed amplitude $\omega \Delta r_i <<< c$ (like kind of Doppler effect). Such bandwidths can be measured (for many specific colors) on the Equator and somewhere far from Equator (as well as from some satellite observatory), and we should notice the differences between corresponding bandwidths. Since here we are talking about modulated and standing waves motions (between two masses), we can apply generally-valid relations between group and phase velocity, where: group velocity (of a relevant gravitational wave) is $v = c_{gr} = v_{gr}$, the phase velocity is $u = u_{gr}$, modulating planetary oscillating speed is $\Delta v = \omega \Delta r_i <<< c$, and mean group and phase velocities are $\overline{v} = \overline{c}_{gr} = \overline{v}_{gr}$, $\overline{u} = \overline{u}_{gr}$.

This would give us an idea of how to establish relations between relevant frequency and wavelengths bandwidths, as follows,

$$\left\{ \begin{array}{l} v = u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda} = u + p\frac{du}{dp} = \frac{d\omega}{dk} = \frac{d\tilde{E}}{dp} = H\frac{df}{dp} = \frac{df}{df_s} = \frac{2u}{1 + \frac{uv}{c^2}}, \\ u = \lambda f = \frac{\omega}{k} = \frac{\tilde{E}}{p} = \frac{Hf}{p} = \frac{f}{f_s} = \frac{v}{1 + \sqrt{1 - \frac{v^2}{c^2}}} = \frac{E_k}{p},\ f_s = k/2\pi, \\ \Rightarrow 0 \le 2u \le \sqrt{uv} \le v \le c \end{array} \right\} \Rightarrow$$

All over this book are scattered small comments placed inside the squared brackets, such as: [♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

$$\Rightarrow\begin{Bmatrix} d\tilde{E} = Hdf = mc^2 d\gamma, \quad \frac{df}{f} = (\frac{dv}{v})\cdot\frac{1+\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{v^2}{c^2}} \\ (v\pm\Delta v) = (u\pm\Delta u) - (\lambda\mp\Delta\lambda)\frac{d(u\pm\Delta u)}{d(\lambda\mp\Delta\lambda)} \end{Bmatrix}\Rightarrow\begin{Bmatrix} \frac{\Delta f}{\bar{f}} = (\frac{\Delta v}{\bar{v}})\cdot\frac{1+\sqrt{1-\frac{\bar{v}^2}{c^2}}}{1-\frac{\bar{v}^2}{c^2}} \cong \frac{2\Delta v}{\bar{v}} = \frac{\Delta v}{\bar{u}}, \\ \frac{du}{\Delta u} = \frac{d\lambda}{\Delta\lambda} = 2\frac{du}{\Delta v} = \frac{dv}{\Delta v}, \\ \bar{v} \cong 2\bar{u} = 2\bar{\lambda}\cdot\bar{f},\ \Delta v \cong 2\Delta u, \\ 2\frac{\Delta u}{\bar{v}} = \frac{\Delta v}{\bar{v}} = \frac{\Delta v}{2\bar{u}} = \frac{\Delta f}{2\bar{f}} = \frac{\Delta v}{2\bar{\lambda}\cdot\bar{f}}, \bar{\lambda}\cdot\Delta f = \Delta v \end{Bmatrix}. \quad (2.11.14\text{-}22)$$

If we continue developing similar ideas about standing waves communications between masses, we should be able to explain “redshifts and blue shifts” of the light spectra from deep and remote cosmic areas, captured by astronomic observatories on our planet.

No doubt that here we are faced with an oversimplified and accelerated mathematical and brainstorming conceptualization which is mostly useful as the first step towards familiarization with gravitational standing waves as an explanation of the nature of attractive gravitational force. Taking and proving-valid such an approach will have consequences on a better understanding of origins of Gravitation, and nature of all “temporal-spatial” motions in our Universe (see more of familiar elaborations in Chapter 10.).

2.3.3-4 Central Forces, Newton and Coulomb Laws

Next challenging question here is why central forces, like those that Newton and Coulomb's laws are describing, are inversely dependent from the square of the relevant distance, $F(r) = \frac{C}{r^2}, C = const.$? We can indirectly explain such situation ($F(r) = \frac{C}{r^2}$) by analyzing force components involved in orbital motions under a central force. Since in cases of central forces, relevant orbital momentum is constant $\frac{d\vec{L}}{dt} = 0 \Rightarrow \vec{L} = \vec{r}\times\vec{p} = \overrightarrow{const.}$, we can conclude that vector $\vec{L}$ is perpendicular to the plane defined by the vector $\vec{r}$ and the momentum $\vec{p}$. The fact that $\vec{L}$ remains constant is saying that relevant plane ($\vec{r}$, $\vec{p}$) also remains constant (or stable), and that every orbital motion (on such plane) under central force is a stable, planar, and two-dimensional motion (which can naturally host standing waves structures without big need to give probabilistic or stochastic meaning to any of such waves). This is very much like astronomic observations documenting that many solar systems are planar, facilitating involved mathematical processing, for example,

$$L = L(r,\theta) = mr^2\frac{d\theta}{dt} = const., \vec{F}(r,\theta) = \vec{F}(r) + \vec{F}(\theta) = \vec{F}(r) = \overrightarrow{(m\frac{d^2r}{dt^2})} = m\vec{a}_r + m\vec{a}_\theta = \overline{\left[m\frac{d^2r}{dt^2} - mr(\frac{d\theta}{dt})^2\right]},$$

$$a_r = \frac{d^2r}{dt^2} - r(\frac{d\theta}{dt})^2,\ a_\theta = r\frac{d^2\theta}{dt^2} + 2\frac{dr}{dt}\cdot\frac{d\theta}{dt}, mr\frac{d\theta}{dt} + 2m\frac{dr}{dt}\frac{d\theta}{dt} = 0 \Rightarrow \frac{d\theta}{dt} = \frac{L}{mr^2} = \frac{L}{m}\rho^2,\ \rho = \frac{1}{r},$$

$$\frac{d^2\rho}{d\theta^2} + \rho = -\frac{m}{L^2}\cdot\frac{1}{\rho^2}F(\frac{1}{\rho}), F(\frac{1}{\rho}) = F(r) = \frac{C}{r^2} = C\cdot\rho^2 \Rightarrow \frac{d^2\rho}{d\theta^2} + \rho = -\frac{mC}{L^2} \Rightarrow \rho = A\cos(\theta-\theta_0) - \frac{mC}{L^2},$$

$$r = \frac{1}{A\cos(\theta-\theta_0) - \frac{mC}{L^2}}.$$

The last equation is describing conic curves $r = r(\theta)$, such as ellipse, parabola, and hyperbola, depending on constants $A, \theta_0, m, C, \vec{L}$. If we chose the reference coordinates where $\theta_0 = 0$, we will get for a planetary and satellite orbits $r = \frac{1}{A\cos\theta - \frac{mC}{L^2}}$ that is a conic section, which can also be transformed to $r = r_0 \frac{1+e}{1+e\cdot\cos\theta}$, where **e** presents eccentricity of the orbit. For $\mathbf{e} < \mathbf{1}$, the orbit is an ellipse, for $\mathbf{e} = \mathbf{1}$, the orbit is a parabola, for $\mathbf{e} > \mathbf{1}$ the orbit is a hyperbola, and for $\mathbf{e} = \mathbf{0}$ the orbit is a circle (covering all cases of planetary, asteroids and satellite orbits). ------------------------------ ♣]

3. POSSIBILITIES FOR GENERALIZATION OF FARADAY-MAXWELL THEORY

The author of this chapter argues that Maxwell-Faraday Electromagnetic Theory (ET) is an elegant, pragmatic, and stable field theory that has yet to be upgraded and fully integrated with Relativity Theory (RT). The current boundaries where RT and ET operate without doubt could be expanded, and Maxwell-Faraday's theory, with its ontological dominance and conceptual richness, may serve as a model for redeveloping, refining and improving RT. The author suggests that while RT may still be seen as somewhat challenging, controversial, or maybe slightly incomplete in some fundamental respects, Maxwell-Faraday's theory should be considered a superior foundation for developing the new and improved RT. Deep ontological roots that support and explain gravitation are most probably of electromagnetic nature embedded in matter structure, or emanating from atoms.

This chapter also explores possible optimizations of contemporary Maxwell theory and addresses emerging domains between Maxwell ET and RT. New experimental evidence regarding electrons, electric currents, and magnetic fields, especially in the context of rotational motions, indicates that contemporary mathematical concepts in this area require updating. However, mainstream leaders in electromagnetic theory have been hesitant to engage in such analyses (see [63], [68], [88], [89], [91], and [117]).

The implication here is that contemporary Quantum and Relativistic Electrodynamics (QRE), despite being experimentally and theoretically well-supported, may not represent the final, best, or most natural merging of Quantum Theory (QT), Electromagnetism (EM), and RT. Additionally, neither Electromagnetic nor Relativity theory, in their current forms, are fully self-sufficient or ready for a higher-level unification (see [113], Hristopoulos, Demetris; "Beyond Electromagnetic Theory"). Although current QRE models are constructed to align with known experimental facts, conservation laws, and segments of quantum, electromagnetic, and relativity theories, they lack smooth development, simplicity, internal compatibility, natural unity, and conceptual elegance. The author draws here a parallel with the Ptolemaic geocentric theory, which was mathematically sufficient for its time but conceptually fundamentally flawed and resistant to change due to ideological biases.

The author further notes that the terminology frequently used in contemporary physics, such as relativistic, non-relativistic, classical-mechanical, quantum-mechanical, and quantum-electrodynamic reflects the segmented progress and occasional errors in our global understanding of Physics. This segmentation stems from independent advances in different fields, incomplete mathematical models, and a lack of unifying interdisciplinary concepts. Eventually, a new, unified theory should replace these fragmented perspectives, addressing them consistently using a common mathematical framework. Some or all the existing theories may be partially incorrect, incomplete, or only conditionally valid, highlighting the need for a general, unifying theory (see [36], "Immediate Distant Action and Correlation in Modern Physics: The Balanced Universe").

One limitation of Maxwell's theory is its lack of invariance across all inertial systems in relative motion. Currently, Maxwell's equations are only valid within the laboratory coordinate system where observations and calculations are performed (see more on this from Thomas E. Phipps, Jr., in [35]).

The author aims to correct this non-invariance and suggests that an updated Maxwell theory could lead to a new and better RT, offering improved foundations for understanding gravitation. Particularly, Minkowski's 4-vector energy-momentum formalism would be further clarified and generalized through integration with Complex and Hypercomplex Analytic Signals and Phasors modeling, which are common in electric circuits theory. This integration could provide a more sophisticated mathematical framework for mastering electromagnetism, wave-particle duality, wave motions, field theories, mechanics, and gravitation (see more in Chapters 4.0, 4.3 and 10).

The author also contends that Maxwell-Faraday's EM theory remains a practical model for analogically developing other field theories in physics. This book begins by exploring analogies and symmetries as a platform for generating new ideas, before moving on to more advanced mathematical modelling to develop a more comprehensive Electromagnetic and Gravitation theory that aligns with known experimental facts and conservation laws (see [34], [71], [99], and [152]). The author proposes hypothetical extensions of Maxwell-Faraday's theory, based on analogies and symmetries, considering an intrinsic connection between rectilinear motion and by matter-waves associated spinning, which could enhance our understanding of gravitation and wave-particle duality. This concept is analogous to the complementary relationship between electric and magnetic fields. The author suggests that all fields, forces, and matter-wave functions can be mathematically modeled using Complex Analytic Signals and Phasors, where, in case of electromagnetism, electric and magnetic fields are represented as mutually coupled components of a Complex Analytic Signal (see Chapters 4.0, 4.3 and 10). This approach could offer a more natural and logical representation of electromagnetic waves and provide a unified model for addressing various natural phenomena, including matter-waves, mechanics and gravitation.

In subsequent chapters, the author introduces innovative contributions to electromechanical analogies and proposes upgraded Maxwell equations, building on the conceptual foundations laid in the first chapter, where analogical comparisons and symmetries of physical laws, different electromechanical moments and energies are analogically summarized under "T.1.8 Generic Symmetries and Analogies of the Laws of Physics."

3.1. Modification of Maxwell Equations

Non-relativistic expressions of Lorentz, electric and magnetic forces (in homogeneous and isotropic space, mutually orthogonal coordinates of "*Euclid-Galilean-Descartes*" space), with relative motions of participants, and combined with involved magnetic and electric fields, makes possible to establish fully mutually symmetrical or analogical forms of electric and magnetic fields ($\mathbf{E}$ and $\mathbf{H}$), and relevant inductions ($\mathbf{D}$ and $\mathbf{B}$), as being functions of involved *mechanical-motion velocity* $\mathbf{v}$**,** (see more in **[4]**, **[34], [35], [46]**, **[47], [49]**, **[99]**),

$$\vec{E}(v) = \vec{E}_0 \pm \vec{v} \times \vec{B}_0 = \vec{E}_{stat.} \pm \vec{E}_{dyn.},\ \vec{E}_{stat.} = \vec{E}_0,\ \vec{E}_{dyn.} = \vec{v} \times \vec{B}_0,$$
$$\vec{H}(v) = \vec{H}_0 \mp \vec{v} \times \vec{D}_0 = \vec{H}_{stat.} \mp \vec{H}_{dyn.},\ \vec{H}_{stat.} = \vec{H}_0,\ \vec{H}_{dyn.} = \vec{v} \times \vec{D}_0, \quad (3.1)$$
$$\left\langle \vec{B}_0 = \mu\vec{H}_0,\ \vec{D}_0 = \varepsilon\vec{E}_0,\ \vec{B}(v) = \mu\vec{H}(v),\ \vec{D}(v) = \varepsilon\vec{E}(v) \right\rangle,$$

Here, index "**0**" indicates that certain field exists in a static case when **v** = 0 (meaning in a state of relative rest, where applied indexing means: $\langle v = 0\rangle \Rightarrow \langle E(v) = E(0) = E_0 = E_{stat.},\ H(v) = H(0) = H_0 = H_{stat.}\rangle$). The field members, indexed by "**dyn.** (=) dynamic", are representing fields influenced by motion of interaction-participants (being velocity dependent). Lorentz forces, the laws of electromagnetic induction, and the principle of action and reaction may provide a foundation for extending electromagnetism to enhance our understanding of gravitation. This is because electromagnetically neutral masses in relative motion, including oscillations, internally consist of electromagnetic charges, dipoles, and multipoles. These charges, whether free or coupled, internally generate electric and magnetic fields, forces, voltages, and currents, depending on their relative motion. As a result, these electromagnetic interactions can influence mechanical motions and forces, including those related to gravitation.

The ideas mentioned, which are based on Lorentz electric and magnetic forces, offer a potential explanation for the origins of gravitation and its relationship with Maxwell's equations, as discussed in [34], [152], and in the second chapter of this book.

Additionally, it is well-established that electrically and magnetically charged masses, both at the microscopic and macroscopic levels, can produce attractive and repulsive effects. These phenomena are like gravitational forces, with one significant difference: we have not yet achieved mastery over repulsion, or anti-gravitational effects, with electromagnetically neutral masses.

Mutually complementary structure (and full mathematical symmetry and analogy) between equations of electric and magnetic fields could be initially expressed (at this time still as brainstorming and somewhat hypothetically) by transforming (or extending) the present explicit system of almost independent equations (3.1) into new, implicit, mutually linked and dependent system of equations (3.2). Electric and magnetic fields (related to the same process) are anyway and always, naturally, fully united and coupled (see later (3.4)), and should be fully mutually dependent and symmetrical, as follows,

$$\left.\begin{aligned} \vec{E}(\vec{v}) &= \vec{E}_0 \pm \vec{v} \times \vec{B}\left[\vec{E}(\vec{v}), \vec{B}_0\right] = \vec{E}\left[\vec{H}(\vec{v}), \vec{E}_0\right] = \frac{1}{\varepsilon}\vec{D}\left[\vec{H}(\vec{v}), \vec{D}_0\right] = \vec{E}_{stat.} \pm \vec{E}_{dyn.}, \\ \vec{H}(\vec{v}) &= \vec{H}_0 \mp \vec{v} \times \vec{D}\left[\vec{H}(\vec{v}), \vec{D}_0\right] = \vec{H}\left[\vec{E}(\vec{v}), \vec{H}_0\right] = \frac{1}{\mu}\vec{B}\left[\vec{E}(\vec{v}), \vec{B}_0\right] = \vec{H}_{stat.} \mp \vec{H}_{dyn.} \end{aligned}\right\} \Rightarrow \quad (3.2)$$
$$(\text{Lorentz's force law}) \Rightarrow F = q_{el.}\left\{\vec{E}_0 \pm \vec{v} \times \vec{B}\left[\vec{E}(\vec{v}), \vec{B}_0\right]\right\} + q_{mag.}\left\{\vec{H}_0 \mp \vec{v} \times \vec{D}\left[\vec{H}(\vec{v}), \vec{D}_0\right]\right\}$$

We can now analyze different solutions of equations (3.2) by introducing various initial and boundary conditions related to the nature of the involved motions and propagation media. This involves exploring the relationships between the electric field and electric induction, as well as the magnetic field and magnetic induction, while ensuring they satisfy the well-known natural conservation laws and the principles of Faraday-Maxwell theory.

The goal is to develop an updated, structurally symmetric, and inertial system-invariant, unified set of Maxwell's equations. Such a concept would describe perpetually self-interacting, self-regenerating, and mutually coupled electromagnetic fields in both their integral and local forms, starting from the implicit relations outlined in equations (3.2).

Since the specific form of the implicit functions and relations in (3.2) has not been precisely defined, there is considerable flexibility in adjusting and refining proposed initial mathematical model. This flexibility will be especially useful if we accurately represent electric and magnetic fields as components of an analytic signal.

By doing so, we could achieve a higher degree of mathematical symmetry between the mutually interacting and coupled electric and magnetic fields. This would also allow us to analyze new, as-yet-unexplained or hypothetical experiments and other complex situations within this domain, which are closely related to gravitation.

There is a strong possibility that each pair of mutually dependent electric and magnetic field components or waveforms behaves like the real and imaginary parts of an analytic signal. These components would be phase-shifted by 90° and mutually transformable using the Hilbert transform. However, in specific environments (involving electric and magnetic fields, currents, and voltages), different phase shifts could occur, depending on the involved electromagnetic impedances and load configurations.

This approach to mathematical modeling could significantly enhance electromagnetic theory and offer new insights into the understanding of gravitation (see the discussion on electromechanical analogies in the first chapter of this book).

Generally applicable and valid in real material media (not only in a homogeneous, linear, and isotropic or empty space, free of other forces and other interactions), electric and magnetic induction vectors from (3.2) will have the following symmetrical, analogical, and mutually dependent forms such as:

$$\begin{aligned}
&\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 (\vec{E} + \vec{E}_{int.}),\ \vec{E}_{int.} = \frac{\vec{P}}{\varepsilon_0},\ (or\ \vec{D} = \|\varepsilon\|\,\vec{E}\ ),\\
&\vec{B} = \mu_0 (\vec{H} + \vec{M}) = \mu_0 (\vec{H} + \vec{H}_{int.}),\ \vec{H}_{int} = \vec{M},\ (\text{or}\ \vec{B} = \|\mu\|\,\vec{H}\ ),\\
&\vec{D}\left[\vec{H}(\vec{v}), \vec{D}_0\right] = \|\varepsilon\| \cdot \vec{E}(\vec{v}) = \|\varepsilon\| \cdot \vec{E}\left[\vec{H}(\vec{v}), \vec{E}_0\right] = \vec{D}_{int} + \vec{D} = \vec{D}_{stat.} \pm \vec{D}_{dyn.}\\
&\vec{B}\left[\vec{E}(\vec{v}), \vec{B}_0\right] = \|\mu\| \cdot \vec{H}(\vec{v}) = \|\varepsilon\| \cdot \vec{H}\left[\vec{E}(\vec{v}), \vec{H}_0\right] = \vec{B}_{int} + \vec{B} = \vec{B}_{stat.} \mp \vec{B}_{dyn.}\\
&\vec{E}_{int.} = \frac{1}{\|\varepsilon\|}\vec{D}_{int} = \text{internal-polarization electric field inside of media} = \vec{E}_{stat.}, (v = 0)\\
&\vec{H}_{int.} = \frac{1}{\|\mu\|}\vec{B}_{int} = \text{internal-polarization magnetic field inside of media} = \vec{H}_{stat.}, (v = 0)
\end{aligned} \tag{3.3}$$

$\|\varepsilon\| =$ dielectric constant as a tensor,

$\|\mu\| =$ magnetic permeability as a tensor.

The primary mathematical and intuitive strategy in sections (3.2) and (3.3) is to maintain the mutual analogy and symmetry in all mathematical expressions of coupled electric and magnetic fields related to the same situation. These fields should always be formulated as a system of interdependent and implicit equations, which can later be explicitly solved. While making this unifying concept mathematically practical and operational is a separate challenge, where sections (3.2) and (3.3) lay the foundation for what could become the most important framework for a new electromagnetic and gravitation theory. The original modeling concepts found in references [4], [34], [46], [47], and [99] also support and complement this internally symmetrical electromagnetic field concept.

The idea of modifying equations for the magnetic and electric fields to be more symmetrical and mutually coupled is well-established in existing literature. For example, Albert Shadowitz's book *The Electromagnetic Field* (Dover Publications, 1988) discusses similar topics. However, while creating formal mathematical symmetry within electromagnetic fields and Maxwell's equations may seem like a straightforward mathematical exercise, it is only the first necessary step. This initial platform is proposed to explain gravitation, based on the assumption that electric and magnetic fields are always mutually coupled, symmetrical, mutually orthogonal, and interactive, influencing the effects of gravitation through electromechanical actions on masses (both internally and externally). In other words, electromagnetism theoretically underpins mechanics and gravitation (see familiar elaborations in [152]).

It is well known that any electric charge generates an electric field (or electric induction) in its vicinity. Additionally, charges in any kind of motion, including oscillations, create electric currents, which in turn generate magnetic fields around these conductive zones. These electric and magnetic fields are mutually orthogonal, behaving like the real and imaginary parts of a Complex Analytic Signal associated with the electromagnetic field. Furthermore, the particle-wave duality framework (discussed in Chapters 2, 4.1, and 10 of this book) explains that every linear motion has an associated spinning or helical matter-wave field along its path. Analogously, a moving electric charge should create a helical magnetic field structure along its path. The distance between periodically successive helical elements of this spiral magnetic field formation corresponds to the de Broglie matter wavelength, $\lambda_e = h/m_e v_e$ indicating that an electron can be represented as an electromagnetic or photonic matter-wave packet, with its group and phase velocities, frequency, energy, momentum, and magnetic field helicity.

In the context of solar systems, planets rotate around their suns, solar systems orbit galactic centers, and galaxies rotate around larger mass centers. These complex helical or solenoidal motions, along with their associated electromagnetic and spinning field configurations, are omnipresent in the universe.

After analyzing Compton scattering, the Photoelectric effect, and electron diffraction, it becomes clear that the de Broglie wavelength $\lambda_e = h/m_e v_e$ is an essential property of a moving electron or its matter-wave packet. This wavelength allows us to calculate and predict experimental results for these interactions, including the helix-spinning effects and scattering observed in Compton and Photoelectric effects. These analyses rely heavily on Classical particle mechanics, Variation principles, and Minkowski's 4-vector framework.

A surprising and relevant insight is that electromagnetic energy transmitted through wires travels around the wires as a matter-wave or electromagnetic wave, with the wires serving merely as guiding lines. Around these wires, mutually coupled magnetic and electric fields propagate, behaving like the real and imaginary parts of the Complex Analytic Signal that model electromagnetic field-energy transfer (see Chapter 4.0 for more details). When we refer to "electric currents" passing through wires, we are using an alternative, equivalent modelling approach. There is no absolute need for electrons to flow as in a fluidic motion as traditionally imagined.

With a bit of creative imagination, this concept can also explain Nikola Tesla's radiant energy phenomena, particularly when applied to pulse-repetitive DC or AC current trains, such as in Tesla "magnifying transformers". In these systems, one side of the resonant circuit is grounded to Earth, while the other extends like an antenna towards the ionosphere. This antenna creates self-closed, circular magnetic field lines and radially emits Tesla's radiant electric field energy into open space. Both magnetic and electric field lines are mutually orthogonal, behaving as real and imaginary parts of the Complex Analytic Signal of the electromagnetic field. Tesla referred to this spatial configuration of the electromagnetic field as "Scalar Waves."

To further understand the nature of moving electrically charged states, we can analyze the creation of an electron-positron pair from the energy-momentum content of a sufficiently energetic photon. The annihilation of such an electron-positron pair is resulting in two photons. These experimental observations strongly suggest that the internal content of an electron (or positron) is a specific form of stabilized matter-waves of "electromagnetic or photons-energy packing." Such entities naturally obey the conservation of total system energy and relevant angular and linear momentum. A similar process, by analogy and structural symmetry, may also apply to protons (and anti-protons). Since a neutron effectively represents a coupled combination of an electron and a proton (regardless of quark-related options), we can hypothesize that wave-groups of electromagnetic energy (or photons) in different "packing formats" likely account for the mass and atomic diversity in our universe.

More supporting arguments regarding the standing-wave matter structure can be found in Chapters 4.1 and 10.

Richard F. Gauthier [53] publishes the attempt to present electron and positron as a specific superposition of photons; - "a photon is modeled along an open 45-degree helical trajectory. A spatial model of an electron is composed of a charged point-like quantum circulating at an extremely high frequency in a closed, double-looped helical trajectory whose helical pitch is one Compton wavelength h / mc. The two possible helicities of the electron model correspond to the electron and the positron. With these models, an electron is like a closed circulating photon. The electron's inertia is proposed to be related to the electron model's circulating internal Compton momentum mc".

In conclusion, Maxwell-Faraday equations and other relevant formulas that describe the relationships between electric and magnetic fields can, and should, be fully integrated, mutually coupled, inertial-system invariant, symmetrical, and fully interdependent. These equations should also be convertible and analogical regarding the corresponding elements of electric and magnetic fields, as elaborated in [34], [71], [89], [99], [152], and [156]. There remains considerable creative potential to further integrate and unify Maxwell's equations, Ampere's Law, Faraday's Law, Lorentz force, Gauss's Laws of Electrostatics and Magnetostatics, and Lenz's Law.

The current level of mathematical unity and integration within Maxwell's theory and electromagnetic theory is still incomplete. This provides intellectual motivation to reformulate and enhance contemporary electromagnetic theory, aiming for a much higher internal symmetry within the Maxwell equations and laying stronger foundations for the concept of particle-wave duality. This reformulated approach may also offer new insights into the fundamental origins of gravitation.

Furthermore, there is already a significant and striking symmetry between Maxwell's equations and the equations governing fluid flow mechanics. It was through these tangible comparisons and analogies between electromagnetic and fluid dynamics that the creators of electromagnetic theory were able to develop something as meaningful and practical as contemporary electromagnetic theory, culminating in the Maxwell equations. Read the following citation from Tsutomu Kambe [123]:

"Fluid mechanics is a field theory of Newtonian mechanics of Galilean symmetry, concerned with fluid flows represented by the velocity field such as v(x, t) in space-time. A fluid is a medium of continuous mass. Its mechanics is formulated by extending a discrete system of point masses. Associated with two symmetries (translation and space-rotation), there are two-gauge fields: $E \equiv (v \cdot \nabla) v$ and $H \equiv \nabla \times v$, which do not exist in the system of discrete masses. One can show that those are analogous to the electric field and magnetic field in the electromagnetism, and fluid Maxwell equations can be formulated for E and H. Sound waves within the fluid is analogous to the electromagnetic waves in the sense that phase speeds of both waves are independent of wave lengths, i.e., non-dispersive".

[♣ Contrary to the historical development of Maxwell's theory, where fluid dynamics inspired the conceptualization of electromagnetic fields, it is possible to reverse this analogy. We can argue that the principles of fluid mechanics themselves might be explicable through the foundations of a well-established, internally symmetric electromagnetic theory, as initiated in sections (3.1), (3.2), and (3.3). This perspective implies the existence of a subtle, pervasive fluidic medium (an ether) that penetrates all states of matter, including absolute vacuum. In essence, Electromagnetism could underline Mechanics and Gravitation.

Historically, fluid phenomena were more tangible and accessible visually, intuitively and experimentally, making them a natural source for the analogical development of electromagnetic field theory. This historical evolution reinforces the approach advocated in this book: reorganizing and updating physics through indicative analogies and conceptual parallels. Given the current conceptual gaps in electromagnetism and gravitation and acknowledging that fluid dynamics served as a conceptual basis for Maxwell's equations, it is logical to explore these analogies further. This exploration could provide theoretical insights and extend the equations governing both electromagnetism and fluid dynamics, thereby contributing to a better understanding of gravitation.

One might consider the ether, as Nikola Tesla speculated, to be the fluidic medium carrying electromagnetic and maybe other matter-waves. Despite its exclusion from modern physics following Einstein's relativity and the Michelson-Morley experiments, the analogy between fluid mechanics and electromagnetic theory suggests that electromagnetic waves should also have a material, etheric, or fluidic carrier.

A central question addressed in this book is: What the consequences of a fully symmetrical electromagnetic compatibility and unified field theory are, particularly when considering gravitation, mechanics of electromagnetically neutral masses, and electromagnetic interactions among micro and macro particles? The answer lies in the evolving analogies between linear motion and electric fields, and between mechanical rotation, magnetic fields, and the rotation of electric charges, including associated effects of wave-particle duality. Since electric and magnetic fields are inherently coupled, it is reasonable to assume that similar couplings might exist between linear motions and rotations of electromagnetically neutral masses, guiding us towards a deeper understanding of inertia, inertial states, matter waves and gravitation (see more in Chapters 4.1 and 10 about PWDC). In fact laws of conservation of linear and angular moments are always coincidently and synchronously satisfied in any motion, thanks to PWDC.

Several publications, including those by Arbab I. Arbab (see [63]), explore the relationship between electromagnetic and Lorentz forces and gravitation, as updated Maxwell equations suggest. To establish these connections, we must first achieve complete mathematical symmetry between all components of the electric and magnetic fields. This symmetry can then be extended by analogy to the linear and rotational motions of neutral masses, revealing potential complementarities between gravity-related fields.

In summary, the goal here is to establish all possible analogies and symmetries between magnetic fields and rotational motion, and between electric fields and linear motion, as discussed in Chapter 1, section T.1.8 (Generic Symmetries and Analogies of the Laws of Physics). Building on known relations between electric and magnetic fields, we aim to explore possible coupling relations between linear motion and rotational motion of masses. Ultimately, this could lead to a new, well-functioning model that integrates electromagnetic, linear, and rotational motions, including gravitation, as part of the broader context of Particle-Wave Duality.

The natural and wider validity of such analogies is already experimentally verifiable. For instance, high-power mechanical energy, ultrasonic waves, or audio signals can be transmitted through material media using signal modulation techniques applied to laser beams and dynamic plasma states. Similarly, mechanical vibrations can propagate through solid wires in a manner analogous to electromagnetic wave propagation, as discussed in [140] European Patent Application related to MMM technology.

Another important objective here is to identify mechanics and gravity-related entities that could be "analogically equivalent" or symmetrical with the principles established in sections (3.2) and (3.3). This would extend the concept of Maxwell's electromagnetic field equations to encompass mechanics and gravitation. This idea should not be surprising since stable, electrically neutral matter is composed of atoms, which themselves consist of electrons, protons, and neutrons, objects or matter-states intrinsically linked to electromagnetic interactions. Gravitation, therefore, becomes negligible at small scales, where electromagnetic forces dominate.

A key part of this strategy is to demonstrate that the electromagnetic field equations (3.2) naturally generate field vector solutions consistent with Lorentz transformations, without relying on the prescriptive, assumed geometric approach currently used in relativity theory. This could lead to a new version of Special Relativity, derived from the symmetries and interactions within an upgraded electromagnetic theory. As a result, the traditional Special Relativity theory would become obsolete, replaced by a theory grounded in the inherent properties of electromagnetic fields and their interactions with matter.

In parallel, gravitation could be understood as the result of matter-wave interactions via electromagnetic forces and energy-momentum exchanges, aligning with the ideas of Rudjer Boskovic and Nikola Tesla (see [6] and [97]). Moreover, Electromagnetic theory could be conceptualized similarly to Einstein's General Relativity, where spatial-temporal deformations around masses and energy states with electromagnetic charges play a central role.

We must remember that photons, the quanta of electromagnetic energy, possess measurable mass, energy, and mechanical moments. It is conceivable that the entire universe is fundamentally electromagnetic, with photons interacting with atoms and masses that are, in essence, structurally stabilized electromagnetic fields. This perspective allows for precise descriptions of photon interactions with both ordinary masses and charged particles, laying the groundwork for a novel conceptualization of gravitation and mechanics.

Electric and magnetic fields, along with other properties of matter, are always coupled and dependent on motion parameters. In isolated laboratory conditions, we often study magnetic or electric fields in isolation, but they are part of a larger, interconnected system of electromagnetic and mechanical phenomena. Natural forces manifest as effects of electric and magnetic moments, dipoles, and multipoles, all influenced by the motions of interacting particles. This interconnectedness suggests the existence of certain universal medium, a hypothetical fluidic ether that permeates all of space, carrying electromagnetic waves and contributing to the fundamental structure of the universe.

We can extend, unite, and analogically generalize concepts about electromagnetic and gravitational fields, forces, potentials, wave equations, and relativistic aspects of motions if we follow (and later additionally develop) the foundations of such ideas as presented in ***[163]****. Read the following citation from* ***[163],*** *Thomas Minderle, A Brief Introduction to Scalar Physics:*

"The forces of magnetism, electricity, and gravity are distortions of a single primordial field that permeates the universe and comprises the fabric of existence. Vorticity in this field gives rise to magnetic fields. Dynamic undulations give rise to electric fields. Compression or divergence gives rise to gravitational fields. When put into mathematical form, these relations reveal how electric and magnetic fields can be arranged to produce artificial gravity and many other exotic phenomena such as time distortion and the opening of portals into other dimensions".

Practically, it will be necessary to extend and unite basic definitions and relations between electric and magnetic fields, in this chapter summarized within equations (3.1) – (3.8), with familiar concepts and definitions from [163], creatively and analogically. In process, ... working on ♣].

3.2. Generalized view about Currents, Voltages, and Charges

Currents and voltages (or potential differences) present essential engineering and phenomenological, technical items or phenomena in our electromagnetic theory and engineering practices. Let us introduce, and establish, unified and generalized (mutually symmetrical and analog) definitions of electric and magnetic currents $i_{electr.}$, $i_{mag.}$, voltages $u_{electr.}$, $u_{mag.}$, and charges $q_{electr.}, q_{mag.}$, starting from electric and magnetic induction from (3.3), as follows:

$$\left.\begin{array}{l} i_{electr.} = u_{mag.} = -\frac{d\Phi_{electr.}}{dt} = i_{el.stat.} + i_{el.dyn.} = i_{electr.}(x,y,z,t)\ , \\ \Phi_{electr.} = \iint_S \vec{D}d\vec{S} = q_{electr.} = q_{el.stat.} + q_{el.dyn.} = \Phi_{electr.}(x,y,z,t) \\ q_{elect.} = \iiint_\Omega \rho_{el.}\ dV,\ i_{electr.} = \iint_\Sigma \mathbf{J}_{electr.} \cdot d\mathbf{S} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} divD = \nabla D = \rho_{elect.} \\ \nabla \times D = \varepsilon(J_{mag.} - \frac{\partial B}{\partial t}) \end{array}\right\}$$

$$\left.\begin{array}{l} i_{mag.} = u_{electr.} = -\frac{d\Phi_{mag.}}{dt} = i_{mag.stat.} + i_{mag.dyn.} = i_{mag.}(x,y,z,t) \\ \Phi_{mag.} = \iint_S \vec{B}d\vec{S} = q_{mag.} = q_{mag.stat.} + q_{mag.dyn.} = \Phi_{mag.}(x,y,z,t) \\ q_{mag.} = \iiint_\Omega \rho_{mag.}\ dV,\ i_{mag.} = \iint_\Sigma \mathbf{J}_{mag.} \cdot d\mathbf{S} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} divB = \nabla B = \rho_{mag.} \\ \nabla \times B = \mu(J_{elect.} + \frac{\partial D}{\partial t}) \end{array}\right\} \qquad (3.4)$$

In formulations (3.4), there is no attempt to introduce or defend the existence of a simple, independent, natural, unipolar magnetic charge (or magnetic monopole), as it is well-established that such entities cannot exist in ordinary, static, stable, or stationary natural conditions. While magnetic monopoles could be artificially constructed, this process is not straightforward. Specific spatial configurations of magnets can be assembled in a way that exposes only one magnetic pole to the external space, while the opposite pole remains hidden inside the structure.

Instead of using the term "monopoles," we can refer to these as magnetic charges, which can effectively, mathematically, and temporarily exist as certain magnetic flux entities. These also arise in transient situations involving action-reaction forces, currents, voltages, and velocities. In such cases, matter waves combined with mechanical motions and interactions between electromagnetic fields, currents, and voltages are created, becoming complementary parts of a broader intrinsic electromagnetic symmetry and analogy.

....

The power $P(t)$, energy $\tilde{E}$, and equivalent matter-wave mass $\tilde{m}$, of certain electromagnetic field or electromagnetic wave group, in relation to (3.4), can be expressed as:

$$\begin{array}{l} P(t) = i_{electr.} \cdot u_{electr.} = u_{mag.} \cdot i_{mag.} = \frac{d\Phi_{electr.}}{dt} \cdot \frac{d\Phi_{mag.}}{dt} = \oiint_S \vec{\Gamma} \cdot d\vec{S} = \Psi^2(t), \\ \text{where: } \vec{\Gamma} = \vec{\Gamma}(\vec{v}) = \vec{E}(\vec{v}) \times \vec{H}(\vec{v}) = \text{Poynting's vector}, \\ \Psi^2(t) = (\text{square of electromagnetic wave function}) = P(t) = \text{power}, \\ \tilde{E} = \int P(t)dt = \int \Psi^2(t)dt = \text{energy} \Rightarrow \tilde{m} = \frac{\tilde{E}}{c^2} = \text{effective mass}. \end{array} \qquad (3.5)$$

In both (3.4) and (3.5), $\Phi_{electr.}$ and $\Phi_{mag.}$ present electrical and magnetic field fluxes (see important mathematical and analogical elaborations about power and energy of matter waves in Chapter 4.0, around (4.0.82) - (4.0.104) and later in Chapter 10.).

To temporarily avoid all questions and possible dilemmas about the real nature of different forms of electric and magnetic charges, currents, and voltages, we can start with equations (3.4) and (3.5), and replace all corresponding electric and magnetic inductions, or field members, using the expressions from equations (3.2) and (3.3). This way, going backwards (deductively), we can determine (or reinvent) all basic electromagnetic charges. In addition, we shall be able to generalize and unify the meanings of relevant static and dynamic, or transient electromagnetic parameters (such as charges, currents, voltages..., see T.5.2, (5.15) and (5.16)), basically conceptualizing currents and voltages and electric power transfer as an electromagnetic field phenomenon in the vicinity and/or around electric wires.

- **Current Understanding of Electric Charge and its Dynamic Properties**

Traditionally, we define an electric charge as a static or stable quantity measured in Coulombs (or in terms of the number of electrons). However, insights from the first chapter of this book suggest that electric charge may not be a static parameter at all. Instead, it appears to belong to dynamic, motional states, much like mechanical moments. This view challenges conventional understanding and requires further clarification.

If we consider electric charges as active, dynamic variables (particularly when at elevated potentials), they may behave similarly to electromagnetic waves or mass emitters. This perspective implies that they could generate antenna-like emissions, jet-propulsion, or thrust forces, as observed by Nikola Tesla in his experimental research.

- **Electrostatic and Gravitational Forces: A Potential Link**

The attractive Coulomb force between two small charges, -q and +q, separated by a distance r, is well known as $\mathbf{F} = -(\mathbf{1}/\mathbf{4}\pi\varepsilon_0)\cdot \mathbf{q}^2/\mathbf{r}^2$. Remarkably, a similar attractive force arises when a single charge q, with mass m, interacts with an electromagnetically neutral mass m. This occurs because the neutral mass becomes polarized, forming an electric dipole due to the presence of the charge q.

For two equal, electromagnetically neutral masses m and m, the gravitational force between them is described by Newton's law as $\mathbf{F} = -\mathbf{G}\cdot \mathbf{m}^2/\mathbf{r}^2$. Notably, this gravitational force is mathematically identical to the Coulomb electrostatic force between opposite charges +q and -q. This mathematical similarity suggests a deeper connection between gravitation and electromagnetism, hinting that both might share a common electromagnetic nature.

- **Implications for Gravitational Theory**

Given the structural similarities, we might reasonably propose that gravitational interactions arise from the complex, helical, and spinning motions of celestial bodies such as planets, solar systems, and galaxies. This hypothesis is explored further in Chapter 2, specifically around equations (2.4-4.1) to (2.4-4.3).

Consequently, it is conceivable that every mass can be represented mathematically as a constant multiplied by one elementary electron charge, e. This idea offers a unified perspective on the electromagnetic and gravitational phenomena, as follows,

$$\begin{bmatrix} F = -(1/4\pi\varepsilon_0)\cdot q^2/r^2 = -G\cdot m^2/r^2 \\ q_1 = q_2 = q,\ m_1 = m_2 = m,\ (1/4\pi\varepsilon_0)\cdot q^2 = G\cdot m^2 \end{bmatrix} \Rightarrow \begin{bmatrix} m_{1,2} = q_{1,2}/\sqrt{4\pi\varepsilon_0 G} \\ q_{1,2} = N_{1,2}q_e = N_{1,2}e \end{bmatrix} \Rightarrow$$

$$\Rightarrow \begin{bmatrix} F = -G\cdot m_1 m_2/r^2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} = -G\cdot\left(q_1/\sqrt{4\pi\varepsilon_0 G}\right)\cdot\left(q_2/\sqrt{4\pi\varepsilon_0 G}\right)/r^2 = \\ = -\left(N_1 e/\sqrt{4\pi\varepsilon_0 G}\right)\cdot\left(N_2 e/\sqrt{4\pi\varepsilon_0 G}\right)/r^2 = -N_1 N_2 e^2/4\pi\varepsilon_0 r^2, \\ \left(q_1 = q_2 = q = Ne\right), \left(m_1 = m_2 = m\right) \Leftrightarrow \left(N_1 = N_2 = N = const\right) \end{bmatrix} \Rightarrow \tag{3.5-a}$$

$$\Rightarrow F = -G\cdot m^2/r^2 = -N^2e^2/4\pi\varepsilon_0 r^2 = -\frac{q^2}{4\pi\varepsilon_0 r^2} \Rightarrow m^2 = N^2e^2/4\pi\varepsilon_0 G \Rightarrow \boxed{m = \frac{N}{\sqrt{4\pi\varepsilon_0 G}}e}.$$

- **The Bohr Atom Model and Planetary Systems: A Striking Analogy**

Returning to the analogy between Niels Bohr's atom model and planetary or solar systems, as discussed in Chapters 2 and 8 (see "T.2.8. N. Bohr Hydrogen Atom and Planetary System Analogies"), we can see why this comparison is effective. This similarity, highlighted through equations (3.5-a) and (2.4-4.1) to (2.4-4.3), suggests that all masses are predominantly composed of various combinations of electric charges. It further implies that electromagnetic energy, along with the formation of photon structures, is responsible for creating all elementary particles, atoms, and, consequently, the matter and mass around us.

- **Revisiting Maxwell-Faraday Theory: Toward a Unified Framework**

By utilizing the standard framework of Maxwell-Faraday Electromagnetic Theory and expressing all Maxwell equations in both their integral and local forms, using the symmetrical field representations from equations (3.2) and (3.3), we can redefine and expand our understanding of electromagnetic and gravitational charges. This approach allows for the development of more universal, unified, and practical definitions of natural field charges.

Such an improvement would not only make Maxwell's theory more broadly applicable but would also increase its internal symmetry, as proposed in this book.

The biggest obstacles in reconstructing and optimizing contemporary Maxwell electromagnetic theory are related to three major deficiencies of Maxwell's equations, as only partially summarized in the following citation (see [35] from Thomas E. Phipps, Jr.):

• "They are non-invariant at the lowest (first) order of approximation under inertial (Galilean) transformations.
• They are under-parameterized, in the sense that source motions are described, but not sink motions. Such an implicit promotion of the sink to a preferred motional status flouts any conceivable form of the relativity principle. (This is why invariance fails.)
• They misrepresent Faraday's observations of induction through the false implication that $\frac{\partial}{\partial t}$ can treat that for which $\frac{d}{dt}$ is mathematically necessary: *viz.,* description of the *emf* generated by *changing the shape* of a conducting circuit penetrated by magnetic flux".

The rest of the analysis and number of proposals found in the book of Thomas E. Phipps, [35], regarding Maxwell Theory and its upgrading and renovation, are so well elaborated and supported that it is better to read them from the original source, [35].

[♣ COMMENTS & FREE-THINKING CORNER

By William J. Hooper, there are three types of electric and magnetic fields that are still not distinctively and explicitly captured and explained by Maxwell electromagnetic theory.

Citation from: Selected Correspondence on Common Sense Science #1, David L. Bergman, editor, Common Sense Science, P.O. Box 767306, Roswell, GA 30076-7306, E-mail: bergmandavid@comcast.net):

*"The experiments of William J. Hooper as described in his book **New Horizons in Electric, Magnetic and Gravitational Field Theory** identify that all three types of electric and magnetic fields have different empirical properties. For instance, in Chapter 1 Hooper lists 14 empirical properties of E fields. In Table 1 he gives what these properties are for electrostatic E fields, E fields dependent on dA/dt, and E fields dependent on motion V×B. In particular, he notes in property 6 that the motionally caused E fields cannot be shielded".*

Consequently, we can expect big reorganization and updating work within the present electromagnetic theory (see [152]).

Matter is intrinsically unified, both locally and non-locally, through the synchronized existence of all its aspects, forms, charges, fields, and their multilevel and multidimensional interactions. There is little doubt that phenomena observed in one domain of physics have analogous or closely related manifestations in other, conjugate domains. For example, John Stewart Bell's theorem of interconnectedness, established in 1964 in quantum mechanics (CERN, Geneva, Switzerland), highlights this principle.

A deeper parallelism and analogy between Maxwell's electromagnetic theory and gravitation should exist, beyond what is currently understood. Basic constituents of matter, such as atoms composed of electrons, protons, and neutrons, suggest this connection. The fact that a free neutron naturally decays into a proton and an electron implies that all matter, including us, is part of the same electromagnetic framework.

Concepts such as Ampere-Maxwell's Law, Biot-Savart's Law, Lenz's Law, Faraday's Law of induction, and electromagnetic Lorentz forces provide inspiring frameworks for analogous thinking. They can offer richer descriptions of linear and rotational fields or motions. For instance, transforming these electromagnetic laws into analogous expressions for rectilinear and rotational motions could yield significant insights.

It seems that extending Maxwell's electromagnetic theory, potentially through a new strategy involving relativity theory, will require a revitalization and enhancement of Wilhelm Weber's force law. This law offers a natural unification of fundamental laws in classical electrodynamics, including Gauss's laws, Coulomb's law, Ampere's generalized law, Faraday's law, and Lenz's law. See references [28] and [29] for further details.

If we consider that electric and magnetic (vector) fields should always be coupled, and structured as mutually orthogonal (especially in cases of electromagnetic waves propagation), we can try to present the composite vector of such electromagnetic field (or force) as,

$$\begin{gathered}\vec{\mathbf{F}}_{\mathbf{em}}(r, \theta, \phi, t) = \vec{\mathbf{F}}_p(r, \theta, \phi, t) + \vec{\mathbf{F}}_s(r, \theta, \phi, t),\\ (\vec{E} = \vec{E}_1 + \vec{E}_2 , \vec{H} = \vec{H}_1 + \vec{H}_2 = \vec{B}/\mu , \vec{B} = \vec{B}_1 + \vec{B}_2 = \mu\vec{H}),\end{gathered} \tag{3.5.1}$$

*from which the first is **the potential force field**,* $\vec{\mathbf{F}}_p(r, \theta, \phi, t) = \alpha_1\vec{E}_1 + \beta_1\vec{B}_1$

$$\begin{gathered}\nabla \times \vec{E}_1 = 0 , \nabla \cdot \vec{E}_1 = \frac{\rho}{\varepsilon_o} \neq 0\\ \nabla \times \vec{B}_1 = 0 , \nabla \cdot \vec{B}_1 \neq 0,\end{gathered} \tag{3.5.2}$$

*and, the second is **the solenoidal force field**,* $\vec{\mathbf{F}}_s(r, \theta, \phi, t) = \alpha_2\vec{E}_2 + \beta_2\vec{B}_2$

$$\nabla\times\vec{B}_2=\mu_o(J+\varepsilon_o\frac{d\vec{E}}{dt})\neq 0,\ \nabla\cdot\vec{B}_2=0,\ , \qquad \nabla\times\vec{E}_2=-\frac{d\vec{B}}{dt}\neq 0,\qquad \nabla\cdot\vec{E}_2=0, \tag{3.5.3}$$

where vectors of such electric and magnetic fields are,

$$\vec{\mathbf{E}}(r,\theta,\phi,t)=\vec{E}_1+\vec{E}_2,\ \vec{\mathbf{H}}(r,\theta,\phi,t)=\vec{H}_1+\vec{H}_2,\ \vec{\mathbf{B}}_{1,2}=\mu_o\vec{H}_{1,2}$$

$$\vec{\mathbf{F}}_{em}(r,\theta,\phi,t)=\vec{\mathbf{F}}_p(r,\theta,\phi,t)+\vec{\mathbf{F}}_s(r,\theta,\phi,t)=\alpha_1\vec{E}_1+\beta_1\vec{B}_1+\alpha_2\vec{E}_2+\beta_2\vec{B}_2,$$

$$\begin{bmatrix}\nabla\times\vec{\mathbf{F}}_{em}=0\\ \nabla\cdot\vec{\mathbf{F}}_{em}\neq 0\end{bmatrix}\Leftrightarrow\begin{bmatrix}\alpha_1\nabla\times\vec{E}_1+\beta_1\nabla\times\vec{B}_1+\alpha_2\nabla\times\vec{E}_2+\beta_2\nabla\times\vec{B}_2=0\\ \alpha_1\nabla\cdot\vec{E}_1+\beta_1\nabla\cdot\vec{B}_1+\alpha_2\nabla\cdot\vec{E}_2+\beta_2\nabla\cdot\vec{B}_2\neq 0\end{bmatrix}\Leftrightarrow\begin{bmatrix}-\alpha_1\frac{d\vec{B}}{dt}+\beta_1\mu_o(J+\varepsilon_o\frac{d\vec{E}}{dt})=0\\ \alpha_1\frac{\rho}{\varepsilon_o}+\beta_1\nabla\cdot\vec{B}_1\neq 0\end{bmatrix}$$

$$\begin{bmatrix}\nabla\times\vec{\mathbf{F}}_{em}\neq 0\\ \nabla\cdot\vec{\mathbf{F}}_{em}=0\end{bmatrix}\Leftrightarrow\begin{bmatrix}\alpha_1\nabla\times\vec{E}_1+\beta_1\nabla\times\vec{B}_1+\alpha_2\nabla\times\vec{E}_2+\beta_2\nabla\times\vec{B}_2\neq 0\\ \alpha_1\nabla\cdot\vec{E}_1+\beta_1\nabla\cdot\vec{B}_1+\alpha_2\nabla\cdot\vec{E}_2+\beta_2\nabla\cdot\vec{B}_2=0\end{bmatrix}\Leftrightarrow\begin{bmatrix}-\alpha_2\frac{d\vec{B}}{dt}+\beta_2\mu_o(J+\varepsilon_o\frac{d\vec{E}}{dt})\neq 0\\ \alpha_1\frac{\rho}{\varepsilon_o}+\beta_1\nabla\cdot\vec{B}_1=0\end{bmatrix} \tag{3.5.4}$$

In addition, we could (still intuitively and hypothetically) explore the options that solenoidal field component $\vec{\mathbf{F}}_s(r,\theta,\phi,t)$ is equal to Hilbert transform of the potential field component $\vec{\mathbf{F}}_p(r,\theta,\phi,t)$, or that magnetic field $\vec{H}(r,\theta,\phi,t)$ is (on some specific way) product of Hilbert transform of an electric field $\vec{E}(r,\theta,\phi,t)$. Presence of constants or functions $\alpha_{1,2},\beta_{1,2}$ could give us a chance and freedom to adjust and comply with the present (or future) forms of Maxwell equations.

As a continuation of similar explorations, we could develop an analytic signal or complex function (often referred to as a generalized phasor) for the electromagnetic field. This would consist of mutually coupled components, orthogonal and phase-shifted, generated by the Hilbert transform, as discussed in section (3.5.4). For further details on analytic signals, see Chapters 4 and 10. These phase-shifted electric and magnetic fields, currents, and voltages will exhibit phase differences $+\pi/2$ or $-\pi/2$ for inductive and capacitive loads or impedances (like mass and spring loads in mechanics). For resistive and complex impedance loads, the phase shifts will differ (see more on electromechanical analogies in the first chapter of this book).

Additionally, another compelling set of ideas regarding electromagnetic field equations, particularly in their complex form, can be found in Andrija S. Radovic's publication, "Derivation of Maxwell Equations and Their Corrections" (©1996, ©2003, available at http://www.andrijar.com), and in various papers by Prof. Jovan Djuric (referenced in [34] and [71]). These works serve as a rich source of intellectual and creative inspiration.

The next step in the evolution of these ideas could be to conceptualize most physics-related vector fields, such as gravitation and matter wave manifestations, as complementary pairs of potential and solenoidal field vectors. These two components would be interconnected, much like the real and imaginary parts of an analytic signal or complex phasor, drawing an analogy to the relationship between electric and magnetic fields in an electromagnetic wave. For further elaboration, refer to Chapters 4 and 10.

These are preliminary brainstorming proposals that require further refinement and elaboration. Similar concepts, particularly related to Bohr's hydrogen atom model, can be found in the appendix under Chapter 8, within equations (8.69)–(8.73), and in literature reference [63]. ♣**]**

3.3. New, Relativistic-like Formulation of Maxwell Equations

Let us now consider the case of homogeneous and isotropic media (free space and vacuum) and find the simplest solutions for electric and magnetic fields, starting from the system of mutually dependent implicit equations (3.2),

$$\begin{aligned}
&\vec{E}(\vec{v}) = \vec{E}\left[\vec{H}(\vec{v}), \vec{E}_0\right] = \frac{1}{\varepsilon_0}\vec{D}\left[\vec{H}(\vec{v}), \vec{D}_0\right], \ \vec{H}(\vec{v}) = \vec{H}\left[\vec{E}(\vec{v}), \vec{H}_0\right] = \frac{1}{\mu_0}\vec{B}\left[\vec{E}(\vec{v}), \vec{B}_0\right] \\
&\vec{E}(\vec{v}) = \vec{E}_0 \pm \ \vec{v}\times\vec{B}\left[\vec{E}(\vec{v}), \vec{B}_0\right] = \vec{E}_0 \pm \mu_0\,\vec{v}\times\vec{H}\left[\vec{E}(\vec{v}), \vec{H}_0\right] = \vec{E}_0 \pm \mu_0\,\vec{v}\times\vec{H}(v), \\
&\vec{H}(\vec{v}) = \vec{H}_0 \mp \vec{v}\times\vec{D}\left[\vec{H}(\vec{v}), \vec{D}_0\right] = \vec{H}_0 \mp \varepsilon_0\vec{v}\times\vec{E}\left[\vec{H}(\vec{v}), \vec{E}_0\right] = \vec{H}_0 \mp \varepsilon_0\vec{v}\times\vec{E}(v),\ \varepsilon_0\mu_0 = 1/c^2.
\end{aligned} \tag{3.6}$$

One of possible simplified solutions of (3.6) is:

$$\begin{aligned}
&\vec{E}(\vec{v}) = \frac{\vec{E}_0 - \varepsilon_0\mu_0\vec{v}\cdot(\vec{v}\cdot\vec{E}_0)}{1-\varepsilon_0\mu_0 v^2} \mp \frac{\mu_0\vec{v}\times\vec{H}_0}{1-\varepsilon_0\mu_0 v^2} = \frac{\vec{E}_0 - \frac{\vec{v}}{c^2}\cdot(\vec{v}\cdot\vec{E}_0)}{1-\frac{v^2}{c^2}} \mp \frac{\mu_0\vec{v}\times\vec{H}_0}{1-\frac{v^2}{c^2}}, \\
&\vec{H}(\vec{v}) = \frac{\vec{H}_0 - \varepsilon_0\mu_0\vec{v}\cdot(\vec{v}\cdot\vec{H}_0)}{1-\varepsilon_0\mu_0 v^2} \pm \frac{\varepsilon_0\vec{v}\times\vec{E}_0}{1-\varepsilon_0\mu_0 v^2} = \frac{\vec{H}_0 - \frac{\vec{v}}{c^2}\cdot(\vec{v}\cdot\vec{H}_0)}{1-\frac{v^2}{c^2}} \pm \frac{\varepsilon_0\vec{v}\times\vec{E}_0}{1-\frac{v^2}{c^2}}.
\end{aligned} \tag{3.7}$$

From (3.7) we could "experience the picture" of ***mutually dependent, orthogonal and mutually complementing***, repetitively (and perpetually) ***self-generating or self-regenerating*** electric and magnetic fields and charges, when we apply Maxwell equations to (3.7): $\nabla \mathrm{x}\ \vec{H} = \vec{J} + \varepsilon\, \partial\vec{E}/\partial t$, $\nabla \mathrm{x} \vec{E} = -\mu\partial\vec{H}/\partial t$, $\nabla\vec{\mathbf{H}} = \mathbf{0}$, $\nabla\vec{E} = \rho/\varepsilon$.

The next step would be an attempt to find much more generally valid solutions for (3.2), in some similar form like we find in (3.7) and to apply definitions from (3.4) and (3.5) to such solutions. After obtaining more general forms of electric and magnetic currents and charges, we shall be able to go back and *reestablish* updated, more-symmetrical forms of all Maxwell equations compared to the present situation. By exploiting electromechanical analogies (as in the first Chapter of this book), we could go backwards to a new understanding of natural forces, including Gravitation, field charges, and their mechanical and electromagnetic moments.

It is also interesting to notice the validity of the following relations (in connection with understanding spontaneous field vectors bending and rotation):

$$\begin{aligned}
&\left\{\begin{array}{l}\vec{\mathbf{v}}\cdot\vec{\mathbf{E}}(\vec{\mathbf{v}}) = \vec{\mathbf{v}}\cdot\vec{\mathbf{E}}_0 \Leftrightarrow \mathbf{v}\cdot\mathbf{E}(\mathbf{v})\cdot\cos(\vec{\mathbf{v}},\vec{\mathbf{E}}) = \mathbf{v}\cdot\mathbf{E}_0\cdot\cos(\vec{\mathbf{v}},\vec{\mathbf{E}}_0), \\ \vec{\mathbf{v}}\cdot\vec{\mathbf{H}}(\vec{\mathbf{v}}) = \vec{\mathbf{v}}\cdot\vec{\mathbf{H}}_0 \Leftrightarrow \mathbf{v}\cdot\mathbf{H}(\mathbf{v})\cdot\cos(\vec{\mathbf{v}},\vec{\mathbf{H}}) = \mathbf{v}\cdot\mathbf{H}_0\cdot\cos(\vec{\mathbf{v}},\vec{\mathbf{H}}_0)\end{array}\right\} \Rightarrow \\
&\Rightarrow \left\{\begin{array}{l}\mathbf{E}(\mathbf{v})/\mathbf{E}_0 = \cos(\vec{\mathbf{v}},\vec{\mathbf{E}}_0)/\cos(\vec{\mathbf{v}},\vec{\mathbf{E}}) \\ \mathbf{H}(\mathbf{v})/\mathbf{H}_0 = \cos(\vec{\mathbf{v}},\vec{\mathbf{H}}_0)/\cos(\vec{\mathbf{v}},\vec{\mathbf{H}})\end{array}\right\}, \\
&\left\{\begin{array}{l}\vec{v}\cdot\left[\vec{E}(v)-\vec{E}_0\right] = \vec{v}\cdot\Delta\vec{E} = 0 \\ \vec{v}\cdot\left[\vec{H}(v)-\vec{H}_0\right] = \vec{v}\cdot\Delta\vec{H} = 0\end{array}\right\} \Rightarrow \left\{\begin{array}{l}\vec{v} = 0, \ \text{or}\ \ \Delta\vec{E} = 0, \ \ \text{or}\ \ \cos(\vec{v}, \Delta\vec{E}) = 0 \\ \vec{v} = 0, \ \text{or}\ \ \Delta\vec{H} = 0, \ \ \text{or}\ \ \cos(\vec{v}, \Delta\vec{H}) = 0\end{array}\right\},
\end{aligned} \tag{3.8}$$

which are equally applicable to (3.1), (3.2), (3.6) and (3.7).

Another, still hypothetical idea related to the electric and magnetic field vectors (within the same electromagnetic event or matter wave) is that these fields could, or perhaps should, be related like real and imaginary components of an analytic signal, based on the Hilbert transform as established by D. Gabor (see Chapter 4 for more details).

Additionally, we better not overlook the insights and proposals of Thomas E. Phipps, Jr., who highlighted weak areas in Maxwell's theory and relativity that require critical updates. His work, referenced in [35], serves as an important reminder of the need for these necessary revisions.

For instance, Maxwell equations of electromagnetic waves have the same mathematical form of Classical, second order, partial differential wave equation, as known in Classical Mechanics (see (4.9-1) and (4.9-2.1) from Chapter 4.3., and (4.0.82) from Chapter 4.0.), which can be analogically assembled, or mutually connected between electric and magnetic fields (in a shape of the Complex, Analytic Signal), on the following way,

$$
\begin{aligned}
&\Delta \mathbf{E} - \varepsilon_0 \mu_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} = \Delta \mathbf{E} - \frac{1}{u^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = 0, \\
&\Delta \mathbf{B} - \varepsilon_0 \mu_0 \frac{\partial^2 \mathbf{B}}{\partial t^2} = \Delta \mathbf{B} - \frac{1}{u^2}\frac{\partial^2 \mathbf{B}}{\partial t^2} = 0,\ u = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = c = \text{const.} (=) \text{light speed} \\
&\mathbf{E} (=) \text{electric field}, \mathbf{B} (=) \text{magnetic induction},\quad \mathbf{H}\ (=) \text{Hilbert transform} \\
&\overline{\Psi} = \overline{\Psi}(\mathbf{E},\mathbf{B}) = \Psi(\mathbf{E},\mathbf{B}) + \mathbf{I}\cdot\hat{\Psi}(\mathbf{E},\mathbf{B}), \hat{\Psi}(\mathbf{E},\mathbf{B}) = \mathbf{H}\left[\Psi(\mathbf{E},\mathbf{B})\right],\ \mathbf{I}^2 = -1 \\
&\nabla^2\overline{\Psi} - \frac{1}{u^2}\frac{\partial^2 \overline{\Psi}}{\partial t^2} = \Delta\overline{\Psi} - \frac{1}{u^2}\frac{\partial^2 \overline{\Psi}}{\partial t^2} = 0 \Leftrightarrow \begin{Bmatrix} \Delta\Psi - \frac{1}{u^2}\frac{\partial^2\Psi}{\partial t^2} = 0 \rightarrow \Delta\Psi - \frac{1}{c^2}\frac{\partial^2\Psi}{\partial t^2} = 0 \\ \Delta\hat{\Psi} - \frac{1}{u^2}\frac{\partial^2\hat{\Psi}}{\partial t^2} = 0 \rightarrow \Delta\hat{\Psi} - \frac{1}{c^2}\frac{\partial^2\hat{\Psi}}{\partial t^2} = 0 \end{Bmatrix} \\
&\qquad \Psi^2(t,r) = P(t,r) = \frac{d\tilde{E}}{dt} (=) \text{Active Power } (=) \\
&(=) \begin{Bmatrix} i(t)\cdot u(t) & (=) & [\text{Current} \cdot \text{Voltage}], \text{or} \\ f(t)\cdot v(t) & (=) & [\text{Force} \cdot \text{Velocity}], \text{or} \\ \tau(t)\cdot\omega(t) & (=) & [\text{Orb.-moment} \cdot \text{Angular velocity}], \text{or} \\ (\vec{E} \times \vec{H})\cdot\vec{S} & (=) & \left[\overrightarrow{\text{Pointyng Vector}}\right]\cdot\overrightarrow{\text{Surface}} \end{Bmatrix}
\end{aligned}
\tag{3.7-1}
$$

- **Understanding the Constant Speed of Light and Wave Dynamics**

We have the flexibility to define what is meant by a generalized wavefunction for electromagnetic waves $\overline{\Psi}(\mathbf{E},\mathbf{B}) = \Psi(\mathbf{E},\mathbf{B}) + \mathbf{I}\cdot\hat{\Psi}(\mathbf{E},\mathbf{B}),\ \hat{\Psi}(\mathbf{E},\mathbf{B}) = \mathbf{H}\left[\Psi(\mathbf{E},\mathbf{B})\right], \mathbf{I}^2 = -1$ (see equations (4.0.82) in Chapter 4.0 and (4.23) in Chapter 4.3). From Maxwel equations applied on vacuum and equations under (3.7-1), we can conclude that the speed of light, c, is always constant and depends solely on static, fixed, or possibly hidden background properties of the medium, such as the vacuum's dielectric and magnetic permeability parameters.

This means that there is no explicit dependence of c on variable time and length intervals or their mutual contraction in compliance with Lorentz transformations, unless these properties ε, μ themselves depend on velocity, or we treat them separately as electrodynamic constants, where $\mathbf{u} = 1/\sqrt{\varepsilon\mu}$, distinguishing c as the speed of light. Consequently, Albert Einstein, whether intentionally, intuitively, or through postulation, asserted that the speed of light remains constant across all relative motions and inertial frames. Understanding this peculiar property of the Universe remains a challenge in physics.

To describe this universally constant speed of light c more formally, it should be defined as any other physical speed, using:

$$\mathbf{c} = 1/\sqrt{\varepsilon_0\mu_0} = \mathbf{u} = \mathbf{const.} = \frac{\Delta \mathbf{x}_c}{\Delta \mathbf{t}_c} = \frac{\mathbf{k}\cdot\Delta \mathbf{x}_c}{\mathbf{k}\cdot\Delta \mathbf{t}_c} = \frac{\mathbf{p}\cdot\Delta \mathbf{x}_c}{\mathbf{p}\cdot\Delta \mathbf{t}_c} = \ldots \quad (3.7\text{-}2)$$

Here, $\Delta \mathbf{x}_c$ **and** $\Delta \mathbf{t}_c$ are minimal, mutually related spatial and temporal intervals $\Delta \mathbf{x}_c = \mathbf{c}\cdot\Delta \mathbf{t}_c$, that are fundamentally quantized, constant, and cannot be smaller. This conceptualization links the minimal spatial and temporal positions and durations of signals (or matter-wave packets) and offers an additional perspective on relativistic transformations.

The classical second-order wave equations found in Mechanics, Acoustics, and Fluid Dynamics share similar forms with equation (3.7-1), where the phase velocity u is constant and solely dependent on the material properties ε_0, μ_0, independent of spatial and temporal parameters. This opens new avenues for understanding wave motion, Lorentz transformations, and the spatial and temporal durations of moving particles and their corresponding wave groups (see Chapters 4.0, 4.1, and 10 for more on group and phase velocities).

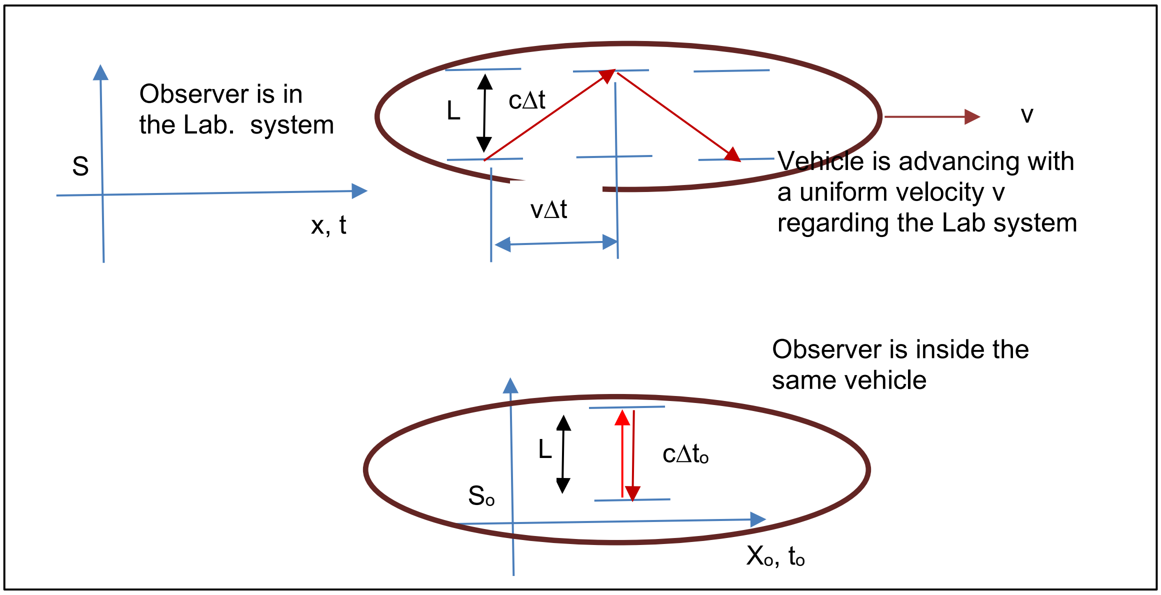
Generally, the relationship between group velocity v and phase velocity u for all types of matter waves (or wave packets) is expressed as:

$$\mathbf{u} = \lambda\cdot\mathbf{f} = \frac{\lambda}{\mathbf{T}} = \frac{\omega}{\mathbf{k}} = \frac{\tilde{\mathbf{E}}}{\mathbf{p}} = 1/\sqrt{\varepsilon\mu}\,,\ \mathrm{v} = \frac{d\omega}{dk} = \frac{dE}{dp} = \frac{d\tilde{E}}{dp} = \frac{dE_k}{dp} = u - \lambda\frac{du}{d\lambda} = u + k\frac{du}{dk} = -\lambda^2\frac{df}{d\lambda}, \quad (3.7\text{-}3)$$

Thus, there is more work to be done to fully understand, unify, and reconcile the relationships between the speed of light, phase velocity, and the spatial and temporal intervals in various wave phenomena.

[♣ COMMENTS & FREE-THINKING CORNER:

Let us imagine that inside a (linearly moving) vehicle a light beam is shot between two mirrors (perpendicularly to the direction of linear vehicle motion), and the distance between mirrors is L. Observer within its reference frame So, linked to the vehicle, will notice or measure only the path of travel of the light-beam equal to $\mathbf{L} = \mathbf{c}\cdot\Delta \mathbf{t}_0$, *where* $\Delta \mathbf{t}_0$ *is the travel-time interval or time duration in So . Another observer in the laboratory coordinate system S will detect the same light-beam travel* $\Delta \mathbf{t}$ *differently, since the system So has the velocity v in the system S. This will result in,* $\mathbf{L}^2 = (\mathbf{c}\Delta \mathbf{t})^2 - (\mathbf{v}\Delta \mathbf{t})^2 = (\mathbf{c}\Delta \mathbf{t}_0)^2$. *See all relevant details in the picture below.*



Time and space intervals of the same event in different referential frames

$$c = 1/\sqrt{\varepsilon_0\mu_0} = \text{const.} = \frac{\Delta x_c}{\Delta t_c} = \frac{k\cdot\Delta x_c}{k\cdot\Delta t_c} = \frac{p\cdot\Delta x_c}{p\cdot\Delta t_c} = \frac{\Delta x_0}{\Delta t_0} = ...$$

$$L^2 = (c\Delta t)^2 - (v\Delta t)^2 = (c\Delta t_0)^2 \Rightarrow \boxed{\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}} = \gamma\cdot\Delta t_0} \Rightarrow$$

$$\begin{bmatrix} \Delta t_0 = k\cdot\Delta t_c = \Delta t/\gamma \\ \Delta x_0 = k\cdot\Delta x_c \\ \\ \frac{\Delta x_0}{\Delta t_0} = \frac{\Delta x_c}{\Delta t_c} = c \end{bmatrix} \Rightarrow \begin{bmatrix} \Delta t = k\cdot\gamma\cdot\Delta t_c = \gamma\Delta t_0 \\ \boxed{\Delta x = v\cdot\Delta t = \gamma\cdot v\cdot\Delta t_0 = \gamma\cdot\frac{v}{c}\Delta x_0} \\ \\ v = \frac{\Delta x}{\Delta t} = \frac{v}{c}\cdot\frac{\Delta x_0}{\Delta t_0} = v \end{bmatrix}$$

$$(\Delta s)^2 = c^2(\Delta t)^2 - (\Delta x)^2 = c^2(\gamma\Delta t_0)^2 - (\gamma\cdot v\cdot\Delta t_0)^2 = (c\Delta t_0)^2 = (\Delta x_0)^2 = L^2$$

$$(\Delta s_0)^2 = c^2(\Delta t_0)^2 - (\Delta x_0)^2 = c^2(\Delta t_0)^2 - (c\Delta t_0)^2 = 0$$

Eventually, in cases when addressing photons, (starting from (3.7-3)), we will get what we expect to have, and what we already know as correct from different points of view, as follows,

$$\begin{bmatrix} u = \lambda\cdot f = \frac{\lambda}{T} = \frac{\omega}{k} = \frac{\tilde{E}}{p} = c = \text{const.},\quad \lambda = \frac{h}{p} \\ v = \frac{d\omega}{dk} = \frac{dE}{dp} = \frac{d\tilde{E}}{dp} = \frac{dE_k}{dp} = u - \lambda\frac{du}{d\lambda} = u + k\frac{du}{dk} = -\lambda^2\frac{df}{d\lambda} \end{bmatrix} \Rightarrow$$

$$\Rightarrow \left[\tilde{E} = cp = c\frac{h}{\lambda} = hf,\ v = \frac{d\tilde{E}}{dp} = -\lambda^2\frac{df}{d\lambda} = u = c\right] \Rightarrow c\frac{d\lambda}{\lambda^2} = -df \Rightarrow \qquad (3.7\text{-}4)$$

$$\Rightarrow \left(\frac{c}{\lambda_2} - \frac{c}{\lambda_1}\right) = (f_2 - f_1),\ \frac{\Delta x_0}{\Delta t_0} = \frac{\Delta x_c}{\Delta t_c} = \frac{\Delta x}{\Delta t} = u = v = c\ .$$

An interesting situation regarding equation (3.7) is that contemporary Maxwell theory, while correctly formulated, has been somewhat artificially integrated into the framework of Relativistic Electrodynamics. This integration successfully aligns with Lorentz transformations of electromagnetic field vectors. While Lorentz electromagnetic field transformations in Relativistic Electrodynamics are accurately supported by experimental and theoretical evidence, there appears to be a difference compared to equation (3.7). Both frameworks use the same Lorentz factor, $(1-\varepsilon_0\mu_0 v^2)=(1-v^2/c^2)$ but there may be fundamental discrepancies, indicating that either equation (3.2), aspects of current Relativistic Electrodynamics, or the forms of Lorentz electric and magnetic forces discussed in earlier works could be incomplete.

It is worth exploring whether Lorentz forces and Lorentz transformations might be partially mis-formulated or placed within an incorrect conceptual framework, rather than being fully integrated with Maxwell's equations as suggested in [34] and [35]. Contemporary relativity theory also has similar weaknesses. An important question is how to successfully unify Maxwell's theory with relativity to create an improved version of Relativistic Electrodynamics, given that both theories have unresolved issues (see [35] for more details on fundamental errors and weaknesses).

Moreover, new electromagnetic forces, not described by the current Maxwell equations, have been experimentally observed and are becoming an increasingly active area of research (see [30]). These new forces suggest that Maxwell's equations may need modification and updating.

The proposal here is to develop a new conceptual framework for the electromagnetic field, combining longitudinal and transverse wave motions with natural couplings to linear and rotational motions within the mathematical framework of analytic signal modeling. Additionally, we should revisit universal force laws formulated by Ampère and Wilhelm Weber. Ampère's law, recently modernized in various ways by several authors, is gaining recognition for its ability to predict or explain experimental results related to forces between current elements and other electrically conductive channels (see references [28], [29], [36], and [63]). ♣]

It is worth to mention that N. Tesla also had number of extraordinary, original, and imaginative ideas, suggestions and experimental results related to electromagnetic phenomenology and Gravitation, certain of them still waiting to be fully understood and decoded (see presentations about N. Tesla ideas, experiments, concepts and results in [81] and [97]).

[♣ COMMENTS & FREE-THINKING CORNER:

Nikola Tesla's legacy *continues to serve as a unifying and inspirational force across various technical, cultural, historical, and national divides. Despite geopolitical and other differences, Serbs, Croats, and Americans all claim Tesla as the source of their national pride and scientific heritage. However, mainstream narratives, especially outside of Serbia, often downplay Tesla's Serbian roots. Some even attempt to portray him as primarily Croatian, particularly those with a Vatican-influenced perspective. Tesla himself addressed these questions directly, affirming his identity as a Serb, with deep Serbian cultural and familial roots. At the same time, he acknowledged that being either Serb or Croat was essentially the same, likely referencing the fact that Croatian identity began to form in regions initially populated by Serbs and influenced by the Vatican. Over generations, many Croats may have forgotten their true origins because of these historical shifts.*

Tesla, who is celebrated globally by Serbs, Croats, Americans and other Slavic nations embodied a cosmopolitan and international spirit that transcends national, ideological, or geographic boundaries. His contributions to global science and technology are what truly matter. Nevertheless, it is important to remember that during World War II, some Croats, deeply indoctrinated and supported by the Vatican and fascists, committed atrocities against hundreds of thousands of Serbs, including numerous members of Tesla's own family.

Even today, the mainstream Western scientific community tends to glorify figures like Edison and Marconi, often overshadowing Tesla's monumental contributions. If Tesla's inventions and engineering *achievements were to suddenly disappear, modern technology would come to a standstill. It's also worth noting the controversial and, at times, unethical practices of Edison and Marconi in relation to Tesla.*

Reflecting on these historical details can deepen our understanding of the complex realities surrounding Tesla's legacy. For neutral and objective observers, the facts about Tesla are clear and unequivocal. However, various authors and interpreters, influenced by their cultural and ideological backgrounds, present differing narratives to either compromise or bolster specific geopolitical or ideological positions. This tendency to manipulate history and facts is as present in the realm of science as it is in everyday life.

Fortunately, modern society has moved away from the most brutal forms of suppression against original thinkers, such as the Inquisition organized by the Vatican in the Middle Ages was. Today, we face subtler and more sophisticated attempts to modify and manipulate history, science, and facts, often through convenient omissions or biased interpretations. This is a reminder to remain vigilant and critical of the narratives presented to us, much like Tesla himself admired Rudjer Boskovic, who faced similar challenges in his time, being still classified as Croat (regardless the fact that when he was alive, Croatia as nation and state did not exist).

....

*Obviously, it is interesting that in (3.7) we can find characteristic Lorentz factor, usual for coordinate transformations in **S**pecial **R**elativity **T**heory (**SRT**),* $\gamma^2 = 1/(1-\varepsilon_0\mu_0 v^2) = 1/(1-v^2/c^2)$, *without applying or imposing anything what "smells" on **SRT**. The author of this book is still not able to offer more precise and final (or better) comments about such insights. Based only on the brainstorming and common-sense intuition we could ask ourselves if today's (Lorentz) relativistic transformations of electric and magnetic field vectors (or applied anywhere else in physics) are so essential, fully correct, complete, and generally applicable to all kinds of particles, fields and wave states and motions? The remaining option is that in our contemporary physics books we still have some particularly valid (well-fitted) cases of artificially created **"relativistic"** field transformations, **"explaining"** well **"mostly already known"** experimental facts, in many cases initially discovered or implemented **without using any of present relativistic or quantum theory background**). ♣]*

Let us consider ideas that could test the general validity of Maxwell's relativistic equations. Magnetic fields are always linked to their complementary electric fields, being mutually coupled and unified through electric and/or magnetic charges, fluxes, and currents.

For example, the magnetic field produced by an electromagnet is generated within and around a ferromagnetic core when the core is placed inside a solenoid with an electric current. The motion of electrons in the solenoid creates a rotational effect, forming a closed electric current loop. This rotation involves non-uniform accelerated motion, which special relativity theory (SRT) does not fully address.

Similarly, regarding a permanent magnet, we can measure its external magnetic field without observing the internal solenoid or some circulation of electrons. It is understood that the permanent magnet's field arises from hidden rotational motion within the spinning electric charges or currents of its atomic-scale magnetic domains.

When electromagnetic waves propagate in free space or a vacuum, we do not see the core material or solenoid, yet we can detect the electromagnetic waves (or photons), which are composed of mutually coupled and rotating electric and magnetic field vectors. Even in a vacuum, there must be some form of rotation or vortex phenomena responsible for generating the magnetic field component associated with moving electric entities.

However, detecting a direct presence of this "carrier-fluid" or space-time texture remains challenging. Fortunately, Maxwell's theory adequately explains the creation and propagation of electromagnetic fields without necessity to explicitly define this "carrier-fluid." The propagation of light or electromagnetic waves can be analogically compared to alternating currents and voltages in capacitors or other complex impedances in various electrical loads.

*[♣ COMMENTS & FREE-THINKING CORNER: **Relativistic** Lorentz transformations (of **SRT**) applied on (visibly or invisibly) rotating and mutually complementary and coupled states, which also perform rectilinear motion, should be (mathematically) more complex than just using simple Lorentz transformations applied only on rectilinear motions (as formally prescribed in present days **SRT**). Most probably that some of missing or limiting aspects of traditionally formulated Maxwell theory and **SRT** (such as not-counted effects of transient situations, accelerated motions and rotation, incomplete mathematical formulation of Faraday's Law of Magnetic Induction, incomplete symmetry in formulating basic Maxwell equations and/or equations of electromagnetic field…), are artificially (or mathematically) compensated by formulating **Q**uantum and **R**elativistic **E**lectrodynamics, (**QRE**). Effectively, **QRE** was constructed and adjusted to satisfy and explain results of (already known) experimental observations. If traditional Maxwell Electrodynamics (formulated in Galilean space) was differently established, we should be able to get relativistic field transformations (as known in present days Relativistic Electrodynamics, similar to (3.7)), without any math-hybridization with **SRT**, or without using or knowing **SRT** (see number of papers indicating such options, from **[23]** to **[26]** and in **[35]**). ♣]*

A significant connection between Maxwell's theory and gravitation is suggested by the fact that a narrow-band wave packet of electromagnetic radiation, or a photon, possesses equivalent dynamic mass hf/c^2*, momentum,* hf/c*, and spin. This is supported by experimental analyses such as Compton and Photoelectric effects. Additionally, it is known that a gravitational field affects the equivalent mass of a photon, exhibiting an attractive influence.*

The rectilinear motion and rotation of particles, along with other energy states, can be viewed as a complementary system like the electric and magnetic fields unified in a photon. The creation of an electron-positron pair from a high-energy γ*-photon, both naturally spinning after creation, further supports the idea that photon (or wave energy) can be transformed into real, rotating particles.*

For instance, equation (3.5) provides the effective or equivalent (static and dynamic) mass and momentum $\tilde{\mathbf{m}} = \tilde{\mathbf{E}}/\mathbf{c}^2 = \mathbf{m}_{\text{stat.}} + \mathbf{m}_{\text{dyn.}}$ *of electromagnetic radiation in terms of electric and magnetic field vectors, as referenced in equations (3.4) and (3.3) and derived from Maxwell's equations. Historically, Albert Einstein, among others, attempted to use Faraday-Maxwell theory as a model for describing gravitation and developing a unified field theory. However, these efforts were largely unsuccessful, likely due to the challenging assumptions and postulates of Special Relativity Theory (SRT), which did not adequately account for rotational and accelerated motions.*

This text proposes a similar idea based on analogical thinking regarding the unification of gravity and electromagnetic fields. Before fully realizing this, we should first establish the active presence and influence of a "field of rotation," which would complement gravitation, analogous to the mutual complementarity of electric and magnetic fields.

By extending and upgrading Maxwell's equations, we aim to make them more compatible with gravitation theory, seeking to demonstrate that gravitation has essential origins in electromagnetic fields and forces, as suggested in references [23] – [26], [63], and [152]. This book will also link the concept of field unification to inertial and reaction forces, "fields of rotation," and de Broglie matter wave phenomenology (see equations (4.18), (4.19), (4.22) – (4.29), (5.15), and (5.16)), with further analysis provided in the subsequent chapters.

[♣ COMMENTS & FREE-THINKING CORNER:

In "*P. W. Bridgman, The Logic of Modem Physics*" we can find the following statement (see in [35]):
"There is no physical phenomenon whatever by which light may be detected apart from the phenomena of the source and the sink ... Hence from the point of view of operations, it is meaningless or trivial to ascribe physical reality to light in intermediate space, and light as a thing traveling must be recognized to be a pure invention".

Photons, or light, are detectable only directly at their source and at their receiver. In the space between, where photons should travel, they are not observable directly. However, if sensors and detectors are placed along the path between the source and the receiver, photons can be detected. Considering that energy-momentum currents, voltages, and electromagnetic motions typically belong to closed circuits or networks with physical endpoints (sources and sinks), we can infer the following:

*a) **Photon Transmission**: Information about an emitted photon is transmitted to its receiver instantly, regardless of the distance between them. However, the physically received photon (or its momentum and energy) will reach the sink only after a certain time due to the finite speed of photons.*

*b) **Entanglement and Synchronization:** Emitters and receivers (or sources and sinks) of photons are likely connected and synchronized through entanglement channels and couplings, due to spatial-temporal interactions and mutual transformability. Atoms (or resonators) involved in photon emission and reception suggest that the universe was once closely and densely unified, with mutual entanglement between atoms, photons, and other particles. This implies that distant receivers can anticipate the arrival of specific photons after a time delay. This conceptualization can be mathematically modeled using complex Analytic signal theory, which describes motions, matter waves, and signals in the universe.*

*c) **Material Media and Cosmic Vacuum:** Since open circuits or networks without sources and sinks do not exist in our universe, there must be some material medium or substance (even in a vacuum) that connects all physical entities. The challenge remains to model and explain this connecting channel, where information is transferred without any time delay.*

Next, let us consider some hypothetical and unconventional ideas regarding contemporary electromagnetic theory. These ideas, while not necessarily correct or incorrect, serve as a basis for exploring extreme and non-standard scientific concepts. Based on Dr. Sorin Cezar Cosofret's proposals [126], these challenging premises are:

1. Electricity and Magnetism: Electricity and magnetism are mutually related but not fully equivalent phenomena.
2. Electromagnetic Waves: Electromagnetic waves (radio, microwaves, terahertz) are phenomenologically different from electricity and magnetism.
3. Nature of Light: Light (IR, visible, UV, and higher-energy photons) or the flux of photons is not an electromagnetic wave.
4. Unification Challenges: Unifying different classes of phenomena may be either impossible or highly problematic.
5. Electric Current Definition: The current definition of electric current is questionable. For example, experiments show that an electric current can pass through a solution without electrode effects, suggesting that the definition of electric current and electrolysis laws may need revision. Additionally, beams of electrons from a cathode ray tube or radioactive source do not produce the same effects as an electrical current in a metal wire from a battery, challenging current explanations of battery function.
6. Magnetism Anomalies: Rational explanations are needed for why Jupiter has a magnetic field thousands of times stronger than the Sun's, why the Sun's magnetic field flips every decade while planetary flips occur less frequently, and why Earth's magnetic field flip patterns are irregular but show certain trends.

By critically examining these unconventional ideas, without preemptively rejecting them, a free-minded and open scientist might find new sources of inspiration and motivation for more fruitful explorations. ♣]

3.4. The Important Extension of Electromechanical Analogies

Since in (3.4) we introduced static and dynamic currents, charges, and fluxes (for both electric and magnetic field), the system of analogies given in T.1.2.1, T.1.3, T.1.4, T.1.5, and 5.4.1 in Chapter 10 can be extended, separating electric and magnetic parameters, towards equivalent (dimensional) analogies, as summarized in T.3.1, T.3.2 and T.3.3:

T.3.1	[E] = [ENERGIES]= = $\int\{[U]\ [I]\}dt = \int[P]dt$	[Q] = [CHARGES] = = [C][U]	[C] = [REACTANCES] = = [Q]/[U]
Electric Field	$[C_{el.}u_{el.}^{2}]$ (=) $[q_{el.}u_{el.}^{1}]$ $[L_{el.}i_{mag.}^{2}]$ (=) $[\Phi_{el.}i_{mag.}^{1}]$	$[C_{el.}u_{el.}^{1}]$ (=) $[q_{el.}u_{el.}^{0}]$ (=) $[q_{el.}]$ (=) $[\Phi_{el}]$	$[C_{el.}u_{el.}^{0}]$ (=) $[q_{el.}u_{el.}^{-1}]$ (=) $[C_{el.}]$ (=) $[L_{el.}]$ (=) [C]
Magnetic Field	$[C_{mag.}u_{mag.}^{2}]$ (=) $[q_{mag.}u_{mag.}^{1}]$ $[L_{mag.}i_{el.}^{2}]$(=) $[\Phi_{mag.}i_{el.}]$	$[C_{mag.}u_{mag.}^{1}]$ (=) $[q_{mag.}u_{mag.}^{0}]$ (=) $[q_{mag.}]$ (=) $[\Phi_{mag}]$	$[C_{mag.}u_{mag.}^{0}]$ (=) $[q_{mag.}u_{mag.}^{1}]$ (=) $[C_{mag.}]$ (=) $[L_{mag.}]$ (=) [L]
Gravitation	$[mv^{2}]$ (=) $[pv^{1}]$	$[mv^{1}]$ (=) $[pv^{0}]$ (=) [p]	$[mv^{0}]$ (=) $[pv^{-1}]$ (=) [m]
Rotation	$[J\ \omega^{2}]$ (=) $[\mathbb{L}\ \omega^{1}]$	$[J\omega^{1}]$ (=) $[\mathbb{L}\omega^{0}]$ (=) $[\ \mathbb{L}\ ]$	$[J\omega^{0}]$ (=) $[\mathbb{L}\omega^{-1}]$ (=) [J]

T.3.2	[U] = [VOLTAGES] = = d[X]/dt	[I] = [CURRENTS]= = d[Q]/dt	[Z] =[IMPEDANCES]=[U]/[I] (= [mobility] in mechanics)
Electric Field	$u_{el.} = d\Phi_{mag.}/dt = i_{mag.}$	$i_{el.} = dq_{el.}/dt = u_{mag.}$	$Z_{el} = u_{el.} / i_{el.} = i_{mag.}/ u_{mag.} = Y_{mag.}$
Magnetic Field	$u_{mag.} = d\Phi_{el.}/dt = i_{el.}$	$i_{mag.} = dq_{mag.}/dt = u_{el.}$	$Z_{mag.} = u_{mag.} / i_{mag.} = i_{el.} / u_{el.} = Y_{el.}$
Gravitation	$v = dx/dt$	$F = dp/dt$	$Z_m = v / F$
Rotation	$\omega = d\alpha/dt$	$\tau = d\mathbb{L}/dt$	$Z_R = \omega / \tau$

T.3.3	[R] = [RESISTANCES]= = $[Z]_{real}$	[X] = [DISPLACEMENTS] = = $\int[U]\ dt$	[P] = [POWER] = d[E]/dt = = [U][I]
Electric Field	$R_{el.}$	$\Phi_{mag} = L_{mag}i_{el} = q_{mag}$	$u_{el.}i_{el.}$
Magnetic Field	$R_{mag.}$	$\Phi_{el.} = L_{el.}\ i_{mag.} = q_{el.}$	$u_{el.}i_{el.}$
Gravitation	R_m	$x = S F$	$v F$
Rotation	R_R	$\alpha = S_R\tau$	$\omega\tau$

Building on the analogies presented in T.3.1, T.3.2, and T.3.3 (with the understanding that terminology and symbols might be refined further), future analyses may either open new hypotheses or reinforce existing ones (see [3]). This exploration could lead to a more general theory of gravitation that incorporates field effects of rotation. It is evident that we are only scratching the surface of potential unification concepts for a higher-level Universal Field Theory, as discussed in T.1.8, "Generic Symmetries and Analogies of the Laws of Physics" in Chapter 1.

Our current focus is on identifying indicative and provocative ideas, facts, and analogies that could provide a foundation for a New Field Theory. A more comprehensive and practical field unification platform will be introduced in Chapters 4 and 5, particularly through the development of equations (4.22) – (4.29) and (5.15) – (5.16). For those interested in exploring challenging and imaginative ideas about the potential shortcomings of Maxwell's electromagnetic theory and proposals for enhanced unification and symmetry, Wim Vegt's work, *150 Years of Physics Based on the Wrong Equation* [156], is highly recommended for open-minded readers.

Analogies can be a powerful tool for unification and prediction, provided they are correctly formulated, and all fundamental elements of the analogy are fully coherent and replaceable. To date, physics has achieved a notable level of coherence and symmetry through established analogies.

Another approach to analogies involves starting from Energy in any of its forms as the fundamental building block of the universe. By expressing all other physics-related quantities—such as charges, reactance, voltages, impedances, resistances, displacements, and power (as detailed in T.3.1, T.3.2, and T.3.3)—in terms of Energy, we could develop more robust analogies and symmetries. This approach may contribute to the formulation of a generally acceptable Unified Field Theory.

[♣ COMMENTS & FREE-THINKING CORNER:

From T.3.2 we can easily notice that the product between electric and magnetic impedances (of the same electric circuit) is equal to 1 ($Z_{el.} \cdot Z_{mag} = \frac{u}{i} \cdot \frac{i}{u} = 1$ *). Let us extend this analogically, searching for mechanical impedance (or mobility) of a linear motion, and for the mechanical impedance of the rotation or spinning that belongs to the same motion, where the product of both would also be equal to 1 (or to some constant). We shall see that this is still not possible to satisfy as a generally valid case since* $Z_m \cdot Z_R = \frac{v}{F} \cdot \frac{\omega}{\tau} \neq \text{const.}$ *(because all the involved variables* v, F, ω, τ *are multidimensional, space-time dependent). If we take the product between relevant mechanical impedance and admittance* $Z_m \cdot Y_R = \frac{v}{F} \cdot \frac{\tau}{\omega}, Y_R = \frac{1}{Z_R} = \frac{\tau}{\omega}$*, we will see that the result could be interesting (most probably invariant to referential system changes).*

As the first example, let us intentionally take the case of a circular mass motion that can be presented as linear particle motion along the circular line or rotational motion around its center of rotation. In both cases, we should get the same mechanical impedance and all other relevant energy members should be mutually equal ($dE = v \cdot dp = \omega \cdot d\mathbb{L}, v = \omega \cdot r$ *).*

$$\left\{\begin{array}{l}\vec{p} = m\vec{v},\ \vec{\mathbb{L}} = \vec{r} \times \vec{p},\ \vec{F} = \frac{d\vec{p}}{dt}, \vec{\tau} = \frac{d\vec{\mathbb{L}}}{dt} = \vec{r} \times \vec{F}, \\ dE = v \cdot dp = \omega \cdot d\mathbb{L}, v = \omega \cdot r\end{array}\right\} \Rightarrow$$

$$\Rightarrow \begin{cases} Z_m = \frac{v}{F} = \frac{dE \cdot dt}{(dp)^2} = \frac{\omega}{\tau} \cdot (\frac{v}{\omega})^2 = \frac{dE \cdot dt}{(d\mathbb{L})^2} \cdot (\frac{v}{\omega})^2 = \frac{\omega}{\tau} \cdot r^2 = Z_r \cdot r^2 = \\ = \frac{\omega}{(\frac{\tau}{r^2})} = \frac{\omega}{\pi(\frac{\tau}{\pi r^2})} = \frac{\omega}{\pi(\frac{\tau}{S})} = \frac{\omega}{\frac{\pi}{S}\tau} = \frac{\omega}{\tau *},\ S = \pi r^2, \\ \tau * = \frac{\pi}{S}\tau = \frac{\pi}{S}\frac{d\mathbb{L}}{dt} = \frac{d\mathbb{L}^*}{dt}, \vec{\mathbb{L}}^* = \frac{\pi}{S}\vec{\mathbb{L}} = \frac{\pi}{S}(\vec{r} \times \vec{p}) = \mathbb{J}\vec{\omega}, \\ Z_r = \frac{\omega}{\tau} = \frac{dE \cdot dt}{(d\mathbb{L})^2} = \frac{v}{F} \cdot (\frac{\omega}{v})^2 = \frac{dE \cdot dt}{(dp)^2} \cdot (\frac{\omega}{v})^2 = \frac{v}{F} \cdot \frac{1}{r^2} = Z_m \cdot \frac{1}{r^2} = \\ = \frac{v}{F} \cdot \frac{\pi}{\pi r^2} = \frac{v}{F} \cdot \frac{\pi}{S} = \frac{v}{\frac{S}{\pi} \cdot F} = \frac{v}{F^*}, Z_m \cdot Z_r = \frac{v}{F} \cdot \frac{\omega}{\tau} \neq 1, \\ F^* = \frac{S}{\pi} \cdot F = \frac{S}{\pi} \cdot \frac{dp}{dt} = \frac{dp^*}{dt}, \vec{p}^* = \frac{S}{\pi}\vec{p} = \frac{S}{\pi}m\vec{v}. \end{cases} \quad (3.9)$$

$$\Rightarrow Z_m \cdot Y_m = \frac{v}{F} \cdot \frac{\tau}{\omega} = \frac{v}{F} \cdot \frac{F}{v} \cdot r^2 = r^2 = \frac{S}{\pi}$$

- ***Exploring the Analogous Mechanical Counterpart to Electromagnetic Fields***

The idea here is to consider the possibility that the electromagnetic field has an analogous yet hypothetical mechanical counterpart—a field that combines linear and spinning motions in a specific *manner. Although this counterpart lacks a formal name, it would represent a unique combination of motion types, like how electromagnetic fields are structured.* $Z_m \cdot Y_R = \frac{v}{F} \cdot \frac{\tau}{\omega} = const.$

The most plausible candidate for this complementary field is the de Broglie matter wave associated with any moving particle. This matter wave would exhibit an axially or helicoidally spinning structure around the particle, making it analogous to a photon. In certain respects, a photon can be treated as a wave-packet, $m_f = hf/c^2, p_f = m_f c = hf/c, \tilde{E}_f = hf, \lambda_f = h/p_f = c/f$ *, that exhibits properties like those of a moving particle (refer to the Compton and Photoelectric effects for detailed analyses).*

From electromagnetic theory, we know that a photon's electric and magnetic field vectors rotate along its propagation path. Extending this concept, it is only a small theoretical step to suggest that a similar rotational property may exist for any particle in relation to its de Broglie matter wave.

3.4.1. The Hyperspace Communications and Light

Imagine hypothetically that light is a manifestation of oscillating phenomena connected to a multidimensional world, one that extends beyond our familiar four-dimensional universe. We might explore communication with this multidimensional realm by manipulating and modulating the dynamic aspects of light (photons and electromagnetic waves) within our 4-dimensional space.

Consider a scenario where we create a spatially closed, toroidal mirror-wall container and introduce a light source inside it. As light reflects, scatters, and interferes within this container, it eventually reaches an equilibrium state. Within this closed system, the light energy is continuously fluctuating. If the container remains stationary, the internal "light energy fluid" could establish a stable center of gravity or center of inertia within the container's space. Some of the light energy would be converted into heat dissipated through the container walls.

Although the container appears as a fully enclosed object in our four-dimensional universe, if light indeed belongs to a higher-dimensional world, the "light-energy fluid" within it might not be truly confined. To generate meaningful information that could potentially bridge our four-dimensional world with this hypothetical multidimensional realm, we could modulate the "light energy carrier fluid" dynamically. This could involve external mechanical, ultrasonic, or electromagnetic oscillations, or applying complex motions or rotations to the mirror walls of the container. Frequency, phase, and amplitude modulation might also produce similar effects. Such dynamic modulation would create local imbalances in energy, potentially allowing part of it to penetrate the multidimensional world, adhering to conservation laws of energy and momentum in these higher dimensions.

A similar system, equipped with sensors, could potentially receive signals from the multidimensional realm into our 4-dimensional world. This proposed experimental setup could serve as a potential link between our universe and higher dimensions, provided the initial hypothesis about multidimensional light is valid. However, this concept remains speculative and oversimplified at this stage.

If light proves to be an unsuitable carrier for multidimensional communication, we might seek alternative phenomena or matter waves within our 4-dimensional world that could interact with higher dimensions. We might explore dynamic, non-stationary modulation processes where the effective center of gravity is unstable, potentially facilitating communication with higher dimensions.

In a more speculative vein, consider Nikola Tesla's "communicating vacuum tubes," which he claimed could connect us with other worlds, planets, or even extraterrestrial civilizations. Tesla reported communication with Martians and beings within Venus. While these claims should not be taken literally, Tesla's surprising and often validated statements suggest that such speculative ideas merit further exploration (see more in [160]).♣]

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4. DE BROGLIE MATTER WAVES and QUANTUM THEORY

4.0 WAVEFUNCTIONS, WAVE VELOCITIES AND UNCERTAINTY RELATIONS

This chapter focuses on wavefunctions that are universally applicable and well-suited for modeling oscillations, waves, and signals. These wavefunctions are natural and rich in detail, making them ideal for deterministic and dimensional processing of all wave phenomena in Physics. They can also be easily transformed into non-dimensional, probabilistic wave functions, as used in contemporary Quantum Theory (QT). This way, QT will get significant mathematical and conceptual modeling upgrade (reducing its exclusive dependence on probabilistic foundations).

In physics, wavefunctions or signals are temporal and spatial variables dependent, such as voltages, currents, velocities, forces, natural fields, power functions, different moments, and displacements. Among these, power-related wavefunctions are particularly important because they allow us to determine the energy of a wave.

The evolution of natural wavefunction modeling in physics begins with simple harmonic, sinusoidal wave functions $\Psi = a \cdot \cos\varphi$, $a(=)\text{amplitude}=\text{const.}, \varphi(=)\text{phase}=\text{const}$. Initially, we understand that amplitude and phase functions $a = a(x,t), \varphi = \varphi(x,t)$ depend on spatial and temporal variables. As our understanding deepens, we recognize that our universe, currently conceptualized as having three spatial dimensions and one temporal dimension, allows for amplitude and phase functions that can evolve or extend within these dimensions, $a = a(x,y,z,t) = a(r,t), \varphi = \varphi(x,y,z,t) = \varphi(r,t), r = r(x,y,z)$.

Mathematically, we discover that various linear and integral combinations, interferences, and superpositions of elementary wave functions also serve as useful and natural wave functions. We then extend this understanding by moving from the real number domain (r, t) of trigonometric wavefunctions to the complex domain (r, φ), eventually adopting Complex Analytic Signal wavefunctions modeling as, $\Psi = a \cdot \cos\varphi + i \cdot b \cdot \sin\varphi = a \cdot e^{i\varphi}, i^2 = -1$.

Building on this, we further realize that ordinary complex functions (with one imaginary unit) can be extended into Hypercomplex Analytic functions with three or more imaginary units, all dependent on spatial and temporal variables, $\Psi = \Psi + i \cdot \Psi_i + j \cdot \Psi_j + k \cdot \Psi_k, (i^2, j^2, k^2 = -1, -1, -1)$. Ultimately, these complex and hypercomplex wavefunctions evolve into four-dimensional (or even multi-dimensional) Complex Analytic Signals or Phasors.

Given that the three spatial dimensions (x, y, z) are mutually orthogonal, we can analogically assume that the temporal dimension t should also be orthogonal to x, y, and z. In the case of complex numbers or functions, the real and imaginary parts are mutually orthogonal. If we place the spatial dimensions r(x, y, z) on the real axis, the temporal dimension t should lie on the imaginary axis or to be captured within the phase function. This approach extends the 3D space into a 4D space in the case of ordinary complex functions with one imaginary unit.

For Hypercomplex functions with three or more imaginary units, we can conceptualize a multidimensional spatial-temporal reality, where all dimensions or axes are mutually orthogonal. The mathematical relationships between the spatial dimensions $r = r(x,y,z)$ and t, or $\varphi = \varphi(r,t)$, and between relevant group and phase velocities, will be defined or deduced from the relevant Complex Analytic Signal wavefunction.

These ideas about dimensional extensions are also reflected (or applied) in the Minkowski space of Relativity theory.

All over this paper are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

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This chapter explores various aspects of "energy flow" in physics, which can be mathematically modeled as currents, motions, oscillations of particles, fluids, wave groups, and wave functions. In mechanics, the focus is on the motion of particles or masses. Other energy flow scenarios involve different kinds of currents, oscillations, signals, and field phenomena, which can generally be modeled as phasors, a concept widely used in electromagnetic science, or through relevant wavefunctions.

Energy flow, or "energy current," can be understood as a power function or as the time derivative of the involved energy $\text{Power} (=) dE/dt$. Power is expressed in different branches of physics as the product of relevant quantities: voltage and current, velocity and force, or mutually complementary field elements (or corresponding complex phasors). All these functions, such as currents, voltages, forces, and velocities can be analogically modeled as phasors, particularly practiced in electrical engineering.

A further level of generalization is based on our understanding of wave-particle duality, which shows that almost any arbitrary function relevant to physics and energy flow analysis can be decomposed or synthesized using elementary sinusoidal wave components. Our universe consistently adheres to this mathematical framework, with phenomena like superposition, interference, synthesis, and decomposition, originally grounded in Fourier analysis.

This chapter further develops mathematical models related to the natural manifestations of wave-particle duality. The use of wave packets, wave groups, or wave functions, which in some respects represent real moving particles, plays a significant role in explaining and understanding this duality.

Uncertainty relations and Parseval's energy identity, which describe relationships between original and spectral (or conjugate) domains of power-related elements, are universally valid and crucial in both micro and macro scales of physics. Since we are addressing motions in both temporal and spatial coordinates, it is essential to accurately describe the velocities of "energy-mass-flow" motions, including wave motions.

The literature on wavefunctions and wave velocities (group and phase velocity) often simplifies or limits comparisons with "equivalent-particle velocity." For example, quantum theory tends to use statistically averaged, non-dimensional, or normalized wavefunctions with reduced ontological content and applicability. Contemporary discussions on wave functions and relevant wave velocities in physics and quantum theory often appear insufficiently general, incompletely defined, or not fully convincing. They may also lack phase function meaning, and present somewhat artificial, averaged results, failing to demonstrate the general applicability of the models to all wave-related phenomena in physics and nature.

This could be because many authors try to align their work with the established norms of orthodox quantum theory (QT) and relativity theory (RT), simplifying their presentations to fit mainstream expectations. However, this chapter aims to rectify and generalize concepts related to wave-motion functions and velocities, particularly concerning particle-equivalent wave-group velocities. This will be done (in this book) using the most natural and universally applicable model: analytic signals (see [57], [109], [110], and [111]).

We will demonstrate that the mathematics of complex analytic signals or phasors is equally applicable to any wave-like propagation or oscillation, whether in the micro or macro universe, or in mechanical and electromagnetic phenomena. This approach contrasts with the contemporary QT trend of tailoring wavefunction and wave equation concepts, especially concerning photons and subatomic particles, to fit statistical and probabilistic models, a consensus established in orthodox or Copenhagen QT foundations.

In modern physics, there is a tendency to emphasize the significance and "eternal validity" of orthodox QT. However, the first significant step in this book regarding matter waves and wave-particle duality is to establish, or adopt, a comprehensive, rich, and universal wavefunction model. This model should encompass all physics-related waves and oscillatory motions in a unified time-space and frequency domain. Although this may seem ambitious, it will be shown to be both realistic and feasible if we apply Dennis Gabor's complex analytic signal modeling, based on the Hilbert transform (see [57], Michael Feldman, [109], Poularikas A. D., including [110], and [111]).

In the study of waves and oscillations, it can be asserted that all such phenomena in physics should adhere to a unified concept, theory, and mathematical modeling. Fourier Signal Analysis clearly demonstrates that all spatial and temporal signals, or their combinations, can be decomposed into elementary simple-harmonic or sinusoidal waves, and vice versa. This suggests that Joseph Fourier can be considered a foundational figure in the development of wave-particle duality, followed with Dennis Gabor, later refining and advancing Fourier's work through his Analytic Signal model.

Luis de Broglie further explored the connections between particles and wave motions, revealing additional aspects of their unity. In physics and mathematics, all types of waves exhibit similar behaviors, such as interference, diffraction, refraction, scattering, and superposition, indicating that they should be described using the same mathematical models and methods. Any particles or energy-momentum states in motion can be represented as a superposition of simple-harmonic or other elementary waveforms, including matter-waves, wave groups, or wave packets.

This chapter seeks to connect and explain the mechanical properties of moving particles or masses with the intrnsically associated matter-wave motions of equivalent wave groups. While probability waves and similar formulations in microphysics and quantum theory (QT) can be mathematically useful to a certain extent, they can also be misleading, unrealistic, and abstract when it comes to non-statistical situations where immediate insights into frequency, phase, and amplitude are necessary.

Waves and oscillations always exhibit linked spatial and temporal characteristics, even when extended to multidimensional universes. To simplify the analysis of wave motions, we can focus on the temporal and/or spatial shapes of waves.

The first significant advancement in modern electrical engineering and circuit theory came when the founders of the field introduced the use of complex numbers and functions to model alternating or sinusoidal currents and voltages (or rotation related motions). This approach replaced the complex trigonometric operations with straightforward algebraic operations, providing a strong and stable mathematical foundation for circuit analysis.

The most advanced and widely recognized mathematical model for representing and analyzing arbitrary-shaped waves in physics, including in electrical engineering, is the Complex Analytic Signal model, which uses a single imaginary unit. This model is naturally connected to de Broglie's matter-waves concept. Future advancements in this direction may involve the use of Hypercomplex Analytic Signals, which would incorporate three or more imaginary units for even more sophisticated modeling.

Download the last version here:
http://www.mastersonics.com/documents/revision_of_the_particle-wave_dualism.pdf

Let's begin with an arbitrary-shaped, energy-finite waveform $\Psi(t)$ represented using the Analytic Signal model, first introduced by Dennis Gabor [57]. This model expresses the waveform $\Psi(t)$ as the product of a specific amplitude and phase function (see equation 4.0.1). The Analytic Signal model is the most natural and ready-made wavefunction model, where the amplitude and phase functions are distinctly separated, like simple harmonic signals in modulation techniques. In this framework, the amplitude function propagates with the group velocity v, while the phase function has its phase velocity u. It is important to emphasize that all energy-finite wavefunctions in physics can be represented using Analytic Signal modeling. Later in this discussion, we will explore why Analytic Signal modeling is considered the optimal, most universal, and most natural approach for waveform modeling.

Despite its advantages, Analytic Signal modeling is still underutilized in the scientific literature on wave motions and wave functions. This presents a limitation in mathematical physics, as different interpretative frameworks are still prevalent, leading to a lack of harmony in essential, multidisciplinary conclusions, unification, and generalization. The superficial differences between Analytic Signal and Fourier Transform analyses, particularly when presenting and analyzing wavefunctions, are not immediately clear without deep mathematical investigation. This may be why the scientific community often continues to rely on traditional methods, such as Fourier analysis, statistics, and probability, which emphasize averaging and offer limited immediate insights.

Many authors active in this field tend to repeat or "creatively" replicate what was published long ago, rather than adopting more advanced mathematical modeling techniques. There is a pressing need to move into a new, combined space-time-frequency framework that leverages dynamic and instantaneous Complex Analytic Signals and Hilbert transform-based waveform modeling and analysis. This approach captures all components, nuances, and benefits of signal analysis and distinguishes itself from the traditional Fourier analysis-based methods. The difference in modeling and applicability between Fourier analysis and Analytic Signal concepts is akin to the difference between operations in the realm of real numbers and those in complex numbers, where the domain of real numbers is merely a small subset of complex number analysis.

In summary, Analytic Signal modeling allows us to extract instantaneous or immediate time-and-frequency-dependent amplitude and phase functions, including instantaneous frequency and power, from any arbitrary energy-finite waveform relevant to physics. These can be presented in their temporal, spatial, and frequency domains, along with analogous amplitude and phase spectral functions in their frequency domains. Moreover, starting from a wavefunction presented as a Complex Analytic Signal, it becomes relatively straightforward to develop differential wave equations in physics, such as the Schrödinger and Klein-Gordon equations, without resorting to unnatural hybridizations or temporary "ad hoc" solutions, as often seen in quantum theory.

Going directly to the most useful analytic signal forms for wavefunctions, let us consider that $\Psi(t)$ is our original, time-domain wavefunction, wave packet (or wave group), and $\hat{\Psi}(t)$ is its Hilbert transform, both being real-value functions,

$$\Psi(t)=\frac{1}{\pi}\int_0^{\infty}\left[U_c(\omega)\cos\omega t+U_s(\omega)\sin\omega t\right]d\omega=\frac{1}{\pi}\int_0^{\infty}\left[A(\omega)\cos(\omega t+\Phi(\omega))\right]d\omega=$$
$$=a(t)\cos\varphi(t)=-H\left[\hat{\Psi}(t)\right],$$
$$\hat{\Psi}(t)=H\left[\Psi(t)\right]=\frac{1}{\pi}\int_0^{\infty}\left[U_c(\omega)\sin\omega t+U_s(\omega)\cos\omega t\right]d\omega=\frac{1}{\pi}\int_0^{\infty}\left[A(\omega)\sin(\omega t+\Phi(\omega))\right]d\omega= \quad (4.0.1)$$
$$=a(t)\sin\varphi(t)=\Psi(t)*\frac{1}{\pi t}=\frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{\Psi(\tau)}{t-\tau}d\tau=\frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{\Psi(t-\tau)}{\tau}d\tau\ ,\ H=H_t\,(=)\text{ Hilbert transform}.$$
$$\int_{-\infty}^{+\infty}\Psi(t)\cdot\hat{\Psi}(t)dt=0,\ \text{Mod}\left[H\right]=1,\ \text{Arg}\left[H\right]=-\pi/2,\ H\left[\text{const}\right]\cong 0.$$

where $\mathbf{a(t)}$ is the instantaneous signal amplitude or signal envelope, $\varphi(t)$ is the signal phase function, and $\omega(t)=d\varphi(t)/dt=2\pi f(t)$ is the instantaneous signal frequency (all in a time domain). Analogical functions in the signal frequency domain are $A(\omega)$, as the signal amplitude, and $\Phi(\omega)$ as the signal phase function. The Hilbert transform $\mathbf{H}$ is a kind of filter, which shifts phases of all elementary (simple harmonic) components of its input ($\Psi(t)$) by $-\pi/2$ (see the picture below). The same time-domain function $\Psi(t)$, transformed into Complex, time-domain Analytic Signal, or complex Phasor function $\overline{\Psi}(t)$ has the following form,

$$\overline{\Psi}(t)=\Psi(t)+j\hat{\Psi}(t)=(1+jH)\,\Psi(t)=\frac{1}{\pi}\int_0^{\infty}U(\omega)e^{j\omega t}\,d\omega=\frac{1}{\pi}\int_0^{\infty}A(\omega)e^{j(\omega t+\Phi(\omega))}\,d\omega,$$
$$=a(t)e^{j\varphi(t)},\ \Psi(t)=\frac{1}{2}\left[\overline{\Psi}(t)+\overline{\Psi}^*(t)\right],\ \overline{\Psi}^*(t)=\Psi(t)-j\hat{\Psi}(t)\ ,\ j^2=-1,$$

$$a(t)=\sqrt{\Psi^2(t)+\hat{\Psi}^2(t)}=e^{R_e\left[\ln\overline{\Psi}(t)\right]},\ \dot{a}(t)=\frac{\Psi(t)\dot{\Psi}(t)-\hat{\Psi}(t)\dot{\hat{\Psi}}(t)}{a(t)}=a(t)\text{Re}\left[\frac{\dot{\overline{\Psi}}(t)}{\overline{\Psi}(t)}\right],$$
$$\varphi(t)=\text{arctg}\frac{\hat{\Psi}(t)}{\Psi(t)}=\text{Im}\left[\ln\overline{\Psi}(t)\right]=\text{instantaneous phase}, \quad (4.0.2)$$
$$\omega(t)=\frac{\partial\varphi(t)}{\partial t}=\dot{\varphi}(t)=\frac{\Psi(t)\dot{\hat{\Psi}}(t)-\dot{\Psi}(t)\hat{\Psi}(t)}{a^2(t)}=\text{Im}\left[\frac{\dot{\overline{\Psi}}(t)}{\overline{\Psi}(t)}\right]=2\pi f(t)=\text{instantaneous angular frequency}.$$

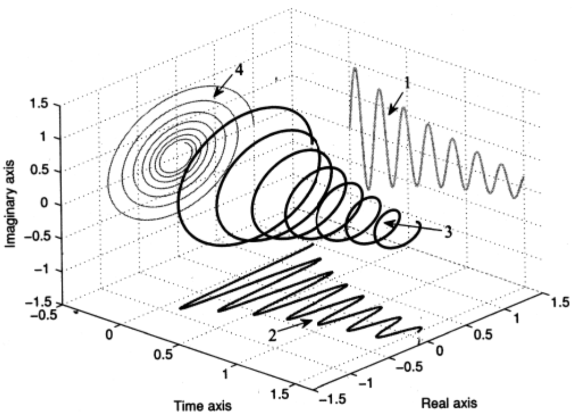


Figure 2.3 The HT projection (1), the real signal (2), the analytic signal (3), and the phasor in complex plain (4) (Feldman, ©2011 by Elsevier)

The frequency-domain, wave functions of an Analytic Signal are,

$$\begin{gathered}
A(\omega)=U(\omega)e^{-j\Phi(\omega)}, \Phi(\omega)=-\arctan\left[U_s(\omega)/U_c(\omega)\right], j=\sqrt{-1} \\
\bar{U}(\omega)=U_c(\omega)-jU_s(\omega)=\int_{-\infty}^{+\infty}\Psi(t)e^{-j\omega t}dt=A(\omega)e^{j\Phi(\omega)}=A(\omega)\cos\Phi(\omega)+jA(\omega)\sin\Phi(\omega), \\
\begin{pmatrix} U_c(\omega)=A(\omega)\cos\Phi(\omega)=-H\left[U_s(\omega)\right], H=H_\omega(=) \text{ Hilbert transform}, \\ U_s(\omega)=H\left[U_c(\omega)\right]=A(\omega)\sin\Phi(\omega)=U_c(\omega)*\frac{1}{\pi\omega}=\frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{U_c(\omega)}{\omega-\Omega}d\Omega=\frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{U_c(\omega-\Omega)}{\Omega}d\Omega. \end{pmatrix}
\end{gathered} \tag{4.0.3}$$

In the analysis of waveform velocities in this book (which follows), we will use all here mentioned, real and complex time-domain and frequency-domain Analytic Signal wave functions, (4.0.1) - (4.0.3).

For completer understanding of Analytic Signal function and its advantages in comparison to all other waveform presentations and analyses, it would be useful to go to already published Analytic Signal and Hilbert Transform related literature listed under [57], [58], [59] and [79] (to focus analyses of this book only to wave functions, wave velocities and closely related topics).

As we can see, Analytic Signal (established by Dennis Gabor, [57]) is a kind of function-shape modeling where we are "casting" an arbitrary-shaped and energy-finite function into a "phasor molds" of sinus and cosines functions. Such (generalized amplitude and phase) functions exist in the real and complex, time, and frequency domains. This offers significant advantages for arbitrary wave-functions analyses and characterizations, in the same way as known for simple harmonic wave functions, and we can easily make analogical comparisons and associations between amplitude, phase and frequency of a simple-harmonic, sinusoidal forms, and generalized amplitude, phase, and frequency of Analytic Signal forms of arbitrary shaped functions. The mathematical power of such modeling is being reinforced by the fact that all waveforms known in physics and nature are composed of elementary sinusoidal functions. The similar (but much more simplified) concept is already very successfully applied as complex ***phasors*** *notation and mathematical operations concerning complex functions of (simple harmonic) currents, voltages, and impedances in electro-technique and electronics, by replacing and enormously facilitating mathematical operations with initially trigonometric functions, replacing them with very simple algebraic, ordinary complex-numbers operations (see more in the same chapter under "4.0.11. Generalized Wave Functions and Unified Field Theory").* ***In this book, Analytic Signal will be considered as the optimal and most natural modeling framework for de Broglie or all matter waves, where real analytic signal component*** $\Psi(t)$ ***describes the state of motion we know, measured, and qualified in our real space-time domain. Associated, phase shifted signal*** $\hat{\Psi}(t)$ ***creates an imaginary component of a relevant analytic signal*** $\bar{\Psi}(t)$***, but both*** $\Psi(t)$ ***and*** $\hat{\Psi}(t)$ ***should be considered as existing, natural and relevant signals (only being time and space, phase-shifted and mutually orthogonal).*** *Most probably that (after applying necessary mathematical processing and dimensional arrangements) we will be able to prove that wave functions of any mutually-coupled electric and magnetic field vectors (like photons or electromagnetic waves) are behaving as a united Analytic signal of an electromagnetic field couple of* $\Psi(t)$ *and* $\hat{\Psi}(t)$*, or effectively being a couple of relevant electric and magnetic field wavefunctions. Here, we should avoid oversimplified speculations with nonrealistic magnetic monopolies.* ***The Universe or Nature is on some realistic and tangible way always synchronously creating both*** $\Psi(t)$ ***and*** $\hat{\Psi}(t)$ ***matter wave functions, and here are the roots, meaning and mathematical modelling explanations of matter waves, particle-wave duality, matter and antimatter states, entanglement and matter synchronization effects known in quantum physics. Since Analytic Signal concept (meaning operating with*** $\Psi(t)$ ***and*** $\hat{\Psi}(t)$ ***functions) is mathematically well defined and being the part of a significant and stable mathematical body of signal analysis, also well and smoothly connected to mathematical waves processing in Physics, we should consider that both*** $\Psi(t)$ ***and*** $\hat{\Psi}(t)$ ***in every case of***

matter-waves are something that synchronously exists, and should be detectable in our Universe. The best example or candidate for supporting such a situation should be the structure of electromagnetic fields and photons. If we know, see or detect only one wave component of certain natural motion and we consider it as being $\Psi(t)$, **we should be sure that sooner or later, corresponding and complementary** $\hat{\Psi}(t)$ ***will be discovered within the same space (what belongs to a deeper meaning of the matter-wave duality).***

The same situation could be particularly interesting in all kinds of transient, switching, pulsing, and wave creating situations concerning electromagnetic currents, voltages, and waves. In mentioned situations, we usually analyze, as real and most relevant signals (meaning currents, voltages, and electric and magnetic fields), conveniently formulated $\Psi(t)$ ***functions, but Nature, or relevant Physics, is immediately creating both*** $\Psi(t)$***, and*** $\hat{\Psi}(t)$ ***(see the works and publications of Rudjer Boskovic, Nikola Tesla, Eric P. Dollard, and Konstantin Meyl in [6], [97], [98] and [99]). Something similar or equivalent should exist as a natural coupling between linear and spinning or angular motions. In addition, we should not forget that the general solution of Classical, second order differential wave equations, always has (at least) two wavefunctions (or two wave groups that can be considered being two complex Analytic Signals). Mentioned wave groups are synchronously propagating in mutually opposed directions, naturally being in mutual synchronization and entanglement relation. This is valid both for*** $\Psi(t)$ **and** $\hat{\Psi}(t)$***), and this should also be valid (or always present) in the real world of all matter-waves situations (being also an explanation or essence of "action-equal-to reaction" induction laws). Anyway, in some cases we cannot easily visualize or explain such waves propagations (since this sometimes looks like one wave component is propagating along positive temporal and spatial direction, and the other is advancing in the negative time and space direction, or in any other imaginable and mathematically possible combination of spatial and temporal directions;*** *- see more in the chapter 4.3-MATTER WAVES AND WAVE EQUATIONS) and in Chapter 10. Here we could draw the philosophic conclusion such as: "In our Universe, past, present and future are mutually balanced, coupled and present synchronously in every moment of our existence".*

The amount of ***Entanglement and quality of resonant couplings*** *between two matter-wave groups, functions, or Analytic Signals, we could express with Coherence factors (see more about coherence factors at the end of this chapter around definitions (4.0.83), (4.0.87) and (4.0.109)). Between two Complex Analytic signals, or wave functions,* $\overline{\Psi}_1(r,t)$ and $\overline{\Psi}_2(r,t)$ *it is possible to find the measure of their mutual spatial-temporal and spectral coherence (or non-coherence), using the following coherence factors:*

$$\overline{K}_{r,t} = \frac{\iiint\limits_{(-\infty,+\infty)} \overline{\Psi}_1(r,t)\cdot\overline{\Psi}_2(r,t)\cdot dr\cdot dt}{\iiint\limits_{(-\infty,+\infty)} \left|\overline{\Psi}_1(r,t)\right|\cdot\left|\overline{\Psi}_2(r,t)\right|\cdot dr\cdot dt},\ \overline{K}_{k,\omega} = \frac{\iiint\limits_{(-\infty,+\infty)} \overline{U}_1(k,\omega)\cdot\overline{U}_2(k,\omega)\cdot dk\cdot d\omega}{\iiint\limits_{(-\infty,+\infty)} \left|\overline{U}_1(k,\omega)\right|\cdot\left|\overline{U}_2(k,\omega)\right|\cdot dk\cdot d\omega}$$

See more about Complex Analytic Signal here (citation):
*"**Analytic signal**, From Wikipedia, the free encyclopedia: https://en.wikipedia.org/wiki/Analytic_signal*
*In mathematics and signal processing, an **analytic signal** is a complex-valued function that has no negative frequency components.[1] The real and imaginary parts of an analytic signal are real-valued functions related to each other by the Hilbert transform.*
*The **analytic representation** of a real-valued function is an analytic signal, comprising the original function and its Hilbert transform. This representation facilitates many mathematical manipulations. The basic idea is that the negative frequency components of the Fourier transform (or spectrum) of a real-valued function are superfluous, due to the Hermitian symmetry of such a spectrum. These negative frequency components can be discarded with no loss of information, provided one is willing to deal with a complex-valued function instead. That makes certain attributes of the function more accessible and facilitates the derivation of modulation and demodulation techniques, such as single sideband.*
If the manipulated function has no negative frequency components (that is, it is still analytic), the conversion from complex back to real is just a matter of discarding the imaginary part. The analytic representation is a

generalization of the phasor concept:[2] while the phasor is restricted to time-invariant amplitude, phase, and frequency, the analytic signal allows for time-variable parameters".

4.0.1. Signal Energy Content

Let us first express the energy $\tilde{E}$ carried by an analytic signal waveform. Generally applicable Parseval's theorem is connecting time and frequency domains of signal's or wavefunctions, and such expression has the meaning of signal's (or wave) motional energy in cases when $\Psi^2(t) = P(t) = d\tilde{E}/dt$ is modeled to present an instantaneous signal power $P(t)$, as follows,

$$\tilde{E} = \int_{-\infty}^{+\infty}\Psi^2(t)dt = \int_{-\infty}^{+\infty}\hat{\Psi}^2(t)dt = \frac{1}{2}\int_{-\infty}^{+\infty}\left|\overline{\Psi}(t)\right|^2 dt = \frac{1}{2}\int_{-\infty}^{+\infty}a^2(t)dt = \int_{-\infty}^{+\infty}\left[\frac{a(t)}{\sqrt{2}}\right]^2 dt =$$
$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty}\left|\overline{U}(\omega)\right|^2 d\omega = \int_{-\infty}^{+\infty}\left|\frac{\overline{U}(\omega)}{\sqrt{2\pi}}\right|^2 d\omega = \frac{1}{\pi}\int_0^{\infty}\left[A(\omega)\right]^2 d\omega = \int_0^{\infty}\left[\frac{A(\omega)}{\sqrt{\pi}}\right]^2 d\omega =$$
$$= \int_{-\infty}^{+\infty}P(t)dt \ (=\tilde{m}c^2=)\ [J](=)[Ws],\ \int_{-\infty}^{+\infty}\Psi(t)\cdot\hat{\Psi}(t)dt = 0 \Rightarrow$$

$$\rho(\tilde{E})_t = \frac{d\tilde{E}}{dt} = \Psi^2(t) = P(t)\ (\Leftrightarrow)\left[\frac{a(t)}{\sqrt{2}}\right]^2 (=)\left[W\right],\ t\in(-\infty,+\infty),$$
$$\rho(\tilde{E})_\omega = \frac{d\tilde{E}}{d\omega} = \frac{\Psi^2(t)}{d\omega/dt}(\Leftrightarrow)\left[\frac{A(\omega)}{\sqrt{\pi}}\right]^2 (=)\left[Js = Ws^2\right],\ \omega\in(0,+\infty),$$
$$\frac{\tilde{E}}{\omega(t)} = \frac{a^2(t)\int_{-\infty}^{+\infty}a^2(t)dt}{2\left[\Psi(t)\dot{\hat{\Psi}}(t) - \dot{\Psi}(t)\hat{\Psi}(t)\right]} = \frac{\left[\Psi^2(t)+\hat{\Psi}^2(t)\right]\int_{-\infty}^{+\infty}\left[\Psi^2(t)+\hat{\Psi}^2(t)\right]dt}{2\left[\Psi(t)\dot{\hat{\Psi}}(t) - \dot{\Psi}(t)\hat{\Psi}(t)\right]}(=)\left[Js = Ws^2\right],$$
$$\text{if}\ \left\{\frac{\tilde{E}}{\omega(t)} = \frac{h}{2\pi} \Rightarrow \tilde{E} = h\overline{f}\right\} \Leftrightarrow \text{narrow-band wave-packet energy.}$$

From the expressions for signal energy (4.0.4), it is evident that a total signal energy content is captured and propagating only by the signal amplitude (or envelope) function $a(t)$ or $A(\omega)$, both in time and frequency domain,

$$\tilde{E} = \int_{-\infty}^{+\infty}\left[\frac{a(t)}{\sqrt{2}}\right]^2 dt = \int_0^{\infty}\left[\frac{A(\omega)}{\sqrt{\pi}}\right]^2 d\omega. \quad (4.0.5)$$

Also, it is evident that signal phase functions, $\varphi(t)$ and $\Phi(\omega)$ do not directly participate in a total signal energy content (meaning do not carry signal energy at all). This is a significant fact to notice to understand the meaning of signal velocities (and this is universally valid for all waves and signals in micro and macro physics). We can also say that the wave *velocity of the signal amplitude (or signal envelope) is the velocity of the total signal energy propagation*. This velocity is known as a group velocity (in particle-wave duality concepts, group velocity corresponds to particle velocity, and a wave packet is the wave-equivalent of a motional particle; -such situations will be analyzed in detail later). The speed of the signal-carrier or phase function is known as a phase velocity, and it should present the signal carrier velocity or velocity of primary simple-harmonic signal elements (which are creating a wave packet). Here, the signal carrier functions in a time-domain are $\cos\varphi(t)$, or $e^{j\varphi(t)}$, and in a frequency-domain $\cos\Phi(\omega)$ or $e^{j\Phi(\omega)}$, and we can use both of them (equally and analogically), depending on preferences and mathematical advantages when operating with trigonometric or complex functions.

Experimentally provable and theoretically well-supported knowledge is that for Physics-related (analytic) signals or wave packets like photons, electrons and many other energy-quanta and elementary-particles, the following expressions, properties, and parameters are well known, applicable and mutually very closely related and coupled, such as:

a) *Important property of Analytic Signals (between their corresponding time and frequency domains) is that all phase functions (*$\cos\varphi(t)$, $\text{sinc}\varphi(t)$, $e^{j\varphi(t)}$, $\cos\Phi(\omega)$, $e^{j\Phi(\omega)}$*), regardless in which domain formulated, have the same phase velocity* $u = \lambda f$*. Also, all amplitude functions (*$\mathbf{a(t)}$, $A(\omega)$*), regardless in which domain formulated, have the same group velocity v. Also, there is a well-known (universally applicable in physics) equation connecting group and phase velocity of wave functions (*$v = u - \lambda du/d\lambda = -\lambda^2 df/d\lambda$*). Analytic Signal modeling presents the best native mathematical environment for conceptualizing de Broglie matter-waves and wave-particle duality. See more in chapters 4.1, 4.3 and 10.*

b) *M. Planck-Einstein, wave-packet energy, valid for frequency narrow-band or band-limited waves and wave groups (like photons) is,*

$$\tilde{E} = hf \ (=) \left[\text{Kinetic energy} = E_k = (\gamma - 1)mc^2\right],$$

c) *Einstein-Minkowsky's energy-momentum, 4-vector relations,*

$$\overline{P}_4 = \overline{P}(p, \frac{E}{c}),\ \overline{P}^2 = \vec{p}^2 - \frac{E^2}{c^2} = -\frac{E_0^2}{c^2},\ \vec{p} = \gamma m\vec{v},\ E_k = (\gamma - 1)mc^2,$$
$$E_0 = mc^2,\ E = \gamma E_0 = \gamma mc^2 = E_0 + E_k \Rightarrow \vec{p}^2 c^2 + E_0^2 = E^2$$

d) *de Broglie matter wave wavelength* $\lambda = h/p = u/f$ *,*

e) *In fact, all of them, a), b), c) and d), when used to describe the same momentum-energy matter-wave state in a joint spatial-temporal universe, are entirely mathematically compatible, complementary, and analytically united by the equation that is connecting relevant group and phase velocity* $v = u - \lambda du/d\lambda = -\lambda^2 df/d\lambda$*, being the core of Particle-Wave Duality (see more of supporting elements later, in the same chapter, around equations (4.0.73) - (4.0.76), and in Chapter 10.).*

f) *Also, we will realize that there is a kind of associated spinning, as a helix-path field around moving particles, which is defining (de Broglie) matter-wave wavelength (and all of that will also be analyzed in detail, later). The best mathematical environment to exploit all here-mentioned signal properties (and to unite matter-waves and particle-wave duality foundations originating from L. de Broglie, M. Planck, A. Einstein, W. Heisenberg, E. Schrödinger...) is the*

Analytic Signal concept. De Broglie matter waves have the frequency and phase described by the corresponding Analytic Signal model, as defined in (4.0.2). From an Analytic Signal frequency and phase, we can determine matter-wave wavelength, $\lambda = h/p = u/f$, *and group and phase velocity relations* $v = u - \lambda du/d\lambda = -\lambda^2 df/d\lambda$ *(see more in Chapters 4.1 and 10.). Relevant Analytic Signal wave functions that naturally describe de Broglie matter waves are power and motional energy-related functions, including corresponding field and force functions. Later, (if desired) by creating normalized, non-dimensional wave functions, we will be able to reproduce Quantum Theory approach to the same problematic. See more in chapters 4.1, 4.3 and 10.*

When discussing the particle-wave duality concept, which represents a moving particle with a certain mass as an equivalent wave packet, a key challenge arises in understanding the difference between the group velocity and phase velocity of the wave packet. For non-relativistic particle motion (v << c), the phase velocity is approximately half of the group velocity. Simultaneously, the particle, which is represented in this way, is considered stable and localized in both time and space, without energy residuals or trailing waves.

The best current understanding of the distinction between a particle and its equivalent matter wave group is that the wave packet's amplitude function carries the entire energy of the wave packet or particle and propagates at the group or particle velocity. This concept is universally valid, as demonstrated by equations (4.0.4) and (4.0.5). The phase component of the wave packet, which lags the particle in space and time with a phase velocity u), does not carry energy and thus does not pose a significant issue for the mathematical modeling of particle-wave duality.

The development of group and phase velocities of a wave packet, along with their interrelations, will be explored in detail later. However, given the importance of this topic for understanding particle-wave duality, it is crucial to emphasize its significance in advance (refer to the development of wave velocities starting from equations (4.0.6) to (4.0.46)).

At the conclusion of this book, we will revisit the structure and properties of wavefunctions in physics, with a detailed examination provided in Chapter 10. The Analytic Signal model is particularly effective for representing arbitrary waveforms, offering a unique opportunity to extract instantaneous signal amplitude and phase functions, as well as instantaneous signal frequency and other signal characteristics in both time and frequency domains. The compelling idea here is to apply the Analytic Signal model to all wave motions, oscillations, wave packets, and signals relevant in physics. This modeling approach is rich, natural, selective, informative, and productive, making it a strong candidate for forming the basis of Quantum Theory wave functions as well.

Building on this foundation, all other wave equations known in Mathematical Physics can be logically and straightforwardly developed using clear, step-by-step mathematical methods—without resorting to ad hoc additions, patchwork solutions, or arbitrary operators and postulates. This approach is demonstrated in Chapter 4.3, *where nearly all wave equations and associated operators in current Quantum Theory are derived from an Analytic Signal wavefunction.*

Given the extensive literature on Hilbert transform and Analytic Signal modeling, we will summarize the most important properties and expressions for representing arbitrary and energy-finite waveforms as Analytic Signals in Table T.4.0.

T.4.0.1

The parallelism between Time and Frequency Domains	Analytic Signal	
	Time Domain	Frequency Domain
Complex Signal	$\overline{\Psi}(t) = a(t)e^{j\varphi(t)}$ $= \Psi(t) + j\hat{\Psi}(t)$ $= \frac{1}{\pi}\int_0^{\infty} U(\omega)e^{j\omega t}d\omega$ $= \frac{1}{\pi}\int_0^{\infty} \overline{U}(\omega)e^{-j\omega t}d\omega$ $= \frac{1}{\pi}\int_0^{\infty} A(\omega)e^{j(\omega t+\Phi(\omega))}d\omega$	$\overline{U}(\omega) = A(\omega)e^{j\Phi(\omega)}$ $= U_c(\omega) - jU_s(\omega)$ $= \int_{-\infty}^{+\infty} \Psi(t)e^{-j\omega t}dt$ $= \int_{-\infty}^{+\infty} \overline{\Psi}(t)e^{j\omega t}dt$ $= \int_{-\infty}^{+\infty} a(t)e^{-j(\omega t+\varphi(t))}dt$
Real and imaginary signal components	$\Psi(t) = a(t)\cos\varphi(t) =$ $= -H\left[\hat{\Psi}(t)\right]$, $\hat{\Psi}(t) = a(t)\sin\varphi(t) =$ $= H\left[\Psi(t)\right]$	$U_c(\omega) = A(\omega)\cos\Phi(\omega) =$ $= -H\left[U_s(\omega)\right]$, $U_s(\omega) = A(\omega)\sin\Phi(\omega) =$ $= H\left[U_c(\omega)\right]$,
Signal Amplitude	$a(t) = \sqrt{\Psi^2(t) + \hat{\Psi}^2(t)}$	$\mathbf{A(\omega) = \sqrt{U_c^{\,2}(\omega) + U_s^{\,2}(\omega)}}$
Instant Phase	$\varphi(t) = \mathrm{arctg}\frac{\hat{\Psi}(t)}{\Psi(t)}$	$\Phi(\omega) = \mathbf{arctan}\frac{\mathbf{U_s(\omega)}}{\mathbf{U_c(\omega)}}$
Instant Frequency	$\omega(\mathbf{t}) = \frac{\partial\varphi(\mathbf{t})}{\partial \mathbf{t}}$	$\tau(\omega) = \frac{\partial\Phi(\omega)}{\partial\omega}$
Signal Energy	$\tilde{E} = \int_{-\infty}^{+\infty} \left\lvert\overline{\Psi}(t)\right\rvert^2 dt =$ $= \int_{-\infty}^{+\infty} \Psi^2(t)dt =$ $= \int_{-\infty}^{+\infty} \hat{\Psi}^2(t)dt =$ $= \int_{-\infty}^{+\infty} \left[\frac{a(t)}{\sqrt{2}}\right]^2 dt$ $\int_{-\infty}^{+\infty} \Psi(t)\cdot\hat{\Psi}(t)dt = 0$	$\tilde{E} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \left\lvert\overline{U}(\omega)\right\rvert^2 d\omega =$ $= \frac{1}{2\pi}\int_{-\infty}^{+\infty} U_c^2(\omega)d\omega =$ $= \frac{1}{2\pi}\int_{-\infty}^{+\infty} U_s^2(\omega)d\omega =$ $= \int_0^{\infty} \left[\frac{A(\omega)}{\sqrt{\pi}}\right]^2 d\omega$ $\int_{-\infty}^{+\infty} U_c(\omega)\cdot U_s(\omega)d\omega = 0$

Central Frequency	$\omega_c = \frac{\int_{[t]} \omega(t)\cdot a^2(t)dt}{\int_{[t]} a^2(t)dt} = 2\pi f_c$	$\omega_c = \frac{\int_{[\omega]} \omega\cdot[A(\omega)]^2 d\omega}{\int_{[\omega]} [A(\omega)]^2 d\omega} = 2\pi f_c$
“Central Time Point”	$t_c = \frac{\int_{[t]} t\cdot a^2(t)dt}{\int_{[t]} a^2(t)dt}$	$t_c = \frac{\int_{[\omega]} \tau(\omega)\cdot[A(\omega)]^2 d\omega}{\int_{[\omega]} [A(\omega)]^2 d\omega}$
Standard Deviation	$\sigma_\omega^2 = \frac{1}{\Delta t}\int_{[t]} \lvert\omega(t)-\omega_c\rvert^2 dt$	$\sigma_t^2 = \frac{1}{\Delta\omega}\int_{[\omega]} \lvert\tau(\omega)-t_c\rvert^2 d\omega$
Uncertainty Relations generally applicable to micro and macro world of Physics	$\pi \le \sigma_\omega\cdot\sigma_t < \lvert\omega(t)\cdot\tau(\omega)\rvert \cong \omega_c\cdot t_c \cong \Omega\cdot T \le \frac{\pi}{2\cdot\delta t\cdot\delta f} = \frac{\pi^2}{\delta t\cdot\delta\omega}$ (!?) $0 < \delta t\cdot\delta\omega = 2\pi\cdot\delta t\cdot\delta f < \pi < \Omega\cdot T \le \frac{\pi}{2\cdot\delta t\cdot\delta f} = \frac{\pi^2}{\delta t\cdot\delta\omega}$	
	$\delta t\cdot\delta f \le 2\cdot\delta t\cdot\delta f\cdot T\cdot F \le 1/2,$ $\delta t \le \frac{1}{2F},\ \delta f \le \frac{1}{2T},\ F\cdot T > \frac{1}{2},$ $\delta t\cdot\delta f < \frac{1}{2}, \delta t\cdot\delta\omega < \pi,\ \Omega\cdot T > \pi$ $0 < \delta t\cdot\delta f < \frac{1}{2} < F\cdot T \le \frac{1}{4\cdot\delta t\cdot\delta f}.$ Shannon-Kotelnikov optimal signal-sampling relations in time and frequency domains: δt = maximal time sampling interval, δf = maximal frequency sampling interval $\Omega = 2\pi F$ and $\mathbf{T}$ are the total, or absolute signal-lengths in its frequency and time domains; ($\omega = 2\pi f, \delta\omega = 2\pi\delta f$).	

Other forms of Uncertainty Relations (see later in this chapter for the complete development of such relations):

$$0 < \delta t\cdot\delta\omega = 2\pi\cdot\delta t\cdot\delta f < \pi \le \sigma_\omega\cdot\sigma_t < \Omega\cdot T \le \frac{\pi}{2\cdot\delta t\cdot\delta f} = \frac{\pi^2}{\delta t\cdot\delta\omega}\ ,\ \lvert\omega(t)\cdot\tau(\omega)\rvert \cong \omega_c\cdot t_c \cong \pi$$

$$0 < \delta t\cdot\delta f < \frac{1}{2} \le \sigma_f\cdot\sigma_t < F\cdot T \le \frac{1}{4\cdot\delta t\cdot\delta f}\ ,\ \lvert f(t)\cdot\tau(\omega)\rvert \cong f_c\cdot t_c \cong \frac{1}{2}$$

$$0 < \delta t\cdot\delta f = \delta x\cdot\delta f_x < \frac{1}{2} \le \sigma_t\cdot\sigma_f = \sigma_x\cdot\sigma_{f\text{-}x} < F\cdot T = F_x\cdot L \le \frac{1}{4\cdot\delta t\cdot\delta f} = \frac{1}{4\cdot\delta x\cdot\delta f_x}\ ,$$

$$0 < \delta t\cdot\delta\tilde{E} = \delta x\cdot\delta p < \frac{H}{2} \le 2\pi\sigma_t\cdot\sigma_{\tilde{E}} = \sigma_x\cdot\sigma_p < \tilde{E}\cdot T = P\cdot L \le \frac{H}{4\cdot\delta t\cdot\delta f} = \frac{H}{4\cdot\delta x\cdot\delta f_x}\ ,$$

H (=) Either equal to Planck constant h for micro world,
or analog H >> h constant for planetary and astronomic systems
("H" is not presenting Hilbert transform, here).

4.0.2. Resume of Different Analytic Signal Representations

In many practical cases related to physics (regarding non-dispersive, band-limited, finite-energy signals), Analytic Signal modeling can present an arbitrary-shaped signal to look like a simple-harmonic signal, which has its frequency and phase functions, for instance,

$$\Psi(t) = a(t)\cos\varphi(t) = a(t)\cos\left[\omega_0 t + \varphi(t)_{residual}\right],\ \varphi(t) = \omega_0 t + \varphi(t)_{residual}\quad ,\quad \omega_0 = const.\quad .$$

We could also exercise with more complex situations regarding signal-phase functions of non-linear and dispersive signals such as,

$$\Psi(t) = a(t)\cos\varphi(t) = a(t)\cos\left[\omega_0 t + \frac{\omega_1}{T_1}t^2 + \frac{\omega_2}{T_2^2}t^3 + ...\varphi(t)_{residual}\right]$$

$$\varphi(t) = \omega_0 t + \frac{\omega_1}{T_1}t^2 + \frac{\omega_2}{T_2^2}t^3 + ...\varphi(t)_{residual}\quad ,\quad \omega_{0,1,2...} = const.\quad T_{1,2...} = Const.\quad .$$

Analytic Signals can be presented in different ways, giving the chance to reveal the "*internal structure of waveforms*" from different points of view. Here are summarized several of such possibilities (mainly as superposition and/or multiplication of elementary simple-harmonic signals; -see below).

$$\overline{\Psi}(t) = \Psi(t) + I\hat{\Psi}(t) = a_0(t)e^{I\varphi_0(t)} = a_0(t)\left[\cos\varphi_0(t) + I\sin\varphi_0(t)\right] =$$
$$= a_0(t)e^{\sum_{(k)} i_k\varphi_k(t)} = \sum_{(k)} a_k(t)e^{i_k\varphi_k(t)} = \sum_{(k)} \overline{\Psi}_k(t)\ ,$$

$$\mathbf{\overline{H}\left[\Psi(t)\right] = \overline{\Psi}(t) = \Psi(t) + I\cdot\hat{\Psi}(t) = a(t)\cdot e^{I\cdot\varphi(t)}}$$
$$\mathbf{\overline{H} = 1 + I\cdot H,\ I^2 = -1}$$

Analytic Signal presented as SUPERPOSITION.	Analytic signal presented as MULTIPLICATION
$\overline{\Psi}(t) = \sum_{(k)}\overline{\Psi}_k(t) = \sum_{(k)}\Psi_k(t) + I\sum_{(k)}\hat{\Psi}_k(t)$	$\overline{\Psi}(t) = \Psi_n(t)\prod_{(i=0)}^{n-1}\cos\varphi_i(t) + I\Psi_n(t)\prod_{(i=0)}^{n-1}\sin\varphi_i(t) = = \Psi_n(t)\prod_{(i=0)}^{n-1}e^{I\cdot\varphi_i(t)}$

4.0.2.1. Relations between Additive and Multiplicative Elements

$$\Psi(t)=\sum_{(k)}\Psi_k(t)=\Psi_n(t)\prod_{(i=0)}^{n-1}\cos\varphi_i(t)=a_0(t)\cos\varphi_0(t)=a(t)\cos\varphi(t)=-H\left[\hat{\Psi}(t)\right]$$

$$\hat{\Psi}(t)=\sum_{(k)}\hat{\Psi}_k(t)=\Psi_n(t)\prod_{(i=0)}^{n-1}\sin\varphi_i(t)=a_0(t)\sin\varphi_0(t)=a(t)\sin\varphi(t)=H\left[\Psi(t)\right]$$

$$\overline{\Psi}_k(t)=\Psi_k(t)+i_k\hat{\Psi}_k(t)=a_k(t)e^{i_k\varphi_k(t)}\ ,\Psi_k(t)=-H\left[\hat{\Psi}_k(t)\right],\ \hat{\Psi}_k(t)=H\left[\Psi_k(t)\right]$$

$$\overline{\Psi}(t)=\Psi(t)+I\hat{\Psi}(t)=\frac{a_n(t)}{2^{n+1}}\left\{\prod_{k=0}^{n}(e^{I\varphi_k(t)}+e^{-I\varphi_k(t)})+\frac{1}{(i)^n}\prod_{k=0}^{n}(e^{I\varphi_k(t)}-e^{-I\varphi_k(t)})\right\}$$

See more about energy related structural hierarchy of matter waves when presented as more elementary wave functions multiplication in Chapter 6., under "6.1. Hypercomplex, In-depth Analysis of the Wave Function").

Brainstorming proposal: Let us imagine that we could present an "atom-field wave function, or force" as the following multiplicative and additive analytic signal:

$$\overline{\Psi}(t)=\Psi_n(t)\prod_{(i=0)}^{n-1}\cos\varphi_i(t)+I\Psi_n(t)\prod_{(i=0)}^{n-1}\sin\varphi_i(t)=\Psi_n(t)\prod_{(i=0)}^{n-1}e^{I\cdot\varphi_i(t)}=$$

$$=\Psi_{atom}(t)\cdot\left[\cos\varphi_{electrons}(t)\right]\cdot\left[\cos\varphi_{protons}(t)\right]\cdot\left[\cos\varphi_{neutrons}(t)\right]+\quad\ldots$$

$$+I\Psi_{atom}(t)\cdot\left[\sin\varphi_{electrons}(t)\right]\cdot\left[\sin\varphi_{protons}(t)\right]\cdot\left[\sin\varphi_{neutrons}(t)\right]=$$

$$=A_{electrons}(t)\cdot e^{I\varphi_{electrons}}+A_{protons}(t)\cdot e^{I\varphi_{protons}}+A_{neutrons}(t)\cdot e^{I\varphi_{neutrons}}$$

This could open new opportunities to model and analyze interatomic and nuclear forces, and interactions between all involved participants, internally and externally.

4.0.2.2. Signal Amplitude or Envelope

$$a(t)=a_0(t)=\left|\overline{\Psi}(t)\right|=\sqrt{\Psi^2(t)+\hat{\Psi}^2(t)}=a_n(t)\prod_{(i=1)}^{n}\cos\varphi_i(t)=\Psi_{n+1}(t)\prod_{(i=1)}^{n}\cos\varphi_i(t)$$

$$a_k(t)=\Psi_{k+1}(t)=\left|\overline{\Psi}_k\right|=\sqrt{\Psi_k^2(t)+\hat{\Psi}_k^2(t)}=a_{k+1}(t)\cos\varphi_{k+1}(t)=a_n(t)\prod_{(i=k+1)}^{n}\cos\varphi_i(t),\ k<n$$

$$a_{n-1}(t)=\Psi_n(t)=\left|\overline{\Psi}_{n-1}(t)\right|=\sqrt{\Psi_{n-1}^2(t)+\hat{\Psi}_{n-1}^2(t)}=a_n(t)\cos\varphi_n(t)=\frac{\Psi_0(t)}{\prod_{(i=0)}^{n-1}\cos\varphi_i(t)}$$

4.0.2.3. Signal Instantaneous Phase

$$\varphi_0(t)=\varphi(t)=\operatorname{arctg}\frac{\hat{\Psi}(t)}{\Psi(t)}=\sqrt{\sum_{(k)}\varphi_k^2(t)},\ \varphi_k(t)=\operatorname{arctg}\frac{\hat{\Psi}_k(t)}{\Psi_k(t)}$$

$$I^2=i_1^2=i_2^2=...=i_n^2=-1\ ,\ i_ji_k=0,\forall j\neq k\ \text{(hypercomplex imaginary units)}$$

$$I\varphi_0(t)=I\varphi(t)=i_1\varphi_1(t)+i_2\varphi_2(t)+...+i_n\varphi_n(t)=\sum_{k=1}^{n}i_k\varphi_k(t)$$

$$e^{i_k\varphi_k(t)}=\cos\varphi_k(t)+i_k\sin\varphi_k(t),\ \ \varphi_0^2(t)=\sum_{k=1}^{n}\varphi_k^2(t)$$

4.0.2.4. Signal Instantaneous Frequency

$$\omega_i(t) = 2\pi f_i(t) = \frac{\partial \varphi_i(t)}{\partial t}, \ \ i = 0,1,2,...k,...n$$

4.0.2.5. More interesting relations

$$\overline{\Psi}_k(t) = a_k(t)e^{i_k\varphi_k(t)} = \Psi_k(t) + i_k\hat{\Psi}_k(t),$$

$$\cos\varphi_k = \frac{1}{2}(e^{I\varphi_k(t)} + e^{-I\varphi_k(t)}), \ \sin\varphi_k = \frac{1}{2i}(e^{I\varphi_k(t)} - e^{-I\varphi_k(t)}),$$

$$\varphi_k(t) = \operatorname{arctg}\frac{\hat{\Psi}_k(t)}{\Psi_k(t)}, \ \varphi_0{}^2(t) = \sum_{(k)}\varphi_k{}^2(t), \ \ \omega_k(t) = \frac{\partial\varphi_k(t)}{\partial t} = 2\pi f_k(t),$$

$$a_k{}^2(t) = a_{k\text{-}1}{}^2(t) + \hat{a}_{k\text{-}1}{}^2(t) = \Psi_{k+1}{}^2(t) = \Psi_k{}^2(t) + \hat{\Psi}_k{}^2(t),$$

$$a_0{}^2(t) = \left|\overline{\Psi}(t)\right|^2 = \Psi^2(t) + \hat{\Psi}^2(t) = \sum_{(k)} a_k{}^2(t) + 2\sum_{(i\neq j)}\Psi_i(t)\Psi_j(t), \ \ \forall i,j,k \in [1,n].$$

$$\Psi(t) = \frac{a_n(t)}{2^{n+1}}\prod_{k=0}^{n}(e^{I\varphi_k(t)} + e^{-I\varphi_k(t)}), \hat{\Psi}(t) = \frac{a_n(t)}{(2i)^{n+1}}\prod_{k=0}^{n}(e^{I\varphi_k(t)} - e^{-I\varphi_k(t)}),$$

$$\overline{\Psi}(t) = \Psi(t) + I\hat{\Psi}(t) = \frac{a_n(t)}{2^{n+1}}\left\{\prod_{k=0}^{n}(e^{I\varphi_k(t)} + e^{-I\varphi_k(t)}) + \frac{1}{(i)^n}\prod_{k=0}^{n}(e^{I\varphi_k(t)} - e^{-I\varphi_k(t)})\right\}.$$

4.0.2.6. Hyper-complex Analytic Signal

(This is only a draft... should be better elaborated and combined with Quaternions concept, later)

$$\overline{\Psi}(r,t) = \overline{\Psi}(x,y,z,t) = \Psi(r,t) + I\cdot H\left[\Psi(r,t)\right] = \Psi(r,t) + I\cdot\hat{\Psi}(r,t) =$$

$$= \Psi(r,t) + i\cdot\hat{\Psi}_x(r,t) + j\cdot\hat{\Psi}_y(r,t) + k\cdot\hat{\Psi}_z(r,t) =$$

$$= \overline{\Psi}_i + \overline{\Psi}_j + \overline{\Psi}_k = \left|\overline{\Psi}(r,t)\right|\cdot e^{I\cdot\varphi(r,t)}, \ \overline{\Psi}_{i,j,k} = \Psi_{i,j,k} + \begin{bmatrix} i \\ j \\ k \end{bmatrix}\cdot\hat{\Psi}_{i,j,k} = \left|\overline{\Psi}_{i,j,k}\right|\cdot e^{\begin{bmatrix} i \\ j \\ k \end{bmatrix}\cdot\varphi_{i,j,k}},$$

$$\Psi(r,t) = \Psi_{x,y,z}(r,t) = \Psi_x(r,t) + \Psi_y(r,t) + \Psi_z(r,t) = \Psi_i + \Psi_j + \Psi_k, \hat{\Psi}_{x,y,z}(r,t) = H\left[\Psi_{x,y,z}(r,t)\right]$$

$$\left|\overline{\Psi}(r,t)\right|^2 = \left[\Psi(r,t)\right]^2 + \left[\hat{\Psi}(r,t)\right]^2 = \Psi^2 + \hat{\Psi}^2 = \left|\overline{\Psi}\right|^2, \varphi(r,t) = \operatorname{arctg}\frac{\hat{\Psi}(r,t)}{\Psi(r,t)} = \varphi,$$

$$\left|\overline{\Psi}_{i,j,k}\right|^2 = \left[\Psi_{i,j,k}\right]^2 + \left[\hat{\Psi}_{i,j,k}\right]^2, \varphi_{i,j,k} = \operatorname{arctg}\frac{\hat{\Psi}_{i,j,k}}{\Psi_{i,j,k}}, \omega_{i,j,k} = \frac{\partial\varphi_{i,j,k}}{\partial t}$$

$$\left|\overline{\Psi}\right|^2 = \left|\overline{\Psi}_i\right|^2 + \left|\overline{\Psi}_j\right|^2 + \left|\overline{\Psi}_k\right|^2 = \Psi^2 + \hat{\Psi}^2 ,$$
$$\Psi = \left|\overline{\Psi}\right| \cdot \cos\varphi, \ \hat{\Psi} = \left|\overline{\Psi}\right| \cdot \sin\varphi = H[\Psi],$$
$$\Psi_{i,j,k} = \left|\overline{\Psi}_{i,j,k}\right| \cdot \cos\varphi_{i,j,k} , \ \hat{\Psi}_{i,j,k} = H\left[\Psi_{i,j,k}\right] = \left|\overline{\Psi}_{i,j,k}\right| \cdot \sin\varphi_{i,j,k} ,$$
$$I \cdot \hat{\Psi}(r,t) = i \cdot \hat{\Psi}_i + j \cdot \hat{\Psi}_j + k \cdot \hat{\Psi}_\kappa = e^{I(\frac{\pi}{2}+2m\pi)} \cdot \hat{\Psi}(r,t),$$
$$I \cdot \varphi(r,t) = i \cdot \varphi_i + j \cdot \varphi_j + k \cdot \varphi_k = e^{I(\frac{\pi}{2}+2m\pi)} \cdot \varphi(r,t) , \ I^2 = i^2 = j^2 = k^2 = -1 ,$$
$$\hat{\Psi} = \left|\overline{\Psi}\right| \cdot \sin\varphi = H[\Psi] = \sqrt{\hat{\Psi}_i^2 + \hat{\Psi}_j^2 + \hat{\Psi}_k^2},$$
$$\Psi_{i,j,k} = \left|\overline{\Psi}_{i,j,k}\right| \cdot \cos\varphi_{i,j,k} , \ \hat{\Psi}_{i,j,k} = H\left[\Psi_{i,j,k}\right] = \left|\overline{\Psi}_{i,j,k}\right| \cdot \sin\varphi_{i,j,k}$$
$$I \sin\varphi = i \cdot \sin\varphi_i + j \cdot \sin\varphi_j + k \cdot \sin\varphi_k ,$$
$$I = i \cdot \frac{\varphi_i}{\varphi} + j \cdot \frac{\varphi_j}{\varphi} + k \cdot \frac{\varphi_\kappa}{\varphi} = i \cdot \frac{\hat{\Psi}_i}{\hat{\Psi}} + j \cdot \frac{\hat{\Psi}_j}{\hat{\Psi}} + k \cdot \frac{\hat{\Psi}_\kappa}{\hat{\Psi}} = e^{I(\frac{\pi}{2}+2m\pi)}, \ \frac{\varphi_i}{\varphi} = \frac{\hat{\Psi}_i}{\hat{\Psi}} , \ \frac{\varphi_j}{\varphi} = \frac{\hat{\Psi}_j}{\hat{\Psi}} , \ \frac{\varphi_k}{\varphi} = \frac{\hat{\Psi}_k}{\hat{\Psi}},$$
$$\frac{\varphi_{i,j,k}}{\varphi} = \frac{\hat{\Psi}_{i,j,k}}{\hat{\Psi}} = \frac{H\left[\Psi_{i,j,k}\right]}{H[\Psi]} = \frac{\operatorname{arctg}\frac{\hat{\Psi}_{i,j,k}}{\Psi}}{\operatorname{arctg}\frac{\hat{\Psi}}{\Psi}} ,$$
$$\left[\text{?! (working on...)} \begin{pmatrix} \sin\varphi = \sqrt{\left[(\sin\varphi_i)^2 + (\sin\varphi_j)^2 + (\sin\varphi_k)^2\right]/3}, \\ \cos\varphi = \sqrt{\left[(\cos\varphi_i)^2 + (\cos\varphi_j)^2 + (\cos\varphi_k)^2\right]/3}, \varphi_i + \varphi_j + \varphi_k = \pi, \\ \Psi = \Psi_i + \Psi_j + \Psi_k \ , \ \Psi^2 = \Psi_i^2 + \Psi_j^2 + \Psi_k^2 \ , \ \Psi_i\Psi_j + \Psi_j\Psi_k + \Psi_i\Psi_k = 0 \ , \\ \hat{\Psi} = \hat{\Psi}_i + \hat{\Psi}_j + \hat{\Psi}_k \ , \ \hat{\Psi}^2 = \hat{\Psi}_i^2 + \hat{\Psi}_j^2 + \hat{\Psi}_k^2 \ , \ \hat{\Psi}_i\hat{\Psi}_j + \hat{\Psi}_j\hat{\Psi}_k + \hat{\Psi}_i\hat{\Psi}_k = 0 \ , \\ \varphi = \varphi_i + \varphi_j + \varphi_k \ , \ \varphi^2 = \varphi_i^2 + \varphi_j^2 + \varphi_k^2 \ , \ \varphi_i\varphi_j + \varphi_j\varphi_k + \varphi_i\varphi_k = 0 . \end{pmatrix} \right] \quad (6.10)$$

In chapters 4.1 and 10, we can find proposals "how to apply Hypercomplex functions or Phasors of energy-momentum vectors" (see equations (4.3-0)-p,q,r,s…). In chapter 4.3, we will exercise how all famous wave equations of Quantum Theory can be naturally, smoothly, and easily developed from Analytic and Hypercomplex representations of wave functions (without implementing or postulating certain "ad hoc patchwork"). In Chapter 6.0 (around equations (6.8) - (6.13)), it will be demonstrated how we can structure multidimensional universe using Hypercomplex functions. ***What here looks familiar to Hypercomplex, or Quaternion concepts is certain kind of 3-dimensional structuring of common imaginary unit*** $I^2 = i^2 = j^2 = k^2 = -1$ ***(what could be endlessly extended, by creating new "imaginary-units' triplets", from every (lower level) imaginary unit, this way introducing new foundations of multidimensionality).***

Remarks: Hypercomplex or Quaternions *wavefunction signal phase function* is also presentable as Hypercomplex function, having its real and imaginary parts, such as,

$$\overline{\Psi}(r,t)=\overline{\Psi}(x,y,z,t)=\Psi(r,t)+I\cdot H\left[\Psi(r,t)\right]=\Psi(r,t)+I\cdot\hat{\Psi}(r,t)=$$
$$=\Psi(r,t)+i\cdot\hat{\Psi}_x(r,t)+j\cdot\hat{\Psi}_y(r,t)+k\cdot\hat{\Psi}_z(r,t)=$$
$$=\overline{\Psi}_i+\overline{\Psi}_j+\overline{\Psi}_k=\overline{\Psi}'_i\cdot\overline{\Psi}'_j\cdot\overline{\Psi}'_k=\left|\overline{\Psi}(r,t)\right|\cdot e^{\overline{\varphi}(r,t)},\ \overline{\Psi}_{i,j,k}=\Psi_{i,j,k}+\begin{bmatrix}i\\j\\k\end{bmatrix}\hat{\Psi}_{x,y,z},$$
$$\overline{\varphi}(r,t)=\varphi_R(r,t)+I\varphi_I(r,t)=\varphi_R(r,t)+i\cdot\varphi_i+j\cdot\varphi_j+k\cdot\varphi_k=\left|\overline{\varphi}(r,t)\right|e^{\overline{\varphi}(r,t)},$$
$$I\cdot\varphi_I(r,t)=i\cdot\varphi_i+j\cdot\varphi_j+k\cdot\varphi_k,\ \ I^2=i^2=j^2=k^2=-1.$$

This will give us more substantial mathematical-modeling freedom, since ***every Hypercomplex function (or number) can be presented either as a summation or as multiplication of ordinary complex functions*** (with only one imaginary unit).

4.0.3. Generalized Fourier Transform and Analytic Signal

The Analytic Signal modeling of the wave function can easily be installed in the framework of the Fourier Integral Transform, which exists based on simple-harmonic functions $\cos\omega t$, as follows,

$$\Psi(t)=a(t)\cos\varphi(t)=\int_{-\infty}^{\infty}U(\frac{\omega}{2\pi})e^{j2\pi ft}df=\int_{-\infty}^{\infty}U(\frac{\omega}{2\pi})\left\{\overline{H}\left[\cos2\pi ft\right]\right\}df=F^{-1}\left[U(\frac{\omega}{2\pi})\right],$$

$$U(\frac{\omega}{2\pi})=A(\frac{\omega}{2\pi})e^{j\Phi(\frac{\omega}{2\pi})}=\int_{-\infty}^{\infty}\Psi(t)e^{-j2\pi ft}dt=\int_{-\infty}^{\infty}\Psi(t)\left\{\overline{H}^*\left[\cos2\pi ft\right]\right\}dt=F\left[\Psi(t)\right],\ \ \omega=2\pi f, \tag{4.0.5-1}$$

where the meaning of symbols is:

F (=) Direct Fourier transform,

$\mathbf{F^{-1}}$ (=) Inverse Fourier transform,

$\mathbf{\overline{H}=1+jH}$ (=) Complex Hilbert transform, $\mathbf{j^2=-1}$,

$\overline{H}^*=1-jH$ (=) Conjugate complex Hilbert transform.

$$\overline{H}\left[\cos\omega t\right]=e^{j\omega t},\quad H\left[\cos\omega t\right]=\sin\omega t,$$
$$\overline{H}^*\left[\cos\omega t\right]=e^{-j\omega t},\quad H\left[\sin\omega t\right]=-\cos\omega t,$$
$$e^{\pm j\omega t}=(1\pm jH)\left[\cos\omega t\right],$$
$$\overline{H}\left[\Psi(t)\right]=\overline{\Psi}(t)=\Psi(t)+jH\left[\Psi(t)\right]=\Psi(t)+j\hat{\Psi}(t),$$
$$\overline{H}^*\left[\Psi(t)\right]=\overline{\Psi}^*(t)=\Psi(t)-jH\left[\Psi(t)\right]=\Psi(t)-j\hat{\Psi}(t).$$

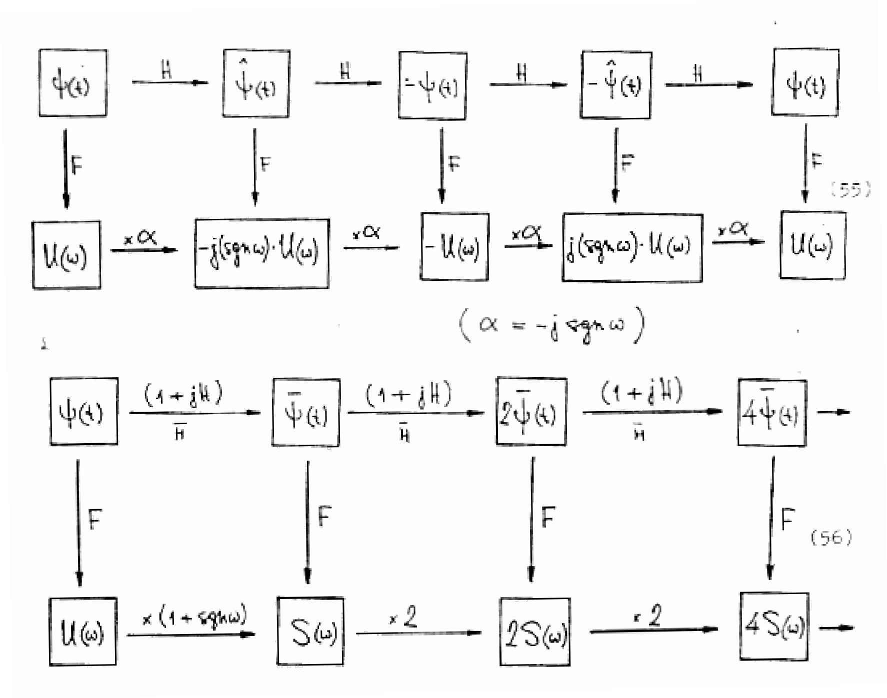
The further generalization of the Fourier integral transformation can be realized by replacement of its simple-harmonic functions basis $\cos\omega t$ by some other, more convenient signal-elements basis $\alpha(\omega, t)$. Now, the wave function (in here generalized framework of Fourier transform) can be presented as,

$$\Psi(t) = \int_{-\infty}^{\infty} U(\frac{\omega}{2\pi})\{\bar{H}[\alpha(\omega,t)]\} df = F^{-1}\left[U(\frac{\omega}{2\pi})\right], \quad U(\frac{\omega}{2\pi}) = \int_{-\infty}^{\infty} \Psi(t)\{\bar{H}^*[\alpha(\omega,t)]\} dt = F[\Psi(t)] . \tag{4.0.5-2}$$

For instance, we could exercise such signals quantization (analysis, decomposition, synthesis, signals reconstruction, and superposition) by using sinc-functions elementary signals such as $\alpha(\omega,t) = \frac{\sin\Omega t}{\Omega t}\cos(\omega t)$, or $\alpha(\omega,t) = e^{-\beta t}\cdot\frac{\sin\Omega t}{\Omega t}\cos(\omega t)$, or using certain of wavelets family similar to Gaussian-Gabor, bell-curve shaped wave packets, solitons, or wave pulses (see later (4.0.33)-(4.0.36) and (10.2-3), being in relation to Kotelnikov-Shannon-Nyquist signal analysis). Gaussian pulses and familiar wave packets like solitons (with bell-curve amplitude-envelope) naturally have the same *amplitude or envelope shape* both in time, and frequency domains, and such shapes are securing optimal (compacted and finite) signal localization both in its time and frequency domains. The idea here is to establish (and shape) elementary basis functions $\alpha(\omega, t)$, which are (on different ways) found as the most natural and generally applicable, elementary wave groups or wave-packets (for instance, applicable for modelling wave functions of photons and other tangible wave-groups from the world of Physics, being part of "*energy-moments*" communications in our Universe).

Here are number of useful relations connecting Hilbert and Fourier transformation:

$$\begin{aligned}
&F[\Psi(t)]\cdot(-j\cdot\operatorname{sgn}\omega) = F\left[\hat{\Psi}(t)\right] = -j\cdot\operatorname{sgn}\omega\cdot U(\omega),\\
&F\left[\hat{\Psi}(t)\right]\cdot(-j\cdot\operatorname{sgn}\omega) = -F[\Psi(t)] = -U(\omega),\\
&F[\Psi(t)]\cdot(1+j\cdot\operatorname{sgn}\omega) = F\left[\bar{\Psi}(t)\right] = S(\omega),\\
&H[\Psi(t)] = \hat{\Psi}(t),\ \Psi(t) = -H\left[\hat{\Psi}(t)\right], (1+jH)\cdot\bar{\Psi}(t) = \bar{\Psi}(t),\ H^2 = -j(Y-H),\\
&H[H(\Psi(t))] = H^2[\Psi(t)] = -\Psi(t),\ H^4[\Psi(t)] = \Psi(t),\\
&\bar{H}\left[\bar{\Psi}(t)\right] = 2\bar{\Psi}(t) = 2\bar{H}[\Psi(t)],\ H\left[\bar{\Psi}(t)\right] = -j\bar{\Psi}(t),\\
&H\cdot\bar{H} = \bar{H}\cdot H = Y,\ H\cdot(\bar{H}\cdot H) = (H\cdot\bar{H})\cdot H,\\
&Y\left[\bar{\Psi}(t)\right] = 2\bar{\Psi}(t) = H\left[\bar{\Psi}(t)\right],\ Y[\Psi(t)] = -j\bar{\Psi}(t),\\
&H[t\cdot\Psi(t)] = t\cdot\hat{\Psi}(t) + \frac{1}{\pi}\int_{-\infty}^{\infty}\Psi(t)dt,\ \bar{H}[t\cdot\Psi(t)] = t\cdot\bar{\Psi}(t) + \frac{1}{\pi}\int_{-\infty}^{\infty}\Psi(t)dt,\\
&H[(t+\tau)\cdot\Psi(t)] = (t+\tau)\cdot\hat{\Psi}(t) + \frac{1}{\pi}\int_{-\infty}^{\infty}\Psi(t)dt,\ \bar{H}[(t+\tau)\cdot\Psi(t)] = (t+\tau)\cdot\bar{\Psi}(t) + \frac{1}{\pi}\int_{-\infty}^{\infty}\Psi(t)dt,\\
&H\left[\frac{d^n\Psi(t)}{dt}\right] = \frac{d^n\hat{\Psi}(t)}{dt},\ \bar{H}\left[\frac{d^n\Psi(t)}{dt}\right] = \frac{d^n\bar{\Psi}(t)}{dt},\\
&\operatorname{Mod}[H] = 1,\ \operatorname{Arg}[H] = -\pi/2,\ H[\text{const}] \cong 0.
\end{aligned} \tag{4.0.5-3}$$



(4.0.5-4)

4.0.4. Meaning of Complex Analytic Signal Functions in Physics

The practice of using complex functions in Physics and Electro-technique has long time been considered mostly as a convenience to simplify complicated mathematical expressions and equations processing (especially applicable for avoiding operations with trigonometric functions). All of that has a much broader and more profound meaning when adequately applied (what has not been quite uniformly and correctly established practice in different analogical domains of Physics, yet). Briefly concluding, the only proper extension and generalization of real-variables wave (or wavefunction) to a corresponding complex variable function (with tangible meaning in Physics) should be formulated as an Analytic Signal function (what is generally still not the case in Physics). Even in electronics, or basic electro-technique, where operating with complex functions such as current and voltage ***phasors,*** *are the common practice, this is still not realized on the grounds of an Analytic Signal modeling, but luckily, what is already realized, it is fully compatible and not-contradictory to such approach (see more at the end of this chapter under "4.0.11. Generalized Wave Functions and Unified Field Theory").*

The fact is that significant arbitrary wave function elements (such as instantaneous amplitude, phase, frequency …) cannot be found if we do not take into account both original wave function Ψ and its Hilbert couple $\hat{\Psi}$, meaning that in reality both of such wave functions should coincidently exist (on some way), explaining the natural meaning of complex Analytic Signals. In other words, *Nature (or the world of Physics) always creates mutually coupled, mirror-states and events (which are mutually orthogonal, phase-shifted functions). In such cases, Hilbert transform, and Complex Analytic Signal model are the right and best mathematical tools to formulate mentioned coupled entities* (however, often we do not notice or realize an intrinsic, or coincidental existence of such coupled states, because in most measurements, or observation cases we see, or detect only one of them). Here is a part of the explanation related to Quantum Theory mathematical structure, which is also (and incompletely) modeling mentioned mutually coupled, dual (or conjugate) matter-wave states in a formally different way that should be isomorphic to an Analytic signal structure (what is still not fully and correctly established). *For instance, well-known, and in some respects unusual or controversial particle-wave duality theory, as elaborated in the contemporary Quantum Theory, is mathematically, verbally, experimentally, stochastically, and philosophically (but not naturally, ontologically, and deterministically) explaining how in some cases objects like photons, electrons, and other elementary particles could behave like waves, and in other cases like particles. Such natural particle-wave duality should also be explicable concerning duality (and complementarity) of an original wave function Ψ and its Hilbert couple $\hat{\Psi}$.* We could conditionally say that if Ψ represents specific ***moving-particle*** model (or wave group), then $\hat{\Psi}$ represents its wave model (and vice versa), since both, Ψ and $\hat{\Psi}$ (as well as their frequency-domain counterparts), exist coincidently, and have the same energy content $\tilde{E}$ (of course, this should be elaborated better), as for instance,

$$\tilde{E} = \int_{-\infty}^{+\infty} \Psi^2(t)dt = \int_{-\infty}^{+\infty} \hat{\Psi}^2(t)dt = \int_{-\infty}^{+\infty} \left|\frac{\overline{\Psi}(t)}{\sqrt{2}}\right|^2 dt = \int_{-\infty}^{+\infty} \left[\frac{a(t)}{\sqrt{2}}\right]^2 dt = \int_{-\infty}^{+\infty} \left|\frac{\overline{U}(\omega)}{\sqrt{2\pi}}\right|^2 d\omega = \int_{0}^{\infty} \left[\frac{A(\omega)}{\sqrt{\pi}}\right]^2 d\omega \cdot \qquad (4.0.4)\ (4.0.5)$$

Here, Ψ and $\hat{\Psi}$ have the same group and phase velocity (to be shown later), being mutually orthogonal (phase shifted for $\pi/2$), and on certain way "*energetically*" mutually exclusive since $\int_{-\infty}^{+\infty} \Psi(t) \cdot \hat{\Psi}(t)dt = 0$. All of that should also belong to the mathematical and natural background of the particle-wave duality and wave functions, and there is still no need to introduce *probability* wave functions when we already have such rich, exact, operational, dimensional, intuitively clear, and explicit wave functions modeling (as operating with Analytic Signals).

Modern electro-technique got its best natural mathematical, engineering and modelling foundations when alternating (simple harmonic or sinusoidal) currents, voltages, impedances, power, and other important parameters of electric circuits started to be presented with Complex numbers, Complex functions, and Complex Phasors (avoiding complicated equations with trigonometric functions).

The next advance in the same direction has been with the Complex *Analytic Signal formulation of arbitrary wave functions being the ultimate generalization of **PHASORS** concept, and there is still a space for the same modelling to be much more extended in Physics, and to everything that is moving, oscillating, and waving in time and/or spatial domains (see more in Chapter 10.). Also, Complex Analytic Signal or Phasors concept is directly applicable in analysis of simple harmonic, alternating currents, and voltages, but in addition applicable when currents and voltages would be arbitrary shaped functions.*

Relativity theory, regarding familiar imaginary and complex entities and functions, is presently still not in a big harmony with here elaborated Complex Analytic Signals and Phasors concepts, but Minkowski-Einstein 4-vectors could also, naturally, and easily be treated as Complex functions, and on some convenient way modified to be as Analytic Signals or Phasors.

For instance, historically original, first-time conceptualized Maxwell theory was established using complex Quaternion functions (with three imaginary units), what is basically modelling using Hypercomplex Analytic signal functions. What will be the next, more general, and more advanced modelling using Complex Wave functions is to apply Hypercomplex Wave functions and Phasors (with three imaginary units) ...

Such new mathematical modelling options are most probably in the background of new imaginative and creative excursions and explanations of matter in motion (where we know how to mutually relate particle and wave properties of the same matter state). For instance, the celebrated physicist Paul Dirac was very imaginative, intellectually vocal, and more convincing in conceptualizing an “ocean” of densely packed negative energy “phantom-like” electrons in a pure vacuum state of matter. This way, he “invented” anti-matter particle named positron, which without such creative imagination could also be found either as certain Ψ-function, or its Hilbert couple $\hat{\Psi}$, or anyway described inside the framework of Analytic Signal functions (and eventually, positron was experimentally discovered, much more by chance and without Dirac’s, and any of here-mentioned theoretical support, but later, backwards it has been associated to Dirac’s prediction). Dirac also gave in the same scientific package his imaginative meaning of complex or imaginary functions concerning relativistic particle energy, what his faithful followers found as an exceptional and brilliant-mind contribution to physics (or better to say to mathematics). This way, he "predicted" the existence of other anti-matter particles, and some of them were, by chance, experimentally (and most probably independently from any Dirac's influence and prediction) discovered a few years later. This prediction has been presented in some books as a brilliant success of the science of 20th century AD, also showing that operational “Ptolemy-type and incomplete theories” can still exist, and that we are in some cases, during certain period, not able to recognize them as such. Ptolemy is mentioned here concerning its geocentric teaching that is conceptually and fundamentally wrong, but practically and mathematically was operational and giving sufficiently correct results (during certain, very long historical period…). Richard Feynman later upgraded and optimized mentioned Dirac's concepts, making them very practical and intuitively useful in the form of Feynman's diagrams. Later, similar imaginative and seducing concepts evolved towards a “Zero Point, Electromagnetic Quantum Vacuum Fluctuations (ZPF)”. Here, we could say that *every real motion-state expressed as certain Ψ-function should have its Hilbert couple $\hat{\Psi}$, which is another and equally real, motional state (existing both mathematically and on some specific, natural way in the real world of physics, like complementing electric and magnetic field components in cases of electromagnetic waves and photons).* The interpretation of how such coupling, mapping, imaging, and phase shifting of complementary and dual wave states manifest in Physics is another question, but it is undoubtedly related to de Broglie matter waves, including Dirac’s, Feynman’s and ZPF concepts. In electronics and electro-technique, when operating with current, voltage, impedance, and power-related **phasors** (meaning with complex and analytic signal functions), we know that real values of mentioned phasors are presenting direct emitted or outgoing energy (or real and active power) flow from its source towards its load or sink. At the same time, imaginary components of the same power-related wavefunctions are presenting reactive or reflected (backward) energy flow. Similar meanings (of direct and reflected energy flow) we can analogically associate to different matter waves in other domains of physics (see much more of familiar problematic at the end of this chapter, under "Extension of Electrical Power Framework to Arbitrary (Non-Sinusoidal) Voltage and Current Signals ..."). This way we will avoid arbitrary and non-natural applications or interpretations of imaginary and complex functions in Physics.

It should not be too far from reality that Ψ and its Hilbert couple $\hat{\Psi}$ are, on some way, in a mutual (and instant or immediate) synchronizing, coupling, and communicating relation of Quantum entanglement (based on signal energy relations (4.04) and (4.05)). At least, we know that entanglement is an experimentally verifiable reality of specifically and coincidently created and by certain natural resonance coupled particle-wave states, being like photons, electrons, protons, atoms, and small atomic clouds (see more about entanglement in [56]).

[♣ COMMENTS & FREE-THINKING CORNER:

Of course, there is no progress and advance without creative imagination, and eventually, wrong concepts, which are on a certain level useful, innovative, and operational, will be gradually replaced or improved by better concepts. In that name we can introduce another of such (still brainstorming) concepts, as follows: *"Since atoms, as "elementary bricks of matter", are composed of electrons, protons, and neutrons, and all of them manifest particle and wave properties, being presentable as wave packets or wavefunctions, we can associate to each of them certain wave function Ψ_i. From the properties of Analytic Signals (as elaborated here), we know that all of them should also have intrinsically coupled and phase-shifted Hilbert wave components $\hat{\Psi}_i$". How such an intrinsic nature of coupled signals behaves inside atoms (or inside planetary or solar systems), is it another question to answer? Is this kind of thinking leading to reinventing positrons, antiprotons, positive, negative, and electro-neutrally charged particles, and maybe "dark energy and matter" states? Another, still challenging aspect of "Hilbert-type matter-wave duality" could be duality of linear and rotational motions, since anyway, every particle (or wave) in linear motion should have its "motional and rotating Hilbert couple", which could be some associated spinning motion (and vice-versa). For instance, the linear motion of an electrically charged particle has its electric field vector, but it should also have an associated, solenoidal magnetic field vector (around the path of its linear motion).* Is de Broglie Particle-Wave Duality describing something familiar?

If we accept the philosophical position that all mathematically consistent, internally coherent, self-standing theories, concepts, and models (based on tangible Physics) are potentially presenting certain reality of our Universe, which may one day be materialized as a verifiable Physics knowledge, we could already see the roots and foundations of Wave-Particle Duality in Physics in relation to Analytic Signal Wave Functions Modeling. Of course, such concepts, models and theories should be well integrated with the overall (and naturally founded) body of relevant Mathematics and Physics, avoiding intellectual creations that are extensively axiomatic, dogmatic, or based on artificial game-theory postulates and assumptions (like happens in the present Relativity and Quantum theory).

What is interesting regarding physics-related and natural, energy-finite signals (as found in our universe) is that such signals are not only *time-dependent functions*, but also *space-time dependent, and in all domains being finite functions*. However, from our point of view (related to measurements and observations in certain space-location) we often experience signals as only time or space-depending functions. Most of the above-presented mathematical relations (starting from (4.0.1) to T.4.0.1) could (equally and reasonably) exist, having physics-related meaning if we replace a time variable $\mathbf{t}$ with corresponding space variable $\mathbf{x}$, for instance, $\mathbf{t} \rightarrow \mathbf{x}$, $\omega = \omega_t = 2\pi f \rightarrow \omega_x = 2\pi f_x$, $\Psi(t) \rightarrow \Psi(x)$. *The reason for such time-space symmetry is that whichever wave function, found as a good model for representing specific sufficiently stable moving particle, energy state or other, relatively stable (non-transient and non-dispersive) wave motion, should intrinsically have a structure that takes care about its space and time parameters matching and integrity. This creates mutual harmony and unity between them (usually expressed by simple mathematical relations between relevant time and space-related parameters).* If this has not been the case in our universe, we would not have temporally and spatially stable objects and recognizable patterns of different motions and astronomic formations, which are mutually analogical, symmetrical, synchronized, and respecting the same fundamental conservation laws of Physics (see (4.0.46)). It will be shown later that mentioned temporal-spatial integrity, stability and synchronization between relevant time, space and their frequency domains are also closely related to relevant group and phase velocity. This is requesting an optimal time and space, mutually dependent signal sampling intervals, known in Signal Analysis in relation to "Shannon-Kotelnikov-Nyquist signal sampling, synthesis and reconstruction" theorems, what associates on quantizing in Physics, essentially explaining how content of our Universe is internally communicating regarding "*energy-moments-matter waves exchanges*" (see more about such situations in Chapter 10). Analyses of that kind will also lead to an extension, revision, and generalization of universally valid Uncertainty Relations, mistakenly and exclusively considered as originating in Physics only from W. Heisenberg, but (in reality) merely taken from universally valid (just mentioned) Signal analysis and associated mathematics.

The unfortunate event (regarding quantum physics) is that Dennis Gabor, the inventor of the Analytic Signal concept, came too late with his mathematical invention, after the stepstones of Quantum Theory have been "*strongly and by consensus established*" and celebrated founders of Quantum Theory were (most probably) already too tired, too much satisfied, and not ready to make (very new) Quantum Theory-house redesign (considering why to change something what already works well). Also, it could probably be embarrassing to admit that few of the Nobel prizes were already awarded for something that maybe was not the best possible, unique, extraordinary, and brilliant creation, as sporadically and vocally announced by QT founders, being (as such) still non-doubtfully maintained by many of their faithful and obedient followers and believers. To keep established harmony of QT as it is, the most appropriate (and essentially wrong) was to continue as nothing serious happened (and, most probably that not many people noticed the relevance of such new mathematical concepts, unintentionally mixing them with similar items from older Fourier Signal Analysis). Also, most of the followers of the Orthodox Quantum Theory teachings are proudly asking themselves and suggesting to others: Why to change foundations (or anything else) in present Quantum Theory? Where the problem is, considering that present-days Quantum Theory is mathematically working very well (in its own self-defined, or postulated frames and assumptions). We should hope that this (about QT) will not become like Ptolemy's geocentric system that unfortunately (and wrongly) resisted as the unique, mainstream-accepted, and most accurate teaching (about Sun and planetary motions) for an exceedingly long time. -- ♣]

4.0.5. Wave Packets and mathematical strategies regarding Wave Velocities

Let us first introduce important wave properties regarding most elementary, simple harmonic waves, as usually practiced in physics of wave motions.

Oscillations at a point (only time-dependent) can be presented as,

$$\Psi(t) = a \cdot \cos\, 2\pi f t = a \cdot \cos \omega t \tag{4.0.6}$$

Traveling, planar waves (in one-dimensional motions) are characterized by,

$$\begin{aligned} &\Psi(t, x) = a \cdot \cos 2\pi\, f(t - \frac{x}{u}) = a \cdot \cos 2\pi (ft - \frac{x}{\lambda}) = a \cdot \cos\, (\omega t - kx) = \\ &= a \cdot \cos k\,(ut - x) = \Psi(x - ut), \\ &\omega = 2\pi f\, (=)\text{ angular frequency}, \; k = \frac{2\pi}{\lambda}\, (=)\text{ wave number}, \\ &u = \lambda f = \frac{\omega}{k}\, (=)\text{ phase velocity}, \end{aligned} \tag{4.0.7}$$

and traveling (also planar) waves as 3-dimensional spatial-temporal motion,

$$\Psi(t, r) = a \cdot \cos\, (\omega t - kr) = \Psi(r - ut), \; \vec{r} = \vec{r}(x, y, z)\; . \tag{4.0.8}$$

In reality (what is often neglected), every wave that has the same wave-shape during its motion (like elementary sinusoidal signals), presented in one-dimension (traveling along the **x** or **r** axis), is usually composed of two waves traveling in mutually opposite directions (below marked as (+) and (-) directions), judging by general mathematical solutions of relevant, differential wave-equations. The wavefunction of such wave motion, also applicable to other arbitrary function shapes, is $\Psi(t, r) = \Psi^{(+)}(r - ut) + \Psi^{(-)}(r + ut)$. It can be proven that for all of particular wave functions such as $\underline{\Psi^{(+)}(r - ut)}$, or $\underline{\Psi^{(-)}(r + ut)}$, or $\underline{\Psi^{(+)}(r - ut) + \Psi^{(-)}(r + ut)}$, (*when the wave propagates in linear media, in both (+ and -) directions, without dispersion, at the same speed independent of wavelength, and independent of amplitude*) is applicable the same partial, differential Wave Equation (known also as Classical and/or d'Alembert, second order differential wave equation), $\frac{1}{u^2}\frac{\partial^2\Psi}{\partial t^2} = \frac{\partial^2\Psi}{\partial x^2} (=) \nabla^2\Psi$. What is even more significant here is that solutions of such Wave Equation can be generalized with a simple, complex form as,

$$\Psi(t, r) = a \cdot e^{i(\omega t - kr)} + b \cdot e^{i(\omega t + kr)} \Leftrightarrow \begin{bmatrix} \Psi(t, r) = a \cdot \cos(\omega t - kr) + b \cdot \cos(\omega t + kr) \\ (=)\; \Psi^{(+)}(r - ut) + \Psi^{(-)}(r + ut) \end{bmatrix},$$

what Schrödinger exploited when he created his wave equation. In this book, similar generalization is also practiced considering that solutions of Wave Equation are Complex, Analytic Signal functions, as in (4.0.2). We will see later that such an approach can effectively produce and explain all essential wave equations known from Quantum Theory, in a much simpler way, compared to contemporary Quantum Theory, ad hock, axiomatic and stochastic, mathematical elaborations.

Of course, the wave velocity (here a phase velocity) **u** will depend on the medium properties through which the wave is propagating.

All other, more complex waveforms and wave packets are presentable as an integral or discrete superposition of elementary, simple-harmonic waves, as shown in (4.0.1), including a superposition of simple waves (4.0.8). Synchronous propagation of two coupled wave components in mutually opposed directions should always be considered as generally present or valid reality for all wave motions (having a much deeper meaning in the world of Physics, than presently seen).

If a wave velocity of the waveform (or a wave packet) is independent of wavelength, each elementary wave (and thus the wave packet) travels at the same speed (what is *valid for propagations in linear media*).

If wave velocity depends on wavelength (*valid for propagations in non-linear media*), each elementary wave travels at a different speed, compared to the wave packet speed (or group speed).

The general condition (regarding an arbitrary waveform) for extracting the waveform phase velocity, **u**, is that signal phase function would become constant. The meaning of that is that we would be able to travel parallel with signal phase, always seeing the same phase point (being linked to the same phase value: or effectively we will not see that signal carrier function $\cos\varphi(t)$ is propagating or being time dependent. Of course, the first step is to present the waveform as an Analytic Signal function). Satisfying this will mean that we are also traveling parallel to the wave in question, with waves phase velocity. Mathematically, this could be summarized as,

$$\left\{\begin{array}{l}\varphi(t) \rightarrow \varphi(t,x) = \text{const.} \\ \omega t \rightarrow \omega t - kx \\ \varphi'(t,x) = \varphi''(t,x) = \text{const.}\end{array}\right\} \Rightarrow \left\{\frac{dx}{dt}, \frac{\Delta x}{\Delta t}\right\} (= u = \text{ phase velocity}). \qquad (4.0.9)$$

Group velocity, **v**, is the velocity of the signal amplitude or its envelope function $\mathbf{a(t)}$. Now the same situation could be visualized if we imagine that we are traveling parallel to signal amplitude and we would always see only one point of the amplitude function (for instance, signal envelope peak value), for instance,

$$\left\{\begin{array}{l}a(t) \rightarrow a(t,x) = \text{const.} \\ \omega t \rightarrow \omega t - kx \\ a'(t,x) = a''(t,x) = \text{const.}\end{array}\right\} \Rightarrow \left\{\frac{dx}{dt}, \frac{\partial x}{\partial t}\right\} (= v = \text{group velocity}). \qquad (4.0.10)$$

We could also create a new analytic signal form, which is equal only to the signal amplitude function, and apply similar method as in case of phase velocity (here we are treating the signal amplitude function as a new wave function, which would have its own, newly calculated phase and amplitude functions).

$$\left\{\begin{array}{l} a(t) \rightarrow a(t, x) = (\sqrt{a^2(t, x) + [H(a(t, x))]^2})\cos\left[\arctan\frac{H(a(t, x))}{a(t, x)}\right] = \\ = a_1(t, x)\cos\varphi_1(t, x);\ \ \varphi_1(t, x) = \text{const.},\quad \omega t \rightarrow \omega t - kx \\ a_1(t, x) = \sqrt{a^2(t, x) + [H(a(t, x))]^2},\ \varphi_1(t, x) = \arctan\frac{H(a(t, x))}{a(t, x)} \end{array}\right\} \Rightarrow \tag{4.0.11}$$

$$\Rightarrow \left\{\frac{dx}{dt}, \frac{\partial x}{\partial t}, \frac{\Delta x}{\Delta t}\right\}_1 \Rightarrow (v = \text{group velocity}).$$

Let us now compare the "**external and internal**" signal structure in a signal time-domain, (4.0.1). *Internally*, (inside of the integration) we have the infinitesimal superposition of elementary waveforms,

$$U_c(\omega)\cdot\cos\omega t + U_s(\omega)\cdot\sin\omega t = A(\omega)\cdot\cos(\omega t + \Phi(\omega)),$$
(4.0.12)

Externally (or after integration), we will get a similar analytic signal form, $\Psi(t) = a(t)\cos\varphi(t)$. In the following steps (1°, 2°, 3° …), it will be shown how group and phase velocities can be found, and what the consequences regarding modeling elementary waveforms that are building elements of other more complex waveforms are. Terms "**internal and external**" signal structures are conditionally and temporarily introduced in this book before we find terminology that is more appropriate.

1°

In both cases (*time-wise and frequency-wise*; as in (4.0.1) – (4.0.3)) there is an amplitude function ($A(\omega)$ or $\mathbf{a(t)}$), and a phase function as an argument of the cosine function ($\cos(\omega t + \Phi(\omega)) = \cos\Phi(\omega,t)$ or $\cos\varphi(t)$), and we could say that both, "**internal and external**" signal waveforms are presenting Analytic Signal forms (also being created on the same way, and looking mutually similar).

For the signal integrity and its existence as the lasting traveling waveform it would be necessary that group and phase velocities of "**internal and external**" wave functions are mutually equal (in other words, here we will analyze only non-dispersive traveling waves).

Since the integration of $A(\omega)\cos(\omega t + \Phi(\omega))$ is made taking into account only frequency ω (not time), it is almost evident that *the wave function* $A(\omega)\cos(\omega t + \Phi(\omega))$ *and wave function* $\mathbf{a(t)\cos\Phi(t)}$ *should have the same velocities*. Of course, all waveforms in question should be conveniently presented in time and space coordinates (briefly, as $\omega t \rightarrow \omega t - kx$), what is applicable in cases of non-dispersive traveling waves.

2°

We can also compare the "**external and internal**" signal structure, (4.0.1) – (4.0.3), in a signal frequency domain to address the group and phase wave velocity differently. This time we will consider complex signal forms to have more condensed mathematical expressions and easier comparison.

$$\begin{aligned} &\overline{U}(\omega) = A(\omega)e^{-j\Phi(\omega)} = U_c(\omega) - jU_s(\omega) = \int_{-\infty}^{+\infty} \overline{\Psi}(t)e^{j\omega t}\,dt \quad = \int_{-\infty}^{+\infty} a(t)e^{j(\omega t+\varphi(t))}\,dt \\ &\overline{\Psi}(t) \ = a(t)e^{-j\varphi(t)} = \Psi(t) + j\hat{\Psi}(t) \quad = \frac{1}{\pi}\int_{0}^{\infty} U(\omega)e^{j\omega t}d\omega \ = \frac{1}{\pi}\int_{0}^{\infty} A(\omega)e^{j(\omega t+\Phi(\omega))}d\omega \end{aligned} \tag{4.0.13}$$

The formal analogy is obvious: both complex signal forms, $\overline{U}(\omega)$ and $\overline{\Psi}(t)$ (one in the frequency domain, and the other in the time domain, representing the same signal or the same wave) should have the same wave velocities; - the phase functions, $\Phi(\omega)$ and $\boldsymbol{\varphi(t)}$, should generate a phase velocity $\mathbf{u}$, and the amplitude functions, $A(\omega)$ and $\mathbf{a(t)}$ should generate a group velocity $\mathbf{v}$. Also, we see again that all "**internal and external**" signal functions are presenting the structure of Analytic Signal forms (of course, differently formulated). Before we start searching for mathematical expressions of wave velocities, all waveforms in question should be extended to have time and space coordinates, or to represent traveling and non-dispersive waveforms (briefly summarizing as, $\omega t \to \omega t - kx$, $\omega = \omega(k)$).

The forms of group and phase wave velocities should equally (and analogically) be presentable regarding ordinary time-space $(\mathbf{t, x})$ variables, and concerning spectral variables $(\omega, \mathbf{k}), \omega = \omega(k) = \omega_t = 2\pi f_t = 2\pi f$, $k = \omega_x = 2\pi f_x = \frac{2\pi}{\lambda}$,

$$\left\{\begin{matrix} \varphi(t,x) = const. \\ \omega t \to \omega t - kx \ , \ \omega = \omega(k) \\ \Phi(\omega, k) = Const. \end{matrix}\right\} \Rightarrow \left\{\frac{dx}{dt}, \frac{\Delta x}{\Delta t}, \frac{\omega}{k}\right\} (=)\, u = \text{phase velocity}, \tag{4.0.14}$$

$$\left\{\begin{matrix} a(t,x) = const. \\ \omega t \to \omega t - kx, \ \omega = \omega(k) \\ A(\omega, k) = Const. \end{matrix}\right\} \Rightarrow \left\{\frac{dx}{dt}, \frac{\partial x}{\partial t}, \frac{d\omega}{dk}\right\} (=)\ v = \text{group velocity}\cdot \tag{4.0.15}$$

3°

Let us now unite the wave-velocities strategies, applicable on "**external and internal**" signal structures, both in time and frequency signal domains. We will now create the simplest possible wave group that is composed only of two elementary waveforms that are mutually infinitesimally close (where closeness is measured by small differences between their space, time, and frequency variables), and find its phase and group velocity. By the nature of the mathematical formulation of all Analytic Signals, we know that amplitude or envelope functions $\mathbf{a(t,x)}$ and $A(\omega, k)$ are placed in the lower frequency spectrum area (being slowly evolving), compared to carrier or phase functions $e^{j(\omega t - kx + \varphi(t,x))}$ and $e^{j(\omega t - kx + \Phi(\omega, k))}$ (and, it is also a time to say, that this is also well-known property of analytic signals). Consequently, we have a chance to find a group and phase velocity in a simple way, since when making a superposition of two infinitesimally close wave elements, we will be able to consider that their amplitude functions remain constant (and that only signal carrier or phase functions are significant variables). Practically, instead of integrating *within total integral limits*, we will take only two of "sub-integral" elementary waveforms (both in time and frequency domains), that are mutually infinitesimally close, and find what their superposition will create,

$$\left\{\begin{array}{l} \overline{y}(t,x) = a(t,x)e^{j(\omega t - kx + \varphi(t,x))} = a(t,x)e^{j\Theta(t,x)} \\ \overline{Y}(\omega,k) = A(\omega,k)e^{j(\omega t - kx + \Phi(\omega,k))} = A(\omega,k)e^{j\Theta(\omega,k)} \\ \Theta\ (t,x) = \omega t - kx + \varphi(t,x) \\ \Theta(\omega,\ k) = \omega t - kx + \Phi(\omega,k) \\ \overline{U}(\omega,k) = A(\omega,k\ )e^{-j\Phi\ (\omega)} = \int_{-\infty}^{+\infty} a(t,x)e^{j(\omega t - kx + \varphi(t))}\,dt = \int_{-\infty}^{+\infty} \overline{y}(t,x)\cdot dt \\ \overline{\Psi}(t,x) = a(t,x)e^{-j\varphi(t)} = \frac{1}{\pi}\int_0^{\infty} A(\omega,k)e^{j(\omega t - kx + \Phi(\omega))}\,d\omega = \frac{1}{\pi}\int_0^{\infty} \overline{Y}(\omega,k)\cdot d\omega \end{array}\right\} \Rightarrow \qquad (4.0.16)$$

$$\begin{array}{l} \overline{\Psi}_{1+2} = \frac{1}{2}\left[\overline{y}_1(t,x) + \overline{y}_2(t,x)\right] = \frac{1}{2}a(t,x)\cdot\left\{ e^{j[\Theta(t,x) - d\Theta(t,x)]} + e^{j[\Theta(t,x) + d\Theta(t,x)]} \right\} \\ \overline{U}_{1+2} = \frac{1}{2}\left[\overline{Y}_1(\omega,k) + \overline{Y}_2(\omega,k)\right] = \frac{1}{2}A(\omega,k)\cdot\left\{ e^{j[\Theta(\omega,k) - d\Theta(\omega,k)]} + e^{j[\Theta(\omega,k) + d\Theta(\omega,k)]} \right\} \end{array} \qquad (4.0.17)$$

$$\begin{array}{l} \overline{\Psi}_{1+2} = \frac{1}{2}a(t,x)e^{j\Theta(t,x)}\cdot\left[e^{-jd\Theta(t,x)} + e^{+jd\Theta(t,x)}\right] = a(t,x)e^{j\Theta(t,x)}\cdot\cos\left[d\Theta(t,x)\right] = \\ = a(t,x)e^{j\varphi(t,x)}\cdot\cos\left[d\Theta(t,x)\right]\cdot e^{j(\omega t - kx)} = \overline{\Psi}(t,x)\cdot\cos\left[d\Theta(t,x)\right]\cdot e^{j(\omega t - kx)} = \\ = \overline{\Psi}(t,x)\cdot\cos\left[d(\omega t - kx)\right]\cdot\cos\left[d\varphi(t,x)\right]\cdot e^{j(\omega t - kx)} \\ ---------------- \\ \overline{U}_{1+2} = \frac{1}{2}A(\omega,k)e^{j\Theta(\omega,k)}\cdot\left[e^{-jd\Theta(\omega,k)} + e^{+jd\Theta(\omega,k)}\right] = A(\omega,k)e^{j\Theta(\omega,k)}\cdot\cos\left[d\Theta(\omega,k)\right] = \\ = A(\omega,k)e^{j\Phi(\omega,k)}\cdot\cos\left[d\Theta(\omega,k)\right]\cdot e^{j(\omega t - kx)} = \overline{U}(\omega,k)\cdot\cos\left[d\Theta(\omega,k)\right]\cdot e^{j(\omega t - kx)} = \\ = \overline{U}(\omega,k)\cdot\cos\left[d(\omega t - kx)\right]\cdot\cos\left[d\Phi(\omega,k)\right]\cdot e^{j(\omega t - kx)} \end{array} \qquad (4.0.18)$$

$$\begin{array}{l} \cos\left[d\Theta(t,x)\right] = \cos\left\{d\left[\omega t - kx + \varphi(t,x)\right]\right\} = \\ = \cos\left[d(\omega t - kx)\right]\cdot\cos\left[d\varphi(t,x)\right] - \sin\left[d(\omega t - kx)\right]\cdot\sin\left[d\varphi(t,x)\right] = \\ = \cos\left[d(\omega t - kx)\right]\cdot\cos\left[d\varphi(t,x)\right] \end{array}$$

$$\begin{array}{l} \cos\left[d\Theta(\omega,k)\right] = \cos\left\{d\left[\omega t - kx + \Phi(\omega,k)\right]\right\} = \\ = \cos\left[d(\omega t - kx)\right]\cdot\cos\left[d\Phi(\omega,k)\right] - \sin\left[d(\omega t - kx)\right]\cdot\sin\left[d\Phi(\omega,k)\right] = \\ = \cos\left[d(\omega t - kx)\right]\cdot\cos\left[d\Phi(\omega,k)\right] \end{array}$$

General conditions to be satisfied (regarding (4.0.18)) in order to find phase velocity are $\boldsymbol{\varphi(t,x) = const.}$ and $\Phi(\omega,k) = Const.$, making that $d\varphi(t,x) = 0$, and $d\Phi\ (\omega\ ,\ k) = 0$, what simplifies the expressions of elementary waveforms in time and frequency domains,

$$\left\{\begin{array}{l}\overline{\Psi}_{1+2}=\overline{\Psi}(t,x)\cdot\cos\left[d(\omega t-kx)\right]\cdot e^{j(\omega t-kx)}\\ ----------------\\ \overline{U}_{1+2}=\overline{U}(\omega,k)\cdot\cos\left[d(\omega t-kx)\right]\cdot e^{j(\omega t-kx)}\end{array}\right\}\Leftrightarrow\left\{\frac{\overline{\Psi}_{1+2}}{\overline{U}_{1+2}}=\frac{\overline{\Psi}(t,x)}{\overline{U}(\omega,k)}\right\} \tag{4.0.19}$$

The only common carrier frequency, or phase function member in both elementary wave functions ($\overline{\Psi}_{1+2}$ and $\overline{U}_{1+2}$) is $\mathbf{e^{j(\omega t-kx)}}$, and their velocities should be the signal phase velocity, found as usually (when the phase is constant),

$$\begin{array}{l}\omega t - kx = const \Leftrightarrow \omega t_1 - kx_1 = \omega t_2 - kx_2 \Leftrightarrow \omega(t_2-t_1) - k(x_2-x_1)=0 \Leftrightarrow\\ \Leftrightarrow \omega\Delta t - k\Delta x = 0 \Leftrightarrow \dfrac{\Delta x}{\Delta t}=\dfrac{\omega}{k}.\end{array} \tag{4.0.20}$$

Since in this analysis we started creating the superposition of two infinitesimally close elementary waves, (4.0.16), it is evident that their phase velocity should be found as,

$$u=(\lim\frac{\Delta x}{\Delta t})_{\Delta t\to 0}=\frac{dx}{dt}=\frac{\omega}{k}=\text{phase velocity}. \tag{4.0.21}$$

From the other joint member, $\cos\left[d(\omega t - kx)\right]$, of two elementary waveforms ($\overline{\Psi}_{1+2}$ and $\overline{\mathbf{U}}_{1+2}$), we would be able to find the group velocity, again using the same constant phase argument as in the case of phase velocity,

$$\begin{array}{l}\omega t-kx=const\Leftrightarrow d(\omega t-kx)=0\Leftrightarrow k(\dfrac{\omega}{k}-\dfrac{dx}{dt})dt-(x-t\dfrac{d\omega}{dk})dk=0\Leftrightarrow\\ \Leftrightarrow\left[\dfrac{dx}{dt}=\dfrac{\omega}{k}=\text{phase velocity}\right]\text{ and }\left[x-t\dfrac{d\omega}{dk}=0\right]\Rightarrow x=t\dfrac{d\omega}{dk}\Rightarrow\\ \Rightarrow v=\dfrac{\partial x}{\partial t}=\dfrac{d\omega}{dk}=\text{group velocity}\end{array} \tag{4.0.22}$$

It is also possible to find the functional connection between phase and group velocity in the following form,

$$v=u+k\frac{du}{dk}=\frac{d\omega}{dk}. \tag{4.0.23}$$

Of course, we could immediately find the elementary sub-integral, waveforms as,

$$\left\{\begin{array}{l}\overline{\Psi}_{1+2}=\overline{\Psi}(t,x)\cdot\cos\left[d(\omega t-kx)\right]\cdot e^{j(\omega t-kx)}=\overline{\Psi}(t,x)\cdot e^{j(\omega t-kx)}=a(t,x)\cdot e^{j\varphi(t,x)}\cdot e^{j(\omega t-kx)}\\ ----------------\\ \overline{U}_{1+2}=\overline{U}(\omega,k)\cdot\cos\left[d(\omega t-kx)\right]\cdot e^{j(\omega t-kx)}=\overline{U}(\omega,k)\cdot e^{j(\omega t-kx)}=A(\omega,k)\cdot e^{j\Phi(\omega,k)}\cdot e^{j(\omega t-kx)}\end{array}\right\}\Leftrightarrow$$

$$\Leftrightarrow\left\{\begin{array}{l}\overline{\Psi}(t,x)=\overline{\Psi}_{1+2}\cdot e^{-j(\omega t-kx)}=a(t,x)\cdot e^{j\varphi(t,x)}\\ \overline{U}(\omega,k)=\overline{U}_{1+2}\cdot e^{-j(\omega t-kx)}=A(\omega,k)\cdot e^{j\Phi(\omega,k)}\end{array}\right\}\Leftrightarrow\left\{\frac{\overline{\Psi}_{1+2}}{\overline{U}_{1+2}}=\frac{\overline{\Psi}(t,x)}{\overline{U}(\omega,k)}\right\},$$

$$\begin{aligned}&\bar{U}(\omega, k) = A(\omega, k)e^{-j\Phi(\omega, k)} = \int_{-\infty}^{+\infty} \bar{y}(t, x)\cdot dt = \int_{-\infty}^{+\infty} \bar{\psi}_{1+2}dt = \int_{-\infty}^{+\infty} \bar{\Psi}(t, x)\cdot e^{j(\omega t - kx)}dt \\ &\bar{\Psi}(t, x) = a(t, x)e^{-j\varphi(t, x)} = \frac{1}{\pi}\int_0^{\infty} \bar{Y}(\omega, k)\cdot d\omega = \frac{1}{\pi}\int_0^{\infty} \bar{U}_{1+2}\cdot d\omega = \frac{1}{\pi}\int_0^{\infty} \bar{U}(\omega, k)\cdot e^{j(\omega t - kx)}\cdot d\omega\ ,\end{aligned} \tag{4.0.24}$$

however, going too fast towards the result, we would not be able to find the expression for a group velocity. The group velocity here sounds a little bit strange, since the amplitude modulating function is $\cos[d(\omega t - kx)] = 1$, making a signal amplitude flat. We should not forget that here we started with the superposition of two infinitesimally close elementary waveforms, what was the essential condition to treat both of their amplitude functions as virtually constant (and if this was not the case some other amplitude modulating function such as $\cos[\Delta(\omega t - kx)]$ would materialize and have a much more significant influence on the signal amplitude). We have here also the proof that the superposition of only two simple waveforms (resulting in $\bar{\Psi}_{1+2}$ and $\bar{U}_{1+2}$) is sufficiently suitable for representing the original waveform when searching for group and phase velocity of the integral waveform. This is possible since the original signal phase and amplitude are not lost in this process (both in time and frequency terms). This way we are also confirming, that "**external and internal**" signal structures, in time and frequency domain, have the same wave velocities (where the original waveform is equal to a superposition of an infinite number of such elementary waveforms).

4°

Since we already know that "sub-integral" elementary waveform would generate the same wave velocities as the original waveform (here we already applied the names: internal and external waveforms), we can construct another way of finding expressions for wave velocities. Let us consider that the wave function $\Psi(t)$ is the narrow frequency-band wave-packet, or wave-group, which should be a wave-model of certain moving-particle. In other words, the wave-packet $\Psi(t)$ presents the (discrete or infinitesimal) superposition of the number of mutually similar and narrow-zone concentrated elementary waves $y(t) = A(\omega)\cos[\omega t + \Phi(\omega)]$. By introducing the assumption that the frequency band of the wave-packet $\omega \in \Omega$ is very narrow, we would be in a position to treat the signal amplitude function $A(\omega)$ as approximately constant, $A(\omega) \cong A(\omega_0) \cong const.$ in the area around central frequency $\omega_0 \in \Omega$. Under the given conditions, we will be able to make the following approximations,

$$\left\{\begin{aligned}&\Psi(t) = \frac{1}{\pi}\int_0^{\infty}[U_c(\omega)\cos\omega t + U_s(\omega)\sin\omega t]d\omega = \\ &= \frac{1}{\pi}\int_0^{\infty}[A(\omega)\cos(\omega t + \Phi(\omega))]d\omega = a(t)\cos\varphi(t)\end{aligned}\right\} \Rightarrow$$

$$\begin{aligned}&\Psi(t)=a(t)\cos\varphi(t)=\frac{1}{\pi}\int_0^{\infty}\left[A(\omega)\cos(\omega t+\Phi(\omega))\right]d\omega\cong\\&\cong\frac{A(\omega_0)}{\pi}\int_{[\Omega]}\left[\cos(\omega t+\Phi(\omega))\right]d\omega\cong\\&\cong\frac{\Omega}{\pi}A(\omega_0)\sum_{[\omega_i\in\Omega]}\cos(\omega_i t+\Phi(\omega_i))=\frac{\Omega}{\pi}A(\omega_0)\sum_{[\omega_i\in\Omega]}\cos\Theta(\omega_i)\\&\Theta(\omega)=\omega t+\Phi(\omega)\end{aligned}\tag{4.0.25}$$

Now, since we already know that the superposition of only two elementary waveforms would carry the information about characteristic signal velocities, we can accelerate this process if the total wave-packet has only two elementary waveforms,

$$\begin{aligned}&\Psi(t)=a(t)\cos\varphi(t)\cong\frac{A(\omega_0)}{\pi}\int_{[\Omega]}\left[\cos(\omega t+\Phi(\omega))\right]d\omega\cong\frac{\Omega}{\pi}A(\omega_0)\sum_{[\omega_i\in\Omega]}\cos\Theta(\omega_i)\Rightarrow\\&\sum_{[\omega_i\in\Omega]}\cos\Theta(\omega_i)\Leftrightarrow\cos\left[\Theta(\omega_0)-\delta\Theta\right]+\cos\left[\Theta(\omega_0)+\delta\Theta\right]=\cos\left[\Theta_0-\delta\Theta\right]+\cos\left[\Theta_0+\delta\Theta\right]=\\&=2\cos\delta\Theta\cdot\cos\Theta_0\ ,\ \frac{\Omega}{\pi}A(\omega_0)=\text{const.}\end{aligned}\tag{4.0.26}$$

Let us now extend the signal to have all other space and frequency related variables,

$$\begin{aligned}&\omega t\to\omega t-kx\,;\ \omega=\omega(k);\ \Theta(\omega)=\omega t+\Phi(\omega)\to\Theta(\omega,k)=\omega t-kx+\Phi(\omega,k)\Rightarrow\\&2\cos\delta\Theta\cdot\cos\Theta_0=2\cos\left\{\delta\left[\omega t-kx+\Phi(\omega,k)\right]\right\}\cdot\cos\left[\omega_0 t-k_0x+\Phi(\omega_0,k_0)\right]=\\&=2\cos\left[k\left(\frac{\omega}{k}-\frac{\delta x}{\delta t}\right)\cdot\delta t+\left(x-t\frac{\delta\omega}{\delta k}\right)\cdot\delta k+\left(\frac{\delta\omega}{\delta k}-\frac{\delta x}{\delta t}\right)\delta k\cdot\delta t\right]\cdot\cos\left[\omega_0 t-k_0x+\Phi(\omega_0,k_0)\right],\end{aligned}\tag{4.0.27}$$

$$\left\{\begin{aligned}&\delta\left[\omega t-kx+\Phi(\omega,k)\right]=(\omega\cdot\delta t+t\cdot\delta\omega)+\delta\omega\cdot\delta k-(k\cdot\delta x+x\cdot\delta k)-\delta k\cdot\delta x+\delta\Phi(\omega,k)=\\&=k\left(\frac{\omega}{k}-\frac{\delta x}{\delta t}\right)\cdot\delta t+\left(x-t\frac{\delta\omega}{\delta k}\right)\cdot\delta\ k+\left(\frac{\delta\omega}{\delta k}-\frac{\delta x}{\delta t}\right)\delta k\cdot\delta t\,;\ \Phi(\omega,k)=\text{const.}\Leftrightarrow\delta\Phi(\omega,k)=0\end{aligned}\right\}$$

The wave function $2\cos\delta\Theta\cdot\cos\Theta_0$ has the form of an amplitude-modulated elementary wave, where $\cos\Theta_0$ presents the carrier-function and $2\cos\delta\Theta$ presents its amplitude function. Both, amplitude, and wave-carrier function have different phases, and consequently, they should have different, but mutually related velocities: where the velocity of the amplitude function will be the group-velocity = **v** and velocity of the carrier function will be the phase-velocity = $\mathbf{u}$. Now we can find the phase velocity, as usual, (when the phase of the carrier-function is constant),

$$\omega_0 t-k_0x+\Phi(\omega_0,k_0)=\text{const.}\Leftrightarrow u=\frac{dx}{dt}=\frac{\omega_0}{k_0}=\frac{\omega}{k}=\text{phase velocity}\ ,\ (\omega,\ \omega_0)\in\Omega\,,\tag{4.0.28}$$

In addition, from the constant phase of the amplitude signal-member, we will be able to find the group velocity as,

$$k(\frac{\omega}{k}-\frac{\delta x}{\delta t})\cdot\delta t+(x-t\frac{\delta\omega}{\delta k})\cdot\delta k+(\frac{\delta\omega}{\delta k}-\frac{\delta x}{\delta t})\delta k\cdot\delta t=\text{Const.}\Leftrightarrow$$
$$\frac{\omega}{k}=\frac{\delta x}{\delta t}=\frac{dx}{dt}=u,\ (x-t\frac{\delta\omega}{\delta k})\cdot\delta k+(\frac{\delta\omega}{\delta k}-\frac{\delta x}{\delta t})\delta k\cdot\delta t=\text{Const.}\Rightarrow \tag{4.0.29}$$
$$\Rightarrow\frac{\delta\omega}{\delta k}=\frac{\delta x}{\delta t}=\frac{dx}{dt}=v=\text{group velocity}$$

5°

What we can conclude and summarize from already presented steps (from 1° to 4°) regarding waveform velocities is:

a) That the most elementary (and non-dispersive) waves, which are building elements of all other waveforms we know in physics, should have forms of specific simple harmonic functions both in time and space coordinates (and being like wavelets belonging to Gaussian pulses and Gabor wavelets. See later (4.0.35) in relation to Kotelnikov-Shannon Theorem). The meaning of energy atomizing, discretization, quantization, and exchanges in Physics and in our Universe should also be closely related to here-elaborated signal analysis and synthesis in the framework of Analytic Signal and Kotelnikov-Shannon theorem.

b) That there is a significant level of structural symmetry and integrity regarding constructing well-operating mathematical models of wave functions in time, space, or joint time-space domains ($\Psi(t),\Psi(x),\Psi(t,x)$), and that similar symmetry is also extending to frequency-momentum domains with spectral functions ($A(\omega),A(k),A(\omega,k)$ and $\Phi(\omega),\Phi(k),\Phi(\omega,k)$). Mentioned symmetry is particularly well-exposed if Analytic Signals modeling is applied, as for instance: Every waveform element, or most elementary signal of other more complex waveforms, "**Internally and Externally**", is carrying all important information regarding waveform velocities, both in time and frequency domains, where terms "**internally and externally**" have the same meaning as already explained in earlier steps. In other words, saying the same, all elementary signals, and other time-space and frequency dependent parameters of relevance for conceptualizing different wave motions in the world of Physics, are mutually dependent, connected, proportional and well-united, for instance: $x=x(t),\ \omega=\omega(k)\ ,\ v\ =\ v(x,\omega)...$

c) That the total signal energy is carried only by the signal amplitude or envelope function either in time or frequency domain, which is propagating with a group velocity.

d) That the only sufficiently narrow, elementary, "*time-space-frequency-energy*" limited and finite, (band-limited in all domains), Gaussian-Gabor signal forms are of the most significant relevance in wave motions analyses. Such building blocks of matter (and our Universe) are synthesizing or decomposing all other more complex waveforms. It seems that the highest preferences of Nature are in communicating between all relatively stable energy-carrying states using such, Gaussian (bell curve shaped amplitude), band-limited elementary waves, because such wave packets are well defined in their temporal and spectral domains. Kotelnikov signal analysis or synthesis addresses such wave packets.

Let us now respect here summarized facts (a, b, c, and d) and construct the following, elementary and band-limited wave-packet or wave group,

$$\Psi(t,x) = a(t,x)\cos\varphi(t,x) = \frac{1}{\pi}\int_{\omega_0-\Delta\omega}^{\omega_0+\Delta\omega}\left[A(\omega)\cos(\omega t - kx + \Phi(\omega))\right]d\omega = \\ = \frac{1}{\pi}\int_{k_0-\Delta k}^{k_0+\Delta k}\left[A(k)\cos(\omega t - kx + \Phi(k))\right]dk \,. \qquad (4.0.30)$$

The most general form and properties of any wavefunction that is not narrow-band limited (valid also for (4.0.30)), is already given at the beginning of this chapter with (4.0.1) – (4.0.5) and T.4.0.1. In other words, elementary wave groups or wave packets (4.0.30) could present kind of the elementary signal basis of any other wave function or signal, meaning that we will be able to use such elementary signals and synthesize or decompose much more complex and even ***non-band-limited*** signals. It will be shown later (see chapter 4.3), that starting from such analytic wave functions and wave packets, (4.0.1) – (4.0.5) and (4.0.30), we will be able to establish generalized forms of Schrödinger wave equation, as complex forms of Classical wave equations (on a different and more elementary, natural, and deterministic way compared to quantum theory practices).

Another challenging situation here, with the narrow-band limited signal like (4.0.30) could be to connect it with a single photon energy $\tilde{E}_o = hf_o$, $h = const.$*, for example,*

$$\tilde{E}_o = \int_{-\infty}^{+\infty}\Psi^2(\omega t - kx)dt = \int_{k_o-\Delta k}^{k_o+\Delta k}\left[\frac{A(\omega t - kx)}{\sqrt{2}}\right]^2 dk = \int_{\omega_o-\Delta\omega}^{\omega_o+\Delta\omega}\left[\frac{A(\omega t - kx)}{\sqrt{\pi}}\right]^2 d\omega = Const\cdot\ \omega_o = (2\pi\cdot Const)\cdot f_o$$

Since here we are analyzing a band-limited and finite, frequency very narrow elementary wave-packet (4.0.30), we can express the dispersion function $\omega = \omega(k)$ in the close vicinity of $\omega_0, k_0;\ \omega_0 = \omega(k_0)$ as,

$$\omega(k) = \omega(k_0) + (k-k_0)(\frac{d\omega}{dk})_{(k=k_0)} + \frac{1}{2}(k-k_0)^2(\frac{d^2\omega}{dk^2})_{(k=k_0)} + ... \cong \\ \cong \omega(k_0) + (k-k_0)(\frac{d\omega}{dk})_{(k=k_0)} \,. \qquad (4.0.31)$$

Now, the elementary and band-limited wave-packet can be formulated as,

$$\Psi(t,x) = \frac{A(k_0)}{\pi}\int_{k_0-\Delta k}^{k_0+\Delta k}\cos\left[\omega t + (k-k_0)(\frac{d\omega}{dk})_0\cdot t - kx + \Phi(k_0)\right]dk = \\ = \frac{2A(k_0)}{\pi}\Delta k\frac{\sin \Delta k\cdot\left[(\frac{d\omega}{dk})_0\cdot t - x + \Phi(k_0)\right]}{\Delta k\cdot\left[(\frac{d\omega}{dk})_0\cdot t - x + \Phi(k_0)\right]}\cos(\omega_0 t - k_0 x) = a(t,x)\cos\varphi(t,x)\,, \qquad (4.0.32) \\ \frac{2A(k_0)}{\pi}\Delta k = constant,\ \Phi(k_0) = \ CONST.,\ \cos(\omega_0 t - k_0 x) = \cos\varphi(t,x).$$

The constant phase of the carrier wavefunction will generate the phase velocity, and the constant phase of the amplitude function will generate the group velocity (as in all the cases presented earlier),

$$\begin{aligned} &\omega_0 t - k_0 x = \text{const.} \Rightarrow u = \frac{dx}{dt} = \frac{\omega_0}{k_0} = \text{phase velocity} \\ &(\frac{d\omega}{dk})_0 \cdot t - x + \Phi(k_0) = \text{Const.} \Rightarrow v = \frac{dx}{dt} = (\frac{d\omega}{dk})_0 = \text{group velocity}. \end{aligned} \qquad (4.0.33)$$

Almost the same example in different and mutually similar variants is very often present in most Quantum Theory books in the chapters explaining group and phase velocity. Here, it is a little bit more convincing, more general, and more evident why, and when, conclusions based on such band-limited elementary signals are correct. We can also show that any other, more complex, or arbitrary (energy-finite) waveform can be presented as the superposition of other elementary signals. In Mathematics, modern Telecommunications Theory, and Digital Signal Processing practice, we can find many methods and formulas for discrete signal representations or signals sampling. This means that time-continuous signals, or wave functions, could be adequately represented (errorless, without residuals) if we implement sufficiently short time-increments of signal sampling, and create discrete series of such signal samples. For instance, if a continuous wave-function, $\Psi(t)$ is a frequency band-limited (and at the same time it should also be an energy-finite function, similar to solitons), by applying Kotelnikov-Shannon sampling theorem, we can express it concerning its samples-values, as summation or superposition of "sinc-functions" (including similar soliton wavefunctions), or on some way discretized wave-groups $\mathbf{\Psi(n \cdot \delta t)}$, as for instance (see **[8]**),

$$\begin{aligned} &\Psi(t) = a(t)\cos\varphi(t) = \sum_{n=-\infty}^{+\infty} \Psi(n \cdot \delta t) \frac{\sin \Omega(t - n \cdot \delta t)}{\Omega(t - n \cdot \delta t)} = \\ &= \sum_{n=-\infty}^{+\infty} a(n \cdot \delta t) \frac{\sin\Omega(t - n \cdot \delta t)}{\Omega(t - n \cdot \delta t)} \cos\varphi(n \cdot \delta t),\ \Psi(n \cdot \delta t) = a(n \cdot \delta t)\cos\varphi(n \cdot \delta t), \end{aligned} \qquad (4.0.34)$$

Where Ω is the highest frequency in the spectrum of $\Psi(t)$, and we could consider that Ω is the total frequency duration of the signal $\Psi(t)$.

Since sampling frequency-domain of signal-amplitude, Ω_L, is always in a lower frequency range than the frequency range of its phase function $\Omega = \Omega_H$, and since the total signal energy is captured only by the signal amplitude-function, we should also be able to present the same signal, again as superposition of "sinc-functions":

$$\Psi(t) = a(t)\cos\varphi(t) = \sum_{n=-\infty}^{+\infty} a(n\cdot\delta t)\frac{\sin\Omega(t-n\cdot\delta t)}{\Omega(t-n\cdot\delta t)}\cos\varphi(n\cdot\delta t) =$$
$$= \left[\sum_{n=-\infty}^{+\infty} a(n\cdot\delta t_L)\frac{\sin\Omega_L(t-n\cdot\delta t_L)}{\Omega_L(t-n\cdot\delta t_L)}\right]\cdot\left[\sum_{n=-\infty}^{+\infty}\cos\varphi(n\cdot\delta t)\frac{\sin\Omega_H(t-n\cdot\delta t)}{\Omega_H(t-n\cdot\delta t)}\right], \quad (4.0.35)$$
$$\overline{\Psi}(t) = \sum_{n=-\infty}^{+\infty} a(n\cdot\delta t)\frac{\sin\Omega(t-n\cdot\delta t)}{\Omega(t-n\cdot\delta t)}e^{j\varphi(n\cdot\delta t)} = a(t)e^{j\varphi(t)},$$

$$a(t) = \sum_{n=-\infty}^{+\infty} a(n\cdot\delta t_L)\frac{\sin\Omega_L(t-n\cdot\delta t_L)}{\Omega_L(t-n\cdot\delta t_L)},\ \cos\varphi(t) = \sum_{n=-\infty}^{+\infty}\cos\varphi(n\cdot\delta t)\frac{\sin\Omega_H(t-n\cdot\delta t)}{\Omega_H(t-n\cdot\delta t)}$$
$$\delta t \le \frac{\pi}{\Omega} = \frac{1}{2F} < \delta t_L\ ,\ \Omega = \Omega_H = 2\pi F = 2\pi F_H,\ \delta t_L \le \frac{\pi}{\Omega_L} = \frac{1}{2F_L}\ ,\ \Omega_L = 2\pi F_L < \Omega. \quad (4.0.36)$$

As we can see by simple comparison, the sampled waveforms under the summation signs, given by (4.0.35) and (4.0.36), have the same (sinc-function) form as an elementary wave-packet (4.0.32), what is giving much more weight and importance to Kotelnikov-Shannon sampling theorem (regarding modeling natural waveforms and energy communications within our Universe). *In fact, (4.0.35) and (4.0.36) are also defining kind of signals atomizing, packing, and formatting, what would become important later when we start making equivalency relations between wave-packets and moving-particles. Planck-Einstein photons or wave-packets quantizing should be related or fully compatible to what we find in (4.0.30) – (4.0.44). See also about PWDC conceptualization in Chapter 10, (PWDC means Particle Wave Duality Concept).*

Now, by applying analogy with already analyzed examples (regarding waveform velocities), we can express the same signal, (4.0.35), as the function of extended time-space variables as,

$$\left\{\begin{array}{l} x = x(t),\ \omega = \omega(k), \\ \omega t \to \omega t - kx,\ \omega\cdot\delta t \to \omega\cdot\delta t - k\cdot\delta x, \\ \Omega t \to \Omega t - Kx,\ \Omega\cdot\delta t \to \Omega\cdot\delta t - K\cdot\delta x \\ \Omega(t-n\cdot\delta t) \to \Omega(t-n\cdot\delta t) - Kx = K\cdot\left[(\frac{\delta\omega}{\delta k})\cdot(t-n\cdot\delta t) - x\right] \\ n\cdot\delta t \to n\cdot\delta t - \frac{K}{\Omega}n\cdot\delta x,\ \frac{\delta\omega}{\delta k} = \frac{\Omega}{K},\ \Omega = 2\pi F,\ K = 2\pi F_x \\ \delta t \le \frac{\pi}{\Omega} = \frac{1}{2F},\quad \delta f \le \frac{1}{2T},\ \delta x \le \frac{1}{2K} \end{array}\right\} \Rightarrow \quad (4.0.37)$$

$$\Psi(t) = a(t)\cdot\cos\varphi(t) = \left[\sum_{n=-\infty}^{+\infty} a(n\cdot\delta t_L)\frac{\sin\Omega_L(t-n\cdot\delta t_L)}{\Omega_L(t-n\cdot\delta t_L)}\right]\cdot\cos\varphi(t) =$$
$$= \left[\sum_{n=-\infty}^{+\infty} a(n\cdot\delta t_L)\frac{\sin\Omega_L(t-n\cdot\delta t_L)}{\Omega_L(t-n\cdot\delta t_L)}\right]\cdot\cos\varphi(t) = \tag{4.0.38}$$
$$= \left[\sum_{n=-\infty}^{+\infty} a(n\cdot\delta t_L)\frac{\sin K_L\cdot\left[(\frac{\delta\omega}{\delta k})\cdot(t-n\cdot\delta t_L)-x\right]}{K_L\cdot\left[(\frac{\delta\omega}{\delta k})\cdot(t-n\cdot\delta t_L)-x\right]}\right]\cdot\cos\varphi(t) ,$$

$$A(f) = A(\frac{\omega}{2\pi}) = \sum_{n=-\infty}^{+\infty} A(n\cdot\delta f_L)\frac{\sin 2\pi T_L(f-n\cdot\delta f_L)}{2\pi T_L(f-n\cdot\delta f_L)} ,\ \delta f_L \leq \frac{1}{2T_L} . \tag{4.0.39}$$

Let us consider that there is a sufficiently high number of samples, N and M, that would adequately represent or reconstruct the same amplitude wave-functions, (4.0.38) and (4.0.39), both in time and frequency domain (to capture the same and total signal energy amount). Kotelnikov-Shannon sampling theorem combined with Parseval's theorem is giving the option to express the signal-energy, and law of energy conservation as,

$$\tilde{E} = \int_{-\infty}^{+\infty}|\Psi(t)|^2 dt = \frac{1}{\pi}\int_0^{\infty}A^2(\omega)\ d\omega = \frac{1}{2}\int_{-\infty}^{+\infty}a^2(t)\cdot dt = \delta t\cdot\sum_{(n)}|\Psi(n\cdot\delta t)|^2$$
$$= \frac{1}{2}\cdot\delta t_L\cdot\sum_{(n)}a^2(n\cdot\delta t_L) = 2\cdot\delta f_L\cdot\sum_{(m)}A^2(m\cdot\delta f_L) = \frac{\delta\omega_L}{\pi}\cdot\sum_{(m)}A^2(m\cdot\delta f_L),$$
$$F_L < F_H,\ \delta t_L > \delta t_H = \delta t\ ,\ n\in[1,2,3...\ N],\ m\in[1,2,3...\ M],$$
$$T_L = N\cdot\delta t_L = \frac{1}{2F_L},\ F_L = \frac{\Omega_L}{2\pi} = M\cdot\delta f_L = \frac{1}{2T_L},\ \Omega_L T_L > \pi\ ,$$
$$\frac{\delta f_L}{\delta t_L} = \frac{\delta\omega_L}{2\pi\cdot\delta t_L} = \frac{N}{M}\cdot\frac{F_L}{T_L} = \frac{1}{2\pi}\cdot\frac{N}{M}\cdot\frac{\Omega_L}{T_L} = \frac{\sum_{(n)}a^2(n\cdot\delta t_L)}{8\pi\cdot\sum_{(m)}A^2(m\cdot\delta f_L)} , \tag{4.0.40}$$
$$\frac{1}{2}\cdot\frac{N}{M}\cdot\frac{\pi}{T_L^2} < \frac{\delta f_L}{\delta t_L} = \frac{\delta\omega_L}{2\pi\cdot\delta t_L} = \frac{N}{M}\cdot\frac{F_L}{T_L} = \frac{1}{2\pi}\cdot\frac{N}{M}\cdot\frac{\Omega_L}{T_L} < \frac{1}{2\pi}\cdot\frac{N}{M}\cdot\frac{\Omega_L^2}{\pi}.$$

Since the *amplitude-function frequency-interval* F_L could be for orders of magnitude lower than *carrier-function frequency-interval* $F_H = F$, it is clear that also many samples necessary to reconstruct the signal amplitude $\mathbf{a(t)}$ could be for an order of magnitude lower than the number of samples which is reconstructing the total wave function $\Psi(t)$. Here it is also the beginning of the explanation of how particles with non-zero rest masses could be created by specific superposition or packing of elementary wave-packets.

If we are interested in exploring the signal-energy options, we can continue developing only the signal amplitude function (as an Analytic Signal function), making the next similar step,

$$\left\{\begin{array}{l} a_0(t) = a(t) = \sum_{n=-\infty}^{+\infty} a(n\cdot\delta t_L)\frac{\sin\Omega_L(t-n\cdot\delta t_L)}{\Omega_L(t-n\cdot\delta t_L)} \Leftrightarrow \\ \Leftrightarrow \tilde{E}_0 = \tilde{E} = \frac{1}{2}\cdot\delta t_{L0}\cdot\sum_{(n)} a^2(n\cdot\delta t_{L0}) = 2\cdot\delta f_{L0}\cdot\sum_{(m)} A^2(m\cdot\delta f_{L0}) \\ \delta t_{L0} = \delta t_L\ ,\ \delta f_{L0} = \delta f_L \end{array}\right\} \Rightarrow \tag{4.0.41}$$

$$\Rightarrow \left\{\begin{array}{l} a_1(t) = \sum_{n=-\infty}^{+\infty} a_1(n\cdot\delta t_{L1})\frac{\sin\Omega_{L1}(t-n\cdot\delta t_{L1})}{\Omega_{L1}(t-n\cdot\delta t_{L1})}, \\ a_1(t) = \sqrt{a_0^2(t) + \left\{H\left[a_0(t)\right]\right\}^2} \Leftrightarrow \\ \Leftrightarrow \tilde{E}_1 = \frac{1}{2}\cdot\delta t_{L1}\cdot\sum_{(n)} a_1^2(n\cdot\delta t_{L1}) = 2\cdot\delta f_{L1}\cdot\sum_{(m)} A_1^2(m\cdot\delta f_{L1}) \\ \Leftrightarrow \tilde{E}_0 = \frac{1}{2^2}\cdot\delta t_{L0}\cdot\delta t_{L1}\cdot\sum_{(n)} a_1^2(n\cdot\delta t_{L1}) = 2^2\cdot\delta f_{L0}\cdot\delta f_{L1}\cdot\sum_{(m)} A^2(m\cdot\delta f_{L1}) \end{array}\right\} \Rightarrow \tag{4.0.42}$$

$$\Rightarrow \left\{\begin{array}{l} a_2(t) = \sum_{n=-\infty}^{+\infty} a_2(n\cdot\delta t_{L2})\frac{\sin\Omega_{L2}(t-n\cdot\delta t_{L2})}{\Omega_{L2}(t-n\cdot\delta t_{L2})}, \\ a_2(t) = \sqrt{a_1^2(t) + \left\{H\left[a_1(t)\right]\right\}^2} \Leftrightarrow \\ \Leftrightarrow \tilde{E}_2 = \frac{1}{2}\cdot\delta t_{L2}\cdot\sum_{(n)} a_2^2(n\cdot\delta t_{L2}) = 2\cdot\delta f_{L2}\cdot\sum_{(m)} A_2^2(m\cdot\delta f_{L2}) \Rightarrow \\ \Leftrightarrow \tilde{E}_0 = \frac{1}{2^3}\cdot\delta t_{L0}\cdot\delta t_{L1}\cdot\delta t_{L2}\cdot\sum_{(n)} a_2^2(n\cdot\delta t_{L2}) = 2^3\cdot\delta f_{L0}\cdot\delta f_{L1}\cdot\delta f_{L2}\cdot\sum_{(m)} A^2(m\cdot\delta f_{L2}) \end{array}\right\} \Rightarrow \tag{4.0.43}$$

…

…

$$\Rightarrow \left\{\begin{array}{l} a_k(t) = \sum_{n=-\infty}^{+\infty} a_k(n\cdot\delta t_{Lk})\frac{\sin\Omega_{Lk}(t-n\cdot\delta t_{Lk})}{\Omega_{Lk}(t-n\cdot\delta t_{Lk})}, \\ a_k(t) = \sqrt{a_{k-1}^2(t) + \left\{H\left[a_{k-1}(t)\right]\right\}^2} \Leftrightarrow \\ \Leftrightarrow \tilde{E}_k = \frac{1}{2}\cdot\delta t_{Lk}\cdot\sum_{(n)} a_k^2(n\cdot\delta t_{Lk}) = 2\cdot\delta f_{Lk}\cdot\sum_{(n)} A_2^2(n\cdot\delta f_{Lk}) \Rightarrow \\ \Leftrightarrow \tilde{E}_0 = \frac{1}{2^{k+1}}\cdot\prod_{n=0}^{k}\delta t_{Ln}\cdot\sum_{(n)} a_k^2(n\cdot\delta t_{Lk}) = 2^{k+1}\cdot\prod_{n=0}^{k}\delta f_{Ln}\cdot\sum_{(n)} A^2(n\cdot\delta f_{Lk}) \end{array}\right\}, \tag{4.0.44}$$

Until the level "k", when we arrive at the most representative amplitude function (since this mathematical process should converge to a specific result, because initially, we assumed that we are dealing only with energy-finite functions). This process, (4.0.40) - (4.0.44), looks very motivating for creative brainstorming. It could be that *here we are just unveiling new signal-atomizing and formatting techniques (being relevant in the world of Physics), and probably coming closer to the explanation why and when Planck's and de Broglie relations regarding wave-packet energy and particle wave-properties are working well (but we should very carefully avoid generalization of energy quanta, as presently used in Quantum Theory).*

4.0.6. Traveling Wave Packets and Waves Dispersion

Velocities of the waveforms (4.0.41) - (4.0.44) are again equal to the already known group and phase velocity (found in the same way as before). Also, we are now able to extract another property of the group velocity, which shows that the same proportionality is conserved between signal's time, space, and frequency-durations (of course, valid only in cases of non-dispersive waves, where signal propagation velocity is frequency independent),

$$v = \frac{\delta\omega}{\delta k} = \frac{\Omega_{Lm}}{K_{Lm}} = \frac{\Omega_{Hm}}{K_{Hm}} = \frac{\delta x}{\delta t} = \frac{dx}{dt} = \frac{d\omega}{dk}\ ,\ m = 0,1,2,3,... \qquad (4.0.45)$$

Here is good place to mention that based on familiar considerations regarding "spatial and time couplings", as elaborated in [36], from Anthony D. Osborne and N. Vivian Pope (Light-Speed, Gravitation, and Quantum Instantaneity), we should be able to recreate and update almost complete Relativity Theory (avoiding using original A. Einstein's concepts).

For the group and phase velocities (as found in all previously shown examples), it seems essential that a waveform time-domain function $\Psi(t)$ should be fully compliant and extendable to an equivalent waveform in a time-space domain $\Psi(t,x)$. In addition, the new *time-space waveform,* $\Psi(t,x)$*, should represent a non-dispersive traveling wave (meaning that signal propagation velocities are not frequency dependent).* As we have seen, there are straightforward rules for such waveform transformations (based on a much more general symmetry of space and time variables, which is in the background, here), that can be described as:

a) *First, an arbitrary waveform* $\Psi(t,x)$ *should be presented as a superposition of elementary waves, where simple harmonic and sinc-functions are involved, such as* $\cos\omega t$, $\sin\omega t$, $\frac{\sin\Omega t}{\Omega t}\cos(\omega t)$, $e^{-\beta t}\cdot\frac{\sin\Omega t}{\Omega t}\cos(\omega t)$... , (being similar to Solitons, Gaussian pulses and Gabor wavelets with bell-curve envelopes)*…*

b) *Time and frequency variables which are arguments of simple-harmonic functions can be spatially extended, as for instance,*

$$\begin{aligned} & x = x(t),\ \omega = \omega(k), \\ & \omega t \to \omega t - kx,\ \omega\cdot\delta t \to \omega\cdot\delta t - k\cdot\delta x, \\ & \Omega t \to \Omega t - Kx,\ \Omega\cdot\delta t \to \Omega\cdot\delta t - K\cdot\delta x\ , \end{aligned}$$

alternatively, *(4.0.46)*

$$\begin{aligned} & x = x(t),\ \omega = \omega(k), \\ & kx \to kx - \omega t,\ k\cdot\delta x \to k\cdot\delta x - \omega\cdot\delta t, \\ & Kx \to Kx - \Omega t,\ K\cdot\delta x \to K\cdot\delta x - \Omega\cdot\delta t\ . \end{aligned}$$

In (4.0.46) all minus "-" signs could be replaced by plus "+" signs, changing only the direction of propagation of a waveform in question. Furthermore, in some cases, we *could combine inwards and outwards signals propagation, without influencing already established conclusions regarding signal integrity and signal velocities. Such simple rules can be well explained and supported by analyzing many elementary wave phenomena known in Physics, where only simple harmonic waveforms and oscillations are involved. Here, we are just generalizing or applying mentioned rules to any other arbitrary waveform, which presents a non-dispersive traveling wave. Effectively, matter-waves and oscillations related to the world of micro-physics, electrons, atoms, and photons… are in most cases presentable as energy-finite functions, composed of simple harmonic and elementary waveforms or wavelets (applicable also in cases of macro-particles motions, which are effectively composed of similar simple harmonic waveforms).*

In fact, instead of extending $\Psi(t) \to \Psi(t,x)$*, we could start from* $\Psi(t,x) \to \Psi(t,0)$ *and go to* $\Psi(t), x = \text{const.}$ *(and then deductively reconstruct steps mentioned above), but for the analysis presented here it was mathematically simpler to start from* $\Psi(t)$*). There are many different strategies (in available literature) regarding waves dispersion, considering many possible situations. Here, we are limiting our framework to items directly applicable to basic particle-wave phenomenology among elementary particles, micro-particles, photons, and de Broglie matter waves (such as different interactions, diffractions, superpositions, and interferences related to elementary-particles, to Compton and Photoelectric Effect, to particles annihilation, or creation, etc.). In all such examples, it looks as if Nature is showing or respecting quite simple interaction rules between wave-packets and particles (where group velocity has its significant place).*

General case of <u>*Dispersion Relation*</u> $\omega = \omega(k)$*, valid for any wave motion, will be addressed later (see (4.0.80) in this chapter) as,*

$$\omega(k) = 2\pi f = ku = \frac{kv}{1+\sqrt{1-v^2/c^2}} = \left(\frac{2\pi}{h}\right)\frac{pv}{1+\sqrt{1-v^2/c^2}} = \\ = \frac{\sqrt{1-v^2/c^2}}{1+\sqrt{1-v^2/c^2}} \cdot \frac{\hbar}{m}k^2 \cong \begin{Bmatrix} \frac{\hbar}{2m}k^2, \text{for } v \cong \frac{\hbar}{m}k = 2u << c, u \cong \frac{\hbar}{2m}k = \frac{v}{2} << c \\ ck, \quad \text{for } v \approx u \approx c \end{Bmatrix}, \tag{4.0.80-1}$$

Consequences concerning (4.0.80-1) are challenging (regarding the relativistic and quantum-mechanical understanding of mass, momentum, and wave propagation vector), as for instance,

$$m(v) = \frac{m}{\sqrt{1-v^2/c^2}} \cong \begin{Bmatrix} m, \quad \text{for } v << c \\ \frac{\hbar k}{c}\sqrt{1-v^2/c^2} = \frac{p}{c}\sqrt{1-v^2/c^2} = \frac{mv}{c} \cong m, \text{ for } v \approx c \end{Bmatrix},$$
$$p(v) = \frac{mv}{\sqrt{1-v^2/c^2}} \cong \begin{Bmatrix} mv, \quad \text{for } v << c \\ \frac{\hbar k}{c}v\sqrt{1-v^2/c^2} = \frac{pv}{c}\sqrt{1-v^2/c^2} = \frac{mv^2}{c} \cong mv \cong mc, \text{ for } v \approx c \end{Bmatrix}, \tag{4.0.80-2}$$
$$k(v) = \frac{2\pi}{\lambda} = \frac{2\pi}{h}p = \frac{2\pi}{h}\frac{mv}{\sqrt{1-v^2/c^2}} \cong \begin{Bmatrix} \frac{2\pi}{h}mv, \quad \text{for } v << c \\ \frac{k}{c}v\sqrt{1-v^2/c^2} = \frac{2\pi}{h}\frac{pv}{c}\sqrt{1-v^2/c^2} = \frac{2\pi}{h}\frac{mv^2}{c} \cong \frac{2\pi}{h}mv \cong \frac{2\pi}{h}mc, \text{ for } v \approx c \end{Bmatrix}.$$

4.0.7. Wave Packets already exercised in Physics

From many different experimental observations (Davisson-Germer, G.P. Thomson, Photo Effect, and Compton Effect…), we are faced with the reality that all microparticles, (regardless of origin) exhibit diffraction and interference effects, and have de Broglie wavelength $\lambda = h/\tilde{p}$, where $\tilde{p} = \sqrt{2mE} = \gamma\, mv$, $h = 6.626176 \cdot 10^{-34}\, J \cdot s$. *Also, the same value for Planck's constant* h *is found applicable for mathematical curve fitting of black body radiation formula, for explaining Compton and Photoelectric effects, for quantifying energy-momentum relations regarding photons, and for processing almost any other kind of sub-atomic, micro, and elementary particles. This looks like introducing some energy atomization, discretization, or quantization in the form of elementary wave packets, wave groups, or energy quanta (*as in here-elaborated signal analysis and synthesis in the framework of Analytic Signal modeling, and Kotelnikov-Shannon theorem*). This has been the reason to exercise the concept where specific wave group or wave packet could replace moving particle (of course only regarding few of mentioned aspects), but we will also find out that similar Kotelnikov-Shannon-Gabor, bell-curve-shaped wave groups are present in macrocosmic "energy-moments" exchanges and communications.*

As we will see, the wave-packet or wave-group is modeled to have its group wave-velocity that should be equal to certain equivalent-particle velocity. Also, it will be shown that the wave-packet energy should be equal to the equivalent-particle kinetic energy. For instance, an electromagnetic wave-packet of energy, or photon, behaving as a wave-packet, is experimentally showing waves and/or particles properties (in different experimental situations). In Physics is demonstrated applicability and compatibility of Planck's narrow-band wave-packet energy as $\tilde{\mathbf{E}} = \mathbf{hf}$ (originally assumed and applied only to photons, and later extended to other wave-groups and microparticles), with Energy and Momentum conservation laws, as well as with de Broglie matter-waves wavelength, $\lambda = h/\tilde{p}$ (where $\tilde{m} = hf/c^2$, $\tilde{p} = hf/c$, $u = \lambda f$), without precisely and completely showing what really makes those relations correct. Mentioned mathematics was working well in explaining many experimental situations, and this has been the justification of Planck's energy formula, and de Broglie's wavelength, but this should not be mixed or treated as a universally meaningful energy quant. Similar wave (or motional) energy relation $\tilde{E} = Hf$ is even applicable to planetary or solar systems, but the constant "H" is no more equal to already known Planck constant "h" (see the second chapter: 2.3.3. Macro-Cosmological Matter-Waves and Gravitation). Moreover, similar relations (with specific H constant) are also applicable to vortex flow meters, related to fluid flow vortices, as speculated in chapter 4.1; -see equations (4.3-0), (4.3-0)-a,b,c,d...

We need the answer to a simple question how and why only one characteristic (central) frequency (multiplied by Planck's constant h, or by another H constant) can represent the motional wave-energy of a narrow-band wave-group (or what means that frequency, and what kind of wave group is behind). The first intuitive and logical starting point is to imagine that this is just a mean frequency, $\bar{f}$, of the corresponding narrow-band, elementary matter-wave-group calculated in relation to its energy (where a wave group, or wave packet, or de Broglie matter-wave is composed from infinite number of elementary waves, covering certain (relatively small) frequency interval: $\mathbf{0 \leq f_{min.} \leq \bar{f} \leq f_{max.}} < \infty$). The energy of such narrow-frequency-band wave-group (established by connecting Parseval's and Planck's energy forms) is,

$$\tilde{E} = \int_{-\infty}^{+\infty} \Psi^2(t)\,dt = \frac{1}{2}\int_{-\infty}^{+\infty} a^2(t)\,dt = \frac{1}{\pi}\int_{0}^{\infty} A^2(\omega)\,d\omega = h\bar{f} = h\frac{u}{\lambda} = pu = \tilde{m}c^2. \tag{4.0.47}$$

Now (by definition) we can find the mean frequency of such wave-packet as,

$$\bar{f} = \frac{\frac{1}{\pi}\int_0^{\infty} f\cdot[A(\omega)]^2 d\omega}{\frac{1}{\pi}\int_0^{\infty}[A(\omega)]^2 d\omega} = \frac{\frac{1}{\pi}\int_0^{\infty} f\cdot[A(\omega)]^2 d\omega}{\tilde{E}} = \frac{\frac{1}{2}\int_{-\infty}^{+\infty} f(t)\cdot a^2(t)\cdot dt}{\frac{1}{2}\int_{-\infty}^{+\infty} a^2(t)\cdot dt} = \frac{\frac{1}{2}\int_{-\infty}^{+\infty} f(t)\cdot a^2(t)\cdot dt}{\tilde{E}}, \tag{4.0.48}$$

$$\omega(t) = \frac{\partial\varphi(t)}{\partial t} = \dot{\varphi}(t) = \frac{\Psi(t)\dot{\hat{\Psi}}(t) - \dot{\Psi}(t)\hat{\Psi}(t)}{a^2(t)} = \mathrm{Im}\left[\frac{\dot{\overline{\Psi}}(t)}{\overline{\Psi}(t)}\right] = 2\pi f(t).$$

Moreover, we can replace it into a wave energy form, (4.0.47),

$$\tilde{E} = \frac{1}{\pi}\int_0^{\infty}[A(\omega)]^2 d\omega = h\bar{f} = h\frac{\frac{1}{\pi}\int_0^{\infty} f\cdot[A(\omega)]^2 d\omega}{\tilde{E}} \Rightarrow \tilde{E}^2 = \frac{h}{\pi}\int_0^{\infty} f\cdot[A(\omega)]^2 d\omega\,. \tag{4.0.49}$$

Using one of the most general formulas valid for all definite integrals (and applying it to (4.0.49)), we can prove that the wave energy (of a wave-group in question) should be equal to the product between Planck's constant (h or H) and mean frequency of the wave group in question, as follows:

$$\left\{\begin{array}{l} \int_a^b f(x)\cdot g(x)dx = f(c)\int_a^b g(x)dx\,, a < c < b\,, g(x) \geq 0\,, \\ f(x) \text{ and } g(x) \text{ are continuous in } [a \leq x \leq b], \\ f(x) = f,\ g(x) = [A(\omega)]^2 > 0, x = \omega \in (0,\ \infty) \end{array}\right\} \Rightarrow$$

$$\tilde{E}^2 = \frac{h}{\pi}\int_0^{\infty} f\cdot[A(\omega)]^2\, d\omega = \frac{h}{\pi}\cdot\bar{f}\cdot\int_0^{\infty}[A(\omega)]^2\, d\omega = hf\cdot\tilde{E} = h\bar{f}\cdot\tilde{E} \Rightarrow \tag{4.0.50}$$

$$\Rightarrow \tilde{E} = h\bar{f} = hf,\ \Delta\tilde{E} = h\cdot\Delta f,\ f = \bar{f}\,.$$

If Planck's energy of a photon or any other equivalent narrow-band wave-group deals with a mean frequency of that wave-group, the same should be valid for de Broglie wavelength, as well as for its phase and group velocities. Consequently, all of them could be treated as mean values, like describing a motion of an "effective center of inertia, or center of gravity" of that wave-group). We do not need to mark them as mean-values, as it was the case with mean-frequency in (4.0.50) since we know that all of them should anyway be mean values $\mathbf{\bar{f} = f,\ \bar{\lambda} = \lambda,\ \bar{u} = u,\ \bar{v} = v}$), and applicable only to narrow frequency band signals (where $\lambda = h/p,\ \tilde{E} = hf,$

$v = u - \lambda du/d\lambda = dE/dp = d\omega/dk,\ u = \lambda f = \tilde{E}/p = \omega/k \ldots$). Also, for sufficiently narrow-banded signals, mean-values of a group and phase velocity should also be presentable as:

$$\bar{v}_g = \bar{v} = \frac{\frac{1}{\pi}\int_0^\infty v\cdot[A(\omega)]^2 d\omega}{\frac{1}{\pi}\int_0^\infty [A(\omega)]^2 d\omega} = \frac{\frac{1}{\pi}\int_0^\infty \frac{d\omega}{dk}\cdot[A(\omega)]^2 d\omega}{\frac{1}{\pi}\int_0^\infty [A(\omega)]^2 d\omega} = \frac{\frac{1}{\pi}\int_0^\infty \frac{d\omega}{dk}\cdot[A(\omega)]^2 d\omega}{\tilde{E}} = \frac{\delta\omega}{\delta k}, \quad (4.0.51)$$

$$\bar{v}_f = \bar{u} = \frac{\frac{1}{\pi}\int_0^\infty u\cdot[A(\omega)]^2 d\omega}{\frac{1}{\pi}\int_0^\infty [A(\omega)]^2 d\omega} = \frac{\frac{1}{\pi}\int_0^\infty \frac{\omega}{k}\cdot[A(\omega)]^2 d\omega}{\frac{1}{\pi}\int_0^\infty [A(\omega)]^2 d\omega} = \frac{\frac{1}{\pi}\int_0^\infty \frac{\omega}{k}\cdot[A(\omega)]^2 d\omega}{\tilde{E}} = \frac{\omega_0}{k_0}. \quad (4.0.52)$$

From the expressions for mean-group and mean-phase velocity (in cases of narrowband waveforms, when we can approximate the *signal frequency-domain amplitude* as a constant) we should also be able to find Planck's formula for the energy of an elementary wave-group, $E = hf = \frac{h}{2\pi}\omega$, as follows,

$$\left\{\begin{array}{l} \bar{v}_g = \bar{v} = \frac{\frac{1}{\pi}\int_0^\infty \frac{d\omega}{dk}\cdot[A(\omega)]^2 d\omega}{\tilde{E}} = \frac{\delta\omega}{\delta k} \Rightarrow \tilde{E} = \frac{1}{\pi}\int_0^\infty \frac{\delta k}{\delta\omega}\cdot\frac{d\omega}{dk}\cdot[A(\omega)]^2 d\omega \Rightarrow \\ \Rightarrow d\tilde{E} \cong \frac{[A(\omega)]^2}{\pi} d\omega = 2[A(\omega)]^2 df \end{array}\right\} \vee$$

$$\vee\left\{\begin{array}{l} \bar{v}_f = \bar{u} = \frac{\frac{1}{\pi}\int_0^\infty \frac{\omega}{k}\cdot[A(\omega)]^2 d\omega}{\tilde{E}} = \frac{\omega_0}{k_0} \Rightarrow \tilde{E} = \frac{1}{\pi}\int_0^\infty \frac{k_0}{\omega_0}\frac{\omega}{k}\cdot[A(\omega)]^2 d\omega \\ \Rightarrow d\tilde{E} \cong \frac{[A(\omega)]^2}{\pi} d\omega = 2[A(\omega)]^2 df \end{array}\right\} \Rightarrow$$

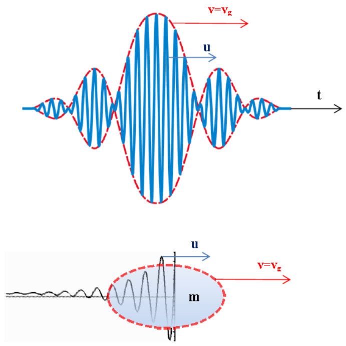
$$\Rightarrow \left\{\begin{array}{l} \delta\tilde{E} = 2[A(\omega_0)]^2\cdot\delta f \\ \tilde{E} = 2[A(\omega_0)]^2\cdot f = hf = \tilde{m}c^2 \\ A(\omega) \cong A(\omega_0) \cong Const. \\ 2[A(\omega_0)]^2 = h = Planck\ constant \end{array}\right\} \Leftrightarrow \left\{\begin{array}{l} \Delta\tilde{E} = h\cdot\Delta f\ ,\ \delta\tilde{E} = h\cdot\delta f \\ \frac{\Delta\tilde{E}}{\Delta f} = \frac{\tilde{E}}{f} = h, \frac{\Delta\tilde{E}}{\tilde{E}} = \frac{\Delta f}{f} \end{array}\right\}. \quad (4.0.53)$$

Given that the conclusions here are based on "narrow band" approximations, they provide insight into the background conditions supporting Planck’s expression for the wave energy of photons. However, it’s important to note that we cannot definitively state that this expression represents an energy quantum. In summary, Planck's energy formula should be applied specifically to narrowband elementary wave packets. The implications of this for wideband, complex spatial, and temporal waveforms, and how it connects to other moving macroparticles with non-zero rest masses, remain to be fully understood.

Fortunately, we already know that Planck's wave-energy packet, when combined with de Broglie's wavelength and relativistic particle-energy expressions, is highly effective or accurate in explaining numerous experiments, such as the Compton and Photoelectric effects (see Chapter 10, “Particles and Self-Closed Standing Matter Waves”). It’s clear that nature employs methods of waveform sampling, analysis, and synthesis, applying them equally to waves and particles. This involves moving specific energy content from one format to another, effectively combining infinitesimal and discrete signal processing approaches. This understanding provides an excellent foundation for further analysis (see sections 4.0.73 to 4.0.76).

The time and frequency domains of wave functions for elementary waves, such as photons and other elementary matter wave packets, belong to a family of finite-energy Gaussian pulses and Gabor wavelets, characterized by bell-shaped envelopes. These waveforms ensure the best possible time-frequency resolution and localization, allowing the same wave packet or signal to be well-defined and localized in both its time and frequency domains.

When it comes to the formation of more complex objects or wave groups (or macroparticles), which do not relate to narrow frequency-banded cases, these can be represented as a superposition of relevant narrowband elementary wave groups, with equations (4.0.50) being applicable (see the simplified illustration below for a conceptual view of particle and wave group equivalency). A highly promising mathematical framework for such signal integration or superposition lies in the application of concepts based on Analytic Signal theory and the “Kotelnikov-Shannon-Nyquist-Whitaker” signal analysis and synthesis theory.



Moving particle and equivalent wave-group

If we agree that an electromagnetic wave-packet or photon is a narrow-band signal, with limited time and frequency durations ($\Delta t = T,\ \Delta f = F = \frac{\Delta\omega}{2\pi}$), it will be,

$$\tilde{E} = \int_{[\Delta t = T]} \Psi^2(t)\,dt = \frac{1}{2}\int_{[\Delta t = T]} a^2(t)\,dt \cong \frac{1}{2}\overline{a}^2 \cdot \Delta t =$$

$$= \frac{1}{\pi}\int_{[\Delta\omega = 2\pi F]} A^2(\omega)\,d\omega \cong \frac{1}{\pi}\overline{A}^2 \cdot \Delta\omega = 2\overline{A}^2 \cdot \Delta f = h\overline{f} \ \Rightarrow$$

$$\left\{\begin{array}{l} d\tilde{E} = \frac{1}{2}\overline{a}^2 \cdot dt = \frac{1}{\pi}\overline{A}^2 \cdot d\omega = 2\overline{A}^2 \cdot df = hdf = \frac{h}{2\pi}d\omega, \overline{a}^2 \cdot T \cong 4\overline{A}^2 \cdot F, \\ \Delta t = T,\ \Delta f = F = \frac{\Delta\omega}{2\pi},\ h = 2\overline{A}^2\frac{\Delta f}{\overline{f}} = \frac{\overline{a}^2 \cdot T}{2\overline{f}} = \text{Constant}, \end{array}\right\} \Rightarrow$$

$$\Rightarrow T = \left(\frac{2\overline{A}}{\overline{a}}\right)^2 F = \frac{1}{2F} = \Delta f = \left(\frac{\overline{a}}{2\overline{A}}\right)^2 T = \frac{1}{2T},\ \boxed{\frac{\Delta f}{\overline{f}} = \frac{F}{\overline{f}} = \frac{h}{2\overline{A}^2}},$$

$$\Delta\tilde{E} \cong h\Delta f = \frac{h}{2\pi}\Delta\omega,\ h\overline{f} = 2\overline{A}^2 F = \frac{\overline{a}^2 \cdot T}{2} = \tilde{E}.$$

For such time and frequency limited-durations signals, where $\Delta t = T$ and $\Delta f = F = \frac{\Delta\omega}{2\pi}$ are its total temporal and frequency durations, it should be valid $TF \cong \frac{1}{2}$, and we will have, $\Delta\tilde{E} \cong hF \cong \frac{h}{2T} \Rightarrow \Delta\tilde{E} \cdot \Delta t \cong \frac{h}{2}$. From Signal and Spectral Analysis we know that the generally valid case is, $TF > \frac{1}{2}$, producing that $\Delta\tilde{E} \cdot \Delta t \geq \frac{h}{2}$. Now we could say that, here, Planck's narrow-band wave-energy formula $\tilde{E} = hf$ is on certain way additionally supported, and proven valid (under described conditions). In other words, Nature is internally communicating by creating and exchanging such finite energy portions (not to be mixed with universal, real, and well-defined, or fixed energy quantization).

Of course, here we could safely say that **h** should be a constant number (what is anyway the case). By intellectual inertia and following analogy from what we know in Quantum Theory, we know that **h** is Planck's constant, and we experience that this is only valid and applicable for atoms, photons, and micro-world of Physics (meaning for relatively stabilized matter states). We would need additional criteria to find numerical and analogical values of such **h** constants that are applicable to different matter-waves situations, and in a macro world of Physics, such as Solar Systems (since $\Delta\tilde{E} = \overline{v} \cdot \Delta\tilde{p} = h \cdot \Delta\overline{f}$, $h = 2\overline{A}^2\frac{\Delta f}{\overline{f}} = \frac{\overline{a}^2 \cdot T}{2\overline{f}} = \frac{\Delta\tilde{E}}{\Delta\overline{f}} = v \cdot \frac{\Delta\tilde{p}}{\Delta\overline{f}} (=) \frac{d\tilde{E}}{d\overline{f}} = v \cdot \frac{d\tilde{p}}{df}$ = Constant). See similar elaborations and problematic or critical conclusions regarding photons and wave energy quantizing in [161], Hidden Variables: The Elementary Quantum of Light. A Significant Question in Quantum Foundations.

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Citation from [79]: "Other essential and significant examples include the Gabor transform, and the wavelet transform both of which transform a signal f(t) in the time-frequency domain (t–w plane). In other words, these new transforms convey essential information about the nature and structure of a signal in the time-frequency domain simultaneously. In 1946, Dennis Gabor, a Hungarian British physicist, an engineer, and a 1971 Nobel Prize winner in physics, introduced the *windowed Fourier transform* (or the *Gabor transform)* of a signal f(t) concerning a window function g, denoted by $\tilde{f}_g(t,\omega)$ and defined by

$$G[f](t,\omega) = \tilde{f}_g(t,\omega) = \int_{-\infty}^{+\infty} f(\tau) \cdot g(\tau - t) e^{-i\omega t} d\tau = \langle f, \overline{g}_{t,\omega} \rangle, \tag{1.2.6}$$

Where f and $g \in L^2(R)$ with the inner product $\langle f, g \rangle$. "

Gabor (1900-1979) first recognized the significant weaknesses of the Fourier transform analysis of signals and realized the great importance of localized time and frequency concentrations in signal processing. All these motivated him to formulate a fundamental method of the Gabor transformation for decomposition of the signal regarding elementary signals (or wave transforms). Gabor's pioneering approach has now become one of the standard models for time-frequency signal analysis. It is also important to point out that the Gabor transform $\tilde{f}_g(t,\omega)$ is referred to as the canonical coherent state representation of f in quantum mechanics. In the 1960s, the term "coherent states" was first used in quantum optics".

*[♣ **COMMENTS & FREE-THINKING CORNER:***
We could additionally test the Planck's radiation law, regarding narrow-band photon energy $\mathbf{E = hf = \frac{h}{2\pi}}\omega$*. It is well proven that a photon has the wave energy equal to the product between Planck's constant* $\mathbf{h}$ *and photon's frequency* $\mathbf{f}$*. Photon is also a wave phenomenon, and it should be presentable using a certain time-domain wave function* $\Psi(\mathbf{t}) = \mathbf{a(t)\cos\varphi(t)}$*, expressed in the form of an Analytic Signal. Since the Analytic Signal presentation gives the chance to extract instantaneous signal amplitude* $\mathbf{a(t)}$*, phase* $\varphi(\mathbf{t})$*, and frequency* $\omega(\mathbf{t}) = \frac{\partial\varphi(\mathbf{t})}{\partial \mathbf{t}} = \mathbf{2\pi f(t)}$*, let us extend and test the meaning of Planck's energy when: instead of constant photon frequency* $\mathbf{f}$ *(valid for a single photon), we take its wave-group, mean wave-frequency,* $2\pi\overline{f} = \overline{\omega}$*, of the time-domain photon wave-function* $\Psi(\mathbf{t})$*. Since we already know that the photon is a "relatively concentrated wave-group", its time-variable frequency-function would also be very narrow band-limited (and could easily be replaced by photon's mean frequency value). However, just for mathematically exercising such an opportunity, we will proceed with this idea, and maybe find some additional conditions applicable to all physics related narrow-band wave-groups, as follows,*

$$d\tilde{E} = \Psi^2(t)dt = \hat{\Psi}^2(t)dt = \left|\overline{\Psi}(t)\right|^2 dt = \left[\frac{a(t)}{\sqrt{2}}\right]^2 dt = \left|\frac{\overline{U}(\omega)}{\sqrt{2\pi}}\right|^2 d\omega = [\frac{A(\omega)}{\sqrt{\pi}}]^2 d\omega =$$

$$= P(t)dt = hdf(t) = \frac{h}{2\pi} d\left[\frac{\Psi(t)\dot{\hat{\Psi}}(t) - \dot{\Psi}(t)\hat{\Psi}(t)}{a^2(t)}\right],\ h = const.,$$

$$\tilde{E} = \int_{[T]} \Psi^2(t)dt = \int_{[T]} [a(t)\cos\varphi(t)]^2 dt = \int_{[T]} a^2(t)dt,$$

$$P(t) = \frac{d\tilde{E}}{dt} = \Psi^2(t)\ (\Leftrightarrow) \left[\frac{a(t)}{\sqrt{2}}\right]^2 (=) [W]\ , t \in (-\infty, +\infty)\ ,$$

$$P(\omega) = \frac{d\tilde{E}}{d\omega} = \frac{\Psi^2(t)}{d\omega / dt} = \left[\frac{A(\omega)}{\sqrt{\pi}}\right]^2 (=) \left[Js = Ws^2\right],\ \omega \in (0, +\infty),$$

$$\Psi(t) = a(t)\cos\varphi(t) = -H\left[\hat{\Psi}(t)\right]\ ,\ \hat{\Psi}(t) = a(t)\sin\varphi(t) = H[\Psi(t)],$$

$$a(t) = \sqrt{\Psi^2(t) + \hat{\Psi}^2(t)}\ ,\ a^2(t) = \frac{\Psi(t)\dot{\hat{\Psi}}(t) - \dot{\Psi}(t)\hat{\Psi}(t)}{\overline{\omega}},\ \varphi(t) = \arctan\frac{\hat{\Psi}(t)}{\Psi(t)},$$

$$f(t) = \frac{\omega(t)}{2\pi} = \frac{1}{2\pi}\frac{\partial\varphi(t)}{\partial t} = \frac{1}{2\pi}\dot{\varphi}(t) = \frac{1}{2\pi}\frac{\Psi(t)\dot{\hat{\Psi}}(t) - \dot{\Psi}(t)\hat{\Psi}(t)}{a^2(t)} = \frac{1}{2\pi}\mathrm{Im}\left[\frac{\dot{\overline{\Psi}}(t)}{\overline{\Psi}(t)}\right].$$

$$\tilde{E} = hf = \frac{h}{2\pi}\omega(t)\ \ (\Leftrightarrow)\ \ \tilde{E} = h\overline{f} = \frac{h}{2\pi}\overline{\omega},$$

$$\omega(t) = \frac{\partial\varphi(t)}{\partial t} = 2\pi f(t)\ \ (\Leftrightarrow)\ \ \overline{\omega}(t) = \left\langle\frac{\partial\varphi(t)}{\partial t}\right\rangle = 2\pi\langle f(t)\rangle\ = 2\pi\overline{f} = \overline{\omega},$$

$$\overline{\omega} = \frac{1}{T}\int_{[T]} \omega(t)dt\ \ (\Leftrightarrow)\ \ \frac{\frac{1}{T}\int_{[T]} \omega(t)\cdot a^2(t)\cdot dt}{\int_{[T]} a^2(t)\cdot dt}\ \ (=)\ \ \tilde{E}\frac{2\pi}{h} \Rightarrow$$

$$\Rightarrow \frac{\tilde{E}}{\overline{\omega}} = \frac{\left[\int_{[T]} a^2(t)\cdot dt\right]^2}{\frac{1}{T}\int_{[T]} \omega(t)\cdot a^2(t)\cdot dt} = \frac{\int_{[T]} a^2(t)\cdot dt}{\frac{1}{T}\int_{[T]} \omega(t)dt} = \frac{a^2(t)\int_{-\infty}^{+\infty} a^2(t)dt}{2\left[\Psi(t)\dot{\hat{\Psi}}(t) - \dot{\Psi}(t)\hat{\Psi}(t)\right]} = \frac{h}{2\pi} = \text{Const.} \quad (4.0.54)$$

More about photon wave function can be found in the Annex at the end of this book (Chapter 9.): ***WAVE FUNCTION OF THE BLACK BODY RADIATION AND A PHOTON****.*

Depending on how we calculate the mean frequency, we should be able to prove at least one of the above-given relations (see the last line). Also, we should be able to find the family of wave functions that are describing photons or any other narrow-band elementary wave-packet in a time domain. In any case, we should be able to see how universal Planck's energy law could be applicable regarding the energy of arbitrary wave functions. See more about a similar problem in the following article from Dr. Juluis S. Bendat: ***THE HILBERT TRANSFORM AND APPLICATIONS TO CORRELATION MEASUREMENTS.***

What is interesting in the analysis of Dr. Bendat, and others is that a group or phase velocity is directly proportional to the square root of signal frequency $\mathbf{v} = \mathbf{v_g} \approx \sqrt{\mathbf{f}}$. This agrees with Planck's radiation-energy expression $\tilde{\mathbf{E}} = \mathbf{hf}$ since we already know that any motional or kinetic energy is proportional to the square of the group velocity, and it should be directly proportional to the dominant wave-packet frequency $\mathbf{E_k} = \tilde{\mathbf{E}} \approx \mathbf{v}^2 = \mathbf{v_g^2} \approx \mathbf{f}$. It should also be a very natural and deterministic mathematical explanation regarding Planck's narrow-band wave-packet energy, and here we are approaching well to it. ♣]

4.0.8 Uncertainty Relations and Waveform Velocities

In Physics, the Uncertainty Principle is usually linked to Heisenberg's Uncertainty Relations. For real, correct, and full understanding of many relations that are linked to the Uncertainty Principles, it would be, for the time being, better to forget that Heisenberg made any invention regarding Uncertainty. In other words, we should know (or learn) that Uncertainty is not married almost exclusively with Quantum Theory and with Heisenberg. It is also the current case that Uncertainty, as presented in contemporary Physics (mostly in Quantum Mechanics), is sporadically (and unintentionally) applied as a useful supporting background for some misinterpretations, and for justifications of some conceptual, and methodological Uncertainties in Physics (author's comment). For better foundations and explanations regarding mathematical and physics related Uncertainty relations or inequalities (what should be the same, united concept or theory), much more space will be devoted in chapter 5. of this book. Here, we will address only generally valid and surrounding grounds of Uncertainty relations that should be universally valid in signal analysis and physics.

A more general approach to Uncertainty relations (in connection with the quantum manifestations of energy-moments formats) should start from the specific finite wave function $\Psi(t)$. Here we are treating the square of the wave function as a power, (4.0.4): $\Psi^2(\mathbf{t}) = \mathbf{Power} = \mathbf{dE}/\mathbf{dt}$, but we could also treat $\Psi(t)$ as a dimensionless function (after proper normalization) without influencing the results of the analysis that follows.

It is an advantage of Analytical Signals that cover only natural domains of real-time and frequency: $-\infty < \mathbf{t} < \infty$, $\mathbf{0} \leq \mathbf{f} < \infty$, opposite to the traditional Signal Analysis (Fourier Analysis), where frequency can also take negative values. In many other aspects, Analytic Signals are giving equivalent results, as in the case of Fourier Signal Analysis, including producing additional, time-frequency dependent, dynamic and spectral signal properties, what Fourier analysis is not able to deliver.

The mutual relations between time and frequency signal-domain-segments and their isolated points of a specific waveform $\Psi(t)$, are not suitable for 1:1 (one-to-one) mapping or imaging. Moreover, they can be mutually related, and precisely localized on both sides, only within certain approximating relations, given by the following Uncertainty relations (or domain-mapping restrictions),

$$
\begin{aligned}
&0 < \delta t \cdot \delta f < \frac{1}{2} \leq F \cdot T \leq \frac{1}{4 \cdot \delta t \cdot \delta f}, (\delta t \leq \frac{1}{2F}) << T, (\delta f \leq \frac{1}{2T}) << F, \\
&0 < \delta t \cdot \delta\omega = 2\pi \cdot \delta t \cdot \delta f < \pi \leq \Omega \cdot T \leq \frac{\pi}{2 \cdot \delta t \cdot \delta f} = \frac{\pi^2}{\delta t \cdot \delta\omega},
\end{aligned}
\tag{4.0.55}
$$

where $\mathbf{T}$ is the total time duration of the signal, $F = \Omega/2\pi$ is its total frequency-duration, δt is the maximal time-sampling interval of the signal, and $\delta f = \delta\omega/2\pi$ is the maximal frequency-sampling interval (all of them compliant to Nyquist and Shannon-Kotelnikov signal sampling theorems; $t \in [\mathbf{T}], 0 \leq \mathbf{T} < \infty$, $-\infty < \mathbf{t} < \infty$, and $\mathbf{f} \in [\mathbf{F}], 0 \leq \mathbf{F} < \infty$, $0 \leq \mathbf{f} < \infty, 2\pi F = \Omega, 2\pi f = \omega$). Of course, only mathematically judging, here we know that absolute time and frequency signal durations (total signal lengths) cannot be precisely found in both time and frequency domains. Only in the frames of capturing the dominant part of the signal energy in both domains (for instance, we can request to have minimum 99 % of a total signal energy in both domains), we can speak about absolute and total signal durations. Also, if a relevant wave or signal functions are presentable like Gaussian-Gabor (bell-curve shaped) signal forms (both in time and frequency domains), we will be able to apply more accurately, uncertainty relations and signal localization. The other fact known from Physics is that Nature (or our universe) is always presenting a kind of attenuating, filtering, and modifying medias for all signals and waves propagation. Thus, even mathematically ideal, unlimited, or infinite spectral durations would be naturally transformed into finite signal-lengths (simply low energy and remarkably high frequency spectral components of the signal would be absorbed, dissipated, or filtered by the media where a signal propagates). Concerning such kind of background, we can use the terms of finite signal durations in all its domains. The crucial mathematical processing dealing with limited and finite, spectral or domain-lengths (in temporal, spatial, and all frequency domains) belongs to signal analysis and synthesis (or to signal sampling rules), as defined by Kotelnikov and Shannon's theorem. This will be addressed again later.

We know that Nature (or everything what is in motion, and what belongs to Physics) also respects mentioned domains mapping and imaging rules. In Physics, such mathematical relations are "domesticated" as Heisenberg Uncertainty Relations and widely applied in different explanations of measurements related data. Since we already know that Nature (and Mathematics) respects Uncertainty Relations, we can

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http://www.mastersonics.com/documents/revision_of_the_particle-wave_dualism.pdf

safely say that Nature also respects our temporal, spatial and frequency signal processing techniques (such as Fourier and Hilbert transform, Nyquist sampling rules, Analytic Signal concept, Kotelnikov-Shannon theorem, etc.). Consequently, we can say that whatever we see as a motional form in our Universe is composed of elementary simple harmonic signal-forms and can be decomposed on such elementary waveforms (but usually being like bell-curve shaped, Kotelnikov-Shannon-Gabor elementary wave groups).

It will be shown later that even stable and virtually non-moving forms (such as particles with non-zero rest masses) were in their past (during creation) assembled by a certain superposition of some specific elementary waveforms.

In such motional particles-related situations, it seems that Nature has conveniently transformed all "*Signal Uncertainties*", applicable to equivalent wave-groups (recognizable by the symbol $\leq$, as found in (4.0.55)), into stabilized "*Waveform Certainties*" (characterized by symbol =). This indicatively describes the formation of stable elementary particles including atoms (which are also stable in states of rest but effectively being objects where internal standing-waves structures are involved as building elements). In all other dynamic cases, where something is moving, Uncertainty relations (4.0.55) are applicable in their original mathematical forms. If we compare the mathematical Uncertainty relations (4.0.55), with similar Uncertainty relations found in Orthodox Quantum Theory, we will notice small differences between them. For instance, in the contemporary Orthodox Quantum Theory, signal durations are treated differently as statistical-deviations intervals. This produces different quantitative uncertainty relations. Instead of $\mathbf{F \cdot T \geq 1/2}$ in Quantum Theory, we can find $\mathbf{F \cdot T} \geq 1$, but both of such uncertainty relations would become mutually identical if the same signal-intervals were taken into consideration. The same problem sometimes gets more complicated in contemporary Quantum Theory, when other assumptions and statistics and probability related criteria are applied.

Since energy-finite signals or waves in real physics always propagate in certain space, during certain times, similar uncertainty relations should exist considering space-related parameters (such as length along the axis of propagation $\mathbf{x}$, and spatial-periodicity $\mathbf{k = 2\pi \cdot f_x}$). Just for making analogies, the ordinary meaning of frequency in a time-domain could also be considered as a time-related-frequency $\omega = 2\pi \cdot f_t = 2\pi \cdot f$, and $\mathbf{k = 2\pi \cdot f_x}$ would be a space-related frequency (see better explanation in Chapter 10.). By analogy with time-frequency-domain Uncertainty Relations (4.0.55), to speed up this process of explanations, we could create similar (but extended) uncertainty relations, where time is analogically replaced by corresponding *signal propagation length*, or *signal spatial duration*, $\mathbf{t \leftrightarrow x, T \leftrightarrow L}$. Moreover, the time-related frequency is replaced by space-related frequency, $\omega \leftrightarrow k, \Omega \leftrightarrow K$, where $\mathbf{L}$ is the total signal length, or total signal, spatial duration, and $\mathbf{K = 2\pi F_x}$ is the total signal spatial frequency interval. Since here we mathematically observe the same waveform $\Psi(t, x)$ in different domains, it is evident that relevant Uncertainty Relations in time and space domains should be mutually analogical, united, and coupled. Thus, such relations describe a stable proportionality between all relevant signal-domain intervals (making that such waveform would also be relatively stable, compact, progressive, and non-dispersive), and in this way, we create an extended Uncertainty Relations chain, such as,

$$0 < \delta t \cdot \delta\omega = 2\pi \cdot \delta t \cdot \delta f = \delta\, x \cdot \delta k = 2\pi \cdot \delta x \cdot \delta f_x < \pi \leq \Omega \cdot T = K \cdot L \ ,$$
$$0 < \delta t \cdot \delta f = \delta x \cdot \frac{\delta k}{2\pi} = \delta x \cdot \delta f_x < \frac{1}{2} \ \Rightarrow\ \overline{v} = \frac{\delta x}{\delta t} = \frac{\delta\omega}{\delta k} \Leftrightarrow \left\{\frac{dx}{dt} = \frac{d\omega}{dk}\right\}. \tag{4.0.56}$$

We can also see that the average velocity $\overline{\mathbf{v}}$ associated to extended Uncertainty relations (4.0.56), at least dimensionally corresponds to the signal's $\Psi(t\,,x)$ group velocity (and that such group velocity is a measure of proportionality between corresponding original and spectral domains durations). Later we will see that the velocity found here really has strong grounds in Physics (meaning that *as long the signal or an equivalent particle (represented by such signal) is in motion, they will have the same group and phase velocity, and relevant wave energy would correspond to the particle kinetic energy*).

Here, to speed up the conclusion making process, we merely applied analogies to create extended uncertainty relations (4.0.56), but in Physics, such relations are an experimentally known and mathematically supported fact. By creating δ -differences, it is evident that we here consider such elementary sampling intervals that are minimal steps defined by Nyquist-Kotelnikov-Shannon signal-sampling rules. By comparing such signal-intervals with total signal-durations in time and frequency domains, we find that δ-differences present minimal signal discretizing steps. By multiplying the last form of Uncertainty Relations (4.0.56) with Planck's constant, to get Planck's energy of a wave packet, and implementing de Broglie wavelength, we can formulate the following Uncertainty Relations,

$$\left\{\begin{array}{l} 0 < h \cdot \delta t \cdot \delta f = h \cdot \delta x \cdot \dfrac{\delta k}{2\pi} = h \cdot \delta x \cdot \delta f_x < \dfrac{h}{2} < h \cdot F \cdot T = h \cdot F_x \cdot L = h \cdot \dfrac{K}{2\pi} \cdot L \ , \\ k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{h} p \ ,\ \lambda = \dfrac{h}{p} \ ,\ h = \text{const.} \ ,\ \tilde{E} = hf \end{array}\right\} \Rightarrow$$
$$\Rightarrow \boxed{0 < \delta t \cdot \delta\tilde{E} = \delta x \cdot \delta p < \frac{h}{2} < \tilde{E} \cdot T = P \cdot L} \tag{4.0.57}$$

In early-days of Quantum Theory when mathematical background of Uncertainty was dominant (what is no more the case in the contemporary Quantum Theory), absolute or total signal durations in all domains (effectively being real signal durations), $\mathbf{F}$, $\mathbf{T}$, $\mathbf{F_x}$, $\mathbf{L}$, $\mathbf{K}$ *were considered as being relevant* Δ *-intervals, where,* $F = \Delta f$, $\tilde{E} = h \cdot F = h \cdot \Delta f$, $T = \Delta t$, $L = \Delta x$, $\frac{h}{2\pi} \cdot K = \Delta p$. *Such early-days, mathematical and Quantum Theory (QT) Uncertainty Relations, initially are known as,* $\Delta\tilde{E} \cdot \Delta t = \Delta p \cdot \Delta x > \frac{h}{2}$ *, what is entirely identical to (4.0.57),* $\tilde{\mathbf{E}} \cdot \mathbf{T} = \mathbf{P} \cdot \mathbf{L} > \frac{\mathbf{h}}{\mathbf{2}}$. *Later, in modern Quantum Theory literature, such absolute or total signal durations are transformed (philosophically, stochastically, and conceptually) into signal standard deviations (since the group of QT founders mutually agreed, by consensus, that everything in the micro world of Physics should be quantifiable and predictable only by probability wavefunctions). The other (right) side of the same inequality has also* *been changed from* $\frac{\mathbf{h}}{\mathbf{2}}$ *to* $\mathbf{h}$ *(what is possible when considering different signal duration intervals, as statistical standard deviations). This way, renewed, probabilistic Uncertainty Relations got their latest and contemporary quantum-mechanical form (becoming less generally valid), such as,*

$$\Delta\tilde{E}\cdot\Delta t=\Delta p\cdot\Delta x\geq h \quad . \tag{4.0.58}$$

In most modern Quantum Theory presentations regarding Uncertainty Relations, and everywhere else in the physics literature, there is an overwhelming tendency of using (in as many situations as possible) the constant $\hbar=\frac{\mathbf{h}}{\mathbf{2\pi}}$ *instead of* $\mathbf{h}$*. This is in some cases not entirely mathematically defendable, but it is somehow "stochastically thinking" and approximately acceptable, being anyway in the same order of magnitude (and this is anyway experimentally not easy to confirm). Here we will try to avoid such a fuzzy approach, remaining less quantum-theory-fashionable.*

In this book, we will keep as relevant to the original, mathematically most robust, and entirely defendable form of Uncertainty (or inequality) Relations, where absolute or total signal durations are explicitly involved, such as,

$$0<\delta t\cdot\delta\tilde{E}=\delta x\cdot\delta p<\frac{h}{2}<h\cdot F\cdot T=\frac{h}{2\pi}\cdot K\cdot L=\tilde{E}\cdot T=P\cdot L \ . \tag{4.0.59}$$

Here (in (4.0.59)), $\tilde{\mathbf{E}}=\mathbf{hF}$ *could be a total wave energy of the signal, and* $\mathbf{P}=\frac{\mathbf{h}}{\mathbf{2\pi}}\mathbf{K}$ *is its total linear moment (analogically concluding). Of course, later, in cases when conditions for statistical conceptualization would be satisfied, the same inequality or Uncertainty Relations can easily be extended and transformed into statistical standard deviations inequality relations (as presently practiced in Quantum Theory).*

What we can see here is that the Physics related formulation of Uncertainty Relations is also related to the Planck's narrow-band wave-packet energy ($\tilde{\mathbf{E}}=\mathbf{hF}$). As we know from earlier wave-packet velocity analyses, such energy relations are well applicable only to "narrow-banded" Gaussian-Gabor waveforms. We also know that such wave-packet concept can be applied as the motional-particle model in two aspects: *when the average group velocity of the wave-packet corresponds to the particle (center of mass) velocity, and when energy of the wave-packet is equal to the kinetic energy of the equivalent-particle,* because in both cases we consider only motional energy. Also, de Broglie wavelength, easily understandable for wave-packets and photons (which do not have rest masses), fits well and very much analogically into the moving particle-equivalent, or particle matter-wave concept (where non-zero rest mass exists). Here are the essential grounds of particle-wave duality in Physics: Motional energy of any origin, belonging to any energy carrying entity, has wave properties defined by de Broglie-Planck-Einstein matter-waves (see much more in Chapter 10.).

It is interesting to find the same Uncertainty Relations (4.0.56) if we analyze a sudden signal duration change for an arbitrary, Δ-signal interval. To address such a question let us start again from total signal durations in its time and frequency domains,

$$T \cdot F = L \cdot F_x > \frac{1}{2}, \; T \cdot \Omega = L \cdot K > \pi. \tag{4.0.60}$$

If a particular transformation, as in (4.0.61), happens to a wave function **Ψ(t),** changing its time and frequency lengths, **T** and **F**, for the (mutually dependent or mutually coupled) amounts **Δt** and **Δf**, such signal transformation will automatically influence all other space, time, and energy-related parameters of **Ψ(x, t)** to change, producing similar Uncertainty relations, as already known. Effective physical signal spatial length **L** and total signal wave-energy $\tilde{\mathbf{E}}$ would also change, for instance:

$$\left\{\begin{array}{l} T \to T \pm \Delta t > 0, \quad F \to F \pm \Delta f > 0, \quad L \to L \pm \Delta x > 0 \quad , \quad K \to K \pm \Delta k > 0 \\ \Delta\tilde{E} = h\,\Delta f = \overline{v}\Delta p = \mathcal{F}\Delta x \quad , \quad \mathcal{F} = \dfrac{\Delta p}{\Delta t} = \text{force} \quad , \\ 0 < \delta t \cdot \delta f = \delta x \cdot \dfrac{\delta k}{2\pi} = \delta x \cdot \delta f_x < \dfrac{1}{2} < F \cdot T = \dfrac{1}{2\pi} \cdot \; K \cdot L = F_x \cdot L \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left\{\begin{array}{l} \tilde{E} \to \tilde{E} \pm \Delta\tilde{E}, \\ \overline{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{\Delta\tilde{E}}{\Delta p} = \dfrac{\Delta\omega}{\Delta k} = \dfrac{\delta x}{\delta t} = \dfrac{\delta\omega}{\delta k} = \dfrac{\delta\tilde{E}}{\delta p} \end{array}\right\}. \tag{4.0.61}$$

In cases when time and frequency changes in (4.0.61) would be either positive or negative, we will have:

$$T \cdot F > \frac{1}{2} \Leftrightarrow \left\{\begin{array}{l} (T + \Delta t) \cdot (F + \Delta f) > \dfrac{1}{2} \\ (T - \Delta t) \cdot (F - \Delta f) > \dfrac{1}{2} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \left[T^2 - (\Delta t)^2\right] \cdot \left[F^2 - (\Delta f)^2\right] > \dfrac{1}{4} \\ T \cdot F + \Delta t \cdot \Delta f > \dfrac{1}{2} \\ T \cdot \Delta f + \Delta t \cdot F > 0 \end{array}\right\} \Leftrightarrow$$

$$\Leftrightarrow \left\{\begin{array}{l} 1 - (\dfrac{\Delta t}{T})^2 - (\dfrac{\Delta f}{F})^2 + (\dfrac{\Delta t}{T})^2 \cdot (\dfrac{\Delta f}{F})^2 > \dfrac{1}{4} \\ 1 + (\dfrac{\Delta t}{T}) \cdot (\dfrac{\Delta f}{F}) > \dfrac{1}{2T \cdot F}, T \cdot F > \dfrac{1}{2} \\ (\dfrac{\Delta t}{T}) + (\dfrac{\Delta f}{F}) > 0 \end{array}\right\}. \tag{4.0.62}$$

By analogy (to avoid lengthy introductions), let us replace the signal total time duration **T** with the signal total spatial length **L**, and signal total frequency duration **F** = $\mathbf{F_t} = \mathbf{\Omega/2\pi}$ by signal total spatial frequency duration $\mathbf{F_x} = \mathbf{K/2\pi}$, and unite results for time and spatial lengths cases,

$$L \cdot F_x > \frac{1}{2} \Leftrightarrow \begin{Bmatrix} (L+\Delta x)\cdot(F_x+\Delta f_x) > \frac{1}{2} \\ (L-\Delta x)\cdot(F_x-\Delta f_x) > \frac{1}{2} \end{Bmatrix} \Rightarrow \begin{Bmatrix} \left[L^2-(\Delta x)^2\right]\cdot\left[F_x^2-(\Delta f_x)^2\right] > \frac{1}{4} \\ L\cdot F_x + \Delta x\cdot\Delta f_x > \frac{1}{2} \\ L\cdot\Delta f_x + \Delta x\cdot F_x > 0 \end{Bmatrix} \Leftrightarrow$$

$$\Leftrightarrow \begin{Bmatrix} 1-(\frac{\Delta x}{L})^2-(\frac{\Delta f_x}{F_x})^2+(\frac{\Delta x}{L})^2\cdot(\frac{\Delta f_x}{F_x})^2 > \frac{1}{4} \\ 1+(\frac{\Delta x}{L})\cdot(\frac{\Delta f_x}{F_x}) > \frac{1}{2L\cdot F_x}, L\cdot F_x > \frac{1}{2} \\ (\frac{\Delta x}{L})+(\frac{\Delta f_x}{F_x}) > 0 \end{Bmatrix} \Rightarrow$$

$$\begin{Bmatrix} 1-(\frac{\Delta t}{T})^2-(\frac{\Delta f}{F})^2+(\frac{\Delta t}{T})^2\cdot(\frac{\Delta f}{F})^2 = 1-(\frac{\Delta x}{L})^2-(\frac{\Delta f_x}{F_x})^2+(\frac{\Delta x}{L})^2\cdot(\frac{\Delta f_x}{F_x})^2 > \frac{1}{4} \\ 1+(\frac{\Delta t}{T})\cdot(\frac{\Delta f}{F}) = 1+(\frac{\Delta x}{L})\cdot(\frac{\Delta f_x}{F_x}) > \frac{1}{2T\cdot F} = \frac{1}{2L\cdot F_x},\ T\cdot F = L\cdot F_x > \frac{1}{2} \\ (\frac{\Delta t}{T})+(\frac{\Delta f}{F}) = (\frac{\Delta x}{L})+(\frac{\Delta f_x}{F_x}) > 0 \end{Bmatrix} \tag{4.0.63}$$

The real waveforms-related domains-uncertainties are given by the set of inequalities in (4.0.63). Now we can extend the *uncertainty relations chain* for a couple more members, which implicitly include all possible Δ -variations found in (4.0.63),

$$\begin{aligned} &0 < \delta t\cdot\delta f = \delta x\cdot\delta f_x < \frac{1}{2} < F\cdot T = F_x\cdot L \leq \frac{1}{4\cdot\delta t\cdot\delta f} = \frac{1}{4\cdot\delta x\cdot\delta f_x}, \\ &0 < \delta t\cdot\delta\tilde{E} = \delta x\cdot\delta p < \frac{h}{2} < \tilde{E}\cdot T = P\cdot L \leq \frac{h}{4\cdot\delta t\cdot\delta f} = \frac{h}{4\cdot\delta x\cdot\delta f_x}. \end{aligned} \tag{4.0.64}$$

The signal durations (or absolute domain lengths) can be changed only for an integer number of (Nyquist) sampling intervals δt, δx, δf and δf_x. To keep the non-dispersive signal integrity and stable mutual-proportionality of selected total signal-durations in all domains (within the same reference or coordinate, inertial system), the average group-velocity found for small sampling intervals should also be equal to the average group-velocity, when much larger space, time and frequency intervals are involved,

$$\begin{aligned} &\bar{v} = \frac{\Delta x}{\Delta t} = \frac{\delta x}{\delta t} = \frac{n\cdot\delta x}{n\cdot\delta t}\ (= \frac{\delta f}{\delta f_x} = \frac{n\cdot\delta f}{n\cdot\delta f_x} = \frac{\delta\omega}{\delta k} = \frac{\Delta\omega}{\Delta k} = \frac{dx}{dt} = \frac{d\omega}{dk}), \\ &\Delta x = n\cdot\delta x,\ \Delta t = n\cdot\delta t,\ n = 1, 2, 3, ... \end{aligned} \tag{4.0.65}$$

The most interesting thing here is that an average group velocity, serving as domains proportionality measure, is not found like any other velocity, and is presently related *only to entire (motional or propagating) signal lengths. Here could be a part of the answer why light or electromagnetic waves velocity in open space and vacuum, $c = 1/\sqrt{\varepsilon_0 \mu_0}$, has a significant and unique place in Physics (like in (4.0.66)). This is probably the case, because such waveforms (or photons) while propagating in an open space, are manifesting as almost continuous motional waveforms. Their space and time domains are permanently (mutually) shifted for certain constant amounts, expressed by Uncertainty relations, producing almost constant group wave-speed. Whatever we try to do to change the speed of light, the same (spatial temporal) proportionality would re-appear and again stabilize the photons' group velocity. Once we succeed in destroying the group-velocity balance between different signal domains, the signal will disappear (being dispersed, absorbed, or dissipated). For instance, continuing the same brainstorming, we could say that since the light speed is the highest signals speed presently known in Physics, the following limitation should be valid,*

$$\overline{v} = \frac{\Delta x}{\Delta t} = \frac{\delta x}{\delta t} = \frac{n \cdot \delta x}{n \cdot \delta t} = \frac{\delta f}{\delta f_x} = \frac{n \cdot \delta f}{n \cdot \delta f_x} = \frac{\delta \omega}{\delta k} = \frac{\Delta \omega}{\Delta k} = \frac{\Omega_L}{K_L} = \frac{\Omega_H}{K_H} \leq c \ , \qquad (4.0.66)$$

*what could additionally be a beneficial relationship to address the mutual coupling and proportionality limits of absolute signal durations in their space, time, and frequency domains. Even if some velocity dependent changes of characteristic signal lengths and intervals would happen, without destroying signal integrity (like in case of Lorentz transformations), the average signal group velocity (or domain lengths proportionality) would be conserved (in cases of non-dispersive signals). This looks like the recognition that spatial and temporal dimensions (signal length or distance **x**, and signal time duration **t**) are mutually related or convertible by a specific constant which has velocity dimension ($\Delta x = \overline{v} \cdot \Delta t,\ \overline{v} = const.$). Of course, there is an analog situation for signal durations in corresponding spectral domains ($\Delta\omega = \overline{v} \cdot \Delta k,\ \overline{v} = const.$). It would be interesting to read publications from Anthony D. Osborne, & N. Vivian Pope, **[36],** where the familiar concept about space-time coupling and proportionality is presented (even on a somewhat simpler level).*

Uncertainty Relations and similar concepts in Physics and Mathematics should be equally valid and applicable across both the micro and macro universes. These concepts need to be fully harmonized and appropriately connected with the frameworks of mathematical or statistical error analysis, as well as with the natural mathematical relations of finite differences. This is crucial when describing and localizing specific particles and their equivalent matter-wave groups across all relevant domains.

The most important and universally applicable are mathematical uncertainty relations, not the Quantum Theory Uncertainty or Heisenberg relations. Mathematical uncertainty relations represent inequality relationships between the absolute, original, and spectral (or total) signal domains durations, and nothing more. These mathematical relations can later be conveniently applied to relevant Quantum Theory concepts. In some probabilistic manner, Quantum Mechanical Uncertainty can be made to comply with mathematical uncertainty, but not the other way around.

The key point here is that the same uncertainty or inequality relations should connect mutually conjugate, original, and spectral domains of a specific signal or wave function, in both

mathematics and physics, and universally across the micro and macro universes, wherever conceptually applicable. To date, Quantum Theory has realized this objective primarily through the platform of Statistics and exclusively within the realm of microphysics. However, this should not be considered the only or final validation, despite what is often asserted in contemporary Quantum Theory publications (see further discussion later in this chapter and in Chapter 5 of this book).

Citation from: https://phys.org/news/2014-12-quantum-physics-complicated.html#jCp "............. An international team of researchers has proved that two peculiar features of the quantum world previously considered distinct are different manifestations of the same thing. The result is published on 19 December in *Nature Communications*. Patrick Coles, Jedrzej Kaniewski, and Stephanie Wehner made the breakthrough while at the Centre for Quantum Technologies at the National University of Singapore. They found that 'wave-particle duality' is simply the quantum 'uncertainty principle' in disguise, reducing two mysteries to one.

"The connection between uncertainty and wave-particle duality comes out very naturally when you consider them as questions about what information you can gain about a system. Our result highlights the power of thinking about physics from the perspective of information," says Wehner, who is now an Associate Professor at QuTech at the Delft University of Technology in the Netherlands. ....

Wave-particle duality is the idea that a quantum object can behave like a wave, but that the wave behavior disappears if you try to locate the object. It is most directly seen in a double slit experiment, where single particles, electrons, say, are fired one by one at a screen containing two narrow slits. The particles pile up behind the slits not in two heaps as classical objects would, but in a stripy pattern like you would expect for waves interfering. At least, this is what happens until you sneak a look at which slit a particle goes through - do that, and the interference pattern vanishes.

The quantum uncertainty principle is the idea that it is impossible to know specific pairs of things about a quantum particle at once. For example, the more precisely you know the position of an atom, the less precisely you can know the speed with which it is moving. It is a limit on the fundamental knowability of nature, not a statement on measurement skill. The new work shows that how much you can learn about the wave versus the particle behavior of a system is constrained in precisely the same way.

Wave-particle duality and uncertainty have been fundamental concepts in quantum physics since the early 1900s. ....

It is possible to write equations that capture how much can be learned about pairs of properties that are affected by the uncertainty principle. Coles, Kaniewski, and Wehner are experts in the form of such equations known as 'entropic uncertainty relations', and they discovered that all the maths previously used to describe wave-particle duality could be reformulated concerning these relations. ....

In earlier papers, Wehner and collaborators found connections between the uncertainty principle and other physics, namely quantum 'non-locality' and the second law of thermodynamics".

4.0.9. Uncertainty Relations in Quantum Theory

Understanding Uncertainty relations in physics (presently still on mathematical level) is mainly related to our choice of signal duration intervals. Until here we have used (or talked about) real, absolute, or total signal interval lengths. Now we will once more extend already established Uncertainty Relations of absolute signal duration intervals, taking into consideration corresponding *signal standard deviations intervals*.

Since the Orthodox Quantum Mechanics mostly deals with statistical distributions and probabilities, interval lengths are represented by signal variance intervals, which are statistical or standard deviations of certain variables around their mean values. Consequently, mathematical expressions of basic Uncertainty Relations, when using variance intervals or statistical deviations, present another aspect of Uncertainty Relations (not mentioned before, but very much present in today's Quantum Mechanics literature). This should be appropriately integrated into a chain of all other, already known Uncertainty Relations. The statistical concept of variance (or standard deviation intervals) is used to measure the signal's energy spreading in time and frequency domains. For instance, for a finite wave function (4.0.1), we can define the following variances (see **[7]**, pages: 29-37, and **[8]**, pages: 273-277):

$$\begin{aligned}
&(\sigma_t)^2 = \Delta^2 t = \frac{1}{\tilde{E}}\int_{-\infty}^{+\infty}(t-\langle t\rangle)^2\left|\overline{\Psi}(t)\right|^2 dt = \int_{-\infty}^{+\infty} t^2\frac{\left|\overline{\Psi}(t)\right|^2}{\tilde{E}}dt - \langle t\rangle^2 < T^2,\\
&(\sigma_\omega)^2 = \Delta^2\omega = \frac{1}{\pi\tilde{E}}\int_{0}^{+\infty}(\omega-\langle\omega\rangle)^2\left|A(\omega)\right|^2 d\omega = \frac{1}{\pi}\int_{0}^{+\infty}\omega^2\frac{\left|A(\omega)\right|^2}{\tilde{E}}d\omega - \langle\omega\rangle^2 < (2\pi F)^2,\\
&\omega = 2\pi f,\ \sigma_\omega = 2\pi\sigma_f,\ \tilde{E} = \left\|\overline{\Psi}(t)\right\|^2 = \int_{-\infty}^{+\infty}\left|\overline{\Psi}(t)\right|^2 dt = \frac{1}{\pi}\int_{0}^{+\infty}\left|A(\omega)\right|^2 d\omega,
\end{aligned} \tag{4.0.67}$$

Where meantime and mean frequency should be found as:

$$\langle t\rangle = \frac{1}{\tilde{E}}\int_{-\infty}^{+\infty} t\left|\overline{\Psi}(t)\right|^2 dt,\ \langle\omega\rangle = \frac{1}{\pi\tilde{E}}\int_{0}^{+\infty}\omega\left|A(\omega)\right|^2 d\omega = 2\pi\langle f\rangle = 2\pi\overline{f}. \tag{4.0.68}$$

If two functions, $\Psi(t)$ and $A(\omega)$, form a Fourier-integral pair, then they cannot both be of short duration. The scaling theorem supports this,

$$\Psi(at) \leftrightarrow \frac{1}{|a|}A(\frac{\omega}{a}), \tag{4.0.69}$$

*where **"a"** is a real constant. The above assertion, (4.0.69), also known as the **Uncertainty Principle**, can be given various interpretations, depending on the meaning of the term "duration".*

Using the time and frequency variances, (4.0.67), as the significant signal duration intervals, found for a finite wave function $\Psi(t)$, it is possible to prove the validity of the following Uncertainty Principle (see **[7]** and **[8]**):

If $\sqrt{t}\Psi(t) \to 0$ for $|t| \to \infty$), then $2\pi TF > TF > \sigma_t\sigma_\omega = 2\pi\sigma_t\sigma_f = \sqrt{(\Delta^2 t)(\Delta^2\omega)} = 2\pi\sqrt{(\Delta^2 t)(\Delta^2 f)} \geq \frac{1}{2}$. (4.0.70)

In the variance relations (4.0.70) we consider as apparent that absolute (or total) time and frequency durations, **T** and **F,** can never be shorter than time and frequency variances, $\boldsymbol{\sigma_t}$ and $\boldsymbol{\sigma_f}$ (and usually, they should be much larger than σ_t and σ_f). It is also clear that statistical, (4.0.70), and Quantum Mechanic's aspect of Uncertainty should be fully integrated with absolute interval values Uncertainty Relations, (4.0.64), for instance,

$$0 < \delta t \cdot \delta f = \delta x \cdot \delta f_x < \frac{1}{2} \leq \sigma_t \cdot \sigma_f = \sigma_x \cdot \sigma_{f\text{-}x} < F \cdot T = F_x \cdot L \leq \frac{1}{4 \cdot \delta t \cdot \delta f} = \frac{1}{4 \cdot \delta x \cdot \delta f_x},$$

$$0 < \delta t \cdot \delta \tilde{E} = \delta x \cdot \delta p < \frac{h}{2} \leq 2\pi\sigma_t \cdot \sigma_{\tilde{E}} = \sigma_x \cdot \sigma_p < \tilde{E} \cdot T = P \cdot L \leq \frac{h}{4 \cdot \delta t \cdot \delta f} = \frac{h}{4 \cdot \delta x \cdot \delta f_x},$$

alternatively, in cases when normal, Gauss distributions are applicable (when 99% of an energy of certain signal is captured within $6 \cdot \sigma_{x,p}$ and/or $6 \cdot \sigma_{t,f}$) it should also be valid,

$$\frac{1}{2} \leq \sigma_t \cdot \sigma_f = \sigma_x \cdot \sigma_{f\text{-}x} < 36 \cdot \sigma_t \cdot \sigma_f = 36 \cdot \sigma_x \cdot \sigma_{f\text{-}x} < F \cdot T = F_x \cdot L,$$
$$\frac{h}{2} \leq 2\pi\sigma_t \cdot \sigma_{\tilde{E}} = \sigma_x \cdot \sigma_p < 36 \cdot 2\pi\sigma_t \cdot \sigma_{\tilde{E}} = 36 \cdot \sigma_x \cdot \sigma_p < \tilde{E} \cdot T = P \cdot L. \quad (4.0.71)$$

To maintain the same signal domains proportionality (regarding non-dispersive wave packets and stable particles), the average group velocity (as concluded before for absolute interval lengths, (4.0.65) - (4.0.66)) should also depend (in the same way) on all relevant signal lengths expressed as standard deviations,

$$\overline{v} = \frac{\Delta x}{\Delta t} = \frac{\delta x}{\delta t} = \frac{\delta\omega}{\delta k} = \frac{\Delta\omega}{\Delta k} = \frac{\Omega_L}{K_L} = \frac{\Omega_H}{K_H} = \frac{dx}{dt} = \frac{d\omega}{dk} \cong \frac{\sigma_x}{\sigma_t} = \frac{\sigma_\omega}{\sigma_k} = \frac{\sigma_f}{\sigma_{f-x}}. \quad (4.0.72)$$

If an overall signal domains proportionality has not been maintained stable (as in dispersive signals situations), we would have a mess or disorder, and stable particles would not be created (see (4.0.73) and (4.0.74)).

Here, as the supporting fact to (4.0.72), is a very convenient place to mention again the well-known and extraordinary expression for electromagnetic waves or photons speed, which is given only concerning static or stationary materials and space properties (or as a function of electric and magnetic permeability). Such relation is showing that dynamic time domain parameters of certain wave-packet (here photon or electromagnetic waves) are perfectly united and synchronized with its space and spectral domain parameters, producing,

$$c = 1/\sqrt{\mu\varepsilon}. \quad (4.0.72\text{-}1)$$

Generalizing the same concept about space-time unity and compactness of finite moving entities (as matter-waves and particles), we become familiar with the idea that even stable and solid particles (without visible waving properties) present specially packed stationary states of relevant (at least) 4-dimensional signals, or wave functions (see more in Chapter 10). Maximal velocity c ***can be considered here as a natural boundary factor (or space-time proportionality factor), which secures relevant space-time signal proportionality, compactness, integrity, and stability.*** *See familiar ides in* ***[36]****, Anthony D. Osborne, & N. Vivian Pope****.***

4.0.10. Wave Group Velocities, Moving Particles and Uncertainty Relations

Let us additionally explore the same idea (about sudden signal duration changes) using the moving particle Energy-Momentum 4-vector from the Minkowski-space of Relativity Theory, by (mathematically) introducing mutually coupled variations of a total system energy and total momentum, applying “discrete, central differentiations” method (based on symmetrical, central differences),

$$\overline{P}_4 = \overline{P}\left[\vec{p} = \gamma m\vec{v}, \frac{E}{c} = \gamma mc\right], \overline{P}^2 = \vec{p}^2 - \frac{E^2}{c^2} = -\frac{E_0^2}{c^2}, E_0 = mc^2, E = \gamma E_0 \Rightarrow \tag{4.0.73}$$

$$\begin{aligned}
&\vec{p}^2 c^2 + E_0^2 = E^2, (p \to p \pm \Delta p) \Leftrightarrow (E \to E \pm \Delta E) \Rightarrow \\
&\left\{\begin{array}{l} (p+\Delta p)^2 c^2 + E_0^2 = (E+\Delta E)^2 \\ (p-\Delta p)^2 c^2 + E_0^2 = (E-\Delta E)^2 \\ \overline{v} = \frac{\Delta x}{\Delta t} = \frac{\delta x}{\delta t} = \frac{\delta\omega}{\delta k} = \frac{\Delta\omega}{\Delta k} \end{array}\right\} \Leftrightarrow \left\{c^2 \cdot p\,\Delta p = E\,\Delta E \Leftrightarrow \frac{\Delta E}{\Delta p} = c^2\frac{p}{E} = \overline{v}\right\} \Rightarrow \\
&\Rightarrow \overline{v} = \frac{\Delta x}{\Delta t} = \frac{\delta x}{\delta t} = \frac{\Delta E}{\Delta p} = c^2\frac{p}{E} = h\frac{\Delta f}{\Delta p} = \frac{\Delta\tilde{E}}{\Delta p} = \frac{\Delta\omega}{\Delta k} = \frac{\delta\omega}{\delta k} \leq c.
\end{aligned} \tag{4.0.74}$$

From equation (4.0.74), we observe that sudden changes in energy-momentum during the motion of a particle are causally linked to its average center-of-mass velocity, which is also the system's average group velocity, as established in equation (4.0.72). This connection suggests a potential relationship between Minkowski 4-vectors and the Analytic Signal concept, which is further explored in Chapter 10. This insight provides a more concrete interpretation of Uncertainty Relations, which needs thorough analysis before drawing further conclusions.

Earlier, in equation (4.0.72), we determined that the signal's group velocity should serve as a dimensional and quantitative proportionality factor for signal domains. The confirmation provided by equation (4.0.74) reinforces this idea (see also [36]).

The core idea is to demonstrate that Uncertainty Relations are causally connected to the propagation velocity of matter waves. This implies that when an object or signal undergoes a sudden change in its "space-time-energy-momentum" parameters, matter waves are instantaneously generated. The results captured by Uncertainty Relations suggest that there is no actual uncertainty in this context; rather, we are not dealing with probabilities or standard deviations of the wavefunctions involved. Additionally, we can express the average group velocity associated with the transformations in equation (4.0.74) as follows:

$$\begin{aligned}
&\left\{\begin{array}{c} u = \frac{\omega}{k}, \ \omega = ku \\ (k \to k \pm \Delta k/2) \Leftrightarrow (u \to u \pm \Delta u/2) \end{array}\right\} \Rightarrow \Delta\omega = (k + \frac{1}{2}\Delta k)(u + \frac{1}{2}\Delta u) - (k - \frac{1}{2}\Delta k)(u - \frac{1}{2}\Delta u) = \\
&= k\Delta u + u\Delta k \Leftrightarrow \overline{v} = \frac{\Delta\omega}{\Delta k} = u + k\frac{\Delta u}{\Delta k} = \left(\frac{\Delta x}{\Delta t} = \frac{\delta x}{\delta t} = \frac{\Delta E}{\Delta p} = c^2\frac{p}{E} = h\frac{\Delta f}{\Delta p} = \frac{\Delta\tilde{E}}{\Delta p} = \frac{\delta\omega}{\delta k}\right) \Leftrightarrow \\
&\Leftrightarrow \left\{v = u + k\frac{du}{dk} = \frac{d\omega}{dk} = \frac{d\tilde{E}}{dp} = \frac{dx}{dt} = \text{immediate group velocity}\right\}.
\end{aligned} \tag{4.0.75}$$

Such group velocity (obtained using central, symmetrical, finite differences) is fully analog to its differential form where infinitesimal signal changes are involved. By merging average group velocity with Uncertainty Relations, we again see that they are mutually compatible,

$$\left.\begin{cases} \overline{v} = u + k\frac{\Delta u}{\Delta k} = \frac{\Delta\omega}{\Delta k} = \frac{\Delta\tilde{E}}{\Delta p} = \frac{\Delta x}{\Delta t} = \text{average group velocity} \\ \textit{and} \\ \left|\Delta x\ \Delta p\right| = \left|\Delta t\ \Delta\tilde{E}\right| = h\left|\Delta t\ \Delta f\right| > h/2,\ \Delta\tilde{E} = h\Delta f\ , \\ 0 < \delta t \cdot \delta f = \delta\ x \cdot \delta f_x < \frac{1}{2} \leq F \cdot T = F_x \cdot L \leq \frac{1}{4 \cdot \delta t \cdot \delta f} = \frac{1}{4 \cdot \delta x \cdot \delta f_x} \end{cases}\right\} \Rightarrow$$

$$\Rightarrow \overline{v} = \frac{\Delta x}{\Delta t} = \frac{\Delta E}{\Delta p} = h\frac{\Delta f}{\Delta p} = \frac{\Delta\tilde{E}}{\Delta p} = \frac{\Delta\omega}{\Delta k} = u + k\frac{\Delta u}{\Delta k} = \frac{\delta x}{\delta t} = \frac{\delta\omega}{\delta k} = \frac{dx}{dt} = \frac{d\omega}{dk}. \quad (4.0.76)$$

An interesting fact regarding average group velocity "$\overline{\mathbf{v}}$" in (4.0.75) and (4.0.76), which could pass unnoticed, is that the full analogy between "Δ-form" for an average group velocity "$\overline{\mathbf{v}}$" and differential or infinitesimal "d-form" for immediate group velocity "v" is not made as an approximation, automatically by formal and simple (mutual) replacing of infinitesimal difference "d" with central, discrete delta-difference "Δ". The development of the average group velocity (as given here) is applying finite symmetrical central differences on basic definitions of group and phase velocity and this way we arrive to entirely correct results without the real need for infinitesimal assumptions like $\Delta \to d\,(\to 0)$. Such methodology must be entirely correct (judging by obtained results), showing that there is a deterministic (as well as analogical) connection between the physics of continuum, and physics of discrete or finite steps (like signals sampling, analysis, and synthesis, based on Nyquist-Kotelnikov-Shannon theorems and concepts). It can be shown that in many equations relevant for mathematical physics it is possible to replace infinitesimal, differential analyses methods with operations based on using central differences. To support such statements, it would be necessary to devote time to learning about properties of central, symmetrical differences. ***We could also say that laws of physics, including equations, and concepts where we have high level of mutual similarity, parallelism, and the same consequences or results when operating with finite Δ-intervals, or with infinitesimal d-intervals, are much stronger, universal and naturally stable, than if infinitesimal d-intervals are not being mathematically (or formally) identical or proportional to similar Δ-relations.***

*[♣ **COMMENTS & FREE-THINKING CORNER:** Here is the place to address another important aspect of United symmetries and analogies between different (mutually coupled) conservation laws of physics, originating from the concept of Minkowski-space 4-vectors used in Relativity Theory. For instance (4.0.73) presents some space-time unification of energy and momentum conservation laws, making that the following expression is always invariant regarding different reference frames which are mutually in a uniform relative motion,*

$$\vec{p}_1^2 - \frac{E_1^2}{c^2} = \vec{p}_2^2 - \frac{E_2^2}{c^2} = ... = \vec{p}_n^2 - \frac{E_n^2}{c^2} = \text{invariant}$$

Also, if the relativistic space-time interval (as presently formulated in Relativity Theory) is in the same way reference-frame invariant, it will be:

$$\begin{aligned}&(\Delta r)_1^2 - c^2(\Delta t)_1^2 = (\Delta r)_2^2 - c^2(\Delta t)_2^2 = ... = (\Delta r)_n^2 - c^2(\Delta t)_n^2 = \text{invariant}\\ &-\left[1 - \frac{1}{c^2}\frac{(\Delta r)_1^2}{(\Delta t)_1^2}\right]c^2(\Delta t)_1^2 = -\left[1 - \frac{v_1^2}{c^2}\right]c^2(\Delta t)_1^2 = ... = -\left[1 - \frac{v_n^2}{c^2}\right]c^2(\Delta t)_n^2 = \text{invariant}\end{aligned}$$

What we can see from here presented invariant expressions, is that only couples which are relevant for (average) group velocity, are involved in such invariant expressions and that analogically we could formulate new reference system invariant expressions such as,

$$\begin{aligned}&(\Delta\omega)_1^2 - c^2(\Delta k)_1^2 = (\Delta\omega)_2^2 - c^2(\Delta k)_2^2 = ... = (\Delta\omega)_n^2 - c^2(\Delta k\,)_n^2 = \text{invariant},\\ &-\left[1 - \frac{v_1^2}{c^2}\right]c^2(\Delta k)_1^2 = ... = -\left[1 - \frac{v_n^2}{c^2}\right]c^2(\Delta k)_n^2 = \text{invariant},\\ &(\Delta\theta)_1^2 - \omega_c^2(\Delta t)_1^2 = (\Delta\theta)_2^2 - \omega_c^2(\Delta t)_2^2 = ... = (\Delta\theta)_n^2 - \omega_c^2(\Delta t)_n^2 = \text{invariant}\ ,\ \omega = d\,\theta/dt\\ &L_1^2 - \frac{E_1^2}{\omega_c^2} = L_2^2 - \frac{E_2^2}{\omega_c^2} = ... = L_n^2 - \frac{E_n^2}{\omega_c^2} = \text{invariant}\ ,\ \omega_c = \text{Const.},\end{aligned}$$

where only absolute signal durations are involved.

On the same way, analogically, it should be possible to formulate many new invariant relations (of course still hypothetically valid until being proven from a more general platform). ♣]

Now is also possible to prove the validity of the following expressions for group and phase velocity,

$$\left\{\begin{aligned}&v = \frac{dE}{dp} = \frac{dE_k}{dp} = \frac{d}{dp}\left[\gamma mc^2\right] = \frac{d}{dp}\left[(\gamma-1)mc^2\right] = \frac{d}{dp}\left[\frac{pv}{1+\sqrt{1-\frac{v^2}{c^2}}}\right],\\ &u = \frac{E_k}{p} = \lambda\cdot f\ ,\quad v = u + p\frac{du}{dp}\ ,\quad p = \gamma mv\ ,\quad \lambda = \frac{h}{p}\ ,\quad \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\end{aligned}\right\} \Rightarrow \tag{4.0.77}$$

$$\Rightarrow \left\{\tilde{E} \Leftrightarrow E_k\right\} \Rightarrow \left\{ \begin{array}{l} u = \dfrac{v}{1+\sqrt{1-\dfrac{v^2}{c^2}}} = \lambda \cdot f = \dfrac{\tilde{E}}{p} \leq c \quad , \\ v = \dfrac{d\omega}{dk} = \dfrac{d\omega/dv}{dk/dv} \quad , \\ k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{h}\gamma mv \quad , \quad \omega = 2\pi f = \dfrac{E_k}{h} \\ E_k = (\gamma - 1)mc^2 \end{array} \right\} , \tag{4.0.78}$$

Here it is directly proven (contrary to the position of contemporary Quantum Theory) that *wave packet energy corresponds only to motional or kinetic particle energy, and that group velocity corresponds to the particle velocity (or that the motional energy is propagating by group velocity). Quantum Theory considers that a wave-packet, which represents a specific particle, relates to a total particle energy, including its rest mass, what is not correct.* The phase velocity found here is only relevant for elementary, simple harmonic wave components propagation and it has the same limits as group velocity, $\mathbf{0 \leq 2u \leq \sqrt{uv} \leq v \leq c}$.

The other useful relations between motional energy, group and phase velocity are,

$$\left\{ \begin{array}{l} \left\{E_k \Leftrightarrow \tilde{E} = pu \,,\, pv = E_k \left[1+\sqrt{1-(\dfrac{v}{c})^2}\right] = \dfrac{\gamma^2 - 1}{\gamma} mc^2 \right\}, p = \gamma mv, \\ \left\{ (\dfrac{u}{v}) = \dfrac{\tilde{E}}{pv} = \dfrac{pu}{E_k\left[1+\sqrt{1-(\dfrac{v}{c})^2}\right]} = \dfrac{1}{1+\sqrt{1-(\dfrac{v}{c})^2}} = \dfrac{\gamma}{\gamma+1} \right\}, \\ \left\{ \lambda = \dfrac{h}{p}, k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{h}p, \omega = 2\pi f \,,\, \dfrac{d\lambda}{\lambda} = -\dfrac{dp}{p} = -\dfrac{dk}{k} = -\dfrac{df}{f}, u = \dfrac{\omega}{k}, v = \dfrac{d\omega}{dk} \right\} \end{array} \right\} \Rightarrow \tag{4.0.79}$$

$$\Rightarrow \left\{ \begin{array}{l} v = u - \lambda\dfrac{du}{d\lambda} = -\lambda^2\dfrac{df}{d\lambda} = u + p\dfrac{du}{dp} = \dfrac{d\omega}{dk} = \dfrac{d\tilde{E}}{dp} = h\dfrac{df}{dp} = \dfrac{df}{df_s} = \dfrac{2u}{1+\dfrac{uv}{c^2}} , \\ u = \lambda f = \dfrac{\omega}{k} = \dfrac{\tilde{E}}{p} = \dfrac{hf}{p} = \dfrac{f}{f_s} = \dfrac{v}{1+\sqrt{1-\dfrac{v^2}{c^2}}} = \dfrac{E_k}{p} \quad \Rightarrow \\ \Rightarrow \quad 0 \leq 2u \leq \sqrt{uv} \leq v \leq c \,, \\ d\tilde{E} = hdf = mc^2 d\gamma = c^2 d\tilde{m}, \quad \dfrac{df}{f} = (\dfrac{dv}{v}) \cdot \dfrac{1+\sqrt{1-\dfrac{v^2}{c^2}}}{1-\dfrac{v^2}{c^2}} \Rightarrow \dfrac{\Delta f}{\bar{f}} = (\dfrac{\Delta v}{\bar{v}}) \cdot \dfrac{1+\sqrt{1-\dfrac{\bar{v}^2}{c^2}}}{1-\dfrac{\bar{v}^2}{c^2}} \\ \omega = 2\pi f = ku = \dfrac{kv}{1+\sqrt{1-v^2/c^2}} = \dfrac{2\pi}{h}\dfrac{pv}{1+\sqrt{1-v^2/c^2}} = \dfrac{\sqrt{1-v^2/c^2}}{1+\sqrt{1-v^2/c^2}} \cdot \dfrac{\hbar}{m}k^2, k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{h}p \end{array} \right\} \tag{4.0.80}$$

$$\frac{df}{f}=\frac{d\tilde{E}}{\tilde{E}}=-(\frac{v}{u})\cdot\frac{d\lambda}{\lambda}=(\frac{dv}{v})(1+1/\gamma)\gamma^2=-(1+\frac{1}{\gamma})\cdot\frac{d\lambda}{\lambda}\cong\begin{cases}2\frac{dv}{v}=-2\frac{d\lambda}{\lambda}, & \text{for } v<<c\\ -\frac{d\lambda}{\lambda}, & \text{for } v\approx c\end{cases}$$

$$\omega(k)=\frac{\sqrt{1-v^2/c^2}}{1+\sqrt{1-v^2/c^2}}\cdot\frac{\hbar}{m}k^2\cong\begin{cases}\frac{\hbar}{2m}k^2, \text{for } v<<c\\ ck, \quad \text{for } v\approx c\end{cases}\Rightarrow$$

$$m(v)=\frac{m}{\sqrt{1-v^2/c^2}}\cong\begin{cases}m, \quad \text{for } v<<c\\ \frac{\hbar k}{c}\sqrt{1-v^2/c^2}=\frac{p}{c}\sqrt{1-v^2/c^2}=\frac{mv}{c}\cong m, \text{ for } v\approx c\end{cases},$$

$$p(v)=\frac{mv}{\sqrt{1-v^2/c^2}}\cong\begin{cases}mv, \quad \text{for } v<<c\\ \frac{\hbar k}{c}v\sqrt{1-v^2/c^2}=\frac{pv}{c}\sqrt{1-v^2/c^2}=\frac{mv^2}{c}\cong mv\cong mc, \text{ for } v\approx c\end{cases},$$

$$k(v)=\frac{2\pi}{\lambda}=\frac{2\pi}{h}p=\frac{2\pi}{h}\frac{mv}{\sqrt{1-v^2/c^2}}\cong\begin{cases}\frac{2\pi}{h}mv, \quad \text{for } v<<c\\ \frac{k}{c}v\sqrt{1-v^2/c^2}=\frac{2\pi}{h}\frac{pv}{c}\sqrt{1-v^2/c^2}=\frac{2\pi}{h}\frac{mv^2}{c}\cong\frac{2\pi}{h}mv\cong\frac{2\pi}{h}mc, \text{ for } v\approx c\end{cases}.$$

$$\Rightarrow\begin{cases}\frac{\tilde{E}}{mc^2}=\frac{hf}{mc^2}=\frac{1}{\sqrt{1-(\frac{v}{c})^2}}-1=\gamma-1=\frac{\tilde{E}}{E_0},\ E_0=mc^2=\text{const.},\\ \frac{\tilde{E}}{\gamma mc^2}=\frac{hf}{\gamma mc^2}=1-\sqrt{1-(\frac{v}{c})^2}=1-\frac{1}{\gamma}=\frac{\tilde{E}}{E_{total}},\ E_{total}=\gamma mc^2=\gamma E_0,\\ \frac{\tilde{E}}{E_k}=\frac{\tilde{E}}{(\gamma-1)mc^2}=\frac{hf}{(\gamma-1)mc^2}=1,\ E_{total}=E_0+E_k=E_t,\\ p^2c^2+E_0^2=E_t^2,\quad p^2v-pE_t+p_0E_0=0,\\ \tilde{E}=pu=-E_0\pm\sqrt{E_0^2+p^2c^2}=E_k\left\{=-E_0+\sqrt{E_0^2+p^2c^2}=E_0\left[\sqrt{1+(\frac{pc}{E_0})^2}-1\right]\right\},\\ \begin{cases}\vec{p}+\tilde{\vec{p}}=\vec{P}=\overrightarrow{\text{const}}\Rightarrow d\vec{p}=-d\tilde{\vec{p}}\\ d\tilde{E}=hdf=d(pu)=dE_k=vdp=c^2d(\gamma m)=-c^2d\tilde{m}=-d(\tilde{p}u)=\\ =-vd\tilde{p}\cdot\cos(\vec{p},\tilde{\vec{p}})=\{vd\tilde{p}\text{ or }-vd\tilde{p}\}=hvdf_s\end{cases}\\ \Rightarrow\Delta E_k=-\Delta\tilde{E},\ \Delta p=-\Delta\tilde{p},\ \Delta L=-\Delta\tilde{L},\ \Delta q=-\Delta\tilde{q},\Delta\dot{p}=-\Delta\dot{\tilde{p}},\Delta\dot{L}=-\Delta\dot{\tilde{L}},...\end{cases}\quad(4.0.81)$$

The *Wave-packet and Particle Equivalency or Analogy* gets clearer when we place corresponding "energy-moments-masses" forms in the same table T.4.0.2, as for instance,

Download the last version here:
http://www.mastersonics.com/documents/revision_of_the_particle-wave_dualism.pdf

T.4.0.2	Wave-Packet		Particle
Motional Energy	$\tilde{E} = \int_{-\infty}^{+\infty} \lvert\Psi(t)\rvert^2 dt = \tilde{m}c^2 = hf =$ $= \hbar\omega = \tilde{m}vu = pu = \tilde{p}u = \tilde{m}c^2 =$ $= \frac{1}{\pi}\int_0^{\infty} A^2(\omega)\, d\omega = \frac{1}{2}\int_{-\infty}^{+\infty} a^2(t)\cdot dt$		$E_k = (\gamma - 1)mc^2$ $(E_{tot.} = \gamma mc^2 = E_0 + E_k,$ $E_0 = mc^2 = const.)$
Mass	**Photons** $\tilde{m} = \frac{\tilde{E}}{c^2} = \frac{1}{c^2}\int_{-\infty}^{+\infty} \lvert\Psi(t)\rvert^2 dt =$ $= \frac{1}{\pi c^2}\int_0^{\infty} A^2(\omega)\, d\omega =$ $= \frac{1}{2c^2}\int_{-\infty}^{+\infty} a^2(t)\cdot dt$	**Any Wave-Packet** $\tilde{m} = \frac{\tilde{E}}{vu} =$ $= \frac{1}{\gamma c^2}\int_{-\infty}^{+\infty} \lvert\Psi(t)\rvert^2 dt$ $= \frac{1}{\gamma\pi c^2}\int_0^{\infty} A^2(\omega)\, d\omega$	$m_{mot.} = \frac{E_k}{c^2} = (\gamma - 1)m$ $(m_{tot.} = \frac{E_{tot.}}{c^2} = \gamma m =$ $= m + m_{mot.},$ $m = const.)$
Momentum	$\tilde{p} = \tilde{m}v = \tilde{E}/u =$ $= \frac{1}{\pi c^2}\int_0^{\infty} \frac{d\omega}{dk}\cdot[A(\omega)]^2 \cdot d\omega =$ $= \frac{1}{2c^2}\int_{-\infty}^{+\infty} v(t)\cdot a^2(t)\cdot dt$		$p = \gamma mv = \frac{h}{\lambda}$
Group Velocity	$v = \frac{d\tilde{E}}{d\tilde{p}} = \frac{d\tilde{E}}{dp} = \frac{d\omega}{dk}$, $\overline{v}_g = \overline{v} = \frac{\int_0^{\infty} \frac{d\omega}{dk}\cdot[A(\omega)]^2\, d\omega}{\int_0^{\infty} [A(\omega)]^2\, d\omega} =$ $= \frac{\int_{-\infty}^{+\infty} v(t)\cdot a^2(t)\cdot dt}{\int_{-\infty}^{+\infty} a^2(t)\cdot dt} = \frac{\delta\omega}{\delta k}$		$v = \frac{dE_k}{dp}$ $(= \frac{dE_{tot.}}{dp})$
Phase Velocity	$u = \frac{\tilde{E}}{\tilde{p}} = \frac{\tilde{E}}{p} = \frac{\omega}{k} = \lambda f$, $\overline{v}_f = \overline{u} = \frac{\int_0^{\infty} \frac{\omega}{k}\cdot[A(\omega)]^2\, d\omega}{\int_0^{\infty} [A(\omega)]^2\, d\omega} =$ $= \frac{\int_{-\infty}^{+\infty} u(t)\cdot a^2(t)\cdot dt}{\int_{-\infty}^{+\infty} a^2(t)\cdot dt} = \frac{\omega_0}{k_0}$		$u = \frac{E_k}{p} = \lambda f$ $(\lambda = \frac{h}{p})$

See later “T.4.0. Photon – Particle Analogies”, in Chapter 4.1, as the supporting and complementary insight to T.4.0.1, T.4.0.2. and T.10.00 from Chapter 10, all of them indicatively addressing Wave-Particle Duality.

4.0.11. Generalized Wave Functions and Unified Field Theory

The wide array of wave phenomena, oscillations, vibrations, and wave-like motions observed throughout physics can be described using a unified mathematical framework of wavefunctions. These wavefunctions fall under the broader concept of matter waves, including those in Quantum Theory. This book primarily focuses on tangible and measurable waves and vibrations within the realm of physics. These wave motions are best represented mathematically as Complex Analytic Signals or Phasors, which are often power-related and modeled as products of coupled Phasors, such as the product of voltage and current across a load.

Complex Phasor modeling was initially developed in electromagnetic theory to simplify operations involving simple harmonic and trigonometric wave functions, like currents and voltages. Over time, this modeling approach has been extended to cover all energy-finite and spectrally band-limited waveforms, particularly when modeled as Complex Analytic Signals (see, for example, (4.0.82)). This extension generates generalized, analytic, and complex wavefunction Phasors, applicable to all types of matter waves.

The products of related (conjugate and generalized) Phasors can form power-related wavefunctions, which adhere to Parseval's theorem regarding mutually conjugate signal domains. By dividing these related Phasors, we obtain complex impedances and admittances, relevant to both electromagnetic and mechanical phenomena, based on electromechanical analogies. When these power-related wavefunctions are normalized, becoming non-dimensional functions, as discussed in Chapter 4.3, section "4.3.3. Probability and Conservation Laws", and integrated into the framework of Signal Analysis, Statistics, and Probability Theory, we can reconstruct all currently recognized Orthodox Quantum-mechanical wave functions and their associated methodologies.

The concept of Particle-Wave Duality, as presented in this book, clearly distinguishes between a stable particle, which has a constant rest mass in its center of mass reference frame, and the wave or particle-wave phenomena that represent different states of motion for the same "energy-mass-momentum state." It is understood that the internal structure of any stable particle is composed of matter-waves and dualistic particle-waves, which are "geared and fitted" together in a stable configuration. Externally, this stable particle appears as an ensemble of oscillators, standing waves, and resonant field structures. Essentially, any particle represents a stable packing of specific matter wave groups (see (4.0.35) - (4.0.44)).

However, the specific conditions under which dynamic wave combinations transform into a stable particle, and when particles disintegrate back into simple harmonic matter waves, remain to be fully explained. The favored concept suggests that relatively free, open-space propagating wave packets can, under the right conditions, close their waveforms and become spatially localized objects (or masses). These objects are internally structured as standing and self-stabilized matter waves, or vortex formations in stable resonance, eventually creating particles with mass. The interaction between internal particle-wave structures and externally detectable matter waves (and associated fields) gives rise to de Broglie matter waves whenever particles are in motion, consistent with the "de Broglie – Planck – Einstein" relations for wavelength and (narrow-band) wave-packet energy.

It is intuitively understood that a stable particle, one with a stable rest mass, must have an internal stationery and wave-like structure arranged as standing waves or stabilized resonant structures. It is also recognized that in many interactions related to motion and scattering, stable particles naturally exhibit particle-wave duality properties or can disintegrate into pure wave-energy components. Furthermore, the appropriate superposition of pure wave elements (which lack rest mass) can result in the creation of a stable particle, as seen in electron-positron creation. Thus, the general case of a stable particle involves internal components composed of well-packed “wave-mass” elements. The proper internal dynamic arrangement of standing waves, ensuring the "gearing and mutual fitting" of all particle constituents, should result in a stable particle with a stable rest mass.

This concept can be further elucidated using Relativity Theory methods, such as the Minkowski-Einstein 4-vectors connection between total energy, rest energy, and particle momentum. This topic is explored in greater detail in Chapter 10, under “10.1 Hypercomplex Analytic Signal Functions and Interpretation of Energy-Momentum 4-Vectors in Relation to Matter-Waves and Particle-Wave Duality.” Additionally, there are intermediate and mixed particle-wave states, which may behave more like particles or waves, depending on how they are structured and spatially arranged relative to the more stable states of surrounding masses and objects.

The way to establish the grounds of a Unified Field Theory (as suggested in this book) will start (based on Parseval’s theorem) from the condition that the square of the wave function should present an active power-function $\Psi^2(t,r) = P(t,r) = d\tilde{E}/dt = -dE_k/dt$ (which carries the same energy amount in all its domains of definition). In other words, if there is some power or energy content in a certain wave function domain, this is good enough condition to have particles, waves, or wave-particle dual matter states there. Since *particles are only particular states of waves in different packing formats*, we would need to have sufficiently good and generally applicable *matter-waves wavefunctions* to conceptualize and model different matter-states and interactions between them. We are already used to expressing power functions as products between corresponding current and voltage functions (in electro-technique), or force and velocity (in mechanics), etc. Here we consider the power-function $\Psi^2(t,r)$, expressed in [W], as one of the most relevant wavefunction forms (see later different power functions in (4.0.82)). This will directly enable us to develop new forms of wave equations (valid for Gravitation, Electromagnetism, Acoustics, and other fields), being formally similar or equivalent to Schrödinger and classical wave equations, but now dealing with velocities, forces, currents, voltages ... (as we can find in the first Chapter about analogies) ... Let us first show what different waveforms regarding creating such wave functions are, by analogically comparing mathematical forms for power (in electric circuits, mechanics, electromagnetic fields etc.). This way we will operate with empirically and dimensionally tangible, clear, and measurable wave functions (and relevant Complex Phasors), being able to explore many analogies between them, and to refer to well-developed and proven methods and theories (of Electromagnetic theory, Mechanics, and Physics), where most of the facts are tangible, non-disputable, and where we really know what we are talking about. In some later steps, such “dimensional” wave functions could easily be normalized (losing dimensionality). Additional modeling can result in getting isomorphic forms of, in Statistics and Probability common functions, like we have in Orthodox Quantum Theory, but such aspect of modified and normalized, or non-dimensional wavefunctions will not be profoundly analyzed here (at least not before we get a clear picture of what we are dealing with, and how further generalizations could be made). In Quantum Theory, the square of a wave function $\Psi^2(t)$ is conveniently modeled as a probability function (where we expect to detect certain matter state), but effectively it behaves in many aspects like normalized and dimensionless Power function. Here, it will be closely related to an effective Power (or power delivered to a load, expressed in watts as its units). An attempt will be made to analogically connect an arbitrary Power Function $\Psi^2(T)$ (which is a product between corresponding current and voltage, or product between force and velocity, or product between any other relevant, mutually conjugate functions creating power), to a wavefunction, as known in Quantum Mechanics. This way (in our thinking and conceptualizing process) we will not go out of tangible, deterministic and dimensional frames of relevant wavefunctions. Analyzed from energy content, any wave propagation in time, space and frequency domains can be mutually (time-space-frequency) related using Parseval’s theorem (4.0.4). Consequently, the instantaneous (time-domain) Power-signal can be presented as the square of the wave function $\Psi^2(t)$, which is always the product of two relevant phasors (see (4.0.82)). The analysis of the optimal power transfer (carried by $\Psi^2(t)$) can be extended to any wave-like propagating field (and to arbitrarily shaped signals), and theoretically profit enormously (in booth, directions) after unifying and analogically generalizing traditional concepts of Active, Reactive and Apparent power (from electro-technique) with the $\Psi^2(t)$ wavefunction mathematics, based on Analytic Signal methodology and Complex PHASORS concept, as analogically known in electrical sciences, and here are the grounds of any future waveform analysis and united Field theory, as follows,

$$\Psi^2(t,r) = P(t,r) = \frac{d\tilde{E}}{dt} (=) \text{Active Power } (=)$$

$$(=)\left\{\begin{array}{lll} i(t,r)\cdot u(t,r) & (=) & [\text{Current}\cdot\text{Voltage}], \text{ or} \\ f(t,r)\cdot v(t,r) & (=) & [\text{Force}\cdot\text{Velocity}], \text{ or} \\ \tau(t,r)\cdot\omega(t,r) & (=) & [\text{Ang.}-\text{moment}\cdot\text{Angular velocity}], \text{or} \\ (\vec{E}\times\vec{H})\cdot\vec{S} & (=) & \left[\overrightarrow{\text{Pointyng Vector}}\right]\cdot\overrightarrow{\text{Surface}} \\ ----- & (=) & --------- \\ s_1(t,r)\cdot s_2(t,r) & (=) & [(\text{signal}-1)\cdot(\text{signal}-2)] \end{array}\right\}. \quad (4.0.82)$$

This wide variety of wave phenomena, oscillations, vibrations, and wave-like motions observed across all domains of physics can be described using a unified mathematical framework of wavefunctions, which all fall within the broader concept of matter waves, including those in Quantum Theory. This book primarily focuses on tangible and measurable waves and vibrations within the realm of physics. The mathematical descriptions of these wave motions are wave functions, best represented as Complex Analytic Signals, or Phasors. These functions are also power-related, often modeled as products of coupled Phasors, such as the product of voltage and current across a load.

The concept of Complex Phasor modeling was initially developed in electromagnetic theory to simplify operations involving simple harmonic and trigonometric wave functions, such as currents and voltages. However, any arbitrary waveform can be composed or decomposed into simple harmonic or sinusoidal elementary waveforms. Over time, this mathematical modeling approach, which creates Phasors, has been extended to cover all energy-finite and spectrally band-limited waveforms, proving particularly useful when modeled as Complex Analytic Signals (see, for example, (4.0.82)). This approach generates generalized, analytic, and complex wavefunction Phasors, applicable to all types of matter waves.

The products of related (conjugate and generalized) Phasors can form power-related wavefunctions, which also comply with Parseval's theorem regarding mutually conjugate signal domains. Dividing these related Phasors produces complex impedances and admittances, relevant to both electromagnetic and mechanical phenomena, based on electromechanical analogies. If these power-related wavefunctions are normalized (becoming non-dimensional functions, as discussed in Chapter 4.3, section "4.3.3. Probability and Conservation Laws") and integrated into the framework of Signal Analysis, Statistics, and Probability Theory, we can reassemble or recreate all currently recognized Orthodox Quantum-mechanical wave functions and their associated methodologies.

The Particle-Wave Duality concept, as presented in this book, clearly distinguishes between a stable particle (which has a constant rest mass in its center of mass reference frame) and the wave or particle-wave phenomena that represent different states of motion for the same "energy-mass-momentum state." It is understood that the internal structure of any stable particle is composed of matter-waves and dualistic particle-waves, which are "geared and fitted" together in a stable configuration. Externally, this stable particle appears as an ensemble of oscillators, standing waves, and resonant field structures. Essentially, any particle represents a stable packing of specific matter wave groups (see (4.0.35) - (4.0.44)).

However, what remains to be explained in this conceptual framework of particle-wave duality is the specific conditions under which dynamic combinations of waves transform into a stable particle, and when particles disintegrate back into simple harmonic matter waves. The favored concept here suggests that relatively free, open-space propagating wave packets can, under the right conditions, close their waveforms and become spatially localized objects (or masses). These objects are internally structured as standing and self-stabilized matter waves, or vortex formations in stable resonance, eventually creating particles with mass. The interaction between internal particle-wave structures and externally detectable matter waves (and associated fields) gives rise to de Broglie matter waves whenever particles are in motion, consistent with the "de Broglie – Planck – Einstein" relations for wavelength and (narrow-band) wave-packet energy.

It is intuitively understood that a stable particle, one with a stable rest mass, must have an internal stationery and wave-like structure, arranged as standing waves or stabilized resonant structures. It is also recognized that in many interactions related to motion and scattering, stable particles naturally exhibit particle-wave duality properties or can be disintegrated into pure wave-energy components. Furthermore, the appropriate superposition of pure wave elements (which lack rest mass) can result in the creation of a stable particle, as seen in electron-positron creation. Thus, the general case of a stable particle is one in which its internal components are composed of well-packed “wave-mass” elements. The proper internal dynamic arrangement of standing waves, ensuring the "gearing and mutual fitting" of all particle constituents, should result in a stable particle with a stable rest mass.

This concept can be explained by applying Relativity Theory methods, such as the Minkowski-Einstein 4-vectors connection between total energy, rest energy, and particle momentum. This is explored further in Chapter 10, under “10.1 Hypercomplex Analytic Signal Functions and Interpretation of Energy-Momentum 4-Vectors in Relation to Matter-Waves and Particle-Wave Duality.” Of course, there are intermediate and mixed particle-wave states, which may behave more like particles or waves, depending on how they are packed and spatially shaped relative to the more stable states of surrounding masses and objects.

The well-established field of Electric Circuits Theory, renowned for its deterministic nature and extensive practical applications, offers valuable insights and methodologies that can be analogously applied to other areas of physics, such as Classical Mechanics, Gravitation, and Quantum Mechanics. This theory, with its concepts of real, imaginary, and complex power, real and complex impedances, and the use of complex phasors or Analytic Signal notation for fundamental electrical values like voltage, current, and impedance, serves as a robust foundation for broader applications.

Our ability to measure electrical currents, voltages, and electric and magnetic fields, along with our capability to transform electrical energy into mechanical motion, oscillations, electromagnetic fields, light, and sound (and vice versa), highlights the rich mathematical modeling developed for managing electromagnetic phenomena. These models and methodologies can be analogously extended to other domains of physics, laying the groundwork for a future Unified Field Theory.

For example, as shown in (4.0.82), electrical power is calculated as the product of current and voltage across the same load or component. A similar principle applies in mechanics and other fields of physics, where mechanical power is the product of a pair of mutually coupled (or conjugate) phasors, such as force and velocity on the same mechanical element. In this book, we propose that the square of the wavefunction corresponds to a relevant power function. Consequently, we suggest modeling Quantum Theory's matter wavefunction as the product of two relevant, mutually coupled (or conjugate) signals. This approach allows for an analogical interpretation of the orthodox quantum theory wavefunction, drawing on the power functions used in electrotechnology and mechanics, which are also represented as the square of a relevant matter-wavefunction (see Chapter 4.3, "4.3.3. Probability and Conservation Laws," and Chapter 10 for further details).

By adopting this perspective, the entire methodology, modeling, and conceptual framework of complex phasors, as known in electrotechnology and electromagnetism, can be naturally and analogously transferred to the quantum realm of wavefunctions. This transformation involves applying relevant normalizations and theoretical adaptations within the frameworks of Signal Analysis, Statistics, and Probability Theory.

Given that Analytic Signal modeling provides a robust mathematical framework for analyzing arbitrary signals and wavefunctions, it should be directly applicable to signals such as current and voltage, or force and velocity. Building on this, we transform all wave and power functions from (4.0.82) into Analytic Signal or Complex Phasor forms, representing them as products of two mutually related phasors. This approach offers the advantage of not being limited to simple harmonic functions; the voltage and current forming the power function can be arbitrarily shaped and wideband in frequency. Simple harmonic waveforms are included as the most straightforward cases.

The complexity and mathematical richness of this approach, particularly related to the complex phasors of electric currents and voltages, reveal numerous advantages and new opportunities for analogous applications in waveform analysis. Since energy and momentum conservation is a fundamental principle in physics, supported by Parseval's theorem, it is reasonable to extend these methods to quantum-mechanical matter and probability waves, albeit with careful consideration of the unique characteristics and constraints of matter-wave phenomena and quantum mechanical, statistical, and probabilistic modeling.

As we advance, it will become evident that this approach offers a promising platform for unifying various fields and wave phenomena. The diverse aspects of matter-wave phenomenology can be effectively modeled within the same superior mathematical framework of Analytic Signals and Complex (or Hyper-Complex) Phasors. In the following sections of this book, we will explore innovative mathematical modeling techniques for analyzing and characterizing wideband and arbitrarily shaped waveforms.

[♣ **COMMENTS & FREE-THINKING CORNER**:

4.0.11.0. Complex Wave Function, Energy, Power, and Impedance

Here we will basically demonstrate how starting from Complex Analytic Signal Waveforms (related to everything motional in Physics, and to alternating currents and voltages in electro-technique), we can simply create Complex Phasors of alternating currents and voltages (created initially and historically without using Analytic Signal modelling).

We can say that, in this book, the wavefunction is related to the power function of certain motion, i.e., that it directly represents specific energy transfer. To determine the power, it is commonly implied in physics that we know the two functions: $s_1(t)$ and $s_2(t)$ (like relevant current and voltage), the product of which defines the power, as mentioned in (4.0.82).

The kind of energy transfer efficiency from its source to certain load depends on the coherence relation between the functions $s_1(t)$ and $s_2(t)$, *which determine the* power. *That is how we can define coherence coefficient between $s_1(t)$* and *$s_2(t)$:*

$$K_t = \frac{\int_{-\infty}^{+\infty} s_1(t)\cdot s_2(t)\cdot dt}{\int_{-\infty}^{+\infty} |s_1(t)|\cdot|s_2(t)|\cdot dt} = \frac{\int_{-\infty}^{+\infty} \Psi^2(t)\cdot dt}{\int_{-\infty}^{+\infty} |\Psi^2(t)|\cdot dt}\,, \quad (4.0.83)$$

And determine their phase functions:

$\varphi_1(t) = \text{arc tang}\left[\hat{s}_1(t)/s_1(t)\right]$, ($s_1(t) = a_1(t)\cos\varphi_1(t) = \frac{1}{\pi}\int_0^{\infty}\left[A_1(\omega)\cos(\omega t + \Phi_1(\omega))\right]d\omega$,

$\hat{s}_1(t) = H[s_1(t)] = a_1(t)\sin\varphi_1(t)$,

$\bar{s}_1(t) = s_1(t) + j\hat{s}_1(t) = a_1(t)\,e^{j\varphi_1(t)}$),

$\varphi_2(t) = \text{arc tang}\left[\hat{s}_2(t)/s_2(t)\right]$, ($s_2(t) = a_2(t)\cos\varphi_2(t) = \frac{1}{\pi}\int_0^{\infty}\left[A_2(\omega)\cos(\omega t + \Phi_2(\omega))\right]d\omega$,

$\hat{s}_2(t) = H[s_2(t)] = a_2(t)\sin\varphi_2(t)$,

$\bar{s}_2(t) = s_2(t) + j\hat{s}_2(t) = a_2(t)\,e^{j\varphi_2(t)}$), (4.0.84)

therefore, it is straightforward to find an additional criterion about the mutual position (coherence) of $s_1(t)$ and $s_2(t)$ by simply forming their phase difference,

$\Delta\varphi(t) = \varphi_2(t) - \varphi_1(t)$. (4.0.85)

It is also possible to find the amplitude spectral functions for the wave function factors $s_1(t)$ and $s_2(t)$ $A_1(\omega)$ i $A_2(\omega)$, *as well as to find the functions of the appropriate phase functions $\Phi_1(\omega)$ and $\Phi_2(\omega)$ using* definitions from *(*4.0.3*)* and T.4.0.1*.*

Now we can draw important conclusions concerning coherence criteria for the wave function factors $s_1(t)$ and $s_2(t)$ and connect them with the optimal energy propagation:

-If $K_t = 1$, $s_1(t)$ and $s_2(t)$ are coherent, or mutually *in phase, $\Delta\varphi(t) = 0$, we have an optimal energy transfe*r. That is the case of active *or resistive* load *impedances.*

-If *f $K_t = 0$*, *$s_1(t)$ and $s_2(t)$ are mutually orthogonal or phase* shifted *for $\pi/2$, $|\Delta\varphi(t)| = \pi/2$, no energy circulation from the source to the* load *is possible (i.e., the* load *does not receive any energy).*

-If $K_t = -1$, $s_1(t)$ and $s_2(t)$ are in the counter-phase, then there is a non-optimal energy transfer, i.e., the energy is completely reflected off the load *and returned to its source (the energy has a "-" sign). That is the case with the reactive (or complex)* load *impedances.*

Power *factor, coherence criterion* between *$s_1(t)$ and $s_2(t)$ can be widened by comparing their phase spectral functions, which are defined by (*4.0.3*), i.e., by forming the difference:*

$\Delta\Phi(\omega) = \Phi_2(\omega) - \Phi_1(\omega)$. (4.0.86)

*It is equally justified to introduce the coherence factor into the spectral domain, using amplitude spectral functions (*4.0.3*), analogous to the definition (*4.0.83*),*

$$K_\omega = \frac{\int_0^{+\infty} A_1(\omega)\cdot A_2(\omega)\cdot d\omega}{\int_0^{+\infty} |A_1(\omega)|\cdot|A_2(\omega)|\cdot d\omega}\,. \quad (4.0.87)$$

While analyzing optimal energy transfer (and introducing the terms like active, reactive, and complex power) great attention should be paid to the fact that, in a general case, Hilbert transformation of wave function elements is not linear, i.e., the following equations are valid:

$$H[\Psi^2(t)] = H[s_1(t)\, s_2(t)] = \mathbf{s_1 s_2} \frac{\mathbf{s_1 \hat{s}_2 + \hat{s}_1 s_2}}{\mathbf{s_1 s_2 - \hat{s}_1 \hat{s}_2}}, \quad (4.0.88)$$

$$H[s_2(t) / s_1(t)] = \left(\frac{\mathbf{s_2}}{\mathbf{s_1}}\right) \frac{\mathbf{s_1 \hat{s}_2 - \hat{s}_1 s_2}}{\mathbf{s_1 s_2 + \hat{s}_1 \hat{s}_2}}. \quad (4.0.89)$$

4.0.11.1. Generalized Complex Impedances

To analyze the problems related to energy transport, it is necessary that an energy source and its consumer or load exist. The term "load impedances" characterizes loads. In physics, electro-technique, electromagnetism, acoustics, and mechanics we can define a universal term "dynamic (time-dependent) load impedance" in the following manner:

$$Z(t) = s_2(t) / s_1(t) = \tilde{S}(t) / s_1^2(t) = s_2^2(t) / \tilde{S}(t) =$$

$$= [\Psi(t) / s_1(t)]^2 = [s_2(t) / \Psi(t)]^2 = \frac{a_2(t)}{a_1(t)} \frac{\cos\varphi_2(t)}{\cos\varphi_1(t)}. \quad (4.0.90)$$

This time it is advisable to think about the way to treat a complex, time-dependent impedance (taking (4.0.89) into account), defined as a quotient of appropriate analytical (complex) functions:

$$\overline{Z}(t) = \overline{s}_2(t) / \overline{s}_1(t) = \frac{a_2(t)}{a_1(t)} e^{j[\varphi_2(t) - \varphi_1(t)]} = \frac{a_2(t)}{a_1(t)} e^{j\Delta\varphi(t)}, \quad (4.0.91)$$

because $\overline{Z}(t) = \overline{s}_2(t) / \overline{s}_1(t) \neq s_2(t) / s_1(t) + j\, H[s_2(t) / s_1(t)]$ (except for simple harmonic waveforms).

In a similar manner, we can define frequency dependent, complex impedances, as a quotient of spectral (co*mplex) functions of the appropriate wave function factors:*

$$Z(\omega) = \overline{Z}(\omega) = \frac{\overline{S}_2(\omega)}{\overline{S}_1(\omega)} = \frac{A_2(\omega)}{A_1(\omega)} e^{j[\Phi_2(\omega) - \Phi_1(\omega)]} = \frac{A_2(\omega)}{A_1(\omega)} e^{j\Delta\Phi(\omega)}. \quad (4.0.92)$$

In the example of passive electric impedances (or networks made of a load *with the resistance R, capaci*tance *C and induc*tance *L), it is possible to present various ways for a more precise defining of the term impedance.* Later, the *same procedure can be extended to other physics disci*plines as well using "current-force" and "voltage-velocity" analogies (see (1.82)), as for instance:

$s_1(t) = i(t) = I(t)\cos\varphi_i(t)$ *(=) current*, $\overline{s}_1(t) = \overline{i}(t) = I(t)\, e^{j\varphi_i(t)}$, $i(\omega) = I(\omega)\, e^{i\Phi_i(\omega)}$

$s_2(t) = u(t) = U(t)\cos\varphi_u(t)$ *(=) voltage*, $\overline{s}_2(t) = \overline{u}(t) = U(t)\, e^{j\varphi_u(t)}$, $u(\omega) = U(\omega)\, e^{i\Phi_u(\omega)}$

$\tilde{S}(t) = d\tilde{E}/dt = \Psi^2(t) = u(t)\, i(t)$ *(=) force.* (4.0.93)

4.0.11.2. Resistive or Active Impedances

Let us assume that the current $i_R(t) = i_R$ runs through the electrical resistance R. In that case, voltage $u_R(t) = u_R$ is formed on the resistance R. It is obvious that the following relations are valid:

$$u_R(t) = Z_R i_R = R\, i_R,\ \tilde{S}_R(t) = u_R(t)\, i_R(t) = R\, i_R^2 = \Psi_R^2(t),$$

$$K_t = 1,\ K_\omega = 1,\ \Delta\varphi(t) = \varphi_u - \varphi_i = 0,\ \Delta\Phi(\omega) = \Phi_u - \Phi_i = 0, \quad (4.0.94)$$

In addition, from this, the resistive dynamic impedance is:

$$Z_R(t) = u_R(t) / i_R(t) = \overline{Z}_R(t) = Z_R(\omega) = u_R(\omega) / i_R(\omega) = R = [\Psi_R(t) / i_R(t)]^2. \quad (4.0.95)$$

4.0.11.3. Inductive Complex Impedances

The previous procedure can be extended analogously. If the current $i_L(t) = i_L$ runs through the inductance L, voltage $u_L(t) = u_L$ will be formed on the inductance L. The validity of the following relations is obvious (or can be proved):

$$u_L(t) = Z_L\, i_L = -L(di_L/dt)\,,\ \tilde{S}_L(t) = u_L(t)\, i_L(t) = -L\, i_L(di_L/dt) = \Psi_L^2(t)\,,$$

a) $U(\omega) = -\omega L I(\omega)$,
$\Delta\Phi(\omega) = \Phi_u - \Phi_i = +\pi/2$,

b) $U(\omega) = +\omega L I(\omega)$,
$\Delta\Phi(\omega) = \Phi_u - \Phi_i = \{+3\pi/2, -\pi/2\}$,

$$\Delta\varphi(t) = \varphi_u - \varphi_i = \operatorname{arctg}\left[\frac{\frac{d\varphi_i}{dt}}{\frac{1}{I(t)}\frac{dI(t)}{dt}}\right] +/- \pi \ , \quad (4.0.96)$$

from this, the inductive impedance is:

$$Z_L(t) = u_L(t) / i_L(t) = \frac{U_L(t)}{I_L(t)}\frac{\cos\varphi_u(t)}{\cos\varphi_i(t)} = (-L / i_L)\,(di_L/dt) = [\Psi_L(t) / i_L(t)]^2$$

$$= (\frac{d\varphi_i(t)}{dt})L\, \operatorname{tg}\varphi_i(t) - \frac{1}{I(t)}\frac{dI(t)}{d(t)}L.$$

$$\overline{Z}_L(t) = \frac{\overline{u}_L(t)}{\overline{i}_L(t)} = \frac{U_L(t)}{I_L(t)}e^{j\Delta\varphi(t)} = -j(\frac{d\varphi_i(t)}{dt})L - \frac{1}{I(t)}\frac{dI(t)}{d(t)}L \ ,$$

$$Z_L(\omega) = u_L(\omega) / i_L(\omega) = \frac{U(\omega)}{I(\omega)}e^{j\Delta\Phi(\omega)} = \{[+j\omega L = +j\frac{U(\omega)}{I(\omega)};\ U(\omega) = -\omega L I(\omega)],$$

ili $[-j\omega L = -j\frac{U(\omega)}{I(\omega)};\ U(\omega) = +\omega L I(\omega)]\}.$ (4.0.97)

4.0.11.4. Capacitive Complex Impedances

Finally, if current $i_C(t) = i_C$ *runs through capacitance C, then voltage $u_C(t) = u_C$ will be formed on capacitance C. The validity of the following relations is obvious (or can be proved):*

$$u_c(t) = Z_c i_c = \frac{1}{C}\int_0^t i_c(t)dt\,,\ \tilde{S}_C(t) = u_C(t)\, i_C(t) = \frac{i_C(t)}{C}\int_0^t i_c(t)dt = \Psi_c^2(t),$$

(4.0.98)

$$U(\omega) = \frac{1}{\omega C} I(\omega)\ ,$$

$$\Delta\Phi(\omega) = \Phi_u - \Phi_i = -\pi/2\,,$$

$$\Delta\varphi(t) = \varphi_u - \varphi_i = -\pi/2$$

from this, capacitance impedances are:

$$Z_c(t) = u_c(t) / i_c(t) = \frac{U_C(t)}{I_C(t)} \frac{\cos\varphi_u(t)}{\cos\varphi_i(t)} = \frac{1}{C i_C(t)} \int_0^t i_c(t)dt = [\Psi_c(t) / i_c(t)]^2 = \ldots,$$

$$\overline{Z}_C(t) = \frac{\overline{u}_c(t)}{\overline{i}_c(t)} = \frac{U_C(t)}{I_C(t)} e^{j\Delta\varphi(t)},$$

$$Z_c(\omega) = u_c(\omega) / i_c(\omega) = \frac{U(\omega)}{I(\omega)} e^{j\Delta\Phi(\omega)} = -j\frac{1}{\omega C} = -j\frac{U(\omega)}{I(\omega)}. \quad (4.0.99)$$

4.0.11.5. R-L-C Complex Impedances of Simple Harmonic, Alternating Currents

If we consider only the cases when through an impedance a simple harmonic stationary current $i(t) = I_0 \cos\omega t$, ($I_0 = const.$) runs, then the previous generalized impedances are simplified to the following:

$$Z_R(t) = \overline{Z}_R(t) = R, \; Z_R(\omega) = R,$$

$$Z_L(t) = \omega L(\operatorname{tg} \omega t) = X_L(\operatorname{tg} \omega t), \; \overline{Z}_L(t) = -j\omega L = -j X_L, \; Z_L(\omega) = j\omega L = jX_L,$$

$$Z_C(t) = (-1/\omega C)(\operatorname{tg} \omega t) = -X_C(\operatorname{tg} \omega t), \; \overline{Z}_C(t) = -j\frac{1}{\omega C} = -jX_C, \; Z_c(\omega) = -j\frac{1}{\omega C} = -jX_C. \quad (4.0.100)$$

In the electro-technique of stationary or simple harmonic currents, we can notice the advantage of introducing complex electric impedances, which simplify time-dependent impedances (4.0.100). If we introduce the transformation of *a real current function into a complex current function*, *like this:*

$$i(t) = I_0\cos\omega t \leftrightarrow I_0 e^{-j\omega t} = \overline{i}(t) = \overline{I}, \quad (4.0.101)$$

and if we use the complex current form (4.0.101) to determine the impedance (4.0.100), we will get so-called complex stationary-current impedance forms, where we (formally) no longer see the difference among so-called time-dependent, complex or frequency dependent impedances (which are evident in (4.0.100)), i.e. for all impedance forms, the following is valid:

$$Z_R = R,$$
$$Z_L = j\omega L = j X_L, \quad (4.0.102)$$
$$Z_C(t) = 1 / j\omega C = -j X_C.$$

The impedance forms are given in (4.0.102), although being practical (or useful) and simple, hide within themselves the real identity of the term "impedance" and most of the related facts about it.

The formal way of imaging (4.0.101), which follows the definition of impedance, is nothing else but a replacement of one real function by an appropriate complex, analytical function. Therefore, if the current function i(t) is arbitrary, and not simple harmonic, we can make the replacement (with an appropriate analytical signal function (4.0.1) and (4.0.2), analogous to (4.0.101)):

$$i(t) \leftrightarrow i(t) + j H[i(t)] = i(t) + j\hat{i}(t) = I_0(t) e^{j\varphi(t)} = \overline{i}(t). \quad (4.0.103)$$

Mathematical formalism (4.0.103), related to simple harmonic (stationary) currents and voltages, is very well developed in electro-technique, whereas the same issue concerning non-periodical and non-harmonic (arbitrary functions of current and voltage) is rather badly dealt with regarding mathematics. It would be good to use the elements of the previously described formalism to analyze the optimal transmission of electrical signals (of the energy, power, currents, etc.) in the cases of their arbitrary, non-periodical waveforms (for arbitrary load impedances). If we do not wish to make a complete redefinition of "complex impedances", and we want that definition to keep its general meaning within the frame previously established (to preserve the continuity with the usual treatment of that term), we must limit the meaning of “complex impedance” only to its appropriate frequency-dependent impedance forms, as given in (4.0.100). It is obvious that time-dependent *impedances (or current and voltage on such impedances) will be changeable according to the modeling factor tanωt (which is not at all practical for any selection or quantification of impedances).*

For now (in the examples given above), we will rely on the methodology established in electro-technique, because that issue is very comprehensive for us, and, besides, it is extremely well developed, in comparison to the other fields in physics. There is only one small step to the analogous broadening and usage of the same way of thinking to other fields, such as acoustics, fluid mechanics, electromagnetic waves, vibrations, quantum wave mechanics, etc.

4.0.11.6. Complex Power Function

The concept of the complex, active and reactive power, which operates well in the electro-technique of stationary (simple harmonic) currents and voltages, is, in the general case of arbitrary forms of currents and voltages, inapplicable (at least not as known so far). Certainly, it is always possible to talk about the active power transfer, when a system operates as an active (resistive) load, and it satisfies the criteria from (4.0.94). All other cases can be treated as cases of the circulation (or transfer) of reactive and complex power. The key criteria for the determination of optimal energy transfer are *coherence relations among the* power *factors (see* (4.0.83*) to* (4.0.99*)). It is good to complement the previous claim with the comparison of the following real and complex wave function forms (by asking the question, which of such relations or parts of the relations, can stand for the complex, and which one/s for the reactive force, and to what extent such a terminology and classification makes sense and practical significance), as follows,*

$$\begin{aligned}
&\Psi^2(t) = s_1 s_2 \ , \\
&\overline{s}_1 \overline{s}_2 = (s_1 s_2 - \hat{s}_1 \hat{s}_2) + j(s_1 \hat{s}_2 + \hat{s}_1 s_2) \ , \\
&\overline{\Psi}^2(t) = s_1 s_2 - \left\{ H\left[\sqrt{s_1 s_2} \right] \right\}^2 + 2j \sqrt{s_1 s_2}\, H\left[\sqrt{s_1 s_2} \right] , \qquad (4.0.104) \\
&\left| \overline{\Psi}(t) \right|^2 = s_1 s_2 + \left\{ H\left[\sqrt{s_1 s_2} \right] \right\}^2 , \\
&s_1 s_2 + jH[s_1 s_2] = \overline{s}_{12} \ .
\end{aligned}$$

It is evident that none of the relations from (4.0.104) presents a direct way of introducing and full understanding of the terms such as complex, active, and reactive power. To have an appropriate way of looking at the issues related to the term ‘power', we must hold on to a standpoint, which defines that, on the resistive load, the current and voltage functions will be entirely coherent, i.e., in-phase. This will be valid for both time and frequency domain of those functions when the power is entirely active. All other cases indicate the presence of complex or reactive power (or the absence of optimal energy transfer). What is also particularly important is that, if there is any other, more general mathematical analysis of the problems mentioned above, it must consider all cases of the classical treatment of active, reactive, and complex power discussed so far. This is a significant advantage and facility in the study of various wave natural phenomena because nature in a way strives towards creating harmonic oscillations, which can often be very well presented by simple harmonic functions (in most of the cases of interest).

The methodology introduced beginning with (4.0.82) to (4.0.104) opens new (generalized) possibilities for the analysis and quantification of the energy transfer from a source to its load and gives the possibility of generalized load-types classification (for arbitrary wavefunctions). It is of special significance that, by applying this methodology (with specific mathematical extra work on previously addressed questions), we can overcome confusions and delusions (about analogically generalized electric and mechanical cases of impedances, power, and energy transfer) that have existed until present.

Naturally, the introduction of the complex functions (and numbers) analysis into the wave motion analysis, besides other options, plays a significant role in the simplification of the mathematical apparatus used, in the way that, instead of solving integral, differential equations, the whole analysis is based on rather elementary algebra operations. ♣]

Analogical Extension of Complex, Electrical Alternating Currents and Voltages, Mathematical Framework to Arbitrary (Non-Sinusoidal) Analytic, Voltage and Current Signals, and Consequences Regarding Novel Understanding of Matter-Wave Complex, Analytic Functions

List of Symbols:

$p(t)$ = Instantaneous Power

$\hat{p}(t)$ = Hilbert Transform of Instantaneous Power

$\overline{p}(t)$ = Instantaneous Power in the form of Complex Analytic Signal

$|\overline{p}(t)|$ = Absolute Value of Instantaneous Power

$P(t)$ = Amplitude function of Instantaneous (real or active) Power

$S(t)$ = Amplitude function of Instantaneous Apparent Power

$Q(t)$ = Amplitude function of Instantaneous Reactive Power

$\overline{u}(t)$ = Instantaneous Voltage in the form of complex Analytic Signal

$u(t)$ = Instantaneous Voltage

$\hat{u}(t)$ = Hilbert Transform of Instantaneous Voltage

$\overline{i}(t)$ = Instantaneous Current in the form of Complex Analytic Signal

$i(t)$ = Instantaneous Current

$\hat{i}(t)$ = Hilbert Transform of Instantaneous Current

$U(t)$ = Amplitude function of Instantaneous Voltage

$I(t)$ = Amplitude function of Instantaneous Current

U_{rms} = Effective, RMS Voltage

I_{rms} = Effective, RMS Current

$\overline{Z} = R - jX$ = Complex Impedance, $|\overline{Z}| = Z = \sqrt{R^2 + X^2}$

R = Resistance, X = Reactance

T = Time interval

$\varphi_p(t)$ = Phase function of the Instantaneous Power

ω_p = Frequency of the Instantaneous Power

φ_u = Phase function of the Voltage signal

ω_u = Frequency of the Voltage signal

φ_i = Phase function of the Current signal

ω_i = Frequency of the Current signal

$j = \sqrt{-1}$ = Imaginary unit

$H[\ ]$ (=) Hilbert Transform

$PF = \cos\theta$ = Power Factor

$\Psi(t)$ = Wave function

$a(t)$ = Amplitude of a Wave function

$\varphi(t)$ = Phase of a Wave function

See more in: [57], Michael Feldman, [109], Poularikas A. D., including [110], and [111].

Download the last version here: http://www.mastersonics.com/documents/revision_of_the_particle-wave_dualism.pdf

Analytic Signal and Electrical Power Characterization (comparative tables); **Extension of the Electrical Power Definition**

Instantaneous Power (Apparent Power & Analytic Signal)	Averaged Complex Power (Averaged Apparent Power & Phasor notation)	Generalized Complex Power (Generalized Phasor Notation)
$\overline{p}(t)=p(t)+j\cdot\hat{p}(t)=\lvert\overline{p}(t)\rvert\cdot e^{j\varphi_p(t)}$, $\varphi_p(t)=arctg\frac{\hat{p}(t)}{p(t)}=\varphi_u(t)+\varphi_i(t)=$ $=\theta(t)+2\varphi_u(t)=2\varphi_i(t)-\theta(t)$, $\theta(t)=\varphi_i(t)-\varphi_u(t),\ \omega_p=\frac{\partial\varphi_p(t)}{\partial t}$, $(j^2=-1)$	$\overline{S}=\frac{1}{2}\overline{U}\cdot\overline{I}^*=P-jQ=Se^{-j\theta}=U_{rms}I_{rms}e^{-j\theta}$ $\theta(t)=\angle[u(t),i(t)]=\varphi_i(t)-\varphi_u(t)=\varphi_i-\varphi_u=$ $=\theta_i-\theta_u=arctg\frac{Q}{P}=\theta,\ \omega=\frac{\partial\varphi_u(t)}{\partial t}=\frac{\partial\varphi_i(t)}{\partial t}$ $(\varphi_u(t)=\omega t+\theta_u, \varphi_i(t)=\omega t+\theta_i)$	$\overline{S}(t)=\frac{1}{2}\overline{u}(t)\cdot\overline{i}^*(t)=P(t)-jQ(t)=$ $=S(t)e^{-j\theta(t)}=\frac{1}{2}U(t)I(t)e^{-j\theta(t)}$ $\theta(t)=\varphi_i(t)-\varphi_u(t)=arctg\frac{Q(t)}{P(t)}$
$p(t)=u(t)\cdot i(t)=\lvert\overline{p}(t)\rvert\cdot\cos\varphi_p(t)=\lvert\overline{p}(t)\rvert\cdot\cos(\varphi_u(t)+\varphi_i(t))$ $=U(t)\cdot I(t)\cdot\cos\varphi_u(t)\cdot\cos\varphi_i(t)$ $\lvert\overline{p}(t)\rvert=\sqrt{p(t)^2+\hat{p}(t)^2}=U(t)I(t)\frac{\cos\varphi_u(t)\cos\varphi_i(t)}{\cos\varphi_p(t)}=$ $=\frac{p(t)}{\cos\varphi_p(t)}=2S(t)\cdot\frac{\cos\varphi_u(t)\cos\varphi_i(t)}{\cos\varphi_p(t)}$ $\hat{p}(t)=H[p(t)]=\lvert\overline{p}(t)\rvert\cdot\sin\varphi_p(t)=p(t)\cdot\tan(\varphi_u(t)+\varphi_i(t))$, $H[\ ]$ (=) **Hilbert transformation**, $u(t)=U(t)\cos\varphi_u(t),\ \overline{u}(t)=u(t)+j\cdot\hat{u}(t)=U(t)\cdot e^{j\varphi_u(t)}$, $U(t)=\sqrt{u(t)^2+\hat{u}(t)^2}$, $\varphi_u(t)=arctg\frac{\hat{u}(t)}{u(t)},\omega_u=\frac{\partial\varphi_u}{\partial t}$, $i(t)=I(t)\cos\varphi_i(t),\ \overline{i}(t)=i(t)+j\cdot\hat{i}(t)=I(t)\cdot e^{j\varphi_i(t)}$, $I(t)=\sqrt{i(t)^2+\hat{i}(t)^2}$, $\varphi_i(t)=arctg\frac{\hat{i}(t)}{i(t)},\omega_i=\frac{\partial\varphi_i}{\partial t}$. $ui+\hat{u}\hat{i}=U(t)I(t)\cos\theta(t),\ u\hat{i}-\hat{u}i=U(t)I(t)\sin\theta(t)$ $ui=U(t)I(t)\cos\varphi_u(t)\cos\varphi_i(t)$, $\hat{u}\hat{i}=U(t)I(t)\sin\varphi_u(t)\sin\varphi_i(t)$	$\overline{U}=\sqrt{2}U_{rms}e^{j\varphi_u},\ \overline{I}=\sqrt{2}I_{rms}e^{j\varphi_i}=\sqrt{2}I_{rms}e^{j(\varphi_u+\theta)}$, $\overline{I}^*=\sqrt{2}I_{rms}e^{-j\varphi_i}=\sqrt{2}I_{rms}e^{-j(\varphi_u+\theta)}$, $P=U_{rms}I_{rms}\cos\theta$ = Active Power (=) [W] $Q=U_{rms}I_{rms}\sin\theta$ = Reactive Power (=) [VAR] $S=\sqrt{P^2+Q^2}=U_{rms}I_{rms}$ (=) [VA] $PF=\frac{P}{S}=\cos\theta$ = Power Factor, $U_{rms}=\sqrt{\frac{1}{T}\int_{[T]}u^2(t)dt}$, $I_{rms}=\sqrt{\frac{1}{T}\int_{[T]}i^2(t)dt}$. $\overline{Z}=\lvert Z\rvert e^{-j\theta}=\frac{U_{rms}}{I_{rms}}e^{-j\theta}=R-jX=$ $=\sqrt{R^2+X^2}\cdot e^{-j\theta}$	$P(t)=S(t)\cos\theta(t)=\frac{1}{2}(ui+\hat{u}\hat{i})$ $Q(t)=S(t)\sin\theta(t)=\frac{1}{2}(u\hat{i}-\hat{u}i)$ $S(t)=\sqrt{P(t)^2+Q(t)^2}=\frac{1}{2}U(t)I(t)=$ $=\lvert\overline{p}(t)\rvert\frac{\cos\varphi_p(t)}{\cos\varphi_u(t)\cos\varphi_i(t)}=$ $=\frac{p(t)}{\cos\varphi_u(t)\cos\varphi_i(t)}=\frac{P(t)}{\cos\theta(t)}$, $PF(t)=\frac{P(t)}{S(t)}=\cos\theta(t)=\frac{ui+\hat{u}\hat{i}}{U(t)I(t)}$ $U(t)=\sqrt{u(t)^2+\hat{u}(t)^2}$, $I(t)=\sqrt{i(t)^2+\hat{i}(t)^2}$ $\overline{u}(t)=U(t)e^{j\varphi_u(t)}$, $\overline{i}(t)=I(t)e^{j\varphi_i(t)}=I(t)e^{j[\varphi_u(t)+\theta(t)]}$ $\overline{Z}(t)=\frac{\overline{u}(t)}{\overline{i}(t)}=\frac{U(t)}{I(t)}e^{-j\theta(t)}=$ $R(t)-jX(t)=\sqrt{R^2(t)+X^2(t)}\cdot e^{-j\theta(t)}$

All over this paper are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

4.0.12. Evolution of the RMS concept

Based on the traditional Complex Phasors modeling of sinusoidal currents and voltages we have:

$$\langle p(t)\rangle = \frac{1}{T}\int_{[T]} p(t)dt = \frac{1}{T}\int_{[T]} u(t)i(t)dt \ (=) \text{ Average Instantaneous Power}$$

$$\langle p(t)\rangle = \frac{{U_{rms}}^2}{R} = \frac{1}{T}\int_{[T]} p(t)dt = \frac{1}{T}\int_{[T]} u(t)i(t)dt = \frac{1}{T}\int_{[T]} u(t)\frac{u(t)}{R}dt =$$

$$= \frac{1}{RT}\int_{[T]} u(t)^2 dt = R{I_{rms}}^2 = \frac{1}{T}\int_{[T]} p(t)dt = \frac{1}{T}\int_{[T]} u(t)i(t)dt =$$

$$= \frac{1}{T}\int_{[T]} Ri(t)i(t)dt = \frac{R}{T}\int_{[T]} i(t)^2 dt \ (=) \text{ Average Active Power} \Rightarrow$$

$$\Rightarrow U_{rms} = \sqrt{\frac{1}{T}\int_{[T]} u^2(t)dt} \ , \ I_{rms} = \sqrt{\frac{1}{T}\int_{[T]} i^2(t)dt}$$

$$P = U_{rms}I_{rms}\cos\theta = \text{Active Average Power}\,(=)[W]$$

$$Q = U_{rms}I_{rms}\sin\theta = \text{Reactive Average Power}\,(=)[VAR]$$

$$S = \sqrt{P^2 + Q^2} = U_{rms}I_{rms} = \text{Average Apparent Power}\ (=)[VA]$$

$$PF = \frac{P}{S} = \cos\theta = \text{Power Factor},$$

$$\overline{U} = \sqrt{2}U_{rms}e^{j\theta_u}, \overline{I} = \sqrt{2}I_{rms}e^{j\theta_i} \ , \overline{I}^* = \sqrt{2}I_{rms}e^{-j\theta_i} \ , \ \overline{S} = \frac{1}{2}\overline{U}\cdot\overline{I}^* \ .$$

It is important to underline that only Active Power P is the power delivered to the load, and that Reactive Power Q is the power reflected from the load (or from its reactive components) and sent back to its energy source. We already know all of that from the basic electro-technique regarding alternative currents and voltages (usually only related to sinusoidal and constant operating frequency signals in electric energy distribution systems). Here we shall extend and generalize the same concept of Active, Reactive and Apparent Power, to the propagation of any, arbitrary shaped and multi-frequency, or large frequency-band signals or wavefunctions. The generalization platform for new Active, Reactive and Apparent power definition will be related to Analytic, complex signal (and Hilbert Transform) that gives the possibility to present an arbitrarily shaped time domain signal into the corresponding Complex and Sinusoidal-like signal.

After replacing the Instantaneous Power with its Analytic Signal form, we find that the traditional concept of RMS currents and voltages remains unchanged:

$$\langle\overline{p}(t)\rangle = \frac{1}{T}\int_{[T]}\overline{p}(t)dt = \frac{1}{T}\int_{[T]}|\overline{p}(t)|e^{j\varphi_p(t)}dt =$$

$$= \langle p(t)\rangle + j\cdot\langle\hat{p}(t)\rangle \ (=) \ \text{Average Instantaneous Complex Power}$$

$$\langle\overline{p}(t)\rangle = \frac{U_{A-rms}^{\ 2}}{R} + j\frac{U_{R-rms}^{\ 2}}{R} = \frac{1}{T}\int_{[T]}\overline{p}(t)dt = \frac{1}{T}\int_{[T]}p(t)dt + j\frac{1}{T}\int_{[T]}\hat{p}(t)dt =$$

$$= \frac{1}{RT}\int_{[T]}u(t)^2 dt \ + j\frac{1}{T}\int_{[T]}\hat{p}(t)dt = \frac{U_{rms}^{\ 2}}{R}(1+j) =$$

$$= R\cdot I_{A-rms}^{\ 2} + jR\cdot I_{R-rms}^{\ 2} = \frac{R}{T}\int_{[T]}i(t)^2 dt \ + j\frac{1}{T}\int_{[T]}\hat{p}(t)dt = RI_{rms}^{\ 2}(1+j) \ \Rightarrow$$

$$\Rightarrow U_{A-rms} = \sqrt{\frac{1}{T}\int_{[T]}u^2(t)dt} = \sqrt{\frac{R}{T}\int_{[T]}p(t)dt} = U_{R-rms} = \sqrt{\frac{R}{T}\int_{[T]}\hat{p}(t)dt} = U_{rms} \ ,$$

$$I_{A-rms} = \sqrt{\frac{1}{T}\int_{[T]}i^2(t)dt} = \sqrt{\frac{1}{RT}\int_{[T]}p(t)dt} = I_{R-rms} = \sqrt{\frac{1}{RT}\int_{[T]}\hat{p}(t)dt} = I_{rms},$$

$$\langle\overline{p}(t)\rangle = \langle p(t)\rangle + j\cdot\langle\hat{p}(t)\rangle = \langle p(t)\rangle(1+j) = \langle\hat{p}(t)\rangle(1+j).$$

Based on complex Analytic Signal forms of voltage and current functions we can now generalize the Phasor notation concept by defining Instantaneous, Complex, Apparent, Active and Reactive Power (to apply to arbitrary signal shapes), as follows:

$$\overline{S}(t) = \overline{u}(t)\cdot\overline{i}^*(t) = P(t) - jQ(t) = S(t)e^{-j\theta(t)} = U(t)I(t)e^{-j\theta(t)} \ ,$$

$$\theta(t) = \varphi_i(t) - \varphi_u(t) = \operatorname{arctg}\frac{Q(t)}{P(t)} = \operatorname{arctg}\frac{\hat{i}(t)}{i(t)} - \operatorname{arctg}\frac{\hat{u}(t)}{u(t)} \ ,$$

$$P(t) = S(t)\cos\theta(t) = \frac{1}{2}(ui + \hat{u}\hat{i}) \ = \text{Power delivered to a load} \ ,$$

$$Q(t) = S(t)\sin\theta(t) = \frac{1}{2}(u\hat{i} - \hat{u}i) \ = \text{Power reflected from a load} \ ,$$

$$S(t) = \sqrt{P(t)^2 + Q(t)^2} = \frac{1}{2}U(t)I(t) = |\overline{p}(t)|\frac{\cos\varphi_p(t)}{\cos\varphi_u(t)\cos\varphi_i(t)} =$$

$$= \frac{p(t)}{\cos\varphi_u(t)\cos\varphi_i(t)} = \frac{P(t)}{\cos\theta(t)} = \frac{Q(t)}{\sin\theta(t)},$$

$$PF(t) = \frac{P(t)}{S(t)} = \cos\theta(t) = \frac{ui + \hat{u}\hat{i}}{U(t)I(t)} \ , \left\{\cos\theta(t) = 1 \ \Leftrightarrow (u\hat{i} = \hat{u}i)\right\}$$

$$U(t) = \sqrt{u(t)^2 + \hat{u}(t)^2} \ \left\{ = u(t)\sqrt{1 + (\frac{\hat{i}}{i})^2} = u(t)\sqrt{1 + (\frac{\hat{u}}{u})^2} \ , \text{for } \cos\theta(t) = 1 \right\},$$

$$I(t) = \sqrt{i(t)^2 + \hat{i}(t)^2} \ \left\{ = i(t)\sqrt{1 + (\frac{\hat{i}}{i})^2} = i(t)\sqrt{1 + (\frac{\hat{u}}{u})^2} \ , \text{for } \cos\theta(t) = 1 \right\},$$

$$\overline{u}(t) = U(t)e^{j\varphi_u(t)}, \ \overline{i}(t) = I(t)e^{j\varphi_i(t)} = I(t)e^{j[\varphi_u(t) + \theta(t)]} \ ,$$

$$\overline{i}^*(t) = I(t)e^{-j\varphi_i(t)} = I(t)e^{-j[\varphi_u(t) + \theta(t)]}$$

$$\overline{Z}(t) = \frac{\overline{u}(t)}{\overline{i}(t)} = \frac{U(t)}{I(t)}e^{-j\theta(t)} = R(t) - jX(t) = \sqrt{R^2(t) + X^2(t)}\cdot e^{-j\theta(t)}$$

$$\left\{ = \frac{u(t)}{i(t)}e^{-j\theta(t)}, \text{ for } \cos\theta(t) = 1 \right\}.$$

In most energy transfer systems, we do not care too much about immediate, transient, time-domain signals since mathematically it could be complicated to use such functions (especially for arbitrary-shaped signals). What counts much more in the practice of power and energy conversion and distribution systems are effective, rms, mean, averaged and other numerically expressed values (applicable to sufficiently characteristic frequency and time signal intervals).

To avoid time dependence of the above expressions, we shall determine all corresponding average values, as follows:

$$\langle p(t) \rangle = \frac{1}{T}\int_{[T]} p(t)dt = \frac{1}{T}\int_{[T]} u(t)i(t)dt = \frac{1}{T}\int_{[T]} |\overline{p}(t)| \cos\varphi_p(t)dt =$$

$$= \frac{1}{T}\int_{[T]} U(t)I(t)\cos\varphi_u(t)\cos\varphi_i(t)dt =$$

$$= \frac{U_{rms}{}^2}{R} = \frac{1}{RT}\int_{[T]} U(t)^2 \cos\varphi^2{}_u(t)dt \ = RI_{rms}{}^2 =$$

$$= \frac{R}{T}\int_{[T]} I(t)^2 \cos\varphi^2{}_i(t)dt \,(=)\ \text{Average Active Power} \Rightarrow$$

$$U_{rms} = \sqrt{\frac{1}{T}\int_{[T]} U(t)^2 \cos\varphi^2{}_u(t)dt} = \sqrt{\frac{1}{T}\int_{[T]} u(t)^2 dt}\ ,$$

$$I_{rms} = \sqrt{\frac{1}{T}\int_{[T]} I(t)^2 (t)\cos\varphi^2{}_i(t)dt} = \sqrt{\frac{1}{T}\int_{[T]} i(t)^2 dt}$$

$$P = U_{rms}I_{rms}\langle \cos\theta(t) \rangle \ = \frac{1}{T}\int_{[T]} P(t)dt =$$

$$= \frac{1}{T}\int_{[T]} S(t)\cos\theta(t)dt = \text{Active Average Power}$$

$$Q = U_{rms}I_{rms}\langle \sin\theta(t) \rangle \ = \frac{1}{T}\int_{[T]} Q(t)dt =$$

$$= \frac{1}{T}\int_{[T]} S(t)\sin\theta(t)dt = \text{Reactive Average Power}$$

$$\overline{S} = P - jQ = Se^{-j\theta} = \text{ Complex Apparent Power}$$

$$\theta = \text{arctg}\frac{Q}{P} = \text{arctg}\frac{\langle \sin\theta(t) \rangle}{\langle \cos\theta(t) \rangle}$$

$$S = \frac{1}{T}\int_{[T]} \sqrt{P(t)^2 + Q(t)^2}\,dt = U_{rms}I_{rms} =$$

$$= \sqrt{P^2 + Q^2} = \text{ Average Apparent Power}$$

Now it will be possible to extend the meaning of average electrical impedance for the general case of arbitrary voltage and current signals, for instance:

$$\overline{S} = P - jQ = Se^{-j\theta} = ZI^2_{rms}e^{-j\theta} = \frac{U^2_{rms}}{Z}e^{-j\theta} = (R - jX)I^2_{rms}$$

$$\overline{Z} = R - jX = Ze^{-j\theta} = \text{Average Complex Impedance}$$

$$Z = \left|\overline{Z}\right| = \frac{U_{rms}}{I_{rms}} = \sqrt{R^2 + X^2} = \sqrt{\frac{\int_{[T]} u(t)^2 dt}{\int_{[T]} i(t)^2 dt}} = \sqrt{\frac{\int_{[T]} U(t)^2 \cos\varphi^2_u(t)dt}{\int_{[T]} I(t)^2 \cos\varphi^2_i(t)dt}},$$

$$\langle\theta\rangle = \theta = \arctan\frac{X}{R} = \arctan\frac{Q}{P} = \arctan\left[\frac{\langle\sin\theta(t)\rangle}{\langle\cos\theta(t)\rangle}\right], \quad \overline{\theta} = \arctan\left[\frac{\overline{\sin\theta(t)}}{\overline{\cos\theta(t)}}\right]$$

$$R = \frac{P}{I^2_{rms}} = Z\cdot\langle\cos\theta(t)\rangle = \text{Average Active (Real) Impedance}$$

$$X = \frac{Q}{I^2_{rms}} = Z\cdot\langle\sin\theta(t)\rangle = \text{Average Reactive (Imaginary) Impedance, or Reactance}$$

$$\frac{X}{R} = \frac{Q}{P} = \frac{\langle\sin\theta(t)\rangle}{\langle\cos\theta(t)\rangle} = \tan\theta = \text{Average Quality Factor}, \quad \tan\overline{\theta} = \frac{\overline{\sin\theta(t)}}{\overline{\cos\theta(t)}}$$

$$\left\{\cos\theta(t), \langle\cos\theta(t)\rangle, \overline{\cos\theta(t)}, \cos\overline{\theta}\right\} \; (=) \; \text{Different Power Factors}$$

Practically, in power management systems (after introducing Analytic Signal methodology) we shall be able to apply innovative concepts based on averaged and rms signal values, easily measurable using existing technology, and valid for arbitrary-shaped signals (without the need to have precise time and frequency expressions). This way, many traditionally known concepts of power and frequency regulation will be generalized and could be significantly optimized. In addition, Quantum mechanical energy exchanges, quantum states and wave functions can be explained using "*active and reactive wave functions*" (like active and reactive power components). Stable atoms could be described as reactive resonant (multi-dimensional or multi-component) circuit structures, where Quality Factor $= \tan\theta$, approaches infinity ($\theta \to \frac{\pi}{2}$), or where all internal, stationary atom waves and other motions behave (analogically) like currents and voltages in simple loss-less capacitive-inductive resonant-oscillating circuits, without resistive components.

4.0.12.0. Wavefunction modelling based on electric power

Here we will attempt to analogically connect an arbitrary Power Function (which is the product between current and voltage, or product between force and speed, or product between any other relevant, mutually conjugate functions creating power), to a wave function, as known in Quantum Mechanics. Energy-wise analyzed, any wave propagation in time and frequency domain can be mutually (time-frequency) correlated using Parseval's theorem. Consequently, the immediate (time-domain) Power-signal can be presented as the square of the wave-function $\Psi^2(t)$*, and analysis of the optimal power transfer can be extended to any waves-propagation field, currents, voltages, forces, velocities etc. (and to arbitrarily shaped signals), and we can profit enormously (in booth, directions) after generalizing concepts of Active, Reactive and Apparent power in Electro-technique with the* $\Psi^2(t)$ *wave-function mathematics, based on Complex Analytic Signal modelling (or complex Phasors of all* *relevant functions creating power, like voltages and currents, or like other relevant power-analogies from Physics, as found in (4.0.82)).*

In Quantum Mechanics the wave function $\Psi^2(t)$ *is convenient (and by the voting consensus decision, meaning administratively) modeled as a probability function, but effectively behaves like a normalized total energy, being dimensionless function. In this book it will be treated as a power delivered to certain load (expressed in Watts as its units). Since here we will operate with Complex Voltage and Current Phasors, also the wavefunction will be presented in the form of the Complex Analytic Signal (but in other Physics or Mechanics related situations we will operate with velocities and forces, or with other power-related phasors as found in (4.0.82))*

Let us analogically create (see (4.0.1) - (4.0.4)) complex wavefunction $\overline{\Psi}(t)$ *presenting electric, complex power (within R-L-C complex impedance circuits and loads), being the product between relevant voltage and current phasors, as follows,*

$$\left\{\begin{array}{l} \overline{\Psi}(t)=\left|\overline{\Psi}(t)\right|\cdot e^{i\varphi(t)}=a(t)\cdot e^{i\varphi(t)}\Rightarrow \overline{\Psi}^2(t)=\left|\overline{\Psi}(t)\right|^2\cdot e^{i\cdot 2\cdot\varphi(t)}=\overline{P}(t) \\ \overline{P}(t)=\overline{U}(t)\cdot\overline{I}(t)=U(t)\cdot I(t)\cdot e^{i\varphi_u(t)}\cdot e^{i\varphi_i(t)}=U(t)\cdot I(t)\cdot e^{i[\varphi_u(t)+\varphi_i(t)]}= \\ =\overline{\Psi}^2(t)=\left|\overline{\Psi}(t)\right|^2\cdot e^{i\cdot 2\cdot\varphi(t)}\quad (=)[W] \\ \overline{P}(t)=P(t)\cdot e^{i[\varphi_u(t)+\varphi_i(t)]},\ P(t)=U(t)\cdot I(t) \end{array}\right\}\Rightarrow$$

$$\Rightarrow\left|\overline{\Psi}(t)\right|^2=U(t)\cdot I(t),\ \varphi_u(t)+\varphi_i(t)=2\cdot\varphi(t)\Rightarrow\varphi(t)=\frac{1}{2}[\varphi_u(t)+\varphi_i(t)]\Rightarrow$$

$$\Rightarrow\overline{\Psi}(t)=\left|\overline{\Psi}(t)\right|\cdot e^{i\varphi(t)}=\left|\overline{\Psi}(t)\right|\cdot e^{i\frac{1}{2}[\varphi_u(t)+\varphi_i(t)]}=\Psi(t)+i\cdot\hat{\Psi}(t)$$

Based on such foundations of wavefunctions (where involved Phasors could also be relevant velocity and force, angular-velocity, and torque, etc., as summarized before in (4.0.82)), the important analogy and connection of power and energy transfer with wavefunctions formalism is established (and can be analogically and indicatively related to wavefunctions practices in Quantum theory). The ontological foundations of any wavefunction formalism, especially in Quantum theory, are essentially based on Parseval's relations (starting from (4.0.4)), or on wavefunction energy in its normalized, non-dimensional form, (which can later be shaped as operations with probability functions, since this also works well, and it has the same meaning and compatibility with the total energy and/or total probability conservation laws; Only phase-function information is lost, or not taken into any account, in such modelling. See more in Chapter 4.3, under "4.3.3. Probability and Conservation Laws", equations (4.20) - (4.21)), as follows,

$$\left\{\begin{array}{l} \tilde{E}=\int_{-\infty}^{+\infty}\Psi^2(t)dt=\int_{-\infty}^{+\infty}\hat{\Psi}^2(t)dt=\frac{1}{2}\int_{-\infty}^{+\infty}\left|\overline{\Psi}(t)\right|^2dt=\frac{1}{2}\int_{-\infty}^{+\infty}a^2(t)dt=\int_{-\infty}^{+\infty}\left[\frac{a(t)}{\sqrt{2}}\right]^2dt= \\ =\int_{-\infty}^{+\infty}P(t)dt\ (=\tilde{m}c^2=)\ [J](=)[Ws] \end{array}\right\}\Rightarrow$$

$$\Rightarrow\frac{\tilde{E}}{\tilde{E}}=1=\frac{1}{\tilde{E}}\int_{-\infty}^{+\infty}\Psi^2(t)dt=\frac{1}{\tilde{E}}\int_{-\infty}^{+\infty}\hat{\Psi}^2(t)dt=\frac{1}{2\tilde{E}}\int_{-\infty}^{+\infty}\left|\overline{\Psi}(t)\right|^2dt=\frac{1}{2\tilde{E}}\int_{-\infty}^{+\infty}a^2(t)dt=\frac{1}{\tilde{E}}\int_{-\infty}^{+\infty}\left[\frac{a(t)}{\sqrt{2}}\right]^2dt\ldots$$

At least, now we know more about internal structure and real content of wavefunctions (as being products of two relevant Phasors), and such $\Psi^2(t)$ can be later conveniently normalized to be treated as a non-dimensional function, in connection with Parseval's identity (associating on Quantum theory, probability-related wavefunctions). Anyway, Quantum theory founders reduced this original and very rich meaning of wavefunctions (like favored in this book) to something being as an arbitrary assumption, at the beginning conceptualized (by consensus, meaning non-scientifically) as a probability and statistics related wavefunction, without any phase and internal-content information. Here, the internal content of wavefunctions is relevant voltage and current, or other power-couples from *(4.0.82), and meaning of* $\Psi^2(t)$ *is that (before normalization) it presents, dimensional Power function.* As the additional support to Wave-Particle Duality and wavefunction modeling, see also "T.4.0. Photon – Particle Analogies", T.4.0.1, T.4.0.2. from Chapters 4.0 and 4.1, and T.10.00. from chapter 10.

.... This part (below, and everywhere in this Chapter) should be additionally updated, better organized, partially corrected, and united with other familiar elaborations from the same Chapter (to be modified and rearranged later), but all of that is already sufficiently clear, informative, and indicative to explain the real meaning of wavefunctions.

Let us continue with the <u>*immediate electrical power*</u> found as a product between corresponding voltage and current signals, where both are arbitrarily shaped (energy, time and frequency limited) functions. We can show that such an active-power function (which transfers power from its source to its load) can be presented as,

$$\Psi^2(t) = \mathbf{P(t)} = \mathbf{S(t)}\cos\theta(\mathbf{t}) = \frac{1}{2}(\mathbf{ui}+\hat{\mathbf{u}}\hat{\mathbf{i}}) = \mathbf{Q(t)}\cdot\mathbf{cotan}\,\theta(\mathbf{t})\ (=)[\mathbf{W}]\ .$$

The power reflected from a load, or Reactive Power, can be given as:

$$\mathbf{Q(t)} = \mathbf{S(t)}\sin\theta(\mathbf{t}) = \frac{1}{2}(\mathbf{u}\hat{\mathbf{i}}-\hat{\mathbf{u}}\mathbf{i}) = \Psi^2(\mathbf{t})\cdot\tan\theta(\mathbf{t}) = \mathbf{P(t)}\cdot\tan\theta(\mathbf{t})\,(=)\,[\mathbf{VAR}]$$

Electric Power and Energy transfer analysis (especially for arbitrary voltage and current signal forms) can be related to a wave-function analysis if we establish the wave function (or more precisely, the square of the wave function) in the following way:

$$\mathrm{P(t)} = \Psi^2(\mathrm{t}) = [\mathrm{a(t)}\cos\varphi(\mathrm{t})]^2 = \text{Wave function}\ ,\ \mathrm{t}\in[\mathrm{T}],$$

$$\Psi(\mathrm{t}) = \mathrm{a(t)}\cos\varphi(\mathrm{t}),\ \hat{\Psi}(\mathrm{t}) = \mathrm{a(t)}\sin\varphi(\mathrm{t}),$$

$$\overline{\Psi}(\mathrm{t}) = \Psi(\mathrm{t}) + \mathrm{j}\hat{\Psi}(\mathrm{t}) = \Psi(\mathrm{t}) + \mathrm{jH}[\Psi(\mathrm{t})] = \mathrm{a(t)}e^{\mathrm{j}\varphi(\mathrm{t})} = \frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty}\mathrm{U}(\omega)e^{-\mathrm{j}\omega\mathrm{t}}d\omega =$$

$$= \frac{1}{\pi^2}\int_{0}^{+\infty}\mathrm{U}(\omega)e^{-\mathrm{j}\omega\mathrm{t}}d\omega = \frac{1}{\pi}\int_{(0,+\infty)}\mathrm{A}(\omega)e^{-\mathrm{j}\omega\mathrm{t}}d\omega,$$

$$\mathrm{U}(\omega) = \mathrm{U_c}(\omega) - \mathrm{j\,U_s}(\omega) = \int_{(-\infty,+\infty)}\overline{\Psi}(\mathrm{t})\,e^{\mathrm{j}\omega\mathrm{t}}d\mathrm{t} = \mathrm{A}(\omega)e^{-\mathrm{j}\Phi(\omega)}\ ,$$

$$\mathrm{U_c}(\omega) = \mathrm{A}(\omega)\cos\Phi(\omega),\ \mathrm{U_s}(\omega) = \mathrm{A}(\omega)\sin\Phi(\omega),$$

$$a(t) = \sqrt{\Psi(t)^2 + \hat{\Psi}(t)^2}\ , \quad \varphi(t) = \operatorname{arctg}\frac{\hat{\Psi}(t)}{\Psi(t)}\,,$$

$$A^2(\omega) = U^2{}_c(\omega) + U^2{}_s(\omega), \quad \Phi(\omega) = \operatorname{arctg}\frac{U_s(\omega)}{U_c(\omega)}\,,$$

$$T\cdot\langle P(t)\rangle = \int_{-\infty}^{+\infty} P(t)dt = \int_{-\infty}^{+\infty} \Psi^2(t)dt = \int_{-\infty}^{+\infty} \hat{\Psi}^2(t)dt = \frac{1}{2}\int_{-\infty}^{+\infty} \left|\overline{\Psi}(t)\right|^2 dt =$$

$$= \frac{1}{2}\int_{-\infty}^{+\infty} \left|\Psi(t) + j\hat{\Psi}(t)\right|^2 dt = T\cdot\langle \hat{P}(t)\rangle = \int_{-\infty}^{+\infty} \hat{P}(t)dt =$$

$$= \frac{1}{2}\int_{-\infty}^{+\infty} a^2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |U(\omega)|^2 d\omega = \frac{1}{\pi}\int_0^{\infty} [A(\omega)]^2 d\omega,$$

As we can see, any physics-related and finite wave function Ψ has its Hilbert couple $\hat{\Psi}$. Both are presentable as a product of two other functions, since to create an instantaneous (short time) power function $\Psi^2(t)$, (see (4.0.4)) it is essential to make the product of two relevant, mutually conjugate signals, like current and voltage, velocity and force, or some other equally important and analogous couple of conjugate signals (see examples in (4.0.82)).

Consequently, short time active power $P(t)$ in electric circuits is,

$$\overline{P}(t) = \frac{1}{2}\overline{u}(t)\cdot\overline{i}(t)\cdot\cos\theta_{Load}(t) \Rightarrow$$

$$P(t) = \frac{1}{2}(ui + \hat{u}\hat{i}) = S(t)\cos\theta_{Load}(t) \ = \Psi^2(t) = \Psi_1{}^2(t) + \Psi_2{}^2(t),\ \Psi_1^2 = \frac{ui}{2},\ \Psi_2^2 = \frac{\hat{u}\hat{i}}{2}.$$

Instantaneous current and/or voltage frequency is (see (4.0.2)),

$$\omega(t) = \frac{\partial\varphi(t)}{\partial t} = \dot{\varphi}(t) = \frac{\Psi(t)\dot{\hat{\Psi}}(t) - \hat{\Psi}(t)\dot{\Psi}(t)}{a^2(t)} = \operatorname{Im}\left[\frac{\dot{\overline{\Psi}}(t)}{\overline{\Psi}(t)}\right] = 2\pi f(t) \Rightarrow$$

$$\omega_i(t) = \frac{\partial\varphi_i(t)}{\partial t} = \dot{\varphi}_i(t) = \frac{i(t)\frac{d}{dt}\hat{i}(t) - \hat{i}(t)\frac{d}{dt}i(t)}{I^2(t)} = 2\pi f_i(t),$$

$$\omega_u(t) = \frac{\partial\varphi_u(t)}{\partial t} = \dot{\varphi}_u(t) = \frac{u(t)\frac{d}{dt}\hat{u}(t) - \hat{u}(t)\frac{d}{dt}u(t)}{U^2(t)} = 2\pi f_u(t),$$

$$\sin(\varphi_u - \varphi_i) = \frac{i(t)\hat{u}(t) - \hat{i}(t)u(t)}{I(t)\cdot U(t)} = \frac{i(t)\hat{u}(t) - \hat{i}(t)u(t)}{\sqrt{i(t)^2 + \hat{i}(t)^2}\cdot\sqrt{u(t)^2 + \hat{u}(t)^2}}.$$

For the same electric circuit with the same source and same load, current and voltage frequencies should be the same.

" Short-Time" Magnitude of the electric-circuit Load Impedance is,

$$M_{stZ(t)} = \frac{U(t)}{I(t)} = \frac{\sqrt{u(t)^2 + \hat{u}(t)^2}}{\sqrt{i(t)^2 + \hat{i}(t)^2}} = \left|\overline{Z}(t)\right|.$$

It should not be too big success to explain quantum-mechanical diffraction causally, or to explain superposition and interference effects, when a "single-wave-object" and/or a single particle (like an electron, or photon) passes a diffraction plate with (at least) two small, diffraction holes, because there is not a single particle or wave object here. There are always minimum two of mutually Hilbert-coupled wave elements and their mixed wavelike products, in some way being “force-moments-energy-coupled” with their environment, this way extending the number of interaction participants, and we need all of them to find resulting amplitude, phase and frequency functions). What could look like a bit unusual quantum interaction, or interference of a single wave or particle with itself, in fact, presents an interaction of at least two wave entities with some other, third object, here with a plate with diffraction holes ($\Psi^2(t) = \Psi_1{}^2(t) + \Psi_2{}^2(t)$). Somehow Nature always creates complementary and conjugate couples of essential wave elements (signals, particles, energy states …) belonging to all kind of matter motions, or we can also say that every object (or energy state) in our universe has its non-separable and conjugate, phase-shifted image (defined by an Analytic Signal concept). Consequently, the quantum-mechanical wave function and wave energy should represent only motional energy (or power) composed of minimum two mutually coupled wave functions (see also relevant comments about the meaning of complex functions, as examples in (4.0.82), and Analytic signal components as found in T.4.0.1).

Here applied mathematics, regarding wave functions $\Psi^2(t) = \Psi_1{}^2(t) + \Psi_2{}^2(t)$, after making convenient normalization/s and generalizations, would start to look as applying Probability Theory laws, like in the contemporary Quantum Theory. Consequently, modern Quantum Theory could also be understood as generalized mathematical modeling of microworld phenomenology, which unifies and respects all conservation laws of physics in a joint, normalized, and dimensionless, mutually well-correlated theoretical platforms, using the framework of Statistics, Probability, and modern Signal Analysis (practically respecting different forms of Parseval’s identity relations, and total probability conservation, what corresponds to total energy conservation). In this way, a new mathematical or Quantum theory is created, that is only a bit unusual and artificial by its appearance, but it is isomorphic to usual mathematical modeling as known in Signal Analysis, and by respecting Conservation Laws from deterministic and classical Physics (and there is no wonder that in such situations, related to conservation laws, QT works very well; -nothing to admire QT so much as something divine, unique, and exceptional; -just interesting, exotic and successful mathematical modelling that beside all good sides, also has number of its structural limitations).

Also, a kind of generalized analogy with Norton and Thevenin's theorems (known in Electric Circuit Theory) should also exist (conveniently formulated) in all other fields of Physics and Quantum Theory related to wave motions, since the cause or source of a particular action is producing specific effect (or an output), and vice versa and such events are always mutually coupled.

See more in: [57], Michael Feldman, [109], Poularikas A. D., including [110], and [111]

[♣ COMMENTS & FREE-THINKING CORNER (working on ...):

We could also address complex power in electric circuits, as a product between two complex phasor functions, as for example,

$$\left\{\begin{array}{l} \overline{P}(t)=\frac{1}{2}\overline{u}(t)\cdot\overline{i}(t),\ \overline{u}(t)=Ue^{j[\varphi(t)+\varphi_u]},\ \overline{i}(t)=Ie^{j[\varphi(t)+\varphi_i]},\ j^2=-1,\ (\varphi_u,\varphi_i)=\text{constants} \\ \overline{P}(t)=\frac{1}{2}\overline{u}(t)\cdot\overline{i}(t)=\frac{1}{2}U\cdot I\cdot e^{j[2\varphi(t)+\varphi_u+\varphi_i]}=\frac{1}{2}U\cdot I\cdot e^{j\theta_{Load}(t)}=\frac{1}{2}U\cdot I\cdot\cos\theta_{Load}(t)+j\frac{1}{2}U\cdot I\cdot\sin\theta_{Load}(t) \\ \theta_{Load}(t)=2\varphi(t)+\varphi_u+\varphi_i,\ S(t)=\frac{1}{2}U\cdot I=\frac{U}{\sqrt{2}}\cdot\frac{I}{\sqrt{2}}=U_{rms}\cdot I_{rms},...\varphi(t)=\omega t\ \ or\ \ \varphi(t,x)=\omega t\pm kx \end{array}\right\},$$

$$P(t)=\frac{1}{2}(ui+\hat{u}\hat{i})=\frac{1}{2}U\cdot I\cdot\cos\theta_{Load}(t)=S(t)\cos\theta_{Load}(t)=\Psi^2(t)=\Psi_u(t)\cdot\Psi_i(t).$$

or to present complex power, alternatively and simply as,

$$\overline{P}(t)=\overline{u}(t)\cdot\overline{i}(t)=U\cdot I\cdot e^{j[2\varphi(t)+\varphi_U+\varphi_I]}=\overline{\Psi}^2(t),\ \overline{P}(t)=\overline{u}(t)\cdot\overline{i}(t)=U\cdot I\cdot e^{j[2\varphi(t)+\varphi_U+\varphi_I]},$$

$\overline{\Psi}(t)=\left|\overline{\Psi}(t)\right|\cdot e^{j\left[\varphi(t)+\frac{\varphi_u+\varphi_i}{2}\right]}$ *since it is obvious here that a phase function* $\varphi(t)$*, and* $\theta_{Load}(t)=\varphi(t)+\frac{\varphi_u+\varphi_i}{2}$ *should be appropriately taken into account … (still working on...).*

Quantum theory effectively and implicitly (or unintentionally and creatively) postulated as the most relevant wavefunction $\left|\overline{\Psi}(t)\right|$***,*** $\left|\overline{\Psi}(t)\right|^2$ ***or ,*** $\left|\overline{\Psi}(r,t)\right|^2$, $r=r(x,y,z)$ ***, without considering the importance of the wavefunction phase*** $\varphi(t)$***, or*** θ_{Load}***, what is, only energetically, based on Parseval theorem or identity, mathematically correct, but incomplete (see (4.0.82)). Then, such wavefunction, dimensionally being a power, was conveniently normalized (losing its dimensionality) and assumed or postulated as being a probability function. This is also possible and correct since the sum of all events' probabilities of certain "energy-moments" state is equal to one (1), and this is mathematically mimic conservation laws of Physics. Complete internal structure of such probabilistic QT wavefunction is simply lost or not considered (analogically on the way here we presented and analyze current, voltage, impedances, power etc.). It is also not correct to say that big statistical sets or ensembles of identical objects, states or particles do not have joint or resulting phase information (since effects of mutual interferences, couplings, synchronizations, and entanglements are anyway working within such statistical sets; -see more in Chapter 10). Phase function is addressing spatial-temporal unity, spectral complexity and dynamic or velocities of involved matter-wave states (and it should not be neglected). After convenient mathematical makeup, Orthodox QT managed to be operational and mathematically predictable, but stayed incomplete, complicated, and unnatural. Too many devoted founders and followers of such QT are honestly and proudly saying that nobody understands such QT, but since it works very well, it should be considered as fully correct. Such QT also surfaces a lot of unnecessary absurdities and dilemmas about our world reality. Many brilliant minds dissipated their intellectual and creative energy as the consequence of improper and unnatural modelling of QT. The message of the author of this book is that we need to apply appropriate remodeling of QT, and make it much more tangible, elegant, powerful, and easier applicable to the world of Physics.***

4.0.12.1. Standard Deviation concerning Load Impedance

*Since the standard deviation is often used to explain Uncertainty relations, let us analyze the meaning of the signal Standard Deviation regarding electric circuits where it is possible to measure voltage, **u**, and current, **i**, on an electric load. An electric load can have an entirely resistive (or active) impedance, **R**, or in general case, it can present the complex impedance, **Z** (as the combination of **R**, **L** and **C** elements).*

The Standard Deviation presents the power of the average signal deviation from the mean power, and since we can measure coincidently (by sampling) the load current and voltage (both mutually dependent), we are in the position to formulate the following generalized expression for Standard Deviation, by multiplying voltage and current deviations,

$$\sigma_g^2 = \frac{1}{N}\sqrt{\sum_{(i)} \left|(u_i - \bar{u})(i_i - \bar{i})\right|^2} = \frac{1}{N}\sqrt{\sum_{(i)} (u_i - \bar{u})^2 (i_i - \bar{i})^2}$$

$$N (=) \text{ number of samples, } \bar{u} = \frac{1}{N}\sum_{(i)} u_i \text{ , } \bar{i} = \frac{1}{N}\sum_{(i)} i_i \qquad (4.0.105)$$

$$\bar{u},\ \bar{i} (=) \text{ runnung mean values (time evolving)}$$

Only in cases when the load is resistive, above-formulated Standard Deviation, (4.0.105) can be transformed into the following expressions (fully equivalent to the contemporary definition of the Standard Deviation),

$$u = Ri,\ \bar{u} = R\bar{i} \Rightarrow \sigma_g^2 \to \sigma^2 \Rightarrow$$

$$\sigma^2 = \frac{1}{N}\frac{1}{R}\sum_{(i)} \left|u_i - \bar{u}\right|^2 = \frac{1}{N}R\sum_{(i)} \left|i_i - \bar{i}\right|^2 = \frac{1}{N}\frac{1}{R}\sum_{(i)} (u_i - \bar{u})^2 = \frac{1}{N}R\sum_{(i)} (i_i - \bar{i})^2$$

$$\sigma^2 = \frac{1}{N}\frac{\bar{i}}{\bar{u}}\sum_{(i)} (u_i - \bar{u})^2 = \frac{1}{N}\frac{\bar{u}}{\bar{i}}\sum_{(i)} (i_i - \bar{i})^2 = \frac{1}{N}\sum_{(i)} \frac{i_i}{u_i}(u_i - \bar{u})^2 = \frac{1}{N}\sum_{(i)} \frac{u_i}{i_i}(i_i - \bar{i})^2 \qquad (4.0.106)$$

$$\sigma^2 = \frac{1}{N}\sqrt{\sum_{(i)} (u_i - \bar{u})^2 \cdot \sum_{(i)} (i_i - \bar{i})^2}$$

*If after sampling of a load current and voltage we find that expressions (4.0.105) and (4.0.106) produce significantly different results, this should mean that the load impedance has non-resistive, complex character (presenting a combination of **R**, **L** and **C** elements). The factor of “Impedance Complexity” concerning measured standard definition can be defined as,*

$$\frac{\sigma^2}{\sigma_g^2} = \sqrt{\frac{\sum_{(i)} (u_i - \bar{u})^2 \cdot \sum_{(i)} (i_i - \bar{i})^2}{\sum_{(i)} (u_i - \bar{u})^2 \cdot (i_i - \bar{i})^2}} \begin{Bmatrix} = 1 \to \text{Resistive or Active Load} \\ \neq 1 \to \text{Complex Load} \end{Bmatrix} \qquad (4.0.107)$$

In other words, when the ratio (4.0.107) is not equal to 1, the contemporary formulation of Standard Deviation (found in all Statistics books) is not the best (and not the most general) representation of the natural signal deviation. If we (analogically) apply the same situation on arbitrary signals, we could say that the complexity of any signal, obtained as a product of two mutually conjugate signals (like found in (4.0.82)), can be tested and additionally classified by (4.0.105), (4.0.106) and (4.0.107). Consequently, Standard Deviation in Quantum Theory and Physics often used in many cases could be very limited (or in some cases wrong), since not all loads are resistive (electrically, or mechanically, or in some other analogical meaning). Many essential laws of Statistics, Thermodynamics, Quantum Theory, etc., such as Normal Gauss distribution, Black body radiation law, Uncertainty Relations, etc. are already formulated using the traditional definition of the Standard Deviation. Such standard deviation is intrinsically limited to what analogically or directly corresponds to active or resistive loads (where functions, defining the signal power, are in phase), what is influencing that some of the conclusions based on such laws could be wrong (the fact never noticed, because of the present incomplete definition of the Standard Deviation). ♣]

4.0.12.2. New definitions of Average and RMS Power functions

Let us take into consideration electric voltage and current on specific load (related to useful measurements and power quantification). Later, when using established systems of electro-mechanical analogies, it will be possible to extend similar definitions to velocity, force, torque, etc. Standard definitions of average and RMS functions are already known from the literature. Current and voltage signals on specific load are mutually dependent (and load impedance dependent), and what should matter in such situations is to consider immediate electric power, since existing definitions of average and RMS values are not taking directly and explicitly an immediate load power. Here are proposals on how to redefine electrical load power, using new RMS functions for voltage and current.

Power Signal	AVERAGE	RMS
$u(t),\ \hat{u}(t) = H[u(t)],$ $\bar{u}(t) = u(t) + j\cdot\hat{u}(t) = U(t)e^{j\varphi_u(t)},$ $U(t) = \sqrt{u(t)^2 + \hat{u}(t)^2}$ (=) voltage, $j^2 = -1$	$\langle u\rangle = \frac{1}{T}\int_{t_0}^{t_0+T} u(t)dt,$ $\langle \hat{u}\rangle = \frac{1}{T}\int_{t_0}^{t_0+T} \hat{u}(t)dt$	$u_{RMS} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} u^2(t)dt},$ $\hat{u}_{RMS} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} \hat{u}^2(t)dt},$ $U_{RMS} = \sqrt{u_{RMS}^2 + \hat{u}_{RMS}^2} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T}\left[u^2(t)+\hat{u}^2(t)\right]dt}$
$i(t), \hat{i}(t) = H[i(t)],$ $\bar{i}(t) = i(t) + j\cdot\hat{i}(t) = I(t)e^{j\varphi_i(t)}$ $I(t) = \sqrt{i(t)^2 + \hat{i}(t)^2}$ (=) current, $j^2 = -1$	$\langle i\rangle = \frac{1}{T}\int_{t_0}^{t_0+T} i(t)dt,$ $\langle \hat{i}\rangle = \frac{1}{T}\int_{t_0}^{t_0+T} \hat{i}(t)dt$	$i_{RMS} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} i^2(t)dt},$ $\hat{i}_{RMS} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} \hat{i}^2(t)dt},$ $I_{RMS} = \sqrt{i_{RMS}^2 + \hat{i}_{RMS}^2} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T}\left[i^2(t)+\hat{i}^2(t)\right]dt}$
$p(t) = u(t)\cdot i(t) = \Psi^2(t),$ $\hat{p}(t) = H[p(t)],$ $\bar{P}(t) = \hat{p}(t) + j\hat{p}(t) = \bar{\Psi}^2(t),$ $p^*(t) = \frac{1}{2}[p(t)+\hat{p}(t)] = \left(\Psi^*(t)\right)^2,$ $\bar{P}^*(t) = \bar{u}(t)\cdot\bar{i}(t) = \left(\bar{\Psi}^*(t)\right)^2 =$ $= U(t)\cdot I(t)e^{j[\varphi_u(t)+\varphi_i(t)]}$	$\langle p\rangle = \langle u\rangle\cdot\langle i\rangle, \langle\hat{p}\rangle = \langle\hat{u}\rangle\cdot\langle\hat{i}\rangle,$ $\langle P\rangle = \frac{1}{2}(\langle p\rangle + \langle\hat{p}\rangle),$ $\langle P^*\rangle = \frac{1}{T}\int_{t_0}^{t_0+T}\left[\frac{u(t)\cdot i(t)+\hat{u}(t)\cdot\hat{i}(t)}{2}\right]dt$	$U_{RMS}\cdot I_{RMS} = \sqrt{\left(u_{RMS}^2+\hat{u}_{RMS}^2\right)\cdot\left(i_{RMS}^2+\hat{i}_{RMS}^2\right)} =$ $= P_{RMS}$ (=) RMS Power

*For an arbitrary-shaped electric signal, **RMS Power** could be an optimal effective, apparent power, such as,*

$$P_{RMS} = U_{RMS}\cdot I_{RMS} = \sqrt{\left(u_{RMS}^2+\hat{u}_{RMS}^2\right)\cdot\left(i_{RMS}^2+\hat{i}_{RMS}^2\right)},$$
$$\left\langle P_{RMS}^*\right\rangle = \frac{1}{T}\int_{t_0}^{t_0+T}\frac{p(t)\cdot\hat{p}(t)}{U_{RMS}\cdot I_{RMS}}dt \ (= \text{average RMS Power}). \qquad (4.0.108)$$

Of course, benefits and applicability of such power definitions should be more rigorously tested.

Coherence factors:

Between two signals, $s_1(t)$ and $s_2(t)$ *it is possible to find the measure of their mutual temporal and spectral coherence, using the following coherence factors definitions:*

$$K_t = \frac{\int_{-\infty}^{+\infty} s_1(t)\cdot s_2(t)\cdot dt}{\int_{-\infty}^{+\infty} |s_1(t)|\cdot|s_2(t)|\cdot dt},\ K_\omega = \frac{\int_0^{+\infty} A_1(\omega)\cdot A_2(\omega)\cdot d\omega}{\int_0^{+\infty} |A_1(\omega)|\cdot|A_2(\omega)|\cdot d\omega},$$

$$\tilde{K}_t = \frac{\int_{-\infty}^{+\infty} \tilde{s}_1(t)\cdot \tilde{s}_2(t)\cdot dt}{\int_{-\infty}^{+\infty} |\tilde{s}_1(t)|\cdot|\tilde{s}_2(t)|\cdot dt},\ \tilde{K}_\omega = \frac{\int_0^{+\infty} \tilde{A}_1(\omega)\cdot \tilde{A}_2(\omega)\cdot d\omega}{\int_0^{+\infty} |\tilde{A}_1(\omega)|\cdot|\tilde{A}_2(\omega)|\cdot d\omega}, \quad (4.0.83), (4.0.87)$$

$$K_t^* = \sqrt{K_t\cdot\tilde{K}_t}, \qquad K_\omega^* = \sqrt{K_\omega\cdot\tilde{K}_\omega}.$$

Consequently, for electric signals, we have,

$$K_t = \frac{\int_{-\infty}^{+\infty} u(t)\cdot i(t)\cdot dt}{\int_{-\infty}^{+\infty} |u(t)|\cdot|i(t)|\cdot dt} = \frac{\int_{-\infty}^{+\infty} p(t)\cdot dt}{\int_{-\infty}^{+\infty} |u(t)|\cdot|i(t)|\cdot dt} \cong \frac{\int_{t_0}^{t_0+T} p(t)\cdot dt}{\int_{t_0}^{t_0+T} |u(t)|\cdot|i(t)|\cdot dt}. \quad (4.0.109)$$

Now we can draw important conclusions concerning coherence criteria for certain signals $s_1(t)$ and $s_2(t)$, or $u(t)$ and $i(t)$ and connect them with the optimal power and energy propagation:

-If $K_t = 1$, $s_1(t)$ and $s_2(t)$ are totally coherent, or mutually *in phase, $\Delta\varphi(t) = 0$. If $s_1(t)$ and $s_2(t)$ are voltage and current on certain electric load, we will have optimal energy (or power) transfer (see (4.0.109)). That is the case of active or resistive load impedances. The total power delivered to such a load is active.*

-If $K_t = 0$, $s_1(t)$ and $s_2(t)$ *are mutually orthogonal or phase* shifted *for $\pi/2$, $|\Delta\varphi(t)| = \pi/2$. If $s_1(t)$ and $s_2(t)$ are voltage and current on certain electric load (see (4.0.109)), no energy circulation from the source to the* load *is possible (i.e., the* load *does not receive any energy).*

-If $K_t = -1$, $s_1(t)$ and $s_2(t)$ are in the counter-phase, $|\Delta\varphi(t)| = \pi$. If $s_1(t)$ and $s_2(t)$ are voltage and current on certain electric load (see (4.0.109)), then there is a non-optimal energy transfer, i.e., the energy is completely reflected off the load and returned to its source (the energy has a "-" sign). That is the case of fully reactive load impedances.

In any other case when K_t is not equal to 1, 0, or -1, part of power is delivered to a load and part reflected towards its source. Those are cases of complex load impedances (with active and reactive loading components).

Such coherence functions or factors are interesting concerning different signal analysis and measurements strategies, where we need to separate or extract signals (from the specific multicomponent, complex signal) that are mutually dependent and correlated, or we can also extract signals that could be very mutually independent. For instance, between two Complex Analytic signals, or wave functions, $\overline{\Psi}_1(x,t)$ and $\overline{\Psi}_2(x,t)$ it is possible (in a similar way as already elaborated) to find the measure of their mutual spatial-temporal and spectral coherence, orthogonality, and entanglement, using the following coherence factors definitions:

$$\overline{K}_{r,t} = \frac{\iiint_{(-\infty,+\infty)} \overline{\Psi}_1(r,t)\cdot\overline{\Psi}_2(r,t)\cdot dr\cdot dt}{\iiint_{(-\infty,+\infty)} |\overline{\Psi}_1(r,t)|\cdot|\overline{\Psi}_2(r,t)|\cdot dr\cdot dt},\ \overline{K}_{k,\omega} = \frac{\iiint_{(-\infty,+\infty)} \overline{U}_1(k,\omega)\cdot\overline{U}_2(k,\omega)\cdot dk\cdot d\omega}{\iiint_{(-\infty,+\infty)} |\overline{U}_1(k,\omega)|\cdot|\overline{U}_2(k,\omega)|\cdot dk\cdot d\omega}$$

*If we give a freedom to certain creative brainstorming and imagination, we could consider coherence factor K_t as a replacement for **Power Factor**, or "cosines-theta" factor (in an analogical relation to a standard, existing definitions of Active, Reactive and Apparent power),*

$$K_t \cong \frac{\int_{t_0}^{t_0+T} p^*(t)\cdot dt}{\int_{t_0}^{t_0+T} |\overline{u}(t)|\cdot|\overline{i}(t)|\cdot dt} (\cong) \cos\theta = PF \Rightarrow$$

$$\Rightarrow \sin\theta \cong \sqrt{1-(\cos\theta)^2} (\cong) \sqrt{1-\left[\frac{\int_{t_0}^{t_0+T} p^*(t)\cdot dt}{\int_{t_0}^{t_0+T} |\overline{u}(t)|\cdot|\overline{i}(t)|\cdot dt}\right]^2} = \sqrt{1-K_t^2} \ . \quad (4.0.110)$$

What is interesting (to explore) in (4.0.109) and (4.0.110), is that Power Factor can take positive values between 0 and 1, and negative values between 0 and -1.

Now, combining Average and RMS Power (4.0.108), and newly introduced Power Factor (4.0.110), we could exercise if a redefinition of Active, Reactive and Apparent power will be defendable, such as,

$$\begin{aligned} \mathbf{P}_{active} &= \mathbf{P}_{RMS}\cdot\cos\theta = \mathbf{U}_{RMS}\mathbf{I}_{RMS}\cdot\cos\theta\ (=)\frac{\mathbf{K}_t}{\mathbf{T}}\int_{t_0}^{t_0+T}\frac{\mathbf{P}^2(t)}{\mathbf{U}_{RMS}\cdot\mathbf{I}_{RMS}}\mathbf{dt}, \\ \mathbf{P}_{reactive} &= \mathbf{P}_{RMS}\cdot\sin\theta = \mathbf{U}_{RMS}\mathbf{I}_{RMS}\cdot\sin\theta\ (=)\frac{\sqrt{1-\mathbf{K}_t^2}}{\mathbf{T}}\int_{t_0}^{t_0+T}\frac{\mathbf{P}^2(t)}{\mathbf{U}_{RMS}\cdot\mathbf{I}_{RMS}}\mathbf{dt}, \\ \mathbf{P}_{apparent} &= \sqrt{(\mathbf{P}_{active})^2+(\mathbf{P}_{reactive})^2} = \mathbf{P}_{RMS} = \mathbf{U}_{RMS}\mathbf{I}_{RMS}\,. \end{aligned} \quad (4.0.111)$$

The meanings of positive and negative power components could be as follows,

-If $\mathbf{P}_{active}$ *is taking positive values between 0 and 1, active power is being delivered from its source to its load.*

-If $\mathbf{P}_{active}$ *is taking negative values between 0 and -1, active power is being reflected or delivered from its load to its source.*

-If $\mathbf{P}_{reactive}$ *is taking positive values between 0 and 1, reactive power is being reflected from its load to its source.*

-If $\mathbf{P}^*_{reactive}$ *is taking negative values between 0 and -1, reactive power is being delivered from its source to its load.*

The variety of positive and negative power components can be explained by considering that both the energy source and its load have complex impedance components. Additionally, when the phasors of electric currents and voltages are arbitrarily shaped and wideband (i.e., non-simple harmonic) signals, this complexity becomes more apparent. Coherence factors—whether in the original or in spectral, temporal, and spatial domains—essentially address resonance effects and the degree of mutual synchronization between two wave functions, signals, or power-function components, such as current and voltage across a load.

These concepts of resonance and synchronization apply not only to simple harmonic signals but also to much more complex, spectrally wideband signals. Furthermore, they are linked to entanglement effects, extending the understanding of entanglement to a broader context. The unity of our Universe and the behavior of all matter wave phenomena within it can be characterized using the coherence factors introduced here.

Theoretical and conceptual analysis of quantum-mechanical wave functions has been significantly "mutilated" and oversimplified in current practices of Quantum Theory, particularly regarding probabilistic wave functions. Neglecting the rich possibilities offered by analytic signals and phasors limits the depth of analysis and qualification. While methods and models from Statistics and Probability Theory can be particularly useful and successful when applied as final steps in process modeling, calculations, and analysis, they should not be the primary tools in the initial stages of modeling. This contrasts with their current use in contemporary Orthodox Quantum Theory, where they are often imposed or practiced as foundational elements.

4.1. MATTER WAVES AND QUANTUM MECHANICS

This chapter builds upon the concepts, ideas, and theoretical foundations of wave-particle duality discussed in earlier sections, particularly in Chapter 2 (e.g., "2.3.2. Macro-Cosmological Matter-Waves and Gravitation") and Chapter 4.0 ("Wave Functions, Wave Velocities, and Uncertainty Relations"). To fully grasp the ideas presented here, it is strongly recommended that readers first familiarize themselves with the basics of particle-wave duality found in Chapters 2, 4.0, 9, and 10. This will make the concepts introduced in this chapter easier to understand and develop.

Chapter 4.0 primarily focuses on the mathematical aspects of wavefunctions and the basic modeling of wave-particle duality. In contrast, Chapter 4.1 will explore how Nature, or our Universe, manifests the dualistic properties of matter-waves and particles. This chapter assembles a variety of simplified, intuitive, and analogy-based speculations and improvisations. These ideas are presented not as definitive answers but as a source of inspiration and challenge for future analyses, recognizing that such a vast and complex topic cannot be fully mastered by a single author.

A key challenge in this chapter is to extend and refine the de Broglie concept of particle-wave duality. The goal is to move beyond the current understanding of duality towards a more unified perspective, extending its applicability from the subatomic micro-world to the macro-world of planets, stars, and galaxies. This approach lays the groundwork for developing a new Unified Field Theory. Additionally, this chapter initiates conceptual upgrade and modification of the foundations of Quantum Theory (QT), which currently serves as the primary framework for scientifically addressing particle-wave duality.

This chapter also investigates the links between wave and particle properties, exploring when, where, and how certain wave groups begin to manifest as particles or stable masses (and vice versa). These foundational ideas are encapsulated in the Particle-Wave-Duality-Concept (PWDC) introduced in this book. This concept also offers a novel approach to understanding Uncertainty and "Certainty" relations, which are further elaborated in this chapter and in Chapters 5 and 10.

As discussed in previous chapters, the main themes of this book involve exploring:

- **Gravity and Particle–Wave Duality**: This is based on specific matter-wave couplings between linear and angular motions (such as rotations and spinning), including associated electric and magnetic fields and charges. These principles apply analogically to both micro and macro-world phenomena because the active natural field charges that influence or produce gravitation are electric and magnetic flux, electric charges, and the linear and angular moments of motional states. It is important to note that electromagnetically neutral, static, or rest masses are not the primary sources of gravitation. Instead, it is the moving and oscillating masses, with associated linear, angular, and electromagnetic moments, that play a crucial role in gravitation.

- **Masses as Electromagnetic Energy Packings**: Masses are fundamentally electromagnetic energy packings of energy-momentum entities, or specifically formatted matter-wave states. These states are self-stabilized agglomerations of spinning and other matter-wave states, along with active natural field charges. These are structured as self-closed, standing-wave resonant states. Static, heavy, electromagnetically neutral, and mechanically stable rest-masses are not primary sources of gravitation. Rather, it is the moving and oscillating masses, associated with linear, angular, and electromagnetic moments, that are sources of gravity.

- **Matter Waves as Motional Energy States:** Matter waves relate to space-time proportionality and the "energy-momentum" exchanges between different motional states of matter. These waves are equally present in both the micro and macro worlds of physics. Stable rest masses do not contribute to matter-wave energy; only the kinetic energy of motional masses (in various forms) constitutes matter-wave energy. All waves and oscillations known in physics, such as acoustic waves, mechanical vibrations, fluid and plasma waves, and electromagnetic waves, are forms of matter-wave states. These should be modeled using a unified mathematical framework based on Complex Analytic Signals and the Classical wave equation. Uncertainty Relations, originating from Mathematics or Signal Analysis (not from Heisenberg), are equally valid and applicable across the micro and macro worlds of Physics (see more in Chapter 5). Self-stabilized (or self-closed and properly packed) matter-waves and resonant configurations, like standing waves, can form solid matter or rest-mass states.

- **Fundamental Forces of Nature:** The contemporary understanding of the four fundamental forces, Electromagnetic, Gravitational, Strong, and Weak Nuclear forces, may one day be conceptually redefined, modified, or even replaced with better concept. These forces might be better explained by interactions within or between matter-waves and standing-wave structures, particularly around nodal or stationary zones of energy and mass-distribution agglomerations or gradients. It is likely that all these forces will ultimately be found to have a fundamentally electromagnetic origin or nature. In this view, particles are seen as mass or energy agglomerations around the nodal zones of relevant standing matter waves.

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[♣ As the historical and state-of-the-art background of matter-waves and wave-particle duality theory, we can read the following citation from [124]:

C Matter waves

"As explained in Introduction, the wave-particle duality is commonly associated with both light and matter, but in the thesis our attention has been restricted to light only. However, in several places (Chapter 3.4 and Chapter 9.3) we are nonetheless forced to refer to the duality of matter. Therefore, for the sake of completeness, in the following appendix we give a strongly abbreviated presentation of the subject, both from the theoretical and from the experimental side.

It was the French physicist and nobleman Louis de Broglie who in his doctoral thesis in 1924 presented the revolutionary idea that all matter had a wavelike nature. This conceptual breakthrough, confirmed in an electron diffraction experiment due to Lester Germer and Clinton Davisson three years later, paved way for the further development of quantum mechanics in the late 20s and the 30s. The so-called de Broglie relations, put in a very simple but strictly mathematical form, assign to every physical particle (like an electron) a wavelength and a frequency. These parameters can then be used to anticipate and describe the diffractive behavior of the particles.

The basic postulate is this: Given a physical object with momentum p and total energy E, we relate to it a wavelength λ and a frequency f given by the formulas [36]:

$$\lambda = h / p \qquad (205)$$

$$f = E / h \qquad (206)$$

The relativistic effects could be taken into account by introducing the Lorentz factor, $\gamma = 1/\sqrt{1 - v^2/c^2}$, and setting $p = \gamma mv$ and $E = \gamma mc^2$.

It is not immediately clear what is meant by “relating wavelength and frequency to a physical object”. We have seen in Chapter 9.2 that within the Copenhagen interpretation of quantum mechanics one simply perceives physical objects themselves as undulatory phenomena (in specific experimental circumstances), while Bohm’s interpretation (see Ch. 9.3) claims that particles are always accompanied by quantum fields responsible for their undulatory behavior.

It is instructive to consider a simple numerical example. An electron with mass $m_e = 9.11\times10^{-31}$ kg and moving with 10% of the speed of light, $v = 0.1c$, has wavelength $\lambda = 2.4\times10^{-11}$m which is comparable with the size of an atom ($\approx 10^{-10}$m). Thus, a slowly moving electron will be able to show a diffractive behavior while interacting with matter. On the other hand a car with mass, say, $m = 1000$ kg and moving with speed $v = 100$ km/h $\approx$ 28m/s has wavelength $\lambda = 2.4 \times 10^{-38}$m which is three orders of magnitude smaller than the Planck length $\ell_P \approx 1.6\times10^{-35}$m. The undulatory aspect of the macroscopic physical objects is therefore unobservable and in everyday life, our senses perceive them just as large “corpuscles”.

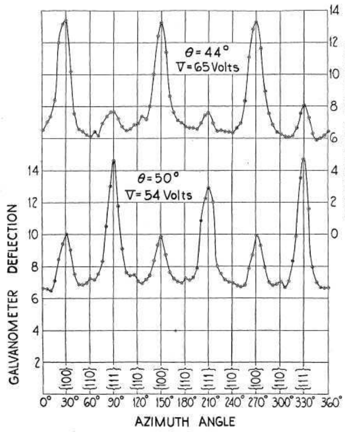

Figure 58: Results of the Davisson-Germer experiment where a block of nickel crystal was bombarded with thermally excited electrons. The crystal scattered the electrons and the authors measured the distribution of the electron intensity behind the target to be periodically dependent on the azimuth angle φ. The diagram shows the measured intensity of the scattered electrons as a function of the angle. Two data series are shown. They differ in the accelerating potential V (which determines the speed of the incident electrons) and the co-latitude of the beam θ. The oscillating pattern, suggesting an interference of some kind, is easily seen. Source: Davisson and Germer [159].

De Broglie’s theoretical suggestion that matter in motion could be perceived as a wave with a well-defined wavelength was confirmed experimentally in 1928 by Davisson and Germer [37] [159], and, independently, by Thomson and Reid [160]. The experiments involved scattering narrow electron beams (cathode rays) from a nickel crystal (Davisson and Germer) and a thin celluloid film (Thomson and Reid). The diffraction pattern obtained in both cases (see Fig. 58 for the results of Davisson and Germer) could be easily explained under the assumption that electrons behaved like waves with wavelength given by Eq. (205), and that these waves interfered during their propagation through material just as an ordinary electromagnetic radiation would do. However, the occurrence of these patterns were not predicted by standard corpuscular model combined with knowledge about the atomic structure inside the target.

For some time afterwards, it was not known whether the analogous diffraction phenomena occur with other elementary particles like neutrons and protons, or even with much larger atoms and molecules The second question was settled already in 1930 by Immanuel Estermann and Otto Stern who diffracted a beam of hydrogen and helium atoms using a lithium fluoride crystal [38]. The validity of Eq. (205) was again confirmed. In 1945 Ernest Wollan and R. B. Sawyer carried out the first neutron diffraction experiments using a beam of “monochromatic” neutrons obtained from an atomic reactor [161]. Soon neutron diffraction proved itself to be a fruitful crystallographic technique for the determination of the structure of various materials.

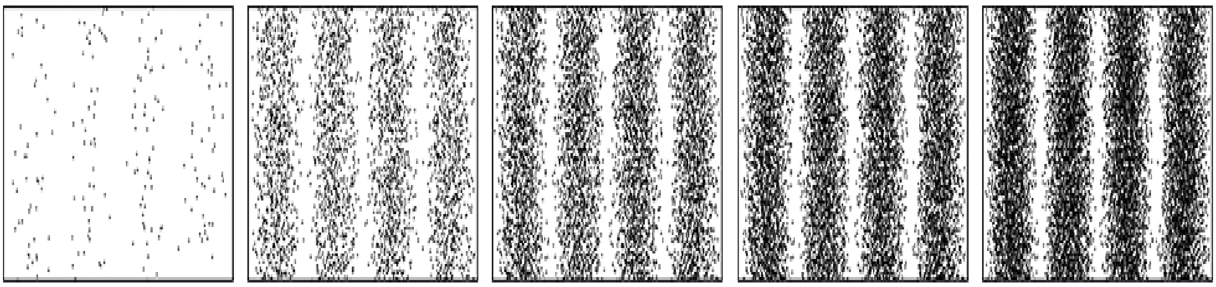

Figure 59 (replaced with a fully equivalent and visually improved picture): The statistical build-up of an interference pattern in the single-electron diffraction experiments due to Tonomura et al. Only the central part of the whole field of view of the detector is shown. The interference fringes are more distinct as the number of single electrons that have hit the detector increases. From left to right, there are respectively 3000, 20000 … and 70000 electron hits. This is a negative of the original picture with increased contrast. Source: Tonomura et al. [5].

A loophole, however, had existed in the matter diffraction experiments so far. In each of them, a continuous flow of particles was considered, and one had to ask whether the diffraction pattern could be explained in terms of some collective behavior of these particles (see the argument from Ch. 3.2 about the corpuscular photons scattering from each other) instead of employing de Broglie waves. The ambiguity would be resolved by performing a diffraction experiment where particles (like an electron or a neutron) travel through the apparatus one by one. If the diffraction pattern would eventually occur, then the case for a matter wave associated with a single particle would be made much stronger.

It was A. Tonomura and his team that in 1989 successfully performed the first precise diffraction experiment with single electrons82 [5]. Moreover, the experiment was also the first exact realization of the famous thought experiment with a single electron passing a double slit (see Introduction) [162]. Tonomura et al. employed an electron microscope equipped with an electron biprism as an equivalent to the double slit, and a position-sensitive electron-counting system as an equivalent to the screen on which the interference pattern could be formed. Fig. 59 presents the pattern they obtained. Their results unambiguously implied that it was a single electron that was able to interfere in a wavelike fashion with itself, and that the phenomenon must not be ascribed to a collective behavior of many electrons propagating together through system.

So far, the largest material objects that has been shown to exhibit interferential behavior are fullerene C_{60}-molecules. A research team led by Zeilinger obtained in 1999 a diffraction pattern by sending a beam of C_{60}-molecules through a diffraction grating consisting of nominally 50 nm wide slits with a 100 nm period [44]. The velocity distribution of the molecules was measured and fitted; the most probable velocity corresponded to a de Broglie wavelength of 2.5 pm, which is approximately 400 smaller than the diameter of C_{60}. It should be stressed the total mass M of one molecule was used in calculating the de Broglie wavelength, i.e., it was assumed that each interfering de Broglie wave corresponded to a single undivided particle of mass M. Furthermore, the observations supported the view that each C_{60}-molecule interferes with itself alone, even though they did not propagate singly through the apparatus.

Many different experiments confirmed the validity of de Broglie's relation between momentum and wavelength of material objects. Although the relation does not make any explicit distinction between the macroscopic and the microscopic level, it has been verified only in the case of the latter. It remains to be seen if analogous results will be obtained for still larger and more complicated molecules. If not, it will be very interesting to see if it is the size, mass or rather the structure of the physical object under examination that decides when the diffractive behavior ceases to occur. It is also conceivable that the diffractive behavior of matter will persist, but that the simple de Broglie relation, Eq. (205), will have to be replaced by some other formula, which maybe will give us a better physical insight into the nature of the phenomenon.

Aside from looking for the upper spatial bound, there is another crucial question that could be answered empirically. Imagine that an experiment similar to that of Zeilinger et al. is performed, but with slits in the diffraction grating being considerably smaller than the size of the material object we wish to diffract. Will the diffractive pattern still be obtained? If no, it would imply that there is something intrinsically solid about the matter (in addition to the de Broglie waves) that does not allow a material object to propagate through a slit, which is smaller than the object itself (the size of the object being determined with help of some different means). The persistence of the diffractive pattern, however, would suggest that – at least at the microscopic level and under particular circumstances – the structure of matter is completely undulatory. ♣]

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82 Earlier experiments of these kind were conducted by Claus Jönsson in 1961 [163] and P. G. Merli, G. F. Missiroli and G. Pozzi in 1974 [164], but they were less exact and used less sophisticated apparatus.

Wave-Particle duality, Matter-Waves conceptualization and Quantum theory are still part of a continuously evolving cognitive process, and we can expect significant updates, and fundamental modifications of the contemporary Orthodox Quantum theory, which already has number of distinctive and useful interpretations. One of such advances will be a creative revitalization and optimization of "de Broglie-Bohm theory" and "Many Worlds Interpretation" (*which can be conveniently updated, approaching, and uniting with the Analytic Signal modelling and Wave-Particle Duality as promoted and favored in this book; -see relevant resume in Chapters 4.3 and 10*).

Another evolving process will be to show *existence of macrocosmic matter-waves and wave-corpuscular duality, analog to original de Broglie and Schrödinger foundations*, where instead of Planck's constant h it will be relevant another macrocosmic constant H >>>h (especially in cases of self-closed, standing matter-waves formations like in solar systems; -see more in Chapters 2 and 10 of this book).

Wave function concept and its mathematical modeling, including Schrödinger equation will also significantly evolve towards better modeling and more tangible, natural and more deterministic (or not exclusively probabilistic) presentation, based on Analytic Signals and Complex Phasors, as elaborated in this book (see more in Chapters 4.3 and 10.).

As an introduction into such kind of thinking and overall Quantum theory advances, let us read the following Citation from,
https://en.wikipedia.org/wiki/Wave%E2%80%93particle_duality#cite_note-6

Main article: de Broglie–Bohm theory

Couder experiments,[17] "materializing" the *pilot wave* model.

De Broglie himself had proposed a pilot wave construct to explain the observed wave-particle duality. In this view, each particle has a well-defined position and momentum, but is guided by a wave function derived from Schrödinger's equation. The pilot wave theory was initially rejected because it generated non-local effects when applied to systems involving more than one particle. Non-locality, however, soon became established as an integral feature of quantum theory and David Bohm extended de Broglie's model to explicitly include it.

In the resulting representation, also called the de Broglie–Bohm theory or Bohmian mechanics,[18] the wave-particle duality vanishes, and explains the wave behavior as a scattering with wave appearance, because the particle's motion is subject to a guiding equation or quantum potential.

This idea seems to me so natural and simple, to resolve the wave–particle dilemma in such a clear and ordinary way, that it is a great mystery to me that it was so generally ignored.[19] - J.S.Bell

Both-particle-and-wave view

The pilot wave model, originally developed by Louis de Broglie and further developed by David Bohm into the hidden variable theory proposes that there is no duality, but rather a system exhibits both particle properties and wave properties simultaneously, and particles are guided, in a deterministic fashion, by the pilot wave (or its "quantum potential") which will direct them to areas of constructive interference in preference to areas of destructive interference. This idea is held by a significant minority within the physics community.[39]

At least one physicist considers the "wave-duality" as not being an incomprehensible mystery. L.E. Ballentine, *Quantum Mechanics, A Modern Development*, p. 4, explains:

When first discovered, particle diffraction was a source of great puzzlement. Are "particles" really "waves?" In the early experiments, the diffraction patterns were detected holistically by means of a photographic plate, which could not detect individual particles. As a result, the notion grew that particle and wave properties were mutually incompatible, or complementary, in the sense that different measurement apparatuses would be required to observe them. That idea, however, was only an unfortunate generalization from a technological limitation. Today it is possible to detect the arrival of individual electrons, and to see the diffraction pattern emerge as a statistical pattern made up of many small spots (Tonomura et al., 1989). Evidently, quantum particles are indeed particles, but whose behavior is quite different from classical physics would have us to expect.

The Afshar experiment[40] (2007) may suggest that it is possible to simultaneously observe both wave and particle properties of photons. This claim is, however, disputed by other scientists.[41][42][43][44]

"Wave nature of large objects

Since the demonstrations of wave-like properties in photons and electrons, similar experiments have been conducted with neutrons and protons. Among the most famous experiments are those of Estermann and Otto Stern in 1929. [21] Authors of similar recent experiments with atoms and molecules, described below, claim that these larger particles also act like waves.

A dramatic series of experiments emphasizing the action of gravity in relation to wave–particle duality was conducted in the 1970s using the neutron interferometer. [22] Neutrons, one of the components of the atomic nucleus, provide much of the mass of a nucleus and thus of ordinary matter. In the neutron interferometer, they act as quantum-mechanical waves directly subject to the force of gravity. While the results were not surprising since gravity was known to act on everything, including light (see tests of general relativity and the Pound–Rebka falling photon experiment), the self-interference of the quantum mechanical wave of a massive fermion in a gravitational field had never been experimentally confirmed before.

In 1999, the diffraction of C_{60} fullerenes by researchers from the University of Vienna was reported. [23] Fullerenes are comparatively large and massive objects, having an atomic mass of about 720 u. The de Broglie wavelength of the incident beam was about 2.5 pm, whereas the diameter of the molecule is about 1 nm, about 400 times larger. In 2012, these far-field diffraction experiments could be extended to phthalocyanine molecules and their heavier derivatives, which are composed of 58 and 114 atoms respectively. In these experiments the build-up of such interference patterns could be recorded in real time and with single molecule sensitivity. [24] [25]

Importance

Wave–particle duality is deeply embedded into the foundations of quantum mechanics. In the formalism of the theory, all the information about a particle is encoded in its wave function, a complex-valued function roughly analogous to the amplitude of a wave at each point in space. This function evolves according to Schrödinger equation. For particles with mass, this equation has solutions that follow the form of the wave equation. Propagation of such waves leads to wave-like phenomena such as interference and diffraction. Particles without mass, like photons, have no solutions of the Schrödinger equation so have another wave".

An excellent resume about Wave-Particle Duality and foundations of QT is given here:
https://en.wikipedia.org/wiki/Wave%E2%80%93particle_duality

Citation from: Scientific American. Ask the Experts: Physics and Math (Kindle Locations 284-286). Scientific American, 2020. Joseph S. Merola, associate dean for research at Virginia Polytechnic Institute, responds:
"There are several different ways of approaching this question, but I won't beat around the bush. The simple answer is that wave-particle duality, as it is called, is present in the macroscopic world — but we can't see it. Scientists have developed a number of indirect methods for observing wave-particle duality. One of the earliest experiments showed that a regular array of atoms could diffract an electron beam. Because diffraction is a property of a wave, this test indicated that particles — electrons in this case — could also behave as waves.
The physicist Louis de Broglie proved that any particle in motion has a wave-like nature. He developed the following relationship: the wavelength of a particle's wave aspect is equal to Planck's constant divided by the momentum of the particle".

In this book it is shown (in the second Chapter, about Gravitation) that macrocosmic objects, like solar or planetary systems can (analogically to micro-world of Physics) manifest a wave-like nature, where the relevant wavelength is equal: to certain constant (much bigger than Planck's constant) divided by the momentum of the orbiting planet. In fact, M. Planck-Einstein wave energy packet (or quant) is a limited validity formula regarding photons quantizing, and when addressing narrow-band wave groups.

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The core understanding of de Broglie matter waves, as presented in this book, can be summarized with a certain level of creative imagination and intellectual flexibility as follows (more detailed support will be provided in Chapter 10):

De Broglie matter waves are not merely strange, only indirectly detectable phenomena; they are fundamentally embedded within the structure of every stable particle and atom. These waves exist as self-sustaining, rotating, stationary, and standing waves, essentially being a stable form of "energy packing." When particles move and interact with their environment, these internally packed de Broglie matter waves "unfold," producing externally detectable manifestations such as wave motions, energy exchange, moments, and mass transfer. In this view, the external manifestations of de Broglie matter-waves are a natural extension of the intrinsic, internal oscillating, spinning, and orbiting structure of particles (or emanating from atoms).

The most direct indicators of such hidden, rotation-related matter-wave behaviors include the spin and orbital moment attributes of all elementary particles, along with their associated electric and magnetic moments, charges, and dipoles. It is noteworthy that celestial bodies like planets, moons, and solar systems also exhibit spinning and rotating behaviors, further emphasizing some intrinsic spatial-temporal periodicity of these phenomena.

If rotation played a role in the creation of elementary particles, atoms and other masses in our Universe, then to satisfy the conservation of total orbital momentum (including spinning states), some permanent, rotation-like characteristics must be intrinsically linked to the properties of any particle, especially those in linear motion. These characteristics are currently described as natural, and intrinsic spin attributes, even though the specific mechanism of this spinning remains unclear.

From a different perspective, when considering pure waveforms, such as photons, which are energy states without rest-masses, experimental evidence suggests that these wave states can "solidify" by becoming integrated into an existing particle structure, which is effectively capturing their "energy-momentum" content. It appears that under certain conditions, photons (when passing near other particles or atoms, and likely influenced by torsional and spinning field components) can be transformed into particles with non-zero rest mass, while still adhering to global energy and moments conservation laws.

In other words, every motional energy represents a specific state of effective spatial mass distribution that propagates as a kind of spinning and oscillating matter wave. When certain conditions are met, wave-packets, peaks, and ripples of this spatially distributed mass and energy can coalesce to form a space-localized, stable particle with non-zero rest mass. This likely occurs when torsional and linear motion components specifically couple and align to create self-closed standing matter waves, which then externally exhibit stable mass properties.

For further supporting arguments, refer to the Appendix in Chapter 10, "Particles and Self-Closed Standing Matter Waves." Here, the concept of particle-wave dualism and the parallelism between different forms of motional energy is explored, with innovative analyses of phenomena such as the Compton Effect, the Photoelectric Effect, and the continuous spectrum of X-rays. These analyses illustrate that only kinetic or motional energy $\tilde{E} = E_k = (\gamma - 1)mc^2$ contributes to matter-wave energy states, challenging the contemporary micro-world physics perspective that total energy $E_t = E_k + E_0 = \gamma mc^2$ entirely belongs to matter-waves states.

Much of the phenomenology associated with particle-wave duality is also closely related to Thermodynamics and Fluid Dynamics, not exclusively to Quantum Mechanics. Thermodynamics should not be limited to dealing with the random motions of isolated particles, as all moving particles exhibit undulatory properties, creating matter waves individually and through mutual synchronization (see more about this universal matter synchronization in Chapter 10). Beyond its application to ideal gases, Thermodynamics should also address the various interactions, secondary emissions, and impacts between particles (such as photons, electrons, ions, atoms, and molecules), all of which, when in motion, exhibit Wave-Particle Duality.

In describing and understanding matter-waves, it is crucial to consider the interactions between fields, forces, and matter waves of interacting entities. Every moving particle exists within a certain "force-field" relationship with its environment and possesses matter-wave attributes, which become particularly significant at elevated temperatures when the mass of photons, electromagnetic waves, and acoustic and mechanical waves are involved.

De Broglie matter waves and all other wave phenomena and oscillations known in Physics belong to the same family of events and should be analyzed using a unified theoretical framework. This approach will necessitate the development of extended and upgraded Thermodynamics, theoretically much more unified with Wave-Particle Duality concepts and Max Planck's Blackbody radiation law. Additionally, all fields and wave phenomena in Physics should be considered natural extensions of the wave-particle duality states of atoms, micro-particles and fluids, extending toward the diverse momentum-energy states found in the macro world.

Waves represent oscillations within a medium, fluid and elastic states of matter where energy fluctuates between its kinetic and potential forms in both temporal and spatial domains. Consequently, even in an absolute vacuum, where electromagnetic waves, neutrinos, solar winds, elementary particles, and various cosmic radiation propagate, some form of exotic, etheric, fluidic matter, as a carrier, must exist, as Nikola Tesla suggested.

In this context, particles can be considered as specifically condensed, packed, or solidified energy states, composed of self-sustaining, internally folded forms of oscillating, rotating, and spinning matter-waves. The process of stabilizing particles is causally related to the formation of self-closed standing waves. This concept is further elaborated in Chapter 10.

Understanding Particle-Wave duality will become much clearer when we apply mathematical structures, Conservation Laws, and particle-wave duality relations to it. For instance, contemporary Thermodynamics defines temperature as directly proportional to the average kinetic energy of involved particles, gases, atoms, and molecules. *In a new, innovative approach to Thermodynamics, we should focus on wave energy rather than purely mechanical kinetic energy, since the total wave energy of a system equals its total motional energy, including electromagnetic contributions.*

This perspective will gradually extend and deepen our understanding of Temperature and Radiant energy. Although contemporary Thermodynamics is already well-developed, tested, and interconnected with other areas of Physics, ***replacing averaged mechanical kinetic energy with generalized and total matter-wave energy*** will primarily impact extreme situations where real gases, atoms, molecules, and other particles are no longer the dominant media or reaction participants.

In line with the ideas presented in this book, atoms should be considered multi-resonant, structured matter states, akin to small multidimensional worlds or micro-cosmic states with perfect unification of electromagnetic, electromechanical, and Wave-Particle Duality properties. These states serve as sources and sink of all-natural energy-flow effects and forces. Although the founders of Classical and contemporary Thermodynamics did not consider these innovative elements related to total wave or motional energy and Wave-Particle Duality, this new approach will enrich our understanding of these phenomena.

The ideas and concepts related to Particle-Wave Duality and Quantum Theory are already a significant part of contemporary micro-world physics, developed from the beginning of the 20th century. Let us pragmatically, empirically, and creatively update the initial foundations for understanding particle-wave duality, better termed unity, through the following exploration:

1. *The initially established concept regarding matter waves was based on de Broglie's explanation of stationary electron orbits of Bohr's planetary atom model. The same wave hypothesis was shown to be essential in explanations of Compton and Photoelectric Effects (combined with Planck and M. Maric-Einstein's formula for the energy of a wave packet or photon). Bragg's X-rays (or photons) diffraction from stable crystals is initially explained considering X-rays only as wave phenomena. Later it has been demonstrated, in many variants, that the same Bragg's diffraction law (or mathematical formula) is applicable for describing similar scattering and diffraction of particles like electrons and neutrons, when we treat them as matter waves having de Broglie wavelengths. Davisson and Germer demonstrated experimentally the wave properties of massive motional particles (in 1927) in their electron-diffraction experiments. Later, 1930, Estermann and Stern demonstrated diffraction of helium atoms and hydrogen molecules from lithium fluoride crystals (effectively starting the field of atom optics, since they were the first to demonstrate the wave-like properties of atoms). See [92] as an excellent historical resume about the foundations of wave-particle duality concepts. Here we need to add that solid states or masses are also internally or structurally, spatially periodical (known as crystalline and fractal structures, internally vibrating, with certain spatial periodicity, homogeneity, uniformity). The same as temporal periodicity of wavefunctions has its spectral or frequency domain, spatial*

periodicity of solid matter structures also has its spatial, mechanical moments related, spectral characteristics (all of that based on Fourier analysis). Such temporal and spatial periodicity-related properties of matter states are especially interesting and relevant in explaining diffraction and scattering phenomenology. Recent development regarding the understanding of particle-wave duality made possible the production of gaseous Bose-Einstein condensates. In such matter-state, macroscopic numbers of atoms occupy the same quantum state, where waves associated with each of the atoms are in phase with one another in a way that is directly analogous to the behaviors of photons in laser devices (see chapter 3 in [27]). If we are convinced that a kind of pure harmonic wave (with predictable and quantifiable wavelength and frequency in relevant time-frequency domains) is associated with particles in motions, we should ask ourselves what and where the source of such waves is. Logically, based on experimental data and applied wave modeling parameters we use in such situations, it should be certain kind of "oscillatory circuit" or "waves generator", or some spinning and helix motion generator, causally linked to a particle in motion (to its linear and angular moments and involved kinetic energy), producing de Broglie matter waves. Such "oscillatory source", spinning or helix wave motion could be partially related to force and field effects between a moving particle and its environment. However, it could also be that de Broglie or matter waves are an intrinsic part of every particle structure, like some internal field-rotation intrinsically associated with moving and oscillating particles, externally creating helix waves around the path of linear particle motion, and here actually, this is the starting platform we will follow in this book. Future thermodynamics should be naturally enriched and extended or united with such matter-waves phenomenology and relevant modelling (based on Fourier and Analytic Signal analysis).

2. *Briefly, we can safely say that any particle motion is unified (associated, followed, coupled, or at least mathematically presentable) with a corresponding spiraling wave motion and vortices (based on de Broglie hypothesis), and that all wave motions, including de Broglie matter waves, are manifestations of different forms of motional, mechanical, and electromagnetic energy (or power). There should exist direct correspondence, mutual dependence, and equivalence between matter wave and particle kinetic energy,* $\tilde{E} \Leftrightarrow E_k$ *. Consequently, every quantitative change or (certain time-space dependent) modulation of motional energy (of any origin) should also be a source of matter waves, and it should be causally related to action-reaction and inertial forces phenomenology (in some way analog to electric and magnetic induction phenomenology).*

3. *The same mathematical modeling (related to Signal Analysis) used to describe different (mechanical, electrical, electromagnetic, etc.) oscillations, wave motions and similar phenomena in Physics, is (almost) universally applicable to all of them, regardless of the nature of wave phenomena and regardless of, if being applied to microcosms of subatomic entities, or to a macro universe of planets, stars, and galaxies. Uncertainty relations in Physics (or Heisenberg relations) are just a product of the same, generally applicable mathematics, regardless of Physics and the scale and size of analyzed phenomena. Waves' synthesis and analysis, as well as waves superposition and interferences are mathematically and generally explicable and applicable to any size of the universe (using the same mathematical modeling). There is nothing in mathematical processing what is giving general advantage and uniqueness to a micro world (regarding signals and waves analysis), as Orthodox Quantum Mechanics implies. Particle-wave duality theory (or its mathematical modeling) starts from the intuitive concept that velocity of a real particle,* v*, should be the same as the wave-group velocity,* $v_g = v$ *, of the wave packet or wave group associated to that particle (based on a habitual modeling of a wave group known from Quantum Mechanics). Since every wave group also has its phase velocity,* $u = \lambda f$ *, and since there is the well-known analytic connection between a group and phase velocity of simple-*

harmonic, modulated sinusoidal waves ($v_g = v = u - \lambda du / d\lambda$ *), there should also exist one consistent mathematical modeling that will unify all elements of a real particle motion with its associated wave-group replacement.*

4. *Moreover, we should not forget that Einstein-Planck's narrow-band wave energy* $\tilde{E} = hf$*, combined with relativistic and mechanistic forms of particle energy, but essentially in its infinitesimal or differential form* $d\tilde{E} = h \cdot df$ *(regarding Photoelectric and Compton effects) produces correct results related to wave-mass and momentum calculations and transformations. For instance, a photon that has energy* $\tilde{E} = hf$*, also has an equivalent (particle-like) momentum* $\tilde{p} = hf / c$*, and equivalent (particle-behaving) mass,* $\tilde{m} = hf / c^2$*, because the total wave (or motional) energy of the photon is equal to its total relativistic energy and its total kinetic energy (since photon has zero rest mass),* $\tilde{E} = hf = \tilde{m}c^2 = \gamma mc^2 = pv / (1 + \sqrt{1 - v^2 / c^2}), v = c, \tilde{p} = p = hf / c$*. At the same time, the photon has its intrinsic, angular momentum or spin equal to* $L_f = \tilde{E} / \omega = hf / 2\pi f = h / 2\pi$ *(see (2.11.3) and T. 4.0). In Chapter 2 ("2.3.2. Macro-Cosmological Matter-Waves and Gravitation") we can also find that analogical conceptualization and mathematics applies to planets and solar systems (except that M. Planck-Einstein quant of wave energy should be re-examined and conceptually re-established (see more in Chapters 9 and 10). Practically, only motional energy of a photon (as a spatially and temporally narrow-band wave packet, analogically corresponds to a motional or kinetic particle energy, and this is one of the intuitive ways for conceptualizing wave-particle duality, but modern Quantum Theory wrongly and ambiguously started from the platform that total particle energy (including rest mass) creates or presents its matter-wave energy (what is correct in some rare cases, but this should be well explained).*

5. *Every particle or wave motion (in fact, a mathematical function describing certain motion) that can be characterized by some spectral distribution, characteristic central frequency, wavelength, oscillating or waving process, etc., should be an integral part of certain visible or hidden rotation, or torsion-field phenomena (in its original or transformation domain). Particles, in the frames of de Broglie Particle-Wave Duality concept, behave (analogically) as photons regarding linear moments, since we know that photons have their intrinsic spinning moments. By applying the same analogy (backward) it should also be valid that all (at least elementary) particles in linear motion should have certain wave equivalent to angular momentum (or spin,* $\mathbf{L = \frac{c^2}{uv}(\frac{h}{2\pi}), \Delta L = \frac{h}{2\pi}}$ *: -see the table T.4.0, Photon – Particle Analogies).* ***<u>In fact, the big secret of particle-wave duality (or unity) is that somehow all forms of wave and/or kinetic energy (or motional energy of elementary particles, quasi-particles, and wave packets) have the same elementary spin, equal to</u>*** $\tilde{L} = \frac{\mathbf{h}}{\mathbf{2\pi}} = \frac{\mathbf{E_{motional}}}{\omega} = \frac{\tilde{\mathbf{E}}}{\omega} = \frac{\mathbf{E_k}}{\omega} = \Delta L$ *(or some integer multiple of* $\frac{h}{2\pi}$ *). Consequently, elements of rotation and linear motion should always be intrinsically coupled in all cases of (electrically charged or neutral) particles and wave motions (like* $\frac{L_0}{p_0} = \frac{\Delta L}{\Delta p} = \frac{L}{p} = \lambda \frac{L}{h} = \frac{c^2}{\omega v} = \frac{c^2}{v^2} \cdot r^* = \frac{c^2}{\omega^2} \cdot \frac{1}{r^*}$ *). For instance, electrons always have their intrinsic magnetic and orbital moments mutually coupled, meaning that something equivalent to rotation naturally embedded in internal electron structure (connecting rotating mass and rotating electric charge, also known in relation to gyromagnetic ratio) should exist. Also, in cases of linear (or circular) macro-motions of electrons, again we should have rotating (or helix) magnetic field components around their paths (see on Internet somewhat similar concept about Henry Augustus Rowland effect of the magnetic field around the rotating conductor, presented by Jean de*

Climont). ***Photons also have certain helicity of associated mutually coupled electric and magnetic field vectors, including resulting spin and angular moments. Consequently, the analogical situation could also be valid for electrically neutral particles, like atoms in linear motion, producing simultaneously spinning and helical effects of certain field components (belonging to their internal, electrically charged constituents).*** *This will eventually (after interferences and superposition) produce externally measurable consequences, such as different and omnipresent rotations in the world of molecules, atoms, and elementary particles, as well as rotations and spinning of astronomic objects. Here is the reason why Gravitation is still not well integrated into the texture of other important theories of Physics, since its coupled, or conjugate field-component, related to certain kind of rotation and/or spinning (coupled with linear mass motion, like coupling between electric and magnetic fields), is still not adequately considered. We could search for such missing rotating components (in the world of Gravitation, and non-charged particle motions) by analyzing Coriolis, Centrifugal and Centripetal forces, and effects associated with gyroscopes and oscillations of a pendulum. We know that accelerated motions of masses are producing effects equivalent to Gravitation. This is easy and simply detectable when we are in a certain elevator (as A. Einstein speculated), but in real cases of gravitation, where we do not have real or imaginary Einstein's elevators, we still need to have certain exotic and invisible, continuous mass flow in one direction that will effectively create reactive gravitational force in the opposite direction. Something similar, N. Tesla conceptualized as his never completely finalized and published Dynamic Theory of Gravitation, and he experimentally measured and produced mentioned associated effects of continuous and somewhat exotic mass flow, being kind of* ***"radiant"*** *electromagnetic and cosmic rays' energy, [97]). Here an elaborate particle-wave duality concept is strongly related to the necessity of theoretical unification of linear and rotational motions (in the domain of Gravitation, Mechanics and Electromagnetism, see (2.3) - (2.4-3,) as the first step in such attempts) on a more profound and explicit level than presently recognized in Physics. Without such specific rotation (or spin, vortex, torsional field components, and eddy currents, associated with linear motions …), existence and creation of stable elementary particles would not be possible. Most probably, the future regarding upgraded theory of Gravitation will deal with couples of mutually conjugated fields (analogically like in electromagnetic field), related to linear and rotational motions. Eventually, we would be able to find that only motional electromagnetically charged or polarized, including vibrating particles, and associated electromagnetic fields (manifesting as currents and voltages) are in the background of all gravitation-related situations. Anyway, electrically neutral masses are internally composed of electrically charged particles (often having internal magnetic domains and electric dipoles). Since there is an enormously big difference between masses of positive and negative electrical charges, it should be easy and natural to experience effects of inertia and forces because of accelerated motions, including effects of electrical and magnetic dipole polarizations and attractions based on such effects (when masses are in a mutually relative angular or orbital motions). Laws of electromagnetic induction, Coulomb-Newton laws, and Lenz law would guide such interactions between mutually moving masses (when they are in a zone of interaction). The situation regarding elementary spin units or orbital and linear moments of photons, atoms, electrons, and other particles could be much more complex than here presented, but what counts here is that linear and rotating or spinning motions (of any kind) are always mutually coupled.*

6. *The creation of an electron-positron pair from the energy-momentum content of a sufficiently energetic photon, and the annihilation of an electron-positron pair that produces two photons are very indicative experimental situations explaining that internal mass content of an electron (or positron) could be just another specific form of* ***resonant photons-energy packing.*** Of course, additional reaction ingredients are also necessary to be present here to satisfy all known conservation laws. In fact, in the mentioned examples, we have specific structuring and

formatting of electromagnetic field energy. *Of course, such events respect the conservation of a total system energy and important orbital and linear moments. Something similar (at least by analogy and symmetry) should also be valid for protons (and anti-protons). Since neutron anyway and dominantly presents specifically coupled combination of one electron and a proton (including additional conditions making that all relevant conservation laws are satisfied), we could conclude that quanta of electromagnetic energy (or photons) in different* ***"packing and resonant formats"*** *(most probably) create overall masses and atoms diversity in our Universe. The hydrogen atom is also the specific association of an electron and proton, somewhat like a neutron by its internal content since both are composed of an electron and a proton on different ways under favorable conditions (meaning being differently coupled and packed). Conceptualizing this way, we could, still conditionally and hypothetically, exercise that all other atoms are in some specific way composed of hydrogen atoms, (meaning from electrons and protons, what also means composed from photons), what is for the time being, at least quantitatively and intuitively close to correct. There are some measurement-related insights that neutrons exist only when being outside of atoms. Here is also an explanation of the nature of particle-wave duality or unity between particles and waves. It could eventually happen that we discover that our universe is composed of a variety of matter-forms, like waves, particles, photons, and their transients, all of them having profound electromagnetic nature (as N. Tesla many times stated, [97]). See also familiar elaborations and concepts in [16, 17, 18, 19, 20, 22, 25, 29, 54, 68, 76, 83, 88, 89, 91, 145], from Bergman & Lucas.*

7. A well-established method for modeling particle-wave duality in a stochastic and pragmatic way already exists as Orthodox Quantum Mechanics (OQM). OQM is based on probabilistic principles and empirical and mathematical data fitting. This is currently the most widely accepted framework in mainstream physics for matter understanding at the quantum level. However, while the mathematical structure of OQM produces accurate results, the conceptual understanding behind it remains incomplete. The world picture it presents is often unclear, lacking a deterministic, intellectually coherent explanation that aligns with natural empirical laws.

   OQM relies heavily on assumptions, postulates, and principles that can appear ad hoc or "magical" in nature, yet the theory works. Its self-correcting methodology produces accurate predictions, even though it sometimes does so without clear explanations or at the expense of conceptual clarity. This approach relates to mass data processing, applying mathematical tools like probability, statistics, and signal analysis, to large sets of similar or identical items, which can be applied to natural and other sciences alike. While this provides useful results, it is not an original, independent theory that offers a deep, experimentally verifiable, and clear understanding of the fundamental work of physics.

   Proponents of OQM often present it as the only viable option for describing the quantum world. They tend to reinforce probabilistic quantum theory through selective examples, guided questions, and answers that seem premeditated to defend the current OQM framework. Yet, there is potential to "dress" such OQM's mathematical modeling in a clearer, more logical, and deterministic conceptual framework. By drawing on known models and analogies from other areas of physics and utilizing improved mathematical models for wave functions, wave motions and energy quantization, it is possible to arrive at a more natural interpretation.

   This text will demonstrate that, in parallel with the probabilistic OQM, there can exist another isomorphic, more general, and more natural level of QM and wave function modeling. This alternative approach would capture more tangible reality, using complex analytic signals and Hilbert integral transforms to describe particles and waves. When normalized or "undressed" (stripped of unnecessary dimensionality), these new models would reduce to those already known in OQM.

The term "Quantum Mechanics" itself is misleading. A more appropriate, descriptive title might be: "Theory of Interactions, Communications, Energy Exchanges, and Structuring within Resonant and Standing-Wave Forms of Matter-Waves and Particle States," (until we find better formulation). Regardless critical remarks, contemporary quantum theory will always retain its place in physics as a valuable mathematical toolbox, rich with imaginative, exotic, and creative concepts. While these ideas are often as simplified visualizations, they provide useful, though imperfect, explanations of the quantum world. Over time, however, much of the theory's substance will need to be refined or reestablished.

8. What should be more general and realistic modeling of particle-wave duality is the framework of Analytic Signal functions (established by D. Gabor; -see much more in chapters 4.0, 4.3 and 10). Using the Analytic Signal model (creatively, and with convenient mathematical and dimensional arrangements), we should be able to present every matter-wave function of certain motion, field and force, as a couple of mutually phase shifted (and Hilbert transform related) wave functions $\Psi(t)$ and $\hat{\Psi}(t)$, which create complex analytic signal function $\overline{\Psi}(t) = \Psi(t) + j\hat{\Psi}(t) = (1 + jH)\,\Psi\,(t)$. Here, both $\Psi(t)$ and $\hat{\Psi}(t)$ present real, natural, and detectable items, like combination and coupling of electric and magnetic field vectors that are creating an electromagnetic field. On a similar way, every linear motion of particles and waves presented with a wave function $\Psi(t)$, should be automatically followed by synchronously created analytic signal couple $\hat{\Psi}(t)$, again creating a complex analytic signal $\overline{\Psi}(t) = \Psi(t) + j\hat{\Psi}(t)$, which has all matter-wave properties that are products of Analytic Signal modeling (such as de Broglie wavelength, frequency, phase, group and phase velocity etc.). Both $\Psi(t)$ and $\hat{\Psi}(t)$ should present detectable material items. For instance, an excellent example of $\Psi(t)$ and $\hat{\Psi}(t)$ coupled matter waves is when $\Psi(t)$ presents specific linear motion, and $\hat{\Psi}(t)$ presents a spiral, spinning or helix, de Broglie matter wave of a field existing in a space around the path of the original linear motion of $\Psi(t)$. De Broglie matter waves have the frequency and phase described by the properties of the corresponding Analytic Signal model, as defined in (4.0.2). From the Analytic Signal frequency and phase, we can determine matter-wave wavelength, $\lambda = h/p = u/f$, and belonging group and phase velocity $v = u - \lambda du/d\lambda = -\lambda^2 df/d\lambda$. Relevant Analytic Signal wave functions that are naturally describing de Broglie or matter waves are power and motional energy-related functions, including corresponding field and force functions. By creating normalized, non-dimensional (power-related) wave functions, we will produce similar Quantum theory approach to the same problematic. Most of the contributions of Louis de Broglie, A. Einstein, Max Planck, E. Schrödinger, and W. Heisenberg, concerning matter waves and particle-wave duality, will be perfectly well and naturally described, and (integrally) considered, when being mathematically modeled with Analytic Signal wavefunctions.

9. In conclusion, the true and most significant historical discovery of matter waves and particle-wave duality can be traced back to Jean-Baptiste Joseph Fourier, a French mathematician and physicist (21 March 1768 – 16 May 1830). Fourier's integral transformation, the predecessor of Dennis Gabor's Analytic Signal (as discussed in Chapter 4.0), demonstrated that any time-domain function or signal can be represented as an integral superposition of simple harmonic waves or components. This implies that matter waves are essentially motional states represented as wave states.

Over time, numerous analyses and experiments, from Fourier's era to the present, have confirmed that nature consistently adheres to Fourier spectral analysis. This means that almost any time-domain signal can be naturally decomposed into simple harmonic wave functions. Dennis Gabor (5 June 1900 – 9 February 1979) further refined Fourier's work by introducing the concept of the Analytic Signal, which allowed for more advanced joint time-frequency analysis. Gabor's approach extracts critical wave parameters, such as amplitude, frequency, phase, power, and impedance, from any wave function, making it applicable even to arbitrary, finite-duration signals, as well as spatial domain functions.

Historical predecessors who contributed to the development of Fourier spectral analysis include Christiaan Huygens (14 April 1629 – 8 July 1695) and Isaac Newton (25 December 1642 – 20 March 1726/27). Newton's work with the optical prism was an early example of spectrum analysis, as he separated white light into its spectral components. Later, Augustin-Jean Fresnel (10 May 1788 – 14 July 1827), a French engineer and physicist, and Thomas Young (13 June 1773 – 10 May 1829), an English polymath and physician, further advanced the understanding of the wave nature of light.

Citation from: https://en.wikipedia.org/wiki/Huygens%E2%80%93Fresnel_principle,
Huygens' Theory and the Modern Photon Wavefunction

Huygens' theory served as a fundamental explanation of the wave nature of light interference and was further developed by Fresnel and Young but did not fully resolve all observations such as the low-intensity double-slit experiment that was first performed by G. I. Taylor in 1909, see the double-slit experiment. It was not until the early and mid-1900s that quantum theory discussions, particularly The Feynman Lectures on Physics as well as early discussions at the 1927 Brussels Solvay Conference, where Louis de Broglie proposed his de Broglie hypothesis that a wave function guides the photon.[7] The wave function presents a much different explanation of the observed light and dark bands in a double slit experiment. Feynman partially explains that a photon will follow a predetermined path, which is a choice of one of many possible paths. These chosen paths form the pattern; in dark areas, no photons are landing, and in bright areas, many photons are landing. The path of the photon, or its chosen wave function is determined by the surroundings: the photons originating point (atom), the slit and the screen, the wave function is a solution to this geometry. The wave function approach was further proven by additional double-slit experiments in Italy and Japan in the 1970s, and 1980s with electrons see [8].
Huygens' principle and quantum field theory

Huygens' principle can be seen as a consequence of the homogeneity of space—the space is uniform in all locations.[9] Any disturbance created in a sufficiently small region of homogenous space (or in a homogeneous medium) propagates from that region in all geodesic directions. The waves created by this disturbance, in turn, create disturbances in other regions, and so on. The superposition of all the waves results in the observed pattern of wave propagation. Homogeneity of space is fundamental to quantum field theory (QFT) where the wave function of an object propagates along all available unobstructed paths. When integrated along all possible paths, with a phase factor proportional to the action, the interference of the wave functions correctly predicts observable phenomena. Every point on the wavefront acts as the source of secondary wavelets that spread out in the light cone with the same speed as the wave. The new wavefront is found by constructing the surface tangent to the secondary wavelets.

It wasn't until the 20th century that key figures in science, such as Louis de Broglie, Niels Bohr, Albert Einstein, Max Planck, Arthur Compton, Werner Heisenberg, and Erwin Schrödinger, emerged and contributed to the development of contemporary concepts of particle-wave *duality and matter waves. Through their theoretical and experimental work, often unintentionally, demonstrated that Jean-Baptiste Joseph Fourier had laid down the foundational mathematical principles of wave-particle duality and time-frequency analysis.*

To summarize and expand upon the significance of Fourier's work, as well as Dennis Gabor's contributions to Analytic Signal and spectral analysis (further discussed in Chapters 4.0 and 10):

A) Any time-dependent signal in its original time domain has a natural, mathematically and physically verifiable spectrum in the spectral domain, and vice versa.

B) Similarly, any spatial-domain signal has a corresponding spectral or spatial-frequency domain, and vice versa, principles widely applied in Statistical and Fourier Optics.

C) The structure of our Universe exhibits a certain symmetry and direct proportionality between its temporal and spatial domains, both in their original and spectral forms. This symmetry forms the broadest and most significant mathematical framework for understanding particle-wave duality and matter waves. Concepts such as uncertainty relations, group and phase wave-packet velocity, and Einstein's theory of relativity support the direct proportionality between these domains. Fourier and Gabor's integral transformations ultimately lead to the definition of the Analytic Signal concept, which is highly applicable to modeling all matter-waves, wave-particle duality signals, wave functions, and motions. This is due to the essential facts of particle-wave duality discovered during the 20th century.

*In this book, these essential facts are summarized under the term **PWDC** (Particle-Wave Duality Code), representing the most valuable insights of contemporary Matter-Waves and Wave-Particle Duality theory (see more about PWDC in the following pages and in Chapter 10).*

While Statistics and Probability theory are often used in the context of wave-particle duality within contemporary Quantum theory, their role is primarily as auxiliary tools for mathematical modeling, applicable only when natural and mathematical conditions permit. It's important to note that Statistics and Probability theory are universally applicable across all sciences and situations in our Universe, provided the conditions for their effective use are met.

4.1.1. Particle-Wave Duality Code, or PWDC foundations

In the early days of discovering Particle-Wave Duality and Matter Waves, the physics community encountered numerous manifestations of these phenomena, such as the Compton and Photoelectric effects, the creation and annihilation of electron-positron pairs, secondary emissions, scattering transformations, and Bragg diffraction of X-rays, electrons, and neutrons. While these observations were mathematically explainable using Einstein's theories, particularly through Minkowski's 4-vectors and mass-energy relations, the community lacked a comprehensive theory that seamlessly integrated these phenomena with Classical Physics, as it was known by the late 20th century.

Similarly, Einstein's Theory of Relativity, introduced alongside Particle-Wave Duality and Matter Waves concepts, remains not fully unified with these ideas. In response to this gap, large-scale theoretical and experimental exploration, and trial-and-error efforts eventually led to the construction of modern Quantum Theory. This theory, though mathematically operational and capable of explaining many matter-wave-related experimental results, has not fully clarified the ontological and deterministic understanding of Particle-Wave Duality and Matter Waves.

In this book, we aim to address this gap by starting from experimental facts and leveraging a well-known mathematical framework, striking analogies, and incremental explanations. Our goal is to provide a more tangible, logical, and natural explanation of Particle-Wave Duality and Matter Waves, as opposed to the artificial, probabilistic, and axiomatic approach of traditional Quantum Theory.

Our search focuses on finding a unifying zone where classical mechanics, wave theory, fluid mechanics, relativity, quantum theory, and electromagnetic theory overlap and converge. This convergence would involve using similar, mutually complementary, and compatible mathematical frameworks and conceptualizations. The term "Quantum Theory" itself is misleading, as it suggests randomness and stochasticity in nature, when in fact, the universe operates under deterministic principles within a broader conceptual framework with clear causal foundations.

Quantization becomes relevant when dealing with spatially confined structures characterized by stable resonant and standing waves, where integer numbers of half-wavelengths or energy amounts can be counted. However, the oversimplified and artificially imposed stochastic quantization cannot serve as the dominant unifying factor. Instead, unification lies in using complementary and equivalent mathematical models and concepts that relate linear, rotational, and spinning motions in Particle-Wave Duality and Matter Waves.

For instance, linear motions create matter waves characterized by phase and group velocities $u = \lambda f$, $v = u - \lambda\left(du/d\lambda\right)$, de Broglie wavelengths $\lambda = h/p$, and Planck-Einstein wave energy $d\tilde{E} = h \cdot df$, comparable to moving particles when using wave packet concepts. The complexity of Particle-Wave Duality Code (PWDC) arises from the interconnectedness of these elements, which are analyzed using modern signal analysis tools. Significant advancements in mathematical modeling can be achieved by transitioning from Fourier-based analysis to Analytic Signal analysis, particularly using the Hilbert transform. However, this shift is not immediately obvious, as the mathematics of Analytic Signals superficially resembles Fourier analysis (see Chapter 4.0 for more on the Analytic Signal concept).

The success of Quantum Mechanics, particularly its predictive power, stems from the proper integration of the essential Particle-Wave Duality Concept with modern Signal Analysis, Statistics, Probability theory, and the foundational equations of Classical and Schrödinger Wave Mechanics. However, this simplified formulation of PWDC and the foundations of Quantum Theory are not commonly used. The central ideas of PWDC were initially and gradually formulated, almost by chance, by key figures such as Louis de Broglie, Max Planck, Niels Bohr, Albert Einstein, Werner Heisenberg, and Erwin Schrödinger, among others.

In this book we will address PWDC in a more natural, slightly upgraded, and more profoundly explained manner than is currently found in contemporary Quantum Theory.

Briefly, PWDC represents a set of common rules, properties, mathematical relations, equations, and formulations that govern how micro and macro particles and matter waves are coupled. PWDC explains the relationships between linear, circular, and torsional motions, implicating how matter waves can transform into particles and vice versa, and how energy and momentum can be distributed between involved particles and waves. The Analytic Signal model is the best mathematical framework for defining, exploring, and uniting all aspects of PWDC, wave functions and wave equations (see Chapter 10 for more on PWDC).

One of the fundamental wave equations, applicable to micro-world matter-wave states and closely related to PWDC, was first formulated (or postulated) by Erwin Schrödinger. In this book, we will smoothly, on a simple way, develop, upgrade, and generalize Schrödinger's equation based on Analytic Signal wave functions (without any probability and statistics related background, ad hock assumptions and by divine inspiration added missing equation members). This will demonstrate a much higher level of applicability for generalized Classical wave equations across all wave phenomena, including electromagnetism, acoustics, hydrodynamics, quantum mechanical waves, and even planetary motions and gravitational phenomena (see more in Chapter 4.3).

Ultimately, it will become evident that Schrödinger's Equation, originally derived from modeling standing waves on an oscillating string, is simply a mathematical consequence of Classical wave equations and generalized, complex analytic signal wave functions, not necessarily tied to Quantum Theory assumptions. The success of the "Schrödinger family of equations" lies in their ability to model condensed matter with stable rest masses based on various standing-wave energy packings.

We will also show that new, universally valid wave equations, such as those introduced in Chapter 4.3 (see equations (4.9) and (4.10)), when merged with PWDC, can describe any wave motion across various fields of physics. This will lead to the formulation of generalized Schrödinger-like wave equations that provide a more conceptually tangible, precise, natural, and enriched understanding of Particle-Wave Duality, surpassing the Orthodox Quantum Mechanical view of the same problematic (see Chapter 10 for further discussion).

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To answer the question of how matter waves have been initially created is one of the objectives in this book (and will be specifically addressed, in the next paragraph, 4.1.2.). As an easy introduction (into conceptual understanding of the **PWDC**), let us start analyzing an idealized, a "sufficiently isolated (self-confined) and mutually interacting, two-body" or two-particle system. In such a two-body system, the first body is an ordinary, known-parameters particle, in a rectilinear motion (still and temporarily considered as being without elements of rotation), having linear momentum p_1. Here, the second "particle" effectively presents surrounding environment or universe, having some resulting linear momentum p_2. We can assume that certain force or interaction (explicable by the existence of an intermediary field, or spatial energy-distribution) should always exist inside and between mentioned particles. Here, all interactions are treated traditionally (from the platform of Classical Mechanics) manifested by forces having the ability to change the magnitude and direction of linear moments of involved moving particles. We still exclude options of having other types of forces manifesting between rotating and spinning objects, that could change orbit and spin moments of particles having such attributes (just to simplify this analysis in its very first steps). Since we intend to treat this case (two particles interaction) like an isolated system, the general law of total momentum conservation (First Newton Law) should be satisfied: $\vec{P} = \vec{p}_1 + \vec{p}_2 = \overrightarrow{const}$. The central forces effectively acting on each entity of the two-body system can be found by applying the Second Newton Law: $\frac{dP}{dt} = \frac{dp_1}{dt} + \frac{dp_2}{dt} = \vec{F}_1 + \vec{F}_2 = 0, \ \frac{dp_1}{dt} = \vec{F}_1 = \vec{F}_{12}, \ \frac{dp_2}{dt} = \vec{F}_2 = \vec{F}_{21}, \vec{F}_{12} = -\vec{F}_{21} \Leftrightarrow \vec{F}_1 = -\vec{F}_2$. The ordinary-like real particle (the first body) is characterized by its linear motion particle momentum p_1 and performs the force $\vec{F}_{12}$, acting on its couple (which is surrounding universe). The second body has its resulting linear moment p_2, and performs an action $\vec{F}_{21}$ on the first particle characterized with p_1. Later in this book, conceptualization of forces and fields between involved particles ($p_1 = p$, and resulting, surrounding-space momentum $p_2 = \tilde{p} \Rightarrow P = p + \tilde{p} = \text{const.}, \ dp = -d\tilde{p}$) will be the starting spot for explanation of the **PWDC**. In other cases, if we were dealing with two (known and mutually interacting), moving particles "immersed" in the surrounding environment (what would effectively become, at least, a three-body system), the previous situation evolves as, $\vec{P} = \vec{p}_1 + \vec{p}_2, \ \frac{d\vec{P}}{dt} = \frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} = \vec{F}_1 + \vec{F}_2,$ $\frac{d\vec{p}_1}{dt} = \vec{F}_1 + \vec{F}_{12}, \ \vec{F}_{12} = -\vec{F}_{21}, \ \frac{d\vec{p}_2}{dt} = \vec{F}_2 + \vec{F}_{21}$. The significant **PWDC** elements (in this new case) would be related to "internal" central forces between particles $\vec{F}_{12} = -\vec{F}_{21}$, and to the work produced by such forces (acting on the path r_{12}, which connects two moving particles): $E_{12} = \int_{(1)}^{(2)} \vec{F}_{12} d\vec{r}_{12} = E_{k1} + E_{k2}, \ \ \vec{F}_{12} = -\overrightarrow{grad(U_{12})}, \ E_{k1} + E_{k2} + U_{12} = 0.$ (see (4.3)-(4.8); in this case forces $\vec{F}_1, \vec{F}_2$ are external forces; -see also (4.8-3) in Chapter 4.2).

The understanding of **PWDC** (regarding forces between mutually approaching particles) could also be complemented with the generalized Newton-Coulomb force laws, as presented in Chapter 2. (see equations from (2.3) to (2.9), and later from (2.11.10) to (2.11.21)). The important message here is that forces between two interacting objects could be composed of different, static, and dynamic force components, since "Newton-Coulomb force laws" are not enough to capture the complexity of all possible forces between two interacting objects.

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In this book, we will demonstrate that the general Two-Body problem has not yet fully explored the coupling between linear motions and the associated rotating, orbital, and spinning motions. This coupling is key to understanding the nature and origins of Particle-Wave Duality in matter (see Chapter 4.1.2: "De Broglie Matter Waves, Unity of Linear and Angular Motions, and Fluid Dynamics"). Matter in our universe consists of atoms and molecules with embedded electric charges, magnetic properties, and spinning states, creating an electromagnetic background that serves as a platform for unifying matter waves, gravitation, and electromagnetic fields.

A critical question we will address is how and why forces between interacting objects create matter waves. Consider two approaching particles without any force-field interaction; they would pass through the interaction zone without affecting each other, continuing into open space. In cases of collisions, classical mechanics predict outcomes based on conservation laws. However, when central forces come into play, introducing elements of angular motion, torsional fields, and spinning or vortex dynamics, the scenario becomes more complex and intriguing. These rotating motion elements generate matter waves, allowing us to associate wavelengths and frequencies with such motions, a concept mathematically mastered by de Broglie, and one we will further elaborate upon in this book.

If the interacting particles already possess linear, orbital, and spinning moments, along with electric charges and magnetic properties, the resulting forces and associated matter waves become richer and more naturally explicable. This book takes the concept further by suggesting that linear and angular moments can act as "active field charges" for these interacting forces, including gravitation (see Chapter 2 for related equations).

We can also imagine that all interacting particles and objects, along with their environment, behave as though connected by invisible springs, forming a multidimensional, elastic, spatial standing-wave structure. This structure, or matrix, oscillates and produces matter waves when any of its nodes or elements are excited or set in motion. PWDC (Particle-Wave Duality Code) describes the essential elements of such wave behaviors and couplings. Moreover, each nodal element, whether a particle, atom, or molecule—represents a stabilized, oscillating, or resonant structure. PWDC provides the framework for understanding how these coupled oscillators are structured, how they interact, and how they synchronize.

This universal, spatial matrix, the ever-present texture of our universe, should behave like an exotic, fluidic medium, sort of ether that carries matter waves. Nikola Tesla also supported a similar concept of space and ether with tangible and measurable electromagnetic properties, such as dielectric constant and magnetic permeability, $\varepsilon, \mu, c = 1/\sqrt{\varepsilon\mu}$.

Contemporary physics often analyzes two-body relations using classical mechanics of rigid particles, which is a limited approach. This book will introduce and explain the broader implications of PWDC. For example, in electric circuit theory, we analyze currents, voltages, and power, applying concepts of active, reactive, and apparent power. Similarly, we should develop analogous concepts for mechanical and other forces, velocities, and power associated with moving particles and matter waves (as introduced in Chapter 1). This approach is not yet widely accepted, but it allows us to introduce concepts like Complex, Active, Reactive, and RMS forces and velocities, as well as Active, Reactive, and Apparent mechanical power (by analogy with electric circuit theory items, as elaborated in Chapter 4.0). Such a framework reveals that any motion in physics has a vibrant balance between its intrinsic, interacting corpuscular and wave properties, a theme explored in the following chapters.

The central message of this book is that "ENERGY IN MOTION," or "CURRENT OF ENERGY," essentially describes the nature of matter waves and de Broglie waves. Here, total fluctuating and alternating POWER (=) dE/dt can encompass electromagnetic, mechanical, linear, rotational, and other coupled components. Later chapters will show that Schrödinger's equation and Quantum Wave Mechanics were effectively formulated using similar principles. By normalizing and averaging power-related wave functions, based on Analytic Signals modeling, applying universally valid Probability Theory, Statistics, and Signal Spectrum Analysis, and respecting all known Conservation Laws, we can derive most of the equations of contemporary Quantum Wave Mechanics.

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The initial **formulation and explanation of the PWDC** we shall start based on a specific set of analogies (whenever applicable). Much more about **PWDC** can also be found in Chapters 9 and 10 of this book. For instance, we can start with "*T.1.8 Generic Symmetries and Analogies of the Laws of Physics*", from the first chapter, combined with principal formulas and results already known, proven, and used in all contemporary presentations of particle-wave duality concepts, (see T.4.0), such as:

A) De Broglie matter-wavelength, $\lambda = h/p = 2\pi/k$, and Einstein-Planck's differential form for wave packet (or photon) energy, $d\tilde{E} = h \cdot df$,

$$\left\{\lambda = \frac{h}{p},\ u = \lambda f,\ \tilde{E} = hf\right\} \Leftrightarrow \left\{\frac{\lambda}{v} = \frac{h}{pv} \Leftrightarrow \frac{\lambda f}{vf} = \frac{u}{vf} = \frac{h}{pv}\right\} \Leftrightarrow \left\{\frac{u}{v} = \frac{hf}{pv} = \frac{\tilde{E}}{pv}\right\}. \quad (4.1)$$

In (4.1), $\mathbf{v}$ and $\mathbf{u = \lambda f}$ are group and phase velocities, $\gamma mv = p = hk/2\pi$ is the particle linear momentum, $\gamma = (1 - v^2/c^2)^{-1/2}$, $f_t = f = \omega/2\pi$ is the time-domain frequency of associated de Broglie wave, $f_s = k/2\pi$ is its space-domain frequency, $\mathbf{h}$ is Planck's constant, and $\tilde{E} = pu = hf \Leftrightarrow E_k$ is the (motional or wave) energy of the involved (narrow-band) mater wave packet, where $E_k = pv/(1 + \sqrt{1 - v^2/c^2})$ is the kinetic energy of the particle in linear motion. We often use relations and forms as given in (4.1) without thinking how de Broglie, A. Einstein and M. Planck came to such simple and useful relations (but also some of them being challenging). *The basic particle-wave duality relations (4.1) are formulated or postulated (or mathematically fitted, but still not completely explained) by searching for the best possible solutions, or missing mathematical links (whatever works well), to explain some dualistic and matter-wave phenomena, experiments, and models in microphysics. This was initially based on analogies between a photon as a matter wave-packet, and an equivalent moving particle; -see more in "4.1.1.1. Photons and Particle-Wave Dualism". For instance, we are using such photon-particle analogies to explain Bohr's hydrogen atom model and thermal radiation of a black body, and to serve similar purposes in explaining many other micro-world phenomena, such as Photoelectric, Compton Effect, Bragg's diffraction (or scattering) of X-rays, the structure of electrons and neutrons, etc. In chapter 2. (concerning Gravitation), it is successfully revealed that similar, or mutually equivalent models, wave equations and concepts from the world of microphysics, are analogically applicable to planetary systems, and motions of macrocosmic objects (without involving Statistics and Probability theory concepts).*

B) For a moment we do not need to state explicitly what are all possible mutual relations (or quantitative connections) between the wave energy $\tilde{\mathbf{E}}$, wave momentum $\tilde{\mathbf{p}}$, kinetic (particle) energy $\mathbf{E_k}$, and its (particle) momentum $\mathbf{p}$, except to consider that specific analogy, equivalency and coupling between them should exists, and that both entities (particle and wave-packet) have the same (group) velocity, $\mathbf{v = v_g}$. Later, the exact mathematical connections between all of them will be found; -see (4.2). Currently, we will show equivalency or direct proportionality relations such as $\tilde{\mathbf{E}} = \mathbf{pu} = \mathbf{hf} \Leftrightarrow \mathbf{E_k}$, since any wave energy is also a kind of kinetic or motional energy). One of the objectives here is to explore if matter-wave entities are something that could exist independently or only associated with particles in motions. Of course, here we are again using A. Einstein and Minkowski 4-vectors relativistic relations between involved masses, moments, and energy. In the explanation of Photoelectric effect, where energy and momentum of particles are analogically comparable with similar photon properties, we also have in mind that, in a larger (mathematical and experimental) frame, similar concepts are successfully applied on explanations of Compton effect, electron-positron creation and annihilation, and Bragg's scattering of X-rays, electrons and neutrons…

For a moving particle, we know the differential of its relativistic kinetic energy in the form $\mathbf{dE_k = vdp}$. By analogy with a photon (see later **T.4.0.**), for wave energy of an associated or equivalent wave-group, similar relation should also be valid: $d\tilde{E} = dE_k = vdp$. Differential kinetic or wave energy amounts should have the same mathematical forms in Classical and Relativistic Mechanics since there is anyway only one, united or the same physics reality (we intend to describe), what is producing,

$$\left\{ \begin{pmatrix} E_k = \frac{1}{2}mv^2 = \frac{pv}{2} \\ m = m_0 = const. \\ p = mv \end{pmatrix} \Rightarrow \boxed{\begin{matrix} \boxed{dE_k = vdp} \\ dE_k = pdv \\ dE_k = \frac{1}{2}(vdp + pdv) \\ vdp = pdv \end{matrix}} \right\} \Leftrightarrow \left\{ \begin{pmatrix} E_k = mc^2(\gamma - 1) \\ m = m_0 = const. \\ p = \gamma mv \\ E_{tot.} = \gamma mc^2 \end{pmatrix} \Rightarrow \boxed{dE_k = vdp} = mc^2 d\gamma = dE_{tot.} \right\}.$$

Consequently, we will use $d\tilde{E} = dE_k = vdp$ in differential equations of interest (for instance in the equation that connects group and phase velocity, (4.2). Practically, we will (analogically and intuitively) assume and accept that both, for classical mechanics and relativistic mechanics cases, the differential of kinetic energy should be $dE_k = v \cdot dp = d\tilde{E} = h \cdot df$, regardless we see that in Classical Mechanics case we also have other options.

C) Let us now try to connect and unify all (above mentioned) energy forms regarding particle and its corresponding wave group. From the last, right-hand part of the equation (4.1), we can come closer to the conclusion that de Broglie wavelength is not the most-significant qualification of particle-wave duality, and that fuller qualification should be the relation between phase and group velocity of de Broglie wave packet. Practically, we shall exploit and merge all direct proportionality relations from (4.1) with $\mathbf{\tilde{E} = hf = pu \Leftrightarrow E_k, dE_k = vdp \Leftrightarrow}$ $\mathbf{d\tilde{E} = hdf = d(pu)}$, and $v = u - \lambda(du/d\lambda)$, and come to conclusions as given in (4.2). It is not necessary, but we could also apply in (4.1) conclusions based on analogies (from the first chapter), using the last part of (1.19), to create more explicit relations between particle and wave characteristics of de Broglie or matter wave packet, as shown in (4.2). Modern Quantum Theory supports that a total particle energy (including its rest mass energy) is the content of an equivalent matter-wave packet or group, what is not the case in this book. Here, we will exclusively identify motional or kinetic, or matter-wave energy as the content of an equivalent, matter-wave object, or wave packet (without rest mass), being especially useful in its differential or infinitesimal form.

Presence of inertial effects (in mechanics and electromagnetism), we can detect whenever some sudden and non-uniform changes of certain energy-flow happen (meaning when some currents, voltages, forces, velocities, and moments of specific stationary state would suddenly change). Our conceptual problem related to mathematical modeling of transient inertial effects is that we understand well what that means in an electromagnetic environment (described under different induction laws), but in mechanics, our deeper understanding of similar problematic stops with Newton laws. Usually, we do not search there for field components that should complement Gravitation, like in mutual relations between electric and magnetic fields. It is also not *excluded that all transient and inertia-related effects (in mechanics) have their deep roots in electromagnetic induction laws (since mass is composed of atoms, and atoms have electrically and magnetically charged content). Anyway, it should be clear that inertia is not only a tendency to keep a steady state of constant velocity of uniform, linear or rectilinear motion. Place for "inertial rotational, and spinning", as well as for other more complex, stable, periodical, and inertial motions (including specific accelerated motions, and standing waves states) should also be properly established in Physics.*

If corpuscular and wave momentum $(\mathbf{p}, \tilde{\mathbf{p}})$ are mutually collinear or parallel (even identical) vectors ($\cos(\mathbf{p}, \tilde{\mathbf{p}}) = \mathbf{1}$ **or** $-\mathbf{1}$) we will find that there is only one consistent and unifying result, given by (4.2), which extends relations (4.1), relevant for the basic understanding of particle-wave duality. Let us support such matter-waves concept by a very simplified mathematical procedure (by listing initial statements and going directly to their implications), what is intuitively improvised, and summarized as follows (see more in [5], and in Chapter 10. Especially under "10.1 Hypercomplex Analytic Signal functions and interpretation of energy-momentum 4-vectors in relation to matter-waves and particle-wave duality"):

$$\left\{\begin{array}{l}\left\{\begin{array}{l}E_k \Leftrightarrow \tilde{E} = pu\,,\; pv = E_k\left[1+\sqrt{1-(\frac{v}{c})^2}\right] = \frac{\gamma^2-1}{\gamma}mc^2\,,\\ E = E_0 + E_k = \gamma mc^2 = mc^2 + \frac{\gamma mv^2}{1+\sqrt{1-(\frac{v}{c})^2}} = mc^2 + \frac{1}{2}mv^2 + \ldots\\ dE_k = vdp = d\tilde{E} = hdf\end{array}\right\},\\ \left\{(\frac{u}{v}) = \frac{\tilde{E}}{pv} = \frac{pu}{E_k\left[1+\sqrt{1-(\frac{v}{c})^2}\right]} = \frac{1}{1+\sqrt{1-(\frac{v}{c})^2}} = \frac{\gamma}{\gamma+1}\right\},\\ \left\{\begin{array}{l}\lambda = \frac{h}{p}, k = \frac{2\pi}{\lambda} = \frac{2\pi}{h}p\,, \omega = 2\pi f\,,\\ \frac{d\lambda}{\lambda} = -\frac{dp}{p} = -\frac{dk}{k} = -\frac{df}{f}, u = \frac{\omega}{k}, v = \frac{d\omega}{dk}\end{array}\right\}\end{array}\right\} \Rightarrow$$

$$\Rightarrow \left\{ \begin{array}{l} v = u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda} = u + p\frac{du}{dp} = \frac{d\omega}{dk} = \frac{d\tilde{E}}{dp} = h\frac{df}{dp} = \frac{df}{df_s} = \frac{2u}{1+\frac{uv}{c^2}}, \\ u = \lambda f = \frac{\omega}{k} = \frac{\tilde{E}}{p} = \frac{hf}{p} = \frac{f}{f_s} = \frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{E_k}{p},\ f_s = k/2\pi \ \Rightarrow \\ \Rightarrow \quad 0 \le 2u \le \sqrt{uv} \le v \le c\,, \\ d\tilde{E} = hdf = mc^2 d\gamma = dE_k = dE, \ \ \frac{df}{f} = (\frac{dv}{v})\cdot \frac{1+\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{v^2}{c^2}} \Rightarrow \frac{\Delta f}{\overline{f}} = (\frac{\Delta v}{\overline{v}})\cdot \frac{1+\sqrt{1-\frac{\overline{v}^2}{c^2}}}{1-\frac{\overline{v}^2}{c^2}} \end{array} \right\} \Rightarrow$$

$$\Rightarrow \left\{ \begin{array}{l} \frac{\tilde{E}}{mc^2} = \frac{hf}{mc^2} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1 = \gamma - 1 = \frac{\tilde{E}}{E_0},\ E_0 = mc^2 = const., \\ \frac{\tilde{E}}{\gamma mc^2} = \frac{hf}{\gamma mc^2} = 1 - \sqrt{1-\frac{v^2}{c^2}} = 1 - \frac{1}{\gamma} = \frac{\tilde{E}}{E_{total}},\ E_{total} = \gamma mc^2 = \gamma E_0 = E, \\ \frac{\tilde{E}}{E_k} = \frac{\tilde{E}}{(\gamma-1)mc^2} = \frac{hf}{(\gamma-1)mc^2} = 1,\ E_{total} = E_0 + E_k = E_t\,, \\ p^2c^2 + E_0^2 = E_t^2, \quad p^2v - pE_t + p_0E_0 = 0, \\ \tilde{E} = pu = -E_0 \pm \sqrt{E_0^2 + p^2c^2} = E_k \left\{ = -E_0 + \sqrt{E_0^2+p^2c^2} = E_0\left[\sqrt{1+(\frac{pc}{E_0})^2} - 1\right]\right\}, \\ \left\{ \begin{array}{l} \vec{p} + \tilde{\vec{p}} = \vec{P} = \overrightarrow{const} \Rightarrow d\vec{p} = -d\tilde{\vec{p}} \\ d\tilde{E} = hdf = d(pu) = dE_k = vdp = c^2 d(\gamma m) = -d(\tilde{p}u) = \\ = -vd\tilde{p}\cdot\cos(\vec{p},\tilde{\vec{p}}) = \{vd\tilde{p} \text{ or } -vd\tilde{p}\} = h\cdot df_s \end{array} \right\} \\ \Rightarrow \Delta E_k = -\Delta\tilde{E},\ \Delta p = -\Delta\tilde{p},\ \Delta L = -\Delta\tilde{L},\ \Delta q = -\Delta\tilde{q},\ \Delta\dot{p} = -\Delta\dot{\tilde{p}},\ \Delta\dot{L} = -\Delta\dot{\tilde{L}},... \end{array} \right\}, \quad (4.2)$$

As we can see (from (4.2), (4.8-3), Chapters 4.2 and 10, and later in this chapter), here we are also dealing with the **generalized concept of "action-equal-to-reaction"**, transient and inertial forces (as an equivalent to the Third Newton Law, being similar to electric and magnetic induction laws: ($\Delta p = -\Delta\tilde{p}$, $\Delta L = -\Delta\tilde{L}$, $\Delta q = -\Delta\tilde{q}$ ...) $\Rightarrow$ ($\Delta\dot{p} = -\Delta\dot{\tilde{p}}$, $\Delta\dot{L} = -\Delta\dot{\tilde{L}}$, $\Delta\dot{q} = -\Delta\dot{\tilde{q}}$ ...)). This becomes explicitly evident after implementing the time differentiation on involved momentum and charge properties, regardless of their field nature, being applicable in mechanics, gravitation, electromagnetic fields, rotation, etc. See more supporting background in Chapter 4.2; -equations from 4.8-2 to 4.8-4.

Later (in Chapters 5. and 10.) we will see that PWDC relations (4.2), and Uncertainty relations (5.1), $\Delta x \cdot \Delta p = \Delta t \cdot \Delta E = h \cdot \Delta t \cdot \Delta f \ge h/2 \Leftrightarrow \Delta x \cdot \Delta\tilde{p} = \Delta t \cdot \Delta\tilde{E} = h \cdot \Delta t \cdot \Delta f \ge h/2$, are effectively describing mutual space-time proportionality, or coupling, since group velocity (of a specific narrow-band signal, or wave group) is also equal to,

$$v = \frac{\Delta x}{\Delta t} = \frac{\Delta\tilde{E}}{\Delta p} = \frac{\Delta\omega}{\Delta k} = h\frac{\Delta f}{\Delta p} \Leftrightarrow v\Delta t = \Delta x = \frac{\Delta\omega}{\Delta k}\Delta t = h\frac{\Delta f}{\Delta p}\Delta t \ .$$

In practical terms, it will become increasingly clear—especially here and in subsequent chapters, that the equations and relations in (4.1) and (4.2) form the core of the Particle-Wave Duality Code (PWDC). For further understanding, refer to (4.3), tables T.4.0 and T.5.3, and the uncertainty relations from Chapters 4.0, 5, and 10, which provide essential complements to the PWDC framework.

The mathematical formalism of the PWDC should also incorporate Einstein-Minkowski 4-vectors, which can be conveniently unified with the concept of the Analytic Signal (discussed in detail in Chapter 10). The most direct and explicit experimental and mathematical confirmation of PWDC can be found in Chapter 4.2 of this book, particularly in section 4.2.2, Example 3: Elastic Collision of a Photon and Electron (or the Compton Effect).

What we define as the PWDC is also inherently linked to the natural properties of a Complex (and hypercomplex) Analytic Signal, or Phasor, which serves as the most general mathematical model for working with matter wavefunctions, such as de Broglie matter waves. This Analytic Signal concept is introduced in Chapter 4.0 of this book.

It is important to note, however, that the general applicability of a wave energy quantum $\tilde{\mathbf{E}} = \mathbf{hf}$ *in its typical form is approximate and limited to narrow-band signals. A more useful expression is a differential or infinitesimal relation for narrow-band wave groups* $\mathbf{d\tilde{E}} = \mathbf{h} \cdot \mathbf{df}$ *.*

4.1.1.1. Photons and Particle-Wave Dualism

We are already familiar with numerous convincing experiments and analyses related to wave-particle duality. These include interactions between electrons, photons, neutrons, and other elementary particles, as seen in phenomena such as the Compton effect, the photoelectric effect, electron-positron creation and annihilation, photon absorption and emission by atoms, X-ray generation, and Bragg diffraction of X-rays, electrons, and neutrons. These experiments provide valuable insights into the nature of photons and the origins of particle-wave duality (see examples and analyses in Chapter 4.2).

The concept of matter waves, formulated by pioneers like L. de Broglie, M. Planck, A. Einstein, and N. Bohr, was initially developed through mechanical analogies between photons and moving particles. However, during the development of these analogies, certain oversights and contradictions, were introduced. These primarily stemmed from improper or incomplete conclusions regarding the relationship between photons, particles, and matter waves.

The most significant issue in modern quantum theory is the analogy drawn between the total energy of a photon and that of a particle ($\mathbf{hf} = \mathbf{mc}^2$ or $\mathbf{hf} = \gamma\mathbf{mc}^2$, $\mathbf{uv} = \mathbf{c}^2$). A photon, having no rest mass, possesses only motional energy, meaning its total energy is purely kinetic. Therefore, a photon should only be compared with the motional energy states of particles, excluding rest mass or rest energy. This fundamental error laid an incomplete and incorrect QT foundation regarding our understanding of matter waves and wave-particle duality, leading to the introduction of additional erroneous concepts to align with experimental observations and always valid natural laws.

For instance, the common relations between particle mass and matter wave properties are not universally applicable or correct, yet they still appear in modern physics texts when explaining particle-wave duality. Furthermore, insufficient attention has been given to the detailed analysis of continuous X-rays emission, and the spectral components of the Compton and photoelectric effects produced by high-speed electrons. A more rigorous analysis reveals that the motional energy and mechanical moment of the interacting or created photons are equivalent to the motional energy of the electrons involved. This energy exchange occurs without the participation of the rest-masses, as will be explicitly shown in Chapter 4.2.

By incorporating rest-mass energy into the description of matter-wave energy, the founders of particle-wave duality and quantum theory, along with their uncritical followers, did not fully describe the nature of matter waves and wave-particle duality. Matter waves (or de Broglie waves) represent only the motional “energy-momentum” states of particles and exclude rest-mass energy. This has yet to be clearly and affirmatively stated in quantum theory, a gap that this book aims to address.

Although, in some situations, photons can fully transform into particles, or particle impacts can generate photons, this is not a general rule. Additionally, the use of probabilities, statistics, and uncertainty relations is not the most natural or optimal way to resolve the foundational errors in the theory, except when applied to large sets of identical items for mathematical modeling or curve-fitting purposes.

Photons also possess structural helicity in the form of their electric and magnetic field vectors, including spin moments. By analogy, particles in linear motion should also exhibit helicity, or spiral solenoidal field effects, like in photon motion. The analogy between photons and moving particles should be valid in both directions, applicable to linear motion and the associated helical or spinning effects. For example, an electron moving in a straight line should generate a helical or spiral magnetic field around its path.

Electrons are known to have intrinsic magnetic and mechanical spin moments, which suggests that an electron is a complex electromagnetic field structure where photons or electromagnetic field vectors are arranged in a specific way, in accordance with conservation laws. Self-contained standing-matter-waves can be linked to the formation of stable elementary particles. A related concept can be found in the Henry Augustus Rowland effect, where a magnetic field surrounds a rotating conductor, as discussed by Jean de Climont (see more in [117].

This helicity analogy should also apply to protons and neutrons. Since atoms and other neutral particles are composed of electrons, protons, neutrons, photons, and structured standing waves, we can infer that electromagnetically neutral particles in motion must also possess associated matter-wave helicity and spin moments. However, contemporary physics, including Faraday-Maxwell theory, does not account for the helical magnetic field components associated with electrons in linear motion. Nor does it properly explain the analogy between photon spin and the helical structure of matter waves around moving electromagnetically neutral particles. This gap in understanding remains unaddressed in modern physics.

We know that photons carry measurable energy and linear momentum (for instance, pressure of photons’ radiation creates mechanical motion or rotation), analogically implicating those photons are comparable to a stream of particles. If photon has certain unknown rest-mass $\mathbf{m_o}$, then its relativistic energy and momentum should be,

$$\begin{bmatrix} \tilde{E}_p = \gamma m_o c^2 = m_o c^2 / \sqrt{1 - v^2/c^2} = hf_p \neq 0 \\ \tilde{p}_p = \gamma m_o v = m_o v / \sqrt{1 - v^2/c^2} \neq 0 \\ \tilde{p}_p / \tilde{E}_p = v/c^2, \ \ v = c \end{bmatrix} \Rightarrow \begin{bmatrix} m_o = 0, \\ \tilde{p}_p = \tilde{E}_p / c = hf_p / c = h/\lambda_p = \tilde{m}_p c, \\ \tilde{E}_p = \tilde{m}_p c^2 = hf, \\ \lambda_p f_p = c \end{bmatrix},$$

and we can find that its rest mass should be equal to zero (or being nonexistent), implicating that any particle with non-zero rest mass can never reach speed v = c.

Since photon also has certain spin-moment, let us additionally exploit such "particle-matter-wave-photon analogy" (see below). Such analogy was not originally considered useful when de Broglie formulated his hypothesis about matter-waves duality, and can be formulated as follows,

Wave-Particle Duality and analogical PHOTON-PARTICLE comparison, extended from linear motion towards associated helicity

$$\left\{\begin{array}{c}\text{PHOTON}\\ \text{(accounting only its linear motion)}\\ \tilde{E}_p = hf_p = \tilde{m}_p c^2 = \tilde{p}_p c,\ \tilde{p}_p = \tilde{E}_p / c = hf_p / c,\\ f = \tilde{p}_p c / h,\ \lambda_p f_p = c \Rightarrow \boxed{\lambda_p = c / f_p = h / \tilde{p}_p}\end{array}\right\} \Rightarrow \left\{\begin{array}{l}\text{Particle}\\ \text{matter}\\ \text{wave}\\ \text{wavelength}\\ \boxed{\lambda = h/p}\end{array}\right\} \Leftrightarrow \left\{\begin{array}{l}\text{PHOTON (acounting its spinning)}\\ \tilde{L}_p = h / 2\pi,\ h = 2\pi \tilde{L}_p,\\ \tilde{E}_p = hf = \omega \tilde{L}_p = \tilde{m}_p c^2 = \tilde{p}_p c,\\ \lambda_p f_p = c \Rightarrow \lambda_p = c / f_p = h / \tilde{p}_p = 2\pi \tilde{L}_p / \tilde{p}_p,\\ \boxed{\tilde{L}_p = \lambda_p \tilde{p}_p / 2\pi = h / 2\pi}\end{array}\right\} \Rightarrow \left\{\begin{array}{l}\text{Particle spin-moment}\\ \text{as a matter wave}\\ \boxed{\begin{array}{l}\tilde{L} = \lambda p / 2\pi = h / 2\pi\\ \tilde{L}\omega = pv\end{array}}\end{array}\right\}$$

Effectively, particles in linear motion should also have certain associated spinning moments and vice versa (by the analogy with a photon properties), and this spinning is related to de Broglie matter waves. ***In other words, spinning object is also (based on analogical conclusions) creating "linear momentum thrust" or force, which could be casually related to the force of Gravitation (of course considering spinning and rotation-associated effects of magnetic field).***

A) To initiate new insights in relation to photons and wave-particle duality, we can again summarize our knowledge about photons and wave-particle duality starting from reasonable assumptions and facts in relation to photons, such as,

1. *What we understand as a photon is certain spatially-&-temporally localizable (and finite), narrow-band wave-group, or soliton structure of an electromagnetic field, which has an energy amount of* $\tilde{E} = h \cdot \overline{f} = \frac{h}{2\pi} \cdot \overline{\omega},\ h = \text{const.}$, *"being much more as a condensed droplet of energy", then a real quantum of electromagnetic field. It is also likely that many other forms of electromagnetic waves are not shaped and do not carry energy like photons.*

2. *Photons have zero rest mass. Photon presents only a wave, and it has motional or kinetic energy, which is at the same time its total energy* $\tilde{E}_t = \tilde{E}_k = \tilde{E} = hf = \tilde{m}c^2$*, and such energy balance relation is much more correct in its differential or infinitesimal form* $d\tilde{E}_t = d\tilde{E}_k = d\tilde{E} = h \cdot df = c^2 \cdot d\tilde{m}$.

3. *Effective motional mass of a photon is* $\tilde{m} = \frac{\tilde{E}}{c^2} = \frac{h \cdot f}{c^2} = \frac{h}{2\pi c^2} \cdot \omega,\ (h, c = \text{consts.})$.

   Linear moment of a photon is $\tilde{p} = \tilde{m}c = \frac{\tilde{E}}{c} = \frac{hf}{c} = \frac{h}{2\pi c}\omega,\ (h, c = \text{consts.})$. *We know that such statements and mathematical expressions are useful since in number of impact and scattering interactions, like Compton and Photoelectric effects are, we are successfully explaining experimentally verifiable results* *using mentioned expressions for photon mass, momentum, and energy … (but what is still challenging here is the meaning of an elementary photon as the quant of energy).*

4. *We could now safely consider that spectrally-temporally and spatially narrow-band photon, when moving in an isotropic and homogenous, non-dissipative and non-dispersive space (like in a vacuum), presents certain kind of uniform and inertial, self-maintaining linear and spinning motion, or motional energy state, and effectively manifests some particle properties (since it is interacting with other real, non-zero rest mass particles, like particle interactions are treated or behaving in mechanics).*

5. *Consequently, photons and moving particles should have limited spatial, temporal, and spectral sizes and lengths (or durations) to be presentable as wave-particle dual objects, as we find in Quantum Theory. For instance, if a photon is propagating along certain linear path s, the relations* $ds = c \cdot dt \Rightarrow \Delta s = c \cdot \Delta t$ are naturally valid, where Δs and Δt are spatial and temporal durations or lengths of the same photon. Since energy of such (band-limited) photon is $\tilde{E} = hf = \tilde{m}c^2$, we can specify a photon size or its finite spectral and energy content as, $d\tilde{E} = h \cdot df = c^2 \cdot d\tilde{m} \Rightarrow \quad \Delta\tilde{E} = h \cdot \Delta f = c^2 \cdot \Delta\tilde{m}$. ***Since photons are obviously space-time-energy limited, energy finite and localizable electromagnetic matter-wave packets in all mentioned domains, we should conclude that amplitude or envelope function of a photon is like Gaussian pulse (because only Gaussian pulses can be well defined in both temporal and spatial domains).*** The next consequence of such a situation is that here we can approximately apply relations of "*elementary certainty*" (5.3) of *matter-states domains*, as described in Chapter 5. of this book, which are here presenting relations between finite (total, non-statistical) durations of certain photon in all its band-limited domains,

$$\left(\Delta q_{mag.} \cdot \Delta q_{el.}\right)_{min.} = \left(\Delta\alpha \cdot \Delta L\right)_{min.} = h \cdot \left(\Delta t \cdot \Delta f\right)_{min.} = \left(\Delta x \cdot \Delta\tilde{p}\right)_{min.} = c\left(\Delta t \cdot \Delta\tilde{p}\right)_{min.} =$$
$$= \left(\Delta t \cdot \Delta\tilde{E}\right)_{min.} = c^2\left(\Delta t \cdot \Delta\tilde{m}\right)_{min.} \geq h/2,\ \Delta\tilde{E} = h \cdot \Delta f = c^2 \cdot \Delta\tilde{m},\ \Delta x = c \cdot \Delta t.$$

Furthermore, in Chapter 9. of this book, under "9.1. Wave Function of a Single Photon", we can find number of additional relations between temporal, spatial and spectral durations, amplitudes, and other important characteristics addressing a single photon wave function.

Consequently, here we could speculate with the concept that by frequency narrow band-limited photons are specifically structured, presenting elementary matter-wave packets and building blocks of all matter in our Universe, and that different matter-states are mutually communicating and interacting involving photons.

B) Based on such initial grounds (from 1. to 5.), we can create number of conclusions and new concepts, such as,

6. *In mechanics, the closest analogical state of motion, or motional energy, analogical to a narrow-band photon, is a spinning body, which could be a disk, gyroscope, certain toroidal form, spheroidal or ellipsoidal body, and which has spinning motional energy expressed as,* $E_s = L \cdot \omega = 2\pi L \cdot f$ *(as gyroscopes). If mentioned spinning body is in a self-maintaining, uniform and inertial motion, we can consider that its angular moment is constant,* $L = const.$ *Now, we are coming to the direct conceptual and mathematical analogy between narrow-band photon and certain spinning (or orbiting) body, since photon energy is* $\tilde{E} = h \cdot f = hf_s = \frac{h}{2\pi} \cdot \omega = \tilde{L} \cdot \omega = \tilde{m} c^2, \tilde{L} = \frac{h}{2\pi} = const.$*, and spinning body energy is also* $E_s = L \cdot \omega = hf_s , L = \overline{const.}$*. Based on such analogy, we could consider that photon presents certain spatial, spinning electromagnetic formation. If there are other, isomorphic, equivalent and 1:1 spatial mapping of photons configurations or topologies, photon could also take forms of such spatial (Gauss and Bell-curve shaped) formations. Consequences of spinning properties of photons (or matter-waves) for macro masses are addressed in Chapter 2 under "2.3. How to Account for Rotation in Relation to Gravitation?"*

7. *Frequency Band-limited Photon energy we can express as,* $\tilde{E} = h \cdot f = \frac{h}{2\pi} \cdot \omega = \tilde{L} \cdot \omega = \tilde{m}c^2, \tilde{L} = \frac{h}{2\pi} = \frac{\tilde{m}c^2}{2\pi f} = \frac{\tilde{E}}{\omega} = const.$ *We could now conclude that not all electromagnetic waves are like photons, and that* $\tilde{E} = h \cdot f = \frac{h}{2\pi} \cdot \omega, h = const.$ *is not applicable to all forms of electromagnetic waves. Only electromagnetic formations that are part of interactions within and between resonant structures as atoms and elementary particles, are conceptually and energetically presentable as spinning formations, or as what we consider being photons (and in some cases behaving like particles). In fact, for electromagnetic waves being like photons, or like other waveforms, there is a decisive question how such waves are being created. L-C antenna-circuits-created radio, telephone and TV electromagnetic waves are most probably different wave formations when compared to spinning photons that are spatially limited and finite by energy. Other forms of electromagnetic waves could have different, not bound spatial shapes (being not analogue to photons). Photons have constant spinning moments,* $\tilde{L} = \frac{h}{2\pi} = const.$*, and other electromagnetic waves (created by L-C antenna circuits), most probably, have variable angular moments.*

8. *If we replace spinning photon with an equivalent rotating toroidal (gyroscope or ring) form with standing waves structure of electromagnetic waves, which could be considered as an isomorphic, 1:1 functional mapping (or transformation from a spinning disk or ring to a spinning torus), we will again have similar angular or orbital mechanical moment quantizing, as for example,* $2\pi r = n\lambda = n\frac{h}{\tilde{p}} \Leftrightarrow \tilde{L} = \tilde{p}r = n\frac{h}{2\pi}, n = 1, 2, 3...$*. Bohr-Sommerfeld quantization and development of Schrödinger equation is also a familiar approach to here-*

initiated torus-geometry quantization based on stable, self-closed standing waves. Uncertainty relations are also valid and applicable here, regardless of Heisenberg formulations, since such relations are the product of generally valid signal analysis and relations between total (non-statistical) durations of mutually original and spectral domains (applicable in any size of natural states). Consequently, particles (with non-zero rest masses) could be created starting from standing-waves electromagnetic formations combined with an inertial rotation, with elements of certain intrinsic periodicity, and with constant spinning or orbital moment properties. Electrons are the example of specific toroidal resonant formations of standing electromagnetic waves. Solid matter, atoms and stable rest masses most probably have photons and electromagnetic-waves formations as their primary constituents being like nucleation states (combined or formatted in a way that all relevant conservation laws are satisfied). For instance, we can show that a single photon has a zero rest-mas $\tilde{m}_0 = 0$ *based on its 4-vector moment, and inertial-systems invariance relation, as follows,*

$$\tilde{P}_4 = (\tilde{p}, \frac{\tilde{E}}{c}) \Rightarrow \tilde{p}^2 - \left(\frac{\tilde{E}}{c}\right)^2 = -\left(\frac{\tilde{E}_0}{c}\right)^2 \Rightarrow 0 = -\left(\frac{\tilde{E}_0}{c}\right)^2 \Rightarrow$$

$$\Rightarrow \tilde{E}_0 = \tilde{m}_0 c^2 = 0 \Leftrightarrow \tilde{m}_0 = 0, \left(\tilde{E} = hf = pc\ , \tilde{m} = \frac{hf}{c^2}, p = \frac{hf}{c}\right).$$

However, we can also show in a similar way that conveniently superimposed groups of non-collinear and favorably spatially focused photons, and/or other matter wave-packets without rest masses, could create a particle with non-zero rest mass M_0*, as follows,*

$$\tilde{P}_4 = (\sum_{(n)} \tilde{p}_i , \frac{\sum_{(n)} \tilde{E}_i}{c}) = (\sum_{(n)} \tilde{p}_i , c \cdot \sum_{(n)} \tilde{m}_i) \Rightarrow \left[\sum_{(n)} \tilde{p}_i\right]^2 - \left[c \cdot \sum_{(n)} \tilde{m}_i\right]^2 = -\left(\frac{\tilde{E}_0}{c}\right)^2 = -\left(M_0 c\right)^2 \Rightarrow$$

$$\Rightarrow M_0 = \sqrt{\left[\sum_{(n)} \tilde{m}_i\right]^2 - \left[\frac{1}{c}\sum_{(n)} \tilde{p}_i\right]^2} = \frac{L_o \omega_0}{c^2} = \frac{Hf_0}{c^2} \neq 0,\ \tilde{m}_i = \frac{hf_i}{c^2},\ \tilde{p}_i = \frac{hf_i}{c},\ H = const.$$

Of course, all conservation laws should be properly addressed if we intend to describe the creation of real particles starting from matter wave-packets without rest masses. Based on similar grounds we could start developing ideas about new mass-propulsion or trust methods, about remote and targeted particles projection, regarding new ways of motion and energy transfer, etc.

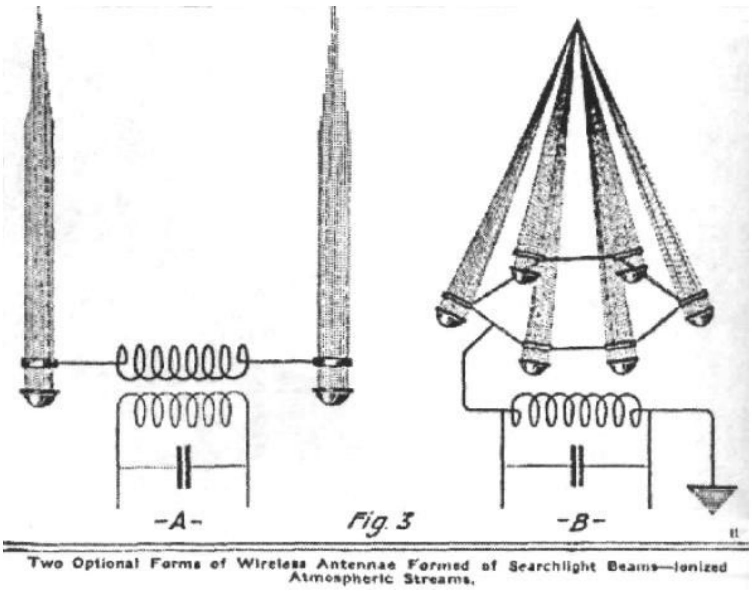
What is essential here are standing waves formations of electromagnetic waves on self-closed spatial structures, orbits or paths, and our expectations that stable, solid, non-zero rest-masses could be created starting from such self-closed, structural formations and combinations of photons and other wave packets.

As an exciting brainstorming and imaginative excursion towards creating real masses from wave packets, we could revamp some familiar, still innovative visions of Nikola Tesla. The biggest ever-born, creative and visionary inventor, as well as a visionary and realistic science fiction dreamer on our planet was Nikola Tesla, who invented in his exotic, magic, and very imaginative way everything that still presents important electromagnetic, electrotechnical and electromechanical grounds of our contemporary *technological civilization. Tesla mostly used pure imagination and spiritual or mental visualization, going almost directly to final inventions formulation, like taking his inventions from certain “universal data bank or common cosmic knowledge library”.*

*Tesla also gave number of descriptions, some of them still not well understood or documented, almost like science fiction predictions and looks-like inventions, where proper theoretical and factual grounds or terminology to give better and more appropriate explanations and formulations have been still missing. Modern science and technological community are presently considering such spiritual inventions indeed as creative but as not well founded, almost as an arbitrary dreaming or brainstorming and magic visions. Tesla has many of such (mental) inventions, visions and patents that are completely verified and present fundaments of our modern planetary and technological civilization. We could now ask ourselves if other Tesla inventions and visions, being still not verified, confirmed, or technically materialized, also have certain predictive merits or potentially significant meaning, if properly addressed, and slightly reformulated with updated modern scientific language, proper mathematics, and with contemporary scientific achievements. One of such, still not materialized, and not well or completely formulated mental invention is about Tesla’s “**death-rays**”. We could imaginatively and creatively, connect or revitalize mentioned “**death-rays**” idea with photons transformation, from zero-rest-mass electromagnetic wave-packets to real non-zero-rest-mass particles (like electrons), as already discussed (see citation below).*

*One of still not realized Tesla’s invention about “**death-rays**”. Citation from internet: Proposing the "death ray" for defense - Philadelphia Inquirer - October 20, 1940:*

*"It is based on an entirely new principle of physics that nobody ever has dreamed of. **It is different from the principle embodied in my inventions relating to the transmission of electrical power from a distance**, for which I hold a number of basic patents."*



*Illustrations from an article in the March 1920 issue of Electrical Experimenter entitled "Wireless Transmission of Power Now Possible". The illustrations show Tesla’s prototype devices for "directed ionized beam transmissions," a "**deathray**—searchlight" device. But* *according to Tesla, the results of tests did not justify the hope of important practical applications in large distances.*

Comment from the author of this book: The same Tesla's invention, when slightly modified and combined with proper geometry of electrodes and well-selected light sources to stimulate Photoelectric and Compton effects, has big chances to produce results expected by Nikola Tesla.

Let us now compare, analogically, dimensionally, and mathematically, essential characteristics, equations, and formulas applicable to a photon (which here we consider as a ***Gaussian-envelope narrow-band wave-packet***, of course without rest mass), with a moving particle that has a non-zero rest mass. The comparison photon-particle will be presented in table T.4.0, based on data from (2.11.3), (4.1), (4.2) and T.1.8. Specific formulas and symbols, found in T.4.0, are introduced there only to make simpler and more indicative (dimensionally correct) mathematical analogies and comparisons, to initiate and support new ideas about wave-particle duality that would appear in this book later. ***It will become apparent, concluding based on analogical comparisons found in T.4.0, and in T.4.0.2, that photon properties can be compared only with motional or kinetic energy states of a particle (or with a particle-equivalent matter-wave packet).*** Rest mass is not the part of matter waves (except in situations with annihilation of matter and antimatter particles, and in certain impact interactions in high-energy particle accelerators when rest masses are being decomposed or annihilated). Similar conclusions can be obtained when we selectively analyze several simple interactions, mentioned earlier (see also in this chapter: "4.1.3.1. Example 2: X-ray Spectrum and Reaction Forces" and see familiar examples from Chapter 4.2).

Since photon has its spin, by analogy, an equivalent matter wave (or motional particle) should also have a spin (because good and complete analogy should work in both directions, considering all photon properties).

Citation from [147] : **Electrons are spin 1/2 charged photons generating the de Broglie wavelength**
Richard Gauthier. Proceedings Volume 9570, The Nature of Light: What are Photons? VI; 95700D (2015). 10 September 2015. https://doi.org/10.1117/12.2180345 Event: SPIE Optical Engineering + Applications, 2015, San Diego, California, United States

Abstract
The Dirac equation electron is modelled as a helically circulating charged photon, with the longitudinal component of the charged photon's velocity equal to the velocity of the electron. The electron's relativistic energy-momentum equation is satisfied by the circulating charged photon. The relativistic momentum of the electron equals the longitudinal component of the momentum of the helically circulating charged photon, while the relativistic energy of the electron equals the energy of the circulating charged photon. The circulating charged photon has a relativistically invariant transverse momentum that generates the z-component of the spin ħ / 2 of a slowly moving electron. The charged photon model of the electron is found to generate the relativistic de Broglie wavelength of the electron. This result strongly reinforces the hypothesis that the electron is a circulating charged photon. Wave-particle duality may be better understood due to the charged photon model—electrons have wavelike properties because they are charged photons. New applications in photonics and electronics may evolve from this new hypothesis about the electron.

INTRODUCTION
In his Nobel Prize lecture Paul Dirac[1] said: "It is found that an electron which seems to us to be moving slowly, must actually have a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us. As a result of this oscillatory motion, the velocity of the electron at any time equals the velocity of light. This is a prediction which cannot be directly verified by experiment, since the frequency of the oscillatory motion is so high, and its amplitude is so small. But one must believe in this consequence of the theory,

since other consequences of the theory which are inseparably bound up with this one, such as the law of scattering of light by an electron, are confirmed by experiment."

One surprising result of the Dirac equation was that the electron moves at the speed of light and has a characteristic associated length $R_o = \hbar / 2mc = 1.93 \times 10^{-13}$m, where $\hbar = h / 2\pi$ and h / mc is the Compton wavelength $\lambda_{Compton} = 2.426\times10^{-12}$m. A second and related surprise was that the electron has a "trembling motion" or *zitterbewegung* in addition to its normal linear motion. Dirac's finding of light-speed for the electron is particularly problematical because electrons are experimentally measured to travel at less than the speed of light. Dirac did not offer a spatially extended model of the electron to correspond to these results, though his results did contain the characteristic length R_o.

Various researchers such as Hestenes[2], Williamson and van der Mark[3], Rivas[4], Hu[5], and Gauthier[6-8] have suggested spatially-extended electron models where the internal energy of the electron circulates at light-speed with the characteristic Dirac radius $R_o = \hbar / 2mc$. Both the Dirac equation's light-speed motion and its *zitterbewegung* are reflected in these models.

Hestenes and Rivas independently analyzed the Dirac equation for spatial and dynamical characteristics of the electron's motion. Based on these analyses, they independently proposed that the trajectory of a moving free electron is a helix along which the electron's charge moves at light-speed. When the linear speed and momentum of the electron are zero, the helix becomes a circle of radius Ro = ħ / 2mc. Neither author associates this circulating light-speed electric charge with a photon.

Another challenging idea regarding the comparison between a photon and a real particle in motion is to show that internal (or intrinsic) particle-wave properties (such as its characteristic wavelength and frequency) are related to its external matter-wave attributes (such as de Broglie wavelength and frequency), originating from certain associated spinning. Here we will exercise the vision that ontologically, particles are electromagnetic fields-folded, self-stabilized standing matter-waves structures, or saying the same more imaginatively and hypothetically, particles (including electrons, protons, and neutrons) are entities assembled from photons (or from electromagnetic wave packets). All of that is also a kind of analogical and indicative conceptualization of **PWDC** (see more about **PWDC** in T.4.0.1, T.4.0.2. from Chapters 4.0 and 4.1, and T.10.00. from Chapter 10), serving here to initiate and stimulate imaginative and creative thinking.

T.4.0. Photon – Particle Analogies

	PHOTON	MOVING PARTICLE		
	Wave-Packet energy-mass-moment forms	State of rest	Corpuscular motional energy-mass forms	Total energy-mass content
Energy	$\tilde{E} = hf$	$E_0 = mc^2$	$E_k = (\gamma - 1)mc^2 = \dfrac{pv}{1+\sqrt{1-v^2/c^2}}$	$E_t = E_0 + E_k = \gamma mc^2$
	$d\tilde{E} = hdf$	$dE_0 = 0$	$dE_k = vdp = hdf = d\tilde{E}$	$dE_t = dE_k = vdp = hdf$
Frequency	$f = \dfrac{\tilde{E}}{h}$ Mean frequency of the photon wave group	$f_0 = \dfrac{E_0}{h} = \dfrac{mc^2}{h}$	$f = \dfrac{E_k}{h} = \dfrac{\tilde{E}}{h} = f_t - f_0 = (\gamma-1)f_0 = \dfrac{\gamma(\Delta f)^*}{\gamma - 1} = \dfrac{c^2}{uv}(\Delta f)^*$	$f_t = f_0 + f = \dfrac{E_t}{h} = \dfrac{\gamma mc^2}{h} = \gamma f_0$

	$df=\frac{d\tilde{E}}{h}$	$df_0=0$	$df=\frac{dE_k}{h}=\frac{d\tilde{E}}{h}$	$df_t=df=$ $=\frac{dE_t}{h}=\frac{dE_k}{h}$
Mass	$\tilde{m}=\frac{\tilde{E}}{c^2}=\frac{hf}{c^2}$	$m=\frac{E_0}{c^2}$	$m_k=\Delta m=\frac{E_k}{c^2}=$ $=(\gamma-1)m=\frac{\tilde{E}}{c^2}=\tilde{m}$	$m_t=\frac{E_t}{c^2}=\gamma m=$ $=m+\Delta m$
	$d\tilde{m}=\frac{d\tilde{E}}{c^2}=\frac{hdf}{c^2}$	$dm=0$	$dm_k=\frac{dE_k}{c^2}=d\tilde{m}$	$dm_t=\frac{dE_t}{c^2}=$ $=\frac{dE_k}{c^2}=dm_k$
	$\frac{\tilde{E}}{\tilde{m}}=c^2$	$\frac{E_0}{m}=\frac{E_k}{\Delta m}=\frac{E_t}{m_t}=\frac{dE_k}{dm_k}=\frac{dE_t}{dm_t}=\frac{d\tilde{E}}{d\tilde{m}}=c^2$		
Linear Momentum & Energy	$\tilde{p}=\frac{\tilde{E}}{c^2}c=\frac{h}{\lambda}=$ $=\frac{hf}{c}=\tilde{m}c$	$p_0=\begin{cases}\frac{E_0}{c^2}v=mv, \text{ or} \\ \frac{h}{\lambda_0}=\frac{E_0}{c}=mc\frac{h}{\lambda_0}= \\ =\frac{E_0}{c}=mc\end{cases}$	$\Delta p=\frac{E_k}{c^2}v=(\Delta m)\cdot v=\frac{h}{\Delta\lambda}=(\gamma-1)mv$	$p=\frac{E_t}{c^2}v=\frac{h}{\lambda}=$ $=\frac{\Delta p}{\gamma-1}=$ $=p_0+\Delta p=$ $=\gamma p_0=\gamma mv$
	$\lambda=\frac{h}{\tilde{p}}=$ $=\frac{h}{\tilde{m}c}=\frac{c}{f}$	$\lambda_0=\frac{h}{p_0}=\frac{h}{mc}=\frac{u_0}{f_0}$	$(\Delta\lambda)^*=\frac{h}{\Delta p}=\frac{h}{(\Delta m)\cdot v}$ $=\frac{\lambda_0}{\gamma-1}=\frac{\lambda\lambda_0}{\Delta\lambda}=(\frac{c}{f})(\frac{c}{v})=$ $=\frac{u}{(\Delta f)^*}=\lambda\frac{f}{(\Delta f)^*}$	$\lambda=\frac{h}{p}=\frac{h}{\gamma mv}=$ $=\frac{\lambda_0}{\gamma}=\frac{u}{f}$
		$\frac{1}{\lambda}=\frac{1}{\lambda_0}+\frac{1}{(\Delta\lambda)^*}$		
	$u=\lambda f=c=v$	$u_0=\lambda_0 f_0=c$	$u^*=(\Delta\lambda)^*(\Delta f)^*=u$	$u=\lambda f$
	$uv=c^2$	$u_0 v_0=c^2$	$uv=c^2(1-\sqrt{1-v^2/c^2})=c^2(1-1/\gamma)\le c^2$ $0\le 0.5v\le u<v\le c$	
	$\tilde{E}=\tilde{m}uv=$ $=\tilde{m}c^2=hf$	$E_0=mu_0v_0=mc^2$	$\tilde{E}=m^*uv=\gamma muv=E_k=$ $=(\gamma-1)mc^2=hf$	$E_t=\gamma mc^2$
	$d\tilde{E}=hdf=$ $=c^2d\tilde{m}=cd\tilde{p}$	$dE_0=0$	$d\tilde{E}=dE_k=dE_t=c^2d\tilde{m}=hdf=vdp$	

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

<table>
<tr>
<td></td>
<td>$\tilde{E} = hf =$
$= \tilde{m}c^2 = \tilde{p}c$
$(c = \lambda f = u = v),$
$\tilde{E} \cdot \Delta t = \tilde{p} \cdot \Delta s = h,$
$u = v = \Delta s / \Delta t = c$
(Photon is Gaussian- or Bell-curve shaped envelope, narrow band wave-group)</td>
<td colspan="3">$p^2c^2 + E_0^2 = E_t^2 = (E_0 + E_k)^2, \quad p = h/\lambda \quad , \quad \lambda = hc/\sqrt{E_t^2 - E_0^2}$
$\gamma = (1 - v^2/c^2)^{-1/2} = (1 - uv/c^2)^{-1}, \quad u = v/(1 + 1/\gamma) = \lambda f$
$(\frac{\gamma - 1}{\gamma})\lambda_0 f_0 = (\Delta\lambda)^*(\Delta f)^* = \lambda f = u \quad , \quad \frac{(\Delta f)^*}{f} = \frac{uv}{c^2} = \frac{\gamma - 1}{\gamma} = \frac{\lambda}{(\Delta\lambda)^*}$
$p\lambda = p_0\lambda_0 = (\Delta p)(\Delta\lambda)^* = h \quad , \quad \frac{p}{p_0} = \frac{\lambda_0}{\lambda} = \gamma \quad ,$
$\Delta\lambda = \lambda_0 - \lambda = \lambda(\gamma - 1) = \frac{\lambda\lambda_0\Delta p}{h} = \frac{h\Delta p}{pp_0} = \lambda\frac{\Delta p}{p_0} = \frac{\lambda\lambda_0}{(\Delta\lambda)^*}$</td>
</tr>
<tr>
<td rowspan="2">Angular Momentum

see (2.9.5)</td>
<td>$\tilde{L} = \frac{\tilde{E}}{\omega} =$
$= \frac{hf}{\omega} = \frac{h}{2\pi}$</td>
<td>$L_0 = \frac{E_0}{\omega} = \frac{h}{2\pi}$</td>
<td>$\Delta L = \frac{E_k}{\omega} = \frac{h}{2\pi}$
$\tilde{E} = \frac{L\omega}{1 + \sqrt{1 - v^2/c^2}}$</td>
<td>$L = \frac{E_t}{\omega} = L_0 + \Delta L =$
$= \frac{c^2}{uv}(\frac{h}{2\pi}) = n(\frac{h}{2\pi})$</td>
</tr>
<tr>
<td>$\frac{\tilde{L}}{\tilde{p}} = \frac{\lambda}{2\pi} = \frac{c}{\omega} = \lambda\frac{\tilde{L}}{h}$
$\tilde{L}\omega = \tilde{p}c$</td>
<td colspan="3">$\frac{L_0}{p_0} = \frac{\Delta L}{\Delta p} = \frac{L}{p} = \lambda\frac{L}{h} = \frac{v}{\omega} \leq \left(\frac{c^2}{v^2} \cdot r^* = \frac{c^2}{\omega^2} \cdot \frac{1}{r^*}\right), \; L\omega = pv$</td>
</tr>
<tr>
<td colspan="5">The meaning of indicative analogies and terms found in this table T.4.0. will be deeply explained and analyzed later: see (4.1) - (4.8). The most important background for conceptual understanding and modeling matter waves, photons and wave-particle duality phenomenology is elaborated in Chapter 10. PARTICLES AND SELF-CLOSED STANDING MATTER WAVES (especially specifically in the “10.00 DEEPER MEANING OF PWDC”). We still need to work on creating better (or more complete) unification of mechanistic, undulatory and wave-particle duality of matter. Presently it looks that all matter forms in our Universe could be synthesized and assembled from photons and electromagnetic-related phenomenology (using as the most important wavefunction modeling the Analytic Signal concept established by Denis Gabor; -see much more about such wave function modeling in Chapter 4.0). This table, T.4.0., of kinematic, wave mechanical, photon-particle dynamic states addressing only motional properties of photons when we like to treat them as moving particles (having only kinetic energy, being without rest mass), also showing analogies and connections of moving particles with matter waves. We also like to know the shape, size, and relations between photons in original spatial-temporal and similar properties in relevant spectral domains. This part of the photon’s nature (and way of modeling) will be addressed much deeper in Chapter 10.</td>
</tr>
</table>

Here is the resume additionally describing photons:

1. Photon is a frequency narrow-band electromagnetic wave packet, composed of simple-harmonic elementary (sinusoidal) electromagnetic waves, concentrated around certain mean frequency f and it has the energy equal to $\tilde{E} = E_p = hf = \tilde{m}c^2$ (h = Planck constant). It is not correct to say that such a photon presents an energy quant. It is also correct and maybe better to define photon energy differentially, such as $d\tilde{E} = dE_p = h \cdot df = c^2 \cdot d\tilde{m} = v \cdot dp$.

2. Photon propagates linearly, while oscillations of electric and magnetic field components of a photon are transversal.

3. Photon has finite energy and finite spatial and temporal durations in both original and conjugate spectral domains, meaning that its amplitude envelopes are Gaussian curves, both in its original and spectral domains (also associating on soliton wavefunctions). Single photon wave function could be formulated as a sinc function in its spatial-temporal domain as, $\overline{\psi}(x,t) = \left|\overline{\psi}(x,t)\right| \frac{\sin(\underline{\Delta\omega}\, t - \underline{\Delta k}\, x)}{(\underline{\Delta\omega}\, t - \underline{\Delta k}\, x)} e^{I(\omega t \pm kx)}$.

4. When photon is propagating trough an isotropic space with dielectric constant ε, and magnetic permeability μ, its propagation, group, and phase speed are mutually equal and constant, and can be calculated as $c = 1/\sqrt{\varepsilon\mu}$. Photon group and phase velocity (v and u) in isotropic and non-dispersive media are always mutually related or connected as, $\mathbf{v = u - \lambda du / d\lambda = -\lambda^2 (df / d\lambda) = c,\ u = \lambda f = c}$, $c = 1/\sqrt{\varepsilon\mu}$.

5. Photon is like a moving particle without rest mass, since it has all mechanical moving-particle properties, such as motional energy $\tilde{E} = E_p = hf = \tilde{m}c^2$, linear momentum $\tilde{p} = hf/c = \tilde{m}c$, and an effective motional mass $\tilde{m} = \tilde{p}/c = hf/c^2$. Photon also has a spinning moment $\tilde{L}_f = \tilde{E} / \omega = hf / 2\pi f = \tilde{m}c^2 / 2\pi f = h / 2\pi$, $\tilde{L}_f / \tilde{p} = c / \omega$.

6. Photon is an energy-finite and spatially-temporally concentrated and limited wave-group within its domains, like real particles are, and interacts (mechanically and electro-mechanically) with other particles. Mechanical moment and mass of a photon changes during interactions with other particles, and motional mass of a photon does not have the property of inertia (see similar situation regarding an equivalence between a spinning object and "***linear-moment thrust force, produced by spinning***", $L\omega = pv$, around equations (2.11-4), in Chapter 2, and in "10.02 Meaning of natural forces", in Chapter 10.).

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Taken from the Internet: http://en.wikipedia.org/wiki/Matter_creation .

"Because of momentum conservation laws, the creation of a pair of fermions (matter particles) out of a single photon cannot occur. However, matter creation is allowed by these laws when in the presence of another particle (another boson, or even a fermion) which can share the primary photon's momentum. Thus, matter can be created out of two photons.

The law of conservation of energy sets minimum photon energy required for the creation of a pair of fermions: this threshold energy must be greater than the total rest energy of the fermions created. To create an electron-positron pair, the total energy of the photons must be at least $2m_ec^2$ = 2 × 0.511 MeV = 1.022 MeV (m_e is the mass of one electron and c is the speed of light in vacuum), an energy value that corresponds to soft gamma-ray photons. The creation of a much more massive pair, like a proton and antiproton, requires photons with an energy of more than 1.88 GeV (hard gamma ray photons).

Lev Landau did first calculations of the rate of e^+-e pair production in the photon-photon collision in 1934.[1] It was predicted that the process of e^+–e^- pair creation (via collisions of photons) dominates in the collision of ultra-relativistic charged *particles—because those photons are radiated in narrow cones along the direction of motion of the original particle greatly increasing photon flux.*

In high-energy particle colliders, matter creation events have yielded a wide variety of exotic massive particles precipitating out of colliding photon jets (see two-photon physics). Currently, two-photon physics studies the creation of various fermion pairs both theoretically and experimentally (using particle accelerators, air showers, radioactive isotopes, etc.).

As shown above, to produce ordinary baryonic matter out of a photon gas, this gas must not only have a very high photon density but also be very hot – the energy (temperature) of photons must exceed the rest mass energy of the given matter particle pair. The threshold temperature for production of electrons is about 10^{10} K, 10^{13} K for protons and neutrons, etc. According to the Big Bang theory, in the early universe, photons and fermions (massive particles of matter) would inter-convert freely. As photon gas expanded and cooled, some fermions would be left over (in tiny amounts ~10^{-10}) because low energy photons could no longer break them apart. Those leftover fermions would have become the matter we see today in the universe around us".

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This is a good opportunity to emphasize that photons, electrons, and positrons share a fundamental electromagnetic-based similarity and complement each other in several keyways:

- Photon-Electron Interactions: Photons absorbed by an atom can move electrons or electron wave groups within the atom, altering the energy states of electrons. In phenomena like the photoelectric effect, Compton scattering, and secondary emission, photons directly interact with electrons and positrons, indicating that electrons may behave as specific electromagnetic wave groups. The absorption and emission of photons accompany the energy transitions of electrons within atoms.

- Photon Conversion: A sufficiently high-energy photon, when passing near an atom, can be converted into an electron-positron pair. Similarly, collisions between high-energy photons can produce matter-wave energy-momentum states, including elementary particles and quasi-particles. These effects are facilitated by more complex interactions involving the spin and torsional effects of local electromagnetic fields.

- Synchrotron Radiation: Spinning and spiraling electrons in strong magnetic fields generate synchrotron radiation, emitting gamma photons. This implies that electrons possess a structured photonic energy content.

- Bremsstrahlung Radiation: When electrons pass near another electrically charged particle, they emit gamma photons through a process known as bremsstrahlung. This occurs as electrons lose energy, suggesting that photons are inherently part of the electron's energy content and are naturally emitted when conditions are met.

- Inverse Compton Effect: The inverse Compton effect occurs when a low-energy photon collides with a high-energy electron, producing gamma photons. This demonstrates the natural interactions and recombination between photons, electrons, and positrons.

- Electron-Positron Annihilation: When an electron and positron collide, they annihilate each other, producing high-energy photons and various matter-wave energy-momentum states with linear and angular momentum properties. The kinetic energy of the particles before the collision contributes to the energy of the photons produced.

- Spin Characteristics: All these complementary particles, photons, electrons, and positrons, exhibit spin, along with other particles such as protons and neutrons, as well as various energy-momentum states recognized in microphysics.

- Wave-Particle Duality: Electrons, positrons, and photons are examples of wave packets or wave groups, behaving either as particles or waves depending on how they are used, observed, or detected. This duality is likely applicable to protons and neutrons as well. Notably, neutrons are considered (though still officially unrecognized) as a tightly bound electron-proton pair. This idea is fiercely rejected by proponents of contemporary quantum theory.

- Bragg Scattering: Bragg scattering provides evidence of analogous matter-wave behavior between electrons and neutrons, as de Broglie wavelengths and Bragg's diffraction formula are successfully applied in both cases.

- Electron Microscopy: The electron microscope serves as a powerful example of how high-speed electrons, functioning as de Broglie matter waves or photon-like beams, achieve short wavelengths (much shorter than those in light-based optical microscopes), offering exceptional magnification and resolution. This confirms, in a sense, that electrons are uniquely structured or "packaged" photons.

Electron microscope is an excellent example or direct confirmation of Wave-Particle Duality. We initially consider electrons as particles, and such accelerated particles should have matter waves, or de Broglie waves properties. Since we know that electron microscopes are perfectly working, producing results of magnification as any other optical or photon-microscope, but with enormously higher resolution, we can conclude that accelerated electrons are behaving as waves or photons with noticeably short wavelengths, respecting de Broglie wavelength formula.

Electron microscope: *Citation from Wikipedia: https://en.wikipedia.org/wiki/Electron_microscope . Electron microscope constructed by Ernst Ruska in 1933.*

*An **electron microscope** is a microscope that uses a beam of accelerated electrons as a source of illumination. As the wavelength of an electron can be up to 100,000 times shorter than that of visible light photons, electron microscopes have a higher resolving power than light microscopes and can reveal the structure of smaller objects. A scanning transmission electron microscope has achieved better than 50 pm resolution in annular dark-field imaging mode[1] and magnifications of up to about 10,000,000× whereas most light microscopes are limited by diffraction to about 200 nm resolution and useful magnifications below 2000×.*
Electron microscopes use shaped magnetic fields to form electron optical lens systems that are analogous to the glass lenses of an optical light microscope.
Electron microscopes are used to investigate the ultrastructure of a wide range of biological and inorganic specimens including microorganisms, cells, large molecules, biopsy samples, metals, and crystals. Industrially, electron microscopes are often used for quality control and failure analysis. Modern electron microscopes produce electron micrographs using specialized digital cameras and frame grabbers to capture the images.

- Short-lived and Transient Particles: Most exotic particles known from the Standard Model of Quantum Theory and from high-energy impact reactions are short-lived, unstable, and transient. These particles are dualistic matter-wave packets or quasi-particles, sometimes existing as conditional, mathematical constructs rather than directly observable entities (quarks, for instance). Further arguments regarding wave-particle duality, matter waves, and photons are discussed in the Appendix (Chapter 10: *Particles and Self-Closed Standing Matter Waves*, particularly in “10.00: Deeper Meaning of PWDC”).

- Photon Structure and Electron-Positron Creation: Photons are specific combinations of rotating electric and magnetic field vectors, which are mutually orthogonal and also orthogonal to the direction of photon propagation. When a photon is influenced by external electric or magnetic fields, it can transform into an electron-positron pair, suggesting that electrons and positrons are also specific electromagnetic field configurations, like real and imaginary part of complex analytic signals. This idea, however, is still not recognized in modern Quantum Theory.

- Proton Structure: Protons, like electrons and positrons, likely possess an internal electromagnetic field structure. All elementary particles exhibit dualistic wave-particle behavior, although photons have no rest mass. In contrast, electrons, positrons, protons, and neutrons have effective rest masses due to their total electromagnetic energy content and internal structure. The way electric and magnetic field vectors are coupled and packed inside these particles (and potentially inside atoms and molecules) gives rise to the property we describe as rest mass.

- Material Analysis through Electromagnetic Emissions: Modern material analysis, using techniques like electromagnetic emissions from electric discharges or high-temperature excitation, reveals that each atom or molecule has a unique, repeatable spectral content. This indicates that atomic and molecular structures are unique combinations of electromechanical elements with resonant or oscillatory matter-wave states and harmonics. The resonant nature of these states suggests that they are fundamentally electromagnetic due to the involvement of photons and charged particles. These states represent dualistic wave-particle objects with motional properties, supporting the concept of self-closed, standing matter-wave formations, which underline modern microphysics and quantum theory. Historical grounds for these concepts were initially laid by Rudjer Boskovic (see Chapter 8 for further elaboration).

- Light-by-Light Scattering: As noted in quantum electrodynamics (QED), light-by-light scattering is a rare phenomenon where two photons interact to produce another pair of photons, a process forbidden by classical physics (such as Maxwell's electrodynamics). This phenomenon was experimentally confirmed during the Large Hadron Collider's (LHC) second data run. The lead ion collisions at the LHC provided a unique environment to observe this interaction, supporting QED predictions. [Cited from CERN] (https://home.cern/news/news/physics/atlas-observes-light-scattering-light).

- Spatial-Temporal Nature of Photons and Particles: It is important to remember that photons and elementary particles are spatial-temporal formations of standing matter waves (or electromagnetic waves). These waves are velocity-dependent, meaning that scattering, diffraction, superposition, and interference of these objects depend on their wave-packet geometry, duration, and spatial environment.

- Electromagnetic Analogies Between Particles: There is a deep electromagnetic analogy between photons, electrons, positrons, and other motional particles. These particles share common properties that apply to both wave and particle motion. Only the moving or kinetic energy of a particle fully corresponds to the creation of an appropriate matter-wave packet. Modern quantum theory, however, includes the energy content of rest masses, which this perspective does not. From a mathematical standpoint, matter waves should exhibit finite spatial, temporal, and frequency localizations, often represented by Gaussian wave packets or pulses. These can be processed using windowed Fourier or Gabor transforms (see [79] for more details).

- Conclusion: All matter is composed of resonant or oscillatory, vibrating and spinning matter-wave states, forming the building blocks of elementary particles. This approach is like string theory concepts, though in this case, elementary particles are seen as specific combinations of the simplest resonating strings.

Citation from https://en.wikipedia.org/wiki/String_theory. In physics, ***string theory*** *is a theoretical framework in which the point-like particles of particle physics are replaced by one-dimensional objects called strings. It describes how these strings propagate through space and interact with each other. On distance scales larger than the string scale, a string looks just like an ordinary particle, with its mass, charge, and other properties determined by the vibrational state of the string. In string theory, one of the many vibrational states of the string corresponds to the graviton, a quantum mechanical particle that carries gravitational force. Thus string theory is a theory of quantum gravity.*

String theory is a broad and varied subject that attempts to address a number of deep questions of fundamental physics. String theory has been applied to a variety of problems in black hole physics, early universe cosmology, nuclear physics, and condensed matter physics, and it has stimulated a number of major developments in pure mathematics. Because string theory potentially provides a unified description of gravity and particle physics, it is a candidate for a theory of everything, a self-contained mathematical model that describes all fundamental forces and forms of matter. Despite much work on these problems, it is not known to what extent string theory describes the real world or how much freedom the theory allows in the choice of its details.

String theory was first studied in the late 1960s as a theory of the strong nuclear force, before being abandoned in favor of quantum chromodynamics. Subsequently, it was realized that the very properties that made string theory unsuitable as a theory of nuclear physics made it a promising candidate for a quantum theory of gravity. The earliest version of string theory, bosonic string theory, incorporated only the class of particles known as bosons. It later developed into superstring theory, which posits a connection called supersymmetry between bosons and the class of particles called fermions. Five consistent versions of superstring theory were developed before it was conjectured in the mid-1990s that they were all different limiting cases of a single theory in eleven dimensions known as M-theory. In late 1997, theorists discovered an important relationship called the AdS/CFT correspondence, which relates string theory to another type of physical theory called a quantum field theory.

Let us now creatively address "double-slit diffraction" experiments that are too often being taken as a proof or strongest argument confirming probabilistic and wave-particle duality nature of photons and other elementary and micro particles, including atoms and molecules.

<u>Explanations of double-slit diffraction experiments</u> from the point of view favored in this book:

Citation (lit. [124]): "The famous double-slit experiment, different versions of which we will come back to in the course of the thesis, serves as the <u>canonical illustration of the wave-particle duality</u>. Let us here present its simplified description: A light beam emerges from a source, propagates through two very small slits and impinges on a screen. We can reduce the intensity of the beam in such a way that according to a standard concept of quantum mechanics there will be only one quantum of light (photon) present in the apparatus at any given time. If we now place a detector behind each slit, we will see that they do not respond simultaneously, and thus we will be led to the conclusion that the photons behave like tiny corpuscles moving through either the first or the second slit. However, if we choose not to disturb the light with measurement before it reaches the screen, an interference pattern will emerge on it. This pattern is most easily predicted and explained by claiming that light is in fact an electromagnetic wave. The double-slit experiment can be also conducted with electrons (or other material particles) instead of light, and the same conclusions would be reached. In the words of Richard Feynman, this extraordinary phenomenon "is impossible, absolutely impossible, to explain in any classical way, and (...) has in it the heart of quantum mechanics. In reality, it contains the only mystery" [4]. Thus, claimed Feynman, the wave-particle duality problem is one of the central features of quantum mechanics".

We start with clarifications and characterization regarding most frequent experimental participants and practices of double-slit experiments, gradually introducing new, imaginative, hypothetical, and other still not well exploited options. The intention of the author here is to rectify and modify present interpretations of wave-particle duality.

1. Photon is always and only a limited-energy, narrow-frequency-band, Gaussian or Bell-curve-envelope, electromagnetic wave-group, or wave-packet, agglomerated around certain mean frequency (in all its joint temporal-spatial, original, and spectral domains). In other words, photon presents limited-bandwidth superposition of elementary sinusoidal, electromagnetic wave elements (as known from spectral measurements and from "Kotelnikov-Shannon-Whittaker-Nyquist-Fourier-Dennis Gabor" signal analysis and signals reconstruction). For such photon we can say that it has certain absolute and definite time duration T, certain spatial length, width or duration L, certain total, motional or wave energy $\tilde{\mathrm{E}}$, and linear and angular moments $\tilde{\mathrm{P}}$ and $\tilde{\mathrm{L}}$. Relation between all total (and absolute) domains-durations of a photon is $\mathrm{T} \cdot \tilde{\mathrm{E}} = \mathrm{L} \cdot \tilde{\mathrm{P}}$ (see more in Chapter 10.). Analogically, and respecting de Broglie matter waves concept regarding wave-particle duality, we can give the same description for a moving particle, electrons, protons, atoms, and molecules (which are also narrow frequency band superpositions of elementary sinusoidal matter-wave components). On the similar way as a total photon-energy $\tilde{\mathrm{E}}$ can be presented as superposition of energies of sinusoidal or "sinc functions" of time-dependent wave components, its total momentum $\tilde{\mathrm{P}}$ can be presented as a superposition of elementary moments being sinusoidal wave components dependent of relevant spatial coordinate x, (as for example,

$$\tilde{\mathbf{E}} = \int_{[T]} \mathbf{d}\tilde{\mathbf{E}} = \int_{[T]} \psi^2(\mathbf{t}) \cdot \mathbf{dt} = \int_{[T]} \tilde{\mathbf{P}}(\mathbf{t}) \cdot \mathbf{dt}, \quad \tilde{\mathbf{P}} = \int_{[T]} \mathbf{d}\tilde{\mathbf{p}} = \int_{[L]} \frac{\mathbf{1}}{\mathbf{v}} \mathbf{d}\tilde{\mathbf{E}} = \int_{[L]} \frac{\psi^2(\mathbf{x})}{\mathbf{v}^2} \cdot \mathbf{dx} = \frac{\mathbf{d}\tilde{\mathbf{E}}}{\mathbf{dt}},$$

$$\psi(x,t) = a(x,t) \frac{\sin(\underline{\Delta\omega}\ t \pm \underline{\Delta k}\, x)}{(\underline{\Delta\omega}\ t \pm \underline{\Delta k}\, x)} \cos(\omega t \pm kx)).$$

2. The same spectral, geometrical, or spatial relationships seen in time-domain phenomena apply to single, double, or multiple slits diffraction and interference experiments. Similar wave patterns, sinusoidal components, and spatial periodicity can be associated with the geometry of diffraction plates. In other words, the temporal periodicity of incident particles like photons, electrons, or atoms, interacts with the spatial periodicity of stationary diffraction plates, as the temporal and spatial domains are inherently coupled, phase-shifted, and mutually transformable (see Chapter 10 for more on spatial-temporal proportionality). Double or multiple slits diffraction presents an interference between temporal diffraction of its (time dependent) sinusoidal components, and spatial diffraction of its (spatial or distance dependent) sinusoidal components created by geometry of the involved interaction participants.

In essence, temporal and spatial periodicities (or spectral signatures) are always interconnected and synchronously present with spectral characteristics in both domains. When particles, such as electrons or photons, are directed toward a double-slit diffraction plate, the plate's spatial spectral components are activated, generating new matter waves that interfere with the initial beams. This entire setup, source, diffraction plate, back screen, and everything else, is immersed in a material-fluid or ether (whatever that may be).

For a photon traveling in the x-direction, temporal-spatial coupling can be described by a certain proportionality as $dx = c \cdot dt$. Furthermore, we should not overlook the possible "extended meaning of entanglement" and universal resonant synchronization between incident particles (photons, electrons, etc.) and the surrounding environment, including the diffraction plate. Resonant synchronization and coupling between the spectral components of the incident particles and the spatial resonant states of the diffraction plate may play a key role (see Chapter 10).

Orthodox Quantum Theory tends to oversimplify the double-slit experiment. For example, when there is no observer, the diffraction pattern behaves as expected for typical wave diffraction. However, when an observer or detector is introduced, the diffraction pattern disappears, leaving only particle-like imprints on the back screen. This happens because the observer, a physical body with mass and geometry, disrupts the wave-particle system, altering its spatial and spectral patterns.

It's important to note that the involved wave groups, such as photons or electrons, often behave like Gaussian or Bell-curve envelopes, meaning they are well-defined and finite in both time and spatial domains. Such matter waves should propagate through a material medium, even if we don't fully understand its nature yet.

For instance, an absolute vacuum still has dielectric and electromagnetic properties that define the maximum speed of light, c. The seemingly "magical" wavefunction collapse in quantum mechanics is simply a result of energy and momentum conservation laws. However, Orthodox Quantum Theory doesn't consider the possibility of spontaneous communication and synchronization between elements of condensed matter, resulting in phenomena that appear mysterious but are explainable within the framework of fluid dynamics (see Chapter 10).

3. When a photon, electron, or atom interacts with a static object (like a narrow-slits plate) whose physical size exceeds the photon's wavelength, the particle tends to behave more like a classical particle when passing through the slit.

4. Any particle, such as a photon or electron, traveling towards a physical object like a double-slit plate becomes part of a "two-body system." This interaction creates mechanical, electromechanical, and electromagnetic field relations, and the electromagnetic coupling between the particle and the slit plate must be considered. As the particle approaches the back screen, a center-of-mass system emerges that becomes more relevant than the observer's laboratory frame of reference. This center-of-mass system, defined both mechanically and electromagnetically, evolves as the particle moves, leading to interference or superposition effects on the back screen.

   We should always consider energy flow as part of a larger, closed system, with source and load elements (as explained in Chapter 1). Open-ended, unclosed circuits will not fully explain the results of diffraction experiments. Additionally, if we acknowledge the existence of a material fluid or ether that carries electromagnetic and matter waves, the explanation for diffraction phenomena becomes more deterministic and logical, resembling wave behavior in fluids.

5. Most double-slit experiments have been performed using photons, electrons, atoms, molecules, or nanoparticles. Photons are electromagnetic wave packets, while electrons are structured electromagnetic entities. Both exhibit wave-particle duality, as do atoms, which are composed of protons, neutrons, and electrons. Neutrons are essentially made of an electron and a proton, and protons themselves may be structured standing waves. Thus, moving atoms could create dipole-structured charges, contributing to wave-particle behavior similar to waves

in fluids. When analyzing diffraction, we should also account for external electromagnetic forces, cosmic rays, and neutrinos, as they influence dualistic matter-wave interactions.

6. If we detect a photon just after it passes through a narrow slit, using a sensor or detector much larger than the photon's wavelength, the photon will register as a particle. This is because photons are Gaussian or Bell-curve wave packets, finite in both time and space.

7. Every wave packet has a phase characteristic dependent on spatial and temporal coordinates. Thus, the phase of a photon when passing through a slit will influence interference patterns observed on the back screen. Even photons from the same source will have different phase values upon arrival, creating oscillatory interference patterns over time. Current Quantum Theory doesn't fully utilize Complex Analytic Signal modeling for wavefunctions, which accounts for the phase of wave motions, unlike the traditional probability wave function. In contrast, the Analytic Signal model includes both real and imaginary components, supporting more comprehensive wave behavior.

8. In the absence of a detector, the system will exhibit the wave-packet properties of the reduced mass of the photon, leading to the emergence of interference patterns on the back screen. The photon's interaction with the diffraction plate, through its modulated central mass, creates favorable conditions for interference effects.

9. The two-slit diffraction experiment involving a single particle, such as a photon or electron, exemplifies de Broglie matter waves. The particle passes through one slit, while the interaction field and associated matter waves span both slits. This is like the pilot-wave model developed by Louis de Broglie and expanded by David Bohm, but here effectively very familiar with Complex Analytic Signal, matter-wave modeling (see more in chapters 4.3 and 10).

Citation from Pilot wave theory - Wikipedia: … "Its more modern version, the de Broglie–Bohm theory, interprets quantum mechanics as a deterministic theory, avoiding troublesome notions such as wave–particle duality, instantaneous wave function collapse, and the paradox of Schrödinger's cat. To solve these problems, the theory is inherently nonlocal. The de Broglie–Bohm pilot wave theory is one of several interpretations of (non-relativistic) quantum mechanics".

10. Electromagnetic Nature and Diffraction Interactions

The electromagnetic nature of particles and their time-space-dependent phase functions continuously evolve as part of two-body systems. This evolution causes a single particle, or its spectral and radiative energy components, to interact with both diffraction slits (or holes) well before it reaches the opposite side of the plate. Once the particle reaches the screen, the interaction field (or etheric oscillations) guides and influences the particle's spatial-temporal phase. This is due to the inherent temporal-spatial proportionality and the conservation of energy and momentum within the two-body system.

In diffraction experiments, we observe at least a two- or three-body interaction: the incident particle, initially acting independently, gradually becomes part of a larger system involving the diffracting plate. As this interaction develops, the system's center of mass and reduced mass takes on increasing importance in the near-field interaction region, where synchronization effects dominate. Thus, the stochastic behavior of particle distributions in diffraction experiments arises from the interaction fields created between the particle and the diffraction plate, long before, and continuing after the actual diffraction event. These fields exhibit axial and rotational components, manifesting naturally in the etheric fluid, which behaves as a medium for electromagnetic wave propagation.

11. Multi-component Dipole Polarization

During these interactions, a moving particle generates "multi-component electromagnetic dipole-like polarizations." In a state of rest, a particle's various properties (center of mass, electric charge, orbital moments, etc.) are stable. However, when a particle accelerates, it experiences dynamic dipole-like forces that interact with the environment. This interaction creates new de Broglie matter waves, rooted in extended Newton-Coulomb force laws (referenced in equations 2.3, 2.4-3, and 2.9 in Chapter 2).

Moreover, spatial periodicity and atomization (standing-wave quantization) naturally occur in relation to both linear and angular motions, along with their associated electromagnetic fields. In some cases, moving microparticles cannot freely select any angular position in front of them, due to these standing matter-waves.

12. Quantum Mechanics and Wave-particle Interactions

Quantum mechanics provides a statistical approach to modeling phase-unpredictable interactions, using probability theory to predict the distribution of particles over time ($-\pi \leq \varphi(t, x) \leq +\pi$). Though mathematically effective, this model sacrifices the real-time spatial-temporal identity of interaction participants. In truth, knowing all essential elements of a wave motion should allow us to predict when, where, and how particle interactions will occur (see equations 4.42-4.45).

If we accept that the entire diffraction setup (source, diffraction plate, and screen) is immersed in a fine material fluid or ether (as proposed by Nikola Tesla), the explanation of diffraction patterns becomes more deterministic and logical. The ether connects all interacting entities, causing real-time agitation of diffraction slits by wave packets and particles. This means there's no need to imagine a single electron or photon passing through multiple slits simultaneously.

13. Wave-particle Duality in Double-slit Experiments

The dual nature of photons is well-established through both experimental and theoretical evidence: photons exhibit both wave and particle characteristics. Their Gaussian or Bell curve-shaped envelopes confirm that they are narrow-band, limited-duration wave packets (see Chapter 10 for further details).

When substituting photons with electrons, protons, neutrons, or atoms, we observe similar dual behavior, because these particles also consist of electromagnetic energy and exhibit matter-wave characteristics. Although heavier particles behave more like particles than waves, the logic behind interference and diffraction effects remains consistent.

The confusion surrounding wave-particle duality arises from oversimplified models and a failure to fully conceptualize the nature of particles as narrow-band electromagnetic wave packets. Under certain experimental conditions, photons or electrons may behave more like particles, and under others, more like waves. The entire setup is submerged in an ether-like fluid, which acts as the medium for electromagnetic and matter waves. Incident particles continuously generate secondary waves, synchronously agitating the diffraction slits, leading to deterministic, logical diffraction results.

This eliminates the need for probabilistic interpretations, such as wavefunction collapse, commonly used in modern quantum mechanics.

- **Summary of Key Concepts**

1. Electromagnetic Interactions: Particles interact with diffraction slits through electromagnetic fields long before passing through them, influencing their phase and motion.

2. Multi-component Dipole Polarization: Moving particles generate dynamic electromagnetic dipoles that interact with the environment and produce new matter-waves.

3. Quantum Mechanics' Statistical Approach: The current quantum model uses probability to predict particle distributions over time, sacrificing real-time interaction identity, or relevant phase functions.

4. Wave-particle Duality: Photons, electrons, and other particles exhibit both wave and particle characteristics, with diffraction experiments confirming this behavior.

5. Ether and Diffraction: A material ether or fluid provides a natural medium for electromagnetic waves, making diffraction effects more deterministic and understandable without needing complex quantum interpretations.

There is another, very new or emerging methodology with innovative experimental practices, originally addressing double slit diffraction and interference experiments (see [154], Dr. Hui Peng), where we can find clarifications showing that traditional or old Orthodox Quantum theory, with purely probabilistic double slit explanations, is going to be updated with new experimentally confirmable situations, or even to be replaced with better, more tangible, and much more deterministic explanations.

The slow erosion of the probabilities related interpretation of Quantum theory can also be found in [155]. See the citation below,

Citation: "The probabilities in quantum mechanics arise only for the observer's perspective due to the nature of doing experiments but they do not exist in Nature. In other words, quantum mechanics itself is not a probabilistic theory. This viewpoint is proved by deriving the Born's rule by showing it to be equal to the experimentally observed relative frequencies. Also, it is shown that the classical and quantum mechanical times are one and the same. By identifying the al- ready inherently existing mechanism for the collapse of wave function within the quantum formalism, the present interpretation naturally proves why the Copenhagen interpretation is very successful in explaining the experimental outcomes. Young's double-slit experiment, the Wheeler's delayed choice experiment and the Afshar's experiment are unambiguously explained at a single quantum level".

Most analyses and experiments related to double or multiple slit diffraction (as seen in various publications) typically involve photons and electrons. In these cases, electrons are often considered as ordinary particles with non-zero rest mass, which naturally exhibit wave-like behavior in scenarios such as diffraction, superposition, interference, impacts, and scattering. However, the first and most significant conceptual error in this view is the treatment of the electron as an ordinary particle. An electron is not simply a particle but an active electromagnetic resonator or standing-wave structure. It can be thought of as a toroidal, disk-like, or ring-shaped body composed of electromagnetic waves. Essentially, the electron is a matter-wave formation built from photons, and it naturally has multiple resonant frequencies and spectral characteristics.

This understanding brings us back to the discussions and conclusions regarding single-photon diffraction, which also involves synchronization and entanglement effects between resonators with similar or identical frequencies. The effects of extended entanglement and resonant synchronization become clearer when we model matter waves using complex analytic signal functions. The same principles apply to more complex microparticles such as atoms and molecules, which, although externally electromagnetically neutral, still contain internal electromagnetic structures and couplings.

When particles are in motion, particularly when emitted toward diffraction slits, they can generate electric and magnetic dipole polarization, resulting in spatially separated electromagnetic resonators, such as electrons and protons. These natural resonators follow the classical second-order partial differential wave equation, and their excited states always produce pairs of coupled waves propagating in opposite directions. The analytic signal wavefunction model, which describes these states, consists of two wavefunctions that are phase-shifted relative to one another. This complexity adds to the spatial-temporal evolution of interactions and enriches the two-body problem with entanglement and resonance, leading to unusual and fascinating diffraction effects.

The peculiar, stochastic behavior observed in diffraction and interference patterns involving photons and microparticles has led quantum mechanics to focus on the probabilistic nature of matter waves. However, the underlying reason for these statistically predictable wave-like patterns, such as the positions of particles on a screen, lies in the interaction between rectilinear motion and mass-spinning, which creates matter waves. This interaction, which begins long before a particle reaches the diffraction slits, is akin to the pilot-wave theory proposed by Louis de Broglie and later expanded by David Bohm into hidden variable theory. These interactions involve coupling and guiding effects between particles and the diffraction slits, additionally driven by resonant synchronization and entanglement effects. By considering the presence of a revitalized ether or fluid, the creation of matter waves becomes more evident, and it is even possible to hypothesize the involvement of higher dimensions in the formation of these waves. ***Pilot-wave modeling is very close to Analytic Signal or matter-wave modeling, especially interesting since Schrödinger equation should not be artificially postulated. It can be simply and smoothly developed and united with universally valid Classical, second order, partial, differential wave equation (see more in chapters 4.3 and 10).***

As a particle's wave phase (–π ≤ φ(t, x) ≤ +π) passes through the diffraction slits, the resulting stochastic and wave-like scattering patterns appear on the back screen. These patterns, characterized by concentric circles, black and white zones, or other periodic shapes, emerge after a significant number of particles have passed through the slits. Because we cannot directly measure the local time-domain wave-phase of a particle, it initially appears that the particle randomly chooses its destination on the diffraction screen. This assumption of randomness, combined with the omission of spinning field components in quantum mechanical models, led to the establishment of probabilistic interpretations in Orthodox Quantum Theory (QT). The theory mathematically compensates for this missing knowledge about immediate motional field-phase components, which helps explain the accuracy of its predictions, even though it may be incomplete.

Orthodox QT's success in predicting and modeling experimental results has resulted in a rigid adherence to its probabilistic foundations. This has influenced how scientists analyze diffraction experiments, with the interpretation being limited to probabilistic outcomes rather than considering the possibility of deterministic underlying mechanisms. Many interpretations also invoke simplified and misleading experimental setups, involving overly simplistic models of the participants as independent, stable entities. Moreover, the widespread use of metaphors like "Bob and Alice" in these scenarios further detracts from the underlying physics, replacing proper wavefunction models with discussions about imaginary communications between observers. This tendency, compounded by an over-reliance on probabilistic and statistical explanations, has obscured deeper physical principles like entanglement, resonant synchronization, and the possible role of an ether.

Despite its success, contemporary quantum theory represents a quiet victory for abstract, game-theory-like concepts that unintentionally mix metaphysical ideas with physical ones, often leading to interpretations that defy causality and objectivity. In Western scientific thought, particularly, there seems to be an implicit acceptance of randomness and miracles, concepts that have no place in natural sciences grounded in determinism. Many scholars, philosophers, and other thinkers have embraced these ideas, leading to the publication of numerous works that mistakenly blend physics with mysticism under the guise of objective science.

However, the impressive mathematical tools used within quantum theory, such as Parseval's theorem and probabilistic models, have made the theory highly effective in producing experimentally verifiable results. Yet this mathematical success should not blind us to the theory's philosophical shortcomings. By reintroducing concepts such as ether and reconsidering wave-particle interactions in a deterministic, causal framework, we can achieve a clearer and more comprehensive understanding of diffraction and interference phenomena.

In this book, we present an alternative framework that moves away from the ontological probability assumptions of Orthodox QT. Our approach is based on solid principles of physics, conservation laws, and healthy mathematical models grounded in physical reality. The participants in double-slit experiments, for example, are seen not as independent, isolated entities, but as interconnected objects whose spatial, temporal, and spectral properties evolve during the interaction. This perspective offers a more deterministic and causal explanation of the experimental results, rooted in the deeper connections between the entities involved and their environment.

*Citation from Wikipedia: "Double Slit experiment as (probabilistically) explained in the Orthodox Quantum Theory (**OQT**) still presents the crucial foundation and experimental proof for establishing probabilistic interpretation of **OQT**. Based on available facts, way of imposed presentation, and on the consensus (by voting) of involved founders, it has been accepted and postulated that Double slit experiment presents the grounds of **OQT**. Since that time, too many of devoted followers of **OQT** are defending such postulations by any means. This is like Virgin Mary dogma regarding foundations of Christianity, meaning and dictating that real believers should accept (without doubts and posing questions) offered explanations about the Virgin Mary. It is a time to review and update (in fact abandon and scientifically replace) such practices (Read Citation from https://en.wikipedia.org/wiki/Mary,_mother_of_Jesus: "… Both the New Testament[c] and the Quran describe Mary as a virgin. According to Christian theology, Mary conceived Jesus through the Holy Spirit while still a virgin, and accompanied Joseph to Bethlehem, where Jesus was born.[7] …").*

This book offers another, more deterministic and more scientifically founded, ontologically non-probabilistic way regarding Wave-Particle Dualism and Quantum Theory foundations (see more in Chapter 10.).

On a similar way, we could modify or update our understanding of **Photoelectric effect**.

*Citation (taken from https://en.wikipedia.org/wiki/Photoelectric_effect): "The **photoelectric effect** is the emission of electrons when electromagnetic radiation, such as light, hits a material. Electrons emitted in this manner are called photoelectrons. The phenomenon is studied in condensed matter physics, solid state, and quantum chemistry to draw inferences about the properties of atoms, molecules and solids. The effect has found use in electronic devices specialized for light detection and precisely timed electron emission".*

Photoelectric Effect Overview:

1. *First, we acknowledge the well-established, mainstream explanation of the photoelectric effect, as developed by Mileva Maric, A. Einstein, Max Planck, and L. de Broglie. These foundations provide practical and reliable insights, which we will not challenge here.*

2. ***Modeling the Internal Structure of Metals and Materials***
Next, let's imagine that metals (or other materials) capable of emitting photoelectrons when exposed to energetic photons are internally structured as spatially distributed sets of electromagnetic or electromechanical resonators, each covering certain frequency bands. In atomic models, these resonators are typically referred to as electron clouds, electron states, or electron orbitals. These "electron resonators" generate specific electromagnetic wave patterns.

   In addition to electron resonators, atoms and materials also contain another set of resonators with opposite electric-charge polarity, representing the atomic nucleus. These nucleus resonators operate at significantly higher frequencies or energy ranges compared to electron resonators.

3. ***Energy Binding in Resonators***
Both the electron and nucleus resonators are bound together by a certain amount of electromagnetic energy, which keeps these resonators spatially distributed in defined electron states and nucleus states. This energy plays a critical role in maintaining the structure of the atom.

4. ***Interaction with Low-Energy Photons***
When low-energy photons strike these electromagnetic resonators, if the photon energy (or frequency) is lower than the range of the internal electron states, no excitation occurs, and no photoelectrons are emitted.

5. ***Threshold for Photoelectron Emission***
However, when incident photons have frequencies above a certain threshold, within the frequency range of the internal electron states, photoelectron emission begins. This indicates that electron states are essentially electromagnetic wave formations (or specific photon configurations). When these resonators absorb an incident photon, they become electromagnetically excited or perturbed. After a brief period, the resonator stabilizes by expelling an electron. Mileva Maric and A. Einstein already addressed the energy and momentum balance involved in this interaction.

   Electromagnetic and electromechanical resonators, including mechanical ones, are most sensitive to excitement at their natural resonant frequencies. Since the frequencies of the nucleus states are much higher, it is unlikely that the photoelectric effect can occur via interactions with nucleus states.

6. ***Analogy to Double-Slit Experiments***
This situation is somewhat analogous to the double-slit experiment. When incident photons', matter-waves, or particles have a lower de Broglie frequency (longer wavelengths and durations) relative to the natural resonant frequency or mean wavelength of the interacting object, the interaction will predominantly exhibit wave-like behavior.

   Conversely, when the incident matter-waves (such as photons, electrons, or other particles) have higher frequencies, these wave packets tend to exhibit particle-like behavior. Such packets are characterized by mean frequency, mean wavelength, and total temporal and spatial durations.

7. ***Interaction Based on Resonant Properties***
The interacting object, whether it has internal resonant properties or is simply a static body with a specific geometry (such as a crystalline structure), has a spatial spectrum with distinct periodicities, which correspond to certain distances or spatial frequencies, like the spacing between atoms in a crystal lattice. In these cases, the mean wavelength of the incident wave packet is compared with the spatial periodicities of the interacting object.

If the incident wave packet has a mean wavelength larger than the mean spatial periodicity of the interacting object, it behaves primarily as a wave. If the wavelength is much shorter than the periodicities or size of the object, it behaves more like a particle (as discussed in Chapter 10).

8. ***No Mystery in Duality***
There is no inherent mystery here, except in cases where the wavelengths and sizes of the interacting entities are similar, in which mixed or ambiguous effects may occur. This simple relationship between wave-particle behavior and the resonance or geometry of interacting objects clarifies the nature of such phenomena.

The unity of corpuscular and undulatory or matter-waves nature of moving objects, rather than mutual exclusivity, is already becoming an experimentally confirmed fact (contrary to the position of the contemporary Quantum theory.

See the following Citation from [124], Borys Jagielski:

"The aforementioned events stimulated the interest in the general properties of light, but several experiments conducted quite recently touched directly upon the problem of the wave-particle duality. In 1989 the team of Akira Tonomura conducted for the first time the famous double-slit experiment with electrons in carefully controlled laboratory conditions (see Appendix C) [5]. In 1999 Anton Zeilinger and his colleagues performed another experiment of the same kind, but with fullerene molecules instead of electrons [44]. The interference pattern was again obtained, confirming that also particles much larger than electrons can behave in accordance with de Broglie relations.

The somewhat controversial Afshar experiment was one of the most recent words uttered on the matter of the wave-particle duality of light [45]. It was first conceived and carried out by Shahriar Afshar in 2005 (but later repeated in an improved form). The experiment is assumed by some to demonstrate a paradox of the wave-particle duality, because it seemingly allows to observe both the corpuscular and the undulatory behavior of light at once [46]. However, a consensus has not been yet reached. The Afshar experiment and its possible implications will be analyzed in Chapter 8".

References:
[5] A. Tonomura, J. Endo, T. Matsuda, T. Kawasaki, H. Ezawa, Demonstration of single-electron buildup of an interference pattern, American Journal of Physics 57 (1989), pp. 117-120.

[44] M. Arndt, O. Nairz, J. Vos-Andreae, C. Keller, G. van der Zouw, A. Zeilinger, Wave-particle duality of C60 molecules, Nature 401, pp. 680-682 (1999).

[45] Shahriar Afshar, Sharp complementary wave and particle behaviours in the same "welcher Weg" experiment, Proceedings of SPIE 5866, pp. 229-244 (2005)

[46] Shahriar Afshar, Eduardo Flores, Keith McDonald, Ernst Knoesel, Paradox in Wave-Particle Duality, Foundation of Physics 37, pp. 295-305 (2007)

See also in [124], Chapter 8, The Afshar experiment, and under C; -Matter waves; -pages 179 – 182.

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Citation (from [91], Common Sense Science, Charles W. Lucas, Jr. Statements related to some ideological items in the same text are omitted merely):

Experimental Evidence to Support Boscovich's Atomic Model. ... It started with the discovery of soliton ... Solitons are long-lasting semi-permanent standing wave structures with a stable algebraic topology. [9] The soliton can exist in air or water as a toroidal ring. Solitons in water are usually formed in pairs known as a soliton and anti-soliton. Their structure is weak, and they decay after 10 or 20 minutes.

Bostick's Plasmons. Winston Bostick (1916-1991), the last graduate student of Nobel Prize winner Arthur Compton (1892-1962), experimentally discovered how to make "plasmons" or solitons from the electromagnetic field within an electromagnetic plasma. [10] These structures were robust, compared to solitons in air and water. They had long lives and could not be destroyed by normal processes in nature. Bostick proposed that electrons were quite simple solitons and positrons were contrary or anti-solitons. All other elementary particles were built of more complex, geometrical structures such as dyads, triads, quatrads, etc. All plasmons or solitons in the electromagnetic field are of the same shape, i.e. a toroidal ring. The plasmon was extremely high strength. Bostick tried to create a bottle of plasmons to hold controlled thermo-nuclear fusion. All materials known to man up to that time slowly disintegrated when exposed to controlled thermonuclear fusion. Only the plasmon was strong enough, but Bostick failed to succeed in building a bottle of plasmons...

Hooper's Electromagnetic Field Experiments. The nature of the plasmon, electromagnetic soliton was more completely revealed by another modern-day scientist, William J. Hooper [11]. He discovered that charged elementary particles, such as the electron, were not only made from the electromagnetic field, but variations in the field around them due to their structure extending to great distances. This same feature is also observed about solitons in water.

Hooper [11] also discovered that there are three types of electric and magnetic fields. One of these types is due to velocity effects from Lenz's Law causing it to have property that cannot be shielded. Thus, portions of the electromagnetic field exist everywhere in the universe.

References:

9. May, J. P., **Stable Algebraic Topology**, p. 1 (1966). http://www.math.uchicago.edu/~may/PAPERS/history.pdf

10. Bostick, Winston H., "Mass, Charge, and Current: The Essence and Morphology," **Physics Essays, Vol. 4, No. 1**, pp. 45-59 (1991).

11. Hooper, W. J., **New Horizons in Electric, Magnetic, and Gravitational Field Theory** (Electrodynamic Gravity, Inc., 543 Broad Blvd., Cuyahoga Falls, OH 44221, 1974), preface. http://www.rexresearch.com/hooper/horizon.htm

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The message the author conveys in this book is the following:
"If a theory, concept, or abstract mathematical construction begins to generate numerous strange, unanswered questions, dilemmas, contradictions, or ambiguous and multi-plausible explanations—particularly when these are supported by artificially created axiomatic statements, imaginative labels without proper explanation, postulations, or heavy reliance on probabilistic and statistical models, it is often wiser, more natural, and scientifically productive to seek new theories or improved conceptual frameworks."

Some prominent examples of such imaginative, irrational, and exotic constructs include Schrödinger's cat paradox, the concepts of Dark Energy and Dark Matter, unexplained notions behind nuclear forces and gravitation, assumptions about Neutron Stars, speculations about Black Holes, the idea of Wave Function Collapse, and the "magical" influence of the observer on measurements and reality.

The author expects that these and other similarly abstract concepts will eventually evolve or be replaced with significantly updated and more scientifically grounded ideas.

....

Motional or kinetic particle energy that should be causally related to matter-wave energy (and analogically matching photon energy), could be treated as usual (in mechanics), having any positive (velocity dependent) value, if this energy is measured "externally", in the space where a particle is in motion. If we attempt to solve the relativistic equation that connects all energy aspects of a single and moving particle ($p^2c^2 + E_0^2 = E_t^2 = (E_0 + E_k)^2$), we will find that one of the solutions for kinetic energy could be the negative energy amount that here corresponds to the particle rest mass energy, for instance,

$$\left\{\begin{array}{l} E^2 = E_0^2 + p^2c^2 = (E_0 + E_k)^2 = E_0^2 + 2E_0E_k + E_k^2 \ , \\ E_0 = mc^2 \ , \quad E = \gamma mc^2, \quad E_k = E - E_0 = (\gamma - 1)mc^2 \ , \quad \gamma = (1 - v^2/c^2)^{-1/2} \end{array}\right\} \Rightarrow$$

$$E_k^2 + 2E_0E_k - p^2c^2 = 0 \ \Rightarrow$$

$$E_k = -E_0 \pm \sqrt{E_0^2 + p^2c^2} = -E_0 \pm E \Rightarrow$$

$$\Rightarrow E_k = \left\{\begin{array}{l} +E_k \\ -E_0 \end{array}\right\} = \left\{\begin{array}{l} (\gamma - 1)mc^2 \\ -mc^2 \end{array}\right\} \quad (=)$$

$$(=) \ \left\{\begin{array}{l} \text{motional particle energy "in external space"} = (\gamma - 1)mc^2 \\ \text{motional particle energy internaly captured or "frozen" by its rest mass} = -mc^2 \end{array}\right\}.$$

Such a result ($E_k = -E_0$) could look illogical and could be neglected or considered unrealistic. If we consider that internal particle structure (that creates its rest mass) is also composed of motional-field energy components (or well packed, self-stabilized and internally closed, standing matter waves), we could consider that negative motional energy belongs to the ordinary motional energy that is internally “frozen” or captured by the particle rest mass.

*[♣ **COMMENTS & FREE-THINKING CORNER**: **How** to address properly rotating motions, spin, and orbital moment's characteristics (concerning matter-wave properties) is important for understanding the **PWDC**.*

To get an idea of how to express linear motion energy components and orbital, rotational, or spin-related energy components let us again use analogical thinking.

1. *For instance, if a moving particle is only in a linear motion (without having mechanical elements of rotation), its relativistic kinetic energy is:*

$$\left\{\begin{array}{l} E_k = mc^2(\gamma - 1) \\ m = m_0 = \text{const.} \\ p = \gamma mv \\ E_{tot.} = \gamma mc^2 \end{array}\right\} \Rightarrow \left\{dE_k = vdp = mc^2 d\gamma = dE_{tot.} = d\tilde{E}\right\}$$

The same conclusion regarding the infinitesimal value of kinetic energy should be valid for classical-mechanic kinetic energy, since we already demonstrated that one of the possible options is,

$$\left\{\begin{array}{l} E_k = \frac{1}{2}mv^2 = \frac{1}{2}pv \\ m = m_0 = \text{const.} \\ p = mv \end{array}\right\} \Rightarrow \left\{dE_k = pdv = d\tilde{E}\right\}.$$

2. *By the same analogy, if the same particle only spins (no linear or other motion), we will see that differential of its kinetic energy is,* $dE_k = \omega\, dL_s$, *since we already established that it should be analogically valid,* $\{dE_k = vdp\}_{\text{linear-motion}} \Leftrightarrow \{dE_k = \omega dL_s\}_{\text{spinning}}$. *Here we apply the following analogies:* ***linear velocity** replaced by **angular, spinning velocity***, $v \leftrightarrow \omega$ *and **linear moment** replaced by **spinning angular moment***, $dp \leftrightarrow dL_s$.

3. *By extending the same analogies (as under 1 and 2) to an orbital motion when a small mass* m *is orbiting a much bigger mass, while not spinning, (where the radius of such orbiting is very long compared to both particle diameters), we will have,*

$$\left\{\begin{array}{c} E_k = mc^2(\gamma - 1) = \dfrac{pv}{1+\sqrt{1-v^2/c^2}} = \dfrac{L\omega}{1+\sqrt{1-v^2/c^2}} \\ dE_k = vdp = \omega dL = d\tilde{E} \end{array}\right\} \Rightarrow E_k = \left\{\begin{bmatrix} \left(\dfrac{pv}{2} = \dfrac{L\omega}{2}\right) \Leftrightarrow (pv = L\omega), v << c \\ (pv = L\omega), v \approx c \end{bmatrix}\right. \Leftrightarrow \begin{pmatrix} pv = L\omega \\ \text{always} \end{pmatrix}$$

$$\Rightarrow dE_k = \begin{cases} \dfrac{1}{2}(pdv + vdp) = \dfrac{1}{2}(Ld\omega + \omega dL), (v << c) \to (vdp = pdv = \omega dL = Ld\omega) \\ pdv + vdp = Ld\omega + \omega dL = vdp = \omega dL, (v \approx c) \to pdv = Ld\omega = 0,\ v = const.,\ \omega = const. \end{cases}$$

where L and ω are orbiting particle momentum and angular velocity, being mutually parallel, while $pv = L\omega$ is always valid since we present the same kinetic energy of the same particle on two different ways.

4. *Now, let us imagine that the same small particle in orbital motion also has specific spinning. This will produce the following equivalency between involved linear and orbital motions,*

$$\left\{\begin{array}{c} E_k = m^*c^2(\gamma^* - 1) = \dfrac{p^*v^*}{1+\sqrt{1-v^{*2}/c^2}} = \dfrac{(L+L_s)\omega^*}{1+\sqrt{1-v^{*2}/c^2}} \\ dE_k = v^*dp^* = \omega^* d(L+L_s) = d\tilde{E} \end{array}\right\} \Rightarrow (p^*v^* = (L+L_s)\omega^*) ,$$

where symbols with asterisks indicate that particle spin is changing particle linear and angular velocity.

5. *Here we could also hypothesize with the concept that resulting, effective spinning motion (of the number of elementary mass constituents) is somehow captured and "frozen" by a total particle rest mass what (hypothetically) indicates that rest mass is created from some properly packed spinning and torsional-motion energy domains. What is important here is to start familiarizing yourself with the idea or concept that every motion has linear and torsional (or spinning) kinetic energy components but mentioned spinning in cases of particles could be hidden by the formation of particles rest masses. Internal and intrinsic matter-waves rotation and spinning, is captured by a particle rest mass (becoming externally "non-visible" and not measurable because the number of elementary domains inside a macro-particle is randomly packed and mutually coupled, neutralizing the total, resulting spin moment, as well as compensating and neutralizing distributed electromagnetic moments and dipoles). Of course, we know that real macro-particles have many atoms, and that each atom has the number of more elementary particles (all of them have constant and closely related magnetic, spin and orbital moments). All that makes this situation more complicated, but the conceptual understanding would stay simple and clear as, Number of mutually coupled elementary domains that create rest mass, captures internal, hidden, and spinning-related motional matter-waves components. Later we will have enough mathematical and modeling margins to be able to pay attention to details and proper equations in mentioned conceptualization. For better understanding see (2.5-1) to (2.11), (4.18), (4.5-1) - (4.5-3), (4.42) to (4.45) and (5.4.1) to (5.10), (4.3-0), (4.3-0) -a,b,c,d,e,f,g,h,i… and Fig.4.1.1, Fig.4.1.1a, Fig.4.1.2, T.4.4, dealing with the unity of linear and rotational motions. ♣]*

The much more natural and conceptually more precise explanation of **PWDC** (compared to earlier formulated, analogical, and mathematical merging of particle and wave properties (4.1), (4.2) and T.4.0) can be formulated as:

a) First, we should know (or accept) that there is not a case of a single particle (within our universe), fully isolated and without any interaction with its vicinity. Any single particle or energy state belongs to a much more general case of (at least) two-body relations, where the first body is that single particle, and the second body is its vicinity (or surrounding environment). This way (in any possible interaction, or when describing any possible particle or quasi-particles states), we effectively deal with at least a two-body system (or with interactions between two bodies). The primary, natural, and very tangible matter-waves related conceptualizations of two-body, or binary systems properties, are already elaborated in the second chapter and should be taken as an introduction in any **PWDC** foundation (see: "2.3.2. Macro-Cosmological Matter-Waves and Gravitation", -equations from (2.11.21) to (2.11.21)). When we analyze the general two-body problem traditionally (or only mechanically), we can formulate the most important relations describing such interaction, given by results from (4.5) until (4.8). Effectively, by analyzing how we came to all results and relations (4.5) - (4.8), we can conclude, later, that **PWDC** from (4.1) and (4.2), is an equivalent mathematical modeling way to explain the general two-body problem *by creating direct parallelism between particle (or its mechanical) and wave properties*. Differently formulated, we will see that energy-momentum complexity of two-body or two-particle relations presents the real source of **PWDC** (which will become more understandable after we make an equivalent wave presentation of the mechanical two-body situation). When analyzing the two-body problem in the light of **PWDC** we can also formulate the generalized, Newton-like "action-reaction", inertial forces (valid for all possible field situations: in gravitation, in electric and magnetic fields, etc.). Of course, the statements above are still a bit speculative before we see the development of all mentioned relations, (4.3) - (4.8), which is just temporarily postponed before creating a more natural, clear, and still introductory **PWDC** formulation.

b) The second, also an essential element of particle-wave duality is just coming from the fact that two bodies, mentioned in a), mutually interact even before any mechanical contact between them is established (again, see the development of (4.3) - (4.8) …, as a support). We can call this interaction a "field-channel, or particle-wave field connection", ***and this is just a modus of conserving (balancing and redistributing) energy and momentum (both linear and orbital, including possible spinning and wave energy of participants)***. By introducing the idea that a specific field or wave connection should always exist (in any two or multiple-body situation), we can mathematically analyze what happens when two (or much bigger number of) elementary waves interact. For instance, this is the example of a simple superposition of elementary (sinusoidal) waves to create the wave group, or a wave packet model, well analyzed in almost any modern Physics and Quantum Mechanics book). What we create this way is called the wave packet or group. For such waving form, we are in the position to find its group and phase velocity, and the essential relation between them, given by the equation, $v = u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda}$. When we analyze this situation (see also (4.0.73) - (4.0.76) from the chapter 4.0), we can find that de Broglie wavelength $\lambda = h/p$, Einstein-Planck's relation for wave energy $\tilde{\mathbf{E}} = \mathbf{hf}$, and Einstein's mass-energy relation, $\mathbf{E_{total}} = \gamma \mathbf{mc^2}$ are simply (mathematically) integrated in the structure of the equation that connects group and phase velocity (this way supporting and/or deriving every one of them using the combination of others), and all of them are shown to be mutually compatible, complementary and essential to explain (non-mechanically) the two-body situation a), this way automatically explaining the **PWDC** relations from (4.1) and (4.2), **as well as putting new light into a necessity of novel understanding of Max Planck's law of black-body thermal radiation, as briefly summarized, below**:

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

$$\left\{\begin{matrix}\left[\lambda=\frac{h}{p},k=\frac{2\pi}{\lambda}=\frac{2\pi}{h}p,\right]\\ \left[\tilde{E}=hf\right]\\ \left[\begin{matrix}\overline{P}_4=(p,\frac{E}{c})\Rightarrow p^2-\frac{E^2}{c^2}=-m^2c^2\\ p=\gamma mv,\gamma=(1-v^2/c^2)^{-0.5}\\ dE_k=vdp=mc^2d\gamma=dE_{tot}=d\tilde{E}\\ E=E_{tot}=\gamma mc^2,\ E_k=(\gamma-1)mc^2\end{matrix}\right]\end{matrix}\right\}\Leftrightarrow\left\{\begin{matrix}dE=dE_k=d\tilde{E}=vdp=d(pu)=pdu+udp=hdf=c^2d(\gamma m)=mc^2d\gamma,\\ E_k=hf=pu,\omega=2\pi f,\ \lambda\frac{du}{d\lambda}=-p\frac{du}{dp}=-k\frac{du}{dk},\frac{dp}{p}=\frac{dk}{k}=-\frac{d\lambda}{\lambda},\\ v=u-\lambda\frac{du}{d\lambda}=-\lambda^2\frac{df}{d\lambda}=\frac{dE}{dp}=\frac{d\omega}{dk}=u+p\frac{du}{dp}=u+k\frac{du}{dk},\\ u=\lambda f=\frac{\omega}{k}=\frac{E_k}{p}=\frac{\tilde{E}}{p}=\frac{v}{1+\sqrt{1-v^2/c^2}},\ \frac{d\lambda}{\lambda}=-\lambda\frac{df}{v}\\ 0\le 2u\le\sqrt{uv}\le v\le c\end{matrix}\right\}\Leftrightarrow$$

$$\Leftrightarrow\left\{\begin{matrix}\lambda=\frac{h}{\tilde{p}}=\frac{h}{p},\ \tilde{E}=E_k=hf=\hbar\omega=m^*vu=pu=h\frac{\omega}{2\pi}=\frac{h}{\tau}=(\gamma-1)mc^2,\ k=\frac{2\pi}{\lambda}=\frac{2\pi}{h}p,\\ \omega=\frac{2\pi}{T}=\frac{2\pi}{h}\tilde{E},\ \ u=\lambda f=\frac{\tilde{E}}{p}=\frac{\omega}{k},v=v_g=u-\lambda\frac{du}{d\lambda}=\frac{d\tilde{E}}{dp}=\frac{d\omega}{dk},m^*=\gamma m,p=\gamma mv=\tilde{p}\end{matrix}\right\}\Rightarrow$$

$$\Rightarrow\left\{\begin{matrix}dE=d(pu)=pdu+udp=pd\left(\frac{v}{1+\sqrt{1-v^2/c^2}}\right)+\frac{vdp}{1+\sqrt{1-v^2/c^2}}=\\ =pd\left(\frac{v}{1+\sqrt{1-v^2/c^2}}\right)+\frac{dE}{1+\sqrt{1-v^2/c^2}}\Rightarrow\\ \left(\frac{\sqrt{1-v^2/c^2}}{1+\sqrt{1-v^2/c^2}}\right)dE=pd\left(\frac{v}{1+\sqrt{1-v^2/c^2}}\right)\Leftrightarrow\\ \Leftrightarrow\left(\frac{v\sqrt{1-v^2/c^2}}{1+\sqrt{1-v^2/c^2}}\right)\frac{dp}{p}=d\left(\frac{v}{1+\sqrt{1-v^2/c^2}}\right)\Rightarrow\\ \Rightarrow\boxed{\frac{dp}{p}=\frac{dk}{k}=-\frac{d\lambda}{\lambda}=\frac{d\left(\frac{v}{1+\sqrt{1-v^2/c^2}}\right)}{\left(\frac{v\sqrt{1-v^2/c^2}}{1+\sqrt{1-v^2/c^2}}\right)}}=\left(\frac{1}{1+\sqrt{1-v^2/c^2}}\right)\frac{df}{f}=\\ =\frac{\lambda}{v}df=\frac{u}{v}\frac{df}{f}=\frac{hdf}{pv},\ \boxed{\frac{df}{f}=\frac{\left(1+\sqrt{1-v^2/c^2}\right)^2}{v\sqrt{1-v^2/c^2}}d\left(\frac{v}{1+\sqrt{1-v^2/c^2}}\right)}\end{matrix}\right\}\Rightarrow$$

$$\Rightarrow\left\{\begin{matrix}\frac{dp}{p}=\frac{dk}{k}=-\frac{d\lambda}{\lambda}=\frac{d\left(\frac{v}{1+\sqrt{1-v^2/c^2}}\right)}{\left(\frac{v\sqrt{1-v^2/c^2}}{1+\sqrt{1-v^2/c^2}}\right)}=\left(\frac{1}{1+\sqrt{1-v^2/c^2}}\right)\frac{df}{f}\cong\left\{\begin{matrix}\frac{dv}{v}=\frac{1}{2}\frac{df}{f},\ \text{for }v<<c\\ \frac{df}{f},\qquad\text{for }v\approx c\end{matrix}\right.\\ \frac{df}{f}=\frac{d\tilde{E}}{\tilde{E}}=-(\frac{v}{u})\cdot\frac{d\lambda}{\lambda}=(\frac{dv}{v})(1+1/\gamma)\gamma^2=-(1+\frac{1}{\gamma})\cdot\frac{d\lambda}{\lambda}=\frac{\left(1+\sqrt{1-v^2/c^2}\right)^2}{v\sqrt{1-v^2/c^2}}d\left(\frac{v}{1+\sqrt{1-v^2/c^2}}\right)\cong\\ \cong\left\{\begin{matrix}2\frac{dv}{v}=-2\frac{d\lambda}{\lambda},\quad\text{for }v<<c\\ -\frac{d\lambda}{\lambda},\qquad\text{for }v\approx c\end{matrix}\right\}\end{matrix}\right\}\Rightarrow\qquad(4.2\text{-}1)$$

$\Rightarrow$(After integration) $\Rightarrow$

$$\Rightarrow \left\{ \begin{array}{l} p = p(v) = m\gamma v,\ k = k(v) = \frac{2\pi}{h} p(v) = \frac{2\pi}{h} m\gamma v, \\ \lambda = \lambda(v) = \frac{h}{p(v)} = \frac{h}{m\gamma v}, u(v) = \lambda(v) f(v) = \frac{v}{1+\sqrt{1-v^2/c^2}} \end{array} \right\} \Rightarrow \boxed{f = \frac{u}{\lambda} = \frac{up}{h} = f(v)} .$$

We know that by explaining all experimental situations regarding particle-photon interactions (such as Compton, Photoelectric, electron-positron creation from a high-energy photon, or their annihilation, Bragg's diffraction of X-rays, electrons and neutrons, and other familiar effects, such as examples in Chapter 4.2), the equality between wave and particle energy ($\tilde{E} = hf = \tilde{m}c^2 = E_k = mc^2(\gamma - 1),\ p = \tilde{p} = \frac{hf}{c},\ \lambda = \frac{h}{\tilde{p}}$), is entirely provable, and related only to motional energies of all interacting participants. We can find familiar elaborations of **PWDC** situations including rectifications of original de Broglie foundations of wave-particle duality in [105]).

In other words, if we accept such approach, the real sources of particle-wave duality become much more realistic (tangible and deterministic) than a contemporary probability platform of Quantum Theory in explaining the same phenomena (for more complementary explanation of **PWDC** see also the chapter 5, especially T.5.3, and chapter 10).

In conclusion, moving particles analogically behave as photons (or waves), and vice versa, related to their "linear and angular or spinning moments", kinetic energy, and masses. Presently we only exploit the analogy between particle's linear moment and photon. Since photon in addition has an intrinsic angular or spinning moment ($L = \frac{c^2}{uv}(\frac{h}{2\pi})$), analogically this *should also be valid for a particle in linear motion (and this is what creates wave-particle duality and de Broglie matter-waves)*. ***Consequently, the mentioned analogies between photons and moving particles should be naturally applicable in both directions***. It is also indicative that photon spin, and the spin belonging to a moving particle, are both equal to $\frac{h}{2\pi}$ (see (2.11.3), T 4.0 and (4.8)). Now we can say that rotation (or spinning) ***and linear motion (of the same object) should always be intrinsically coupled in all cases of particle motions***. For instance, moving electrons create rotating (or helix) magnetic field around their paths of propagation (*see on Internet very similar concept about Henry Augustus Rowland effect of magnetic field of electrons around rotating metal conductors, as presented by Jean de Climont*). Something similar should also be valid for all cases of electromagnetically neutral particles that perform linear motion. In such situations, we should (analogically and coincidently) expect the effects of rotation (or helix spinning) of specific matter-wave field components, what is eventually producing de Broglie matter waves, and other measurable consequences such as omnipresent rotations in the world of atoms, elementary particles, and rotations of astronomic objects. For instance, planetary rotation around the sun could be quantifiable as: $L = \frac{c^2}{uv}(\frac{H}{2\pi}) \cong \frac{2c^2}{v^2}(\frac{H}{2\pi}),\ u \cong \frac{v}{2} << c, h \to H = const.$). Whenever we have certain dynamic coupling of linear motion with rotation, this means that associated matter waves have been created along specific helix-line of two complementary field components, what presents the content of a particle motional energy. Here, de Broglie wavelength and frequency should be in direct relation with mentioned spinning-helix field structure.

The most direct externally measurable signs regarding the existence of natural rotation-related behaviors of all elementary particles should be their gyromagnetic, spin and orbital moment attributes. In cases of stationary (or stabilized) inter-atomic, self-closed and circular motions, the much more general picture (regarding the same situation) is given by Wilson-Sommerfeld rules (see 5.4.1).

c) Matter Waves & Rotation:
In cases of rotation, it is important not to confuse a particle's angular velocity (i.e., its mechanical rotation around a center) with the angular velocity of the corresponding matter waves. These two frequencies are related by a velocity-dependent function, where both the group velocity and phase velocity must be considered. For example, in analyses of spin and orbital moments, many authors fail to clearly differentiate between these frequencies, leading to conceptual confusion about particle-wave duality (or unity).

d) Understanding the Particle-Wave Duality Concept (PWDC):
A key part of understanding PWDC is the recognition that it can be modeled using an isomorphic mathematical structure, which "on average" effectively represents the deterministic concepts discussed earlier. This has been successfully achieved within the framework of Probability, Statistics, and Signal Analysis, as realized in contemporary Quantum Theory.

This idea of isomorphic mathematical modeling, however, is not officially acknowledged in Orthodox Quantum Mechanics. The theory has constructed an imposing mathematical framework that works exceptionally well, largely due to the uncritical acceptance of its structure by the mainstream scientific community. Despite its effectiveness, alternative voices are often suppressed. The book will argue that, had a more natural and general wave-function modeling approach been adopted at the inception of Quantum Theory, many ad hoc discoveries and rules would have simply been mathematical consequences of a better model, such as using Complex Analytic Signal Phasors.

For instance, Schrödinger's wave equation and other key results of Orthodox Quantum Theory can be derived from a more general model of the Complex Classical Wave Equation, interpreted using the Hilbert transform. Furthermore, all forms of Heisenberg's Uncertainty Principle can be seen as straightforward products of Signal Analysis theory, equally valid in both the micro and macro realms. Probability and stochastic modeling, in this view, are secondary tools, useful for fitting curves or providing "in-average" evidence, but they are non-ontological and mathematically operational.

It is natural that all fundamental laws of physics, present and future, should apply uniformly to all scales of the universe, though this is not yet the case in contemporary physics. Orthodox Quantum Theory, while mathematically sound, could be reinterpreted as an alternative mathematical model that, with immense effort, replaced the more tangible particle-wave platform mentioned earlier. One of this book's objectives is to demonstrate how such equivalent modeling has been realized and to suggest a more advanced conceptual framework.

The current mainstream in physics tends to present Probabilistic Quantum Theory as the most general, correct, and unchangeable vision of the micro-world. In contrast, this book argues that Statistics and Probability theory should only play an essential role in analyzing vast sets or events, such as in Thermodynamics, where probability plays a deterministic role. In contrast, Quantum Theory has a "probabilistically non-deterministic" nature. While unifying Quantum Theory and Thermodynamics into a single theory may be premature, it is time to critically examine why both are so deeply entangled with Statistics and Probability.

Unlike Quantum Theory, Thermodynamics has clear, experimentally verifiable reality, further emphasizing the conceptual gaps in Quantum Mechanics. Although Quantum Theory works effectively after years of intense mathematical development, it remains overloaded with artificial complexity. This book proposes a new conceptual approach to particle-wave duality, offering a neo-deterministic alternative to the traditional teaching of Quantum Theory.

e) Comparing New and Old Theories:
In the early stages of building this new wave-particle duality theory, it is inevitable that we compare it to Orthodox Quantum Theory. While defenders of contemporary Quantum Theory may dismiss such efforts as unnecessary or futile, claiming that everything is already known, the objective of this book is to open the possibility of a new approach.

f) Elements of PWDC in Existing Theories:
Elements of the Particle-Wave Duality Concept (PWDC) can already be found in Niels Bohr's hydrogen atom model, though with artificial and challenging assumptions (see Chapter 8). In Chapter 2, we will also explore how PWDC concepts apply to describing planetary systems (see "2.3.3. Macro-Cosmological Matter-Waves and Gravitation").

Wave-particle duality has been observed and analyzed in various phenomena involving photons, electrons, positrons, protons, and neutrons. Effects like the photoelectric and Compton effects, as well as electron-positron creation and annihilation, suggest that electrons and photons can transform into each other or are composed of similar electromagnetic matter. Bragg's diffraction experiments show that photons behave as waves, and similarly, the diffraction of particles such as electrons and neutrons can be explained using the same formula, further supporting de Broglie's matter-wave theory.

This raises the possibility of understanding the neutron as an "exotic atom," consisting of an electron shell and a proton core, with electron spin playing a dominant role (see David L. Bergman's work, "Notions of a Neutron"). In other words, wave-particle duality is observed primarily when electromagnetic energy formations with spin and magnetic moment properties are involved.

Thus, rather than applying wave-particle duality to matter in motion, we could argue that photons, electrons, protons, and neutrons are specific electromagnetic energy packets responsible for a whole matter in the universe. These packets possess specific wavelengths and energies, as described by the same de Broglie and Einstein-Planck equations.

In conclusion, wave-particle duality is inherently related to electromagnetic and mechanical energy manifestations. No electromagnetic fields or charged entities with magnetic spin means no matter waves, a principle that applies equally to microscopic particles and astronomical objects (see “2.3.3. Macro-Cosmological Matter-Waves and Gravitation”).

Quantum Theory posits that total particle energy can be fully represented by an equivalent matter-wave packet, which is philosophically correct but only in specific cases. However, when considering kinetic energy, modern Quantum Theory often treats low-velocity particles (v << c), which leads to phase velocities higher than the speed of light, an illogical and incorrect result (see Chapter 4.3 for further discussion).

What follows in the next part of this chapter (and later) is kind of step-by-step explanation of a), b), c), d), ... statements, or a deeper explanation of the **PWDC** and its integration with generalized wave equations, *effectively transforming the Particle-Wave Duality concept into Particle-Wave Unity concept*.

[♣ COMMENTS & FREE-THINKING CORNER:

4.1.1.1. The resume regarding PWDC

A brief and simplified summary of all the above-given explanation/s regarding ***PWDC*** *can also be presented in the following way:*

1. *Relativity theory shoved or implicated that there is a simple relation of direct proportionality between any mass and its total energy that could be produced by fully transforming that mass into radiation,* $E_0 = mc^2$, $E_{tot.} = \gamma mc^2 = E_0 + E_k = E_0 + (\gamma - 1)mc^2 = \sqrt{E_0^2 + p^2c^2}$.

2. *The most important conceptual understanding of frequency dependent matter wave energy, which is entirely equivalent to particle motional, or kinetic energy, is related to the fact that total photon energy can be expressed as the product of the Planck's constant and frequency of the photon wave packet,* $E_f = hf$. *Consequently (since there is a known proportionality between mass and energy), the photon momentum was correctly found as* $p_p = hf / c = m_p c$ *(and proven applicable and correct in analyzes of different interactions between photons as waves, or quasi-particles and real particles).*

3. *Since a photon has certain energy, we should be able to present this energy in two different ways, for instance,* $E_p = hf = \sqrt{E_{0p}^2 + p_p^2c^2} = p_p c = E_{kp}$. *Since photon rest mass equals zero, there is only a photon kinetic energy* $E_p = hf = p_p c = E_{kp} = m_p c^2$, *and in many applications, this concept (and all equivalency relations for photon energy and momentum) showed to be correct.*

4. *Going backward, we can apply the same conclusion, or analogy, to any real particle (which has a rest mass), accepting that particle kinetic energy is presentable as the product between Planck constant and characteristic particle's matter wave frequency* $E_k = (\gamma - 1)mc^2 = \tilde{E} = hf$. *Doing it that way, we can find the frequency of de Broglie matter waves as,* $f = E_k / h = (\gamma - 1)mc^2 / h = \tilde{E} / h$. *Now, we can find the phase velocity of matter waves as* $u = \lambda f = \frac{h}{p} f = \frac{E_k}{p} = \frac{(\gamma - 1)mc^2}{\gamma m v} = \frac{v}{1 + \sqrt{1 - v^2/c^2}}$.

5. *The relation between phase and group velocity of a matter wave packet is also known in the form* $v = u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda}$. *Combining two latest forms of phase and group velocities, we can get* $v = u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda} = u(1 + \sqrt{1 - v^2/c^2})$, *implicating validity of the following differential relations:* $d\tilde{E} = d\left[(\gamma - 1)mc^2\right] = mc^2 d\gamma = hdf = vdp = d(pu)$, *and practically confirming mathematical*

*consistency of all above introduced equivalency and analogy based relations (named in this book as **PWDC** = **P**article-**W**ave **D**uality **C**ode).*

6. It is even more interesting to notice that in the same framework (from 1 to 5) the spin of motional energy (photon spin and kinetic energy spin, T 4.0) is always equal to $\frac{\tilde{E}}{\omega} = \frac{E_k}{\omega} = \frac{h}{2\pi}$ *(or to a multiple of* $\frac{h}{2\pi}$*, see (4.8)).*

7. Since the above addressed particle wave equivalency relations are found valid only if we use the wave packet model (as a replacement for a particle in motion), consequently, we have an argument more to say that matter waves (or corresponding wave functions) should exist as forms of harmonic, modulated sinusoidal signals (naturally satisfying the rules of Fourier signal and spectrum analysis). It is of essential importance to notice that a rest mass (or rest energy) does not belong directly to matter-wave energy (opposite to many current presentations in modern physics books, regarding matter wave properties), but certainly, when in motion, rest mass presents the source of matter waves. Analyzing Compton Effect and many other elementary interactions known in Quantum Mechanics, we can easily prove the statement that only kinetic or motional energy presents the active matter wave energy. The next consequence of this concept is that the rest mass of the particle itself should present (internally and intrinsically) a "passive matter wave energy" in the form of a closed, self-sustainable, stationary, and standing wave structure, which only externally looks like a stable and closed shell particle. This internally closed matter wave energy would become externally measurable (directly or indirectly) in all cases when a particle interacts with its environment participating in any motion.

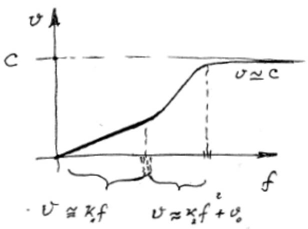
*8. We can also rethink the meaning of phase velocity **u**, based on (4.2), because modern interpretations of this velocity in most Quantum Mechanics and Physics books are different from (4.2), (however, in some basic literature we can also find the relation **v = 2u**, for **v** << **c**, which agrees with (4.2)). For instance, thermal blackbody radiation can also be addressed using group and phase velocity relations from (4.2). We can try to estimate what happens inside a black body cavity where we have a complex, random motion of hot gas particles, atoms and molecules, and random light emission, absorption, and photons scattering. We only know from Planck's blackbody radiation formula the resulting spectral distribution of outgoing light emission, in the case when we make a small hole on the surface of the black body, and let photons be radiated and detected in an external free space of the blackbody. Such external light radiation is characterized (in Planck's modeling) by free photons (with standing waves formations) where each photon has (almost) the same phase and group velocity **v = u = c = constant**. This is not the case inside the black-body cavity, since inside there are many mechanical and field interactions between photons, gas particles, electrically and magnetically charged particles, particles with spinning and magnetic properties, matter waves and cavity walls, and there we should have a broad range of spectral dependency between group and phase velocities,* $0 \le 2u \le \sqrt{uv} \le v \le c$. *A significant number of wave packets (de Broglie matter wave groups with mutually united or coupled mechanical and electromagnetic properties) inside the blackbody cavity permanently interact (among themselves, as well as with the cavity and gas particles) and we cannot consider them being free (mutually independent) wave groups, or stable and/or standing wave formations. It is logical (as the starting point in an analysis of such case), to imagine that the mean particle or group velocity of such wave groups is directly proportional to the blackbody temperature. The average kinetic energy of involved particles is proportional to certain temperature, and when the gas temperature (inside a blackbody radiator) is relatively low, then we should dominantly have motions with non-relativistic particle velocities (* $v << c \Rightarrow v \approx 2u$ *). When the temperature is sufficiently (or remarkably) high, we should dominantly have the case of relativistic particle motions with high velocities (* $v \approx c \Leftrightarrow v \approx u$ *), according to (4.2). **Planck's blackbody radiation formula (see chapter 9.) is very useful and correct, but the way in which it is assembled (or fitted) is not sufficiently correct.** Let us create the following table, T.4.1., comparing relativistic and non-relativistic wave-group energies of an internal blackbody situation, searching for a better background and development of the blackbody radiation formula, and exercising the practical meaning of the **PWDC**, as established in (4.1) and (4.2).*

T.4.1. Interacting and coupled wave groups inside the black body cavity

$$\left\{ \begin{array}{l} v = u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda}, \\ u = \frac{v}{1+\sqrt{1-v^2/c^2}} = \frac{u - \lambda \frac{du}{d\lambda}}{1+\sqrt{1-v^2/c^2}} = \frac{-\lambda^2 \frac{df}{d\lambda}}{1+\sqrt{1-v^2/c^2}} \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} \frac{du}{u} = -\left(\frac{d\lambda}{\lambda}\right)\sqrt{1-v^2/c^2} = \frac{df}{f} \cdot \frac{\sqrt{1-v^2/c^2}}{1+\sqrt{1-v^2/c^2}} \\ \frac{df}{f} = -\left(\frac{d\lambda}{\lambda}\right)(1+\sqrt{1-v^2/c^2}) \end{array} \right\}$$

Nonrelativistic group velocities: $v << c \Rightarrow v \cong 2u,\ \sqrt{1-v^2/c^2} \cong 1$ (Lower temperatures)	Relativistic group velocities: $v \approx c \Rightarrow v \approx u \approx c,\ \sqrt{1-v^2/c^2} \cong 0$ (Very high temperatures)
$\left\{\frac{du}{u} \cong -\frac{d\lambda}{\lambda} \cong \frac{df}{2f}\right\} \Rightarrow \left\{\ln\left\|\frac{u}{u_0}\right\| \cong -\ln\left\|\frac{\lambda}{\lambda_0}\right\| \cong \frac{1}{2}\ln\left\|\frac{f}{f_0}\right\|\right\}$ $\Rightarrow v \cong 2u = 2\lambda f \cong \frac{2u_0\lambda_0}{\lambda} = \frac{C_1}{\lambda}, f \cong \frac{C_2}{\lambda^2}$ $\left[\tilde{E}\right]_{\text{Low temp.}} = hf \approx \frac{C_3}{\lambda^2} = C_4\frac{f^2}{u^2} \approx C_5\left(\frac{f}{v}\right)^2,$ $\left[\frac{d\tilde{E}}{d\lambda}\right]_{\text{Low temp.}} \approx -\frac{C_6}{\lambda^3} = -C_7\frac{f^3}{u^3} \approx -C_8\left(\frac{f}{v}\right)^3,$ $C_i = \text{constants}$ $\left\{\begin{array}{l}\text{Vortex flowmeter relation:} \\ \text{If } v \cong k \cdot f \Rightarrow \left[\tilde{E}\right]_{\text{Low temp.}} = hf \approx \frac{C_5}{k^2} = \text{const.}\end{array}\right\}$	$\left\{\frac{du}{u} \cong 0,\ \frac{df}{f} \cong -\frac{d\lambda}{\lambda}\right\} \Rightarrow$ $\Rightarrow \left\{\ln\left\|\frac{u}{u_0}\right\| \cong 0,\ -\ln\left\|\frac{\lambda}{\lambda_0}\right\| \cong \ln\left\|\frac{f}{f_0}\right\|\right\} \Rightarrow$ $\Rightarrow (v \cong u = \lambda f \cong u_0 = \lambda_0 f_0) \cong c, f \cong \frac{c}{\lambda} \Rightarrow$ $\left[\tilde{E}\right]_{\text{High temp.}} = hf \approx h \cdot \frac{c}{\lambda} = \frac{hf}{u} \cdot c \approx hc\left(\frac{f}{v}\right) \geq \frac{hf}{c} \cdot c,$ $\left[\frac{d\tilde{E}}{d\lambda}\right]_{\text{High temp.}} = -h \cdot \frac{c}{\lambda^2} \approx -hc\left(\frac{f}{v}\right)^2 \leq -\frac{hf^2}{c^2} \cdot \text{const.}$

It is difficult to say what the real situation is inside a black body cavity, but we can imagine that numbers of Secondary Emission, Photoelectric and Compton events are permanently created, and combined with thermal motion, and mutual electromagnetic interferences, and couplings of gas particles, etc. The analyzed situation is not directly comparable with external blackbody radiation (Planck's result), since (in the above-given table) we have only internal wave energy states in the black body cavity, but the results are very indicative and provoking. For instance, we see that in the case of lower temperatures, we have the wave energy function $\left[\tilde{E}\right]_{\text{Low temp.}} \approx A\left(\frac{f}{v}\right)^2,\ v << c,\ A = \text{const.}\ \left[\frac{d\tilde{E}}{d\lambda}\right]_{\text{Low temp.}} \approx -C_8\left(\frac{f}{v}\right)^3$ *, and in the case of very high temperatures, the same energy is* $\left[\tilde{E}\right]_{\text{High temp.}} \approx B\left(\frac{f}{v}\right), v \to c, B = hc = \text{const.}$ $\left[\frac{d\tilde{E}}{d\lambda}\right]_{\text{High temp.}} \approx -hc\left(\frac{f}{v}\right)$*, meaning that internal wave groups are decreasing energy with temperature increase (which is not contradictory to the conclusions of Planck's radiation law). Here we see that there is a big difference between free wave groups, like free photons in open space, and mutually interacting (de Broglie) matter waves and particles (inside of a limited space of a cavity). On the contrary, in most analyses of similar situations in modern Quantum Mechanics, we do not find that such differentiation is explicitly underlined and adequately addressed (mostly we find that de Broglie matter waves are treated similarly to free photons or some other free wave groups, or artificial probability waves). Also, in mathematical development of Planck's radiation law, we can only find specific particularly suitable (idealized, poor, and artificial) modeling and curve fitting situations where the phase velocities of black-body (internal) photons always have the velocity of free (externally radiated) photons,* $u = \lambda f = v = c = \text{Constant}$*. The results are given in table T.4.1. are also applicable to any other situation when we analyze relativistic or non-relativistic motions. Especially interesting is to clarify group- velocity-frequency relation (based on results found in T.4.1.). Presently we do not have enough arguments to make such conclusions, but we could estimate that for a small particle, or group velocities, velocity-frequency curve is linear, for middle range velocity is second order (parabolic) curve, and for remarkably high velocities is asymptotically approaching to the speed of light c (see the picture below).*



Estimated velocity-matter-waves-frequency curve

Another instructive exercise (to understand the meaning of matter waves) is to exploit the symmetry with mutual replacements of corpuscular and wave momentum and motional and wave energy, ($\vec{p} \leftrightarrow \tilde{p}$*), (* $E_k \leftrightarrow \tilde{E}$ *) presented in the following table, T 4.1.1.*

T 4.1.1

Symmetry-based on equivalence: $\vec{p} \leftrightarrow \tilde{p}$	
$E_k = \frac{\vec{p}\vec{v}}{1+\sqrt{1-\frac{v^2}{c^2}}}$	$\tilde{E} = pu$
$\tilde{E} = \tilde{p}u$	$E_k = \frac{\tilde{p}\vec{v}}{1+\sqrt{1-\frac{v^2}{c^2}}}$
$\Rightarrow \left\{ \begin{array}{l} (\frac{\tilde{E}}{E_k})^2 = \left[\frac{\tilde{p}}{p}\cos(\vec{p},\tilde{p})\right]^2 = (\frac{p}{\tilde{p}})^2 = (\frac{m}{\tilde{m}})^2 = \\ = \left[\frac{u}{v}(1+\sqrt{1-\frac{v^2}{c^2}})\right]^2 = \left[\frac{v\cos(\vec{p},\tilde{p})}{u(1+\sqrt{1-\frac{v^2}{c^2}})}\right]^2 = \cos(\vec{p},\tilde{p}) \end{array} \right\}$	
$\Rightarrow \left\{ \lambda = \frac{h}{p} = (\frac{h}{\tilde{p}})\frac{v}{u(1+\sqrt{1-\frac{v^2}{c^2}})} = (\frac{h}{\tilde{p}})\frac{u(1+\sqrt{1-\frac{v^2}{c^2}})}{v\cos(\vec{p},\tilde{p})} = (\frac{v}{f})\frac{\sqrt{\cos(\vec{p},\tilde{p})}}{1+\sqrt{1-\frac{v^2}{c^2}}} \right\}$	
$\{\cos(\vec{p},\tilde{p})=1\} \Rightarrow \left\{ \begin{array}{l} (\frac{\tilde{E}}{E_k})^2 = (\frac{p}{\tilde{p}})^2 = (\frac{m}{\tilde{m}})^2 = \left[\frac{u}{v}(1+\sqrt{1-\frac{v^2}{c^2}})\right]^2 = \left[\frac{v}{u(1+\sqrt{1-\frac{v^2}{c^2}})}\right]^2 = 1, \\ E_k = \tilde{E},\ p = \tilde{p},\ u = \frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}},\ \lambda = \frac{h}{p} = \frac{h}{\tilde{p}} \end{array} \right\},$	$\{\cos(\vec{p},\tilde{p})=-1\} \Rightarrow \left\{ \begin{array}{l} (\frac{\tilde{E}}{E_k})^2 = (\frac{p}{\tilde{p}})^2 = (\frac{m}{\tilde{m}})^2 = \left[\frac{u}{v}(1+\sqrt{1-\frac{v^2}{c^2}})\right]^2 = \left[\frac{v}{u(1+\sqrt{1-\frac{v^2}{c^2}})}\right]^2 = -1 \\ E_k = -j\tilde{E},\ p = j\tilde{p},\ u = \frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}},\ j^2 = -1 \end{array} \right\}$

From table T 4.1.1 we can conclude that corpuscular and wave momentum of a moving particle (which is also represented as a matter-wave object) should act in the same direction, since the only logical and reasonable result is that the angle between them should stay in the following limits: $0 < \cos(\vec{p},\tilde{p}) \le 1 \Rightarrow -\frac{\pi}{2} \le [\angle(\vec{p},\tilde{p})] \le \frac{\pi}{2}$.

The conclusion we could draw from all the elaborations in this chapter (starting from (4.1) until T4.1. and T4.1.1) is that group, and phase velocities of any wave motion (or wave group) can be presented on two mutually equivalent (not contradictory, and mixed) ways, from two very different theoretical and conceptual platforms (see A and B, below). The first of the mentioned platforms (A) almost exclusively belongs to the ordinary wave-motions physics from Classical Mechanics and Acoustics (as wholly explained in Chapter 4.0. around equations (4.0.6) until (4.033)), such as,

(4.1.1.1.)-a

A) $$v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda},\ u = \lambda f = \frac{\omega}{k}.$$

The second point of view or platform (regarding group and phase velocity), not in conflicts with A), is on some way related to Relativity theory, producing the following results,

(4.1.1.1.)-b

B) $$u = \frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}},\ v = \frac{2u}{1+\frac{uv}{c^2}},\ 0 \le 2u \le \sqrt{uv} \le v \le c,$$

Also, at the same time both (A & B) have mutually mixed (not contradictory) uniting relations, and relations with energy concepts of Quantum Mechanics, such as,

(4.1.1.1.)-c

(A & B)

$$v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} = u + p\frac{du}{dp} = \frac{d\omega}{dk} = \frac{d\tilde{E}}{dp} = h\frac{df}{dp} = \frac{df}{df_s} = \frac{2u}{1+\frac{uv}{c^2}},$$

$$u = \lambda f = \frac{\omega}{k} = \frac{\tilde{E}}{p} = \frac{hf}{p} = \frac{f}{f_s} = \frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{E_k}{p},\ f_s = k/2\pi,$$

$$d\tilde{E} = hdf = c^2 d(\gamma m) = mc^2 d\gamma,\quad \frac{df}{f} = (\frac{dv}{v})\cdot\frac{1+\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{v^2}{c^2}} \Rightarrow \frac{\Delta f}{\bar{f}} = (\frac{\Delta v}{\bar{v}})\cdot\frac{1+\sqrt{1-\frac{\bar{v}^2}{c^2}}}{1-\frac{\bar{v}^2}{c^2}},$$

$$\left(\lambda = \frac{h}{p},\ u = \lambda f,\ \tilde{E} = hf,\ \frac{u}{v} = \frac{hf}{pv} = \frac{\tilde{E}}{pv},\ \gamma = 1/\sqrt{1-\frac{v^2}{c^2}}\right).$$

The fact is that one quantum of elementary wave-packet energy is equal to $\tilde{E} = hf$. *This relation is correct, but it is in some way only intuitively and empirically postulated, or fitted to get results in relation to Blackbody radiation, and later successfully used to explain Photoelectric and Compton Effect, Bragg's diffraction, etc. Here, we see that* $\tilde{E} = hf$ *it is a consequence, and part of relations found in **(4.1.1.1.)-a,** and **(4.1.1.1.)-b**, meaning that it also has specific much more significant theoretical background than just useful assumption.*

*When presenting specific particle in motion, the most significant and probably only relevant links to Relativity theory (without covariant or Lorentz invariant misrepresentations), are definitions of particle "**proper time** τ", "**proper mass** m_0" and "**proper energy** E_0", such as,*

(4.1.1.1.)-d

C)

$$dt = \gamma d\tau,\ d\tau^2 = dt^2 - dr^2/c^2 = \text{invariant}$$

$$(dr^2 = dx^2 + dy^2 + dz^2)$$

$$\begin{bmatrix} m = \gamma m_0 \\ E = \gamma E_0 = \gamma m_0 c^2 \\ p = \gamma p_0 = mv = \gamma m_0 v \end{bmatrix} \Rightarrow p^2 - \frac{E^2}{c^2} = \text{invariant} = -\frac{E_0^2}{c^2}.$$

Proper time τ *(measured with a single co-moving clock, linked to the particle in question) has the same meaning as well elaborated and clarified by Thomas E. Phipps, Jr. in [35], and proper mass and proper energy* m_0, E_0 *are only analogical names introduced here to underline the analogy with proper time (* $dt = \gamma d\tau \leftrightarrow m = \gamma m_0, E = \gamma E_0$*), but in reality meaning rest mass and rest energy. Everything else what we know from Einstein-Minkowski 4-vectors formalism should be explicable based on here introduced "**proper parameters**". Inertial "frame time"* t *is the time measured by a spatially extended set of clocks at rest in that inertial frame (and something similar is valid for inertial "frame mass" and inertial "frame energy",* m *and* E *(see T.2.2-3 and (2.4-17.1) in the second chapter of this book).*

....

The equation that relates group and phase velocity, referenced as (4.1.1.1)-b, was originally developed by analyzing the superposition of simple harmonic sinusoidal waves (see equations 4.06 to 4.0.33). However, a similar relation can also be derived from a more "corpuscular and mechanical" perspective, as elaborated in Chapter 4, particularly around equations 4.0.73 to 4.0.76. Moreover, as discussed in Chapter 2, the same relations between group and phase velocity found in (4.1.1.1)-c are fully applicable *to planetary systems (see "2.3.3. Macro-Cosmological Matter-Waves and Gravitation"). Importantly, all of this holds without any need to involve stochastic or probabilistic concepts.*

These platforms and areas, summarized under (4.1.1.1)-a, b, c, d, provide a foundation for understanding wave-particle duality or unity. Even if there are conceptual challenges within Relativistic and Quantum theory, the relations between group and phase velocity remain valid and function correctly (see further discussion in Chapter 2, around equations 2.4-11 to 2.4-17.1). These mixed and mutually reinforcing relations also support the introductory concepts of energy and wave packets in Quantum Mechanics. Given that the Classical Mechanical case of group and phase velocity is always valid, simple, and clear, we can conclude that the extension and connection of these ideas within Relativistic and Quantum theory are sufficiently well formulated.

Notably, we have not addressed statistical and probabilistic concepts here, as they are not yet necessary for this discussion. It is also important to acknowledge that the premises of Relativistic theory and Classical Mechanics regarding velocities and motions are fundamentally different, and sometimes challenging or unclear in their distinctions (especially in Relativistic theory). Nonetheless, through this discussion (with A), B), and C), we have demonstrated a specific analogical and mathematical compatibility that works sufficiently well within the framework of Physics, despite the inherent differences between the two theories.

While one might criticize these explanations as somewhat disorganized or based on simplified analogical conclusions, this critique is valid only if the reader is not willing to be intellectually flexible and open-minded. Regardless, the conclusions and results presented here are indicative and highly motivating for further exploration of matter-wave velocities and related properties. This opens the door for future projects to address these issues more thoroughly.

To conceptualize matter waves, we could say that all waves and oscillations known in Physics, including acoustic and electromagnetic waves, are matter waves. This goes beyond the traditional understanding of wave-like functions as mere probabilities, possibilities, or event distributions. Mathematics used to describe all matter waves belonging to the domain of Analytic Signal functions (see more in Chapter 4). Particles themselves are stabilized formations of matter waves. The author of this book believes that a necessary ingredient for stable particle creation, arising from specific combinations of complex matter waves, is the involvement of electromagnetic components. Electromagnetic matter waves act as a "binding" or "gluing" medium, connecting other non-electromagnetic energy-momentum entities in the creation of stable particles.

One of the significant misunderstandings within Relativity theory is the treatment of the speed of light, \(c\), in a vacuum as a unique and fixed constant. The reasons for this misunderstanding are thoroughly explained in references [35], [80], and [81]. In brief, Relativity theory assumes that the speed of a photon in a vacuum is constant regardless of the motion of its source or observer, which is not universally correct. The resulting mathematics, including velocity addition in Relativity theory, is also not universally valid. Light waves or wave packets, such as photons, behave like other waves, with their velocities depending on the motion of the source and observer, and subject to certain limits of variability.

To illustrate this point, consider a case where a light source is approaching a receiver with velocity v_s, and compare it to a case where the same light source moves in the opposite direction with the same velocity. According to Relativity theory, in both cases, the photons should have the same group velocity, c. However, this assumption is not universally applicable and requires reconsideration.

If this is correct, we should have,

$$\left\{\begin{pmatrix} v = u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda} = u + p\frac{du}{dp} = h\frac{df}{dp} \\ u = \lambda f = \frac{\omega}{k} = \frac{hf}{p} \end{pmatrix} \wedge \begin{bmatrix} (v + v_s) = (v - v_s) = c \\ v_s = const. << c \end{bmatrix}\right\} \Rightarrow$$

$$\Rightarrow \left[u + v_s - \lambda \frac{d(u + v_s)}{d\lambda} = u - v_s - \lambda \frac{d(u - v_s)}{d\lambda} \right] \Leftrightarrow v_s = 0, v \neq c$$

and

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

$$\left\{\begin{pmatrix} v = \dfrac{2u}{1+\dfrac{uv}{c^2}} \\ u = \dfrac{v}{1+\sqrt{1-\dfrac{v^2}{c^2}}} \end{pmatrix} \wedge \begin{bmatrix} (v+v_s) = (v-v_s) = c \\ v_s = const. << c \end{bmatrix}\right\} \Rightarrow$$

$$\left[\frac{v+v_s}{1+\sqrt{1-\frac{(v+v_s)^2}{c^2}}} + v_s = \frac{v-v_s}{1+\sqrt{1-\frac{(v-v_s)^2}{c^2}}} - v_s\right] \Leftrightarrow [v+2v_s \cong v-2v_s] \Leftrightarrow v_s = 0, v \neq c \qquad \textbf{(4.1.1.1.)-e}$$

The conclusion from (4.1.1.1.)-e is clear and distinct: only when source velocity v_s is equal to zero mentioned light, or any wave group velocities are mutually equal.

We can now test the statement that all monochromatic photons (with different frequencies; -for instance f_1 and f_2, $f_1 \neq f_2$*) should have the same speed equal to **c**, as follows,*

$$\begin{pmatrix} v = -\lambda^2 \dfrac{df}{d\lambda}, \; u = \lambda f \\ \wedge \\ v_1 = v_2 = v, \; u_1 = u_2 = u \end{pmatrix} \Rightarrow \left(-\lambda_1^2 \frac{df_1}{d\lambda_1} = -\lambda_2^2 \frac{df_2}{d\lambda_2}, \; \lambda_1 f_1 = \lambda_2 f_2\right) \Rightarrow \frac{d\lambda_2}{\lambda_2^2} = \frac{df_2}{df_1}\frac{d\lambda_1}{\lambda_1^2}, \; \frac{1}{f_1^2}\frac{df_1}{d\lambda_1} = \frac{1}{f_2^2}\frac{df_2}{d\lambda_2} \Rightarrow \qquad \textbf{(4.1.1.1.)-f}$$

$$\Rightarrow \left(\frac{\lambda_2^2\lambda_1^2}{f_1^2}\frac{df_1}{d\lambda_1} = \frac{\lambda_1^2\lambda_2^2}{f_2^2}\frac{df_2}{d\lambda_2}\right) \Rightarrow \left(\frac{\lambda_2^2}{f_1^2} v_1 = \frac{\lambda_1^2}{f_2^2} v_2\right) \Leftrightarrow \left(u_2^2 v_1 = u_1^2 v_2\right) \Leftrightarrow f_1 = f_2 = f, \; \lambda_1 = \lambda_2 = \lambda.$$

The result of (4.1.1.1.)-f is that such a generally valid solution does not exist, and that only mutually identical, monochromatic photons (or other matter-wave groups) could have the same group and phase velocity.

*Now we can safely say that all different photons have a different group and phase velocities, and what we consider as the universal physical constant, **c = 299792458 m/s**, is the only specific mean value of the "white" light speed in a vacuum, but anyway extremely relevant in our world of physics. From experimental practices, we also know that light speed standard deviation or variance, considering all different light sources and photons (in a vacuum), is small. Real universal constant **c** should be more adequately linked only to vacuum properties, since* $c = 1/\sqrt{\varepsilon_0\mu_0}$ *, and consequences of such situation are that speed of light in a vacuum has very narrow velocity distribution curve around* $c = 1/\sqrt{\varepsilon_0\mu_0}$. *Repeated astronomic and laboratory measurements of light speed **c** are showing that it is not possible to find non-doubtful, clear, unique, stable, and constant value, regardless statements that measured deviations are expected as methodological and instrumentation related errors. The same situation, with similar measurements' facts and conclusions, is also applicable and valid for Newton gravitational constant G. Both **c** and G are not entirely constant, stable, and independent from dynamic and geometry conditions related to measurements, and Relativity theory postulates about **c** are not entirely correct (but numerical variations in different measurement conditions are sufficiently small so we can say that **c** and G are constants, at least within our solar system).*

*There should be a certain much stronger and more essential (ontological) relation and direct coupling between **c** and G, still not exposed in contemporary physics. Let us try to create such G and **c** hypothetic connections using analogies exercised in the first and second chapter of this book (see T.1.2 until T.1.8, from the first chapter, and T.2.2-1, T.2.2-2, T.2.8., (2.4-13), (2.4-5.1), (2.11.13-1)-(2.11.13-5), (2.11.23), (2.11.24), (2.11.14-4), from the second chapter).*

$$\left\{\begin{array}{l} F_{e/1,2} = k\frac{q_1q_2}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2} \\ F_{g/1,2} = G\frac{m_1m_2}{r^2} = \frac{1}{4\pi g_0}\frac{m_1m_2}{r^2} = \frac{1}{4\pi g_0^{'}}\frac{m_1c \cdot m_2c}{r^2} = \frac{1}{4\pi g_0^{'}}\frac{p_1^* \cdot p_2^*}{r^2} = \frac{c^2}{4\pi g_0^{'}}\frac{m_1m_2}{r^2}, \\ q_{1,2}\left(\begin{array}{c}\leftrightarrow\\ \text{analog to}\end{array}\right) p_{1,2}^* = m_{1,2}c,\ k = \frac{1}{4\pi\varepsilon_0}\left(\begin{array}{c}\leftrightarrow\\ \text{analog to}\end{array}\right) G = \frac{1}{4\pi g_0} = \frac{c^2}{4\pi g_0^{'}}, \\ \left[c = 1/\sqrt{\varepsilon_0\mu_0}\left(\begin{array}{c}\leftrightarrow\\ \text{analog to}\end{array}\right) c = 1/\sqrt{g_0\rho_0}\right] \Rightarrow \varepsilon_0\mu_0 = g_0\rho_0, \varepsilon_0 = \frac{1}{\mu_0c^2}, \end{array}\right\} \Rightarrow \quad \textbf{(4.1.1.1.)-g}$$

$$\Rightarrow g_0 = \frac{1}{4\pi G} = \frac{g_0^{'}}{c^2} = \frac{1}{\rho_0c^2}, G = \frac{1}{4\pi g_0} = \frac{\rho_0c^2}{4\pi} = \frac{\rho_0}{4\pi\varepsilon_0\mu_0} = \frac{c^2}{4\pi g_0^{'}}, \rho_0 = \frac{4\pi G}{c^2} = \frac{1}{g_0c^2}$$

*Also, here is a place to give expressions for the "fine structure constant" α and to ask the question about possible relations between α, **c** and G,*

$$\alpha = \frac{2\pi ke^2}{hc} = \frac{e^2}{2\varepsilon_0 hc} = \frac{\mu_0 ce^2}{2h} = \frac{e^2}{2h}\sqrt{\frac{\mu_0}{\varepsilon_0}} \cong \frac{1}{137}$$

$$\text{e-electron charge}, k = \frac{1}{4\pi\varepsilon_0}, h = \text{Planck constant}.$$

Of course, here we are dealing with intuitive and brainstorming, hypothetical options, expecting that we will create specific physics-relevant concepts and theory as a better replacement. ♣**]**

4.1.2. Matter Waves Unity and Complementarity of Linear, Angular and Fluid Motions

To introduce the concept of matter waves in relation to rotation, it is essential to first examine the analysis of two-body binary systems, as discussed in Chapter 2 (see equations from 2.4-11 to 2.4-18 and “2.3.2. Macro-Cosmological Matter-Waves and Gravitation” from 2.11.11 to 2.11.23). The author of this book asserts that linear particle motion, particularly at the micro and elementary particle level, likely includes elements of rotation, spinning, and torsion field components. These rotational elements could be accompanied by vortices, turbulence, and electromagnetic field manifestations. The torsion field components may have diverse origins and are present in various ways, including the spinning of particles. This concept will be explored further.

Given the analogy between electric and gravitational fields, as well as between magnetic fields and the "field of mass rotation" (see T.1.6, T.1.8, T.2.2, [3], and [4]), there is potential to develop a Maxwell-like General Theory of Gravitation. This idea, while still in its early stages, opens new possibilities for understanding gravitational phenomena.

The well-known phenomenon of de Broglie’s waves confirms that any rectilinear motion of a micro-particle with momentum can be "dualistically" represented as a wave, with its wavelength directly related to the particle's linear momentum. Waves, including matter waves, are generally connected to some background rotation, which may be associated with the wave source. However, this rotation is not always evident in matter waves. Indirectly, we can infer that any rectilinear motion is likely accompanied by a "field of effective mass and electromagnetic energy rotation, including spinning." In some instances, elements of mechanical mass rotation should be observable, since de Broglie matter waves are oscillations linked to linear motion.

This section will explore the origins of such hidden spinning phenomena. It suggests that some form of distributed mass rotation follows any particle in linear motion. While this rotation may not always be detectable, it exists on a phenomenological or space-time dependent energy-distribution level. This lack of clarity often arises because de Broglie waves produce measurable and mathematically predictable experimental results, yet the associated rotational effects remain elusive. One hypothesis is that our universe has more dimensions than we currently detect, and the associated wave components may be linked to this multidimensional framework. However, this explanation might not be the most productive path forward at present.

What is known is that all subatomic particles and quasi-particles in motion exhibit real, measurable spin and orbital moment characteristics. These characteristics are linked to electromagnetic moments and dipoles and behave according to the principles of de Broglie matter waves. Quantum theory often avoids addressing actual particle rotation or spinning, instead using terms like intrinsic spin, orbital moments, electron clouds, energy states, and statistical distributions. However, rotation and spinning are ubiquitous in planetary, solar system, and galactic motions, suggesting that rotation is a fundamental aspect of nature rather than a random occurrence.

While there are chaotic, non-circular motions in the universe due to impacts, scattering, and explosions, relatively stable and periodic motions, such as those found in planetary and solar systems, tend to produce circular and elliptical orbits. Matter waves can be detected in the motion of fluids or particles within a fluid, manifesting as vortices, turbulence, and spinning effects. These effects are often accompanied by complex electromagnetic phenomena. On a macro scale, the frequency of these matter waves, with their spinning properties, could define a local "time clock" tied to a dominant center of mass. For example, Earth's dominant center of mass is our planet itself, and our most relevant time scale is related to Earth's rotation around the Sun.

Modern Quantum Mechanics typically treats de Broglie waves as probability or "possibility" waves, a concept that is often criticized for lacking tangible meaning. While this is still one of the most successful mathematical models of the micro-world, the author of this book suggests that de Broglie matter waves have a more general, empirically tangible, and deterministic nature. This view is supported by the fact that linear particle motions can be seen as segments of larger orbital motions (see [36], Anthony D. Osborne, and N. Vivian Pope). Matter waves, in this sense, are extensions of energy-momentum exchanges, balancing the orbital and spinning moments of interacting particles.

In almost all known situations when de Broglie wave phenomenology has been considered being a relevant event (directly or indirectly measured or detected), we can find the interactions of two bodies, particles/quasi-particles in a mutually relative motion, where one (test) moving particle or its energy mass-equivalent, $\mathbf{m_1 = m}$, is significantly smaller than the other particle, $\mathbf{m_2 = M >> m}$. We could also say that in a laboratory coordinate system $\mathbf{m_1 = m}$ has velocity v_1 and $\mathbf{m_2 = M}$ has velocity v_2, where $v_1 >>> v_2$. A "bigger particle", $\mathbf{M}$, is usually a scattering target, a diffraction plate, atom nucleus, instrument in a specific laboratory, etc., (to be closer to many commonly known experimental situations regarding de Broglie matter waves).

Let us now analyze the same situation in its center of mass coordinate system. It can be shown that we can always (dynamically and mathematically) transform rectilinear motion of the test particle $m_1 = m$ to a kind of equivalent, a rotation-similar motion of a reduced mass, $\mu = m_r = mM/(m+M) \approx m$ around its center of gravity (that has a significantly more significant mass, $m_c = m + M \approx M >> m_r \approx m$). The rotating circle-radius of $\mathbf{m_r}$ around $\mathbf{m_c}$, will be equal to the real distance between interacting particles, $\mathbf{r_{12}}$. As a general case, $\mathbf{r_{12}}$ would have a space-time evolving value, until eventually stabilizing on certain closed orbit, respecting Kepler, and Newton-Coulomb laws, including possible electromagnetic interactions and couplings (when this is applicable, and when optimal energy packing rules based on standing-waves relations can be satisfied; -see more in chapter 10). The natural and spontaneous tendency of a small mass $\mathbf{m_r}$ to develop its motion towards certain spiral or eventually circular motion, around bigger mass $\mathbf{m_c}$, (especially indicative in their center of mass reference system), is familiar to an analogy and unity between Newton and Kepler's laws of gravitation. This could be the consequence of global orbital and spin momentum conservation. Here it is particularly useful to see the brainstorming example in the second chapter with expressions (2.4-11) - (2.4-18), which can be considered as a natural and straightforward introduction to matter waves understanding. To support the same idea better, we can notice that Newtonian attractive force, between two masses, is equally valid, both in Laboratory and in Center of Mass System. This is leading to an equal force expressions (in cases of non-relativistic motions), for instance, $F_{12} = G\frac{m_1 m_2}{r_{12}^2} = G\frac{m_c m_r}{r_{12}^2} \Leftrightarrow m_1 m_2 = m_c m_r = mM$. Kepler laws are also the consequence of Newton laws (regarding gravitation) and vice versa, and Kepler laws are describing circular and elliptic (orbital or rotational) motions. Here is a part of the understanding why there should exist a natural tendency of a small particle (which could also be some planet) to rotate around a big particle (which could be its local Sun), and to have at the same time certain self-spinning motion. Tendency towards creating stable circular, orbital and spinning motions is well known in micro-world of atoms and elementary particles where electromagnetic forces are dominant (especially concerning innovative atom and elementary particles modeling from Bergman, Lucas, and others; - see [16] to [25]). Here (in relation to Gravitation) we are specifically addressing electrically (and magnetically) neutral masses, but in cases if specific free or dipole-like electromagnetic charges are present (in two-body interactions), the same rotation-related tendency should be even more present (supported by direct and striking analogies between Newton and Coulomb's laws; - see the second chapter: equations (2.4-7) to (2.4-10) and "*2.3.3. Macro-Cosmological Matter-Waves and Gravitation*", around equations (2.11.13-1) - (2.11.13-5)). Based on analogies "**Photon-Moving-Particle**" (see "*4.1.1.1. Photons and Particle-Wave Dualism,* and T.4.0, at the beginning of this chapter), we could conclude that particles in linear motion should have associated helical or spinning matter-waves components. It is also interesting to see ideas about intrinsic nature of all motions to be globally presentable as combinations of rotating and spinning motions in: [36], Anthony D. Osborne, & N. Vivian Pope, "An Angular Momentum Synthesis of 'Gravitational' and 'Electrostatic' Forces".

Until the present (respecting all valid approximations, and mentioned analogies), we can express the balance of total kinetic energy of two particles (or two mutually interacting bodies) in question as,

$$E_{k1} + E_{k2} = E_{km} + E_{kM} = E_{kc} + E_{kr} = \frac{1}{2}m_c v_c^2 + \frac{1}{2}m_r v^2, E_{kr} = \frac{1}{2}m_r v_r^2 \cong \frac{1}{2}mv^2 = E_{k1} \cong \frac{1}{2}\mathbf{J}\omega_m^2,$$

$$\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}Mv_2^2 = \frac{1}{2}m_c v_c^2 + \frac{1}{2}m_r v_r^2 \cong \frac{1}{2}Mv_c^2 + \frac{1}{2}mv^2 \left\{= \frac{1}{2}Mv_c^2 + \frac{1}{2}\mathbf{J}\omega_m^2\right\},$$

or (analogically and by extrapolation concluding, "velocity-upwards", based on always applicable approximations when $v_i << c$) it should also be valid (for higher and relativistic velocities),

$$\begin{Bmatrix} \left(E_{ki} = \frac{1}{2}m_i v_i^2 = \frac{1}{2}p_i v_i\right)_{v\uparrow} \cong \frac{\gamma_i m_i v_i^2}{1+\sqrt{1-v_i^2/c^2}} = \frac{p_i v_i}{1+\sqrt{1-v_i^2/c^2}}, p_i = \gamma_i m_i v_i \\ \left(E_{kr} = \frac{1}{2}m_r v_r^2\right)_{v\uparrow} \cong \frac{p_r v_r}{1+\sqrt{1-v_c^2/c^2}} = \frac{m_r v_r^2}{1+\sqrt{1-v_c^2/c^2}}, \; p_r = m_r v_r, \; \gamma_r = 1 \end{Bmatrix} \Rightarrow$$

$$\Rightarrow \frac{\gamma_1 m v_1^2}{1+\sqrt{1-v_1^2/c^2}} + \frac{\gamma_2 M v_2^2}{1+\sqrt{1-v_2^2/c^2}} \cong \frac{\gamma_c m_c v_c^2}{1+\sqrt{1-v_c^2/c^2}} + E_{kr} \left\{\cong \frac{\gamma_c M v_c^2}{1+\sqrt{1-v_c^2/c^2}} + \frac{\mathbf{J}\omega_m^2}{1+\sqrt{1-v_c^2/c^2}}\right\}.$$

Another analogical and equally possible strategy would be to account for all relevant, total energies, for instance,

$$\left(mc^2 + \frac{\gamma_1 m v_1^2}{1+\sqrt{1-v_1^2/c^2}}\right) + \left(Mc^2 + \frac{\gamma_2 M v_2^2}{1+\sqrt{1-v_2^2/c^2}}\right) = \left(m_c c^2 + \frac{\gamma_c m_c v_c^2}{1+\sqrt{1-v_c^2/c^2}}\right) + \left(m_r^* \cdot c^2 + E_{kr}\right) \Leftrightarrow$$

$$\Leftrightarrow \gamma_1 mc^2 + \gamma_2 Mc^2 = \gamma_c m_c c^2 + \left(m_r^* \cdot c^2 + E_{kr}\right) \Rightarrow m_r^* = m_{or} = \gamma_1 m + \gamma_2 M - \gamma_c m_c - \frac{E_{kr}}{c^2} \cong 0,$$

$$E_{kr} = \frac{m_r v_r^2}{1+\sqrt{1-v_c^2/c^2}} = \frac{p_r v_r}{1+\sqrt{1-v_c^2/c^2}},\ \gamma_i = \frac{1}{\sqrt{1-v_i^2/c^2}},\ p_i = \gamma_i m_i v_i, p_r = m_r v_r, \gamma_r = 1.$$

Obviously that here we could also speculate with an existence of certain initial-velocities dependent, distributed and *"**hidden rest or reduced mass**"* $m_{ro} = m_r^* \neq 0$, which in some way could have specific internally balanced dynamics, but macroscopically it has resulting zero linear and zero angular moments, because $\vec{p}_m + \vec{p}_M = \vec{p}_1 + \vec{p}_2 = \gamma_1 m\vec{v}_1 + \gamma_2 M\vec{v}_2 = \vec{p}_c = \gamma_c m_c \vec{v}_c = \gamma_c (m+M)\vec{v}_c$.

This is only possible if spatially distributed linear and angular moments belonging to m_r are reacting as couples of mutually opposite and equal-intensity rotating vectors (this way generating certain self-balanced torque-couple, and performing kind of self-compensated rotation around m_c, with mutually opposite (and mutually canceling) spin moments, directly related to creation of de Broglie matter waves. Practically, a particle m_r is adequately represented by two matter waves propagating in mutually opposite ways. We know that the general solution of Classical Wave equations is always two wave functions propagating in mutually opposite directions. *Here we could also think about finding possible analogical connections of rotating and spinning m_r with Falaco Solitons; -see [15] and [40]).*

Here we are intentionally introducing somewhat hypothetical, dynamically equivalent, or possible and effective rotational or orbital motion, causally related to the kinetic energy of the reduced mass m_r. ***Such mass has only its kinetic energy (since its rest mass does not exist), meaning that this is something like a pure wave, or like a photon.*** For better conceptual understanding, see more in the second chapter, around equations (2.11.13-1) - (2.11.13-5)) to describe de Broglie matter waves, where specific frequency and wavelength should start to be relevant associated parameters (see later (4.3) for more details). Of course, we know that m_r and m_c are physically artificial and invisible (but effectively or mathematically existing) masses, being mathematical products related to Center of Mass reference system. We always know mathematically where such masses are effectively placed, we know their active energies, moments and velocities, and the main idea here is to show that such artificial masses configuration is the generator or source of matter waves (being extraordinarily significant and noticeable when mentioned conceptualization and approximations are applicable, especially in cases if interacting masses are electromagnetically charged).

In our laboratory system, we can see only initial masses m_1 and m_2, but we also know (mathematically) the exact positions where m_r and m_c should effectively exist. In such zones of spatial and temporal (active) presence of m_r and m_c we could expect real and measurable existence of specific fields, forces, and energy-momentum (phantom) matter waves states that could have an influence on motions of nearby passing objects and waves (since both linear and angular or spinning moments should be coincidently conserved).

It will be shown that kinetic energy belonging to m_r is the energy of the relevant (associated) matter wave (see later, at the end of this chapter "4.1.4. Matter Waves and orbital motions", where the same rotation related concept is becoming clearer and more comfortable to accept). If initial two-body situation participants already have certain angular and orbital moments before interaction (presenting some orbital motion in a larger scale), when they start creating two-body coupled pair, whatever created that way (during and after interaction), would also have certain angular, orbital and/or spinning moment, to satisfy global angular moments and energy conservation laws. Such orbital and spinning moments are elements of de Broglie matter waves.

Practically, we are gradually elaborating that ***matter wave nucleus or source is created from the kinetic energy belonging to a reduced mass*** (in any two-body-problem situation when one mass is much smaller than the other mass), what we should be able to present as,

$$\left(E_{kr} = \frac{p_r v_r}{1+\sqrt{1-v_c^2/c^2}} = \tilde{E} = hf\right) \Rightarrow \begin{pmatrix} dE_{kr} = v_r dp_r = d\tilde{E} = hdf \\ v_1 dp_1 + v_2 dp_2 = v_r dp_r + v_c dp_c \end{pmatrix}$$

$$\left(u_r = u = \lambda_r f_r = \lambda f,\ \lambda = \frac{h}{p}\right) \Rightarrow du = fd\lambda + \lambda df = fd\lambda + \lambda\frac{v_r dp_r}{h} = fd\lambda + v_r\frac{dp_r}{p_r} = u\frac{d\lambda}{\lambda} + v_r\frac{dp_r}{p_r} \Rightarrow$$

$$\frac{du}{u} = \frac{d\lambda}{\lambda} + \frac{v_r}{u_r}\frac{dp_r}{p_r} = \frac{d\lambda}{\lambda} + \frac{v}{u}\frac{dp_r}{p_r} = \frac{d\lambda}{\lambda} + \left(\frac{v}{u}\right)\frac{d\left(\frac{h}{\lambda}\right)}{\left(\frac{h}{\lambda}\right)} = \frac{d\lambda}{\lambda} - \left(\frac{v}{u}\right)\frac{d\lambda}{\lambda} = \left(1-\frac{v}{u}\right)\frac{d\lambda}{\lambda},$$

$$\left((u,v) << c,\ v = 2u \Rightarrow \frac{du}{u} \cong \frac{d\lambda}{\lambda} + 2\frac{dp_r}{p_r} = \frac{dp_r}{p_r}\right), \left((u,v) \approx c \Rightarrow \frac{du}{u} \cong \frac{d\lambda}{\lambda} + \frac{dp_r}{p_r} = 0\right)$$

Using simplified terms, here we are hypothesizing that within any of possible "Two-Body Relations" mass m_r has a natural tendency to enter in a stable, inertial, and orbital motion around m_c. From Chapter 2., we already know that natural, stable, non-forced orbital motions are inertial motions, potentially hosting stable, standing matter waves; -see familiar elaborations around equations (2.11.13-1) - (2.11.13-9) from chapter 2). Here we neglected the possibility that Two-Body interacting participants could initially have angular, mechanical, and electromagnetic, spin and dipole moments, what should be the case facilitating creation (and understanding) of matter waves.

It is well established that all stable, uniform, stationary, and linear (or inertial) motions can be viewed as cases of circular or orbital motions with arbitrarily large radii of rotation. This applies to motions in a central force field as well. As a result, all stable, inertial, and linear motions can be interpreted as orbital in nature, implicitly suggesting the existence of a specific center of rotation. Additionally, such motions can be spatially stabilized when combined with gyroscopic stabilizers or self-spinning, gyroscopic effects.

In summary, the Universe exhibits both holistic and localized (or microscopic) rotation and spinning in various ways, which involve accelerated motions. These motions lead to effects such as centrifugal mass separation and the polarization of associated electric and magnetic dipoles. Due to spatial organization, the forces between these dipoles tend to be of the Coulomb (or Newtonian) attractive type. This is particularly evident in atoms where the mass difference between electrons and protons, coupled with accelerated motion, results in spatial deformation.

This leads to the hypothesis that all stable, inertial, linear, and orbital motions inherently possess self-spinning moments, or matter-wave characteristics. These motions may generate helix- or spiral-shaped matter waves, creating a spatial-temporal structure with periodic, repetitive (spinning) elements. From this concept of stabilized, standing matter waves, we can deduce insights into entanglement, matter-wave wavelengths, energy, and resonant quantization.

The most appropriate mathematical framework for modeling such matter waves already exists in the form of Complex and Hyper-complex Analytic Signals (for further details, see Chapters 4.0 and 10).

Let us now try to visualize de Broglie's matter waves as an associated rotating field manifestation (that has circular and spiral or spinning wave shape around the particle path) present in the space around mowing particle $m_1 \cong m_r \cong m$, which has the speed $\vec{v}_r = \vec{v}_1 - \vec{v}_2 \cong \vec{v}_1 = \vec{v}$. De Broglie wavelength could be visualized as certain (periodically repetitive) length measured along that spiral or helix line, and as a periodical interval between any of neighboring quasi-circles (see illustrations on Fig.4.1.2, Fig.4.1.3, Fig.4.1.4, Fig.4.1.5, equations under (4.3) and later).

The present concept found in Quantum Theory is that de Broglie matter waves are only indirectly detectable, and there is no explicit statement about the nature of such waves. Matter waves (in a micro world) are generally related to relative motions, impacts, diffractions and different interactions between electrons, protons, neutrons, atoms molecules, and other subatomic and micro-particles. Most of the mentioned particles also have measurable spin and orbital moment properties (but nobody is seriously considering that such spinning properties could have any relation to de Broglie matter waves). Such situation (about spin and orbital moment properties) is indicative, supporting the hypothesis that some associated form of spinning field (like the helix path on Fig.4.1.2, Fig.4.1.3, and Fig.4.1.4) naturally accompany every linear motion (much in the same way as magnetic fields accompany electrical currents and fields). We know that in many cases of matter forms charge-to-mass and gyro-magnetic ratios are or should be constant numbers (see chapter 2, equations (2.4-6) – (2.4-10)). Consequently, we could be very sure that de Broglie matter waves are closely related to some form of electromagnetic fields and waves, of course, coupled with associated inertial, spinning, and orbiting effects. As we know, photons also have spin and helical wave properties and shapes.

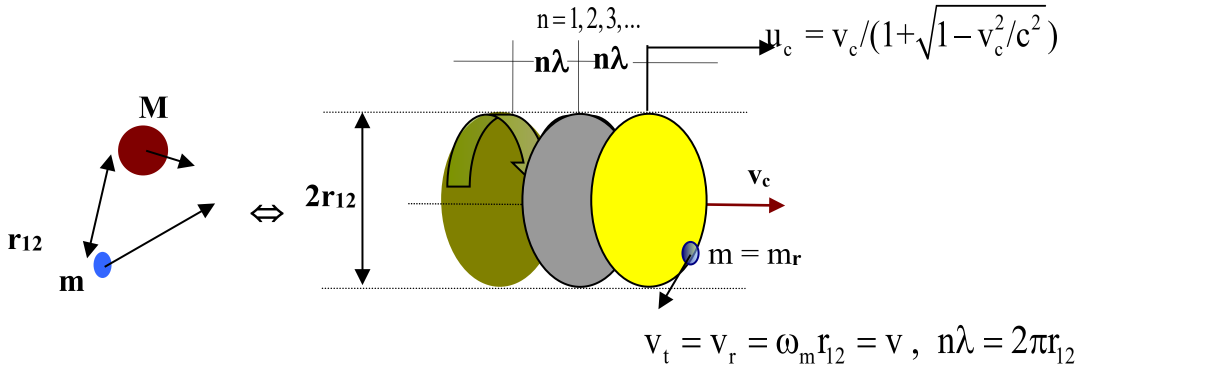


Fig.4.1. The two-body problem presented in the equivalent center of mass system

Let us now analyze in details the above-introduced common sense concept of de Broglie waves (as presented with Fig.4.1 and based on equations (4.1) and (4.2)), by analyzing the dynamically equivalent quasi-rotation of a small particle ($\mu = m_r = mM/(m+M) \approx m$) around a much bigger particle (**M**), or around its center of mass ($\mathbf{m_c} = m+M \approx M$). ***We will effectively analyze the kind of elastic impact situation between two mutually interacting masses with evolving elements of angular motions and rotation (appearing in a suitable center-of-mass reference system). In the same transitory process, we will also get certain spatial (or micro-volumetric) electromagnetic charges and dipoles polarization where Coulomb forces could be relevant.*** In the Laboratory System of coordinates, practically only mass **m** with velocity **v** moves towards mass **M**, and that mass **M** is almost standstill in the same system. The results of the same analysis will not change if we consider that the Laboratory System in question is fixed to the big mass **M**. To comply with de Broglie matter-waves hypothesis (or to rediscover expression for de Broglie wavelength), we should be able to show that de Broglie matter-wave wavelength (in the Laboratory System) originates from the effective or dynamically equivalent particle rotation around its center of mass (in its Center of mass system). This agrees with, $\lambda = \frac{h}{p} = \frac{h}{\gamma mv} = \frac{2\pi r_{12}}{n}$, $n = 1, 2, 3, ...$. Here we (hypothetically) say that the small particle $\mu = \mathbf{m}_r \approx \mathbf{m}$ effectively rotates in its Center of Mass System (around mass **M**) because from the analysis of the two-body problem we know that the total, (internal) angular momentum of such system exists, and can be expressed as: $\mathbf{L} = \mathbf{L}_m = \mathbf{J}_m\omega_m = m_r r_{12} \times v_{12} \cong m_r r_{12} \times v \cong m r_{12} \times v$. Now it is possible to find the value that should be equivalent to the particle angular velocity $\omega = \omega_m = v_{12}/r_{12} \cong v/r$ (valid for the Center of Mass System). Since the small "rotating particle", **m** at the same time makes:

a) **Linear motion in its Laboratory System,**
b) **As well as a kind of (at least one revolution) circular path motion in its Center of Mass System** (having in both situations the same linear speed equal $\mathbf{v}$, because of valid approximations and initial conditions: $m_r = mM/(m+M) \approx m$, $v_1 >>> v_2$, and because mass $\mathbf{M}$ is considered almost standstill),

we could express the particle kinetic energy in two usual ways for linear and rotational motions. For instance, in Classical Mechanics, the kinetic energy of linear particle motion is given by $E_k = \frac{1}{2}mv^2$, and if a particle is rotating, we have an analog kinetic energy expression $E_k = \frac{1}{2}\mathbf{J}\omega_m^2$, where ω_m is the circular, "mechanical" frequency (of rotation), and $\mathbf{J}$ is particle moment of inertia. Since in this example, the same particle is in linear motion and a kind of dynamically equivalent rotation (depending on the point of view), and because of valid approximations, which make particle kinetic energy in a Laboratory and Center of mass system quantitatively (almost) equal, we will have,

$E_k = \frac{1}{2}mv^2 = \frac{1}{2}pv = \frac{1}{2}\mathbf{J}\omega_m^2 = \frac{1}{2}\mathbf{L}\omega_m$, $\Rightarrow mv^2 = \mathbf{J}\omega_m^2 \Leftrightarrow \mathbf{J} = \frac{mv^2}{\omega_m^2} = \frac{m(\omega_m r_{12})^2}{\omega_m^2} = mr_{12}^2$ (see also similar elaborations around equations (2.11.13-1) - (2.11.13-9) in Chapter 2). In a differential form relationship between the linear and rotational aspect of the same motion should be $dE_k = vdp = \omega_m d\mathbf{L} = hdf = c^2 d(\gamma m)$. *Also, because of mentioned approximations and considering only cases when r_{12} is significantly larger than any other space dimension of the rotating particle in question, we have seen that particle's important moment of inertia is $\mathbf{J} = mr_{12}^2$ (which is the result also known for any single rotating particle).* ***We can also get the same result if the rotating particle is (mathematically) replaced by certain closed-space distributed mass, the thin-walls rotating torus of an equivalent wave energy formation.*** *All of that confirms* that linear motion of a certain particle, relative to another particle, could be effectively presented as rotation around their common center of mass. The other message from here elaborated concept is that ***matter waves are consequence or products of interactions between (at least two) masses. If interacting masses are mutually surrounded (and communicating) with electromagnetic fields, created matter waves will also have an electromagnetic nature. If there is another field around the mentioned two or multi-body interacting states, we will have matter waves of other nature.*** Here, we still assume that test-particle is not spinning (it is only making rotational, or orbital motion about much bigger particle). The same kinetic energy equivalency between linear and rotational nature of a test particle motion in question can be (analogically) expressed or extended as:

$$E_k = E_{km} = \tilde{E} = hf = pu = \begin{Bmatrix} \frac{1}{2}mv^2 \\ \frac{1}{2}J\omega_m^2 \end{Bmatrix}_{v<<c} \Leftrightarrow \begin{Bmatrix} \frac{mv^2}{1+\sqrt{1-\frac{v^2}{c^2}}} \\ \frac{J\omega_m^2}{1+\sqrt{1-\frac{v^2}{c^2}}} \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} \frac{pv}{1+\sqrt{1-\frac{v^2}{c^2}}} \\ \frac{L\omega_m}{1+\sqrt{1-\frac{v^2}{c^2}}} \end{Bmatrix} \Rightarrow$$

$$\begin{Bmatrix} E_{km} = \frac{pv}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{L\omega_m}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{J\omega_m^2}{1+\sqrt{1-\frac{v^2}{c^2}}} = (\frac{J4\pi^2 f_m^2}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{J4\pi^2}{1+\sqrt{1-\frac{v^2}{c^2}}}\cdot\frac{v^2}{(2\pi r_{12})^2} \\ = \frac{Jv^2}{1+\sqrt{1-\frac{v^2}{c^2}}}\cdot\frac{1}{(r_{12})^2} = \frac{\gamma m r_{12}^2 v^2}{1+\sqrt{1-\frac{v^2}{c^2}}}\cdot\frac{1}{(r_{12})^2} = \frac{\gamma mv^2}{1+\sqrt{1-\frac{v^2}{c^2}}}) = (\frac{2\pi L}{1+\sqrt{1-\frac{v^2}{c^2}}})\cdot f_m = (\frac{2\pi L}{n})\cdot f \Rightarrow \\ \Rightarrow \frac{2\pi L}{n} = h \Leftrightarrow L = L_m = n\frac{h}{2\pi} = \frac{E_{km}}{\omega_m}\left(1+\sqrt{1-\frac{v^2}{c^2}}\right) \end{Bmatrix} \Leftrightarrow$$

$$\Leftrightarrow \begin{Bmatrix} J = \gamma m r_{12}^2 = J_m,\ v = \omega_m r_{12},\ \gamma = (1-v^2/c^2)^{-0.5}, \\ 2\pi r_{12} = n\lambda = n\frac{h}{p} = \frac{v}{f_m} = n\frac{u}{f},\ n = 1,2,3,...,p = \gamma mv = h\frac{k}{2\pi} = \hbar k,\ k = \frac{2\pi}{\lambda},\ \hbar = \frac{h}{2\pi}, \\ \tilde{E} = hf = \hbar\omega = h\frac{\omega}{2\pi} = pu = \gamma mvu = E_k, u = \lambda f = v/(1+\sqrt{1-v^2/c^2}) = \frac{\omega}{k}, \\ dE_k = vdp = \omega_m dL = hdf = c^2 d(\gamma m). \end{Bmatrix}, \quad (4.3)$$

$$f_m = \omega_m/2\pi = \frac{f}{n}(1+\sqrt{1-\frac{v^2}{c^2}})\ (=)\ \text{frequency of mechanical rotation},\ f_m \neq f,$$

$$f = \omega/2\pi = u/\lambda = nf_m/(1+\sqrt{1-v^2/c^2})\ (=)\ \text{de Broglie} - \text{matter} - \text{wave frequency},$$

$$1 \leq \left[\frac{v}{u} = \frac{pv}{\tilde{E}} = \frac{n\cdot f_m}{f} = 1+\sqrt{1-\frac{v^2}{c^2}}\right] \leq 2,\ n = 1,2,3,...\ ,$$

$$\frac{1}{2} \leq \left[\frac{u}{v} = 1+\frac{\lambda}{v}\frac{du}{d\lambda} = 1+(\frac{h}{pv})\cdot(\frac{du}{d\lambda}) = \frac{hf}{pv} = \frac{\hbar\omega}{pv}\right] \leq 1 \Rightarrow$$

In cases when test particle is making orbital motion and spinning (in the same time), where E_{km} is kinetic, orbital or rotational, mechanical particle energy (like energy of linear motion), E_{ks} is the spinning kinetic particle energy, and S is the test particle spin moment, analogically concluding (as in (4.3)), we will have,

$$\left\{\begin{array}{l} E_k = E_{k(m+s)} = \tilde{E} = h \cdot f = \dfrac{(\vec{L}+\vec{S})\vec{\omega}_m}{1+\sqrt{1-\left(\frac{v^*}{c}\right)^2}} = (\dfrac{2\pi\left|\vec{L}+\vec{S}\right| \cdot \cos\varphi}{1+\sqrt{1-\left(\frac{v^*}{c}\right)^2}}) \cdot f_m = (\dfrac{2\pi\left|\vec{L}+\vec{S}\right|}{n}) \cdot f = \\ = \dfrac{\vec{p}^*\vec{v}^*}{1+\sqrt{1-\left(\frac{v^*}{c}\right)^2}} = hf^*,\ \vec{p}^* = \gamma(m+\Delta m)\vec{v}^*,\ \Delta m = \dfrac{E_{ks}}{c^2} = \dfrac{S\omega_s}{1+\sqrt{1-\left(\frac{v^*}{c}\right)^2}} \end{array}\right\} \Rightarrow \qquad (4.3)^*$$

$$\Rightarrow \frac{2\pi\left|\vec{L}+\vec{S}\right|}{n} = h \Leftrightarrow \left|\vec{L}+\vec{S}\right| = L_{(m+s)} = n\frac{h}{2\pi},\ \lambda^* = \frac{h}{p^*},\ f = \frac{\tilde{E}}{h},\ u^* = \frac{\tilde{E}}{p^*} = \frac{v^*}{1+\sqrt{1-\left(\frac{v^*}{c}\right)^2}}.$$

Here is a big part of the explanation regarding the real (orbital and spinning motions related) origins of Planck's energy formula $\tilde{E} = hf$, which should be complemented with (4.0.50) - (4.0.53) from Chapter 4.0. Practically, mechanical particle spinning is changing its matter-wave frequency and wavelength (meaning instead of $\lambda = \frac{h}{p}$ we will have $\lambda^* = \frac{h}{p^*}$).

We could also dynamically present the motion of the big mass $\mathbf{M}$ as performing a kind of rotation (with a much smaller radius than r_{12}) around the same center of mass, in its Center of mass system, similarly as we did for the small mass $\mathbf{m}$. However, considering already accepted approximations this would not bring us any new conceptual or mathematical benefit in the case analyzed here.

In most of the analyses of two-body situations, we are merely neglecting (or omitting) that participants (m_1, m_2), or (m, M) may carry initial, orbital and spin moments, as well as be electromagnetically dipole-polarized, or carry certain amounts of free electrical charges. Considering such (new) elements in two-body problem analysis will directly generate conclusions about the necessity of matter waves rotation and spinning. ***Of course, if initial spinning (of moving particles participants) does not exist, we will still have (self-generated) matter waves spiral spinning. This way we justify matter-waves wavelength and frequency, with (at least) two mutually canceling spinning moments (propagating in opposite directions, very much associating on solutions of second order Classical and Matter Waves differential equation), since conservation-laws are always valid.***

See later (in this chapter): Fig.4.1.1, Fig.4.1.1a, Fig.4.1.2, Fig.4.1.3 and Fig.4.1.4, the equations (4.3-0), (4.3-0) -a,b,c,d,e,f,g,h..., (4.3-1) until (4.3-3), and equations from (5.4.1) until (5.4.10) from the Chapter 5, where similar and equivalent concept of matter waves is additionally elaborated from a little bit different perspective. Particle-Wave Duality of the matter is so important that it should be completely explained and demystified. As long we have only complicated, unclear, probabilistic, artificial, and non-tangible concepts (regardless of how well mathematically operational), this cannot be a good explanation or picture about certain Physics related phenomenology (this could only be a sufficiently acceptable, or good modeling and fitting with flexible mathematical processing).

As we can see, from (4.3), combined with results from (4.2) and later from (4.3-1), we do not even need to have a real, visible (continuous and full-circle) rotation of two particles in the same plane (in the Laboratory System) to “generate” de Broglie matter-wave with specific wavelength and frequency. Any of two (somehow interacting) particles (**m** and **M**, where **m** could be a photon), in linear motion, can be presented as a sort of rotation of a mass $m_r = mM/(m+M)$ and mass $\mathbf{m_c} = \mathbf{m} + \mathbf{M}$ around their common center of mass. Effectively this kind of quasi-rotation is on some ways linked to associate de Broglie matter waves since it shows that the following fundamental relation could be valid: $2\pi r_i = n\lambda_i = n\frac{h}{p_i} = \frac{v_i}{f_m} = 2\pi\frac{v_i}{\omega_m}$ (what secures structural and orbital standing-waves stability and continuity of described motion). Of course, mentioned particle rotation, visible in the plane of the Center of mass (perpendicular to the velocity of the center of mass), can also be certain angular swing, without creating full circle in a Laboratory System, where kind of associated spinning field and wave motion around it will create matter waves in question. If **M** presents the big *mass in our Laboratory System*, and if $(\mathbf{M} >>> \mathbf{m}) \Rightarrow (\mathbf{m_r} \cong \mathbf{m})$, consequently, the motion of every small particle **m** in such system can be treated (approximately) as the motion of $\mathbf{m_r} \approx \mathbf{m}$ in the Center of mass system, satisfying relations given in (4.3). **This way, it becomes clear that the binding and surrounding (mutually interacting) fields in the space between m and M create de Broglie matter waves as a way of energy-momentum exchange and coupling between them**, also satisfying the following differential energy balance: $dE_k = vdp = \omega_m dL = hdf = c^2 d(\gamma m)$ (see the second chapter: Tables T.2.4, T.2.5 and T.2.6 and equations from (2.1) to (2.11.5)). Now it also becomes clear when and why de Broglie relation $\lambda = h/p$ is valid and applicable. We need to be careful in noticing a significant difference between mechanical rotating or revolving frequency of a rotating particle and spinning matter waves frequency that is a field related parameter (see (4.3) and (4.3-1)).

Now is the right place to mention an analogy between here-introduced concepts (of couplings and equivalency between linear and angular motions), as presented in Chapter 2. around equations (2.11-4), and on illustrations on Fig.4.1, Fig.4.1.2, Fig.4.1.3, Fig.4.1.4, Fig.4.1.5, and equations under (4.3) and later in this chapter, and prof. Eric's Laithwaite demonstrations of unusual and extraordinary spinning gyroscope effects (ref. [102]).

We should also underline that for the time internally interacting nature of structural elements of a particle is entirely neglected regarding its intrinsic orbital and linear moments (and regarding all other electric and magnetic properties, standing waves, spinning, and rotating states; -see equations (2.11.3), (2.11.4) and (2.11.5) from the second chapter).

De Broglie matter waves should belong to all other wave phenomena already known in Physics, including some of (hypothetical) fields proposed in the second chapter of this book by force laws (2.1) and (2.2). The challenging question that appears here is whether all matter waves (of different nature, regarding how we see and measure them) have their profound, hidden, or recognizable origins in the world of electromagnetism. **Since Maxwell equations (analogically) are the ones of hydrodynamics, fluid-flow type of equations, this should lead to the conclusion that theory dealing with electromagnetic phenomena is necessarily a part of classical mechanic's concepts.** We are now starting to see and know, or at least have a clear concept regarding **where, when, why and how** de Broglie matter waves are produced (see: Fig.4.1.2, T.4.4 and equations (2.5.1), (4.18), (4.5-1) - (4.5-3), (4.3-0), (4.3-0)-a,b,c,d,e,f,g,h,i… dealing with unity of linear and rotational motions).

We can now find the quantitative meaning of de Broglie wavelength concerning center-of-mass axial (or linear) motion. Let us determine the shortest axial distance, Δx, between two successive quasi-circles (on the helix line) from the Fig. 4.1, as $\Delta S = v_c \cdot \Delta t \cong \frac{mv}{M} \cdot \frac{1}{f_m} \cong \frac{mv}{M} \cdot \frac{1}{f_m} \cong \frac{mv}{M} \cdot \frac{n\lambda}{v} = n\frac{m}{M}\lambda <<< \lambda$, $n = 1, 2, 3, \ldots M >>> m$. It is clear that de Broglie matter waves are related only to a (kind of) circular motion since for the distance ΔS in the axial direction (when particle m would make one full circle, during the time interval $\Delta t = 1/f_m$) we shall get the length that is much shorter than actual de Broglie wavelength ($n\frac{m}{M}\lambda <<< \lambda$).

The "laboratory" or environment where de Broglie matter waves are generated is described through the interaction of "virtual objects" in the Center of Mass System. Physics and the Universe regard such systems as principal and dominant. The real interaction participants, masses m_1 = m and m_2 = M, are perceptual or laboratory parameters. However, in the Center of Mass System, these masses are replaced by "virtual reaction participants" m_r and m_c, because it is only in this system that rotational motion can be associated with the interaction. In this scenario, m_r rotates around m_c, or more precisely, both rotate around their common center of mass.

After introducing this concept of "virtual or equivalent rotation," which is mathematically and in terms of conservation laws equivalent to the real interaction, we can determine the frequency and wavelength associated with such a rotation. Subsequently, we find that de Broglie matter waves share the same frequency and wavelength, which supports the creation of matter waves being influenced by the complex interplay of forces and fields between approaching objects in their Center of Mass System (see Chapter 2 for more details). These objects, even before any physical impact or scattering, generate mutual couplings and new interaction participants. This becomes mathematically explainable when analyzing the two-body problem in the Center of Mass System, a topic covered in detail later in this chapter.

At this point, it would be useful to consult Fig. 4.1.3 and T.4.4 to explore new perspectives. One could imagine, for instance, that our reality is part of a multidimensional universe with phase-shifted universes. However, while intriguing, this remains speculative and akin to science fiction. Similarly, in two-body interactions, there may be exotic, virtual "magic and rare reactions" phenomena that Quantum Mechanics describes in terms of probabilities, where certain energy-momentum states can appear and disappear unexpectedly.

Another significant theoretical approach that could provide insight into two-body and multi-body problems comes from analogies with Electrostatics and Magnetostatics. The "method of mirror imaging" for electric and magnetic charges, fields, and currents has been successfully legitimized in solving many electromagnetic problems. In this method, a real electromagnetic charge interacts with its mirror image as if synchronously entangled, like how entangled photons behave. Although a mathematical construct, the method yields correct results when solving real-world problems. We could extend this analogy to consider that any motion or particle might have a "mirror image" and entanglement connection with its surrounding environment.

Applying this concept to two- and multi-body interactions or atomic structures could significantly enhance our understanding of Wave-Particle Duality and Matter Waves. For example, the well-known two-slit interference and diffraction experiments (with photons, electrons, and other particles) could be treated as mirror-imaging or entanglement scenarios. In these cases, all interacting objects are synchronously and permanently connected long before the actual interference occurs. This aligns with the two-body concept, incorporating both laboratory and Center of Mass System mathematics (see similar ideas in [36], *Immediate Distant Action and Correlation in Modern Physics*).

Moving forward, it will be necessary to explore how the mirror-imaging concept applies to entities that can generate useful mirror images. In addition to electromagnetic charges, matter states with spinning properties may be well-suited for such mathematical strategies, especially those based on action-reaction, induction effects, and conservation of moments.

The relationship between the center of mass and laboratory systems, as described here, is only an approximation. While macroscopic masses may appear electromagnetically neutral, internally, they contain molecules, atoms, electrons, protons, and neutrons with uneven distributions of internal electric and magnetic fields, forces, and dipoles. These internal components are not globally neutralized at every point within the mass. We can locate the centers of electric and magnetic neutrality within these internal dipoles, but they rarely coincide with the center of mass, center of inertia, or center of self-gravitation.

This discrepancy can naturally create specific torques and angular moments between masses, potentially supporting the global rotation of the Universe. Future theoretical advances will need to redefine the universal centers of mass, angular momentum, and electromagnetic neutrality, particularly in cases where masses also have uncompensated electromagnetic charges, fluxes, and spin moments. The fact that these centers do not perfectly overlap and involve different dipoles, moments, and mass centers is likely to be closely related to the nature of gravitation (for similar ideas, see [33], from Dr. Jovan Djuric).

We could connect the total angular moment of the two-body configuration in the Laboratory System, with the angular moment of the same configuration found in the Center of mass system, as follows:
$\mathbf{L}_{Lab.} = \mathbf{L}_m + \mathbf{L}_c = \mathbf{J}_m \omega_m + \mathbf{J}_c \omega_c = \mathbf{L}_m + m_c \mathbf{r}_c \times \mathbf{v}_c$. This approach could lead to recognition of de Broglie matter wave frequency and wavelength, as we have found in the case of

equations (4.3), where principal rotation was linked only to an angular movement of the reduced mass in its Center of mass system.

We know that all elementary particles and photons possess intrinsic spin characteristics, even if we do not observe visible mechanical rotation. Throughout this book, we have emphasized that, despite the absence of observable rotation, something equivalent to spinning must exist internally within the particle. This rotation is tied to the particle's internal structure and its associated electromagnetic properties. Essentially, stable particles (and quasiparticles) can be thought of as specific "packing" formats of fields, manifesting as self-contained, rotating standing waves. The internal rotation of these matter waves is what we measure externally as the particle's spin attribute. Without this spin or matter-wave rotation, particles would lack stability. This is because some force must bend and constrain these waves into a self-contained circular area, creating stable standing waves (see Chapter 2, equations (2.11.3), (2.11.4), and (2.11.5)).

The same principles regarding particle rotation and spin characteristics are well explained by Ph. M. Kanarev, C. Lucas, David L. Bergman, and others in the field of "Common Sense Science" [44, 16–22]. Additionally, the helical electron model proposed by Oliver Consa provides an excellent conceptual framework for understanding these dynamics [108].

We also understand that any electrically charged particle moving in a magnetic field follows a spiral or helical path, as demonstrated in Fig. 4.1, due to the Lorentz force. This behavior applies similarly to both charged and neutral particles, with charged particles experiencing stronger field interactions. As such, the mathematics and logic applied here are broadly relevant. The primary constituents of atoms, electrons and protons, are electrically charged, and their interactions lead us to conclude that the dominant forces responsible for creating de Broglie matter waves likely have an electromagnetic origin. The most appropriate frame of reference for modeling these interactions is the Center of Mass System.

This quasi-rotational movement and rotation-like field structure can, under certain conditions, become a self-sustaining object, such as a rotating ring or toroidal structure (see [94], *Classical Mechanics*, Chapter 14). Such forms possess rest mass and spin as inherent characteristics, dependent on the kinetic energy and relative positions, paths, and forces of the interacting components. It's important to emphasize that the motion of mutually approaching objects, whether particles, quasiparticles, or waves, naturally generates additional energy and rotating elements with associated orbital moments in the interaction zone. This, in turn, creates the conditions for forming new particles or waves and gives rise to various interference and diffraction effects.

If the interacting objects already possess specific spin characteristics before the interaction, the emergence of additional rotating elements becomes even more evident (see T.2.4, T.2.5, and T.2.6, and equations (2.11.3), (2.11.4), (2.11.5), and (2.11.13-1) to (2.11.13-5)). Once the necessary energy threshold is reached in an interaction, we expect the creation of particles like electrons, positrons, and photons, an observation well-supported by experimental evidence in particle physics.

It's likely that the presence of radial or central attractive forces between interacting elements is balanced with centrifugal forces when objects are sufficiently close. This balance results in stable, self-contained, spinning structures, such as electrons, positrons, and protons, where $2\pi r_i = n\lambda_i = n\frac{h}{p_i} = \frac{v_i}{f_m} = 2\pi\frac{v_i}{\omega_m}$. These systems often involve complementary fields, such as electric and magnetic fields, which interact to form stable, self-sustaining structures known as standing waves, commonly referred to as de Broglie matter waves.

Models of elementary particles based on rotating rings or toroidal field structures are well-established, as explained in sources like [94], *Classical Mechanics* by Tom W.B. Kibble and Frank H. Berkshire (Chapter 14). These models have been mathematically tested and produce precise results, which were previously known only through experimental measurements in Quantum Mechanics. For further elaboration, see the work of C. Lucas, David L. Bergman, and colleagues [16–22] in *Common Sense Science*.

It is not mere coincidence that everything we observe in the Universe exhibits some form of rotation, whether it be galaxies, stars, solar systems, planets, fluid motions, atoms, or elementary particles. Even in cases where objects appear to move only linearly, such as rockets traveling through space, there is internal rotation within the solid mass structure, maintaining stability in the object's external frame.

By understanding where rotation hides in rectilinear motion, we can better conceptualize de Broglie wave phenomena (see Fig. 4.1 and equations (4.2) and (4.3)). This understanding also opens the door to introducing the concept of "Fields of Rotation" or Torsion fields within a unified Theory of Gravitation and Electromagnetism (building on the Faraday-Maxwell framework). The extended table of analogies in T.4.2, based on de Broglie wavelengths of linear particle motion, further supports these ideas conceptually, illustrating the intrinsic coupling and periodic, standing wave relationships between linear motion and spinning.

These concepts can also be extended to electromagnetic fields (see Chapter 5, especially T.5.3 and equations (5.1) to (5.4-1)), and further supporting ideas can be found in the Appendix (Chapter 10: *Particles and Self-Closed Standing Matter Waves*).

T.4.2. Wavelength analogies in different frameworks

Matter Wave Analogies	Linear Motion	Rotation	Electric Field	Magnetic Field
Characteristic Charge	**Linear Momentum** **p**	**Orbital Momentum** $L = pR$	**Electric Charge** $q_e = q$	**"Magnetic Charge"** $q_m = \Phi$
Matter Wave Periodicity	***Linear Path Periodicity*** $\lambda = \frac{h}{p}$ **(Linear motion Wavelength)**	***Angular Motion Periodicity*** $\theta = \frac{h}{L}$ **(Angular motion Wavelength)**	***"Electric Periodicity"*** $\lambda_e = \frac{h}{q_e} = q_m$	***"Magnetic Periodicity"*** $\lambda_m = \frac{h}{q_m} = q_e$
Standing Waves on a circular self-closed zone	$n\lambda = 2\pi R$, $p = n\frac{h}{2\pi} \cdot \frac{1}{R}$	$n\theta = 2\pi$, $L = n\frac{h}{2\pi}$	$\lambda_e\lambda_m = q_e q_m = h$	
	$\theta L = \lambda p = h$, $\theta = \frac{\lambda}{R} = \frac{2\pi}{n}$			

(Periodicity – here invented, unifying formulation, $q_m = \Phi$ is not a free and independent magnetic charge)

Simplifying the same situation, we can say that any rectilinear motion of a particle should always be accompanied by certain angular, rotating and oscillating (like helix vortices) field components (including belonging electromagnetic field components). In many cases, a certain level of real mass rotation could also appear (for instance: rotation and spinning of planets around the sun, rotation of galaxies, rotation and spinning of electrons and other elementary particles, etc.). Since a single, isolated, and very free particle cannot exist, practically we always have a two-particle system: a test particle, **m**, and the rest of its surrounding universe, $\mathbf{M} >> \mathbf{m}$. Even particles like the planet Earth are tiny comparing to our Sun, and practically, we can also say that in our universe, there is no pure linear and straight-line uniform motion, except in some more or less approximate and limited (laboratory or mathematical) conditions (see [4]).

We should not immediately conclude that the frequency of mechanical rotation, $\mathbf{f_m}$, of the particle **m** (meaning number of full particle revolutions per second) directly corresponds to associate de Broglie wave frequency, **f**, since any rotating particle, surrounded by specific fields, presents the source of matter waves, making wave-like perturbations (or wave groups) in its vicinity. When we come to waves propagation, we should not forget that every wave-group has its group and phase velocity, and that mathematical connection between group and phase velocity is given by the following non-linear, mathematical relations, found in (4.2), $v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} = u + p\frac{du}{dp} =$ $= \frac{d\omega}{dk} = \frac{d\tilde{E}}{dp} = h\frac{df}{dp} = 2u/(1 + \frac{uv}{c^2})$. This is producing the clear difference between (a

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

mechanical) frequency of the particle rotation f_m and frequency of its (associated, de Broglie) rotational field components, f, such as: $1 \leq n\frac{f_m}{f} \leq 2$, (see (4.3)).
From another perspective, every instance of linear motion by a specific particle can be viewed as an approximation of orbital rotation with an arbitrarily long radius. This falls under the more general category of "quasi-elastic collision" between the particle and its surroundings. In elastic collisions, the system's rest mass remains unchanged, and the conservation of kinetic energy holds true. In such cases, the interaction between a moving particle and its surrounding environment can be understood as an elastic collision system. Here, the particle's motion, seen in the local Center of Mass System, is accompanied by a rotation along its path. This coupling of motion and rotation results in the generation of de Broglie matter waves.

Based on experimental evidence in particle physics, we know that all elementary particles, atoms, and molecules exhibit both particle and wave behavior. They all have intrinsic wave structures, as well as spin attributes. De Broglie waves and torsion fields are natural extensions of these intrinsic properties. Any change in particle motion perturbs the internal (and elastic) structure of its intrinsic field. This perturbation manifests externally as spinning de Broglie waves, which are detectable in experiments (see T. 5.3 for a broader understanding of de Broglie waves and elementary particles).

The most logical conclusion is that de Broglie matter waves always exist, forming a natural, like internally rotating structure of self-contained standing waves when particles are at rest. These waves become externally measurable when particles interact, change their states of motion, or exchange energy. In essence, the "packing" or folding of de Broglie standing waves constitutes the internal structure of stable particles. When particles move, their internal waves begin to unfold, radiating de Broglie waves and facilitating energy exchange, force coupling, and communication between interacting particles.

Classical Mechanics often neglects the internal wave structure of particles and the associated external manifestations of de Broglie waves. In this book, we aim to address this gap. To simplify, stable particles at rest (with non-zero rest mass) can be thought of as "parking" or folding domains for their internal de Broglie waves. These waves manifest as resonating, rotating, and standing wave formations of electromagnetic energy. When particles interact, their internal de Broglie waves unfold and reveal their external effects.

This book supports the idea that the rotational or spinning motion of a particle is directly related to its de Broglie matter waves. Put differently, de Broglie waves reflect the intrinsic coupling between a particle's linear and rotational motion and the associated electromagnetic phenomena. The total kinetic energy of a particle is thus equivalent to its matter wave energy, excluding the energy locked in the particle's rest mass.

For instance, we know that moving particle, which has linear motion moment $\mathbf{p}$*, also has de Broglie wavelength* $\lambda = h/p$*, phase velocity* $u = \lambda f = \frac{E_k}{p}$*, group velocity*

$v = u - \lambda \frac{du}{d\lambda} = \frac{dE_k}{dp} = -\lambda^2 \frac{df}{d\lambda}$ *and angular velocity* $\omega = 2\pi f = 2\pi \frac{E_k}{h}$ *, and complementary rotational particle motional energy components should also be related to the same (matter wave) parameters. We should be cautious not to immediately identify matter waves frequency* $\omega = 2\pi f = 2\pi \frac{E_k}{h}$ *with a mechanical revolving frequency of a particle* $\omega = L/J_0 = \omega_m$*, (around its center of rotation). Such frequencies are mathematically connected with certain velocity dependent function, and generally not equal, because one of them is typical mechanical rotation (around the certain center) and the other is a spinning frequency around particle's path of propagation. Also, the specific functional relationship between the involved group and phase velocity is making this situation more complex. This will be explained later much better (see (4.3-0), (4.3-0) - a,b,c,d,e,f,g,h,i… and* ***PWDC*** *with equations (4.2) and (4.3)).*

The origins of mechanical rotation of astronomic size macro-objects are only an extension of rotation and torsion field properties of its micro-particle constituents, and well explained as strongly coupled to atomic scale magnetic moment perturbations caused by the attractive force of gravitation inside of big astronomic masses. For more information, see works of P. Savic and R. Kasanin, **[51],** published by Serbian Academy of Sciences, SANU, Department for Natural and Mathematical Sciences, in the period 1960-1980, and summarized in the book: “Od atoma do nebeskih tela”, author Pavle Savic, publisher “Radnicki univerzitet Radivoj Cirpanov”, Novi Sad, 1978, Yugoslavia - Serbia).

[♣ COMMENTS & FREE-THINKING CORNER*:*

The information and conceptual background that is explaining and defending the unity of linear motion, mechanical rotation and associated matter waves spinning (as presented on Fig.4.1., around equations (4.3)), should also be included in the equation that is connecting group and phase velocity.

Let us imagine that specific particle is rotating (around its center of rotation ***C****), having tangential velocity* $v = v_g = \omega_g R$*, where the radius of rotation is* ***R****, and* ω_g *is the mechanical, angular particle velocity (number of full, mechanical rotations per second around the center of rotation* ***C****). Here vectors* $\vec{v}_g, \vec{\omega}_g$ *are mutually orthogonal, and particle velocity is equal to its group velocity (*$v = v_g$*), if we associate wave group to such particle. Let us find all particles and matter-wave parameters for such a moving particle. Here, we will apply the following indexing (to make intuitive associations to mechanical particle motions and equivalent wave motions):* ***m*** *(=) mechanical motion,* ***p*** *(=) phase,* ***g*** *(=) group,* ***c*** *(=) center of rotation,* ***s*** *(=) self-rotation or spinning,* ***gc*** *(=) wave-group related to center* ***C****, …*

Practically, the same concept of a wave packet, which has its group and phase velocity, in cases of linear motions should be analogically extendable to the rotating wave packet that has the group and phase angular velocity (v and u*), for instance,*

$$\left\{ v = v_{gc} = u_c - \lambda \frac{du_c}{d\lambda} = -\lambda^2 \frac{df}{d\lambda} = \omega_{gc} R = \frac{d\omega_s}{dk_s} (=) \text{wave group velocity} (=) \text{particle, tangential velocity} \right\} \Leftrightarrow$$

$$\left\{ \begin{array}{l} \omega_{gc} = \frac{v_{gc}}{R} = \frac{u_c}{R} - \frac{\lambda}{R} \frac{du_c}{d\lambda} = -\frac{\lambda^2}{R} \frac{df}{d\lambda} = \frac{1}{R} \frac{d\omega_s}{dk_s} = 2\pi f_{gc} = \left(\omega_p - \lambda \frac{d\omega_p}{d\lambda} \right)_{R=const.}, u_c = \lambda f = u, \omega_p = 2\pi f = \frac{u_c}{R}, \omega_s = 2\pi f_s \\ f_{gc} = \frac{\omega_{gc}}{2\pi} = -\frac{\lambda^2}{2\pi R} \frac{df}{d\lambda} = \frac{1}{2\pi R} \frac{d\omega_s}{dk_s} = \frac{v_{gc}}{2\pi R} (=) \text{frequency of mechanical particle rotation around center C.} \end{array} \right\} \Rightarrow$$

$$f_{gc} = f - \lambda \frac{df}{d\lambda} - \frac{\lambda f}{R} \frac{dR}{d\lambda} = -\frac{\lambda^2}{2\pi R} \frac{df}{d\lambda} = \frac{1}{2\pi R} \frac{d\omega_s}{dk_s} = \frac{1}{R} \frac{df_s}{dk_s} = -\frac{\lambda_s^2}{2\pi R} \frac{df_s}{d\lambda_s} \Rightarrow$$

$$u = u_c = \lambda f = \lambda_s f_s = \frac{\omega_s}{k_s} = \omega_p R = \omega_{gc} R = v / (1 + \sqrt{1 - \frac{v^2}{c^2}}),$$

$$k_s = \frac{2\pi}{\lambda_s} = \frac{2\pi}{H} p (=) \text{ wave vector, } H = const.,$$

$$\Rightarrow f = \frac{\lambda f}{R} \frac{dR}{d\lambda} + \lambda (1 - \frac{\lambda}{2\pi R}) \frac{df}{d\lambda} = \left(\frac{\lambda f}{R} \frac{dR}{d\lambda} \right)_{\lambda = n\lambda_s = 2\pi R}, \tilde{E} = Hf (=) \text{wave-group energy},$$

$$\text{If } \{\lambda = 2\pi R = n\lambda_s\} \Rightarrow \omega_p = \frac{\omega_s}{n} = 2\pi f = \frac{2\pi f_s}{n}, n = 1, 2, 3...,$$

$$\omega_p = 2\pi f = \frac{2\pi R}{\lambda} \frac{\omega_{gc}}{1 + \sqrt{1 - \frac{v^2}{c^2}}} = \frac{2\pi R}{\lambda} \frac{\omega_{gc}}{1 + \sqrt{1 - \frac{\omega_{gc}^2}{\omega_c^2}}} = \left(\frac{\omega_{gc}}{1 + \sqrt{1 - \frac{v^2}{c^2}}} \right)_{\lambda = n\lambda_s = 2\pi R},$$

$$\frac{\omega_{gc}}{\omega_p} = \frac{\lambda}{2\pi R} \left(1 + \sqrt{1 - \frac{v^2}{c^2}} \right) = \frac{\lambda}{2\pi R} \left(1 + \sqrt{1 - \frac{\omega_{gc}^2}{\omega_c^2}} \right) = \frac{\lambda}{2\pi R} \frac{v}{u},$$

$$\vec{p} = \frac{\omega_s}{v} \vec{L}_s = \gamma m \vec{v}, \vec{L}_s = \frac{v}{\omega_s} \vec{p} = \frac{v}{\omega_s} \gamma m \vec{v},$$

$$\begin{pmatrix} 0 \le v \le c \\ \lambda = 2\pi R = n\lambda_s \end{pmatrix} \Rightarrow 1 \le \frac{\omega_{gc}}{\omega_p} = \frac{f_{gc}}{f} = n \frac{f_{gc}}{f_s} = \frac{v}{u} \le 2$$

$$\begin{pmatrix} v << c \\ \lambda = 2\pi R = n\lambda_s \end{pmatrix} \Rightarrow v = 2u \Rightarrow \frac{\omega_{gc}}{\omega_p} = \frac{f_{gc}}{f} = n \frac{f_{gc}}{f_s} = 2$$

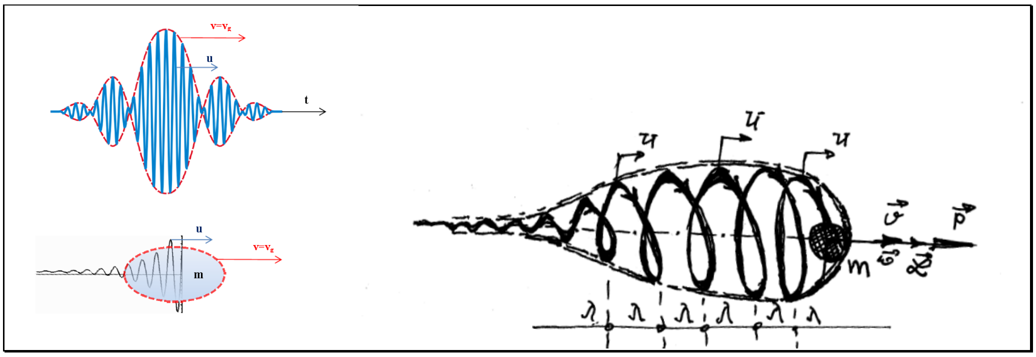
$$\begin{pmatrix} v \cong c \\ \lambda = 2\pi R = n\lambda_s \end{pmatrix} \Rightarrow (v = u) \cong c \Rightarrow \frac{\omega_{gc}}{\omega_p} = \frac{f_{gc}}{f} = n \frac{f_{gc}}{f_s} = 1.$$

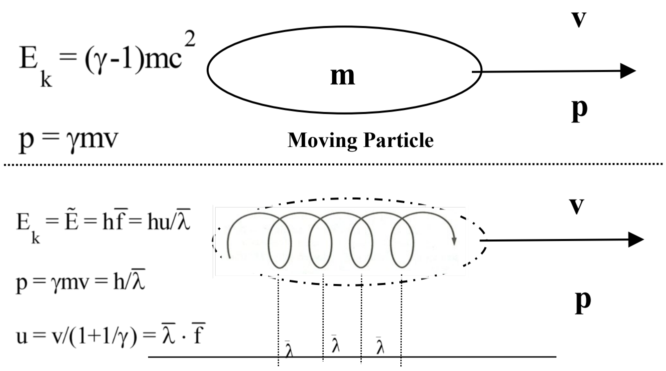
Matter waves associated with any particle in motion are defined by PWDC relations (equations (4.2) – (4.3)). However, they are distinct from the particle's mechanical rotation parameters. It is crucial to distinguish between mechanical rotation (i.e., the angular speed of a particle around a specific center of rotation) and the orbital frequency of its associated spinning or helical matter waves.

This distinction is often overlooked, which has led to the acceptance of ad hoc postulates and questionable, theory-correcting statements within Orthodox Quantum Mechanics. These include issues related to the gyro-magnetic ratio, spin attributes, the correspondence principle, and orbital and magnetic moments. In cases where the rotation radius is not constant, the situation becomes more complex and must be modified according to the Wilson-Sommerfeld quantization rules (see (5.4.1)).

Further exploration and elaboration of these concepts could yield valuable insights into the behavior of elementary particles and atomic structures. ♣].

Based on the above-presented intrinsic nature of matter related to strong couplings between linear motion and rotation (including spinning), and coupling between electric and magnetic fields, we are now in the position to propose a significantly simplified conceptualization of the particle-wave duality, illustrated on Fig.4.1.1, applied on a moving particle (also familiar to the situation presented on the Fig.4.1). If we accept that some matter-wave field is rotating (or spinning) around a particle in a linear motion (to have a chance to visualize the process in a simplified way), all elements of Particle-Wave duality will mutually fit into the united and simple picture, as illustrated on Fig.4.1.1. Particle and its wave-group envelope would have the same group velocity v, and helix tail, or spirally rotating field path (behind such energy state) would have phase velocity u. *Effectively, we will now conceptualize motional particle (or its mass) as an energy packing or formatting state that is realized by specific rotation or spinning (expressed by equivalency of kinetic energies of the associated liner and rotational motions; -see (4.3-0)).* This also presents the content and meaning of already introduced **PWDC** (**P**article-**W**ave-**D**uality-**C**ode).





Wave Group or Wave Packet, as an equivalent of moving the particle

Fig.4.1.1. Different ways of presenting a moving particle and its matter wave

The chain of conclusions, after such conceptualization (supported by many elaborations introduced earlier in the same chapter; -see (4.3)) follows,

$$\left\{\begin{array}{l} u=\frac{v}{1+\sqrt{1-v^2/c^2}}=\lambda f=\frac{h}{p}f \\ v=u-\lambda\frac{du}{d\lambda}=\frac{d\tilde{E}}{dp}=\frac{\tilde{E}}{p}-\frac{h}{p}d(\frac{\tilde{E}}{p})/d(\frac{h}{p}) \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \text{Particle, or its matter-wave packet in linear motion} \\ E_k=(\gamma-1)mc^2=\frac{pv}{1+\sqrt{1-v^2/c^2}}=hf=\tilde{E} \end{array}\right\} \Rightarrow$$

$$\left\{\begin{array}{l} \text{Orbiting particle about its center of rotation} \\ v<<c \Rightarrow E_k=\frac{mv^2}{2}=\frac{pv}{2}=\frac{I\omega_m^2}{2}=\frac{L_m\omega_m}{2} \end{array}\right\} \Rightarrow \{pv=L_m\omega_m\} \Leftrightarrow \left\{\vec{p}=\frac{\omega_m}{v}\vec{L}_m=\gamma m\vec{v}, \vec{L}_m=\frac{v}{\omega_m}\vec{p}=\frac{v}{\omega_m}\gamma m\vec{v}\right\} \Rightarrow$$

$$\left\{\begin{array}{l} \text{Particle motional energy as: linear motion, orbiting motion, and matter-wave packet} \\ E_k=(\gamma-1)mc^2=\frac{pv}{1+\sqrt{1-v^2/c^2}}=\frac{L_m\omega_m}{1+\sqrt{1-v^2/c^2}}=\frac{2\pi L_m f_m}{1+\sqrt{1-v^2/c^2}}=hf=hf_{mw}=\tilde{E} \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left\{\frac{h}{2\pi}\leq L_m\leq\frac{h}{\pi}, 0\leq v\leq c\right\},$$

$$\omega_m=2\pi f_m - \text{orbital, mechanical, angular velocity, } f=f_{mw} - \text{matter-wave, spinning frequency.} \tag{4.3-0}$$

In equation (4.3-0), we consider a particle rotating around its center at an angular speed ω_m. However, the associated matter wave may have a different angular or spinning speed $\omega_{mw}=2\pi f_{mw}$, which is distinct from ω_m and does not correspond to simple mechanical rotation around the particle's center. In this book, we conceptualize the associated matter wave as a state of spinning motion or a formation of spinning waves surrounding the particle. Since the particle is in an orbital motion, its matter-wave state effectively creates a helically shaped tail behind it. We can now define the wavelength, group velocity, and phase velocity of this state, as presented in equation (4.3-0).

This concept of spinning and helical matter waves will be further elaborated around equations (4.3-0 a, b, c...h). The oscillations and spins involved in these situations—often not externally visible—are related to the orbital and spin angular momenta and the associated electromagnetic dipole moments of the moving particle. These are also linked to vortex and turbulence effects in the surrounding medium. The internal structure of matter constituents, such as atoms, has a significant electromagnetic nature, with internal electric charges rotating and spinning on stationary orbits, creating electric dipoles and magnetic moments.

Additionally, the large mass difference between electrons and protons encourages the creation of electric dipoles in curvilinear, accelerated motions. The external matter-wave field surrounding a particle's motion must synchronize with the internal spinning and orbital moments. This behavior is adequately described by de Broglie matter waves. Even if a particle lacks initial spin, spinning matter waves will still self-generate, though angular momentum conservation will result in at least two mutually cancelling wave states.

The simplest and most fundamental instance of helically spinning matter waves can be detected around electrons and all electrically charged particles in linear motion, manifesting as spinning magnetic fields or electromagnetic waves. This phenomenon is still not fully addressed or experimentally verified within current electromagnetic theory, as cannot be predicted by existing equations and laws (see a similar concept regarding the Henry Augustus Rowland effect, discussed by Jean de Climont).

These fundamental matter waves are the principal sources of all other matter-wave manifestations and are integral to the concepts promoted in this book.

Another key idea in this book is that the particle-wave duality of matter waves reflects the fact that they are real physical waves, not merely probability or virtual constructs. These waves are ubiquitous and can be especially observed in elastically deformable fluids and plasma states of matter. Matter waves manifest as electromagnetic waves, acoustic waves, mechanical vibrations, waves in plasma, different forms of radiation, and cosmic and particle beams.

The success of particle-wave analogies, especially regarding the dual nature of photons and particles in relation to Minkowski Energy-Momentum 4-vectors, suggests that whole matter in the universe could be composed of photons or specific agglomerations and formations of electromagnetic and photonic "energy-moments."

Mathematically, the most general and unified framework for describing matter waves is the Analytic Signal (or complex Phasor) function. Given a specific state of motion with its energy, momentum, and wave function in the spatial and time domain, the associated matter wave can be described by applying the Hilbert transform, which produces the imaginary component of the relevant analytic signal or phasor (see chapter 4.0 for more on analytic signals). The vector fields corresponding to the real and imaginary components of the analytic signal behave as elements of a complex field, with one potential and one solenoidal vector field (see equations (3.5.1) – (3.5.4) in chapter 3 and see the picture below).

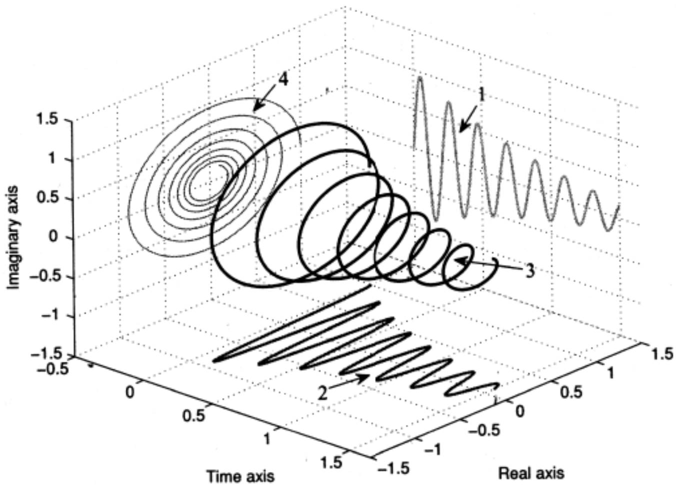


Figure 2.3 The HT projection (1), the real signal (2), the analytic signal (3), and the phasor in complex plain (4) (Feldman, ©2011 by Elsevier)

Spiraling and vortex-like de Broglie matter waves, which arise in various forms and from different fields, should, in theory, be detectable, directly or indirectly, in the medium where these waves are created and propagated. Such a medium, likely fluid-like in nature, must exist as the carrier of matter waves. Although we may not fully understand the exact nature of this background medium, we can measure its electromagnetic properties, such as dielectric permeability and magnetic susceptibility, indicating its influence. This medium was once speculated to be "ether," a concept introduced long ago to explain the texture of space-time, though its precise nature remains unclear.

Fortunately, we can observe the effects of matter waves in more familiar environments, such as liquids. Recent research shows that it is possible to induce various acoustic, mechanical, and electromagnetic excitations in liquids, thereby generating waves within them. When these agitated liquids are frozen or solidified, unique crystalline formations become visible. Each type of liquid agitation produces a distinct crystalline structure, which is experimentally verifiable and repeatable, suggesting that liquids have a form of structural memory. This memory enables the liquid to "record" the agitation, and upon freezing, the corresponding crystals become visible, depending on the type of agitation.

Thus, even though the concept of ether remains elusive, liquids provide a more accessible medium for observing the imprints of matter waves. These crystalline or molecular formations serve as indirect evidence of the underlying structure of matter waves, as encoded and preserved in the liquid's memory.

4.1.2.1 Matter Waves and Vortex Flow Meter

Let us now try to materialize (or verify) the introduced matter-waves concept by analyzing a specific sufficiently convincing example. Every particle submersed in some fluid, which is in relative motion related to that fluid, should also manifest (or produce) some helix-like rotating tail, or vortices on the downstream side (for instance phenomenon known as Karman, Vortex Street). This is well known (but not sufficiently theoretically explained) effect used in vortex flow meters (here is recommended to read readily available literature about flow metering based on vortex shedding effects to get familiar with such practices and terminology). The frequency f_v *of such vortex shedding (in a tube which has internal diameter* $D_c = \text{const.}$*) is experimentally related to the fluid flow velocity* v *(or here, to particle velocity, which is, here, the relative velocity between a particle and fluid) respecting empirically established formula,* $f_v = \frac{1}{D_c} S \cdot v \cong \text{Const} \cdot v$. *Here* $\mathbf{S}$ is *the Strouhal number, which is about constant across a wide range of the Reynolds' numbers (10*^{2} *~ 10*^{7}*). The intention here is to show that matter-waves (associated with motional particles), in the form of some helix vortices, could be detected in liquids. Let us verify this using expression for motion, or wave energy from* (4.3-0).

$$\left\{\begin{array}{l} f_v \cong \frac{1}{D_c} S \cdot v \cong \text{Const} \cdot v \\ \tilde{E} = hf = \frac{pv}{1+\sqrt{1-v^2/c^2}} = E_k \\ f = f_v ,\ \lambda = \frac{h}{p},\ v << c \end{array}\right\} \Rightarrow \left\{\begin{array}{l} f = f_v \cong \frac{p}{2h} \cdot v = \frac{v}{2\lambda} \\ \frac{S}{D_c} = \frac{p}{2h} = \frac{1}{2\lambda} \end{array}\right\} \Leftrightarrow \left\{\begin{array}{l} f_v \lambda = u = \frac{v}{2} \\ \lambda = \frac{2\pi}{k} = \frac{D_c}{2S} \\ \omega_v = 2\pi f_v = \frac{v}{2} k \end{array}\right\}. \qquad (4.3\text{-}0)\text{-a}$$

The result of such unusual (and innovative) analogical associations is that we got well known, correct relation between group and phase velocity (for non-relativistic fluid velocities) $u = \frac{v}{2}$, *which is confirming that matter waves tail, familiar to one illustrated on Fig.4.1.1., should exist. Of course, the real place and significance of Planck constant **h** in flow-metering is doubtful (probably nonexistent), but we can safely say and verify that instead of Planck constant "**h**" in (4.3-0)-a we should anyway have specific analogous constant* H, *and come to the same conclusions, for example,*

$$\left\{\begin{array}{l} f_v \cong \frac{1}{D_c} S \cdot v \cong \text{Const} \cdot v \\ \tilde{E} = Hf = \frac{pv}{1+\sqrt{1-v^2/c^2}} = E_k \\ f = f_v ,\ \lambda = \frac{H}{p},\ v << c \\ h \to H = \text{Const.} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} f = f_v \cong \frac{p}{2H} \cdot v = \frac{v}{2\lambda} \\ \frac{S}{D_c} = \frac{p}{2H} = \frac{1}{2\lambda} \end{array}\right\} \Leftrightarrow \left\{\begin{array}{l} f_v \lambda = u = \frac{v}{2} \\ \lambda = \frac{D_c}{2S} = \frac{2\pi}{k} = \frac{H}{p} \\ p = \frac{2HS}{D_c} \\ \omega_v = 2\pi f_v = \frac{v}{2} k \end{array}\right\}. \qquad (4.3\text{-}0)\text{-b}$$

Effective expression for "fluidic linear momentum" p*, related to a liquid flow in pipelines (from (4.3-0)-b), is* $p = \frac{2HS}{D_c} = \frac{\text{const.}}{D_c}$ *, meaning that if an internal pipe diameter* D_c *is* *smaller, useful fluid momentum is more significant, what looks logical (since fluid velocity will increase, and everything else will stay unchanged).*

Fluid masses, when in relative motion to a specific particle, can act as sensitive sensors or "antennas" for detecting and visualizing phenomena such as matter waves, vortices, spinning, and turbulence. This concept even extends to the potential detection of astronomical-scale matter waves, for instance, considering the ocean as a sensor for macro matter waves.

In the field of physics, the particle's surrounding environment is always composed of fluids, fields, waves, and other particles. However, in some cases, we lack a complete understanding of how to describe certain fluid states. Current fluid dynamics models, such as those based on the Navier-Stokes equations, could benefit from incorporating the manifestations of matter waves as discussed here.

Turbulence and vortices in fluid motion should conceptually be connected to the de Broglie matter wave framework. Furthermore, the application of matter-wave principles can be extended to larger systems, such as the orbital motions of planets in solar systems. This is explored in greater detail in Chapter 2 (Gravitation), specifically in Section 2.3.3 on Macro-Cosmological Matter Waves and Gravitation, where equations (2.11.13-1) through (2.11.13-5) show how matter waves are generated in two-body problems.

Until here, regarding fluids (Fig.4.1.1 and equations (4.3-0)-a, (4.3-0)-b), we could say that relative motion of the specific particle has been considered in different ways, to show that fluids could detect imprints and paths of associated matter waves.

Let us now more systematically decompose the same relative motion situation in **three steps** of understanding and conclusions, as follows:

FIRST, we imagine that the same linear motion is a segment of larger scale rotational motion (around the specific center of rotation C, as initiated in (4.3-0)), where a radius of rotation R can be arbitrarily large (see Fig.4.1.1a). We will practically extend and elaborate the concept of the unity of different motions, as summarized in (4.3-0). This gives the chance to present kinetic energy of the same particle in different terms. For instance, the rotating particle has kinetic energy expressed regarding liner-motion parameters, which is

$$E_k = (\gamma - 1)mc^2 = \frac{pv}{1 + \sqrt{1 - v^2/c^2}} . \tag{4.3-0)-c}$$

SECOND, the assumption is that the same particle is anyway, always rotating (mechanically revolving around specific center C, with an arbitrary, sufficiently large radius of rotation). Such particle has its moment of inertia $J_m = mR^2$, and orbital moment $\vec{L}_m = J_m \vec{\omega}_m$ (being perpendicular to $\vec{V}$ and $\vec{R}$) about the center of rotation C. Now we can add another expression for the same kinetic (now revolving or orbiting) energy of the same particle,

$$E_k = \frac{L_m \omega_m}{1+\sqrt{1-v^2/c^2}} = \frac{pv}{1+\sqrt{1-v^2/c^2}}, \vec{v} = \vec{\omega}_m \times \vec{R} . \quad (4.3\text{-}0)\text{-d}$$

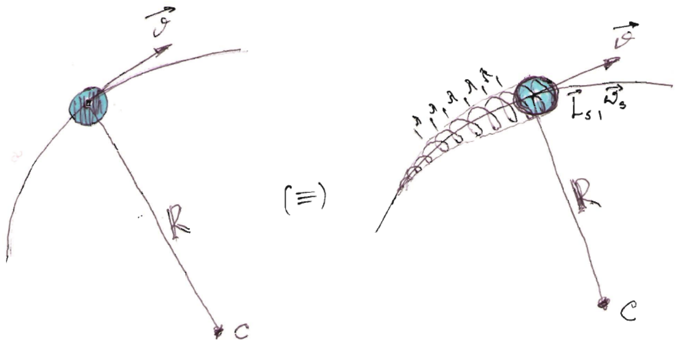


Fig.4.1.1a

The THIRD step is to creatively apply the conceptualization elaborated by Fig.4.1.1 and equations (4.3-0), (4.3-0)-a, (4.3-0)-b. Let us find another, **third** (and somewhat hypothetical) possibility, to express the same kinetic energy of the same particle by introducing an equivalent self-spinning particle which still has the same kinetic energy (see also equations (2.9.5-9) from the second chapter). We will say that particle is not only rotating around its center of rotation C; -it is also in some way spinning around its propagation path (around its velocity vector $\vec{v}$), as illustrated on Fig.4.1.1 and Fig.4.1.1a. Here $\vec{\omega}_{mw}$ is certain *matter-waves spinning,* angular, or spinning velocity, collinear with particle velocity $\vec{v}$, and $\vec{L}_{mw}$ is its *matter-wave spinning* moment, collinear with linear particle moment $\vec{p}$ (a bit analogical to eddy currents and Lenz law situations in electromagnetism). Since this still has speculative and hypothetic meaning, we can say that, at least, introduced *matter-wave spinning* or helix motion (whatever it is) should have the same kinetic energy as one the particle had before,

$$E_k = \frac{\vec{L}_{mw}\vec{\omega}_{mw}}{1+\sqrt{1-v^2/c^2}} = \frac{\vec{L}_m\vec{\omega}_m}{1+\sqrt{1-v^2/c^2}} = \frac{\vec{p}\vec{v}}{1+\sqrt{1-v^2/c^2}},$$
$$\vec{p} = \left(\frac{\omega_{mw}}{v}\right)\frac{\cos(\vec{L}_{mw},\vec{\omega}_{mw})}{\cos(\vec{p},\vec{v})}\vec{L}_{mw} = \gamma m\vec{v}, \vec{L}_{mw} = \left(\frac{v}{\omega_{mw}}\right)\frac{\cos(\vec{p},\vec{v})}{\cos(\vec{L}_{mw},\vec{\omega}_{mw})}\vec{p} . \quad (4.3\text{-}0)\text{-e}$$

Here, we can say (or hypothesize) that such imaginative helix and spinning motion should be linked to de Broglie matter waves that represent moving particles in question. ***Whatever is about wave motion around the particle in question, is a certain kind of kinematic equivalent, or replacement, or specific waveform behaving as an equivalent wave-group or wave-packet. When solving Classical Wave equation, we will always get two wave functions propagating in mutually opposite directions, and here we should have a similar situation, meaning that two matter-wave functions, or wave groups (with energy*** $E_k = \tilde{E}$ ***) should be on some way present, having mutually opposed (and mutually canceling) angular spin moments. Consequently, a total orbital particle moment will stay equal to*** $\vec{L}_m = J_m\vec{\omega}_m$***, and two opposed vectors*** L_{mw} ***(of helicoidally matter-wave spin***

moments) will be mutually canceled. Such helix waveforms should have their group and phase velocity. Group velocity will naturally be the same as the particle velocity. The universally applicable equation that is connecting such group, $\mathbf{v}$, and phase velocity, $\mathbf{u}$, is already known as,

$$v = u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda}, u = \lambda f,\ \lambda = \frac{H}{p}, H = \text{const.} \qquad (4.3\text{-}0)\text{-f}$$

Now we can again express the same kinetic or motional energy (of the same particle) on three different ways including new ***spinning wave-motion*** terms as,

$$\begin{aligned}
&E_k = \tilde{E} = Hf_{mw} = Hf = \frac{2\pi \mathbf{L}_{mw}}{1+\sqrt{1-v^2/c^2}} f_{mw} = \frac{\mathbf{L}_{mw}\omega_{mw}}{1+\sqrt{1-v^2/c^2}} = \frac{\mathbf{L}_m \omega_m}{1+\sqrt{1-v^2/c^2}} = \frac{pv}{1+\sqrt{1-v^2/c^2}},\\
&\vec{p} = \left(\frac{\omega_{mw}}{v}\right)\frac{\cos(\vec{\mathbf{L}}_{mw},\vec{\omega}_{mw})}{\cos(\vec{p},\vec{v})}\vec{\mathbf{L}}_{mw} = \gamma m\vec{v},\ \vec{\mathbf{L}}_{mw} = \left(\frac{v}{\omega_{mw}}\right)\frac{\cos(\vec{p},\vec{v})}{\cos(\vec{\mathbf{L}}_{mw},\vec{\omega}_{mw})}\vec{p},\ \mathbf{L}_{mw} = \gamma m v R = \mathbf{J}_{mw}\omega_{mw},\ \vec{v} = \vec{\omega}_m \times \vec{R},\\
&v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda}, u = \lambda f,\ \lambda = \frac{H}{p}, H = \text{const.}, (\text{for micro and subatomic world}: H \to h = \text{Planck const.}),\\
&\frac{H}{2\pi} \le \mathbf{L}_{mw} = \left\{\frac{H}{2\pi}(1+\sqrt{1-v^2/c^2}) = \frac{H}{2\pi}\frac{v}{u}\right\} \le \frac{H}{\pi},\ (\vec{\omega}_m \text{ and } \vec{\omega}_{mw} \text{ are mutually orthogonal vectors}),\\
&\boxed{pv = \mathbf{L}_m\omega_m = \mathbf{L}_{mw}\omega_{mw}},\ \boxed{dE_k = vdp = \omega_m d\mathbf{L}_m = \omega_{mw} d\mathbf{L}_{mw} = d\tilde{E}},\\
&\mathbf{J}_m = \gamma m v R = \frac{H}{2\pi}\cdot\frac{d\mathbf{L}_m}{d\mathbf{L}_{mw}}(1+\sqrt{1-v^2/c^2}) = \frac{H}{2\pi}\cdot\frac{\omega_{mw}}{\omega_m}(1+\sqrt{1-v^2/c^2}).
\end{aligned} \qquad (4.3\text{-}0)\text{-g}$$

Effectively, all initial, analogical, intuitive, and mathematical elaborations introduced in the very beginning of this chapter, and especially in "**4.1.1. Particle-Wave Duality Code**", around equations (4.1), (4.2), including analogies T.4.0., is getting its full and richer meaning in connection with this **THIRD** step until formulating (4.3-0)-g. *What has been the most important and essential here is to understand that motional energy has different forms and different ways to manifest and to be presented, and that matter wave's energy is the same motional or kinetic energy (see the more supporting background in chapter 4.2). Our Universe or Nature is presenting its unity by different and coincidental manifestations of motional energy states (while mathematically we make appropriate distinctions).*

Specific analyses of Compton, Photoelectric effects, Creation of electron-positron couple from high-energy photon, or Annihilation of an electron-positron couple and creation of photons (as in examples given in Chapter 4.2), can explicitly and without any doubt confirm that Kinetic particles energy is fully transformable to, or equivalent to matter-waves motional energy, and vice versa. The process which is opposite to (or inverse of) Compton Effect is the continuous spectrum of X-rays (or photons) emission, caused by impacts of electrons (accelerated in the electrical field between two electrodes) with the anode as their target. The emission of x-ray photons starts when the electrons are abruptly stopped on the anode surface. If the final impact electron speed is non-relativistic, $\mathbf{v} << \mathbf{c}$, *the maximal frequency of the emitted X-rays is found from the relation:* $\mathbf{hf}_{max.} = \frac{1}{2}\mathbf{mv}^2$, *and in cases of relativistic electron velocities, we have* $\mathbf{hf}_{max.} = (\gamma - 1)\mathbf{mc}^2$ (and both are experimentally confirmed to be correct). *If we now consider electrons (before the impact happen) as matter waves, where the electron matter wave energy corresponds only to kinetic electron energy, without rest-*

mass energy content, we will be able to find de Broglie, matter wave frequency of such electrons (just before their impact with the anode). With impact realization, the electrons are entirely stopped, and the energy content of their matter-waves is fully transformed and radiated in the form of X-ray photons (or into another form of waves), whose frequency corresponds to the matter wave frequency of electrons in the moment of the impact. This equality of the frequencies of radiated X-ray photons and electron matter-waves (in the moment of impact) explains to us the essential nature of electron matter waves (eliminating the possibility that the rest mass belongs to matter-wave energy content). In all such analyses, we are using relations, which agree with ***PWDC****; -see "4.1.1. Particle-Wave Duality Code", around equations (4.1), (4.2) from the beginning of this chapter.*

Let us go back to the bottom-line basics, related to fluids-flow and associated matter waves manifestations. We know (presently only experimentally) that in cases of vortex flow meters, fluid speed and frequency of generated vortices are mutually directly and linearly proportional. Nobody developed such relations mathematically, systematically, and starting from more elementary and generally valid step stones. Strouhal, only empirically, established principal flow metering relation, $f_v \cong Const \cdot v$, between relevant fluid velocity $\mathbf{v}$ and frequency $\mathbf{f_v}$ of Karman Vortex Street, and this relation has been successfully tested, and since very long time utilized in producing vortex flow meters (and looks like nobody is seriously asking how and why it works).

Here, we have a chance to develop the vortex flow metering, Strouhal relation mathematically. We can start from matter-waves relations (4.3-0)-a,b,c,d,e,f,g and get,

$$\left\{\begin{array}{l} \vec{p} = \frac{\omega_{mw}}{v}\frac{\cos(\vec{L}_{mw},\vec{\omega}_{mw})}{\cos(\vec{p},\vec{v})}\vec{L}_{mw} = \gamma m\vec{v}, \\ \vec{L}_s = \frac{v}{\omega_{mw}}\frac{\cos(\vec{p},\vec{v})}{\cos(\vec{L}_{mw},\vec{\omega}_{mw})}\vec{p} \\ vp = \omega_{mw}L_{mw} \\ vdp = \omega_{mw}dL_{mw} \end{array}\right\}_{\substack{initial\\relations}} \Rightarrow \left\{\frac{v}{\omega_s} = \frac{L_s}{p} = \frac{dL_s}{dp}\right\} \Rightarrow_{\substack{after\\integration}} \left\{\begin{array}{l} \frac{L_s}{L_{so}} = \frac{p}{p_o} \Rightarrow L_{mw} = L_{mwo}\frac{p}{p_o} = \frac{v}{\omega_{mw}}p \\ (L_{mwo}, p_o)(=)constants \end{array}\right\} \Leftrightarrow \quad (4.3\text{-}0)\text{-}h$$

$$\Leftrightarrow \omega_{mw} = 2\pi f_{mw} = \frac{p_o}{L_{mwo}}v \Leftrightarrow \underline{f_{mw} = \frac{p_o}{2\pi L_{mwo}}v = Const \cdot v} .$$

As we can see in (4.3-0)-h, the result of combined initial relations is $f_{mw} = \frac{p_o}{2\pi L_{mwo}}v = Const \cdot v$, which is the same vortex-flowmeters' relation, initially and still known only empirically. The conceptual picture of de Broglie matter waves (exercised by (4.3-0), (4.3-0) -a,b,c,d,e,f,g,h) is gradually getting stronger grounds, becoming self-supporting and internally consistent (and supportable from different points of view). Of course, we still need to "digest" unusual analogies between micro and macro quantum worlds (as we can find in chapter 2. *Gravitation;* ***2.3.3. Macro-Cosmological Matter-Waves and Gravitation***), but initial results are challenging.

Especially fruitful relations for future elaborations (from many aspects) are $\{vp = \omega_{mw} \mathbf{L}_{mw}, vdp = \omega_{mw} d\mathbf{L}_{mw}\}$.

In simple terms, we are drawing an analogy between de Broglie matter waves associated with a moving particle and the vortex waves generated in a fluid when it moves relative to an obstruction, such as a flow-barrier inside a vortex flowmeter. In the case of de Broglie matter waves, the moving particle can be thought of as moving relative to a certain fluid-like medium, much like the fluid in the vortex flowmeter. While the exact nature of this "fluid" in the context of de Broglie waves remains unclear, since different fields and forces exhibit different behaviors, the analogy with fluid dynamics allows us to develop equations for vortex flow metering, which have so far only been empirically established.

If we extend this analogy further, we can imagine that planets in a solar system, moving in their orbits, are also moving relative to a surrounding fluid-like medium or space. As a result, we would expect to find vortex formations or "planetary Karman streets" trailing behind planets, like the vortices observed in fluid dynamics. This analogy has already been explored in Chapter 2, where the existence of macro-cosmological matter waves, which adhere to de Broglie wavelength quantization (as in Bohr's atomic model), is documented. By knowing the orbital velocities of planets, we can calculate the frequencies of these "planetary vortex waves" $f_{mw} = \text{Const} \cdot v$, $vp = \omega_{mw} \mathbf{L}_{mw}, vdp = \omega_{mw} d\mathbf{L}_{mw}$ (see equations 2.11.13-1 to 2.11.13-5 and 2.11.14-a in Chapter 2. Gravitation; 2.3.3. Macro-Cosmological Matter Waves and Gravitation).

This same conceptual framework applies to the micro-world of subatomic and elementary particles, although Quantum Theory currently explains and formulates it differently. Many seemingly distinct phenomena in physics can be seen as analogically and analytically linked through similar underlying principles, yet we have traditionally treated them as unrelated. The broader mathematical framework behind these connections between linear and rotational motion likely involves Möbius transformations and Riemann sphere geometry, where such couplings are natural.

A simple, everyday example of this intrinsic coupling between linear and rotational motion is the familiar vortex created in a liquid as it drains from a tank with a circular outlet. No matter how perfectly symmetrical and polished the outlet is, the liquid will always rotate and form a helicoidal funnel as it drains. This indicates that linear motion is intrinsically coupled with rotational or spinning motion. Many turbulent or vortex phenomena in fluid dynamics can be explained by this coupling, from hurricanes to galactic rotations. Fluids are ideal for creating and detecting these couplings between linear and spinning motions, with phenomena such as "Falaco vortex solitons" providing examples.

It is likely that new sensors for detecting gravitational and inertial effects, including spatial linear and torsional accelerations, could be developed by studying matter-wave phenomena in fluids. Additionally, in Rainer W. Kühne's work on the gauge theory of gravity, the necessity of coupling between linear, torsion, and spinning motions is also discussed, further supporting this concept. Fluid dynamics, including the Navier-Stokes framework, could be significantly enhanced by incorporating the matter-wave phenomena described here.

The matter-wave concept explored in equations (4.3-0)-a through h suggests that linear motions, when viewed on a large scale as components of rotational motions, are associated with multiple levels of spinning and rotating motions. Extending this idea, each successive level of spinning could exhibit higher-frequency helical spinning around its primary path, converging into increasingly fine, spatially packed structures, much like twisted, toroidal solenoids with overlapping layers. This structural multilevel quantization means that each new level of spinning involves higher angular frequencies until the maximal tangential velocity of the finest helical path is reached. Turbulence in fluids can be seen as spatial perturbation along these different levels of associated spinning paths.

An additional layer of complexity in this discussion is the understanding that any linear motion can be viewed as a reasonable approximation of specific rotational or orbital motion with an arbitrarily large radius of rotation R. In this context, we can consider the conservation of associated orbital and spin moments as a globally valid principle (see [36], Anthony D. Osborne, & N. Vivian Pope, “An Angular Momentum Synthesis of ‘Gravitational’ and ‘Electrostatic’ Forces”). For instance,

$$\oint p dR = n_R H,\ \oint L_c d\alpha = n_\alpha H,\ \oint L_s d\varphi = n_\varphi H,\ H = const., \left\{n_R, n_\alpha, n_\varphi\right\} \in [1, 2, 3, ...). \quad (4.3\text{-}0)\text{-i}$$

This perspective allows us to apply generalized Arnold-Sommerfeld quantization rules to these arbitrary circular and closed orbits, as illustrated in equation (4.3-0)-i.

However, it is important to note that the micro-world Planck constant h is not universally applicable to all macro motions. The Planck constant is primarily relevant to the micro-world of atoms and subatomic particles, which adhere to distinct self-closed matter-wave formations. In contrast, in macro-world scenarios involving stable orbital motions, it appears that a series of larger constants, denoted as H, become more relevant, depending on the specific situation. For further elaboration on this topic, refer to equations (2.9.1) and (2.9.2) in Chapter 2. From this line of reasoning, we can conclude that an innovative conceptual framework is needed to provide a more intuitive understanding of matter waves.

Let us now go to another extreme, starting with kinetic energy equivalency relations (4.3-0)-g. We can exercise that rest mass, as an energy packing format, could be created (once in its past) when a certain amount of kinetic energy was packed and stabilized that way (with significant participation of spinning). This is producing the following chain of results,

$$\left\{\begin{array}{l} E_k = \tilde{E} = Hf_{mw} = HF = \frac{L_{mw}\omega_{mw}}{1+\sqrt{1-v^2/c^2}} = \frac{pv}{1+\sqrt{1-v^2/c^2}} = \\ = mc^2(\gamma-1),\ E_{tot.} = \gamma mc^2 = mc^2 + E_k, \\ \vec{p} = \left(\frac{\omega_{mw}}{v}\right)\frac{\cos(\vec{L}_{mw},\vec{\omega}_{mw})}{\cos(\vec{p},\vec{v})}\vec{L}_{mw} = \gamma m\vec{v},\ \vec{L}_{mw} = \left(\frac{v}{\omega_{mw}}\right)\frac{\cos(\vec{p},\vec{v})}{\cos(\vec{L}_{mw},\vec{\omega}_{mw})}\vec{p}, \vec{L}_{mw} = \mathbf{J}_{mw}\vec{\omega}_{mw}, \\ v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda}, u = \lambda f,\ \lambda = \frac{H}{p},\ H = const., \\ \frac{H}{2\pi} \le L_{mw} = \frac{H}{2\pi}\frac{v}{u} \le \frac{H}{\pi} \end{array}\right\}_{\left(\substack{v\to c\\ H\to h}\right)} \Rightarrow$$

$$\Rightarrow \left\{\begin{array}{l} mc^2 = \mathbf{J}_c\omega_c^2 = L_c\omega_c, \\ \lambda_c = \lambda_c = \frac{h}{mc}, \omega_c = 2\pi f_c, \lambda_c f_c = u_c = c \\ ------------------ \\ mv^2 = \mathbf{J}_{mw}\omega_{mw}^2 = L_{mw}\omega_{mw}, \\ \lambda = \frac{h}{\gamma mv} \cong \frac{h}{mv},\ v << c \end{array}\right\} \Rightarrow \left\{\begin{array}{l} L_{mwc} = \frac{h}{2\pi} \\ ----- \\ L_{mw} = \frac{h}{2\pi}\frac{v}{u} \end{array}\right\} \Rightarrow m = \frac{hf_c}{c^2} = \frac{h}{c\lambda_c}. \quad (4.3\text{-}0)\text{-j}$$

With (4.3-0)-j we have an essential part of explanation what Compton wavelength $\lambda_c = h/mc$ presents. Until here, Compton wavelength was only a significant or indicative numerical quantification without real conceptual meaning (but appearing in many analyses related to particle interactions). More of the common supporting arguments can be found in the Appendix, Chapter 10. PARTICLES AND SELF-CLOSED STANDING MATTER WAVES.

In cases of inertial motions, we will have the following situation,

$$\left\{\begin{array}{l} E_k = \frac{L_{mw}\omega_{mw}}{1+\sqrt{1-v^2/c^2}} = \frac{pv}{1+\sqrt{1-v^2/c^2}} = \frac{L_m\omega_m}{1+\sqrt{1-v^2/c^2}} \\ \vec{p} = \frac{\omega_{mw}}{v}\vec{L}_{mw} = \gamma m\vec{v},\ \vec{L}_{mw} = \frac{v}{\omega_{mw}}\vec{p} = \frac{v}{\omega}\gamma m\vec{v} \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left\{\begin{array}{l} pv = L_{mw}\omega_{mw} = L_{mwc}\omega_{mwc} = L_m\omega_m, \frac{\omega_{mw}}{v} = \frac{p}{L_{mw}} \\ vdp = \omega_{mw}dL_{mw} = \omega_m dL_m \\ F = \frac{dp}{dt} = \frac{\omega_{mw}}{v}\frac{dL_{mw}}{dt} = \frac{\omega_m}{v}\frac{dL_m}{dt}(=)\,force \\ \tau_{mw} = \frac{dL_{mw}}{dt} = \frac{v}{\omega_{mw}}\frac{dp}{dt} = \frac{\omega_m}{\omega_{mw}}\frac{dL_m}{dt} = \frac{\omega_m}{\omega_{mw}}\tau_m(=)\,torque \end{array}\right\} \Rightarrow$$

$$\Rightarrow \{\text{INERTIAL STATE}\} \Leftrightarrow \left\{\begin{matrix} F=0 \\ \tau=0 \end{matrix}\right\} \Leftrightarrow \left\{\begin{matrix} p = \text{const.} \\ L_{mw} = \text{Const.},\ L_m = \text{constant} \\ \frac{v}{\omega_{mw}} = \frac{L_{mw}}{p} = \frac{dL_{mw}}{dp} = \text{Constant} \\ \frac{v}{\omega_m} = \frac{L_m}{p} = \frac{dL_m}{dp} = \text{Const.} \end{matrix}\right\}. \quad (4.3\text{-}0)\text{-k}$$

Naturally, from (4.3-0)-k we can conclude that constant linear velocity is not the best and unique identifier of inertial motions, and that **rotational or spinning inertial states are among other inertial states** (see in the second chapter "*2.3.1. Extended Understanding of Inertia*").

Unity of Linear and Orbital or Angular motions should also be presentable in a Minkowski 4-vectors framework of Relativity Theory. Let us briefly summarize Momentum 4-vectors of a specific moving particle in linear, circular motion in two cases:

Case A) Particle is in linear motion and rotating around a remote fixed point, and

Case B) Particle is in linear motion (as in Case A)), rotating around a fixed point, and at the same time mechanically spinning (with angular spinning moment $\vec{L}_s = \vec{S}$) around the specific axis (belonging to the same particle).

For **Case A)** we will have the following situation (based on already elaborated expressions from (4.3-0)-c until (4.3-0)-j):

$$\overline{P}_4 = \left(\vec{p}, \frac{E}{c}\right) = \left(\frac{\omega_{mw}}{v}\vec{L}_{mw}, \frac{E}{c}\right)$$

$$\vec{p} = \gamma m\vec{v},\ E = \gamma mc^2 = E_0 + E_k,\ E_0 = mc^2$$

$$E_k = \frac{L_{mw}\omega_{mw}}{1+\sqrt{1-v^2/c^2}} = \frac{L_c\omega_c}{1+\sqrt{1-v^2/c^2}} = \frac{L_m\omega_m}{1+\sqrt{1-v^2/c^2}} = \frac{pv}{1+\sqrt{1-v^2/c^2}} = Hf_s,$$

$$L_{mw}\omega_{mw} = L_c\omega_c = L_m\omega_m = pv, \vec{p} = \frac{\omega_{mw}}{v}\vec{L}_{mw} = \gamma m\vec{v},\ \vec{L}_{mw} = \frac{v}{\omega_{mw}}\vec{p},$$

$$\lambda_{mw} = \frac{H}{p} = \frac{h}{\gamma mv},\ u = \lambda_{mw}f_{mw} = \frac{v}{1+\sqrt{1-v^2/c^2}},$$

$$\vec{L}_{mw} = \left(\frac{H}{2\pi}\right)\cdot(1+\sqrt{1-v^2/c^2}),\ \frac{H}{2\pi} \le L_{mw} = \frac{H}{2\pi}\frac{v}{u} \le \frac{H}{\pi}.$$

For **Case B)** we will have an additional kinetic energy member coming from mechanical spinning (here indexed with "$_s$", as in (4.3)). Particle linear and orbital moments will also change (here marked with asterisks "*"), as follows,

$$\overline{P}_4^* = \left(\vec{p}^*, \frac{E^*}{c}\right) = \left(\frac{\omega_{mw}^*}{v^*}\vec{L}_m^*, \frac{E^*}{c}\right), \vec{L}_m^* = \vec{L}_m + \vec{L}_s, v \to v^*$$

$$\vec{p}^* = \gamma^*(m+\Delta m)\vec{v}^*, E^* = \gamma^*(m+\Delta m)c^2 = E_0 + E_k + E_s, E_0^* = mc^2 + E_s,$$

$$\Delta m = \frac{E_s}{c^2}, E_s = \frac{L_s\omega_s}{1+\sqrt{1-v^2/c^2}}$$

$$E_k = \frac{L_{mw}^*\omega_{mw}^*}{1+\sqrt{1-v^2/c^2}} = \frac{L_m^*\omega_m^*}{1+\sqrt{1-v^2/c^2}} = \frac{p^*v}{1+\sqrt{1-v^2/c^2}},$$

$$L_{mw}^*\omega_{mw}^* = L_c^*\omega_c^* = p^*v^* = \gamma(m+\Delta m)v^2, \vec{p}^* = \frac{\omega_{mw}^*}{v}\vec{L}_{mw}^* = \gamma(m+\Delta m)\vec{v}^*, \vec{L}_{mw}^* = \frac{v^*}{\omega_{mw}^*}\vec{p}^*,$$

$$\omega_{mw}^* = \frac{2\pi}{H}\cdot\frac{\gamma^*(m+\Delta m)v^{*2}}{1+\sqrt{1-v^{*2}/c^2}} = \frac{2\pi}{h}\cdot\frac{p^*v^*}{1+\sqrt{1-v^{*2}/c^2}} = 2\pi f_{mw}^*,$$

$$\lambda_{mw}^* = \frac{H}{p^*} = \frac{h}{\gamma^*(m+\Delta m)v^*}, u^* = \lambda_{mw}^* f_{mw}^* = \frac{v^*}{1+\sqrt{1-v^{*2}/c^2}} = \lambda_{mw}f_{mw} = \lambda_m f_m = \frac{v}{1+\sqrt{1-v^2/c^2}} = u,$$

$$\vec{L}_{mw}^* = \left(\frac{H}{2\pi}\right)\cdot(1+\sqrt{1-v^2/c^2}), \frac{H}{2\pi} \le L_{mw}^* = \frac{H}{2\pi}\frac{v^*}{u} \le \frac{H}{\pi}.$$

As we can see, in the case **under B)**, particle matter wave wavelength λ_{mw}^* is shorter (compared to the case **under A)**), and matter wave spinning frequency f_{mw}^* is higher, but their product, or phase velocity $\mathbf{u}$, did not change. Mechanical spinning ($\vec{L}_s = \vec{S}$), which is motional energy E_s, is merely contributing to the rest-mass of the moving particle. The significance of presented cases A) and B) is to exercise how spinning is intrinsically (or naturally) already present and integrated into a rest mass, in the framework of Minkowski 4-vectors of Relativity Theory, showing one of the aspects how unity between linear and orbital motions is working. Similar conclusions or concepts could be drawn from Analytic Signal modeling and even much better from the unification between Minkowski 4-vector and Analytic Signal concepts (see more in chapter 10).

To support the picture about the unity of linear and rotational (or orbital) inertial motions additionally, let us create table T.4.2.1 with mutually analogical elements of linear and rotational motions seen from an observer in a convenient Laboratory System of coordinates. Certain analogical expressions in T.4.2.1 are unusual and created only to satisfy mathematical symmetry, respecting Mobility table of mutually analogical values (see about such analogies in T.1.2, T.1.4, and T.1.8 from the first chapter). We will accept that all inertial motions (in larger picture observations) are also belonging to rotational, orbital motions, as postulated in [36], by Anthony D. Osborne, & N. Vivian Pope. In case when we analyze (or describe) inertial motions of number of moving particles (or bodies), mutually related (like specific planetary or solar system), we will also consider that each particle could perform mechanical spinning (and be in a linear motion that is just a reduced-picture part of a larger scale rotation). In such situations, it will become evident that strong coupling between all orbital and spinning moments (of participants) should be globally (holistically) satisfied. For instance, on T.4.2.1 we can find a total angular moment $\vec{L}_{ct}$ (as the sum of rotating orbital and mechanical spinning moments of participants),

$$\vec{L}_{ct} = \sum_{(i)} (\vec{L}_i + \vec{L}_{i-s}) = \vec{L}_c + \vec{L}_{c-s} = \text{const} \;\Big|\; \underset{\text{differentiation}}{\Rightarrow} \;\; \dot{\vec{L}}_c + \dot{\vec{L}}_{c-s} = \vec{\tau}_c + \vec{\tau}_{c-s} = \vec{0},$$

meaning that spinning forces, or torques, are mutually coupled and real-time balanced with orbiting forces ($\vec{\tau}_c = -\vec{\tau}_{c-s}$), where index "s" is indicating mechanical spinning. In other words, planets and moons of a specific solar system are in permanent orbital (and inertial) rotation around its sun, but mutually communicating mechanical rotations and spinning of planets and all satellites secure the stable balance. Any sudden change in orbital or spin moments will be detected and balanced in real time by an equivalent change of opposite sign in the rest of the system. Since all linear motions can be viewed as segments of larger-scale rotational motions, these conclusions are not limited to solar systems. They hold universally for any system of mutually approaching and interrelated particles and bodies, especially considering the coupling of electromagnetic and gravitational properties.

In summary, rotational motions are balanced by the spinning of the participants. Even in cases where conventional mechanical spinning is not observable, there is still a spinning effect of matter-waves around moving particles. Thus, our universe continuously generates matter waves, extending beyond the realm of microphysics.

The phenomenon known in quantum theory as entanglement is related to this global and immediate balancing between the orbital and spin moments of specifically coupled systems of particles and matter waves (see Chapter 4.3, including equations (4.10-12)).

T.4.2.1 Analogies Between n-Body Coupled Inertial Motions in a Laboratory System

Linear Inertial Motion	**Orbital inertial rotation around center C**	**Inertial spinning** (in addition to orbital rotation)
Center of mass $\vec{r}_c = \frac{\sum_{(i)} m_i \vec{r}_i}{\sum_{(i)} m_i} = \frac{\sum_{(i)} m_i \vec{r}_i}{m_c}$	**Effective orbital phase** $\theta_c^* = \frac{\sum_{(i)} J_i \theta_i^*}{\sum_{(i)} J_i} = \frac{\sum_{(i)} J_i \theta_i^*}{J_c}$?!	**Effective mechanical spinning phase** $\theta_{c-s}^* = \frac{\sum_{(i)} J_{i-s} \theta_{i-s}^*}{\sum_{(i)} J_{i-s}}$?!
Center of mass velocity $\vec{v}_c = \dot{\vec{r}}_c = \frac{\sum_{(i)} m_i \dot{\vec{r}}_i}{\sum_{(i)} m_i} = \frac{\sum_{(i)} m_i \vec{v}}{m_c}$	**Center of Inertia angular velocity** $\vec{\omega}_c = \dot{\vec{\theta}}_c^* = \frac{\sum_{(i)} J_i \vec{\omega}_i}{\sum_{(i)} J_i} = \frac{\sum_{(i)} \vec{L}_i}{J_c}$	**Effective mechanical spinning, inertial velocity** $\vec{\omega}_{c-s} = \dot{\vec{\theta}}_{c-s}^* = \frac{\sum_{(i)} \vec{L}_{i-s}}{\sum_{(i)} J_{i-ss}} = \frac{\vec{L}_{c-s}}{J_{c-s}}$
Central mass $m_c = \sum_{(i)} m_i$	**Central moment of Inertia** $J_c = \sum_{(i)} J_i$	**Effective mechanical spinning, a moment of Inertia** $J_{c-s} = \sum_{(i)} J_{i-s}$

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

<table>
<tr>
<td>Linear moment
$\vec{p}_c = \sum_{(i)} \vec{p}_i = \sum_{(i)} m_i \vec{v}_i =$
$= m_c \vec{v}_c = Const.$
$\vec{F}_c = \dot{\vec{p}}_c = \vec{0}$</td>
<td>Angular moment
$\vec{L}_c = \sum_{(i)} \vec{L}_i = \sum_{(i)} J_i \vec{\omega}_i$
$= \sum_{(i)} \vec{r}_i \times \vec{p}_i = I_c \vec{\omega}_c$
$= \vec{r}_c \times \vec{p}_c$</td>
<td>The effective mechanical spinning moment
$\vec{L}_{c-ms} = \sum_{(i)} \vec{L}_{i-s} = \sum_{(i)} J_{i-s} \vec{\omega}_{i-s}$
$= (\sum_{(i)} \vec{r}_{i-s} \times \vec{p}_{is} = J_{c-s} \vec{\omega}_{c-s}$
$= \vec{r}_{cs} \times \vec{p}_{cs} = \vec{r}_c \times \vec{p}_{cs})?!$</td>
</tr>
<tr>
<td></td>
<td colspan="2">$\vec{L}_{ct} = \sum_{(i)} (\vec{L}_i + \vec{L}_{i-s}) = \vec{L}_c + \vec{L}_{c-s} = const,\ \dot{\vec{L}}_c + \dot{\vec{L}}_{c-s} = \vec{\tau}_c + \vec{\tau}_{c-s} = \vec{0}$</td>
</tr>
<tr>
<td>Kinetic Energy
$E_k = \frac{p_c v_c}{1+\sqrt{1-v_c^2/c^2}} \cong \frac{p_c v_c}{2}$ (for v<<c)</td>
<td>Angular Kinetic Energy
$E_{kr} = \frac{L_c \omega_c}{1+\sqrt{1-v_c^2/c^2}} \cong \frac{L_c \omega_c}{2}$ (for $\omega_c \ll \omega_{c-max.}$)</td>
<td>Mechanical Spinning Motional Energy
$E_{k-ms} = \frac{L_{c-s} \omega_{c-s}}{1+\sqrt{1-v_c^2/c^2}} \cong \frac{L_{c-s} \omega_{c-s}}{2}$ (for $\omega_{c-s} \ll \omega_{c-s-max.}$)</td>
</tr>
<tr>
<td colspan="3">Total Kinetic Energy = Orbiting + Mechanical Spinning Energy
$E_k = E_{kr} + E_{k-s}\ ,\ p_c v_c + L_{c-s}\omega_{c-s} = (L_c \omega_c + L_{c-s}\omega_{c-s}),\ p_c v_c = L_c \omega_c = L_{cs}\omega_{cs}\ ,\ \vec{L}_{total} = \vec{L}_c + \vec{L}_{c-s}$
$dE_k = dE_{kr} + dE_{k-s} = v_c dp_c + \omega_{c-s} dL_{c-s} = \omega_c dL_c + \omega_{c-s} dL_{c-s} = \omega_{cs} dL_{cs} + \omega_{c-s} dL_{c-s}$
$v_c dp_c = \omega_c dL_c = \omega_{cs} dL_{cs}$</td>
</tr>
</table>

- $\theta_c^*, \theta_{cs}^*$ (=) effective, composite, spatial angular values (composed of 2 angles), when the analyzed system of orbiting particles is not in the same plane.

- Index "ʃ" (=) mechanical spinning.
- ?! (=) the value that has only formal, dimensional meaning to satisfy used analogical presentation.
- L_{cs}, ω_{cs} (=) spinning matter wave angular moment and angular velocity when orbiting around C.

To focus our attention on Energy-Momentum conservation laws, let us address the same n-Body inertial situation from T.4.2.1 using 4-vectors (of Minkowski space) in the same Laboratory System. The first case, (4.3-0)-I, will be when a system of n particles is performing only mechanical orbital (and linear) motions, without mechanical self-spinning of participants (just to start with, for simplicity and more natural understanding of the next case, (4.3-0)-m, when participants are mechanically spinning). We will at the same time attempt to define particle orbital, 4-vector momentum:

$$\left\{ \begin{array}{l} \overline{P}_{4-i} = \left(\vec{p}_i, \frac{E_i}{c}\right),\ \overline{P}_4 = \sum_{(i)} \overline{P}_{4-i} = \left(\sum_{(i)} \vec{p}_i, \frac{\sum_{(i)} E_i}{c}\right) \Rightarrow \\ \vec{p}_i^2 - \frac{E_i^2}{c^2} = -\frac{E_{0i}^2}{c^2},\ \left(\sum_{(i)} \vec{p}_i\right)^2 - \frac{\left(\sum_{(i)} E_i\right)^2}{c^2} = -\frac{\left(\sum_{(i)} E_{0i}\right)^2}{c^2} \\ \vec{p}_i = \gamma_i m_i \vec{v},\ E_i = \gamma_i m_i c^2 = E_{0i} + E_{ki},\ E_{0i} = m_i c^2 \end{array} \right\} \Rightarrow$$

$$\left\{\begin{array}{l} \overline{L}_{4-i} = \vec{r}_i \times \overline{P}_{4-i} = \vec{r}_i \times \left(\vec{p}_i, \frac{E_i}{c}\right) = \left(\vec{r}_i \times \vec{p}_i, \frac{E_i}{c}\vec{r}_i\right) = \left(\vec{L}_i, \frac{E_i}{c}\vec{r}_i\right), \\ \overline{L}_4 = \sum_{(i)} \overline{L}_{4-i} = \left(\sum_{(i)} \vec{L}_i, \frac{\sum_{(i)} E_i \vec{r}_i}{c}\right) = \vec{r}_c \times \overline{P}_4 = \left(\vec{r}_c \times \sum_{(i)} \vec{p}_i, \frac{\sum_{(i)} E_i}{c}\vec{r}_c\right) \end{array}\right\} \Rightarrow \tag{4.3-0)-l}$$

$$\left\{ \vec{L}_i^2 - \frac{E_i^2}{c^2}\vec{r}_i^2 = -\frac{E_{0i}^2}{c^2}\vec{r}_i^2, \ \left(\sum_{(i)} \vec{L}_i\right)^2 - \frac{\left(\sum_{(i)} E_i \vec{r}_i\right)^2}{c^2} = -\frac{\left(\sum_{(i)} E_{0i} \vec{r}_i\right)^2}{c^2} \right\}$$

The same case when particles are (also) mechanically spinning ($\vec{L}_{i-ms} \neq \vec{0}$), having certain motional, spinning energy ($E_{i-s} = m_{i-s}c^2$), will become,

$$\left(\begin{array}{l} \vec{p}_i = \gamma_i m_i \vec{v} \rightarrow \gamma_i (m_i + m_{i-s})\vec{v} = \vec{p}_i + \vec{p}_{i-s} = \vec{p}_i^* \\ \vec{L}_i \rightarrow \vec{L}_i + \vec{L}_{i-s} = \vec{L}_i^* \\ E_i = \gamma_i m_i c^2 \rightarrow E_i + E_{i-s} = E_{0i} + E_{ki} + E_{i-s} = E_{0i} + E_s + E_{i-s} = E_i^* \\ E_{0i} = m_i c^2 \rightarrow E_{0i} + E_{i-s} = (m_i + m_{i-s})c^2 = E_{0i}^* \\ E_{ki} = (\gamma_i - 1)m_i c^2 = E_s, E_{i-s} = \text{mechanical spinning energy} \end{array}\right) \Rightarrow$$

$$\Rightarrow \left\{\begin{array}{l} \overline{P}_{4-i} = \left(\vec{p}_i^*, \frac{E_i^*}{c}\right), \ \overline{P}_4 = \sum_{(i)} \overline{P}_{4-i} = \sum_{(i)} \left[\vec{p}_i^*, \frac{\sum_{(i)} E_i^*}{c}\right] \Rightarrow \\ (\vec{p}_i^*)^2 - \frac{(E_i^*)^2}{c^2} = -\frac{(E_{0i}^*)^2}{c^2}, \ (\sum_{(i)} \vec{p}_i^*)^2 - \frac{\left(\sum_{(i)} E_i^*\right)^2}{c^2} = -\frac{\left(\sum_{(i)} E_{0i}^*\right)^2}{c^2} \end{array}\right\}. \tag{4.3-0)-m}$$

Mechanical particle spinning ($\vec{L}_{i-s}$) from (4.3-0)-m should not be mixed with de Broglie, matter-waves helix spinning ($\vec{\mathbf{L}}_{mw} = \frac{v}{\omega_{mw}} \frac{\cos(\vec{p}, \vec{v})}{\cos(\vec{\mathbf{L}}_{mw}, \vec{\omega}_{mw})} \vec{p}, \vec{\mathbf{L}}_{mw} = \mathbf{J}_{mw}\vec{\omega}_{mw}$), which is directly and only related to linear particle motion (when a particle has certain linear momentum $\vec{\mathbf{p}}$, collinear with matter waves spin moment $\vec{\mathbf{L}}_{mw}$). Mechanical self-spinning vector ($\vec{L}_{i-s}$) could have any spatial orientation around the specific axis of the moving body (not related to linear momentum $\vec{\mathbf{p}}$).

New, total orbital or angular moment (analog to 4-vector (4.3-0)-l), which considers additional mechanical self-spinning of participants, is:

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

$$\left\{\begin{array}{l}\vec{L}_{4-i}=\left(\vec{L}_i+\vec{L}_{i-s},\frac{E_i^*}{c}\vec{r}_i\right),\ \sum_{(i)}(\vec{L}_i+\vec{L}_{i-s})=\vec{r}_c\times\sum_{(i)}\vec{p}_i^*,\frac{\sum_{(i)}E_i^*\vec{r}_i}{c}=\frac{\sum_{(i)}E_i^*}{c}\vec{r}_c\\ \vec{L}_4=\sum_{(i)}\vec{L}_{4-i}=\left(\sum_{(i)}(\vec{L}_i+\vec{L}_{i-s}),\ \frac{\sum_{(i)}E_i^*\vec{r}_i}{c}\right)=\vec{r}_c\times\vec{P}_4=\left(\vec{r}_c\times\sum_{(i)}\vec{p}_i^*,\ \frac{\sum_{(i)}E_i^*}{c}\vec{r}_c\right)\end{array}\right\}\Rightarrow \tag{4.3-0)-n}$$

$$\left\{(\vec{L}_i+\vec{L}_{i-s})^2-\frac{(E_i^*)^2}{c^2}\vec{r}_i^2=-\frac{(E_{0i}^*)^2}{c^2}\vec{r}_i^2,\ \left(\sum_{(i)}(\vec{L}_i+\vec{L}_{i-s})\right)^2-\frac{\left(\sum_{(i)}E_i^*\vec{r}_i\right)^2}{c^2}=-\frac{\left(\sum_{(i)}E_{0i}^*\vec{r}_i\right)^2}{c^2}\right\}.$$

The conclusion from (4.3-0)-l and (4.3-0)-n shows that *mechanical spinning energy contribution is effectively entering or increasing the rest mass of the system, while any spin moment is increasing total orbital moment (as a vector)*. The same situation can be easily extended to account the angular velocity of matter waves spinning (ω_{is} , ω_{cs}) if we conveniently (and creatively) apply (4.3-0)-n, combined with (4.3-0)-m,

$$p_i v_i = (L_i\omega_i + L_{i-s}\omega_{i-s}) = L_{is}\omega_{is} \text{ and } p_c v_c = (L_c\omega_c + L_{c-s}\omega_{c-s}) = L_{cs}\omega_{cs}. \tag{4.3-0)-o}$$

Elaborations from and around equations (4.3-0)-l until (4.3-0)-o are still presenting kind of preliminary brainstorming and somewhat loosely imaginative thinking, but touching particularly challenging problematic, which will evolve towards more natural and richer concepts.

Let us now examine the linear motion of an electron (or any electrically charged particle) within an electric field. It is important to note that the electron is not only moving linearly but is also spinning, possessing an intrinsic angular momentum. Current electromagnetic theory does not account for the presence of a helical magnetic (or electromagnetic) field structure around moving electrons. However, the concept of helical matter waves, as discussed here, suggests that we should be able to detect a specific helical field structure surrounding a moving electron. This is akin to the phenomenon depicted in Fig. 4.1.1 and resembles the Henry Augustus Rowland effect, which describes the magnetic field generated around a rotating conductor, as presented by Jean de Climont.

We are already somewhat familiar with the association of de Broglie matter-wave wavelengths, $\lambda_e = h/p = h/m_e v_e$, with a moving electron. This concept is experimentally verifiable through experiments involving the diffraction of electron beams passing through crystalline structures. However, there has been little exploration into linking this wavelength to the periodicity of the electron's helical (and electromagnetic) wave structure. This connection between helix waves and their implications will remain a topic for further experimental investigation in the future.

Before any of such future revelations, we already know that the force law *between equidistant, parallel paths of uniformly moving electrical charges* (q_1 , q_2), *while being at the same height*, is going out of the frames of Coulomb law predictions (see (2.4-4) in the second Chapter of this book). Such force is:

$$F_{1,2} = \frac{\mu}{4\pi}\frac{(q_1 v_1)\cdot(q_2 v_2)}{r^2} = K\frac{(q_1 v_1)\cdot(q_2 v_2)}{r^2},\ K = \frac{\mu}{4\pi} = \text{const.}. \tag{2.4-4}$$

Here, the original Coulomb's law (like in cases of static electric charges) cannot directly explain magnetic force between them. For the same sign on the products $(q_1 v_1)$ and $(q_2 v_2)$ the charges are drawn closer (attractive force), and for opposite signs the charges are drawn apart (repulsive force). The most likely, still intuitive explanation is that moving charged particles are creating helicoidally spinning electromagnetic fields around paths of motion. Where such electromagnetic fluxes are mutually not cancelling, electric charges will experience an attractive force, and in cases of mutually opposed magnetic fields, the repulsive force between them will appear. Experimental verification of such spiral field phenomenology could be expected in electric plasma discharges in rarefied gases.

Direct analogical (and still hypothetical) force expression between two moving masses, based on (2.4-4) and on Mobility analogies from the first chapter is,

$$\begin{aligned} F_{12} &= K'\frac{(p_1 v_1)\cdot(p_2 v_2)}{r^2} = K''\frac{E_{k1}\cdot E_{k2}}{r^2} = K''\frac{(E_1 - E_{01})\cdot(E_2 - E_{02})}{r^2} = \\ &= \frac{K''}{r^2}\left(E_1E_2 - E_1E_{02} - E_{01}E_2 + E_{01}E_{02}\right) = \frac{K''}{r^2}\left(\gamma_1\gamma_2 m_1 m_2 C^4 - \gamma_1 m_1 m_2 C^4 - \gamma_2 m_1 m_2 C^4 + m_1 m_2 C^4\right) = \\ &= K'''\left(\gamma_1\gamma_2\frac{m_1 m_2}{r^2} - \gamma_1\frac{m_1 m_2}{r^2} - \gamma_2\frac{m_1 m_2}{r^2} + \frac{m_1 m_2}{r^2}\right) = \boxed{K'''\frac{m_1 m_2}{r^2}}\cdot(\gamma_1\gamma_2 - \gamma_1 - \gamma_2 + 1) = \boxed{\gamma^*\cdot K'''\frac{m_1 m_2}{r^2}} \end{aligned} \tag{2.4-4-1}$$

To support the existence of such attractive force, that will be equal to Newton force of gravitation, it will be necessary that masses $\mathbf{m_1}$ and $\mathbf{m_2}$ are in permanent motion or rotation,

$$\begin{aligned} &\boxed{K'''\frac{m_1 m_2}{r^2}}\cdot(\gamma_1\gamma_2 - \gamma_1 - \gamma_2 + 1) = \gamma^*\cdot K'''\frac{m_1 m_2}{r^2} \cong G^*\frac{m_1 m_2}{r^2} \Rightarrow \gamma^* = (\gamma_1\gamma_2 - \gamma_1 - \gamma_2 + 1) > 0, \\ &G^* = G(\gamma_1, \gamma_2) = G(v_1, v_2) \cong \text{Const.},\quad \gamma_{1,2} = (1 - v_{1,2}^2/c^2)^{-0.5}. \end{aligned} \tag{2.4-4-2}$$

We could imagine that our Universe is in permanent (holistic and intrinsic) motion or rotation to support the existence of Newtonian gravitation. Here we can also see why Newton constant of gravitation can never be measured very accurately, and be always the same, constant number, since it is related to involved velocities of relative motions between mutually attracting masses.

[♣ COMMENTS & FREE-THINKING CORNER: still in preparation...

4.1.2.1. Hypercomplex functions interpretation of energy-momentum vectors and time

In parallel to 4-vectors (in the Minkowski-Einstein space of Relativity Theory), we could try to formulate equivalent Hypercomplex Phasor functions or vectors (see chapter 4.0 where Analytic Signal, complex and Hypercomplex functions are introduced, and Chapter 10, where the same idea is much more elaborated as the support to de Broglie mater waves hypothesis). As the example of the linear momentum vector, it is possible to demonstrate how we can create the Hypercomplex momentum and its complex Phasor, as follows,

$$\overline{P}_4 = \left(\vec{p}, \frac{E}{c}\right) \Leftrightarrow \left(\vec{p}_x, \vec{p}_y, \vec{p}_z, I\frac{E}{c}\right) = \left(\vec{p}, I\frac{E}{c}\right) \Rightarrow \vec{p}^2 - (\frac{E}{c})^2 = -(\frac{E_0}{c^2})^2 \Leftrightarrow E_0^2 + p^2c^2 = E^2 = (E_0 + E_k)^2 \Rightarrow$$

$$\Rightarrow \left[\begin{array}{l} \overline{E} = E_0 \pm I \cdot pc = E \cdot e^{\pm I\theta} = (E_0 + E_k) \cdot e^{\pm I\theta} = \sqrt{E_0^2 + p^2c^2} \cdot e^{\pm I \cdot arctg\frac{pc}{E_0}} = \gamma mc^2 \cdot e^{\pm I\theta} = \gamma \overline{m} c^2 = \\ = E \cdot \cos\theta \pm I \cdot E \cdot \sin\theta,\ E \cdot \sin\theta = H[E \cdot \cos\theta],\ H(=) \text{Hilbert transform}, \\ E_0 = mc^2,\ \overline{m} = m \cdot e^{\pm I\theta} = m \cdot \cos\theta \pm I \cdot m \cdot \sin\theta = m_r \pm I \cdot m_i,\ I^2 = -1, \\ \gamma mc^2 = \sqrt{E_0^2 + p^2c^2} = E_0 + E_k = E \Leftrightarrow (\gamma m)^2 = (\frac{E_0}{c^2})^2 + (\frac{p}{c})^2, \gamma = 1/\sqrt{1-(\frac{v}{c})^2}, \\ \theta = arctg\frac{pc}{E_0} = arctg(\gamma\frac{v}{c}) = arctg\frac{\frac{v}{c}}{\sqrt{1-(\frac{v}{c})^2}},\ 0 \le \theta \le \frac{\pi}{2},\ \overline{m} = \begin{cases} m, & \theta = 0 \\ (1 \pm I)\frac{\sqrt{2}}{2} m = m \cdot e^{\pm I\pi/4}, & \theta = \frac{\pi}{4} \\ \pm I \cdot m, & \theta = \frac{\pi}{2} \end{cases} \end{array}\right] \Rightarrow$$

$$\Rightarrow \overline{P} = \gamma \overline{m} \vec{v} = \gamma m \vec{v} \cdot e^{\pm I\theta} = \vec{p} \cdot e^{\pm I\theta} \Rightarrow$$

$$\Rightarrow \begin{cases} \overline{L}_{mw} = \frac{v}{\omega_{mw}} \overline{P} = \frac{v}{\omega_{mw}} \vec{p} \cdot e^{\pm I\theta} = \vec{L}_{mw} \cdot e^{\pm I\theta} = \mathbf{J}_{mw} \vec{\omega}_{mw} \cdot e^{\pm I\theta} = \overline{\mathbf{J}}_{mw} \vec{\omega}_{mw}, \\ \vec{L}_{mw} = \frac{v}{\omega_{mw}} \vec{p} = \mathbf{J}_{mw} \vec{\omega}_{mw},\ \omega_{mw} = 2\pi f_{mw}, \lambda = H/p, u = \lambda f_{mw} \end{cases} \quad (4.3\text{-}0)\text{-}p$$

The same case as in (4.3-0)-p, when the motional particle will get certain mechanical spinning moment ($\vec{L}_{i-s} \neq \vec{0}$) will become (like (4.3-0)-m),

$$\left\{\begin{array}{l} \left(\begin{array}{l} \vec{p} = \gamma m \vec{v} \to \gamma(m + m_s)\vec{v} = \vec{p} + \vec{p}_s = \vec{p}^* \\ \vec{L} \to \vec{L} + \vec{L}_s = \vec{L}^* \\ E\ = \gamma mc^2 \to E + E_s = E_0 + E_k + E_s = E_0 + E_{mw} + E_s = E^* \\ E_0 = mc^2 \to E_0 + E_s = (m + m_s)c^2 = E_0^* \\ E_k = (\gamma - 1)mc^2 = E_{mw}, E_s = \text{mechanical spinning energy} \end{array}\right) \Rightarrow \\ \overline{P}_4 = \left(\vec{p}, \frac{E}{c}\right) \to \left(\vec{p}^*, \frac{E^*}{c}\right) \Leftrightarrow \left(\vec{p}^*, I\frac{E^*}{c}\right) \Rightarrow (\vec{p}^*)^2 - (\frac{E^*}{c})^2 = -(\frac{E_0^*}{c^2})^2 \Leftrightarrow (E_0^*)^2 + (\vec{p}^*)^2c^2 = (E^*)^2 = (E_0^* + E_k^*)^2 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left[\begin{array}{l} \overline{E}^* = E^* \cdot e^{\pm I\theta^*} = (E_0^* + E_k^*) \cdot e^{\pm I\theta^*} = \sqrt{(E_0^*)^2 + (\vec{p}^*)^2c^2} \cdot e^{\pm I \cdot arctg\frac{p^*c}{E_0^*}} = \gamma m^* c^2 \cdot e^{\pm I\theta^*} = \gamma \overline{m}^* c^2, \\ E_0^* = m^*c^2,\ \overline{m}^* = m^* \cdot e^{\pm I\theta},\ I^2 = -1, \\ \gamma m^* c^2 = \sqrt{(E_0^*)^2 + (\vec{p}^*)^2c^2} = E_0^* + E_k^* = E^*, \\ \theta^* = arctg\frac{p^*c}{E_0^*} = arctg(\gamma\frac{v}{c}) = arctg\frac{\frac{v}{c}}{\sqrt{1-(\frac{v}{c})^2}} = \theta,\ 0 \le (\theta = \theta^*) \le \frac{\pi}{2} \end{array}\right] \Rightarrow \quad (4.3\text{-}0)\text{-}q$$

$$\Rightarrow \overline{P}^* = \gamma \overline{m}^* \vec{v} = \gamma m^* \vec{v} \cdot e^{\pm I\theta^*} = \vec{p}^* \cdot e^{\pm I\theta^*} = \vec{p}^* \cdot e^{\pm I\theta}$$

Hypercomplex Phasors interpretation of energy-momentum vectors is revealing energy-momentum phase function "θ". The same Phase function will naturally define or support de Broglie matter waves wavelength and frequency (see kore in Chapter 10.).

Here, it looks like we come closer to understanding a natural and proper time flow (time scale, or time dimension) concerning a mass in motion and its associated helix matter-wave, if we find a connection between energy-momentum phase function "θ" and its proper, local time "t". Here, certain (local and dominant) matter-wave, spinning frequency $\overline{\omega_s}$, associated to linear momentum of a motional particle, could serve as the time clock, or time-reference signal for registering (measuring or representing) real-time flow. *For instance, (advancing with a loosely presented brainstorming exercise), we could explore the following possibility,*

$$\theta = \operatorname{arctg}\frac{pc}{E_0} = \operatorname{arctg}(\gamma\frac{v}{c}) = \operatorname{arctg}\frac{\frac{v}{c}}{\sqrt{1-(\frac{v}{c})^2}} \ (=)\ \omega_s t \mp kx,$$

$$\vec{L}_{mw} = \frac{v}{\omega_{mw}}\vec{p} = I\vec{\omega}_{mw}, \overline{L}_{mw} = \frac{v}{\omega_{mw}}\overline{P} = \frac{v}{\omega_{mw}}\vec{p}\cdot e^{\pm I\theta} = \vec{L}_{mw}\cdot e^{\pm I\theta} = \vec{L}_{mw}\cdot e^{\pm I\vec{\omega}_{mw} t}, \quad (4.3\text{-}0)\text{-}r$$

$$\vec{p} = \frac{\omega_{mw}}{v}\vec{L}_{mw}\cdot e^{\pm I\vec{\omega}_{mw} t}, \ \overline{P} = \gamma\overline{m}\vec{v} = \gamma m\vec{v}\cdot e^{\pm I\theta} = \vec{p}\cdot e^{\pm I\vec{\omega}_{mw} t}, \ \overline{P}_4 = (\vec{p}, I\frac{E}{c}),$$

$$I^2 = -1,\ I = (i,j,k),\ i^2 = j^2 = k^2 = -1, ij = k,\ jk = i, ki = j...$$

*In (4.3-0)-p,q,r we could recognize certain combinations between complex and vector functions. Spinning circular frequency $\vec{\omega}_s$ is a vector collinear with the particle linear moment $\vec{p}$, and hyper-complex imaginary unit **I** also has the structure similar to vectors, since it is composed of three, mutually orthogonal, more elementary imaginary units **i, j, k** (see more in the chapter 6., around equations (6.8) – (6.13) and in Chapter 10.). Ordinary vectors can naturally be related (or fixed) to specific observer system of reference, but hyper-complex imaginary units, here naturally coupled with linear particle momentum and associated spinning, could get other dynamic and structural assignments and meanings, and have specific couplings with observer's system of reference.*

Much more promising evolution of energy-momentum 4-vector from Relativity theory is to extend it (analogically, towards Hypercomplex and Phasor functions), and at the same time to stay very close to present formulation of 4-vectors, as follows:

$$\left\{\begin{array}{l} \overline{P}_4 = \left(\vec{p},\frac{E}{c}\right) \Leftrightarrow \left(\vec{p}, I\frac{E}{c}\right) = \left(\vec{p}_i, i\frac{E_i}{c_i}\right) + \left(\vec{p}_j, j\frac{E_j}{c_j}\right) + \left(\vec{p}_k, k\frac{E_k}{c_k}\right) \Leftrightarrow \\ \Leftrightarrow \left(\vec{p}, I\frac{E}{c}\right) = \left(\vec{p}_i+\vec{p}_j+\vec{p}_k, \frac{iE_i + jE_j + kE_k}{c_i}\right) = \left(\vec{p}_i+\vec{p}_j+\vec{p}_k, I\frac{E}{c}\right) \\ \overline{P}_4 = \left(\vec{p},\frac{E}{c}\right) = \text{invariant} \Rightarrow \vec{p}^2 - (\frac{E}{c})^2 = -(\frac{E_0}{c^2})^2 \Leftrightarrow E_0^2 + p^2c^2 = E^2 = (E_0 + E_k)^2, \\ \vec{p} = \vec{p}_i + \vec{p}_j + \vec{p}_k = \vec{p}_t, E = E_{0t} + E_{kt}, \\ E_i = E_{0i} + E_{ki}, E_j = E_{0j} + E_{kj}, E_k = E_{0k} + E_{kk}, \\ I^2 = i^2 = j^2 = k^2 = -1, ij = k,\ jk = i, ki = j, ji = -k, kj = -i, ik = -j \end{array}\right\} \Rightarrow (\vec{p}_i + \vec{p}_j + \vec{p}_k)^2 - (\frac{E}{c})^2 = -(\frac{E_0}{c^2})^2$$

$$\left\{\begin{array}{l} I\frac{E}{c} = i\frac{E_i}{c_i} + j\frac{E_j}{c_j} + k\frac{E_k}{c_k}, \\ \left(\frac{E}{c}\right)^2 = \left(\frac{E_i}{c_i}\right)^2 + \left(\frac{E_j}{c_j}\right)^2 + \left(\frac{E_k}{c_k}\right)^2 \\ \left(\frac{E_0}{c}\right)^2 = \left(\frac{E_{0i}}{c_i}\right)^2 + \left(\frac{E_{0j}}{c_j}\right)^2 + \left(\frac{E_{0k}}{c_k}\right)^2 \\ \left(\frac{E_{kt}}{c}\right)^2 = \left(\frac{E_{ki}}{c_i}\right)^2 + \left(\frac{E_{kj}}{c_j}\right)^2 + \left(\frac{E_{kk}}{c_k}\right)^2 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} p^2 = \frac{E^2 - E_0^2}{c^2} = \frac{E_i^2 - E_{0i}^2}{c_i^2} + \frac{E_j^2 - E_{0j}^2}{c_j^2} + \frac{E_k^2 - E_{0k}^2}{c_k^2} = \\ = \vec{p}_i^2 + \vec{p}_j^2 + \vec{p}_k^2 \end{array}\right\} \quad (4.3\text{-}0)\text{-}s$$

In (4.3-0)-s, constants c_i, c_j, c_k that have dimensions of speed (like the universal speed constant $c \cong 3\cdot 10^8 m/s$), could also be equal to $c = c_i = c_j = c_k$ (but involved mathematics is giving chances for other options). Energies indexed with "0" and "k" in (4.3-0)-s are effectively paving the ways to explain creations of significant numbers of products in impact reactions (zeros are indicating particles with rest masses, and k-indexing stands for kinetic energy states). Such extended energy-momentum framework can be later merged with universal Complex and Hypercomplex Analytic Signal representation of wave functions (leading to all famous wave equations of Quantum Theory), and with different foundations of multidimensional Universe (see Chapter 4.3 and chapter 6, equations (6.10) - (6.13)). The presence of three imaginary units in (4.3-0)-s is intuitively igniting ideas about mutually *coupled energy triplets such as three quarks, three anti-quarks, neutrinos etc., what could create another, more general and more precise concept of modeling of Super-symmetry in the world of microphysics (and significantly or fundamentally enrich and simplify the Standard Model). See much more about similar items in chapter 10.*

Regarding the time reference signal, as initiated in (4.3-0)-r, we can understand it in the following way: For humans and all living species on Earth, the dominant and natural flow of time is closely related to the orbital motion of our planet around the Sun, as well as its rotation around its own axis and local center of mass.

The Earth is simultaneously self-rotating and exhibits a specific helical matter-wave characterized by a spiraling frequency $\vec{\omega}_s$. This frequency, along with the associated wavelength, serves as parameters for the de Broglie matter-waves (for more details, see Chapter 10). Given that the mass of humans and other living organisms is negligible compared to that of the Earth and its center of mass, it follows that our perception of time is primarily influenced by the time flow associated with our planet.

Furthermore, we recognize that our SI units of time have long been linked to the parameters of the Earth's orbital and rotational motions. ♣]

Let us now briefly address the relevant wave functions that can be associated to rotational motions as presented on Fig.4.1.1, Fig.4.1.1a, and elaborate around equations (4.3.0) and (4.3.0)-a,b,c,d,e,f,g,h. Based on factorized, Analytical wave function illustrated in chapter 6, around equations (6.18) until (6.23), we will be able to give an idea how different levels of rotation and spinning could be mathematically modeled, as for instance,

$$\begin{aligned}
&E_k = \tilde{E} = \frac{pv}{1+\sqrt{1-v^2/c^2}} = \frac{L_c\omega_c}{1+\sqrt{1-v^2/c^2}} = \frac{L_{mw}\omega_{mw}}{1+\sqrt{1-v^2/c^2}} = E_c = E_s \\
&\tilde{E} = \int_{-\infty}^{+\infty}\Psi^2(t)dt = \int_{-\infty}^{+\infty}\hat{\Psi}^2(t)dt = \int_{-\infty}^{+\infty}\left|\frac{\overline{\Psi}(t)}{\sqrt{2}}\right|^2 dt = \int_{-\infty}^{+\infty}\left[\frac{a(t)}{\sqrt{2}}\right]^2 dt = \int_{-\infty}^{+\infty}\left|\frac{\overline{U}(\omega)}{\sqrt{2\pi}}\right|^2 d\omega = \int_0^{\infty}\left[\frac{A(\omega)}{\sqrt{\pi}}\right]^2 d\omega, \\
&\Psi(t) = a_0(t)\cos\varphi_0(t) = a_1(t)\cos\varphi_1(t)\cos\varphi_0(t) = a_2(t)\cos\varphi_2(t)\cos\varphi_1(t)\cos\varphi_0(t), \\
&E_c = \frac{L_c\omega_c}{1+\sqrt{1-v^2/c^2}} \to \Psi_c^2 = \frac{dE_c}{dt} = \left[a_0(t)\cos\varphi_0(t)\right]^2 = \left[a_0(t)\cos(\omega_c t + \varphi_c(t))\right]^2, \\
&E_{mw} = \frac{L_{mw}\omega_{mw}}{1+\sqrt{1-v^2/c^2}} \to \Psi_c^2 = \frac{dE_c}{dt} = \left[a_1(t)\cos\varphi_1(t)\cos\varphi_0(t)\right]^2 = \left[a_1(t)\cos(\omega_s t + \varphi_s(t))\cdot\cos(\omega_c t + \varphi_c(t))\right]^2.
\end{aligned} \tag{4.3-0)-t}$$

*[♣ **COMMENTS & FREE-THINKING CORNER:** There could be one great comparison between photon (or some wave group) and fluid vortex-shedding phenomenology, known from fluid flow measurements, that supports the existence of torsion field components in connection with the linear motion. It is experimentally found that fluid velocity is directly proportional to the vortex-shedding frequency **f** (when a non-moving obstacle or "bluff-body" is placed in a moving fluid). In such a case the fluid flow velocity can be measured based on Strouhal relation $\mathbf{v_{fluid} = s \cdot f}$, where the proportionality parameter, $\mathbf{s}$ = **constant** (**dimensionally equal to certain wavelength**), is essentially constant over wide velocity ranges and independent of fluid density (see **[12]**). Since motional or kinetic energy is directly proportional to the square of relevant velocity $E_k \sim mv^2$, then vortex-shedding waves should have an energy proportional to their squared frequency s^2f^2. There are wave-phenomena, like flexural waves, where wave group velocity is proportional to the square root of appropriate frequency, $v \approx \sqrt{f}$, what agrees with Planck wave energy that is proportional to frequency $\tilde{E} = hf \ (\approx v^2)$. Other matter-waves creating (analogical) situations are waves on a quiet water surface created by some moving object (a boat), where the water surface is visualizing matter waves associated with a moving object. Something similar regarding matter waves understanding could be associated with a pendulum motion if we observe the pendulum from another inertial reference system that is in relative motion to the pendulum*

*system of reference. Let us imagine (by analogy with the above-described situation of vortex shedding), that any object from our 4-dimensional universe is "immersed" or moving in some fluid. Mentioned fluid presents for us still non-detectable hyperspace, or multidimensional universe, but it is conceptually useful since we could easily create an association with appearance of de Broglie matter waves, by comparing them to the vortex-shedding phenomenology of real objects (from our detectable universe), being in an unknown and still undetectable "fluid". An interesting reference regarding similar subjects (vortices in fluids and Schrödinger Equation) can be found in the article [**15**] written by R. M. Kiehn. It is also evident that wave energy (or waves velocities) in cases of different waves, could be on a different way dependent on relevant waves' frequency, proportional either to* f or f^2*, or also to* f/v or f^2/v^2*, like shown in T.4.1. The micro-world of atoms, subatomic particles and states, and photons are dominantly respecting Planck's, Einstein, de-Broglie energy and wavelength formulations, where* $\tilde{E} = hf,\ \lambda = h/p,\quad u = \lambda f = \omega/k,\ v = d\omega/dk$*, and other wave phenomena from a Marco-objects world could have different wave energy to frequency relations. The intention here is to initiate thinking that for macro-objects, like planets and other big objects, certain relevant and characteristic wavelengths also exist, analog to de Broglie matter-waves wavelengths, but no more proportional to Planck's constant. The problem in defining de Broglie type of wavelength for macro-objects is that such wavelength (*$\lambda = h/p$*) is meaningless and extremely small, and since macro-objects in motion should also create associated waves, like any other micro-world object, the macro-wavelength in question should be differently formulated. ♣].*

In brief, this book favors the conceptual model that any motional, or Particle-Wave object could have, a) elements of linear motion (as a moving particle), b) elements of spinning around the line of its linear motion, and c) associated de Broglie matter waves. Associated forms of matter-waves are kind of helix and oscillating field perturbation, which is composed of axial and torsion field components, where torsion field components are coupled with particle rotation (where the particle kinetic or motional energy corresponds to the energy of a matter wave associated with that particle). If we do not directly see, measure, or detect de Broglie waves in the space around moving Particle-Wave object, this only means that de Broglie matter waves are still inside the internal and intrinsic waving structure of that object (and will become externally measurable in case of interaction, impact, diffraction, scattering, interference, etc.).

Matter waves and particle-wave duality concept presented in this book precisely states that kinetic or motional energy (of microparticles and/or matter waves) can be expressed in two different and mutually fully equivalent ways, such as $\mathbf{E_k = (\gamma - 1)mc^2 = hf}$ ***(while using the wave packet model as an equivalent replacement for a moving particle, apart from the particle rest mass). The analogy of the statement above with properties of stationary electron states in an atom is almost total. When an electron's stationary wave changes its state from a higher energy level,*** $\mathbf{E_{s1}}$***, state (1), to a lower energy level,*** E_{s2}***, state (2), the surplus of energy will be radiated as a photon that will have the frequency,*** $\mathbf{f_{1-2}}$***, equal to a frequency difference between corresponding frequencies of electron stationary waves:*** $E_{s1} - E_{s2} = E_{k1} - E_{k2} = hf_1 - hf_2 = h\Delta f = hf_{1-2}$***.***

Practically, a very similar situation happens (regarding de Broglie waves) when any moving particle (previously) in an energy state $\mathbf{E_1}$***, passes to another energy state*** $\mathbf{E_2}$***, where*** $\mathbf{E_1}$ ***and*** $\mathbf{E_2}$ ***could be both total or kinetic particle energy (because differences between total or only kinetic energy states of the same particle are mutually equal):***

$$\Delta E = E_1 - E_2 = E_{total-1} - E_{total-2} = E_{k1} - E_{k2} = hf_1 - hf_2 = h\Delta f = hf_{1-2} = c^2\Delta m$$
$$\lambda_1 = h / p_1 , \lambda_2 = h / p_2 , \quad \lambda_{1,2} \cdot f_{1,2} = u_{1,2} = v_{1,2} / (1 + \sqrt{1 - v_{1,2}^2 / c^2})$$
$$E_{k(1,2)} = (\gamma_{1,2} - 1)mc^2 = hf_{1,2} \ , E_{total-1,2} = \gamma_{1,2}mc^2 \ .$$

The macro-universe of particles reacts to de Broglie matter waves in a manner akin to the micro-universe of stationary states of elementary particles. However, unlike the micro-world, the realm of macroparticles is not limited to stationary or standing matter waves. This observation may serve as a foundation for a new understanding of the quantum nature of matter, fields, and waves in our universe. Specifically, "quantum properties should reflect optimal integer-number packing rules related to stable objects, standing waves (resonant structures), and their energy-momentum 'communications.'" Additionally, there exists a wide array of non-quantum phenomena related to forces, matter, and waves in our universe.

For example, the parameters of shape and size for an electron within an atom differ from those of a relatively free-moving electron in open space. Nonetheless, we can associate different de Broglie matter wavelengths and frequencies with both cases. The primary distinction lies in that a stable inter-atomic state of an electron possesses stable and constant matter wave parameters, while a freely moving electron exhibits time-evolving matter wave characteristics. In essence, when waves fold, particles can emerge; similarly, wave sources are often linked to the unfolding of waves captured by “particle shells.”

An important question arises in this context: Are we encountering some currently undetectable multidimensional medium (such as a fluid or ether) in which de Broglie waves are a natural phenomenon? Mathematically, this undetectable field or fluid is often modeled as the distribution of "possibility" or "probability" for the realization of specific events. Orthodox Quantum Mechanics has effectively utilized this modeling approach to address phenomena that we cannot directly observe or measure. However, this reliance leads to a significant loss of any common-sense conceptual framework.

If we do not recognize rotation, spinning, and associate rotational fields as inherent characteristics of all particles in linear motion, and if such phenomena always exist regardless of our awareness, we encounter a substantial conceptual issue. This oversight results in a missing link in our explanations of particle interactions, an issue that often goes unnoticed because we cannot apply our common-sense logic to phenomena we do not observe. Consequently, we interpret the outcomes of particle interactions, such as diffraction and interference, only as random distributions, disregarding a critical aspect of particle motion: its intrinsic rotation. Additionally, we may lack an understanding of the immediate real-time particle phase function, expressed as $\{-\pi \leq [\varphi(t, x) = \varphi(\omega t - kx)] \leq +\pi\}$.

To bridge this gap (the absence of rotation and phase), we often assume that over time, following numerous experimental interactions or diffractions, a smooth and wave-like probability distribution or interference pattern will emerge. This assumption underpins the framework of Orthodox Quantum Mechanics. It appears miraculous that we can statistically predict the resulting diffraction pattern while remaining unable to determine the immediate real-space position of a single event. Quantum Mechanics posits that the nature of the micro-world is primarily governed by probability distributions rather than real-time distributions, often represented in wave form. This perspective is substantiated by a wealth of experimental evidence.

Consequently, the founders of Quantum Theory have convinced their uncritical followers that the peculiar assumptions and mathematics of Quantum Mechanics are accurate, even when lacking a comprehensive common-sense explanation. Closely related to these concepts are the Uncertainty Relations, which, if misunderstood or incorrectly applied, can further complicate the Particle-Wave Duality picture (see Chapter 5 of this book).

A thorough analysis of the multi-faceted nature of Quantum Reality beyond contemporary Quantum Mechanics is presented in [13]. If a more common-sense logic had been applied from the beginning, as proposed here, many aspects of this complex reality would likely lose their mystique.

Anyway, until the present, all over this book, it has been attempted to explain the unknown origins of the associated effects of effective mass rotation and its direct connection to de Broglie matter waves. In other words, a mass in linear motion should have some hidden spinning (matter wave) properties, like a photon (see T.4.0.). We can show that, if kinetic particle-energy is equal to its (hypothetical) matter-wave spinning energy $E_{mw} = E_k$, and if such hidden spinning energy is equal to $E_{mw} = \frac{J\omega^2}{\sqrt{1 - v^2/c^2}} = L\omega = E_k$, then we will get the same results as already found in (4.2), (4.3), (4.3-0)-a,b,c,d, …. Consequently, a particle in motion can be modeled as a wave-packet that has wave energy equal to a particle kinetic energy:

$$\begin{Bmatrix} E_k = (\gamma - 1)mc^2 = \frac{pv}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{mv^2}{(1+\sqrt{1-\frac{v^2}{c^2}})\cdot\sqrt{1-\frac{v^2}{c^2}}} = hf = \tilde{E}\ , \\ E_{mw} = \frac{J\omega^2}{\sqrt{1-\frac{v^2}{c^2}}} = L\omega = E_k,\ \lambda = \frac{h}{p},\quad \omega = 2\pi f,\ L = \frac{J\omega}{\sqrt{1-\frac{v^2}{c^2}}}. \end{Bmatrix} \Leftrightarrow$$

$$\Leftrightarrow \frac{E_k}{hf} = \frac{\frac{pv}{hf}}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{\frac{v}{u}}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{L\omega}{hf} = \frac{E_{mw}}{hf} = \frac{\tilde{E}}{hf} = \frac{L}{(\frac{h}{2\pi})} = 1 \Rightarrow \tag{4.3-1}$$

$$\Rightarrow E_k = hf = \tilde{E},\ L = \frac{h}{2\pi},\ u = \frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}} = \lambda f = \frac{\omega}{k},\ v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} = \frac{d\omega}{dk}$$

$$\omega = \frac{\sqrt{1-\frac{v^2}{c^2}}}{1+\sqrt{1-\frac{v^2}{c^2}}}\cdot\frac{\hbar}{m}k^2 = 2\pi f,\ p = \frac{h}{\lambda} = \frac{h}{2\pi}k = \hbar k.$$

First, here we need to consider that any linear motion is just a particular case of specific rotational (or orbital) motion where the radius of rotation could be appropriately large. For idealized rectilinear motion radius of rotation will be infinite. The meaning of (4.3-1) is that any particle in linear motion should have some associated ***rotating (or spinning) field****, and energy of such* ***spinning field*** *is equal to the kinetic particle* *energy.* ***Waves created by such spinning fields (toroidal, helix or spiral wave structure caused by linear particle motion) are de Broglie matter waves*** *(see Fig.4.1.1, Fig.4.1.1a and Fig.4.1.2). The same concept is already introduced in the second chapter of this book; -see equations from (2.11.2) to (2.11.22)* and very indicative examples in the second chapter, around equations (2.11.13-1) - (2.11.13-5), where it is clearly shown how matter waves in two-body problems are being created. *Here we should make a difference between the two kinds or forms of rotational or spinning motions. The kinetic energy of a particle in linear motion can be presented as* $E_k = (\gamma - 1)mc^2 \cong \left(\frac{1}{2}mv^2\right)_{v<<c}$ *. If we could say that the same particle is in the same time making specific circular (or orbital) motion around specific center of such rotation (having a radius of rotation* r*) we can present the same kinetic energy as* $E_k = E_{rot.} \cong \left(\frac{1}{2}J_r\omega_r^2\right)_{v<<c}, v = \omega_r r$, *where moment of inertia of the particle is* J_r *(related to mentioned center of rotation). This is the first, natural or ordinary, visible mechanical particle rotation, which has not too much to do with de Broglie matter waves. The second (hidden) rotational or spinning motion associated with the same particle is a matter-waves', spiral or helical motion surrounding the particle along the particle orbit (see Fig.4.1.4). Again, we can introduce another alternative formulation for the same particle kinetic energy (as before),* $E_k = E_{spinning} \cong \left(\frac{1}{2}J_{mw}\omega^2\right)_{v<<c} = E_{mw} = \tilde{E}$. *Of course, ordinary mechanical rotation frequency* $\omega_r = 2\pi f_r$ *will not be comparable to matter waves spinning frequency* $\omega = 2\pi f$ *, and anyway here we have mutually orthogonal vectors (*$\omega_r \neq \omega$*). Now we can unite all the introduced kinetic energy aspects and start conceptualizing particle-wave duality such as,*

$$\left\{\begin{array}{l} E_k \cong \left(\frac{1}{2}mv^2 = \frac{1}{2}pv\right)_{v<<c} = \\ = E_{rot.} = \left(\frac{1}{2}J_r\omega_r^2 = \frac{1}{2}L_r\omega_r\right)_{v<<c} = \\ = E_{spinning} = \left(\frac{1}{2}J_{mw}\omega^2 = \frac{1}{2}L_{mw}\omega\right)_{v<<c} = E_{mw} = \tilde{E}, v = \omega_r r \end{array}\right\} \Rightarrow \tag{4.3-1.1}$$

$$\Rightarrow mv^2 = J_r\omega_r^2 = J_{mw}\omega^2 = pv = L_r\omega_r = L_{mw}\omega$$

In cases of high-speed motions (relativistic motions), we could establish analogical formulations for more generally valid kinetic energy expressions, for instance,

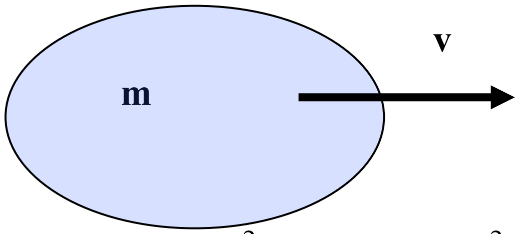
$$\left\{\begin{array}{l} E_k = \frac{mv^2}{\sqrt{1-\frac{v^2}{c^2}}(1+\sqrt{1-\frac{v^2}{c^2}})} = \frac{m^*v^2}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{p^*v}{1+\sqrt{1-\frac{v^2}{c^2}}} \cong \left(\frac{1}{2}mv^2 = \frac{1}{2}pv\right)_{v<<c} \\ m \to m^* = \frac{m}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma m, p \to p^* = m^*v = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma mv, \frac{1}{2} \to \frac{1}{1+\sqrt{1-\frac{v^2}{c^2}}}, \\ mv^2 = J_r\omega_r^2 = J_{mw}\omega^2 = pv = L_r\omega_r = L_{mw}\omega, \ \omega = 2\pi f. \end{array}\right\} \Rightarrow$$

$$\left\{\begin{array}{l} J_r \to J_r^* = \frac{J_r}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma J_r, J_{mw} \to J_{mw}^* = \frac{J_{mw}}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma J_{mw}, \\ L_r = J_r\omega_r \to L_r^* = J_r^*\omega_r = \frac{J_r\omega_r}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma J_r\omega_r = \gamma L_r, L_{mw} = J_{mw}\omega \to L_{mw}^* = J_{mw}^*\omega = \frac{J_{mw}\omega}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma J_{mw}\omega = \gamma L_{mw} \end{array}\right\} \Rightarrow$$

$$\left\{\begin{array}{l} E_k = \frac{p^* v}{1+\sqrt{1-v^2/c^2}} = \frac{L_r^*\omega_r}{1+\sqrt{1-v^2/c^2}} = \frac{L_{mw}^*\omega}{1+\sqrt{1-v^2/c^2}} = E_{rot.} = E_{spinning} = E_{mw} = \tilde{E} = hf^* = p^*u^*, \\ pv = L_r\omega_r = L_{mw}\omega = mv^2 = J_r\omega_r^2 = J_{mw}\omega^2, p^*v = L_r^*\omega_r = L_{mw}^*\omega = m^*v^2 = J_r^*\omega_r^2 = J_{mw}^*\omega^2, \\ v = \omega_r r, \lambda \to \lambda^* = h/p^* = \frac{h}{L_{mw}^*}(\frac{v}{\omega}) = u^*/f^*, p^* = L_r^*\frac{\omega_r}{v} = L_{mw}^*\frac{\omega}{v} = m^*v = J_r^*\frac{\omega_r^2}{v} = J_{mw}^*\frac{\omega^2}{v}, \\ u = \lambda^* f^* = v/(1+\sqrt{1-v^2/c^2}),\ L_{mw}^* = \frac{h}{2\pi}(1+\sqrt{1-v^2/c^2}) \end{array}\right\} \quad (4.3\text{-}1.2)$$

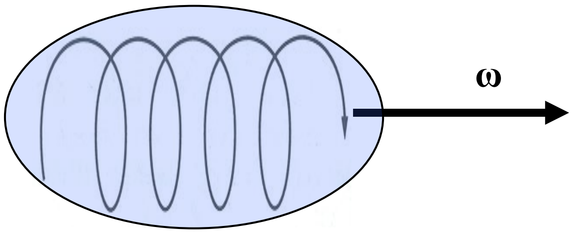
What is interesting here is that $\vec{p}^*$ and $\vec{L}_{mw}^*$ are mutually collinear vectors $\left(\vec{p}^* = \vec{L}_{mw}^*\frac{\omega}{v}\right)$, but in cases when moving particle is additionally spinning (this time mechanically), we should consider that new, resulting orbital moment of such particle will change. ***Also, we could conclude that many misinterpretations of contemporary particle-wave duality are related to improper linking and mixing between spinning $\vec{\omega} = \vec{\omega}_s$ and orbiting $\vec{\omega}_r$ (that are mutually orthogonal vectors).***

If we now imagine that matter waves are created as stable, stationary, and standing waves on a closed circular path (see Fig.4.1.4), we could start modeling periodical, cosmological or atom-world orbital motions and structures of elementary particles. For additional conceptual understanding see (2.11.10) – (2.11.23) from the second chapter, as well as books from Ph. M. Kanarev [44], C. Lucas and David L. Bergman dealing with innovative modeling of subatomic and elementary particles [16] - [22]; - "Common Sense Science".

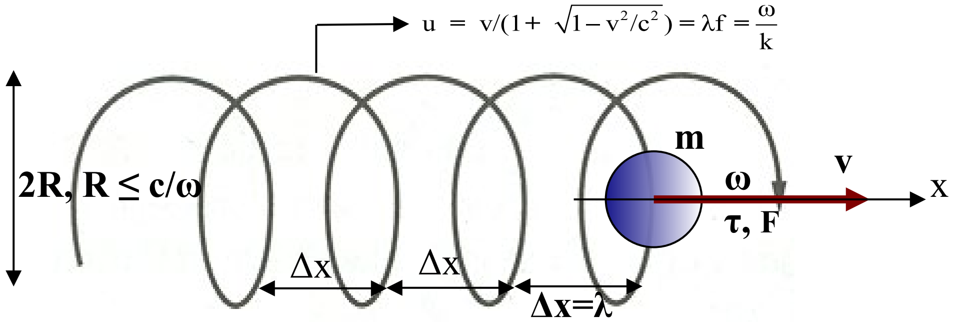


Moving particle: $E = mc^2 + E_k = \gamma mc^2$

⇓ (Equivalent) ⇑



Progressive matter-wave packet: $E = mc^2 + \tilde{E} = \gamma mc^2$



$$E_k = E_k(p,v) = \frac{pv}{1+\sqrt{1-v^2/c^2}},\ p = \gamma mv,\ \gamma = \frac{1}{\sqrt{1-v^2/c^2}},$$

$$\tilde{E} = \tilde{E}(\mathbf{L},\omega) = hf = h\frac{\omega}{2\pi} = \frac{\mathbf{L}\omega}{1+\sqrt{1-v^2/c^2}} = \frac{pv}{1+\sqrt{1-v^2/c^2}} = E_{mw} = E_k,$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{h}p,\ \omega = 2\pi f = ku = \frac{kv}{1+\sqrt{1-v^2/c^2}} = \frac{2\pi}{h}\frac{pv}{1+\sqrt{1-v^2/c^2}} = \frac{\sqrt{1-v^2/c^2}}{1+\sqrt{1-v^2/c^2}}\cdot\frac{\hbar}{m}k^2,$$

$$\mathbf{L} = \mathbf{L}_{mw} = \frac{h}{2\pi}(1+\sqrt{1-v^2/c^2}) = \mathbf{J}\omega = \frac{pv}{\omega} = \frac{\gamma mv^2}{\omega},\ \mathbf{J} = \frac{pv}{\omega^2} = \frac{\gamma mv^2}{\omega^2},\ \omega = \mathit{spinning}.$$

Fig.4.1.2 Moving-particle and de Broglie matter wave

In (4.3-1) and on Fig.4.1.2 particle linear velocity $\mathbf{v}$ *and angular velocity* ω*, which belongs to its associated, spinning matter-waves field,* ***should be mutually collinear***

***vectors**, and any (axial) force F acting on the same moving particle should be collinear to the torque τ related to the same spinning, as shown in (4.3-2).*

$$\begin{gathered}\frac{dE_k}{dt} = v\frac{dp}{dt} = \omega\frac{dL}{dt} = v\cdot F = \omega\cdot\tau = \psi^2\ (=)\ [W],\ p = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{h}{\lambda}, L = \frac{J\omega}{\sqrt{1-\frac{v^2}{c^2}}},\\ dE_k = vF\cdot dt = \omega\tau\cdot dt = F\cdot dx = \tau\cdot d\theta = h\cdot df = vdp = \omega dL,\ Z_m = \frac{v}{F} = \frac{\omega}{\tau}(\frac{dL}{dp})^2,\\ F = \frac{\omega}{v}\cdot\tau = \frac{v}{\omega}(\frac{dp}{dL})^2\cdot\tau = \frac{d\theta}{dx}\cdot\tau = h\cdot\frac{df}{dx} = \frac{1}{v}\cdot\frac{dE_k}{dt} = \frac{dp}{dt} = -\frac{h}{\lambda^2}\frac{d\lambda}{dt}\ (=)\ [N],\\ \omega = \frac{2\pi m}{h}\frac{v^2}{(1+\sqrt{1-\frac{v^2}{c^2}})\sqrt{1-\frac{v^2}{c^2}}} = \frac{2\pi m}{h}\frac{uv}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{2\pi}{h}E_k = 2\pi f = \frac{2\pi}{T} \le \frac{c}{R},\\ \Rightarrow \left\{\begin{array}{l} dL = \frac{h}{2\pi}\cdot\frac{df}{f} = \frac{h}{2\pi}\cdot\frac{dE_k}{E_k} = \frac{v}{\omega}dp \Rightarrow L = L_c - \frac{h}{2\pi}\cdot\left(\frac{E_c}{E_k}\right)^2 = L_c - \frac{h}{2\pi}\cdot\left(\frac{F_c}{f}\right)^2, *?!?!* \\ (E_c, F_c, L_c) = \text{constants},\ h = E_c/F_c = \text{Planck constant.}\ , \text{*to be more analyzed*} \end{array}\right\}.\end{gathered} \tag{4.3-2}$$

If we consider that maximal tangential velocity of the spinning matter waves v_t (which is perpendicular to the particle linear velocity v) should not be higher than c, $v_t = \omega R \le c$, where R is the "activity radius" captured by the spinning field (Fig.4.1.2). From (4.3-2) we can find R as,

$$\left\{\begin{array}{l} v_t T = 2\pi R = \frac{v_t}{f} = \omega R\cdot\frac{1}{f},\ E_k = \frac{mv^2}{(1+\sqrt{1-\frac{v^2}{c^2}})\sqrt{1-\frac{v^2}{c^2}}} = hf, \\ \Delta x = uT = \frac{u}{f} = \frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}}\cdot\frac{1}{f} = \frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}}\cdot\frac{h}{E_k} = \frac{h}{p} = \lambda \\ v_t = \omega R = 2\pi R f = 2\pi R\cdot\frac{E_k}{h} = \frac{2\pi R m}{h}\cdot\frac{v^2}{(1+\sqrt{1-\frac{v^2}{c^2}})\sqrt{1-\frac{v^2}{c^2}}} \le c \end{array}\right\} \Rightarrow$$

$$\begin{gathered}\Rightarrow R \le \frac{hc}{2\pi m v^2}(1+\sqrt{1-\frac{v^2}{c^2}})\sqrt{1-\frac{v^2}{c^2}} = \lambda\frac{c}{2\pi v}(1+\sqrt{1-\frac{v^2}{c^2}}) \Rightarrow \\ \Rightarrow \left(\begin{array}{l} v << c,\ u \cong \frac{v}{2},\ R \cong \frac{hc}{\pi m v^2} = \frac{\lambda}{\pi}\frac{c}{v} = \frac{\Delta x}{\pi}\frac{c}{v}, uv \cong \frac{h}{mT}(1-\frac{1}{2}\frac{v^2}{c^2}) \\ \quad v \to 0,\ u \cong \frac{v}{2} \to 0,\quad R \to \infty,\ uv \to \frac{v^2}{2} = \frac{h}{mT} \\ \quad v \to c,\ u \to c,\quad R = \frac{\lambda}{2\pi} = \frac{\Delta x}{2\pi} \to 0 \end{array}\right).\end{gathered} \tag{4.3-3}$$

*Interesting conclusion from (4.3-3) is that **for a particle which has remarkably high linear speed,** $v \to c$**, de Broglie matter waves are converging inside of a particle,** since $\mathrm{Lim} R|_{v \to c} = 0$, as illustrated on the Fig.4.1.3.*

$$\Delta x = \lambda = \frac{h}{p} = \frac{2\pi}{k},$$
$$v \to c,\ \Delta x \to 0,\ R \to 0$$

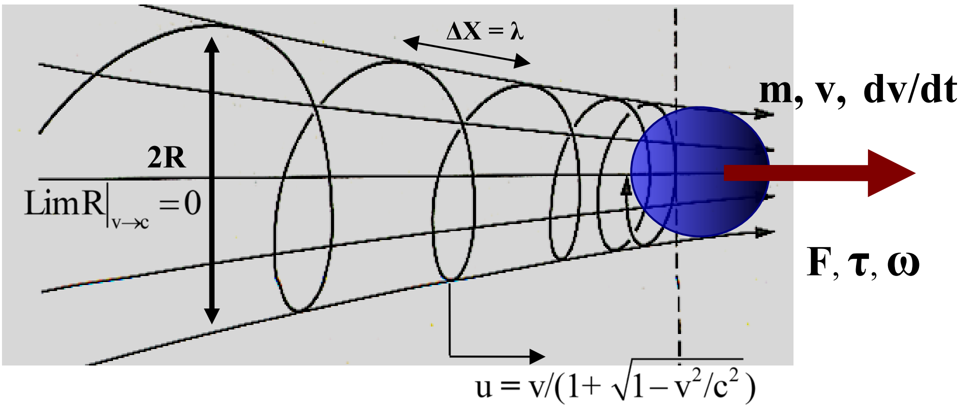


Fig.4.1.3 De Broglie matter waves around accelerating particle

The next task is to unify the concepts illustrated in Figures 4.1, 4.1.2, 4.1.3, and 4.1.4, as well as the equations (4.3), (4.3-0)-a,b,c,d,…, (4.3-1), (4.3-2), and (4.3-3), while addressing any discrepancies between them. It is becoming increasingly clear that de Broglie matter waves are less mysterious than previously thought (refer also to Chapter 2, specifically equations (2.5.1-4) to (2.5.1-6), which discuss the role of spinning in total particle energy).

Drawing an analogy from electromagnetic theory, which presents a specific unification of electric and magnetic field components, we can propose that every mechanical linear motion should be similarly coupled with a corresponding spinning motion. We can now understand that this associated spinning field component is causally linked to de Broglie matter waves, as illustrated in Figure 4.1.2 and expressed in relations (4.3-1) to (4.3-3). This associated matter wave field is expected to exhibit an axially spinning field structure around the particle. This concept parallels the behavior of a photon, which can be represented and treated, in relevant interactions with other particles or energy-momentum states, as both a moving particle and an equivalent wave packet, $(m_p = hf/c^2, p_p = m_p c = hf/c, \tilde{E}_p = hf, \lambda_p = h/p_p = c/f)$. Additionally, electromagnetic theory informs us that a photon is rotating; more precisely, its mutually complementary and orthogonal electric and magnetic field vectors rotate along the common path of propagation.

Particularly intriguing scenarios arise when a particle (or matter wave with effective mass m) undergoes orbital motion, creating a stationary and standing wave (toroidal) field structure along its rotational path, as illustrated in Figure 4.1.4. This perspective allows us to conceptualize the structure of atoms, elementary particles, and planetary motions within a specific solar system in relation to surrounding matter waves, a notion that is theoretically supported (see [94], Classical Mechanics; Chapter 14). For instance, if we consider the mass m) in Figure 4.1.4 to represent our planet Earth, its surrounding toroidal (or helical) matter-wave complement could relate to the lunar path of our Moon.

In this astronomical framework, where relevant objects are significantly larger than atoms and other subatomic entities, the analogous relationships regarding matter wave wavelength, energy, and so forth may differ from those associated with the original conceptualizations of de Broglie, Max Planck, Schrödinger, and Einstein in the micro-world (refer to Chapter 2, particularly section 2.3.2 on “Macro-Cosmological Matter-Waves and Gravitation,” and equations (2.11.10) to (2.11.20)).

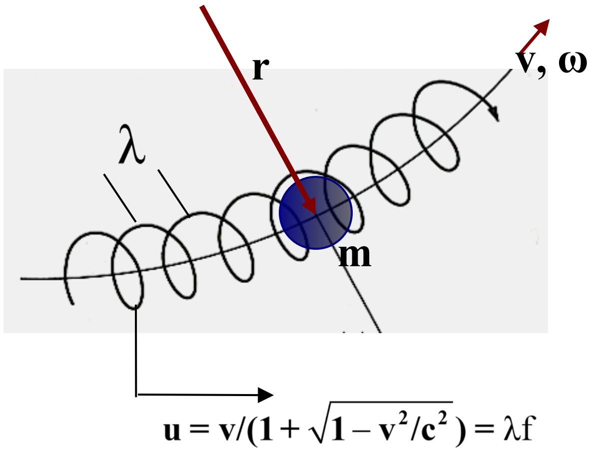


Fig.4.1.4 De Broglie matter wave on a circular path

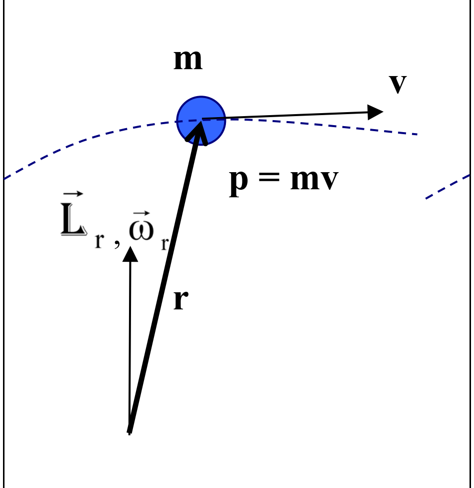
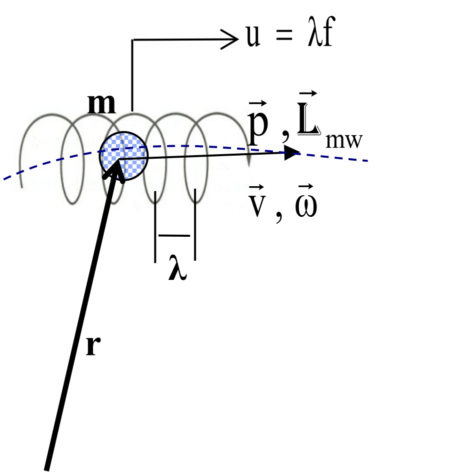
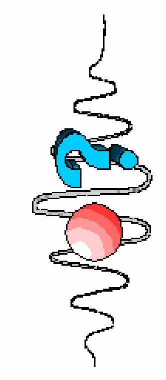
Particle-wave duality is fundamentally linked to how we represent a particle or energy state in motion. This dualistic representation primarily pertains to the formulation of motion or kinetic energy. The choice between a corpuscular or wave-motions methodology often depends on the convenience of analyzing specific problems.

To illustrate this point, consider two particles: one is a specific non-zero rest mass particle in a state of linear motion, while the other is in similar linear motion but also undergoing mechanical (externally visible or measurable) spinning. We will compare the different energy states of these moving particles while emphasizing their dualistic particle-wave presentations.

Historically, considerable effort has been made to demonstrate that matter waves (or de Broglie waves) represent only motional energy, with their structure resembling a helix spinning in a specific (hidden) field form. In our example, the second particle, which is in linear motion and visibly spinning, should exhibit a coupling between its mechanical spinning and the associated matter wave’s spinning field.

Table T.4.3 summarizes the most relevant comparisons between corpuscular and dualistic (particle-wave) characteristics of these moving particles (see also Figure 4.1.2 and further elaborations in Chapter 10).

Comparative particle-wave presentations of a motional particle

<table>
<tr><td></td><td>The particle is only in linear motion,
(which is a particular case of certain rotating or orbital motion)</td><td>The same particle is presented as a matter wave train (where matter wave is spinning around the particle)</td></tr>
<tr><td>Simplified conceptualization of corpuscular and wave nature related to the same particle</td><td>
</td><td>
</td></tr>
<tr><td>Particle-wave duality relations
</td><td colspan="2">

$$E = mc^2 + E_k = mc^2 + \tilde{E} = \gamma mc^2,$$

$$E_k = E_{mw} = E_k(p,v) = \frac{pv}{1+\sqrt{1-v^2/c^2}} = (\frac{v}{c})\frac{\sqrt{\tilde{E}(\tilde{E}+2E_0)}}{1+\sqrt{1-v^2/c^2}} =$$

$$= \frac{\mathbb{L}_r\omega_r}{1+\sqrt{1-v^2/c^2}} = \frac{\mathbb{L}_{mw}\omega}{1+\sqrt{1-v^2/c^2}} = \tilde{E} = hf = h\frac{\omega}{2\pi},$$

$$P_4^2 = (\vec{p}, \frac{E}{c})^2 = inv. \Leftrightarrow p^2c^2 - \frac{E^2}{c^2} = -\frac{E_0^2}{c^2}, p^2c^2 + m^2c^4 = E^2,$$

$$dE_k = vdp = \omega_r d\mathbb{L}_r = \omega d\mathbb{L}_{mw} = hdf = d\tilde{E} = dE_{mw}, v = \omega_r r,$$

$$p = \gamma mv = \mathbb{L}_{mw}\frac{\omega}{v} = \mathbb{L}_r\frac{\omega_r}{v} = \frac{\mathbb{L}_r}{r} = \pm\frac{1}{c}\sqrt{E^2 - E_0^2} =$$

$$= \pm\sqrt{(E-E_0)(E+E_0)}/c = \pm\sqrt{\tilde{E}(\tilde{E}+2E_0)}/c,$$

$$\mathbb{L}_{mw}\omega = \mathbb{L}_r\omega_r = pv, \mathbb{L}_{mw} \neq \mathbb{L}_r, \omega \neq \omega_r, \mathbb{L}_{mw}d\omega = \mathbb{L}_r d\omega_r,$$

$$\mathbb{L}_{mw} = \mathbb{L} = \mathbb{L}_r\frac{\omega_r}{\omega} = p\frac{v}{\omega} = \frac{\gamma mv^2}{\omega} = \frac{h}{2\pi}(1+\sqrt{1-v^2/c^2}) = \mathbb{J}\omega,$$

$$\left\{\begin{matrix} d\mathbb{L} = \frac{h}{2\pi}\cdot\frac{df}{f} = \frac{h}{2\pi}\cdot\frac{dE_k}{E_k} = \frac{v}{\omega}dp \Rightarrow \mathbb{L} = \mathbb{L}_c - \frac{h}{2\pi}\cdot\left(\frac{E_c}{E_k}\right)^2 = \mathbb{L}_c - \frac{h}{2\pi}\cdot\left(\frac{F_c}{f}\right)^2, *?!?!* \\ (E_c, F_c, \mathbb{L}_c) = \text{constants}, \ h = E_c/F_c = \text{Planck constant.}, \text{*to be more analyzed*} \end{matrix}\right\},$$

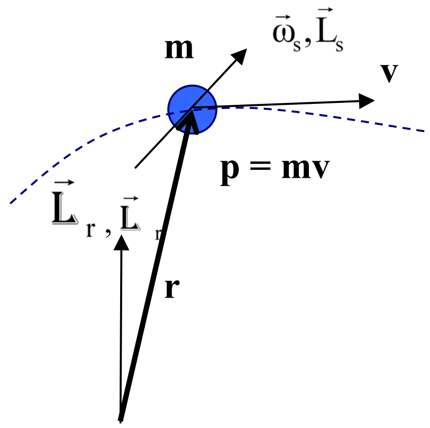
$$\mathbb{L}_r = \gamma L_r = p\frac{v}{\omega_r} = pr = \mathbb{L}_{mw}\frac{\omega}{\omega_r} = \frac{\gamma mv^2}{\omega_r},$$

$$\lambda = \frac{h}{p} = \frac{h}{\gamma mv} = \left(\frac{h}{\mathbb{L}_{mw}}\right)\frac{v}{\omega} = \left(\frac{h}{\mathbb{L}_r}\right)\frac{v}{\omega_r} = \left(\frac{h}{\mathbb{L}_r}\right)r, \left(\frac{h}{\mathbb{L}_i}\right) = \theta_i,$$

$$u = \lambda f = \frac{v}{1+\sqrt{1-v^2/c^2}}, \ v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda},$$

$$\Psi^2 = \frac{d\tilde{E}}{dt} = v\frac{dp}{dt} = \omega\frac{d\mathbb{L}_{mw}}{dt} = \omega_r\frac{d\mathbb{L}_r}{dt} = vF = \omega_r\tau_r = \omega\tau.$$

</td></tr>
</table>

<table>
<tr><td colspan="2">The particle is in linear motion and mechanically spinning
(having externally visible or measurable spinning: $\vec{\omega}_{ms}, \vec{L}_{ms}$)</td></tr>
<tr><td></td><td>
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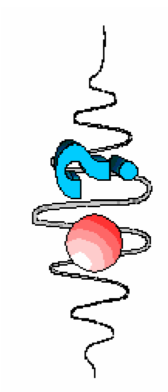</td><td>
$E = mc^2 + E_k = mc^2 + \tilde{E} = \gamma Mc^2, p^2c^2 + M^2c^4 = E^2, M = m + \frac{E_s}{c^2},$

$E_k = E_{mw} = E_k(p,v) + E_k(L_s,\omega_s) = \frac{pv + L_s\omega_s}{1+\sqrt{1-v^2/c^2}} = \frac{\mathbb{L}^*_{mw}\omega_{mw}}{1+\sqrt{1-v^2/c^2}} =$

$= \frac{\mathbb{L}_r\omega_r + L_s\omega_s}{1+\sqrt{1-v^2/c^2}} = \frac{\mathbb{L}_{mw}\omega_{mw} + L_s\omega_s}{1+\sqrt{1-v^2/c^2}} = \tilde{E} = h(f + f_s) = h\frac{\omega}{2\pi},$

$dE_k = vdp + \omega_s dL_s = \omega_r d\mathbb{L}_r + \omega_s dL_s = \omega d\mathbb{L}_{mw} + \omega_s dL_s = hd(f + f_s) = d\tilde{E} = dE_{mw},$

$E_{ms} = E_k(\mathbb{L}_s,\omega_s) = (M - m)c^2, v = \omega_r r, p = \gamma Mv, \mathbb{L}_r = \frac{pv}{\omega_r} = pr,$

$\omega_r d\mathbb{L}_r = \omega d\mathbb{L} = vdp, \mathbb{L}_{mw}\omega = \mathbb{L}_r\omega_r = pv, \mathbb{L}_{mw} \neq \mathbb{L}_r, \omega \neq \omega_r, \mathbb{L}_{mw}d\omega = \mathbb{L}_r d\omega_r,$

$\lambda = \frac{h}{p}, u = \lambda f = \frac{v}{1+\sqrt{1-v^2/c^2}}, v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda}, \omega_{mw} = \omega = 2\pi f,$

$\Psi^2 = \frac{d\tilde{E}}{dt} = v\frac{dp}{dt} + \omega_s\frac{dL_s}{dt} = \omega\frac{d\mathbb{L}_{mw}}{dt} + \omega_s\frac{dL_s}{dt} = \omega_r\frac{d\mathbb{L}_r}{dt} + \omega_s\frac{dL_s}{dt} =$

$= vF + \omega_s\tau_s = \omega_r\tau_r + \omega_s\tau_s = \omega\tau + \omega_s\tau_s .$
</td></tr>
</table>

Our universe hosts various inertial and uniform motions, a phenomenon partially explained by Newton's Law of Inertia, which relates to the conservation of linear momentum and force. However, many familiar or complementary motions that align with the primary concept of inertial events are still awaiting recognition as inertial motions. These include various rotating or spinning motions within planetary systems and atoms, angular and spin moments associated with elementary particles, states characterized by stable gyromagnetic ratios, and phenomena described by Faraday-Lenz laws of electromagnetic induction.

It is natural to generalize and unify our understanding of inertia and inertial motions through the lens of particle-wave duality concepts.

If we consider linear motion as a specific case of rotation with a large (or infinite) radius, then the law of conservation of orbital or angular momentum should encompass and represent the conservation of linear momentum. To illustrate this, we will apply conservation laws to a specific example of uniform and inertial particle motion, which incorporates elements of both linear and rotational motions. This exploration will utilize the particle-wave duality concepts elaborated upon earlier, referring to the relevant mathematical expressions from equations (4.3-1) to (4.3-3) and Table T.4.3.

If we (also) imagine that uniform state of motion (as one from **T.4.3.**) has many mutually related particles (and other energy states), and if we attempt to apply linear and orbital momentum conservation laws strictly, we will get,

$$\left\{\begin{array}{l} \vec{P}=\sum_{(i)}\vec{p}_i=\sum_{(i)}\overrightarrow{\left(L_{mw}\frac{\omega}{v}\right)_i}=\sum_{(i)}\overrightarrow{\left(L_r\frac{\omega_r}{v}\right)_i}=\sum_{(i)}\left(\frac{\vec{L}_r}{r}\right)_i=\overrightarrow{Const.}, \\ \vec{L}_r=\sum_{(i)}\overrightarrow{\left(p\frac{v}{\omega_r}\right)_i}=\sum_{(i)}\left(r\times p\right)_i=\sum_{(i)}\overrightarrow{\left(L_{mw}\frac{\omega}{\omega_r}\right)_i}=\sum_{(i)}\overrightarrow{\left(\frac{\gamma m v^2}{\omega_r}\right)_i}=\overrightarrow{const.}, \\ \vec{L}_{mw}=\sum_{(i)}\overrightarrow{\left(L_r\frac{\omega_r}{\omega}\right)_i}=\sum_{(i)}\overrightarrow{\left(p\frac{v}{\omega}\right)_i}=\sum_{(i)}\overrightarrow{\left(\frac{\gamma m v^2}{\omega}\right)_i}=\overrightarrow{CONST.}, \\ P_4^2=(\vec{P},\frac{E}{c})^2=inv.\Leftrightarrow P^2-\frac{E^2}{c^2}=-\frac{E_0^2}{c^2}, E_k=E-E_0=\tilde{E}=... \end{array}\right\}. \quad (4.3\text{-}3.1)$$

Demonstrating whether, how, and when all linear and orbital moments from (4.3-3.1), which are mutually dependent and related to the same inertial motion, can be conserved, for instance as, $\vec{P}=\overrightarrow{Const.}$, $\vec{L}_r=\overrightarrow{const.}$, $\vec{L}_{mw}=\overrightarrow{CONST.}$, is undoubtedly a challenging task. This exploration will rely on certain assumptions and approximations, like the elaboration found in Chapter 2.3.1, which discusses the extended meaning of inertia as shown in equations (2.9.1) and (2.9.2), among others.

If we assume that the relevant set of objects in (4.3-3.1) is in a specific, complex state of stable, inertial, uniform, circular, and stationary motion, like the planets in a solar system, and that all involved orbits host stable, standing matter-wave formations, then we can satisfy the conservation of all involved linear and orbital moments. For a more comprehensive understanding of this concept, refer to Chapter 2.3.1, where we apply the extended meaning of inertia in our analyses of various interactions and motions.

The conceptualization of matter waves, as presented in Table T.4.3 and Figures 4.1.2, 4.1.3, and 4.1.4, may require further support and refinement. This process will benefit from open-mindedness, creativity, and intellectual flexibility, yet the fundamental and ontological essence of matter waves is becoming increasingly clear.

We are now closer than ever to understanding what de Broglie or matter waves truly are. All currently known wave phenomena in our universe are, in some respect, manifestations of de Broglie matter waves. The challenge lies in our ignorance, misinterpretation, or incomplete and sometimes skewed modeling, which obscures our ability to recognize de Broglie matter waves as integral to the broader framework of physics.

Another contributing factor to this confusion is the adequacy of our current mathematical processing and engineering of micro-world problems, which yields statistically acceptable results. However, these results often present a superficial view by focusing on averages and probabilities, thereby neglecting the physical and conceptual dimensions of the issue at hand. The prevailing tendency in contemporary Quantum Theory is to assert its mathematical framework as the ultimate and sole acceptable perspective, a stance that may persist for some time before significant corrections are made.

4.1.2.1. Example 1: Bohr's Hydrogen Atom Model

As an illustration of the here introduced platform for treating de Broglie waves, let us briefly analyze a hydrogen atom in the center-of-mass coordinate system (not going too far from the original Bohr's model). Let us apply (4.1), (4.2) and (4.3) to describe the movement of an electron and a proton around their common center of gravity (in the center of mass system), and to describe their associated de Broglie waves, assuming that the electron and atom nuclei are treated as real rotating charged bodies (slightly modified Bohr's model). The following terms are involved here:

$\mathbf{m_e}$ -electron mass,

$\mathbf{m_p}$ -the nucleus or proton mass,

$\mathbf{v_e = \omega_{me} r_e}$ -electron velocity around the common center of gravity,

$\mathbf{v_p = \omega_{mp} r_p}$ -proton velocity around the common center of gravity,

$\mathbf{r_e}$ -the radius of the revolving electron,

$\mathbf{r_p}$ -the radius of the revolving proton,

$\omega_{me} = \omega_{mp} = \omega_m = \mathbf{2\pi f_m}$ -the mechanical, revolving frequency of the electron and the proton,

$\lambda_e = h/\gamma_e m_e v_e = h/p_e$ -de Broglie wavelength of the electron wave,

$\lambda_p = h/\gamma_p m_p v_p = h/p_p$ -de Broglie wavelength of the proton wave,

$\mathbf{f_e}$ -de Broglie frequency of the electron wave,

$\mathbf{f_p}$ -de Broglie frequency of the proton wave,

$\mathbf{u_e = \lambda_e f_e}$ -the phase velocity of de Broglie electron wave and

$\mathbf{u_p = \lambda_p f_p}$ -the phase velocity of de Broglie proton wave.

In order to satisfy structural stability and non-dissipative nature of a hydrogen atom, the internal angular momentum of the electron and proton orbital motion should also be conserved: $\gamma_e m_e r_e^p \omega_m = \gamma_p m_p r_p^2 \omega_m \Rightarrow \gamma_e m_e v_e r_e = \gamma_p m_p v_p r_p$, and electron and proton stationary wave structure should be stable and satisfy relations, $\mathbf{n\lambda_e = 2\pi r_e}$, $\mathbf{n\lambda_p = 2\pi r_p}$ (see **[4]** regarding the same situation). We can also imagine that an electron and proton have the forms of rotating, electrically charged rings, with spiral current paths on their toroid, like solenoids because of similar reasons already described with results (4.3) and

Fig. 4.1. This will only proportionally change their moments of inertia, for specific multiplicative constant on both sides of equation $\left(J_e\omega_{me} = A\cdot\gamma_e m_e r_e^2\omega_m = J_p\omega_{mp} = A\cdot\gamma_p m_p r_p^2\omega_m\ ,\ A = Const.\right)$, not changing the results (see (4.4)). In a few steps, implementing the conditions mentioned above in the framework of the original Bohr's atom model (and using data from (4.1) - (4.3)), we can find:

$$\sqrt{\frac{\gamma_p m_p}{\gamma_e m_e}} = \frac{r_e}{r_p} = \frac{\lambda_e}{\lambda_p} = \frac{p_p}{p_e} = \frac{\gamma_p m_p v_p}{\gamma_e m_e v_e} = \sqrt{\frac{\gamma_p}{\gamma_e}1836.13} = 42.8503386217\sqrt{\frac{\gamma_p}{\gamma_e}},$$

$$f_m = f_{me} = f_{mp} = f_{m(e,p)} = \omega_m / 2\pi,\quad \gamma_{e,p} = (1 - v_{e,p}^2/c^2)^{-0.5}$$

$$\text{or for } (v_e, v_p, u_e, u_p << c) \Rightarrow$$

$$\sqrt{\frac{m_p}{m_e}} \cong \frac{r_e}{r_p} = \frac{\lambda_e}{\lambda_p} = \frac{p_p}{p_e} \cong \frac{m_p v_p}{m_e v_e} \cong \frac{v_e}{v_p} \cong \frac{u_e}{u_p} \cong \sqrt{1836.13} = 42.8503386217$$

$$f_m \cong \frac{m_e e^4}{4n^3h^3\varepsilon_0^2},\quad \frac{v_p}{v_e}\sqrt{\frac{m_p}{m_e}} \cong \frac{E_{kp}}{E_{ke}} = \frac{\tilde{E}_p}{\tilde{E}_e} = \frac{m_p u_p}{m_e u_e} = 1,$$

$$f_e = n\frac{f_m}{2}(1+\frac{u_e^2}{c^2}) \cong n\frac{m_e e^4}{8n^3h^3\varepsilon_0^2}(1+\frac{u_e^2}{c^2}) \cong n\frac{m_e e^4}{8n^3h^3\varepsilon_0^2} \cong f_p,$$

$$f_p = n\frac{f_m}{2}(1+\frac{u_p^2}{c^2}) \cong n\frac{m_e e^4}{8n^3h^3\varepsilon_0^2}(1+\frac{u_p^2}{c^2}) \cong n\frac{m_e e^4}{8n^3h^3\varepsilon_0^2} \cong f_e,$$

$$\frac{f_e}{f_p} \cong 1 + 1836.13\frac{u_p^2}{c^2} \cong 1$$

$$1 < \frac{v_e}{u_e} = n\cdot\frac{f_m}{f_e} = \frac{2}{(1+\frac{u_e^2}{c^2})} = 1 + \sqrt{1 - \frac{v_e^2}{c^2}} \le 2,$$

$$1 < \frac{v_p}{u_p} = n\cdot\frac{f_m}{f_p} = \frac{2}{(1+\frac{u_p^2}{c^2})} = 1 + \sqrt{1 - \frac{v_p^2}{c^2}} \le 2,\ n = 1,2,3,... \tag{4.4}$$

It is important to underline that the revolving mechanical frequency of the electron and the proton around their common center of gravity, $\omega_{me} = \omega_{mp} = \omega_m = 2\pi f_m$ should not be mixed with de Broglie wave frequency of stationary electron and proton waves, $\omega_m = 2\pi f_m \neq (\omega_e = 2\pi f_e,\ \omega_p = 2\pi f_p)$, and that relationship between them is given by $f_{e,p} \le n\cdot f_{m\text{-}e,p} = f_{e,p}\cdot(1+\sqrt{1 - v_{e,p}^2/c^2}) \le 2f_{e,p}$.

From (4.4) we can also conclude that (wave) energy of a stationary electron wave ($hf_e = (\gamma_e\text{-}1)m_e c^2 = \gamma_e m_e v_e u_e = p_e u_e = E_k$) is fully equal to electron's motional or kinetic energy, meaning that the rest electron mass or its rest energy has no direct participation in this part of the energy (see much more about Bohr's atom model in Chapters 2., and 8).

*[♣ **COMMENTS & FREE-THINKING CORNER**:*

The relations equivalent to (4.4) should also apply to planets rotating around their suns. However, in this context, the dominant fields will be gravitational rather than electromagnetic, and the relevant mass ratios will differ. This indicates that planets within their solar systems also possess associated de Broglie waves. For example, our planet Earth not only rotates around the Sun but also spins on its axis. Additionally, our entire solar system orbits the center of our galaxy, which is itself in rotation.

Bohr's model of the hydrogen atom is straightforward and has been extensively tested, proving applicable within its defined framework, despite its known limitations. By combining Bohr's planetary model with the concept of de Broglie waves introduced here (as illustrated in Fig. 4.1 and supported by equations (4.1), (4.2), and (4.3)), we indirectly validate the hypothesis of this book. This hypothesis posits that every rectilinear motion is accompanied by rotation, which naturally generates de Broglie waves and yields the correct results as shown in (4.4). Furthermore, Bohr's hydrogen atom model can be utilized to demonstrate all elements of the Particle-Wave Duality Code (PWDC) outlined in equations (4.1) to (4.3).

Wave theory emerged from the need to explain Bohr's postulates, serving as its initial meaningful application. The stability of atoms and the experimentally verifiable spectral series of the hydrogen atom prompted modifications to Rutherford's dynamic atom model, culminating in Bohr's model. As the founder of Quantum Theory concepts, Bohr addressed the practical need to explain the structure of the orbital hydrogen atom and the nature of quantized emissions and absorptions of electromagnetic energy. He introduced specific postulates without providing a deeper explanation for them.

A key unresolved issue in Bohr's model is why and how an electron does not emit or absorb light while rotating in its stationary orbits. To create a workable mathematical formulation, electron momentum had to be quantized, and a logical connection was established between the frequency of periodic orbital electron motion and the frequency of the emitted or absorbed photons. In classical electrodynamics, these two frequencies are equal; however, in Bohr's hydrogen atom model, they differ, except for orbits with integer principal quantum numbers, where the frequencies are approximately equal. This discrepancy is further supported by the formulation of the "correspondence principle."

Photons can only be emitted or absorbed when electrons transition between two stationary orbits. The energy of such photons, which is directly proportional to their frequency multiplied by Planck's constant, is equal to the difference in energy between the corresponding orbital electron states. This legitimizes the Planck expression for narrow-band photon energy. The total energy of an electron in a stationary orbit is the sum of its kinetic and potential energies. It appears that Bohr simply merged concepts from Quantum Theory, Classical Mechanics, and Electrodynamics to explain already known phenomena, without deeply elaborating on his theory.

Luis de Broglie's electron wavelength fits seamlessly into the concept of stationary electron orbits, providing a straightforward explanation for atomic stability. It suggests that the perimeter of a stationary orbit must equal an integer multiple of the electron's matter-wave wavelength, akin to standing waves on a string. Thus, one could argue that Bohr inadvertently applied the PWDC without fully realizing its implications.

Unfortunately, Bohr's hydrogen atom model and the planetary atom concept now seem outdated compared to contemporary atomic modeling approaches, such as those proposed by Kanarev, C. Lucas, David L. Bergman, and others from the "Common Sense Science" movement. Furthermore, the understanding of the electron and its associated magnetic field requires significant updating. For innovative concepts related to the magnetic field around a rotating conductor, refer to the work of Henry Augustus Rowland as presented by Jean de Climont. *♣]*

4.1.3. Matter Waves and Conservation Laws

De Broglie relation, $\lambda = h/p$, for matter-wave wavelength, (4.1), is not fully explained and systematically developed starting from energy and momentum conservation laws. De Broglie found it by fitting the most logical solution that makes orbital electron wave stable, by supporting postulates of Bohr's hydrogen atom model (and creating self-closed standing waves). Later, the same de Broglie relation was successful in supporting the wave properties of different particles (diffraction and interference experiments, Compton, and Photoelectric effect…). Consequently, nobody asked the question of how this relation could generally be proved valid and developed from a more independent platform (than Bohr's hydrogen atom model, which was in certain aspects fundamentally wrong; -see Kanarev [44], and Bergman-Lucas [16] - [22]). The same situation was with Planck's expression for the photon energy $\tilde{\mathbf{E}} = \mathbf{hf}$, (4.1). **The proper expression for wave-packet energy was found (mathematically combined with certain assumptions) by the best curve fitting to explain measured data regarding the spectrum of the black body radiation.** Later, the same relation, combined with de Broglie wavelength, and energy-momentum conservation laws, was quietly generalized and accepted to become valid for any matter wave. This was successfully applied in explaining many quantum interactions (Compton Effect, Photoelectric Effect, etc. See more in chapter 10. Of this book).

Let us briefly exercise compatibility between de Broglie wavelength and Planck's wave energy with energy and momentum conservation laws, analyzing the situation on Fig.4.1.

We will start from a two-body interaction when two particles $\mathbf{m_1}$ and $\mathbf{m_2}$, ($m = m^{rel.} = m_o/\sqrt{1-v^2/c^2} = \gamma m_0$, $m_0 = const.$) move relative to each other, respectively having velocities $\mathbf{v_1}$ and $\mathbf{v_2}$, linear moments $\mathbf{p_1}$ and $\mathbf{p_2}$, orbital moments $\mathbf{L}_{c1} = \mathbf{L}_1$ and $\mathbf{L}_{c2} = \mathbf{L}_2$, and kinetic energies $\mathbf{E_{k1}}$ and $\mathbf{E_{k2}}$. By presenting them in the Laboratory and Center of mass systems, and applying total Energy and Momentum conservation laws (using already established analogies in earlier chapters and the same symbolic definitions and designations as in (4.1) - (4.4) and on Fig.4.1, where $\mathbf{E_{oi}}$ and $\mathbf{m_{oi}}$ are particle energies and masses in the state of rest) we will have:

$$E_{tot.} = E_{01} + E_{k1} + E_{02} + E_{k2} = E_{0C} + E_{kc} + E_{kr},$$
$$E_0 = E_{01} + E_{02} = E_{0C},\ E_{k1} + E_{k2} = E_{kc} + E_{kr}, E_{01} = m_{01}c^2, E_{02} = m_{02}c^2,\ E_{oc} = m_{oc}c^2,$$
$$m_{oc} = m_1 + m_2,\ \mu = m_r = \frac{m_1 m_2}{m_1 + m_2},\ E_{12} = \int_{(1)}^{(2)} \vec{F}_{12} d\vec{r}_{12} = E_{kr}, F_{12} = F_r = \frac{dp_r}{dt},$$

We already know from (4.1) and (4.2) that de Broglie wavelength and Planck wave packet energy in connection with group and phase velocity equation are fully mutually integrated, cross-related, and compatible, for instance,

$$\left\{\lambda = \frac{h}{p}, u = \lambda f, \tilde{E} = hf\right\} \Leftrightarrow \left\{\frac{u}{v} = \frac{hf}{pv} = \frac{\tilde{E}}{pv}\right\} \Leftrightarrow \left\{\begin{array}{l} v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} = u + p\frac{du}{dp} = \frac{d\omega}{dk} = \frac{d\tilde{E}}{dp} = \frac{2u}{1+\frac{uv}{c^2}}, \\ u = \lambda f = \frac{\omega}{k} = \frac{\tilde{E}}{p} = \frac{hf}{p} = \frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{E_k}{p} \\ \text{For microworld: } h = \text{Planck constant} \\ \text{For macroworld: } h \to H = \text{constant} \end{array}\right\}.$$

Since only motional (or kinetic) energy could be equal to (de Broglie) matter-wave energy, $E_k = \vec{p}\vec{u} = Hf = \tilde{E}$, there is no other possibility than treating de Broglie matter waves as motional energy forms. Also, matter-waves-wavelength cannot be conceptually and logically explained if we do not consider that all particle motions are cases of certain orbital motions, having associated spinning fields (see (4.3-0)-g). Taking all of that into account, we have,

$$\left\{\begin{array}{l} \bar{P}_4 = (\vec{P}, \frac{E}{c}) \Rightarrow \vec{P}^2 - \frac{E^2}{c^2} = \vec{P}'^2 - \frac{E'^2}{c^2} = -\frac{E_0^2}{c^2} = -\frac{(E_{01}+E_{02})^2}{c^2} = \text{invariant}. \\ \vec{P} = \vec{p}_1 + \vec{p}_2 = \gamma_1 m_1\vec{v}_1 + \gamma_2 m_2\vec{v}_2 = \vec{p}_c = \frac{E}{c^2}\vec{v}_c = \frac{E_1+E_2}{c^2}\vec{v}_c = \frac{E_c+E_r}{c^2}\vec{v}_c = \gamma_c m_{oc}\vec{v}_c, \\ E = E_{tot.} = E_1 + E_2 = \gamma_1 m_1 c^2 + \gamma_2 m_2 c^2 = E_c + E_r = E_{01} + E_{k1} + E_{02} + E_{k2} = E_{0C} + E_{kc} + E_{kr}, \\ \vec{v}_c = \frac{c^2}{E}\vec{p}_c = \frac{c^2(\gamma_1 m_1\vec{v}_1 + \gamma_2 m_2\vec{v}_2)}{\gamma_1 m_1 c^2 + \gamma_2 m_2 c^2} = \frac{\gamma_1 m_1\vec{v}_1 + \gamma_2 m_2\vec{v}_2}{\gamma_1 m_1 + \gamma_2 m_2}, \vec{v}_r = \vec{v}_1 - \vec{v}_2, u_i = \frac{v_i}{1+\sqrt{1-v_i^2/c^2}} = \lambda_i f_i, \\ E_{ki} = (\Delta m_i)c^2 = \tilde{E}_i = Hf_{si} = \frac{L_{si}\omega_{si}}{1+\sqrt{1-v_i^2/c^2}} = \frac{L_{ci}\omega_{ci}}{1+\sqrt{1-v_i^2/c^2}} = \frac{p_i v_i}{1+\sqrt{1-v_i^2/c^2}} = p_i u_i, \\ \vec{p} = \frac{\omega_{si}}{v_i}\frac{\cos(\vec{L}_{si},\vec{\omega}_{si})}{\cos(\vec{p}_i,\vec{v}_i)}\vec{L}_{si} = \gamma m\vec{v}_i, \vec{L}_{si} = \frac{v_i}{\omega_{si}}\frac{\cos(\vec{p}_i,\vec{v}_i)}{\cos(\vec{L}_{si},\vec{\omega}_{si})}\vec{p}_i, v_i = u_i - \lambda_i\frac{du_i}{d\lambda_i} = -\lambda_i^2\frac{df_i}{d\lambda_i}, u_i = \lambda_i f_i, \lambda_i = \frac{H}{p_i}, \\ \frac{H}{\pi} \leq L_{si} = \left\{\frac{H}{2\pi}(1+\sqrt{1-v_i^2/c^2}) = \frac{H}{2\pi}\frac{v_i}{u_i}\right\} \leq \frac{H}{2\pi}, \\ (\vec{\omega}_{ci} = 2\pi f_{ci} \text{ and } \vec{\omega}_{si} = 2\pi f_{si} \text{ are mutually orthogonal vectors}). \end{array}\right\}$$

$$\Leftrightarrow \left\{\begin{array}{l} \begin{bmatrix} \tilde{E}_1 + \tilde{E}_2 = \tilde{E}_c + \tilde{E}_r \\ \vec{v}_c = \frac{\gamma_1 m_1\vec{v}_1 + \gamma_2 m_2\vec{v}_2}{\gamma_1 m_1 + \gamma_2 m_2} \end{bmatrix} \Rightarrow \begin{bmatrix} [f_1 + f_2 = f_c + f_r], \\ |\vec{v}_r| = |\vec{v}_1 - \vec{v}_2|, |\vec{u}_r| = |\vec{u}_1 - \vec{u}_2| = \lambda_r f_r \end{bmatrix} \Rightarrow \\ \Rightarrow \left[|\vec{u}_c| = \left|\frac{\gamma_1 m_1\vec{u}_1 + \gamma_2 m_2\vec{u}_2}{\gamma_1 m_1 + \gamma_2 m_2}\right| = \lambda_c f_c = \frac{v_c}{1+\sqrt{1-v_c^2/c^2}}\right] \end{array}\right\} \Rightarrow \tag{4.5}$$

$$\Rightarrow \left\{\begin{array}{l} \lambda_1 = \frac{H}{p_1}, \lambda_2 = \frac{H}{p_2}, \lambda_c = \frac{H}{p_c}, \lambda_r = \frac{H}{p_r} = \lambda_1\frac{v_1}{v_r} + \lambda_2\frac{v_2}{v_r}, v_i = u_i - \lambda_i\frac{du_i}{d\lambda_i} = -\lambda_i^2\frac{df_i}{d\lambda_i} \\ (\vec{p}_1 + \vec{p}_2 = \vec{p}_c) \Leftrightarrow (\frac{\vec{p}_1}{H} + \frac{\vec{p}_2}{H} = \frac{\vec{p}_c}{H}) \Rightarrow \begin{bmatrix} \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2} + 2\frac{\cos(\vec{p}_1,\vec{p}_2)}{\lambda_1\lambda_2} = \frac{1}{\lambda_c^2} \\ \lambda_2^2\lambda_c^2 + \lambda_1^2\lambda_c^2 + 2\lambda_1\lambda_2\lambda_c^2\cos(\vec{p}_1,\vec{p}_2) = \lambda_1^2\lambda_2^2 \end{bmatrix} \\ \frac{Hv_1}{\lambda_1(1+\sqrt{1-v_1^2/c^2})} + \frac{Hv_2}{\lambda_2(1+\sqrt{1-v_2^2/c^2})} = \frac{Hv_c}{\lambda_c(1+\sqrt{1-v_c^2/c^2})} + E_r, \\ \vec{v}_c = \frac{\gamma_1 m_1\vec{v}_1 + \gamma_2 m_2\vec{v}_2}{\gamma_1 m_1 + \gamma_2 m_2} = \frac{\gamma_1 m_1\vec{u}_1 + \gamma_2 m_2\vec{u}_2}{\gamma_1 m_1 + \gamma_2 m_2} - \lambda_c\frac{d\left(\frac{\gamma_1 m_1\vec{u}_1 + \gamma_2 m_2\vec{u}_2}{\gamma_1 m_1 + \gamma_2 m_2}\right)}{d\lambda_c} (=) -\lambda_c^2\frac{df_c}{d\lambda_c} \end{array}\right\}. \tag{4.6}$$

Motional energies and group and phase velocities of mutually approaching objects (as well as all values in (4.5) and (4.6)) are coupled, time and position dependent, and continuously evolving, (since objects somehow communicate by the presence of surrounding fields. See later (4.5-3)).

More informative and useful relations unifying energy and momentum conservation (and avoiding differences between Relativistic and Classical Mechanics mass interpretation) can be given in the following differential form (as in (4.2)),

$$\left\{\begin{array}{l} E_{k1}(t,x)+E_{k2}(t,x)=E_{kc}(t,x)+E_{kr}(t,x) \Leftrightarrow \tilde{E}_1+\tilde{E}_2=\tilde{E}_c+\tilde{E}_r, \\ dE_{k1}+dE_{k2}=dE_{kc}+dE_{kr} \Leftrightarrow d\tilde{E}_1+d\tilde{E}_2=d\tilde{E}_c+d\tilde{E}_r, dE_{kr}=v_r dp_r=\omega_r dL_r \\ v_i=v_i(t,x)=u_i-\lambda_i\frac{du_i}{d\lambda_i}=-\lambda_i^2\frac{df_i}{d\lambda_i}=\frac{d\tilde{E}_i}{d\tilde{p}_i},\ u_i=u_i(t,x)=\lambda_i f_i \end{array}\right\} \Rightarrow \quad (4.7)$$

$$\Rightarrow \left\{\begin{array}{l} d\tilde{E}_i=Hdf_i=v_i dp_i=d(p_i u_i)=dE_{ki}=c^2 d(\gamma m_i)=-c^2 d\tilde{m}_i=-d(\tilde{p}_i u_i)= \\ =-v_i d\tilde{p}_i\cdot\cos(\vec{p}_i,\tilde{p}_i)=\left[v_i d\tilde{p}_i \text{ or } -v_i d\tilde{p}_i\right] \end{array}\right\},$$

since after applying integration (when solving such equations) we can take into consideration boundary conditions and all stationary, motional, or state of rest parameters of specific interaction (see also (4.9-0)).

Here we can also address the idea of how to treat forces acting between two mutually approaching particles ($\vec{F}_1,\vec{F}_2,\vec{F}_c,\vec{F}_r$), for instance:

$$\begin{array}{l} \left\{dE_{k1}+dE_{k2}=dE_{kc}+dE_{kr}\right\}/dt \Rightarrow \vec{v}_1\frac{d\vec{p}_1}{dt}+\vec{v}_2\frac{d\vec{p}_2}{dt}=\vec{v}_c\frac{d\vec{p}_c}{dt}+\vec{v}_r\frac{d\vec{p}_r}{dt} \Leftrightarrow \\ \Leftrightarrow \vec{v}_1\vec{F}_1+\vec{v}_2\vec{F}_2=\vec{v}_c\vec{F}_c+\vec{v}_r\vec{F}_r\ ;\ \vec{F}_1+\vec{F}_2=\vec{F}_c\ ,\ E_{12}=\int_{(1)}^{(2)}\vec{F}_{12}d\vec{r}_{12}=\int_{(1)}^{(2)}\vec{F}_r d\vec{r}_{12}=E_{kr}. \end{array} \quad (4.7.1)$$

By considering the existence of initial orbital moments (including spinning) of interaction participants, we can analogically (just for brainstorming exercising) create another torques-balancing equation,

$$\begin{array}{l} \left\{dE_{k1}+dE_{k2}=dE_{kc}+dE_{kr}\right\}/dt \Rightarrow \vec{\omega}_1\frac{d\vec{L}_1}{dt}+\vec{\omega}_2\frac{d\vec{L}_2}{dt}=\vec{\omega}_c\frac{d\vec{L}_c}{dt}+\vec{\omega}_r\frac{d\vec{L}_r}{dt} \Leftrightarrow \\ \Leftrightarrow \vec{\omega}_1\vec{\tau}_1+\vec{\omega}_2\vec{\tau}_2=\vec{\omega}_c\vec{\tau}_c+\vec{\omega}_r\vec{\tau}_r\ ;\ \vec{\tau}_1+\vec{\tau}_2=\vec{\tau}_c\ ,\ E_{12}=\int_{(0)}^{(2\pi)}\vec{\tau}_{12}d\theta=\int_{(0)}^{(2\pi)}\vec{\tau}_r d\theta=E_{kr}. \end{array} \quad (4.7.1\text{-}1)$$

Particularly interesting cases are when the force $\vec{F}_r=\vec{F}_{12}$ between mutually approaching objects becomes balanced with the centrifugal force of quasi-rotational movement of the same objects in their Center of mass system, $F_r=\frac{dp_r}{dt}=\frac{m_r v_r^2}{r_{12}}(=G\frac{m_r m_c}{r_{12}^2}\pm\ ...\ ?!)$. This could create conditions for stable and self-sustaining helix spinning-toroid formation, leading eventually to stable particle/s formation. See also force expressions (2.1) to (2.9) from the second chapter, to understand conceptually that such forces should have several static, dynamic, and mixed, linear, and rotational or spinning components. Also, see very indicative examples in the second chapter, around equations (2.11.13-1) - (2.11.13-5), where it is clearly shown how matter waves in two-body problems are created). Giving a little bit more freedom to our thinking and imagination, we could make direct relations and analogies between Ruđer Josip Bošković Force, (see literature references under [6]), and two-body particle-matter-wave interactions around equations from (4.5) to (4.7.1-1).

***[♣ COMMENTS & FREE-THINKING CORNER**:*

- *Ruđer Josip Bošković (18 May 1711 – 13 February 1787) was born in the Republic of Ragusa (now Dubrovnik, Croatia) during a time when the modern state of Croatia did not exist. The Republic of Ragusa was a multi-ethnic Adriatic city-state located within a region historically known as Dalmatia, a province of the Roman Empire. This region was not exclusively connected to the Croatian people as geopolitically conceived today. In fact, during the medieval period, Dubrovnik was part of the Serbian state in the 11th century and was largely populated by Serbs at the time.*

*The etymology of the term "Croatia" is debated, with some claims tracing it to the word *cratkia*, which meant "short" in Old Slavic. The region of Dalmatia, including its Slavic population, was influenced by various migrations and invasions, including the Ottoman Turks. The local Slavic population, predominantly Serbian, gradually converted from Orthodox Christianity to Roman Catholicism over centuries due to social and political pressures. This shift was driven by the benefits of assimilation into Catholic-dominated societies, leading to significant cultural and religious transformation.*

- *Croatian Identity and Historical Controversies: The creation of a distinct Croatian national identity in the modern sense is often viewed as a byproduct of geopolitical and ideological processes, particularly those influenced by Western powers, including the Vatican and Germanic states. These influences, alongside local historical circumstances, contributed to the development of a Croatian identity that, in many ways, diverged from its shared Slavic roots. The controversial policies and atrocities committed during World War II by the Ustaše regime further complicated the region's ethnic and political dynamics. During this period, hundreds of thousands of Serbs were exterminated, further deepening divisions between Serbs and Croats.*

Today, many Croats speak a language that is linguistically almost identical to Serbian, a testament to their shared cultural and historical heritage. While modern Croatian linguistics has sought to differentiate the language from Serbian, the mutual intelligibility between the two remains high.

- *Ruđer Bošković's Legacy: Born to an Italian mother and a Serbian father, Bošković was a product of this complex cultural and ethnic milieu. His education in Catholic schools exposed him to Western European intellectual traditions, though his identity as a "Croat" would have been largely anachronistic in his time. Regardless, Bošković's contributions to science—particularly in physics and cosmology—transcended national or ethnic boundaries. His work remains a testament to the universal nature of scientific inquiry, detached from the geopolitical conflicts of his era.*

- *Historical Revisionism and National Identity: Many of the historical claims about the origins of the Croats, Serbs, and other Slavic peoples are often rooted in ideological narratives that have been shaped over centuries. This includes attempts by various powers, particularly the Vatican and Germanic states, to reshape the national identities of these peoples. This manipulation, along with later geopolitical strategies, has led to the formation of new national identities in the Balkans and Eastern Europe, often at the expense of a shared Slavic heritage.*

The manipulation of history, language, and cultural identity for political purposes is a well-documented phenomenon in the region. This can be seen in the way languages, once united under a common Slavic or Serbian umbrella, have been deliberately divided and reclassified. The same process occurred in various other regions, where national and linguistic identities were reshaped through historical and geopolitical interventions.

- *Conclusion: Bošković's legacy, alongside that of other prominent figures like Nikola Tesla, Milutin Milanković, and Mihailo Pupin, underscores the potential for intellectual achievement to rise above the divisions of national or ethnic identity. In a globalized world, where cultural and ideological borders are increasingly porous, it is important to acknowledge the shared heritage of humanity while recognizing the forces that have historically sought to divide us. The focus should be on fostering unity and understanding, rather than perpetuating the artificial divisions that have plagued the Balkans and other parts of the world for centuries.* ♣]

Let us imagine that only one particle moves relatively towards the other (that is in a state of rest) and that the moving particle is much smaller than another particle (see equations (4.3), (4.5) and (4.6)). We can demonstrate that the rotation of a reduced mass $\mathbf{m_r}$ in the Center of mass system is responsible for creating de Broglie matter waves.

$$m_1 = m << m_2 = M,\ v_1 = v, v_2 = 0, E_{k1} = \tilde{E}_1 = \tilde{E}_r = pu = hf,\ E_{kc} \cong 0,$$

$$m_{oc} = m + M \cong M,\ m_r \cong m_1 = m,\ v_c \cong \frac{m}{M} v \cong 0, v_r = v_1 = v, u_c \cong \frac{m}{M} u \cong \frac{1}{2} v_c \cong 0,$$

$$(\vec{p}_1 + \vec{p}_2 = \vec{p}_c) \Leftrightarrow (\frac{\vec{p}_1}{h} + \frac{\vec{p}_2}{h} = \frac{\vec{p}_c}{h}) \Rightarrow \begin{bmatrix} \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2} + 2\frac{\cos(\vec{p}_1, \vec{p}_2)}{\lambda_1 \lambda_2} = \frac{1}{\lambda_c^2} \\ \lambda_2^2 \lambda_c^2 + \lambda_1^2 \lambda_c^2 + 2\lambda_1 \lambda_2 \lambda_c^2 \cos(\vec{p}_1, \vec{p}_2) = \lambda_1^2 \lambda_2^2 \end{bmatrix},$$

$$p = \gamma mv = p_1 \cong p_r, p_2 = 0, \cos(\vec{p}_1, \vec{p}_2) = \cos\pi = -1,$$

$$u_r = u = u_1 = \lambda f = \frac{hf}{p} = \frac{v}{1 + \sqrt{1 - v^2/c^2}}, u_2 = 0,$$

$$f = f_1 \cong f_r = \frac{\mathbf{J}_r \omega_{mr}^2}{h(1 + \sqrt{1 - v_c^2/c^2})} = \frac{nf_m}{1 + \sqrt{1 - v_c^2/c^2}} \cong \frac{nf_m}{2}, \omega_{mr} = 2\pi f_m,$$

$$f_m = nh/4\pi^2 \mathbf{J}_r, \mathbf{L}_r = \mathbf{J}_r \omega_m = nh/2\pi = n\hbar, n \in N \Leftrightarrow \{1,2,3...\}, \tag{4.8}$$

$$f_2 \cong f_c \cong \frac{m}{M} f \cong 0, \lambda_1 = \frac{h}{p_1} = \frac{h}{\gamma mv} = \frac{h}{p} = \lambda \cong \lambda_r \cong \lambda_c, \lambda_2 \cong \frac{1}{2}\lambda.$$

What is interesting in (4.8) is that wavelength $\lambda_2 \cong \frac{1}{2}\lambda = \frac{1}{2}\lambda_1$ also exists, even $\mathbf{p_2} = \mathbf{0}$, probably as the consequence of a kind of "mirror imaging effect" of the incident particle $\mathbf{m}$ that creates coupling and interactive field with its (big mass) target, meaning that specific oscillating perturbation should also be measurable on/in the mass $\mathbf{M}$.

Since in the Center of mass system both moving particles $\mathbf{m_r}$ and $\mathbf{m_c}$ can also be presented as rotating around their common center of inertia, this "rotation" directly creates associated de Broglie waves. Such waves are mathematically recognizable on an energetic or spectral level, and "visible" in the Center of mass system, but not necessarily and directly recognizable and "visible" in the original time-space domain, in the Laboratory System. The above-analyzed example, summarized by (4.8), can also be applicable in the case of hydrogen Bohr's atom model (4.4), where an electron rotates around atom nucleus.

Here we have been talking more about transient, motional elements like a mechanical rotation (in the Center of a mass system), than about real, full-circle rotational movement (of interacting particles). When particles are in linear motion and approaching each other (without previously having their orbital moments), it is imaginable that specific transitory angular and vortex field components could be created between them (producing de Broglie matter waves), and both particles will feel (and get) equal, mutually opposite (mechanical) orbital moments. Consequently, the resulting orbital moment (of all mutually interacting objects) equals zero, but energy (or spectral) component associated with such orbital moments could be higher than zero. Generalizing this situation, we can always say that every single particle in linear motion should have a certain level of associated rotational components (orbital moments, spin, torsion field structure, etc.) since it always creates a two-body system with the rest of the surrounding universe. Implicitly, here we always assume that between mutually interacting particles there should exist specific field, force and certain material wave carrier, or some physical medium, even though we are sometimes not able to detect or explain what kind of material wave carrier we are dealing with.

This situation can also be modeled as a dynamic and transient "dipole-formation" (between the moving particle and its vicinity, or its target), where such dipoles could have electrical, magnetic, gravitational, inertial, or some other composite nature. The above-mentioned "dipole states" effectively rotate or produce transitory torque swings in a local Center of mass system, because the observed particle moves, and consequently produces angular and vortex field components.

The next consequence could be that Einstein Special Relativity Theory (SRT) is much more limited than it is currently considered to be (valid only under certain assumptions and for uniform, non-accelerated and rectilinear motion, which effectively does not exist without elements of rotation), and that something similar should also be valid for Maxwell electromagnetic field components. It becomes evident that SRT, Gravitation and linear motions should be upgraded for the missing rotational (or spinning) field components. Typical examples of situations that are sources of torsion field components should be all cases of elastic collisions (of course, any other collision types should also create torsion field components and de Broglie matter waves).

When we consider initial particles attributes, long before the interaction, and final attributes long after interaction happen, we address the totality of possible interactions between mutually approaching objects $\mathbf{m_1, m_2}$, or more correctly between their moments $\mathbf{p_1, p_2}$. We should also consider specific coupling (or binding) energy $\mathbf{U_{12}}$, or potential energy $\mathbf{U(r_{12})}$ between them, especially in cases of plastic collisions when after collision we get only one object. Until the point when a collision starts and during the short process of collision (before colliding objects separate again or stay united) we can treat all collisions in the same way. Later, if objects separate, this could be the case of elastic collisions, but if objects stay fully united, we shall have an ideal plastic collision (and we could also have some other intermediary cases).

This way, the ideal plastic collision (realized or not realized) becomes like an asymptotic guiding and modeling frame for treating all collision types (as well as for treating all other interactions between two objects). Here, the idea favored is that the most essential and decisive elements of one collision process are parameters of that process ($\mathbf{m_c, m_r, v_c, v_r}$), related to its Laboratory and Center of Inertia or Center of Mass reference system. The mutually closer interacting objects with masses (m_1, m_2) and moments ($\mathbf{p_1, p_2}$) are, the more dominant, and more relevant become (new and calculated) equivalent parameters ($\mathbf{m_c, m_r, v_c, v_r}$) = (central mass, reduced mass, center mass speed, reduced mass speed). The two-body problem between mutually non-interacting, neutral objects (like between two neutral masses) in this book is also treated as kind of interaction between them, underlining that in the near proximity of mutually approaching objects certain specific, dynamic, and transitory conditions are created which are making such bodies as mutually interacting (or energy exchanging and generating matter waves).

[♣ COMMENTS & FREE-THINKING CORNER:

Effectively, in case of plastic collision of two mutually non-interacting, electrically neutral particles (if we take care only about input-output energy balance), the equations (4.5) - (4.8) could be modified accordingly, for instance:

$$E = E_{tot.} = E_1 + E_2 = E_{01} + E_{k1} + E_{02} + E_{k2} = E_{oc} + E_{kc} + E_{kr},$$
$$E_{01} + E_{02} = E_{oc} = (m_1 + m_2)c^2 = m_{oc}c^2,\ E_k = E_{k1} + E_{k2} = E_{kc} + E_{kr}, \quad (4.8\text{-}1)$$
$$E_{01} = m_1c^2, E_{02} = m_2c^2,$$
$$E_{kr} = E_{k1} + E_{k2} - E_{kc}.$$

Let us briefly mention the number of mathematical options related to well-known or most probable formulations of energies involved and moments (to initiate thinking about the same problem from different platforms).

$$E_{k1} = \frac{\gamma_1 m_1 v_1^2}{1+\sqrt{1-\frac{v_1^2}{c^2}}} = \frac{(p_1)^2}{\gamma_1 m_1\left[1+\sqrt{1-\frac{v_1^2}{c^2}}\right]} = \frac{p_1 p_c}{\gamma_1 m_{oc}\left[1+\sqrt{1-\frac{v_1^2}{c^2}}\right]} + \frac{p_1 p_r}{\gamma_1 m_1\left[1+\sqrt{1-\frac{v_1^2}{c^2}}\right]} = E_{k1c} + E_{k1r} =$$
$$= (\gamma_1 - 1)m_1c^2 = p_1v_1 / \left[1+\sqrt{1-\frac{v_1^2}{c^2}}\right] = p_1c\sqrt{\frac{\gamma_1-1}{\gamma_1+1}},\ p_1 = \gamma_1 m_1 c\sqrt{\gamma_1^2-1} = \gamma_1 m_1 v_1,$$

$$E_{k2} = \frac{\gamma_2 m_2 v_2^2}{1+\sqrt{1-\frac{v_2^2}{c^2}}} = \frac{(p_2)^2}{\gamma_2 m_2\left[1+\sqrt{1-\frac{v_2^2}{c^2}}\right]} = \frac{p_2 p_c}{\gamma_2 m_c\left[1+\sqrt{1-\frac{v_2^2}{c^2}}\right]} - \frac{p_2 p_r}{\gamma_2 m_2\left[1+\sqrt{1-\frac{v_2^2}{c^2}}\right]} = E_{k2c} - E_{k2r} =$$
$$= (\gamma_2 - 1)m_2c^2 = p_2v_2 / \left[1+\sqrt{1-\frac{v_2^2}{c^2}}\right] = p_2c\sqrt{\frac{\gamma_2-1}{\gamma_2+1}},\ p_2 = \gamma_2 m_2 c\sqrt{\gamma_2^2-1} = \gamma_2 m_2 v_2,$$

$$E_{kc} = \frac{\gamma_c m_{oc} v_c^2}{1+\sqrt{1-\frac{v_c^2}{c^2}}} = \frac{(p_c)^2}{\gamma_c m_{oc}\left[1+\sqrt{1-\frac{v_c^2}{c^2}}\right]} = \frac{p_1 p_c}{\gamma_c m_{oc}\left[1+\sqrt{1-\frac{v_c^2}{c^2}}\right]} + \frac{p_2 p_c}{\gamma_c m_{oc}\left[1+\sqrt{1-\frac{v_c^2}{c^2}}\right]} = E_{kc1} + E_{kc2} =$$

$$= \frac{\gamma_1}{\gamma_c}\cdot\frac{m_1}{m_c}E_{k1} + \frac{\gamma_2}{\gamma_c}\cdot\frac{m_2}{m_c}E_{k2} + \frac{p_1p_2}{\gamma_c m_c} = (\gamma_c - 1)m_{oc}c^2 = p_cv_c / \left[1+\sqrt{1-\frac{v_c^2}{c^2}}\right] = p_cc\sqrt{\frac{\gamma_c-1}{\gamma_c+1}},$$
$$p_c = \gamma_c m_c c\sqrt{\gamma_c^2-1} = \gamma_c m_{oc} v_c = |\vec{p}_1 + \vec{p}_2|,\ \vec{p}_c + \vec{p}_r = \vec{p}_c = \vec{p}_1 + \vec{p}_2,$$

$$E_{kr} = \frac{p_r v_r}{1+\sqrt{1-\frac{v_c^2}{c^2}}} = J_r\omega_r^2 / \left[1+\sqrt{1-\frac{v_c^2}{c^2}}\right] = \frac{\gamma_2}{\gamma_c}\cdot\frac{m_2}{m_{oc}}E_{k1} + \frac{\gamma_1}{\gamma_c}\cdot\frac{m_1}{m_{oc}}E_{k2} - \frac{p_1p_2}{\gamma_c m_{oc}} \quad ?!$$

$$p_r = \|\vec{p}_r\|_{eff.} = m_r v_r \ (= \text{effective value}), ?!$$
$$\vec{p}_r = \iiint d\vec{p}_r = \vec{0} \ (= \text{total, resulting vectorial field, ??!!}), ?!$$

$$E_{k1} + E_{k2} = E_{kc} + E_{kr} \Rightarrow v_1dp_1 + v_2dp_2 = v_cdp_c + v_rdp_r,$$

$$\Leftrightarrow \frac{\gamma_1 m_1 v_1^2}{1+\sqrt{1-\frac{v_1^2}{c^2}}} + \frac{\gamma_2 m_2 v_2^2}{1+\sqrt{1-\frac{v_2^2}{c^2}}} = \frac{p_c v_c}{1+\sqrt{1-\frac{v_c^2}{c^2}}} + \frac{p_r v_r}{1+\sqrt{1-\frac{v_c^2}{c^2}}}, ?!$$

$$\Leftrightarrow E_{k-Lab.} = [E_{k1} + E_{k2}]_{Lab.} = [E_{k1}]_{Lab.} + [E_{k2}]_{Lab.} =$$
$$= [E_{k\text{-Translat.}}] + [E_{k\text{-Rotat.}}] = \frac{(p_1 + p_2)v_c}{1+\sqrt{1-\frac{v_c^2}{c^2}}} + \frac{J_r\omega_r^2}{1+\sqrt{1-\frac{v_c^2}{c^2}}},$$
$$[E_{k\text{-Translat.}}] = \frac{(p_1 + p_2)v_c}{1+\sqrt{1-\frac{v_c^2}{c^2}}} = \frac{J_c\omega_c^2}{1+\sqrt{1-\frac{v_c^2}{c^2}}}, \tag{4.5-1}$$
$$[E_{k\text{-Rotat.}}] = \frac{J_r\omega_r^2}{1+\sqrt{1-\frac{v_c^2}{c^2}}} = \frac{p_r v_r}{1+\sqrt{1-\frac{v_c^2}{c^2}}},$$

$$2\pi r_i = n\lambda_i = n\frac{h}{p_i} = \frac{v_i}{f_m} = n\frac{u_i}{f_i},\ u_i = \lambda_i f_i,\ \omega_r = \omega_c = 2\pi f_m ,$$

where indexing "**Lab.**" represents energy states in a Laboratory coordinate system, "**Translat.**" energy states of translation (or linear motion) and index "**Rotat.**" represents energy states of rotation. Based on (4.5-1) we could also upgrade (4.6) - (4.8) similarly. In the process of particles' mutual approaching and an impact, and just after the impact happens, all energies and moments from (4.5-1) should be presented by time-space evolving functions.

*What is very characteristic in (4.5-1) is that every particle (m_1, m_2, m_{oc}) in the Laboratory System can have certain non-zero momentum $\vec{p}_1, \vec{p}_2, \vec{p}_c$ (in a vector form), except the particle $\mathbf{m_r}$. The resulting linear macro moment of $\mathbf{m_r}$, as a vector equals zero $\vec{p}_r = 0$ (in the Laboratory System), but its effective (**eff.**) non-vector moment (that makes contribution in energy E_{kr}) is different from zero, $p_r = \|\vec{p}_r\|_{eff.} \neq 0$... (?!). This implicates that $\mathbf{m_r}$ should be distributed around m_{oc} (in some cases maybe like a toroid) performing rotation, which would create $\vec{p}_r = \vec{0}$. What should be distributed around m_{oc} is an inertial and waiving field that has torsion components and motional energy $\mathbf{E_{kr}}$.* It is also important to notice that energy conservation of two-body interactions, like (4.5-1), in the physics of particle interactions, is usually analyzed without highlighting its direct relation to Particle-Wave Duality and Torsion Fields. Certain expressions in (4.5-1) should still be considered only as "temporarily valid" (as a starting brainstorming initiation). Relevant elaborations should be reconfirmed and developed from a much more general platform, such as the one given by T.4.4 See, also, very indicative examples in the second chapter around equations (2.11.13-1) - (2.11.13-5), where it is clearly shown how matter waves in two-body problems are being created.

In parallel with here given conceptualization, we could also say that de Broglie matter waves could be presentable as products of certain "equivalent to antenna, or resonant circuit oscillations", where moving mass, force-coupled with its environment, intrinsically creates a kind of mass-spring or inductance-capacitance oscillating circuit (where missing oscillatory circuit elements belong to the particle environment). Here we could also apply a much wider analogy with electric or mechanical oscillatory circuits to deduce what should be the unknown oscillatory circuit elements that complement the motion of the known mass since we already know some of the important parameters of de Broglie waves. In other words, we know certain results, and we would search what produces such results. We know from electrical oscillatory circuits that total energy circulates between inductive and capacitive elements (following certain sinusoidal function, and periodically being either electrical or fully magnetic, but having constant, a total amount of energy. Analogically thinking, we would be able to conclude that total motional particle energy should fluctuate from the kind of linear motion kinetic energy to its complementary rotational motion energy and vice versa. This could be the reason why de Broglie waves are detectable only as consequences or final acts of certain interactions of particles (behaving as well-hidden waves). This theoretical matter wave concept regarding "equivalent oscillatory, resonant and antenna type circuits" should be much better elaborated later and connected with concepts and results from (4.3).

If we now imagine (staying in the frames of Newtonian mechanics) that between interacting objects could exist specific coupling or binding energy U_{12}, and that there should be specific energy exchange between energy states of translation and rotation (for the amount $\delta m \cdot c^2 = U_{12} / c^2 \Rightarrow E_{Lab.} = E_{CI} = [E_c]_{Translat.} + [E_r]_{Rotat.} \cong \{[E_c]_{Translat.} - \delta m \cdot c^2\} + \{[E_r]_{Rotat.} + U_{12} / c^2\}$), instead of (4.5-1) we can create the following (very much speculative, and unusual to think about) energy conservation balance:

$$E_{tot.} = E_1 + E_2 = E_c + E_r = (E_c - \delta m \cdot c^2) + (E_r + \delta m \cdot c^2), E_c = E_{oc} + E_{kc}, E_r = E_{0r} + E_{kr},$$
$$E_{tot.} = (E_{01} + E_{k1}) + (E_{02} + E_{k2}) = (E_{oc} + E_{kc}) + (E_{0r} + E_{kr}),$$

$$E_{01} + E_{02} = E_{oc} + E_{0r},\ E_{k1} + E_{k2} = E_{kc} + E_{kr},$$
$$E_{01} = m_1 c^2, E_{02} = m_2 c^2, E_{oc} = (m_{oc} - \delta m_c) c^2,\ E_{0r} = (m_r + \delta m_r) c^2 = \delta m_c c^2,$$

$$E_{k1} = (\gamma_1 - 1) m_1 c^2 = E_1 - E_{01}, E_{k2} = (\gamma_2 - 1) m_2 c^2 = E_2 - E_{02},$$
$$E_{kc} = (\gamma_c - 1)(m_{oc} - \delta m_c) c^2, E_{kr} = (\gamma_r - 1)(m_r + \delta m_r) c^2 = (\gamma_r - 1) \delta m_c c^2,$$

$$\frac{\delta m_c}{\delta m_r} = (\frac{v_r}{v_c})^2,\ \delta m_r = m_r / \left[(\frac{v_r}{v_c})^2 - 1 \right] = \frac{v_c}{v_r} \cdot \frac{U_{12}}{c^2} = \delta m_c - m_r \cong \frac{v_c}{v_r} \cdot \delta m,$$

$$\delta m_c = m_r (\frac{v_r}{v_c})^2 / \left[(\frac{v_r}{v_c})^2 - 1 \right] = \frac{v_r}{v_c} \cdot \frac{U_{12}}{c^2} = m_r + \delta m_r \cong \frac{v_r}{v_c} \cdot \delta m, \tag{4.5-2}$$

$$m_{oc} = m_1 + m_2 = \frac{m_1 m_2}{\delta m} \cdot (\frac{v_r v_c}{v_r^2 - v_c^2}), \delta m \cong \sqrt{\delta m_r \delta m_c} = \frac{v_r}{v_c} \delta m_r = U_{12}/c^2,$$
$$m_r = m_1 m_2 / (m_1 + m_2) = \frac{U_{12}}{c^2} \cdot (\frac{v_r^2 - v_c^2}{v_r v_c}) = \delta m \cdot (\frac{v_r^2 - v_c^2}{v_r v_c}) = \delta m_c - \delta m_r.$$

If we now compare the first relation from (4.4) for non-relativistic velocities, $\frac{m_p}{m_e} = (\frac{v_e}{v_p})^2 = (\frac{r_e}{r_p})^2 = (\frac{\lambda_e}{\lambda_p})^2 = (\frac{u_e}{u_p})^2$, with similar mass relation from (4.5-2), $\frac{\delta m_c}{\delta m_r} = (\frac{v_r}{v_c})^2$, it becomes obvious where (hidden) elements of rotation associated to linear motions (of mutually attracting or interacting objects) are.

If we now consider that mases m_1 and m_2 are electrically charged particles (for instance an electron and a proton, or any other combination of $+/-q_1$ and $-/+q_2$), and at the same time we know that involved attractive forces between them would be for too many orders of magnitude stronger compared to gravitational attraction between them), we will again have similar "center of mass, two-body relations", like between masses m_1 and m_2 , only being replaced by $+/-q_1$ and $-/+q_2$ (see familiar elaborations in the same chapter, under "4.1.2. Matter Waves Unity and Complementarity of Linear, Angular and Fluid Motions"). The direct proportionality (and some kind of formal analogy) between electric charges and their relevant mases is also elaborated in Chapter 1., regarding Analogies, under (1.14) – (1.16), in Chapter 2., regarding Gravitation, within equations (2.4-4.1) – (2.4-4.3), (2.4-4) – (2.4-8), and table "T.2.8. N. Bohr hydrogen atom and planetary system analogies", and in Chapter 3. under (3.5-a). All that gives us chances to analogically treat two-body mechanical systems, as two-body electric, or electromagnetic systems (when electromagnetic charges are significantly involved), profiting from already well-developed analyses of two-body mechanical systems.

To reveal the secret about what could happen in the close vicinity of approaching objects is not an easy task because the full picture can be created only if we consider that specific (known or unknown) carrier medium (fluid or coupling field) should exist between them. In such carrier-medium, particle-wave phenomenology should be detectable. This can be analyzed by solving characteristic wave equations describing such a process (see (4.25) - (4.37)). Before we develop universal wave equations (suitable for treating such situations), we will try to use simplified modeling and to create clear conceptual picture related to collision processes. The principal objective in presenting different expressions for energy conservation in (4.5) - (4.8), (4.5-1) and (4.5-2) is to show that all two-body interactions create a particular near field, a transitory interaction zone, where all interacting members (real and virtual) mutually "communicate" producing inertial and particle-wave duality effects (being especially significant when electromagnetic charges are involved).

The wave-to-particle transformation, or particle creation, should be a process related to two-body, mutually approaching objects interactions. Their relative energy (in their center of a mass system), $E_{kr} = E_{12} = \int_{(1)}^{(2)} \vec{F}_{12} d\vec{r}_{12} = p_r u_r$*, would become a part of internal energy (and rest mass) of the unified object* $\mathbf{m_c}$*, in case of an ideally plastic impact. The other possibility is that before such impact happens, the same relative energy* $\mathbf{E_{kr}}$ *will reach certain energy level (and satisfy other necessary conditions, related to relevant conservation laws), sufficient for generating new particles (which are initially not present in the same interaction). The typical example of such interactions is when a very high-energy photon passes close to an atom, generating a couple of electron-positron particles (practically transforming the quasi-rotating wave energy content* $\mathbf{E_{kr}}$*, of a high-energy incident photon, into (new) real particles with non-zero rest masses). ♣]*

Let us again summarize two-body relations and underline how and where matter waves are being created (see the illustration on Fig. 4.1.5. and details from T.4.4).

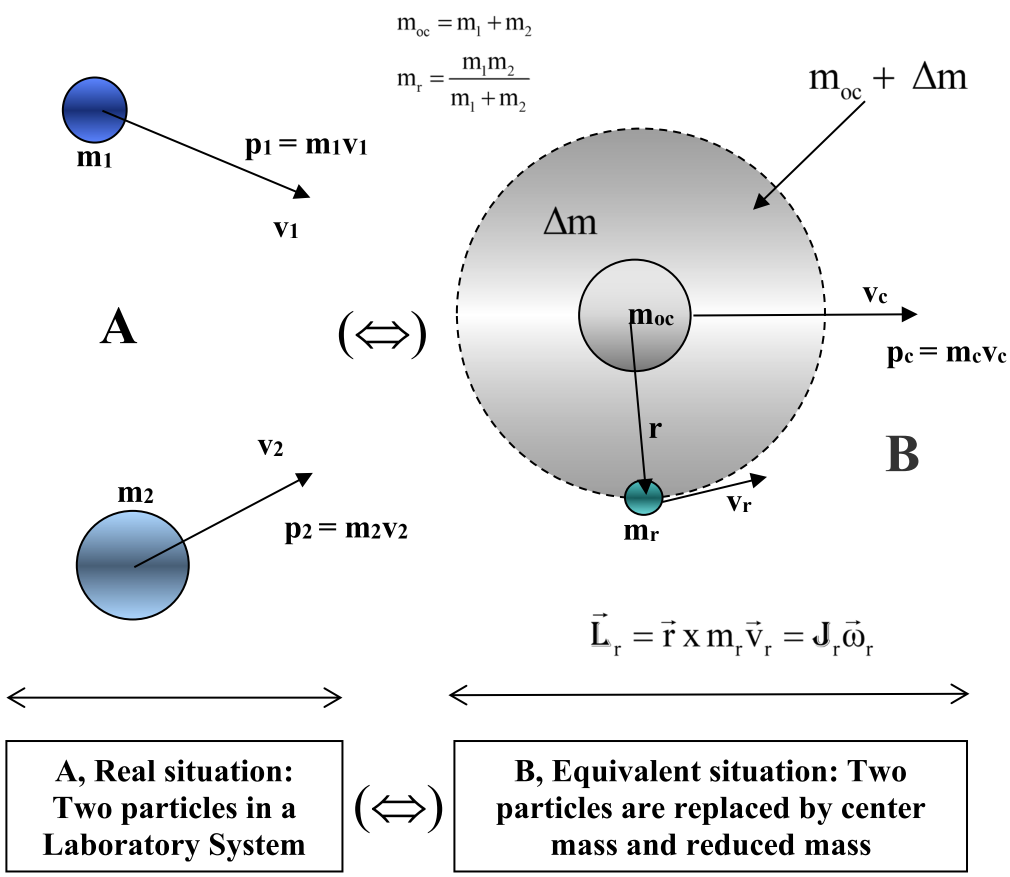

Fig.4.1.5 Mutually equivalent presentations of a two-body system: A ⇔ B
A (=) before interacting, B (=) in the process of interacting
(The plane where m_r performs rotation-like motion around m_c should be considered being perpendicular to the center of mass velocity v_c; -see T.4.4

The same two-body system from Fig.4.1.5, which is equivalent to the situation from Fig.4.1, can be analyzed (or described) from energy and momentum conservation laws, as follows (see the table below; T.4.4). Under two-body interactions, here we would understand: Elastic and/or Inelastic Impacts, Particle/s Creation and/or Disintegration/s, Annihilation, Compton, and Photoelectric effect etc. See, also, very indicative examples in the second chapter, around equations (2.11.13-1) - (2.11.13-5), where it is clearly shown how matter-waves in two-body problems are created.

From the total energy conservation (comparing the states in a Laboratory system given under **A** and **B**), the two-body situation from Fig. 4.1.5 could be described as,

$$E_{total} = E = E(A) = E_0(A) + E_k(A) = (m_1 + m_2)c^2 + (\gamma_1 - 1)m_1c^2 + (\gamma_2 - 1)m_2c^2 =$$
$$= E(B) = E_0(B) + E_k(B) = m_cc^2 + E_{0r} + (\gamma_c - 1)m_cc^2 + E_{kr} = m_cc^2 + m_rc^2 + (\gamma_c - 1)m_cc^2 + E_{kr} =$$
$$= \gamma_1m_1c^2 + \gamma_2m_2c^2 = \gamma_cm_cc^2 + E_r = \gamma_cMc^2 \Rightarrow$$
$$M_{total} = M = m_c + \frac{E_r}{\gamma_cc^2} = \frac{\gamma_1m_1 + \gamma_2m_2}{\gamma_c}, \ (m_r = \frac{m_1m_2}{m_1 + m_2}, \ m_c = m_1 + m_2),$$
$$E_r = E_{0r} + E_{kr} = m_rc^2 + E_{kr} = \gamma_c(M - m_c)c^2 = (\gamma_1m_1 + \gamma_2m_2 - \gamma_cm_c)c^2, E_{0r} = m_rc^2,$$
$$E_{kr} = [\gamma_c(M - m_c) - m_r]c^2 = \left[\gamma_c(\frac{M - m_c}{m_r}) - 1\right]m_rc^2 = (\gamma_1m_1 + \gamma_2m_2 - \gamma_cm_c - m_r)c^2,$$
$$E_c = \gamma_cm_cc^2 = E_{oc} + E_{kc} = m_cc^2 + (\gamma_c - 1)m_cc^2, E_{oc} = m_cc^2, E_{kc} = (\gamma_c - 1)m_cc^2.$$

Since situations A and B describe two mutually (dynamically) equivalent states, it should be valid,

$$E_0 = Mc^2 = \left(\frac{\gamma_1m_1 + \gamma_2m_2}{\gamma_c}\right)c^2 = \left(m_c + \frac{E_r}{\gamma_cc^2}\right)c^2 = (m_c + \Delta m)c^2 = E_0(A) \ = \ E_0(B),$$
$$E_0(A) = E_{01} + E_{02} + \Delta E_A = m_1c^2 + m_2c^2 + \Delta E_A = m_cc^2 + \Delta E_A = Mc^2,$$
$$E_0(B) = E_{0r} + E_{oc} + \Delta E_B = m_rc^2 + m_cc^2 + \Delta E_B = Mc^2,$$
$$\Delta E_A = \left[\frac{\gamma_1m_1 + \gamma_2m_2}{\gamma_c} - (m_1 + m_2)\right]c^2 = c^2\Delta M_A,$$
$$\Delta E_B = \left[\frac{\gamma_1m_1 + \gamma_2m_2}{\gamma_c} - (m_r + m_c)\right]c^2 = c^2\Delta M_B, \ \Delta m = \frac{E_r}{\gamma_cc^2}.$$

$$E_k = (\gamma_c - 1)Mc^2 = (\gamma_c - 1)(m_c + \frac{E_r}{\gamma_cc^2})c^2 = (\gamma_c - 1)(\frac{\gamma_1m_1 + \gamma_2m_2}{\gamma_c})c^2 = E_k(A) \ = \ E_k(B),$$
$$E_k(A) = E_{k1} + E_{k2} + \delta E_A = (\gamma_1 - 1)m_1c^2 + (\gamma_2 - 1)m_2c^2 + \delta E_A,$$
$$E_k(B) = E_{kc} + E_{kr} + \delta E_B = (\gamma_c - 1)m_cc^2 + E_{kr} + \delta E_B = [(\gamma_1m_1 + \gamma_2m_2) - (m_r + m_c)]c^2 + \delta E_B,$$
$$\delta E_A = (\gamma_c - 1)(\frac{\gamma_1m_1 + \gamma_2m_2}{\gamma_c})c^2 - [(\gamma_1 - 1)m_1 + (\gamma_2 - 1)m_2]c^2 = c^2\delta M_A,$$
$$\delta E_B = (\gamma_c - 1)(\frac{\gamma_1m_1 + \gamma_2m_2}{\gamma_c})c^2 - [(\gamma_1m_1 + \gamma_2m_2) - (m_r + m_c)]c^2 = c^2\delta M_B.$$

$$\left\{\begin{array}{l} \left[\vec{P}(A)\right]^2 - \frac{1}{c^2}[E(A)]^2 = \left[\vec{P}(B)\right]^2 - \frac{1}{c^2}[E(B)]^2 = invariant = M^2c^2, \\ \vec{P}(A) = \vec{p}_1 + \vec{p}_2 = \gamma_1m_1\vec{v}_1 + \gamma_2m_2\vec{v}_2, \\ \vec{P}(B) = \vec{P}(A) = \gamma_cm_c\vec{v}_c + \delta\vec{p} = \gamma_c(m_c + \delta m)\vec{v}_c, \\ E(A) = \gamma_1m_1c^2 + \gamma_2m_2c^2, \ E(B) = \gamma_cm_cc^2 + \delta E = \gamma_cm_cc^2 + E_r. \end{array}\right\} \Rightarrow$$
$$\Rightarrow [E(A)]^2 = [E(B)]^2 = M^2c^4 \Rightarrow [\gamma_1m_1c^2 + \gamma_2m_2c^2]^2 = [\gamma_cm_cc^2 + \delta E]^2 = M^2c^4 \Rightarrow \quad (4.5\text{-}3)$$
$$\Rightarrow (\gamma_1m_1c^2)^2 + (\gamma_2m_2c^2)^2 + 2(\gamma_1m_1c^2)(\gamma_2m_2c^2) = (\gamma_cm_cc^2)^2 + 2(\gamma_cm_cc^2)(\delta E) + (\delta E)^2 = M^2c^4$$

Here we are paving or testing the concept that in the near zone of interaction, at least certain time, interacting objects would create a virtually united object **m_c** with energy coupling and energy-exchange events (here presented with: $\Delta E_A, \Delta E_B$, $\delta E_A, \delta E_B$, $c^2\Delta m$, δE, $\delta\vec{p}$), where the matter waves are being created. Another message in formulating (4.5-3), and later (4.5-4), regardless of some future mathematical revision (4.5-3) and (4.5-4) would be corrected or upgraded, is to show that the part of the motional energy of $\mathbf{m_r}$ is "effectively injected" into the "effective rest mass" of the central mass, as $m_c + \Delta m = m_c + \frac{E_r}{\gamma_c c^2}$. Motional energy associated with reduced mass $\mathbf{m_r}$ is dynamically equivalent to the energy of rotation (where effectively $\mathbf{m_r}$ is rotating around $\mathbf{m_c}$). Externally (from the Laboratory system) we can see only a linear particles motion, as a motion of their common center of mass (since rotational motions would be "mathematically captured" by internal content of the equivalent rest mass; - For additional conceptual clarification see chapter 2, equations (2.5.1-4) until (2.5.1-6), (2.11.1) until (2.11.9), and T.2.4, T.2.5, and T.2.6). For instance, if the specific particle is spinning around its axis and performing a rectilinear motion at the same time, it looks evident that its total motional energy should have two different components: $E_{rot.} + E_{linear-motion} = E_{rot.} + E_k$. In order to follow the message from (4.5-3) and to be more explicit, we could say that the same particle without having any element of rotation (or spinning) would have the total and kinetic energy equal to: $E_{tot.} = \gamma mc^2$, $E_k = (\gamma - 1)mc^2$, and if elements of rotation or spinning are present the total and kinetic energy would become $E_{tot.} = \gamma(m_0 + \frac{E_{rot.}}{c^2})c^2$, $E_k = (\gamma - 1)(m_0 + \frac{E_{rot.}}{c^2})c^2$, $\mathbf{m = m_0 + \frac{E_{rot.}}{c^2}}$. In other words, here, all elements of spinning are treated as a specific equivalent contribution to the rest-mass. In cases when we have many particles passing from one complex motional state (state 1) to the other (state 2), where particles could have linear and rotational motion components, the same situation would be presentable as given in (4.5-4),

$$\begin{aligned}
&\overline{P}^2 = p_1^2 - \frac{E_1^2}{c^2} = p_2^2 - \frac{E_2^2}{c^2} = -m^2c^2 = \text{Invariant} \ , \\
&\vec{L} = J\vec{\omega} = \vec{L}_1 = J_1\vec{\omega}_1 = \vec{L}_2 = J_2\vec{\omega}_2 \ \ (= \text{total orbital momentum conservation}), \\
&p_{1/2} = \gamma_{1/2} m_{1/2} v_{1/2} = \gamma_{1/2}(m_{0\text{-}1/2} + \frac{E_{rot\text{-}1/2}}{c^2})v_{1/2} \ , \ E_{rot\text{-}1/2} = E(\vec{L}_{1/2}) \\
&E = \gamma_{1/2} m_{1/2} c^2 = \gamma_{1/2}(m_{0\text{-}1/2} + \frac{E_{rot\text{-}1/2}}{c^2})c^2, \\
&m_{1/2} = m_{0\text{-}1/2} + \frac{E_{rot.\text{-}1/2}}{c^2} \ .
\end{aligned} \tag{4.5-4}$$

In all cases, given by expressions in T.4.4 and (4.5-1) - (4.5-3), the real and initial interaction participants (**m_1**, **m_2**) have only linear motion moments (no rotation, no spinning). When applying the law of orbital moments conservation, it should be clear that the sum of all (initial) orbital moments before interaction will stay equal to the sum of all orbital moments appearing after interaction (in this case equal zero).

*There is only a transitory period in the near zone of interaction when two interacting or mutually approaching bodies (**m_1, m_2**) effectively create additional elements of rotation, such as **m_r** rotates around **m_c** or **m_1** and **m_2** both rotate around their common center-of-mass point. Such additional elements of rotation should also be balanced, producing that important, total orbital moment (including spinning) in every moment during the interaction equals the total, initial orbital moment. In other words, if the initial total orbital moment of (**m_1, m_2**) equaled zero (measured from the Laboratory System, before the interaction started), in the transitory, near zone of interaction we should have only interaction products or participants with mutually balanced orbital* *moments that as vectors cancel each other. This is extremely important to consider if we want to understand the nature of rotation associated to the Center of the mass system (that is at the same time the source of de Broglie matter waves). In cases when initial particles (m_1, m_2) have non-zero orbital moments and spin attributes, like in (4.5-4), the same situation becomes much more complex and mathematically more productive (because we need to apply the Orbital Moments Conservation Law and find all possible distributions and redistributions of orbital moments and spinning during the process of interaction, and after interaction).*

This time we did not address the possibility that between two initial masses m_1, m_2 (entering interaction) exist some electromagnetic or other binding energy couplings (such as U_{12} in (4.5-1)), what would make previous mathematical elaboration more complicated, but without diminishing conclusions regarding motional or rotational energy transformation into a total, equivalent rest mass. Here could also be a part of the answer about understanding hidden or "dark matter" of our universe (what is a question of proper mathematical interpretation of known conservation laws).

Effectively, any two-body situation in the process of interaction evolving creates a kind of transitory, compound system where the resulting (and equivalent) central rest mass m_c is increased for the rest mass amount of Δm, becoming $m_c + \Delta m$ (where $\Delta m = \frac{E_r}{\gamma_c c^2}$). This way, the "rotation-like motion" of a reduced mass m_r around m_c is effectively considered by the amount of Δm, and a new, transitory compound system is presented only as a linear motion of the mass $m_c+\Delta m$ with the velocity v_c. If Δm eventually became a real particle with a rest mass, the final rest mass increase or reduction (after the interaction is ended) would depend on many other factors, still not introduced here, to give the advantage to clear, global, and conceptual thinking (without too many details). The mass Δm is presented only in the function to show how rotation related motional energy component could be "mathematically injected in or extracted" from the rest mass. This situation would become more challenging if interacting particles with masses m_1 and m_2 present one real particle (with non-zero rest mass) and a photon (for instance), or if both are photons. Here we are paving the way to a new understanding of how particles are created (or disintegrated), and where the place of rotation in such a process is (regardless that what we have here could not be only an ordinary kind of rotation, but it is a state that can have orbital and magnetic moments). De Broglie, matter waves are created inside the interaction zone between m_c and m_r, and such matter waves present the "communicating channel" for all energy exchanges and mass transformations that would happen there. The parameters of mentioned matter waves, de Broglie wavelength, and frequency are also products of the same interacting zone between m_c and m_r. Both objects (m_c and m_r) could effectively be presented as having some aspects of rotational motions (for instance, as rotating around their common center of inertia, having orbital moments: see also Fig.4.1 and equations (4.3), and complementary elaborations in chapter 10. Of this book).

[♣ COMMENTS & FREE-THINKING CORNER:

The real origins of matter waves in this book are related to energy-momentum coupling forces or fields between (at least) two bodies that are mutually in relative motion. In the following table (that has many possible and critical items, still in development, but well enough for initiating productive brainstorming) we will present natural kinetic, and total energy balance for a two-body system.

<table>
<tr><td></td><td>E_{k1}, E_1
(Particle m_1)</td><td>E_{k2}, E_2
(Particle m_2)</td><td>E_{kc}, E_c
($m_{co} = m_1 + m_2$)</td><td>E_{kr}, E_r
($m_r = \frac{m_1 m_2}{m_1 + m_2}$)</td></tr>
<tr><td rowspan="2">Kinetic Energy</td><td>$E_{k1} = \frac{m_1 v_1^2}{2}, v_1 \ll c$</td><td>$E_{k2} = \frac{m_2 v_2^2}{2}, v_2 \ll c$</td><td>$E_{kc} = \frac{m_{co} v_c^2}{2}, v_c \ll c$</td><td>$E_{kr} = \frac{m_r v_r^2}{2}, v_r \ll c$</td></tr>
<tr><td>$E_{k1} = (\gamma_1 - 1) m_1 c^2$</td><td>$E_{k2} = (\gamma_2 - 1) m_2 c^2$</td><td>$E_{kc} = (\gamma_c - 1) m_{co} c^2$</td><td>$E_{kr} = \frac{m_r v_r^2}{1 + \sqrt{1 - \frac{v_c^2}{c^2}}}$</td></tr>
<tr><td rowspan="2">Total Energy</td><td>$E_1 = E_{k1} + m_1 c^2$
$= \gamma_1 m_1 c^2$</td><td>$E_2 = E_{k2} + m_2 c^2$
$= \gamma_2 m_2 c^2$</td><td>$E_c = E_{kc} + m_{co} c^2$
$= \gamma_c m_{co} c^2$</td><td>$E_r = E_{kr} = \frac{p_r v_r}{1 + \sqrt{1 - \frac{v_c^2}{c^2}}}$</td></tr>
<tr><td colspan="4">$E = E_1 + E_2 = E_c + E_r \Leftrightarrow \gamma_1 m_1 c^2 + \gamma_2 m_2 c^2 = \gamma_c m_{co} c^2 + E_r \Rightarrow E_r = \gamma_1 m_1 c^2 + \gamma_2 m_2 c^2 - \gamma_c m_{co} c^2 = cp_r$
$E_r = E_{kr}, dp = h\frac{d\lambda}{\lambda^2}, E_{k1} + E_{k2} = E_{kc} + E_{kr} \Rightarrow v_1 dp_1 + v_2 dp_2 = v_c dp_c + v_r dp_r \Rightarrow v_1 \frac{d\lambda_1}{\lambda_1^2} + v_2 \frac{d\lambda_2}{\lambda_2^2} = v_c \frac{d\lambda_c}{\lambda_c^2} + v_r \frac{d\lambda_r}{\lambda_r^2}$
$\vec{p}_c = \frac{E}{c^2}\vec{v}_c = \frac{E_1 + E_2}{c^2}\vec{v}_c = \frac{E_c + E_r}{c^2}\vec{v}_c = \gamma_c m_{co}\vec{v}_c = \vec{p}_1 + \vec{p}_2 = \vec{p}_{c,r} = \vec{p}_c = \vec{p},$
$\vec{v}_c = \frac{c^2}{E}\vec{p}_c = \frac{c^2(\gamma_1 m_1 \vec{v}_1 + \gamma_2 m_2 \vec{v}_2)}{\gamma_1 m_1 c^2 + \gamma_2 m_2 c^2}, v_i \ll c \Rightarrow \gamma_i \cong 1 \Rightarrow E_i = m_i c^2, \vec{p}_i = m_i \vec{v}_i \Rightarrow$
$\vec{v}_c = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2}, m_c = m_1 + m_2, m_r = \frac{m_1 m_2}{m_1 + m_2}, E_r \cong \frac{1}{2} m_r v_r^2, E_c = m_c c^2 + \frac{1}{2} m_c v_c^2.$</td></tr>
</table>

4-vectors **First option (in the process !?)**	$\overline{P}_1 = (\vec{p}_1, \frac{E_1}{c}),$ $\vec{p}_1 = \gamma_1 m_1 \vec{v}_1,$ $\overline{P}_1^2 = \vec{p}_1^2 - \frac{E_1^2}{c^2} = -m_1^2c^2$	$\overline{P}_2 = (\vec{p}_2, \frac{E_2}{c}),$ $\vec{p}_2 = \gamma_2 m_2 \vec{v}_2,$ $\overline{P}_2^2 = \vec{p}_2^2 - \frac{E_2^2}{c^2} = -m_2^2c^2$	$\overline{P}_c = (\vec{p}_c, \frac{E_c}{c}),$ $\vec{p}_c = \gamma_c m_{co} \vec{v}_c,$ $\overline{P}_c^2 = \vec{p}_c^2 - \frac{E_c^2}{c^2} = -m_{co}^2c^2$	$\overline{P}_r = (\vec{p}_r, \frac{E_r}{c}), m_{ro} = 0,$ $\overline{P}_r^2 = \vec{p}_r^2 - \frac{E_r^2}{c^2} = 0 \Rightarrow E_r = cp_r =$ $= \gamma_1 m_1 c^2 + \gamma_2 m_2 c^2 - \gamma_c m_{co} c^2 \Rightarrow$ $p_r = \gamma_1 m_1 c + \gamma_2 m_2 c - \gamma_c m_{co} c = \frac{E_r}{c}.$
	$(\overline{P}_1+\overline{P}_2)^2 = (\overline{P}_c+\overline{P}_r)^2 = -m_{co}^2c^2 \Leftrightarrow \overline{P}_1^2 + 2\overline{P}_1\overline{P}_2 + \overline{P}_2^2 = \overline{P}_c^2 + 2\overline{P}_c\overline{P}_r + \overline{P}_r^2 = -m_{co}^2c^2 \Rightarrow \overline{P}_c\overline{P}_r = 0,...$ $\left\{ \begin{matrix} \overline{P}_i = (\vec{p}_i, \frac{E_i}{c}), \overline{P}_i^2 = \vec{p}_i^2 - \frac{E_i^2}{c^2} = -m_i^2c^2, \vec{p}_i = \gamma_i m_i \vec{v}_i, \\ \overline{P}_i \cdot \overline{P}_j = \vec{p}_i \cdot \vec{p}_j - \frac{E_i \cdot E_j}{c^2}, \overline{P}_1 \cdot \overline{P}_2 = \overline{P}_c \cdot \overline{P}_r \end{matrix} \right\} \Rightarrow (\vec{p}_1, \frac{E_1}{c}) \cdot (\vec{p}_2, \frac{E_2}{c}) = (\vec{p}_c, \frac{E_c}{c}) \cdot (\vec{p}_r, \frac{E_r}{c}) \Leftrightarrow$ $\Leftrightarrow \vec{p}_1 \cdot \vec{p}_2 - \frac{E_1 \cdot E_2}{c^2} = \vec{p}_c \cdot \vec{p}_r - \frac{E_c \cdot E_r}{c^2} = \vec{p}_c \cdot \vec{p}_r - \frac{\gamma_c m_{co} c^2 \cdot cp_r}{c^2} = \vec{p}_c \cdot \vec{p}_r - \gamma_c m_{co} \cdot (\gamma_1 m_1 c^2 + \gamma_2 m_2 c^2 - \gamma_c m_{co} c^2)$ $\vec{p}_1 \cdot \vec{p}_2 - \frac{E_1 \cdot E_2}{c^2} = \vec{p}_c \cdot \vec{p}_r - \gamma_c m_{co} \cdot (\gamma_1 m_1 c^2 + \gamma_2 m_2 c^2 - \gamma_c m_{co} c^2) \Rightarrow \vec{p}_c \cdot \vec{p}_r = \vec{p}_1 \cdot \vec{p}_2 - \frac{E_1 \cdot E_2}{c^2} + \gamma_c m_{co} \cdot (\gamma_1 m_1 c^2 + \gamma_2 m_2 c^2 - \gamma_c m_{co} c^2)$ $\overline{P}_4^2 = (\gamma_c m_c \vec{v}_c)^2 - \frac{(E_c + E_r)^2}{c^2} = (\vec{p}_1 + \vec{p}_2)^2 - \frac{(E_1 + E_2)^2}{c^2} = inv., \vec{p}_c = \gamma_c m_c \vec{v}_c, E_c = \gamma_c m_c c^2, m_c = m_1 + m_2, m_r = \frac{m_1 m_2}{m_1 + m_2},$ $E_r = \sqrt{(\gamma_c m_c \vec{v}_c)^2 c^2 - (\vec{p}_1 + \vec{p}_2)^2 c^2 + (E_1 + E_2)^2} - \gamma_c m_c c^2, E_c + E_r = \gamma_c (m_c + \Delta m) c^2, E_r = \gamma_c (\Delta m) c^2 = \gamma_r^* m_r c^2,$ $\Delta m = m_c \left(\sqrt{\frac{\vec{v}_c^2}{c^2} - \frac{(\vec{p}_1 + \vec{p}_2)^2}{m_c^2 c^2} + \frac{(E_1 + E_2)^2}{m_c^2 c^4}} - 1 \right) = \frac{\gamma_r^*}{\gamma_c} m_r, \gamma_r^* = \gamma_c \frac{m_c}{m_r} \left(\sqrt{\frac{\vec{v}_c^2}{c^2} - \frac{(\vec{p}_1 + \vec{p}_2)^2}{m_c^2 c^2} + \frac{(E_1 + E_2)^2}{m_c^2 c^4}} - 1 \right).$			

<table>
<tr>
<td rowspan="2">4-vectors

Second option (in the process !?)</td>
<td>$\overline{P}_1 = (\vec{p}_1, \frac{E_1}{c}),$
$\vec{p}_1 = \gamma_1 m_1 \vec{v}_1,$
$\overline{P}_1^2 = \vec{p}_1^2 - \frac{E_1^2}{c^2} = -m_1^2 c^2$</td>
<td>$\overline{P}_2 = (\vec{p}_2, \frac{E_2}{c}),$
$\vec{p}_2 = \gamma_2 m_2 \vec{v}_2,$
$\overline{P}_2^2 = \vec{p}_2^2 - \frac{E_2^2}{c^2} = -m_2^2 c^2$</td>
<td>$\overline{P}_c = (\vec{p}_c - \vec{p}_r, \frac{E_c}{c}),$
$\vec{p}_c = \gamma_c m_{co} \vec{v}_c,$
$\overline{P}_c^2 = (\vec{p}_c - \vec{p}_r)^2 - \frac{E_c^2}{c^2} = -m_{co}^2 c^2$</td>
<td>$\overline{P}_r = (\vec{p}_r, \frac{E_r}{c}), m_{ro} = 0,$
$\overline{P}_r^2 = \vec{p}_r^2 - \frac{E_r^2}{c^2} = 0 \Rightarrow E_r = cp_r =$
$= \gamma_1 m_1 c^2 + \gamma_2 m_2 c^2 - \gamma_c m_{co} c^2 \Rightarrow$
$p_r = \gamma_1 m_1 c + \gamma_2 m_2 c - \gamma_c m_{co} c = \frac{E_r}{c}.$</td>
</tr>
<tr>
<td colspan="4">$(\overline{P}_1 + \overline{P}_2)^2 = (\overline{P}_c + \overline{P}_r)^2 = -m_{co}^2 c^2,$
xxxxxxxxx
$\left\{\begin{matrix} \overline{P}_i = (\vec{p}_i, \frac{E_i}{c}), \overline{P}_1^2 = \vec{p}_i^2 - \frac{E_i^2}{c^2} = -m_i^2 c^2, \vec{p}_i = \gamma_i m_i \vec{v}_i, \\ \overline{P}_i \cdot \overline{P}_j = \vec{p}_i \cdot \vec{p}_j - \frac{E_i \cdot E_j}{c^2}, \overline{P}_1 \cdot \overline{P}_2 = \overline{P}_c \cdot \overline{P}_r \end{matrix}\right\} \Rightarrow$
$\Rightarrow (\vec{p}_1, \frac{E_1}{c}) \cdot (\vec{p}_2, \frac{E_2}{c}) = (\vec{p}_c - \vec{p}_r, \frac{E_c}{c}) \cdot (\vec{p}_r, \frac{E_r}{c}) \Leftrightarrow$
$\Leftrightarrow \vec{p}_1 \cdot \vec{p}_2 - \frac{E_1 \cdot E_2}{c^2} = (\vec{p}_c - \vec{p}_r) \cdot \vec{p}_r - \frac{E_c \cdot E_r}{c^2} = (\vec{p}_c - \vec{p}_r) \cdot \vec{p}_r - \frac{\gamma_c m_{co} c^2 \cdot cp_r}{c^2} = (\vec{p}_c - \vec{p}_r) \cdot \vec{p}_r - \gamma_c m_{co} \cdot (\gamma_1 m_1 c^2 + \gamma_2 m_2 c^2 - \gamma_c m_{co} c^2)$
$\vec{p}_1 \cdot \vec{p}_2 - \frac{E_1 \cdot E_2}{c^2} = (\vec{p}_c - \vec{p}_r) \cdot \vec{p}_r - \gamma_c m_{co} \cdot (\gamma_1 m_1 c^2 + \gamma_2 m_2 c^2 - \gamma_c m_{co} c^2) \Rightarrow$
$\Rightarrow (\vec{p}_c - \vec{p}_r) \cdot \vec{p}_r = \vec{p}_1 \cdot \vec{p}_2 - \frac{E_1 \cdot E_2}{c^2} + \gamma_c m_{co} \cdot (\gamma_1 m_1 c^2 + \gamma_2 m_2 c^2 - \gamma_c m_{co} c^2)$</td>
</tr>
</table>

(in the process !?)	$\overline{P}_{1,2} = \overline{P}_1 + \overline{P}_2 = (\vec{p}, \frac{E}{c}) = (\vec{p}_1 + \vec{p}_2, \frac{E_1 + E_2}{c})$ $\vec{p} = \vec{p}_1 + \vec{p}_2,$ $\overline{P}_{1,2}^2 = \left(\overline{P}_1 + \overline{P}_2\right)^2 = \overline{P}_1^2 + \overline{P}_2^2 + 2\overline{P}_1 \cdot \overline{P}_2 =$ $= \vec{p}_1^2 - \frac{E_1^2}{c^2} + \vec{p}_2^2 - \frac{E_2^2}{c^2} + 2\vec{p}_1 \cdot \vec{p}_2 - 2\frac{E_1 \cdot E_2}{c^2} =$ $= -m_{co}^2 c^2$	$\overline{P}_{c,r} = (\vec{p}_{c,r}, \frac{E_{c,r}}{c}) = (\vec{p}_c + \vec{p}_r, \frac{E_c + E_r}{c}),$ $\vec{p}_{c,r} = \vec{p}_c + \vec{p}_r = \vec{p}_1 + \vec{p}_2 = \gamma_c m_{co} \vec{v}_c = \vec{p},$ $\overline{P}_{c,r}^2 = \left(\overline{P}_c + \overline{P}_r\right)^2 = \overline{P}_c^2 + \overline{P}_r^2 + 2\overline{P}_c \cdot \overline{P}_r =$ $= \left(\vec{p}_c^2 - \frac{E_c^2}{c^2}\right) + \left(\vec{p}_r^2 - \frac{E_r^2}{c^2}\right) + 2\left(\vec{p}_c \cdot \vec{p}_r - \frac{E_c \cdot E_r}{c^2}\right) =$ $= \left(\vec{p}_r^2 + 2\vec{p}_c \cdot \vec{p}_r + \vec{p}_c^2\right) - \left(\frac{E_c^2}{c^2} + 2\frac{E_c \cdot E_r}{c^2} + \frac{E_r^2}{c^2}\right) =$ $= \left(\vec{p}_r + \vec{p}_c\right)^2 - \left(\frac{E_r}{c} + \frac{E_c}{c}\right)^2 = -m_{co}^2 c^2 \Leftrightarrow$ $\Leftrightarrow \left(\vec{p}_1 + \vec{p}_2\right)^2 - \left(\frac{E_r + E_c}{c}\right)^2 = -m_{co}^2 c^2 \Leftrightarrow \vec{p}^2 - \frac{E}{c}^2 = -m_{co}^2 c^2$

4-Vectors Momentum & Energy during and after interaction (in the process !?)	$\overline{P}_{1,2} = (\vec{p}, \frac{E}{c}) = (\vec{p}_1 + \vec{p}_2, \frac{E_1 + E_2}{c}) \Leftrightarrow \overline{P}^2_{1,2} = \vec{p}^2 - \frac{E^2}{c^2} = -(m_1 + m_2)^2 c^2$, $\overline{P}^2_{1,2} = (\vec{p}_1 + \vec{p}_2)^2 - \frac{(E_1 + E_2)^2}{c^2} = -m^2_{co} c^2$, $E^2 = \vec{p}^2 c^2 + m^2_{co} c^4 = (\vec{p}_1 + \vec{p}_2)^2 c^2 + m^2_{co} c^4 = (\gamma_1 m_1 + \gamma_2 m_2)^2 c^4 = \gamma_c^2 (m_1 + m_2 + \Delta m)^2 c^4$, $E^2 = [E_{k1} + m_1 c^2 + E_{k2} + m_2 c^2]^2 = (\vec{p}_1 + \vec{p}_2)^2 c^2 + m^2_{co} c^4$ $E = E_1 + E_2 = E_c + E_r = \gamma_c (m_{co} + \Delta m) c^2 = \gamma_c m_c c^2, \vec{p} = \vec{p}_1 + \vec{p}_2$, $m_c = m_{co} + \Delta m$, $\gamma_c \Delta m = \gamma_1 m_1 + \gamma_2 m_2 - \gamma_c (m_1 + m_2)$, $E_k = E_{k1} + E_{k2} = E_{kc} + E_{kr} = \left[1 \pm 2\sqrt{1 - \left(\frac{p}{m_{co} c}\right)^2}\right] m_{co} c^2 = (\gamma_c - 1) m_{co} c^2 + \gamma_c \Delta m \cdot c^2 = E_{kc} + \Delta E$ $\left\{ = \frac{(\vec{p}_1 + \vec{p}_2)^2}{(\gamma_c + 1)(m_{co} + \Delta m)} \right\}$, $E_r = E_{kr} = \gamma_1 m_1 c^2 + \gamma_2 m_2 c^2 - \gamma_c m_{co} c^2 = (\gamma_c - 1) m_{co} c^2 + \gamma_c \Delta m \cdot c^2 - E_{kc} = \gamma_c \Delta m \cdot c^2 = \Delta E$, $\{ = (\gamma_r - 1) m_r c^2 \}$, $\overline{P}_c = \overline{P}(\vec{p}_c, \frac{E_c}{c}), \Leftrightarrow \overline{P}^2_c = \vec{p}^2_c - \frac{E^2_c}{c^2} = -m^2_{co} c^2$, $\overline{P}_{c,r} = (\vec{p}_{c,r}, \frac{E_{c,r}}{c}) \Leftrightarrow \overline{P}^2_{c,r} = \vec{p}^2_{c,r} - \frac{E^2_{c,r}}{c^2} = -(m_{co} + \Delta m)^2 c^2$, $\overline{P}^2_{c\text{-}r} = [\gamma_c (m_{co} + \Delta m) \vec{v}_c]^2 - \frac{[\gamma_c (m_{co} + \Delta m) c^2 + (E_r - \Delta E)]^2}{c^2} = -(m_{co} + \Delta m)^2 c^2$, $E^2_{c,r} = [\gamma_c (m_{co} + \Delta m) c^2 + (E_r - \Delta E)]^2 = [\gamma_c (m_{co} + \Delta m) \vec{v}_c]^2 c^2 + (m_{co} + \Delta m)^2 c^4 = (E_1 + E_2)^2$, $\vec{p}_{c,r} = \gamma_c (m_{co} + \Delta m) \vec{v}_c$, $E_{c,r} = \gamma_c (m_{co} + \Delta m) c^2 + (E_r - \Delta E)$, $\Delta E = \gamma_c \Delta m \cdot c^2$.

As another way of analyzing the same situation, let us make mutually equivalent and analog mathematical descriptions of the same two-body system (moving masses $\mathbf{m_1}$ and $\mathbf{m_2}$, which are electrically and magnetically neutral, or without other active charges) in two different systems of references: - Laboratory System and Center of mass system (A and/or B, and A* and/or B*). This will be presented in four different ways, or four mutually linked (or mutually dependent) systems of references. The principal (original, real) and initial "mass-momentum players" in A are particles $\mathbf{m_1}$ and $\mathbf{m_2}$. The dynamic (or mathematically) equivalent, "mass-momentum players" in B, and B* will be new "virtual particles" $\mathbf{m_r}$ and $\mathbf{m_c}$. In fact, parallel to real interaction participants $\mathbf{m_1}$ and $\mathbf{m_2}$, that are introduced in A (where everything is conceptually obvious and clear), we are introducing additional and a little bit artificial (but dynamically equivalent), "virtual two-body situations", placed in A* and B*, or B, with mutually interacting masses $\mathbf{m_r}$ and $\mathbf{m_c}$, which have known mathematical relations with $\mathbf{m_1}$ and $\mathbf{m_2}$. A* and B*, as well as A and B, are mutually linked Laboratory and Center of mass systems (where A and A* are dealing with $\mathbf{m_1}$ and $\mathbf{m_2}$, and B, and B* are dealing with $\mathbf{m_r}$ and $\mathbf{m_c}$). The objective here is to present all possible mathematical relations between moving objects $\mathbf{m_1}$, $\mathbf{m_2}$, $\mathbf{m_r}$ and $\mathbf{m_c}$ in different referential systems A, B, A* and B*, by respecting relevant conservation laws (See table T.4.4). The idea behind all of that is to show that moving objects are entering certain "mass-momentum communication" initiating (dynamically equivalent) elements of rotation, which are sources of matter-waves phenomenology. From the Laboratory System, we see (mutually approaching) objects $\mathbf{m_1}$ and $\mathbf{m_2}$, but how $\mathbf{m_1}$ is noticing $\mathbf{m_2}$ and vice versa is related to "energy-momentum" coupling/s between them, and this part of the analysis should explain the background of matter-waves and particle-wave duality. Let us first make more precise descriptions or definitions of all referential systems, A, B, A* and B*, as:

1° Laboratory System **A**: Is presented with moving and Real Interaction Participants $\mathbf{m_1}$ and $\mathbf{m_2}$.

2° Laboratory System **B**: Is presented with moving and "Virtual Interaction Participants" such as, Center mass $\mathbf{m_c}$, Reduced Mass $\mathbf{m_r}$, Center Mass Velocity $\mathbf{v_c}$, and Reduced Mass Velocity $\mathbf{v = v_r}$, etc., all of them measured by the observer from the Laboratory System. Here we need to imagine that the same observer, as in the case of Laboratory system A, would start seeing only motions of $\mathbf{m_c}$ and $\mathbf{m_r}$, instead of seeing $\mathbf{m_1}$ and $\mathbf{m_2}$, and all mathematical relations should be established to make presentations in A and B mutually equivalent.

3° Center of mass system **A***: The observer is linked to the center of mass, seeing only two primary and initial particles $\mathbf{m_1}$ and $\mathbf{m_2}$, which are also found in Laboratory System A, now having different velocities and moments. Here, system A* is moving with relative velocity $\mathbf{v_c}$, measured from A.

4° Center of mass system **B***: The observer is linked to the center of mass, seeing only "Virtual Interaction Participants" such as: Center mass and Reduced mass, $\mathbf{m_c}$ and $\mathbf{m_r}$, analog to the situation in the Laboratory System under B. This time only $\mathbf{m_r}$ is moving around $\mathbf{m_c}$ and $\mathbf{m_c}$ is in the state of rest (from the point of view of the observer in B*). Here we need to imagine that the same observer, as in the case of the Center of mass system A*, would start seeing only motions of $\mathbf{m_c}$ and $\mathbf{m_r}$, instead of seeing $\mathbf{m_1}$ and $\mathbf{m_2}$, and all mathematical relations should be established to make presentations in A* and B* mutually equivalent.

The main idea here is to show that particle-wave duality should have its roots and explanation in relation to Laboratory System B and Center of mass system B*. It has already been explained (in the beginning of this chapter) that the "kitchen" where matter waves (de Broglie wavelength and frequency) are created is causally related to what happens between $\mathbf{m_r}$ and $\mathbf{m_c}$ in B and B*, since this is the best and maybe the only way to show that $\mathbf{m_r}$ performs rotation around $\mathbf{m_c}$. Of course, here we are talking about something that is mathematically presentable as equivalent to the rotation while paying attention to satisfy all conservation laws and to make mutually similar or mutually compatible descriptions between states of motions of real particles ($\mathbf{m_1}$, $\mathbf{m_2}$) and their effective replacements ($\mathbf{m_c}$, $\mathbf{m_r}$). The concept of rotation is causally linked to the concept of frequency. There is just a small step to imagine the creation of a certain kind of wave, which would have a specific wavelength (that is de Broglie, matter waves wavelength). If we can find elements of rotation (frequency and wavelength) related to virtual particles ($\mathbf{m_c}$, $\mathbf{m_r}$), it would be necessary to take just a small step to determine how such elements would appear to an observer from the Laboratory System. This is what L. de Broglie, A. Einstein, M. Planck, and other founders of Wave Quantum Mechanics established, apparently using different methodology; in this book, we are using the abbreviated name **PWDC** = **P**article – **W**ave – **D**uality – **C**ode, to encircle the same domain).

1° With the data presented in T.4.4, **as the first step**, we intend to make this situation mathematically and conceptually much clearer. Table T.4.4 is created by exploiting the complete, formal, or mathematical symmetry for all expressions that are related to energies and moments of ($\mathbf{m_c}$, $\mathbf{m_r}$), by making them look like analogous expressions of energies and moments of ($\mathbf{m_1}$, $\mathbf{m_2}$), in all systems of reference (A, A*, B, B*). We will soon realize by analyzing mutual mathematical consistency and compatibility of data from T.4.4 that certain energy-momentum relations in T.4.4 are mathematically non-sustainable and not compatible. Especially challenging are expressions in connection with reduced mass $\mathbf{m_r}$. The biggest mathematical and conceptual challenge in realizing here elaborated strategy would be the question how to address or associate quantity of motion (linear momentum) to the Reduced Mass $\mathbf{m_r}$, and to the Center mass $\mathbf{m_c}$, or saying differently, what would mean "vector" quantities $\vec{p}_r = m_r\vec{v}_r$ and $\vec{p}_c = \gamma_c m_c \vec{v}_c$, found in T.4.4. The most probable case is that the total initial quantity of motion, or linear motion momentum of both particles ($\mathbf{m_1}$, $\mathbf{m_2}$), would be "given" only to $\mathbf{m_c}$. This way, since $\mathbf{m_r}$ has a certain amount of motional energy, it would be shown that this is only the rotational-motional energy (and that $\mathbf{m_r}$ has nothing related to linear motion; consequently, $\mathbf{m_r}$ should only have the specific orbital moment or spin).

2° Then, **as the second step**, for the Laboratory System B, we would introduce the assumption that $\mathbf{m_c}$ should be the carrier of the total quantity of rectilinear motion, and that $\mathbf{m_r}$ has only a certain amount of rotational motional energy, without having any rectilinear motion momentum, $\vec{\mathbf{p}}_\mathbf{r} = \mathbf{0}$ (in other words, $\mathbf{m_r}$ can only make rotation or spinning around $\mathbf{m_c}$). Doing this way, we should be able to correct/modify all mutually not-compatible expressions in T.4.4, and exactly explain the origin and meaning of de Broglie matter waves, and the nature of unity between linear and rotational motions (obviously, this would be a voluminous mathematical task, well started but still not finalized in this book).

T.4.4. Laboratory System (A)	Laboratory System (B)	Center of the mass system (A*)	Center of the mass system (B*)
$\mathbf{m_1}$ $\mathbf{m_2}$	$\mathbf{m_c = m_1 + m_2}$ $\mathbf{m_r = \frac{m_1 m_2}{m_1 + m_2}}$	$\mathbf{m_1}$ $\mathbf{m_2}$	$\mathbf{m_c = m_1 + m_2}$ $\mathbf{m_r = \frac{m_1 m_2}{m_1 + m_2}}$
$\vec{\mathbf{v}}_1$ $\vec{\mathbf{v}}_2$	$\vec{\mathbf{v}}_c = \frac{\mathbf{c^2(\vec{p}_1 + \vec{p}_2)}}{\mathbf{E_1 + E_2}}$ $\vec{\mathbf{v}}_r = (\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1) = \vec{\mathbf{v}}$	$\vec{\mathbf{v}}_1^* = \vec{\mathbf{v}}_1 - \vec{\mathbf{v}}_c$ $\vec{\mathbf{v}}_2^* = \vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_c$	$\mathbf{v_c^* = 0}$ $\mathbf{v_r^* = v_r = v}$
$\vec{p}_1 = \gamma_1 m_1 \vec{v}_1$ $\vec{p}_2 = \gamma_2 m_2 \vec{v}_2$ $\vec{P}(A) = \vec{p}_1 + \vec{p}_2$	$\vec{p}_c = \gamma_c m_c \vec{v}_c$ $\vec{p}_r = \vec{0},\ \lvert\vec{p}_r\rvert = m_r v_r$ (?!) $\vec{P}(B) = \vec{p}_c + \vec{p}_r$	$\vec{p}_1^* = \gamma_1^* m_1 \vec{v}_1^* = \gamma_1^* m_1 (\vec{v}_1 - \vec{v}_c) =$ $= -m_r(\vec{v}_2 - \vec{v}_1) = -m_r \vec{v}$ $\vec{p}_2^* = \gamma_2^* m_2 \vec{v}_2^* = \gamma_2^* m_2 (\vec{v}_2 - \vec{v}_c) =$ $= -m_r(\vec{v}_2 - \vec{v}_1) = +m_r \vec{v}$ $\vec{P}(A^*) = \vec{p}_1^* + \vec{p}_2^* = 0$	$p_c^* = 0$ $p_r^* = m_r v_r^*$ $P(B^*) = p_r^*$
$E_1 = E_{01} + E_{k1} = \gamma_1 m_1 c^2$ $E_{01} = m_1 c^2$, $E_{k1} = (\gamma_1 - 1) m_1 c^2$ $E_2 = E_{02} + E_{k2} = \gamma_2 m_2 c^2$ $E_{02} = m_2 c^2$, $E_{k2} = (\gamma_2 - 1) m_2 c^2$ $E(A) = E_1 + E_2 = E(B) =$ $= \gamma_c E(A^*) \geq E(A^*)$ $E_0(A) = E_{01} + E_{02} = E_0(A^*) =$ $= (m_1 + m_2) c^2 = m_c c^2$ $E_k(A) = E_{k1} + E_{k2} =$ $= \gamma_c E_k(A^*) \geq E_k(A^*)$	$E_c = E_{0c} + E_{kc} = \gamma_c m_c c^2$ $E_{0c} = m_c c^2$, $E_{kc} = (\gamma_c - 1) m_c c^2$ $E_r = E_{kr}, E_{0r} = 0$, $E(B) = E_c + E_r = E(A) =$ $= \gamma_c E(B^*) \geq E(B^*)$ $E_0(B) = E_{0c} + E_{0r} = E_0(B^*) =$ $= (m_c + m_r) c^2$ $E_k(B) = E_{kc} + E_{kr} =$ $= \gamma_c E_k(B^*) \geq E_k(B^*)$	$E_1^* = E_{01}^* + E_{k1}^* = \gamma_1^* m_1 c^2$ $E_{01}^* = m_1 c^2$, $E_{k1}^* = (\gamma_1^* - 1) m_1 c^2$ $E_2^* = E_{02}^* + E_{k2}^* = \gamma_2^* m_2 c^2$ $E_{02}^* = m_2 c^2$, $E_{k2}^* = (\gamma_2^* - 1) m_2 c^2$ $E(A^*) = E_1^* + E_2^* = E(B^*) =$ $= E(A)/\gamma_c \leq E(A)$ $E_0(A^*) = E_{01}^* + E_{02}^* = E_0(A) =$ $= (m_1 + m_2) c^2 = m_c c^2$ $E_k(A^*) = E_{k1}^* + E_{k2}^* =$ $= E_k(A)/\gamma_c \leq E_k(A)$	$E_c^* = E_{0c}^* + E_{kc}^* = \gamma_c^* m_c c^2 = m_c c^2$ $E_{0c}^* = m_c c^2$, $E_{kc}^* = (\gamma_c^* - 1) m_c c^2 = 0$ $E_r^* = E_{0r}^* + E_{kr}^*$ $E_{0r}^* = m_r c^2$, $E_{kr}^* =$ $E(B^*) = E_c^* + E_r^* = E(A^*) =$ $= E(B)/\gamma_c \leq E(B)$ $E_0(B^*) = E_{0c}^* + E_{0r}^* = E_0(B) =$ $= (m_c + m_r) c^2$ $E_k(B^*) = E_{kc}^* + E_{kr}^* = E_{kr}^* =$ $= E_k(B)/\gamma_c \leq E_k(B)$
$\gamma_1 = (1 - \frac{v_1^2}{c^2})^{-0.5}$ $\gamma_2 = (1 - \frac{v_2^2}{c^2})^{-0.5}$	$\gamma_c = (1 - \frac{v_c^2}{c^2})^{-0.5}$	$\gamma_{1,2}^* = (1 - \frac{(v_{1,2}^*)^2}{c^2})^{-0.5}$	$\gamma_c^* = 1$

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

The same situation presented with 4 vectors in the Minkowski Space.			
$\overline{P}_1 = \overline{P}_1(\vec{p}_1, \frac{E_1}{c})$ $\overline{P}_2 = \overline{P}_2(\vec{p}_2, \frac{E_2}{c})$ $\overline{P}_{1,2}^2 = p_{1,2}^2 - \frac{E_{1,2}^2}{c^2} = -m_{1,2}^2 c^2$ $\left[\overline{P}(A)\right]^2 = (\overline{P}_1 + \overline{P}_2)^2 = \left[\overline{P}(A^*)\right]^2 =$ $= \left[\overline{P}(A)\right]^2 c^2 - \frac{\left[E(A)\right]^2}{c^2} =$ $= -\frac{\left[E_0(A)\right]^2}{c^2} = -(m_1 + m_2)^2 c^2 =$ $= \overline{P}(A)\overline{P}(B) = \overline{P}(A^*)\overline{P}(B^*) =$ $= \overline{P}(A)\overline{P}(A^*) = \overline{P}(B)\overline{P}(B^*) =$ $= \overline{P}(A)\overline{P}(B^*) = \overline{P}(A^*)\overline{P}(B) = ...$ $\overline{P}_1\overline{P}_2 = \overline{P}_1^*\overline{P}_2^*$	$\overline{P}_c = \overline{P}_c(\vec{p}_c, \frac{E_c}{c})$ $\overline{P}_r = \overline{P}_r(\vec{p}_r, \frac{E_r}{c})$ $\overline{P}_{c,r}^2 = p_{c,r}^2 - \frac{E_{c,r}^2}{c^2} = -m_{c,r}^2 c^2$ $\left[\overline{P}(B)\right]^2 = (\overline{P}_c + \overline{P}_r)^2 = \left[\overline{P}(B^*)\right]^2 =$ $= \left[\overline{P}(B)\right]^2 c^2 - \frac{\left[E(B)\right]^2}{c^2} =$ $= -\frac{E_0(B)}{c^2} = -(m_c + m_r)^2 c^2 =$ $= \overline{P}(A)\overline{P}(B) = \overline{P}(A^*)\overline{P}(B^*) =$ $= \overline{P}(A)\overline{P}(A^*) = \overline{P}(B)\overline{P}(B^*) =$ $= \overline{P}(A)\overline{P}(B^*) = \overline{P}(A^*)\overline{P}(B) = ...$ $\overline{P}_c\overline{P}_r = \overline{P}_c^*\overline{P}_r^*$	$\overline{P}_1^* = \overline{P}_1^*(\vec{p}_1^*, \frac{E_1^*}{c})$ $\overline{P}_2^* = \overline{P}_2^*(\vec{p}_2^*, \frac{E_2^*}{c})$ $(\overline{P}_{1,2}^*)^2 = (p_{1,2}^*)^2 - \frac{(E_{1,2}^*)^2}{c^2} = -m_{1,2}^2 c^2$ $\left[\overline{P}(A^*)\right]^2 = (\overline{P}_1^* + \overline{P}_2^*)^2 =$ $= \left[\overline{P}(A^*)\right]^2 c^2 - \frac{\left[E(A^*)\right]^2}{c^2} = \left[\overline{P}(A)\right]^2 =$ $= -\frac{\left[E_0^*(A)\right]^2}{c^2} = -(m_1 + m_2)^2 c^2 =$ $= \overline{P}(A)\overline{P}(B) = \overline{P}(A^*)\overline{P}(B^*) =$ $= \overline{P}(A)\overline{P}(A^*) = \overline{P}(B)\overline{P}(B^*) =$ $= \overline{P}(A)\overline{P}(B^*) = \overline{P}(A^*)\overline{P}(B) = ...$ $\overline{P}_1\overline{P}_2 = \overline{P}_1^*\overline{P}_2^*$	$\overline{P}_c^* = \overline{P}_c^*(\vec{p}_c^*, \frac{E_c^*}{c})$ $\overline{P}_r^* = \overline{P}_r^*(\vec{p}_r^*, \frac{E_r^*}{c})$ $(\overline{P}_{c,r}^*)^2 = (p_{c,r}^*)^2 - \frac{(E_{c,r}^*)^2}{c^2} = -m_{c,r}^2 c^2$ $\left[\overline{P}(B^*)\right]^2 = (\overline{P}_c^* + \overline{P}_r^*)^2 = \left[\overline{P}(B)\right]^2 =$ $= \left[\overline{P}(B^*)\right]^2 c^2 - \frac{\left[E(B^*)\right]^2}{c^2} =$ $= -\frac{E_0(B^*)}{c^2} = -(m_c + m_r)^2 c^2 =$ $= \overline{P}(A)\overline{P}(B) = \overline{P}(A^*)\overline{P}(B^*) =$ $= \overline{P}(A)\overline{P}(A^*) = \overline{P}(B)\overline{P}(B^*) =$ $= \overline{P}(A)\overline{P}(B^*) = \overline{P}(A^*)\overline{P}(B) = ...$ $\overline{P}_c\overline{P}_r = \overline{P}_c^*\overline{P}_r^*$

T.4.4 is created mostly using mathematical analogies and generally known methodology from 4-vector relativistic relations in the Minkowski space (without paying too much attention to whether all details regarding newly introduced concepts about “virtual interaction participants” are already fully correct and defendable). Also, it would be necessary to double-check and test all the critical relations listed there, to be able to draw final and relevant conclusions. Practically all relevant results, expressions and relations starting from (4.2), (4.3), (4.5) – (4.8) until (4.5-1) and (4.5-2) should be compared with similar, identical, or equivalent results and relations that could be developed from T.4.4, and should be mutually compatible, or if not, should be corrected and made compatible). An early, and still non-finalized “experimental” attempt, without considering 4-vectors in the Minkowski space, to develop a similar concept, as T.4.4 is presently paving, has been initialized by formulating expressions for energies and moments given in (4.5-1) and (4.5-2). The real remaining task and challenge in this situation, which would give the correct picture about the unity of rectilinear and rotational motions, would be to make all colons (of energies, moments, velocities, etc.) found in T.4.4 mutually compatible and correctly formulated in all details, what could still not be the case. Presently, the most important activity here has been to establish the concept of matter-waves creation regarding real and virtual interaction participants in different systems of reference. If we continue such a process, not all possibly missing or incorrect mathematical details will escape being arranged later (*in other words the significance of the concept that is presently being introduced here is much higher than still unfinished mathematical works around it*). We should also not forget that interacting (real and virtual) objects would mutually create certain forces or fields that should agree with generalized Newton-Coulomb force expressions given from (2.4) until (2.4-3); see the second chapter of this book (Gravitation). We should not exclude the possibility of having “exotic” interactions between real ($\mathbf{m_1}$, $\mathbf{m_2}$), and virtual ($\mathbf{m_r}$, $\mathbf{m_c}$) objects (that are presently still brainstorming options). ***......................♣]***

4.1.3.1. Example 2: X-ray Spectrum and Reaction Forces

As an illustration regarding the extension of the particle-wave duality concept, in this example, we shall analyze the generation of X-rays in an x-ray tube. Let us imagine that there is a potential difference $\mathbf{U}$, between two stable metal electrodes (in an x-ray tube). If the potential difference between electrodes is sufficiently high, this will pull and accelerate electrons from the negative electrode towards the positive electrode. Now when the electron leaves the negative electrode (since there is an electrical voltage and field between electrodes, and the electron has its mass and charge), both electrodes will "feel" a certain reactive force in the form of a small "electromechanical" shock/s, and certain transient electric current (or current pulse) will be measured in the external electrode circuit. Also, now when the flying electron strikes the surface of the positive electrode, this will again create a certain electromechanical shock or waving perturbation inside the positive electrode, and X-rays will be radiated from the impact surface. Of course, the negative electrode will also "feel" the impact event because there is an electrical field between the positive and the negative electrode and a certain amount of current will flow. The acoustic activity (mechanical vibrations in electrodes) will also be generated when the electron leaves or strikes an electrode because the electron behaves as a particle that has its mass, spin, and charge. This situation is sufficiently complex to explain the nature and appearance of de Broglie matter waves.

In this process, effectively we have three distinct time intervals with sets of different (waves and particles) energy and momentum states. To make a difference between them (on a time scale), we will introduce the next indexing:
-index “**0**” - will characterize all states of rest (or electrical non-activity) before the electron leaves the negative electrode (and before voltage between electrodes is switched on),
-index “**1**” will characterize all states covering the time interval when the electron flies between the two electrodes, and
-index “**2**” will characterize all states (in both electrodes and between them) after the electron strikes the positive electrode.
-For marking electron states, we shall use index “**e**”,
-for negative electrode states, index “**ne**”,
-for positive electrode states index “**pe**” and
-for X-rays index “**x**”
-adding indexes **0**, **1** or **2** for indicating which time interval we are considering.
-For kinetic energy we will use the index “**k**”, and
-to indicate that a certain state is a kind of wave, vibration, or oscillation (that is, in fact, a state of motional, or kinetic energy), we will use the symbol "**~**".

We will apply the energy and momentum conservation laws, if before an electric field accelerated the electron, we had only the electron in its (relative) state of rest (inside the negative electrode with energy: $\mathbf{E_{eo}} = \mathbf{mc^2}$). When the electrical voltage was switched on, we got a moving electron under the influence of the electrical field (which will give the motional energy to the same electron of, $E_{electric\ field} = -eU = E_{ek1} = (\gamma_1 - 1)mc^2$).

We can also consider that the electron and both electrodes of the X-rays tube, in the state of rest (just until the moment the electron leaves the negative electrode, before the voltage was switched on, in the time interval marked with “**0**”) had negligible amounts of (internal, average, equilibrium state) wave energy and wave momentum

($\tilde{E}_{eo} \approx 0$, $\tilde{p}_{eo} \approx 0$, $\tilde{E}_{neo} \approx 0$, $\tilde{p}_{neo} \approx 0$, $\tilde{E}_{peo} \approx 0$, $\tilde{p}_{peo} \approx 0$). Also, it is obvious that the negative and the positive electrode (both being relatively big masses in the permanent state of rest) cannot have (macroscopically) any kinetic energy, or momentum ($E_{neko} = 0$, $p_{neo} = 0$, $E_{nek1} = 0$, $p_{ne1} = 0$, $E_{nek2} = 0$, $p_{ne2} = 0$, $E_{peko} = 0$, $p_{peo} = 0$, $E_{pek1} = 0$, $p_{pe1} = 0$, $E_{pek2} = 0$, $p_{pe2} = 0$), providing that they are well fixed to the walls of an X-rays tube. This is important to underline, since we have already established (in this book) that every kinetic energy of a particle automatically corresponds to the same amount of its wave energy, and since the electrodes do not move (looking externally from the position of the Laboratory System), there is no wave energy belonging to them, too. Since the electrodes are parts of a closed electric circuit, a certain amount of wave energy could be created internally, inside the electrodes. The same electrodes act as a carrier medium for electric currents (or electric waves and oscillations), and a carrier for mechanical vibrations and such internal electrode states having a certain content of wave energy will be marked using the symbol "**~**".

Let us clarify the same situation more precisely. Usually, when we analyze a moving particle (in free space), its kinetic energy is equal to its wave energy $E_{ek} = (\gamma - 1)mc^2 = \tilde{E} = hf$, which is the case regarding an electron in the state "**1**", flying between two electrodes (accelerated by an electric field). When the same electron strikes the positive electrode (being absorbed in the state "**2**"), we shall say that the electron as a particle is stopped (losing its kinetic energy and momentum: $(E_{ek2} = \tilde{E}_{e2}) \cong 0$, $(p_{e2} = \tilde{p}_{e2}) \cong 0$), but the positive electrode itself becomes the carrier of specific transient electric current pulse, and carrier of certain mechanical vibration, being characterized by non-zero internal wave states $(\tilde{E}_{pe2}, \tilde{p}_{pe2}) = (\tilde{E}_{ne2}, \tilde{p}_{ne2})$. Of course, a similar situation regarding internal electrode states, when the electron flies between them, will also make $(\tilde{E}_{pe1}, \tilde{p}_{pe1}) = (\tilde{E}_{ne1}, \tilde{p}_{ne1})$ because both electrodes are permanently a part of the externally closed electric circuit. To make this situation even more apparent, in the following table, all (particle and wave) energy and momentum states of electrodes, electron, and x-ray photons are classified.

	States just before electron left negative electrode (indexing: 0)	States after electron left the negative electrode, before striking positive electrode (indexing: 1)	States just after electron stroke a positive electrode (indexing: 2)
electron (index: e)	$(E_{ek0} = \tilde{E}_{e0}) \cong 0$, $(p_{e0} = \tilde{p}_{e0}) \cong 0$	$E_{ek1} = \tilde{E}_{e1}$ $p_{e1} = \tilde{p}_{e1}$	$(E_{ek2} = \tilde{E}_{e2}) \cong 0$, $(p_{e2} = \tilde{p}_{e2}) \cong 0$
negative electrode (index: ne)	$(E_{neko} = \tilde{E}_{neo}) = 0$, $(p_{neo} = \tilde{p}_{neo}) = 0$	$E_{nek1} = 0, p_{ne1} = 0$ $\tilde{E}_{ne1}, \tilde{p}_{ne1}$	$E_{nek2} = 0, p_{ne2} = 0$ $\tilde{E}_{ne2}, \tilde{p}_{ne2}$
a positive electrode (index: pe)	$(E_{peko} = \tilde{E}_{peo}) = 0$, $(p_{peo} = \tilde{p}_{peo}) = 0$	$E_{pek1} = 0, p_{pe1} = 0$ $\tilde{E}_{pe1}, \tilde{p}_{pe1}$	$E_{pek2} = 0, p_{pe2} = 0$ $\tilde{E}_{pe2}, \tilde{p}_{pe2}$
x-ray photons	n/a	n/a	$\tilde{E}_{x2} = hf_x$, $\tilde{p}_{x2} = hf_x / c$

All over this book are scattered small comments placed inside the squared brackets, such as: [♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

We are now in the position to generalize and explicitly formulate another aspect of particle-wave duality regarding the internal wave energy content (not discussed in earlier chapters of this book), practically summarizing facts mentioned above, as follows:

In a Laboratory System of coordinates, we can characterize a moving particle by its kinetic energy. At the same time, the same kinetic energy can be conceptually presented in two different ways, such as $E_{ek} = (\gamma - 1)mc^2 = \tilde{E} = hf$, (producing experimentally, directly, or indirectly verifiable effects of de Broglie matter waves, relative to its Laboratory System). If the same body is in a state of relative rest (not moving macroscopically), its kinetic and wave energy (relative to the Laboratory System) are again mutually equal, and equal zero, $E_{ek} = \tilde{E} = \tilde{E}_{external} = 0$ (looking externally). Since the same macro-body (electrodes in this example) presents a complex material structure, it can serve (internally) as the carrier of electric currents, and mechanical signals, meaning that inside the body we could also have a certain kind of wave propagation, or specific wave energy content, which is precisely the case found in this example. This is the reason why total motional energy of a specific body should be presented as the sum of its (external) kinetic or wave energy (if the body moves relative to its Laboratory System), and its internally captured wave energy. This is particularly interesting if somehow this body is excited and becomes the carrier of mechanical, electrical and any other kind of signals (apart from counting rest mass energy as its internal wave energy content).

For mathematical modeling, the appearance of any wave energy (and action-reaction forces) will be generally related to the cases of sudden changes of electron's motional energy. The first time, when the electron leaves the negative electrode, and the second time when the electron strikes the positive electrode, we can expect some transient electric current is waiving and acoustic perturbation and radiation effects on/in electrodes, or the space around them. We also know that when the electron strikes the positive electrode, x-ray photon/s will be emitted from the positive electrode surface ($\tilde{E}_{x2} = hf_x$, $\tilde{p}_{x2} = hf_x/c$), and at the same time the external electrical circuit between the two electrodes will indicate the presence of corresponding, transient current pulse (here represented by internal electrode states with corresponding wave energies and momentum: $\tilde{E}_{ne1}, \tilde{p}_{ne1}$, $\tilde{E}_{ne2}, \tilde{p}_{ne2}$, $\tilde{E}_{pe1}, \tilde{p}_{pe1}$, $\tilde{E}_{pe2}, \tilde{p}_{pe2}$).

*[♣ **COMMENTS & FREE-THINKING CORNER:***

When generalizing the interactions between particles and waves, we can observe that these processes often involve closed circuits of energy flow, like those found in electric circuit analysis. For example, in an x-ray tube, there are two distinct but mutually coupled closed circuits. The first is the electrical circuit, where an external voltage (U) is applied to the electrodes, causing electrons to flow between them. The second circuit is the photonic circuit, which begins when the electrons strike the positive electrode and generate x-ray photons. These photons propagate through space, and in some manner, the electromagnetic energy flow loops back to the x-ray tube, though not necessarily in the form of electromagnetic waves.

In physics, whether we are discussing particle-wave interactions, wave motions, currents, or signal propagation, these phenomena can often be understood as parts of a local or broader system of closed energy-momentum circuits. This concept, familiar in electric circuit theory, can be analogically extended to all forms of motion, oscillation, and interaction involving particles and waves, as well as the forces and torques they generate.

Wave functions provide a useful tool for analyzing energy flow within these closed circuits, regardless of the energy source. However, any theoretical analysis of particle-wave duality that fails to account for the closed nature of the energy flow (and other conservation laws) suggests that the interaction is not fully understood. This is evident in the case of gravitation, where the mechanisms of energy flow remain unclear. Action and reaction forces, as well as concepts of inertia, should be directly connected to the channels that create closed circuits of energy and momentum flow. Without understanding these circuits (see Fig. 4.1.4), theories risk remaining in a "foggy space of uncertainty and probability," providing only-locally accurate models or data fitting for limited scenarios.

When analyzing closed circuits, whether they involve fluids, electricity, or particles, it's important to recognize that the flow of one type of matter is typically coupled with the inertia and/or induction effects of a complementary or conjugate type of matter. This is analogous to the flow of electrically charged particles, which is accompanied by an electric field and characterized by an electric current that, in turn, generates a corresponding magnetic field. These relationships include the transient inertia effects described by Faraday, Maxwell, and Lorenz through the laws of electromagnetic induction. Similarly, Newton's laws of inertia and action-reaction forces can be understood as part of the same universally applicable principles of inertia and induction.

*For illustrating closed circuit concept of an energy-moments flow, let us imagine that a particle, which initially (in its state of rest) has a mass **m**, is moving under an action of certain active force F_a; see Fig. 4.1.6, and read 1.1. in the first chapter of this book (about Inertia, Inertial systems, and Inertial motions). In the space around the moving particle, we could have a flow of other particles, and presence of different fields (waves and forces), making our moving particle affected, irradiated, and internally excited in many ways, increasing its internal energy, temperature, or its rest mass. Such situations are always present in real particle motions and should be considered in some way to supplement our conceptual understanding of particle-wave duality. To have a more straightforward framework, we will for the time being neglect the possible presence of external and internal rotational elements in a particle motion (such as orbital moments of any kind) and consider that our particle is moving by dominant influence of an active force F_a. The energy balance in such a case must account for the existence of the particle initial rest mass and motional (or environmental) contribution to particle rest mass caused by all possible external influences that increase the internal particle energy. In other words, the moving particle, besides its principal and closed energy flow circuit, also has specific additional energy flow (or exchange) because of couplings with its environment, which in many practical situations (regarding calculations) could be neglected but should not be forgotten entirely.*

$\mathbf{F_a} = \mathbf{dp / dt}$ *(=) Principal, active force that makes particle moving (externally),*

$\tilde{\mathbf{F}}_{int.}$ *(=) Forces of external energy flow that internally excite rest mass states, such as heating, various external radiations, vibrations, a flux of elementary particles, etc.*

$\mathbf{E_0 = m_0 c^2}$ *(=) Real, a minimal level of particle rest energy,*

$\tilde{E}_{int.} = \tilde{m}_{int.} c^2 = \int_{[r]} \tilde{F}_{int.} dr$ *(=) Energy of rest mass (internally) excited states, caused by some external influence/s,*

$\mathbf{E_0 + \tilde{E}_{int.} = mc^2 = (m_0 + \tilde{m}_{int.})c^2}$ *(=) Total particle rest energy,*

$E_{total} = E_0 + E_k = \gamma mc^2 = \gamma (m_0 + \tilde{m}_{int.})c^2$ *(=) Total particle energy in motion,*

$E_k = \tilde{E} = (\gamma - 1)mc^2 = (\gamma - 1)(m_0 + \tilde{m}_{int.})c^2 = \int_{[r]} F_a dr$ *(=) Motional particle energy.*

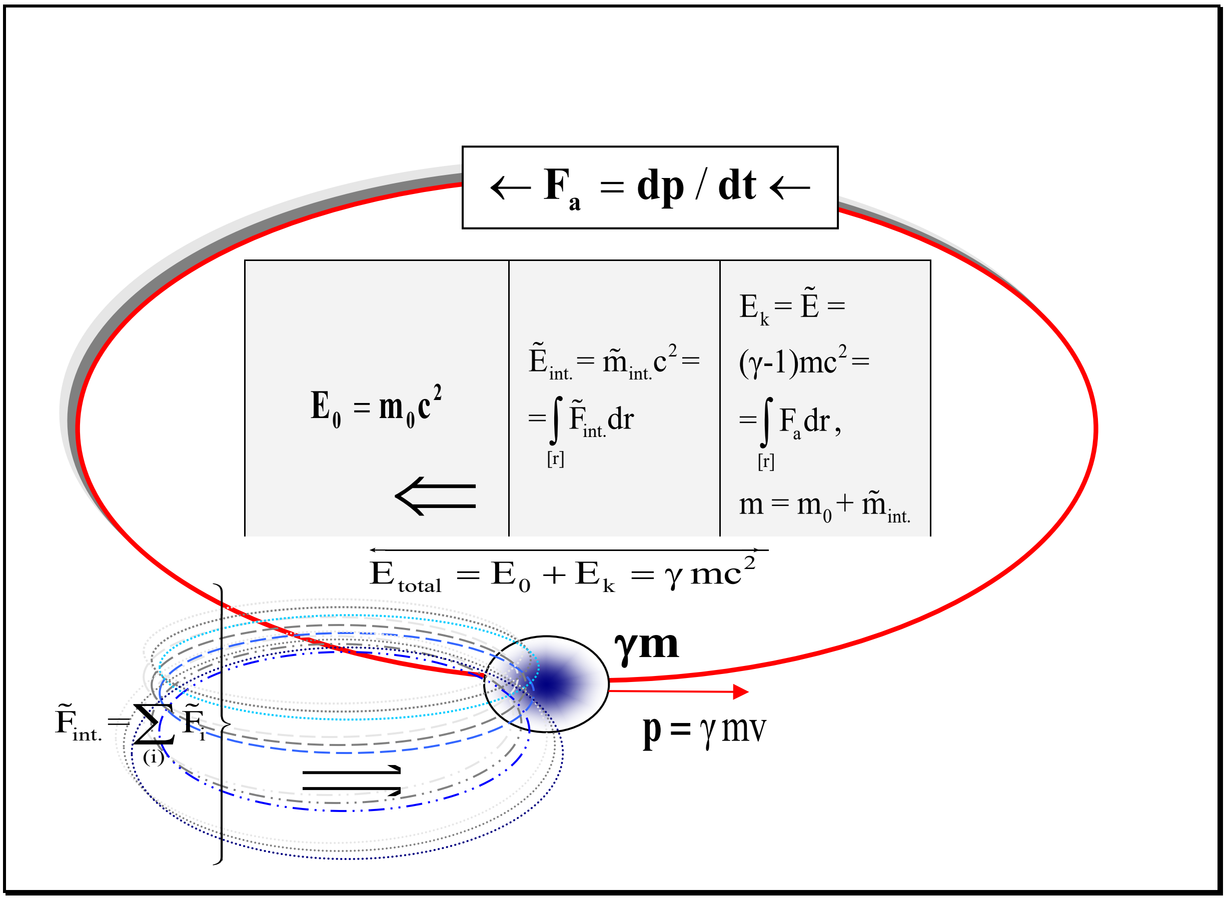

Fig. 4.1.6 An Illustration of the Closed Circuit Energy Flow ♣]

In essence, every system must include entities such as energy or signal sources (e.g., voltage or current sources, velocity or force sources) along with corresponding receivers or loads. These simple source-load circuits can be interconnected and chained with numerous other closed segments, allowing us to identify and characterize them similarly to how we approach electric circuit theory and practice. However, this conceptualization is notably absent in gravitation theory.

Within an atom, the states of electrons and nuclei, along with their bidirectional exchanges of electromagnetic energy or photons, should form fully closed circuits for energy, mass, and momentum flow. This framework paves the way for understanding gravitation in the manner conceptualized by R. Boskovic and N. Tesla (further discussed in Chapters 2, 8, and 9).

The ideas of N. Tesla and R. Boskovic regarding gravitation, universal natural forces, and radiant energy suggest that all atoms in our universe must communicate both externally and internally to maintain closed circuits of energy, mass, momentum, currents, and voltage components. The laws governing Newton's action-reaction forces, along with principles of electromagnetic induction, quantum entanglement effects, and classical wave equations, which demonstrate that opposing waves propagating in different directions are consistently generated, support the concept of closed circuits.

For further exploration of these concepts and analogies, please refer to the first chapter of this book, as well as Chapters 4.3, 8, 9, and 10.

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Let us go back to the previously analyzed example of x-ray radiation. Apparently, in the situation when analyzing X-ray radiation, we have enough tangibles, measurable and visible waving, and radiation (of electrical and acoustic) events. Consequently, we cannot characterize de Broglie electron matter waves only as "phantom probability waves", since here we always have a closed electrical circuit where we are in a natural (and fully deterministic) position to know, see, calculate, and measure what and where really waves and produces electrical currents, voltages, and photons), in real time.

Considering the differences between electron group and phase velocities, when an electron flies between two electrodes, the effects of associated retarded potentials will make this situation a little bit analytically more complex.

Let us now calculate the outgoing kinetic energy $\mathbf{E_{ek1}}$ and speed $\mathbf{v_{e1}}$ of a single electron, $\mathbf{m = m_e}$, $\mathbf{q_e = -e}$ (just now of leaving the surface of a negative electrode; time interval “**1**"). The total energy conservation law applied, in this case, will give,

$$\left\{E_{eo} = E_{ek1} + E_{eo} + E_{\text{electric field}}\right\} \Rightarrow$$
$$\begin{Bmatrix} mc^2 = (\gamma_1 - 1)mc^2 + mc^2 - eU, \\ \gamma_1 = 1 + eU/mc^2 = (1 - v_{e1}^2/c^2)^{-1/2} \Rightarrow v_{e1} = c\left[1 - 1/(1 + eU/mc^2)^2\right]^{1/2}, \\ E_{ek1} = (\gamma_1 - 1)mc^2 = eU = \gamma_1 m v_{e1}^2/\left[1 + (1 - v_{e1}^2/c^2)^{-1/2}\right],\ p_{e1} = mv_{e1} \end{Bmatrix}$$

We know that when the electron leaves the negative electrode (since the electron has a certain mass, moment, spin, and charge), the negative electrode will "feel" small electrical and mechanical shock. A certain amount of energy ($\delta\tilde{E} \geq 0$) will be dissipated in such (transient) process (in a closed electrical circuit), reducing outgoing electron speed (in fact, the outgoing electron speed will be: $v_{e1} \leq c\left[1 - 1/(1 + eU/mc^2)^2\right]^{1/2}$). Applying the law of total energy conservation again, we can consider this correction in the following way:

$$\left\{\left[E_{eo} = E_{ek1} + E_{eo} + E_{\text{electric field}} + \delta\tilde{E}\right], \left[E_{ek1} = (\gamma_1 - 1)mc^2 = \tilde{E}_{e1} = \tilde{p}_{e1} \cdot u_{e1} = -eU\right]\right\} \Rightarrow$$
$$\begin{Bmatrix} mc^2 - \delta\tilde{E} = (\gamma_1 - 1)mc^2 + mc^2 - eU, \\ \gamma_1 = 1 + (eU - \delta\tilde{E})/mc^2 = (1 - v_{e1}^2/c^2)^{-1/2} \\ v_{e1} = c\left\{1 - 1/\left[1 + (eU - \delta\tilde{E})/mc^2\right]^2\right\}^{1/2} \leq c\left\{1 - 1/\left[1 + eU/mc^2\right]^2\right\}^{1/2}, \\ E_{ek1} = (\gamma_1 - 1)mc^2 = -eU - \delta\tilde{E} = \gamma_1 m v_{e1}^2/\left[1 + (1 - v_{e1}^2/c^2)^{-1/2}\right] \leq -eU,\ p_{e1} = mv_{e1} = \tilde{p}_{e1} \end{Bmatrix}$$

Since an external voltage source electrically connects negative and positive electrodes, making a closed electrical circuit, any (wave energy or electrical current) perturbation in one electrode will coincidently produce a similar effect in the opposite electrode ($(\tilde{\mathbf{E}}_{pe1}, \tilde{\mathbf{p}}_{pe1}) = (\tilde{\mathbf{E}}_{ne1}, \tilde{\mathbf{p}}_{ne1}), (\tilde{\mathbf{E}}_{pe2}, \tilde{\mathbf{p}}_{pe2}) = (\tilde{\mathbf{E}}_{ne2}, \tilde{\mathbf{p}}_{ne2})$). Apparently, in this case, energy and momentum conservation laws should be applied in a very general way, considering (internal electrical and acoustical) states in electrodes, flying-electron energy, and energy of x-ray radiation from the positive electrode.

To have a complete energy conservation picture of this process we should apply the principal relations (4.2), between (total) kinetic and wave energy and their momentum,

$\Delta E_k = -\Delta\tilde{E}$ and $\Delta \mathbf{p} = -\Delta\tilde{\mathbf{p}}$. Let us first apply (4.2) between the states **(0 → 1),** when the electron was in the state of rest (on the negative electrode surface) and just after it left the negative electrode,

$$\Delta E_k = -\Delta\tilde{E} \Leftrightarrow \Delta\left[\sum_{(i)} E^i{}_k\right] = -\Delta\left[\sum_{(i)} \tilde{E}^i\right] \Rightarrow$$

$$\left[(E_{ek1} + E_{nek1} + E_{pek1}) - (E_{eko} + E_{neko} + E_{peko})\right] = -\left[(\tilde{E}_{e1} + \tilde{E}_{ne1} + \tilde{E}_{pe1}) - (\tilde{E}_{eo} + \tilde{E}_{neo} + \tilde{E}_{peo})\right] \Rightarrow$$

$$E_{ek1} = -(\tilde{E}_{e1} + \tilde{E}_{ne1} + \tilde{E}_{pe1}) = (\gamma_1 - 1)mc^2 = -eU - \delta\tilde{E} \le -eU \ , \ \delta\tilde{E} \ge 0$$

$$\Delta p = -\Delta\tilde{p} \Leftrightarrow \Delta\left[\sum_{(i)} p^i\right] = -\Delta\left[\sum_{(i)} \tilde{p}^i\right] \Rightarrow$$

$$\left[(p_{e1} + p_{nek1} + p_{pe1}) - (p_{eo} + p_{neo} + p_{peo})\right] = -\left[(\tilde{p}_{e1} + \tilde{p}_{ne1} + \tilde{p}_{pe1}) - (\tilde{p}_{eo} + \tilde{p}_{neo} + \tilde{p}_{peo})\right] \Rightarrow$$

$$p_{e1} = -(\tilde{p}_{e1} + \tilde{p}_{ne1} + \tilde{p}_{pe1}) = mv_{e1}$$

Since the electrical circuit between electrodes, when the electron passes from the negative to the positive electrode, is always closed, internal waving phenomena (or currents) in one electrode will be at the same time present in the opposite electrode. By applying the essential relations between kinetic again and wave energy and their momentum, $\mathbf{\Delta E_k = -\Delta\tilde{E}}$ and $\mathbf{\Delta p = -\Delta\tilde{p}}$, between the states **(1 → 2),** when the electron was flying between two electrodes, and just after it stroke the positive electrode, we get:

$$\Delta E_k = -\Delta\tilde{E} \Leftrightarrow \Delta\left[\sum_{(i)} E^i{}_k\right] = -\Delta\left[\sum_{(i)} \tilde{E}^i\right] \Rightarrow$$

$$\left[(E_{ek2} + E_{nek2} + E_{pek2}) - (E_{ek1} + E_{nek1} + E_{pek1})\right] =$$
$$= -\left[(\tilde{E}_{e2} + \tilde{E}_{ne2} + \tilde{E}_{pe2} + \tilde{E}_{x2}) - (\tilde{E}_{e1} + \tilde{E}_{ne1} + \tilde{E}_{pe1})\right] \Rightarrow$$

$$\Rightarrow \left\{\begin{array}{l} 0 \le \tilde{E}_{x2} = hf_x = E_{ek1} - (\tilde{E}_{ne1} + \tilde{E}_{pe1}) - (\tilde{E}_{ne2} + \tilde{E}_{pe2}) \le E_{ek1} \\ 0 \le \tilde{E}_{x2} - E_{ek1} = hf_x - E_{ek1} = -(\tilde{E}_{ne1} + \tilde{E}_{pe1}) - (\tilde{E}_{ne2} + \tilde{E}_{pe2}) \le 0 \\ -(\tilde{E}_{ne1} + \tilde{E}_{pe1}) \le (\tilde{E}_{ne2} + \tilde{E}_{pe2}) \\ E_{ke1} - (\tilde{E}_{ne1} + \tilde{E}_{pe1}) \le hf_x + (\tilde{E}_{ne2} + \tilde{E}_{pe2}) \end{array}\right\}$$

$$\Delta p = -\Delta\tilde{p} \Leftrightarrow \Delta\left[\sum_{(i)} p^i\right] = -\Delta\left[\sum_{(i)} \tilde{p}^i\right] \Rightarrow$$

$$\left[(p_{e2} + p_{ne2} + p_{pe2}) - (p_{e1} + p_{nek1} + p_{pe1})\right] = -\left[(\tilde{p}_{e2} + \tilde{p}_{ne2} + \tilde{p}_{pe2} + \tilde{p}_{x2}) - (\tilde{p}_{e1} + \tilde{p}_{ne1} + \tilde{p}_{pe1})\right] \Rightarrow$$

$$\left[0 - p_{e1}\right] = -\left[(0 + \tilde{p}_{ne2} + \tilde{p}_{pe2} + \tilde{p}_{x2}) - (\tilde{p}_{e1} + \tilde{p}_{ne1} + \tilde{p}_{pe1})\right], p_{e1} = \tilde{p}_{e1} \Rightarrow$$

$$\tilde{p}_{x2} = hf_x / c = p_{e1} - (\tilde{p}_{ne2} + \tilde{p}_{pe2}) - (\tilde{p}_{e1} + \tilde{p}_{ne1} + \tilde{p}_{pe1}) = -(\tilde{p}_{ne2} + \tilde{p}_{pe2}) - (\tilde{p}_{ne1} + \tilde{p}_{pe1})$$
$$c\tilde{p}_{x2} = hf_x = \tilde{E}_{x2} = -c(\tilde{p}_{ne2} + \tilde{p}_{pe2}) - c(\tilde{p}_{ne1} + \tilde{p}_{pe1}) =$$
$$= E_{ek1} - (\tilde{E}_{ne1} + \tilde{E}_{pe1}) - (\tilde{E}_{ne2} + \tilde{E}_{pe2}) \le E_{ek1} = (\gamma_1 - 1)mc^2 = -eU - \delta\tilde{E} \le -eU$$
$$0 < f_x \le \frac{E_{ek1}}{h} = \frac{(\gamma_1 - 1)mc^2}{h} = \frac{-eU}{h}.$$

Eventually, when the electron (as a particle) strikes the positive electrode, its final kinetic energy ($\mathbf{E_{ek1}}$) will be (partially or fully) transformed into radiation of X-rays (and the part of the same energy would create transient electric current and mechanical oscillations in electrodes circuit). The maximal frequency of radiated x-ray photons will be,

$$(f_x)_{max.} = -eU/h = \frac{(\gamma_1 - 1)mc^2}{h} = \frac{E_{ek1}}{h} > \frac{m_e v_{el}^2}{2}.$$

We got the well-known frequency of X-rays $(\mathbf{f_x})_{max.} = -\mathbf{eU/h}$, explaining this way that (**only and exclusively**) the relativistic motional energy of a particle $\mathbf{E_{ek1}}$, which is equal to the particle-wave energy $\tilde{\mathbf{E}}_{\mathbf{e1}}$, is wholly or partially, radiated in the form of X-rays, $\mathbf{hf_x}$. If accelerated electrons have sufficiently high striking speeds, there is no way to show that x-ray energy could be calculated using classical mechanics kinetic energy expression $\mathbf{m_e v_e^2 / 2}$, which indirectly indicates that traditional (non-relativistic) Schrödinger equation would also be inapplicable to this case (since in Schrödinger's equation, particle kinetic energy is treated as $\mathbf{m_e v_e^2 / 2}$). We also see that the contemporary quantum mechanical concept (or model) of a particle, which includes its rest mass and rest energy as an integral part of its wave packet, is unacceptable (at least in this case). We can calculate and measure that only relativistic motional energy is transformed into X-rays and waving perturbations in electrodes (and all of them belong to de Broglie or matter waves, being easily measurable and with deterministic nature).

The typical X-rays spectrum has a form of continuous spectral distribution because of many reasons such as:

-the negative electrode of the x-ray tube is heated to facilitate electrons emission (modulating the speed of electrons),

-external electrical circuit presents resistive and reactive electric impedance, producing specific energy dissipation and oscillating-current effects,

-there are also associated acoustic phenomena in electrodes, and such process is also different in many other (x-ray tube design) details in comparison with an idealized case of this example.

The only common valid conclusion for all x-ray devices is that the maximal experimentally measured x-ray frequency is precisely equal to the frequency calculated in this example $\mathbf{f_x} \le -\mathbf{eU/h}$, confirming that de Broglie matter waves present only a form of kinetic energy of ordinary vibrations (of electromagnetic, mechanical and/or any other nature), without any participation of rest masses.

4.1.4. Matter Waves and orbital motions

The two-body problem, which has been extensively explored in this context, provides valuable insight into the formation of matter waves. This explanation relies on several assumptions and a degree of creative imagination (see illustrative examples in the second chapter, particularly around equations (2.11.13-1) to (2.11.13-5)).

In most of the literature, including this one, the two-body problem is analyzed through the lens of the relationships and interactions between the linear motions and momenta of the particles involved. However, it is important to recognize that every linear motion represents just a small segment of a broader rotational or orbital motion, where the radius of rotation can be arbitrarily large.

In our universe, there is a natural tendency for all motions to stabilize into some form of rotation or orbital movement. This phenomenon is evident in gravitational interactions described by Kepler's and Newton's laws, as well as in the behavior of atoms and subatomic particles in the micro-world.

This tendency opens an additional avenue for analyzing the two-body problem in relation to matter waves by considering the global conservation of the involved orbital and spin moments.

Let us consider the same two-body situation (see Fig.4.1. and Fig.4.1.5) as already introduced at the beginning of this chapter under 4.1.2., but now, analogically, and equivalently presented from orbital and spin moments conservation. This should always be applicable (see the significant background to the familiar approach regarding consequences of global conservation of orbital and spin moments in [36], Anthony D. Osborne, & N. Vivian Pope), as follows,

$$\left[\begin{array}{l}
\left\{\begin{array}{l}
E_{k1}+E_{k2}=E_{km}+E_{kM}=E_{kc}+E_{kr}=\frac{1}{2}m_c v_c^2+\frac{1}{2}m_r v^2, E_{kr}=\frac{1}{2}m_r v_r^2\cong\frac{1}{2}mv^2=E_{k1}\cong\frac{1}{2}\mathbb{J}\omega_m^2, \\
\frac{1}{2}m_1 v_1^2+\frac{1}{2}m_2 v_2^2=\frac{1}{2}mv_1^2+\frac{1}{2}Mv_2^2=\frac{1}{2}m_c v_c^2+\frac{1}{2}m_r v_r^2\cong\frac{1}{2}Mv_c^2+\frac{1}{2}mv^2\left[=\frac{1}{2}Mv_c^2+\frac{1}{2}\mathbb{J}\omega_m^2\right]
\end{array}\right\}, \\
\text{and/or}\left\{\begin{array}{l}
E_{ki}=\frac{1}{2}m_i v_i^2=\frac{1}{2}p_i v_i\cong\frac{\gamma_i m_i v_i^2}{1+\sqrt{1-v_i^2/c^2}}=\frac{p_i v_i}{1+\sqrt{1-v_i^2/c^2}}, E_{kr}=\frac{1}{2}m_r v_r^2\cong\frac{p_r v_r}{1+\sqrt{1-v_r^2/c^2}}, \\
\frac{\gamma_1 mv_1^2}{1+\sqrt{1-v_1^2/c^2}}+\frac{\gamma_2 Mv_2^2}{1+\sqrt{1-v_2^2/c^2}}\cong\frac{\gamma_c m_c v_c^2}{1+\sqrt{1-v_c^2/c^2}}+E_{kr}\left[\cong\frac{\gamma_c Mv_c^2}{1+\sqrt{1-v_c^2/c^2}}+\frac{\mathbb{J}\omega_m^2}{1+\sqrt{1-v_c^2/c^2}}\right], \\
\left(mc^2+\frac{\gamma_1 mv_1^2}{1+\sqrt{1-v_1^2/c^2}}\right)+\left(Mc^2+\frac{\gamma_2 Mv_2^2}{1+\sqrt{1-v_2^2/c^2}}\right)=\left(m_c c^2+\frac{\gamma_c m_c v_c^2}{1+\sqrt{1-v_c^2/c^2}}\right)+\left(m_r^*\cdot c^2+E_{kr}\right) \\
\Leftrightarrow\gamma_1 mc^2+\gamma_2 Mc^2=\gamma_c m_c c^2+\left(m_r^*\cdot c^2+E_{kr}\right)\Rightarrow m_r^*=\gamma_1 m+\gamma_2 M-\gamma_c m_c-E_{kr}\cong 0, \gamma_i=\frac{1}{\sqrt{1-v_i^2/c^2}}
\end{array}\right\}, \quad \& \\
\text{and}\left\{m_c=m_1+m_2,\ m_r=\frac{m_1 m_2}{m_1+m_2}, \vec{v}_c=\frac{m_1\vec{v}_1+m_1\vec{v}_1}{m_1+m_2}, v_i=\omega_i r_i,\ m_i v_i^2=\mathbb{J}_i\omega_i^2,\ p_i v_i=\mathbb{L}_i\omega_i\right\}
\end{array}\right]$$

$$\& \left\{ \begin{array}{l} \vec{L} = \sum_{(i)} \vec{L}_i = \vec{\omega}_c \sum_{(i)} J_i = \text{const.},\ \vec{\omega}_c = \frac{\sum_{(i)} J_i \vec{\omega}_i}{\sum_{(i)} J_i} = \frac{\sum_{(i)} \vec{L}_i}{\sum_{(i)} J_i} = \frac{J_1\vec{\omega}_1 + J_2\vec{\omega}_2}{J_1 + J_2} = \frac{\vec{L}_1 + \vec{L}_2}{J_1 + J_2}, \\ \vec{L} = \vec{L}_1 + \vec{L}_2 = J_1\vec{\omega}_1 + J_2\vec{\omega}_2 = \vec{\omega}_c (J_1 + J_2), \vec{L}_i = J_i\vec{\omega}_i, J_c = J_1 + J_2, J_r = \frac{J_1 J_2}{J_1 + J_2} \\ \vec{\omega}_r = \vec{\omega}_1 - \vec{\omega}_2 = \vec{\omega}_m,\ \vec{L}_r = \frac{J_1 J_2}{J_1 + J_2}(\vec{\omega}_1 - \vec{\omega}_2) = J_r \vec{\omega}_m = J_r \vec{\omega}_r, \omega_r = \omega_m. \end{array} \right\} \Rightarrow$$

$$\Rightarrow \left\{ \begin{array}{l} E_{k1} + E_{k2} = E_{km} + E_{kM} = E_{kc} + E_{kr} = \\ \frac{1}{2} m_1 v_1 r_1 \frac{v_1}{r_1} + \frac{1}{2} m_2 v_2 r_2 \frac{v_2}{r_2} = \frac{1}{2} m_c v_c r_c \frac{v_c}{r_c} + \frac{1}{2} m_r v_r r \frac{v_r}{r} \\ = \frac{1}{2} J_1 \omega_1^2 + \frac{1}{2} J_2 \omega_2^2 = \frac{1}{2} J_c \omega_c^2 + \frac{1}{2} J_r \omega_r^2 = \\ = \frac{1}{2} L_1 \omega_1 + \frac{1}{2} L_2 \omega_2 = \frac{1}{2} L_c \omega_c + \frac{1}{2} L_r \omega_r,\ E_{kr} = \frac{J_r \omega_r^2}{1 + \sqrt{1 - v_c^2 / c^2}} = hf_s = \tilde{E}_s \end{array} \right\}. \quad (4.1.4\text{-}1)$$

Based on such analogical conceptualization (and accepted assumptions), the two-body interaction participants are rotating or being in specific orbital motion long before interaction starts (both in Laboratory and Center of the mass system). The kinetic energy member (which is a product of such two-body interaction),

$$E_{kr} = \frac{J_r \omega_r^2}{1 + \sqrt{1 - v_c^2 / c^2}} = hf_s = \tilde{E}_s \cong \frac{1}{2} J_r \omega_r^2 = \frac{1}{2} L_r \omega_r = \frac{1}{2} m_r v_r^2, \quad (4.1.4\text{-}2)$$

should present kind of rotation, and at the same time (as promoted in this book) this is the crucial, spinning matter wave energy ($\tilde{E}_s = hf_s$). What is strange regarding such rotation ($\vec{L}_r, \vec{\omega}_r$) is related to the total orbital momentum conservation, which is producing, $\vec{L} = \vec{L}_1 + \vec{L}_2 = \text{const.} \Rightarrow \vec{L}_r = \vec{0}$. Since $E_{kr} = \frac{1}{2} L_r \omega_r \neq 0,\ \vec{\omega}_r = \vec{\omega}_1 - \vec{\omega}_2 \neq 0$ one of the solutions is that $\vec{L}_r$ could present "orbiting-spinning" motion (with spiral, toroidal, or rotating ring envelope; see [94], Classical Mechanics; - chapter 14) composed of minimum two spinning objects with equal and mutually opposed spin moments:

$$\vec{L}_r = \vec{L}_r^+ + \vec{L}_r^- = \frac{\vec{L}_r}{2} - \frac{\vec{L}_r}{2} = \vec{S} - \vec{S} = \vec{0},, \vec{L}_r^+ = +\vec{S} = +\frac{\vec{L}_r}{2},\ \vec{L}_r^- = -\vec{S} = -\frac{\vec{L}_r}{2}$$

$$\Leftrightarrow \vec{L}_r^+ = -\vec{L}_r^- \Rightarrow E_{kr} = \frac{1}{2} L_r \omega_r = \frac{1}{4} L_r^+ \omega_r + \frac{1}{4} L_r^- \omega_r. \quad (4.1.4\text{-}3)$$

Much more about similar items and familiar elaborations, given in chapter 10. Of this book.

[♣ COMMENTS & FREE-THINKING CORNER:

So far, the focus has been on demonstrating that the energy of matter waves (de Broglie waves) is related to the kinetic or motional energy of particles and quasi-particles, including various types of waves and oscillations. However, the rest mass does not directly contribute to matter-wave energy. Rather, within certain limits, the rest mass can act as an absorber or emitter of wave energy.

As we explore deeper into the structure of matter and particles, we encounter increasingly complex field and wave structures, which only conditionally present particles with possible rest mass. By analyzing these structures, we uncover additional energy in the form of waves and fields (often stabilized as standing waves). This raises an important question: What exactly is rest mass if the fundamental building blocks of particles consist of waves, fields, or motional energy in the form of stationary, standing waves, or self-resonant states?

The answer lies in the intrinsic, self-sustaining, rotational field nature of the elementary wave constituents of our universe. Simple matter structures, such as elementary particles (electrons, protons, neutrons, etc.), appear to form stable, closed domains, like toroid or rotating rings, composed of internally rotating, stationary, and standing wave fields. Once such a self-sustaining, space-limited vortex domain is formed, it behaves like an elementary particle. If the "sublimation and solidification" of the wave process is incomplete, the result might be a quasi-particle, such as a photon.

In our universe, it's natural for these closed, rotating wave structures to form and remain stable over time, potentially for extremely long periods. These structures always contain specific energy, which allows us to associate rest mass with them, based on the proportionality between mass and energy from Einstein's theory of relativity.

Observable characteristics such as the orbital moments and spin of elementary particles, as well as phenomena like de Broglie matter waves, spontaneous radioactivity, and the presence of various fields and forces, provide evidence that these rotating structures are a fundamental aspect of reality.

Although this remains an intuitive and speculative concept, it provides a useful starting point for understanding the nature of rest mass. A key mistake in quantum mechanical models of matter waves and wave functions was the unconditional inclusion of a particle's stable rest mass into matter wave modeling. Only a time- and space-variable, non-stationary energy flow gives rise to freely propagating matter waves. ♣]

Download the last version here:
http://www.mastersonics.com/documents/revision_of_the_particle-wave_dualism.pdf

4.2. INTERACTIONS MODELING

Interactions, where involved participants can be micro and elementary particles, quasiparticles, waves, wave-groups, and photons, could be conveniently generalized as cases of scattering (see the picture below, Fig.4.2.0). For instance, a moving particle, or photon, or matter-waves packet arriving from conditions under "Medium 1" collides with the surface of a "Medium 2". In such situation, involved "energy-momentum entity" could be reflected, refracted, diffused, or scattered, producing new particles and/or waves in a "Medium 1", and it could be partially refracted into a "Medium 2" in a form of matter waves (as photons, phonons, some particles, and/or mechanical waves, for instance). Of course, all energy and momentum conservation laws should be satisfied (regarding interaction participants) to correctly describe such events. ***In this chapter, we will address similar interactions among particles and waves, with the intention to show that a total matter-waves energy is equal only to the relevant, incident particle (or wave packet) kinetic, or motional energy. Involved <u>rest-masses energy</u> (if any) will not be a part of the mentioned active, motion-related matter-wave energy of the scattering process (except in cases of nuclear reactions like fission, fusion, and particles annihilations).***

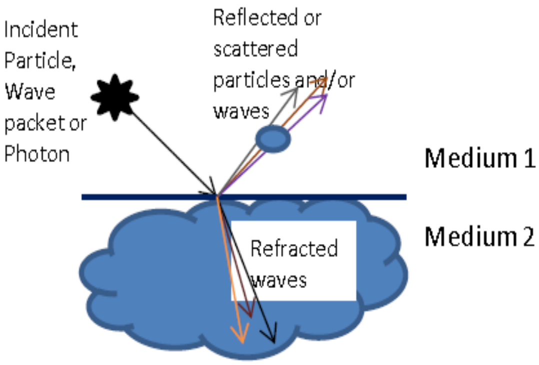


Fig. 4.2.0 Scattering Interactions

Let us now explore the limits of traditional mechanistic analysis regarding conservation laws and interactions where matter wave phenomenology is involved. The most general case of interactions between particles and waves or wave-groups in any mutual combination of participants is presented in table T.4.2.1.

T.4.2.1.

Elements entering interaction	Interaction zone	Resulting elements after interaction
<u>Particles</u>: $\{E_k^i, p^i\}, E_0^i$ $i \in [1,2,3,...P]$	?!?!	**<u>Particles</u>**: $\{E_k^m, p^m\}, E_0^m$ $m \in [1,2,3,...M]$
<u>Waves</u>: $\{\tilde{E}^j, \tilde{p}^j\}$ $j \in [1,2,3,...Q]$		**<u>Waves</u>**: $\{\tilde{E}^n, \tilde{p}^n\}$ $n \in [1,2,3,...N]$

<u>Moving</u> **<u>Particles</u>** are "energy-momentum" dualistic "particle-matter-wave" states with non-zero rest masses, $\{E_k^i, p^i\}, E_0^i > 0$. Not moving (standstill) particles with

All over this paper are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making fast comments, and presenting challenging ideas, that could be some other time developed towards something much more meaningful and more properly integrated with Physics.

non-zero rest masses are pure particle states (inside certain inertial reference system), $\left\{E_k^i = 0,\ p^i = 0\right\},\ E_0^i > 0$.

Pure **Waves** including propagating wave packets are energy-momentum states with zero rest masses (like photons), $\left\{\tilde{E}^j,\ \tilde{p}^j\right\},\ E_0^j = 0$. Energy conservation for cases from T.4.2.1 can be presented as follows:

$$E_{total} = E = E_k + E_0 = \sum_{(i)}(E_k^i + E_0^i) + \sum_{(j)}\tilde{E}^j = \sum_{(m)}(E_k^m + E_0^m) + \sum_{(n)}\tilde{E}^n \Leftrightarrow$$
$$\Leftrightarrow (\sum_{(i)}E_k^i - \sum_{(m)}E_k^m) + (\sum_{(i)}E_0^i - \sum_{(m)}E_0^m) = -\left[\sum_{(j)}\tilde{E}^j - \sum_{(n)}\tilde{E}^n\right] \Leftrightarrow$$
$$\Leftrightarrow \Delta E = \Delta E_k + \Delta E_0 = -\Delta\tilde{E} \Leftrightarrow \Delta E + \Delta\tilde{E} = 0,$$
$$\Delta E_k = \sum_{(i)}E_k^i - \sum_{(m)}E_k^m = \Delta\left[\sum_{(i,m)}E_k^{i,m}\right],\ \Delta E_0 = \sum_{(i)}E_0^i - \sum_{(m)}E_0^m = \Delta\left[\sum_{(i,m)}E_0^{i,m}\right] \quad (4.8\text{-}1)$$
$$\Delta\tilde{E} = \sum_{(j)}\tilde{E}^j - \sum_{(n)}\tilde{E}^n = \Delta\left[\sum_{(j,n)}\tilde{E}^{j,n}\right]$$
$$(E_{total} = E = \gamma mc^2, E_k = (\gamma - 1)mc^2 = E - E_0,\ E_0 = mc^2,$$
$$\tilde{E} = E_k = pu = \gamma mvu = cp\sqrt{\frac{\gamma - 1}{\gamma + 1}} = (\gamma - 1)mc^2 \ (=) \text{ real particles kinetic energy},$$
$$\tilde{E} = hf = pu = cp \ (=) \text{ Photon energy}).$$

Planck constant **h** is applicable and relevant in cases of narrow-band wave-packets, photons and elementary or subatomic microparticles (where conservation of angular moments on closed orbits is satisfied, and where self-closed standing wave structures are hosted). In other cases of bigger microparticles, and/or macro masses, including astronomic masses (like in solar or planetary systems), certain Planck analog, $H >> h$, constant would be applicable when we also have orbital, self-closed and periodical motions; -see more in Chapter 2. about gravitation).

Momentum conservation in cases from T.4.2.1 can be presented as follows:

$$\vec{P} = \sum_{(i)}p^i + \sum_{(j)}\tilde{p}^j = \sum_{(m)}p^m + \sum_{(n)}\tilde{p}^n \Leftrightarrow$$
$$\Leftrightarrow \sum_{(i)}p^i - \sum_{(m)}p^m = -\left[\sum_{(j)}\tilde{p}^j - \sum_{(n)}\tilde{p}^n\right] \Leftrightarrow \Delta p = -\Delta\tilde{p} \Leftrightarrow \Delta p + \Delta\tilde{p} = 0, \quad (4.8\text{-}2)$$
$$\Delta p = \sum_{(i)}p^i - \sum_{(m)}p^m = \Delta\left[\sum_{(i,m)}p^{i,m}\right],\quad \Delta\tilde{p} = \sum_{(j)}\tilde{p}^j - \sum_{(n)}\tilde{p}^n = \Delta\left[\sum_{(j,n)}\tilde{p}^{j,n}\right]$$

$$\vec{P}^2 - \frac{E_{total}^2}{c^2} = \vec{P}'^2 - \frac{E'^2_{total}}{c^2} = \vec{P}''^2 - \frac{E''^2_{total}}{c^2} = ... = inv. = -m^2c^2.$$

If objects entering an interaction have angular moments (either external and tangible macro moments, $\mathbb{L}$ = $\mathbb{L}_{external}$, and/or internal, intrinsic, or spin moments, $\mathbb{L}_0$ = $\mathbb{L}_{internal}$), we can again (analogically) present the conservation of angular moments in a similar way as we did for energy and momentum conservation. By summarizing and analogically generalizing conservation laws (4.8-1) and (4.8-2) in a condensed form, including involved angular moments, the unity and complementarity of particle and wave aspects of certain motion, and *universal action-reaction, induction laws and forces* will be presentable as follows (see also equations (4.2) in Chapter 4.1):

$$\left\{\begin{array}{l}\left[\begin{array}{l}\Delta E = \Delta E_k + \Delta E_0 = -\Delta\tilde{E} \\ \Delta p = -\Delta\tilde{p}\end{array}\right] \Rightarrow \\ \Rightarrow \Delta L = \Delta L_{external} + \Delta L_{internal} = -\Delta\tilde{L} \ \ldots\end{array}\right\} \Leftrightarrow \left\{\Delta X = -\Delta\tilde{X}\right\}. \quad (4.8\text{-}3)$$

If we now consider only the case of a single "energy-moments" state in certain phase of transformation, we can address the meaning of involved inertial and reaction forces by transforming (4.8-3) into (4.8-4), as for instance:

$$\{\Delta \to d\} \Rightarrow \left\{\begin{array}{l} dE = dE_k + dE_0 = dE_k = -d\tilde{E} \\ dp = -d\tilde{p} \\ dL = dL_{external} + dL_{internal} = -d\tilde{L} \\ \ldots\ldots\ldots\ldots \end{array}\right\} \Leftrightarrow \left\{dX = -d\tilde{X}\right\} \Rightarrow$$

$$\Rightarrow \left\{dX = -d\tilde{X}\right\}/dt \Leftrightarrow \frac{dX}{dt} = -\frac{d\tilde{X}}{dt} \Leftrightarrow \{\text{action} \equiv \text{reaction in opposite direction}\},$$

$$\Rightarrow F = \frac{dp}{dt} = -\frac{d\tilde{p}}{dt} (=) \text{Linear force}, \ \tau = \frac{dL}{dt} = -\frac{d\tilde{L}}{dt} (=) \text{Torque, or angular firce},$$

where, based on analogies, we could create relations such as,

$$\left\{\begin{array}{l} v = \frac{dE}{dp} = \frac{d\tilde{E}}{d\tilde{p}} = \frac{\gamma+1}{\gamma} u (\equiv) \text{ linear group velocity}, \\ \left[v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda}, \ \gamma = (1 - v^2/c^2)^{-0.5} \right], \\ u = \frac{\tilde{E}}{\tilde{p}} = \lambda f = c\sqrt{\frac{\gamma-1}{\gamma+1}} = \frac{\gamma}{\gamma+1} v (\equiv) \text{ linear phase velocity}, \ \lambda = \frac{h}{p}, \\ \Rightarrow \left[\begin{array}{l} \omega_g = \frac{dE}{dL} = \frac{d\tilde{E}}{d\tilde{L}} = 2\pi f_g (\equiv) \text{ angular group velocity}, \\ \omega_{ph.} = \frac{\tilde{E}}{\tilde{L}} = 2\pi f_{ph} = 2\pi f (\equiv) \text{ angular phase velocity} \end{array}\right] \Rightarrow \\ \Rightarrow \omega_g = \omega_{ph.} - \lambda_{ph}\frac{d\omega_{ph.}}{d\lambda_{ph}} = -\lambda_{ph}^2\frac{df}{d\lambda_{ph}}, \ \lambda_{ph} = \frac{h}{L} \\ \omega_g = \omega_{ph.} - \frac{h}{L}\frac{d\omega_{ph.}}{d\lambda_{ph}} \leftrightarrow 2\pi f_g = 2\pi f - \frac{h}{L}\frac{d\omega_{ph.}}{d\left(\frac{h}{L}\right)} \leftrightarrow f_g = f + L\frac{df}{dL}. \end{array}\right\}. \quad (4.8\text{-}4)$$

The goal in developing equation (4.8-4), despite using an oversimplified method based on analogies, is to emphasize the unity between the particle and wave nature of all objects or matter-states that carry motional energy and linear or angular momenta. Objects in states of stable, uniform linear and/or angular inertial motions effectively keep both inertial states as mutually coupled, thanks to intrinsically associated matter-waves phenomenology (see more about inertia in chapters 1, 2 and 10). Both particle and wave characteristics coexist simultaneously; -the distinction lies in our interpretation, modeling, perception, used reference system, and the mathematical tools we apply. How we choose to highlight either the corpuscular or wave nature of moving objects also depends on the reference system we adopt, as all motions in our universe are relative (see PWDC foundations in Chapters 4.1 and 10). Effects of forces and currents manifest when inertial states are being changed spatially and/or temporally, during acceleration, or in oscillatory motions.

As we continue this inertia-related analysis, it is crucial to include the associated electromagnetic aspects of an extended meaning of inertia, dealing with different charges, electric dipoles, multipoles, currents, voltages, magnetic and spin moments (see table T.4.2.2. below). This is because angular, orbital, spinning, and circular motions can be directly linked, internally and externally, to relevant electric currents, associated magnetic moments, internal electric polarizations,

electromagnetic fields, electromagnetic inductions and Lorentz forces. Linear or rectilinear motions can also be considered special cases of orbital and angular motions where a radius of rotation is extremely large (refer to the first chapter of this book for more on electromechanical analogies).

T.4.2.2. INERTIA, INERTIAL STATES, INERTIAL MOTIONS			
Mechanics		**Electromagnetism**	
Linear motions	**Angular motions, rotation, spinning**	**Motions and states in electric field**	**Motions and states in magnetic field**
Linear motion and state of rest Inertia	**Rotating and spinning state of Inertia**	**Electric field polarization related Inertia**	**Magnetic field polarization related Inertia**

For example, the principle of "**action equal to reaction**" valid in Mechanics, in Electromagnetism corresponds to Ampere-Maxwell's laws, the Biot-Savart law, Lenz's law, Faraday's law of induction, Electromagnetic Lorentz forces, and Gauss's law for electrostatics and magnetostatics. Additionally, all second-order partial differential wave equations in Physics, including Schrödinger's equation and similar Quantum Theory wave equations, have solutions that include at least two wave groups (or multiple pairs of wave groups) propagating in mutually opposite (spatial and temporal) directions, which also reflect "**action equal to reaction**" and entanglement phenomena.

To address and analyze binary interactions, it is proposed that we conceptualize and study these interactions as happening between two idealized scenarios: a perfect plastic collision, and a perfect elastic collision. These idealized collision models will serve as boundary cases or asymptotic situations. Any real interaction, whether between waves, particles, or any combination of both, should fall within this asymptotic concept in our mathematical modeling and experimental expectations.

The natural, structural, and theoretical frameworks of these boundary cases are governed by all conservation laws and universally applicable principles of physics.

By adopting this approach, we can avoid overly general statistical and probabilistic, averaging strategies that are often used in such analyses. We should use stochastic methods only when they are necessary and useful for better mathematical representation, or for providing instructive, "in-average" modeling of trends and other results, when conditions for a statistical approach are naturally and mathematically defendable, applicable and appropriate.

4.2.1. Elastic Collisions

Let us now analyze an ideally elastic collision of two objects (for instance between two particles, or between one particle and a photon, like in the case of Compton Effect). Since rest-masses of interacting particles do not change, here we only have certain exchange of kinetic energies between collision participants. Because an elastic collision should not be dependent on time-axis direction, we will create the kinetic energy balance in case when particles $\mathbf{m}_1, \mathbf{m}_2$ enter the collision, producing $\mathbf{m}_3, \mathbf{m}_4$ (T.4.3.1), and when particles $\mathbf{m}_3, \mathbf{m}_4$ go backwards, producing $\mathbf{m}_1, \mathbf{m}_2$, (by reversing the time-axis direction, and assuming that we could have the total temporal process reversibility in all of its aspects, as presented with T.4.4).

T.4.3.1.

Elements entering interaction	Interaction zone	Resulting elements after interaction
Object-1 $\{E_{k1}, p_1 = m_1 v_1\}$ or $\{\tilde{E}_1, \tilde{p}_1\}$ →	?!?!	→ **Object-3** $\{E_{k3}, p_3 = m_3 v_3\}$ or $\{\tilde{E}_3, \tilde{p}_3\}$
Object-2 $\{E_{k2}, p_2 = m_2 v_2\}$ or $\{\tilde{E}_2, \tilde{p}_2\}$ →		→ **Object-4** $\{E_{k4}, p_4 = m_4 v_4\}$ or $\{\tilde{E}_4, \tilde{p}_4\}$

T.4.4. **(The same interaction as in T.4.3.1, but time-reversed)**

Elements entering interaction	Interaction zone	Resulting elements after interaction
← **Object-1** $\{E_{k1}, p_1 = m_1 v_1\}$ or $\{\tilde{E}_1, \tilde{p}_1\}$	?!?!	**Object-3** $\{E_{k3}, p_3 = m_3 v_3\}$ or $\{\tilde{E}_3, \tilde{p}_3\}$ ←
← **Object-2** $\{E_{k2}, p_2 = m_2 v_2\}$ or $\{\tilde{E}_2, \tilde{p}_2\}$		**Object-4** $\{E_{k4}, p_4 = m_4 v_4\}$ or $\{\tilde{E}_4, \tilde{p}_4\}$ ←

If we now apply the energy conservation law on T.4.3.1 and T.4.4, for non-relativistic velocities, it will be:

$$\begin{Bmatrix} E_{k1}+E_{k2}=E_{kr-12}+E_{kc-12}=E_{k3}+E_{k4} \\ E_{k3}+E_{k4}=E_{kr-34}+E_{kc-34}=E_{k1}+E_{k2} \end{Bmatrix} \Rightarrow \{E_{kr-12}+E_{kc-12}=E_{kr-34}+E_{kc-34}\} \Rightarrow$$

$$\begin{Bmatrix} \frac{m_1 v^2{}_1}{2}+\frac{m_2 v^2{}_2}{2}=\frac{m_1 m_2}{m_1+m_2}\frac{(\vec{v}_1-\vec{v}_2)^2}{2}+\frac{(m_1\vec{v}_1+m_2\vec{v}_2)^2}{2(m_1+m_2)}=\frac{m_3 v^2{}_3}{2}+\frac{m_4 v^2{}_4}{2} \\ \frac{m_3 v^2{}_3}{2}+\frac{m_4 v^2{}_4}{2}=\frac{m_3 m_4}{m_3+m_4}\frac{(\vec{v}_3-\vec{v}_4)^2}{2}+\frac{(m_3\vec{v}_3+m_4\vec{v}_4)^2}{2(m_3+m_4)}=\frac{m_1 v^2{}_1}{2}+\frac{m_2 v^2{}_2}{2} \end{Bmatrix} \Rightarrow$$

$$\Rightarrow \begin{Bmatrix} E_{kr}=\frac{m_1 m_2}{m_1+m_2}\frac{(\vec{v}_1-\vec{v}_2)^2}{2}=\mu_{r(1-2)}\frac{v^2_{r(1-2)}}{2}=\frac{m_3 m_4}{m_3+m_4}\frac{(\vec{v}_3-\vec{v}_4)^2}{2}=\mu_{r(3-4)}\frac{v^2_{r(3-4)}}{2}, \\ E_{kc}=\frac{(m_1\vec{v}_1+m_2\vec{v}_2)^2}{2(m_1+m_2)}=\frac{(m_3\vec{v}_3+m_4\vec{v}_4)^2}{2(m_3+m_4)}=m_c\frac{v_c^2}{2}=\frac{\vec{P}\vec{v}_c}{2},\ E_{kr}=\mu_r\frac{v_r^2}{2}=\frac{p_r v_r}{2}, \\ v_c=\frac{\vec{P}}{m_c},\ m_c=m_1+m_2=m_3+m_4,\ \mu_r=m_r=\frac{m_i m_j}{m_i+m_j} \\ \vec{P}=\vec{p}_1+\vec{p}_2=\vec{p}_3+\vec{p}_4=m_1\vec{v}_1+m_2\vec{v}_2=m_3\vec{v}_3+m_4\vec{v}_4,\ p_r=\mu_r v_r \\ v^2_{r(1-2)}=(\vec{v}_1-\vec{v}_2)^2,\ v^2_{r(3-4)}=(\vec{v}_3-\vec{v}_4)^2, \\ \frac{m_1 m_2}{m_1+m_2}(\vec{v}_1-\vec{v}_2)^2=\frac{m_3 m_4}{m_3+m_4}(\vec{v}_3-\vec{v}_4)^2,\ \frac{d\vec{p}_1+d\vec{p}_2}{m_1+m_2}=\frac{d\vec{p}_3+d\vec{p}_4}{m_3+m_4}=\frac{d\vec{P}}{m_c} \end{Bmatrix}$$

Alternatively, and analogically created (at least indicative) equivalent mathematical processing in case of relativistic velocities could be:

$$\Rightarrow \begin{Bmatrix} E_{kr}=\frac{\gamma_1\gamma_2 m_1 m_2}{\gamma_1 m_1+\gamma_2 m_2}\left[\frac{(\vec{v}_1-\vec{v}_2)^2}{1+\sqrt{1-v_c^2/c^2}}\right]=\frac{\gamma_3\gamma_4 m_3 m_4}{\gamma_3 m_3+\gamma_4 m_4}\left[\frac{(\vec{v}_3-\vec{v}_4)^2}{1+\sqrt{1-v_c^2/c^2}}\right]= \\ =\frac{\gamma_c\mu_r v_r^2}{1+\sqrt{1-v_c^2/c^2}}=E_{k1}+E_{k2}-E_{kc}=E_{k3}+E_{k4}-E_{kc}=\int_{(1)}^{(2)}\vec{F}_r d\vec{r}_{12}=\int_{(3)}^{(4)}\vec{F}_r d\vec{r}_{34}, \\ v^2_{r(1-2)}=(\vec{v}_1-\vec{v}_2)^2,\quad v^2_{r(3-4)}=(\vec{v}_3-\vec{v}_4)^2,\ \vec{F}_r=\frac{d\vec{p}_{r(1-2)}}{dt}=\frac{d\vec{p}_{r(3-4)}}{dt}, \\ E_{kc}=\frac{P_c v_c}{1+\sqrt{1-v_c^2/c^2}}=\frac{P_c^2}{(1+\sqrt{1-v_c^2/c^2})\gamma_c m_c}=\frac{\gamma_c m_c v_c^2}{1+\sqrt{1-v_c^2/c^2}}= \\ =E_{k1}+E_{k2}-E_{kr}=E_{k3}+E_{k4}-E_{kr} \\ \vec{v}_c=\frac{(\sum\vec{p})c^2}{\sum E}=\frac{\vec{P}}{\gamma_c m_c},\quad \frac{\sum E}{c^2}=\gamma_c m_c=\gamma_1 m_1+\gamma_2 m_2=\gamma_3 m_3+\gamma_4 m_4, \\ \vec{P}=\vec{p}_1+\vec{p}_2=\vec{p}_3+\vec{p}_4=\gamma_1 m_1\vec{v}_1+\gamma_2 m_2\vec{v}_2=\gamma_3 m_3\vec{v}_3+\gamma_4 m_4\vec{v}_4=\gamma_c m_c\vec{v}_c=\vec{P}_c, \\ E_{ki}=\frac{\gamma_i m_i v^2{}_i}{1+\sqrt{1-v_i^2/c^2}}=\frac{p_i v_i}{1+\sqrt{1-v_i^2/c^2}}=\frac{p_i^2}{(1+\sqrt{1-v_i^2/c^2})\gamma_i m_i}=p_i u_i=\tilde{E}_i, \\ \vec{P}^2-\frac{(\sum E)^2}{c^2}=\vec{P}'^2-\frac{(\sum E')^2}{c^2}=\vec{P}''^2-\frac{(\sum E'')^2}{c^2}=inv.=-M^2c^2. \end{Bmatrix} \quad (4.8\text{-}5)$$

As we can see from all energy and momentum conservation relations valid for Classical-Mechanics cases (or for small velocities v << c), and for cases with relativistic velocities (4.8-5), it is relatively complicated to use such mathematical processing and find general solutions for elastic scatterings. The other opportunity is to use energy conservation in its differential form, since mathematically we will have the same and quite simple expressions, both for low and high-velocity (or for non-relativistic and relativistic) motions. Instead of kinetic energy, we can also start from total energy, as for instance:

$$\begin{aligned} & d\{E_{k1}+E_{k2}=E_{kr}+E_{kc}=E_{k3}+E_{k4}\} \Leftrightarrow d\{E_1+E_2=E_{kr}+E_{kc}+E_{oc}=E_3+E_4\} \Leftrightarrow \\ & \Leftrightarrow v_1dp_1+v_2dp_2=v_cdp_r+v_cdp_c=v_3dp_3+v_4dp_4, \\ & dE_{kr}=\vec{v}_rd\vec{p}_r=(\vec{v})_c^2 d(\gamma_c\mu_{r(1-2)})+\vec{p}_{r(1-2)}d\vec{v}_{r(1-2)}=(\vec{v})^2_{r(3-4)}d(\gamma_c\mu_{r(3-4)})+\vec{p}_{r(3-4)}d\vec{v}_{r(3-4)}, \\ & v^2_{r(1-2)}=(\vec{v}_1-\vec{v}_2)^2,\ v^2_{r(3-4)}=(\vec{v}_3-\vec{v}_4)^2, \\ & E_i=E_{0i}+E_{ki}, E_{0i}=const., dE_{0i}=0,\ p_i=\gamma_i m_i v_i\ ,\ \gamma_i=(1-v_i^2/c^2)^{-0.5},\ p_{ri}=\gamma_i\mu_i v_{ri}, \end{aligned} \tag{4.8-6}$$

Doing that way, we can address the forces acting in every phase of scattering,

$$\begin{aligned} & \{v_1dp_1+v_2dp_2=v_rdp_r+v_cdp_c=v_3dp_3+v_4dp_4\}/dt \Rightarrow \\ & \vec{v}_1\frac{d\vec{p}_1}{dt}+\vec{v}_2\frac{d\vec{p}_2}{dt}=\vec{v}_r\frac{d\vec{p}_r}{dt}+\vec{v}_c\frac{d\vec{p}_c}{dt}=\vec{v}_3\frac{d\vec{p}_3}{dt}+\vec{v}_4\frac{d\vec{p}_4}{dt} \Rightarrow \\ & \Rightarrow \vec{v}_1\vec{F}_1+\vec{v}_2\vec{F}_2=\vec{v}_r\vec{F}_r+\vec{v}_c\vec{F}_c=\vec{v}_3\vec{F}_3+\vec{v}_4\vec{F}_4. \end{aligned} \tag{4.8-7}$$

For instance, only mutual forces between moving masses $\mathbf{m_1}$ and $\mathbf{m_2}$, and $\mathbf{m_3}$ and $\mathbf{m_4}$, in their Center of Mass System (including Newton-Coulomb forces) will be:

$$\begin{aligned} & \vec{F}_{r(1-2)}=\frac{d\vec{p}_{r(1-2)}}{dt}=-\vec{F}_{r(2-1)}=\frac{d\vec{p}_{r(2-1)}}{dt}\ ,\ \vec{F}_{r(3-4)}=\frac{d\vec{p}_{r(3-4)}}{dt}=-\vec{F}_{r(4-3)}=\frac{d\vec{p}_{r(4-3)}}{dt}, \\ & E_{kr}=\int_{(1)}^{(2)}\vec{F}_{r(1\text{-}2)}d\vec{r}_{12}=\int_{(3)}^{(4)}\vec{F}_{r(3\text{-}4)}d\vec{r}_{34}. \end{aligned}$$

The force acting in the center of mass (analyzed from a Laboratory System) where the mass $\mathbf{m_c}$ is, will be,

$$\vec{F}_c=\frac{d\vec{p}_c}{dt}=\frac{d\vec{P}}{dt}=\vec{F}_1+\vec{F}_2.$$

And, of course, external forces acting on each particle (from the Laboratory System point of view), will be, respectively,

$$\vec{F}_1=\frac{d\vec{p}_1}{dt}, \vec{F}_2=\frac{d\vec{p}_2}{dt}, \vec{F}_3=\frac{d\vec{p}_3}{dt}, \vec{F}_4=\frac{d\vec{p}_4}{dt}.$$

From (4.8-5) - (4.8-7), we can see that the most important “transient-time interaction place” is the Center of Mass System, regarding evolving transformations of reduced mass $\boldsymbol{\mu_r}$ and relative velocity $\mathbf{v_r}$ is as follows,

$$\vec{v}_r \frac{d\vec{p}_r}{dt} = \vec{v}_r \vec{F}_r = (\vec{v})^2_{r(1-2)} \frac{d(\gamma_c \mu_{r(1-2)})}{dt} + \vec{p}_{r(1-2)} \frac{d\vec{v}_{r(1-2)}}{dt} =$$
$$= (\vec{v})^2_{r(3-4)} \frac{d(\gamma_c \mu_{r(3-4)})}{dt} + \vec{p}_{r(3-4)} \frac{d\vec{v}_{r(3-4)}}{dt} \Leftrightarrow$$
$$\Leftrightarrow \frac{dE_{kr}}{dt} = \vec{v}_r \vec{F}_r = (\vec{v})^2_{r(1-2)} \frac{d(\gamma_c \mu_{r(1-2)})}{dt} + \vec{p}_{r(1-2)} \vec{a}_{r(1-2)} = \tag{4.8-8}$$
$$= (\vec{v})^2_{r(3-4)} \frac{d(\gamma_c \mu_{r(3-4)})}{dt} + \vec{p}_{r(3-4)} \vec{a}_{r(3-4)}$$
$$\vec{p}_r = \gamma_c \mu_r \vec{v}_r \,,\; \vec{a}_{r(i-j)} = \frac{d\vec{v}_{r(i-j)}}{dt} \;(=) \text{acceleration} \;.$$

In reality, reduced mass μ_r is a kind of virtual object, or kind of de Broglie matter wave packet (placed in the space between interacting objects), and presents an evolving wave-energy group in process of transformation ($dE_{kr} = \vec{v}_r d\vec{p}_r = \vec{v}_r \vec{F}_r dt = c^2 d(\gamma_c \mu_r v_r) = h df_r = -d\tilde{E}_r$), created by mutually approaching objects, which has at least two different field or force components, $dE_{kr} = (\vec{v})^2_r d(\gamma_c \mu_r) + \vec{p}_r d\vec{v}_r$. Here is the part of the explanation why and how a single object (electron, or photon, etc.) coincidently passes two slits “making interference and diffraction with itself” on the opposite side of a diffraction plate, being also “energy-momentum” coupled with a diffraction plate, since all the participants here have a joint reduced-mass (without involving any of Orthodox Quantum Mechanical exotics). See more about two slit diffractions in Chapter 4.1. In addition, in cases when energy E_{kr} reaches certain threshold level, we could experience the creation of new particles (not initially entering relevant reaction), but such cases (for the time being) are outside presently analyzed elastic collisions.

It is important to notice that in cases of ideally plastic collisions, after the collision, when initial objects create only one (united) object, the energy E_{kr} will be there as an “injected”, transient matter-wave packet, or certain field energy absorbed by created particle. We will first consider the initial (input) energy in the close temporal and spatial vicinity, just before the act of collision. After the realized plastic collision, mentioned energy E_{kr} is becoming fully absorbed or injected into an internal rest mass, or state of a rest energy of the newly created (single) object, which remains (as a stable product) after the plastic collision (here represented as m_c). This should be kind of “<u>direct wave energy to mass transformation</u>” example ($\mu_r \to m_c$), which has not been elaborated from that point of view in traditional analyses of collisions.

[♣ COMMENTS & FREE-THINKING CORNER:

The relativity theory also implicates that, there is a simple relation of direct proportionality between any mass and total energy that could be produced by fully transforming that mass into radiation $E = mc^2, E_{tot.} = \gamma mc^2 = E_0 + E_k = E_0 + (\gamma - 1)mc^2 = \sqrt{E_0^2 + p^2c^2}$. *The most important conceptual understanding of frequency-dependent matter wave energy, which is fully equivalent to particle motional, or kinetic energy, is related to the fact that total narrow-band photon energy can be expressed as a product between Planck’s constant and a mean frequency of the photon wave packet,* $E_f = hf$. *Consequently (since there is known proportionality between mass and energy), the photon* *momentum was correctly found as* $p_f = hf / c = m_f c$ *(and proven being applicable and correct in analyzes of different interactions). Since a photon has certain energy, we should be able to present this energy in two different ways, like,* $E_f = hf = \sqrt{E_{0f}^2 + p_f^2 c^2} = p_f c = E_{kf}$. *The photon rest-mass equals zero, and there is only photon kinetic energy* $E_f = hf = p_f c = E_{kf} = m_f c^2$. *In number of applications and interactions (such as analyses of Compton and Photoelectric effects), this concept (and all mechanical equivalency relations for photon energy and momentum) proved to be fully correct. Going backward, we can apply the same conclusion, or analogy, to any real particle (which has a rest mass), accepting that particle kinetic energy is presentable as the product between Planck constant and narrow band, mean particle's matter wave frequency* $E_k = (\gamma - 1)mc^2 = \tilde{E} = hf$. *Doing that way, we can find the frequency of de Broglie matter waves as,* $f = E_k / h = (\gamma - 1)mc^2 / h = \tilde{E} / h$. *Now, we can find the phase velocity of matter waves as,* $u = \lambda f = \frac{h}{p} f = \frac{E_k}{p} = \frac{(\gamma - 1)mc^2}{\gamma mv} = \frac{v}{1 + \sqrt{1 - v^2/c^2}}$. *The relation between phase and group velocity of a matter-wave-packet is also known in the form,* $v = u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda}$. *By combining given forms of phase and group velocities we can get:* $v = u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda}$ = $u(1 + \sqrt{1 - v^2/c^2})$, *implicating validity of the following differential relations:* $d\tilde{E} = d\left[(\gamma - 1)mc^2\right] = mc^2 d\gamma$ = $hdf = vdp = d(pu)$, *and practically confirming mathematical correctness and consistency of all above introduced equivalency and analogy-based relations (named in this book as* ***PWDC*** *=* ***P****article-****W****ave* ***D****uality* ***C****ode). Since the above-given equivalency relations are found valid only if we use the wave packet model (as a replacement for a particle in motion), consequently, we have an argument more to say that matter waves (or wave functions) should exist as forms of harmonic, modulated sinusoidal signals (naturally satisfying the framework of Fourier signal and spectrum analysis).*

It is of essential importance to notice that a rest mass (or rest energy) does not belong to matter-wave energy (opposite to many current interpretations in modern physics books, regarding matter-wave properties). Analyzing the Compton Effect and many other elementary interactions known in Quantum Mechanics can easily prove this statement (that only kinetic or motional energy presents the matter wave energy), as follows later in this chapter. ♣]

All objects and energy states in the universe are always in some state of relative motion. For example, one of these objects could be where we are located, with our observatory or laboratory's coordinate system fixed to that position. The other object might be a large single body or a complex system that can be effectively represented as a mathematically defined equivalent position.

When selecting the most relevant or practical coordinate system or observer frame, consider the following options:

a) The coordinate system can be fixed to our location, meaning to one of the bodies in relative motion.

b) Alternatively, it can be anchored to the second body in the binary system we analyze.

c) The system could also be tied to a satellite orbiting one of the bodies within the binary system.

d) Another option is to place the observatory or coordinate system at the center of mass of the binary system, established mathematically.

These observer-related options should be spatially, temporally, and spectrally synchronized using relevant "space-time-velocity" dependent functions or Lorentz transformations, like the practices employed in modern GPS systems. Once a suitable space-time reference point or coordinate system is established, and the relationships between these reference systems are understood (i.e., we know the way they can be connected and synchronized), the analysis of two-body or other binary interactions becomes much more meaningful.

Here we could also address mechanical states of inertia regarding uniform and stable states of rest, linear, and/or rotational motions. Since intrinsic magnetic and spin moments of atoms and its internal elementary particles (like electrons, protons and neutrons), are mutually related to involved mechanical spinning moments (having gyromagnetic ratios that are usually constant numbers), and since all motions of involved objects are also relative, it is imaginable that mentioned mechanical states of inertia in a laboratory observer system, will still have measurable, resulting magnetic spin moments, and certain dominant, internal electric-dipoles polarization. From such point of view, we could extend the meaning of mechanical inertia to causally linked electromagnetic inertial states (see additional elaborations about states of inertia in Chapters 1, 2 and 10).

In practical cases involving observer systems and energy-moments interactions, it's crucial to identify the dominant front-end and last-end elements, as well as the energy sources and sinks of the process in question (as discussed at the end of Chapter 1). Physics literature often may overlook or inadequately address these front and last-end concepts. However, a complete and accurate understanding of any interaction requires addressing these questions, which relate to the closed-loops flow or currents of power, energy and moments, or field-charges.

4.2.2. Example 3: Elastic collision photon-electron (Compton Effect)

The principal objective of this exercise (or Compton Effect analysis) is to show that electrons (as particles) can be equally treated as matter waves, and photons as particles, and vice versa (like elaborated in Chapters 4.1 and 10. of this book). ***In addition, here it will be proven that the rest mass of a stable particle does not belong to its matter-wave energy, and that only motional or kinetic energy is equivalent to matter-waves energy (contrary to what we often find in Orthodox Quantum theory).***

Let us apply the energy and momentum conservation laws (T.4.2.1, T.4.3.1, T.4.4 and equations (4.2), (4.3), (4.8-3), (4.8-5)), in the case when a photon elastically collides with an electron, which is in a state of rest (see Fig.4.2).

A) ***First, we will treat all interaction participants as particles*** (see "T.4.0. Photon – Particle Analogies" in chapter 4.1).

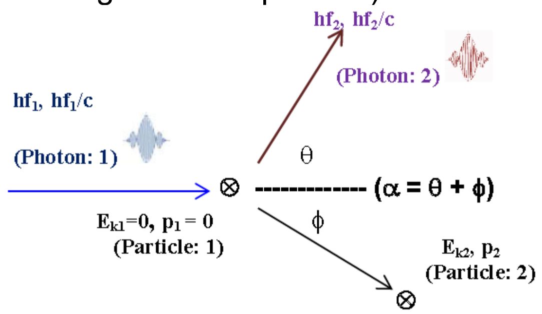


Fig. 4.2 Photon-Particle collision

After an elastic collision, the incident photon **(hf_1, hf_1/c)** loses a part of its energy, being transformed into a new photon **(hf_2, hf_2/c)**, and the electron that initially was in the state of rest (**$E_{k1}=0$, $p_1=0$**) gets certain kinetic energy (**E_{k2}, p_2**). All particle and photon states before collision will be marked using index "1" and after collision using index "2", as presented on Fig.4.2 and in table T.4.5.

T.4.5.

	States long before the collision (indexing: 1)	**States just after the elastic collision (indexing: 2)**
Photon wave energy & wave momentum (for waves in states 1 & 2)	$\tilde{E}_{f1} = hf_1$, $\left[\tilde{p}_{f1} = \frac{hf_1}{c}\right]$	$\tilde{E}_{f2} = hf_2$, $\left[\tilde{p}_{f2} = \frac{hf_2}{c}\right]$
Electron kinetic energy and linear momentum (for particles in states 1 & 2)	$E_{k1} = E_{e1} = 0$ $[p_1 = p_{e1} = 0]$	$E_{ke} = E_{k2} = \underline{(\gamma-1)m_e c^2 = h(f_1 - f_2) = h\Delta f = \tilde{E}_e = p_2 u_2}\Big\|_{v_e \ll c} \cong m_e v_e^2/2,$ $\left[\begin{matrix} \vec{p}_2 = \vec{p}_e = \tilde{p}_2 = \gamma m \vec{v}_2 = \frac{\overrightarrow{hf_1}}{c} - \frac{\overrightarrow{hf_2}}{c} = \underline{\vec{p}_e = \gamma m_e \vec{v}_e}\Big\|_{v_e \ll c} \cong m_e \vec{v}_e \\ (v_2 = v_e \cong 2u_2 = 2u_e) \ll c,\ \gamma = (1 - v_2^2/c^2)^{-0.5} \end{matrix}\right]$
Total motional energy (for electron and photon in states 1 & 2)	$hf_1 + E_{k1} = hf_1$	$hf_2 + E_{k2}$
Total energy, (only for an electron in states 1 & 2)	$E_1' = mc^2$	$E_2' = E_{e2} = \underline{m_e c^2 + hf_1 - hf_2 = \gamma m_e c^2 = E_{e-total}}\Big\|_{v_e \ll c} \cong$ $\cong m_e c^2 + m_e v_e^2/2$
Total energy & momentum (for electron & photon)	$mc^2 + hf_1$ $\left[\frac{\overrightarrow{hf_1}}{c} = \frac{hf_1}{c}\vec{e}_1\right]$	$\gamma mc^2 + hf_2$ $\left[\vec{p}_2 + \frac{\overrightarrow{hf_2}}{c} = \gamma m \vec{v}_2 + \frac{hf_2}{c}\vec{e}_2\right]$
Total energy and linear moments balance between states 1 & 2	$mc^2 + hf_1 = \gamma mc^2 + hf_2$, $hf_1 = hf_2 + E_{k2}$, $\left[\begin{matrix} \frac{hf_1}{c}\vec{e}_1 = m_2\vec{v}_2 + \frac{hf_2}{c}\vec{e}_2, \quad (v_2 = v = v_e, m_2 = m = m_e) \\ \frac{hf_1}{c} = m_2 v_2 \cos\Phi + \frac{hf_2}{c}\cos\theta,\ 0 = m_2 v_2 \sin\Phi + \frac{hf_2}{c}\sin\theta \end{matrix}\right]$	
Differential energy balance between states 1 & 2	$dE_{k1} + d\tilde{E}_{f1} = dE_{kc} + d\tilde{E}_{kr} = dE_{k2} + d\tilde{E}_{f2}$	

From relevant energy-momentum conservation relations in T.4.5 we can find:

$$\left.\begin{bmatrix} h^2(f_1^2 + f_2^2 - 2f_1f_2\cos\theta) = m^2v_2^2c^2 \\ h^2(f_1^2 + f_2^2 - 2f_1f_2) = \frac{1}{4}m^2v_2^4 \\ 2h(f_1 - f_2) = mv_2^2 = m_ev_e^2 \end{bmatrix}\right. \Rightarrow$$

$$2h^2f_1f_2(1-\cos\theta) = m^2v_2^2c^2(1-\frac{v_2^2}{4c^2})\Big|_{v_2<<c} \cong m^2v_2^2c^2 = 2hmc^2(f_1 - f_2) \Rightarrow \tag{4.8-9}$$

$$hf_1f_2(1-\cos\theta) \cong mc^2(f_1 - f_2) = mc^2\Delta f,\ \Delta f \cong \frac{mv_2^2}{2h} = \frac{h}{mc}\frac{f_1f_2}{c}(1-\cos\theta) = \lambda_c\frac{f_1f_2}{c}(1-\cos\theta).$$

B) Let us now consider that ***all interaction participants can be equally treated as matter-waves***, by considering wavelengths of all interaction participants, including de Broglie matter-wave wavelength of the moving electron (analogically, on the same way as summarized in "T.4.0. Photon – Particle Analogies" in chapter 4.1) as,

$$\left\{\begin{array}{l} (\text{Incident photon}), \lambda_1 = h/p_{f1} = h/(\frac{hf_1}{c}) = \frac{c}{f_1}, \\ (\text{Scattered or diffused photon}), \lambda_2 = h/p_{f2} = h/(\frac{hf_2}{c}) = \frac{c}{f_2}, \\ (\text{Moving electron}), \lambda_e = h/p_2 = h/m_2v_2 = h/m_ev_e, \\ (\text{Compton wavelength}), \lambda_c = h/mc = h/m_ec = h/m_ev_e, \\ hf_1f_2(1-\cos\theta) \cong mc^2(f_1 - f_2) = mc^2\Delta f,\ f_c = c/\lambda_c,\ \lambda_c \cdot f_c = c, \\ hf_1 = hf_2 + hf_e \cong hf_2 + \frac{mv_2^2}{2}, \tilde{E}_e = E_{k2} \cong \frac{mv_2^2}{2} = hf_e = h(f_1 - f_2) = h\Delta f \\ \boxed{u = u_2 = u_e = \lambda_e f_e \cong v_e/2},\ u_e = \frac{\omega_e}{k_e} = \frac{E_{ke}}{p_e},\ \omega_e = 2\pi f_e,\ k_e = \frac{2\pi}{\lambda_e}, \\ v_e = u_e - \lambda_e\frac{du_e}{d\lambda_e} = -\lambda_e^2\frac{df_e}{d\lambda_e} = \frac{d\omega_e}{dk_e} = \frac{dE_{ke}}{dp_e}. \end{array}\right\} \Rightarrow \tag{4.8-10}$$

From previous results, we can easily prove the validity of the following relations,

$$h\frac{c^2}{\lambda_1\lambda_2}(1-\cos\theta) \cong mc^2(\frac{c}{\lambda_1} - \frac{c}{\lambda_2}) = mc^3\frac{\lambda_2 - \lambda_1}{\lambda_1\lambda_2} = mc^3\frac{\Delta\lambda}{\lambda_1\lambda_2} \Rightarrow$$

$$\Delta\lambda = \lambda_2 - \lambda_1 \cong \frac{h}{mc}(1-\cos\theta) = \lambda_c(1-\cos\theta) = \frac{c\Delta f}{f_1f_2} = c\frac{f_1 - f_2}{f_1f_2} = c\frac{f_e}{f_1f_2}. \tag{4.8-11}$$

Experimentally measured results for $\Delta\lambda$ are equal to calculated results using the formula,

$$\boxed{\Delta\lambda = \lambda_2 - \lambda_1 \cong \lambda_c(1-\cos\theta) = c\frac{\Delta f}{f_1f_2} = c\frac{mv_2^2}{2hf_1f_2},}$$

and this is exactly confirming (theoretically and experimentally) that we can analyze the same Compton Effect, or elastic scattering situation both as interaction of particles or matter-wave states (including mixed particle-waves states), where ***only***

kinetic or motional energy and linear moment of any interaction participant, at the same time is presenting its matter-wave energy and has wave properties such as wavelength and frequency. After the impact between incident photon and an electron, the electron will get certain velocity and kinetic energy (like in impacts between two particles), and here, we just proved that electron could also be treated as an "***electron matter-wave packet***", having certain mean wavelength and frequency, as for example,

$$\boxed{E_{k2} \cong mv_2^2/2 = hf_e = \tilde{E}_e , u_2 = u_e = \lambda_e f_e \cong v_e/2 , (v_e, u_e) << c , \lambda_e = h/mv_e}. \qquad (4.8\text{-}12)$$

Effectively, it has been proven that in the case of Compton Effect ***photon can be treated as a particle, and an electron treated as a matter wave (and vice versa)***, satisfying and respecting all **PWDC** relations (in both directions) as described in Chapters 4.1 and 10. of this book. In this case, relevant **PWDC** relations are given by (4.8-13) and/or as formulated in Chapter 10., by (10.1),

$$\begin{aligned} & u_e = \frac{\omega_e}{k_e} = \frac{E_{ke}}{p_e} = \lambda_e f_e, \ \omega_e = 2\pi f_e , \ k_e = \frac{2\pi}{\lambda_e}, \lambda_e = h/mv_e \\ & v_e = u_e - \lambda_e \frac{du_e}{d\lambda_e} = -\lambda_e^2 \frac{df_e}{d\lambda_e} = \frac{d\omega_e}{dk_e} = \frac{dE_{ke}}{dp_e}, E_{ke} = \tilde{E}_e = hf_e , \end{aligned} \qquad (4.8\text{-}13)$$

and we can verify that, if (4.8-13) are satisfied, we will get results as in (4.8-12).

We can also exercise (just to satisfy our curiosity and to prove what is correct) that a rest-energy, or total electron energy are creating electron matter wave (what is the contemporary and wrong consideration often practiced in the official Quantum Theory), and we will find that results for both options are either not realistic, being mutually contradictory, or mathematically illogical, as follows,

$$\begin{aligned} & \left[hf_e = \tilde{E}_e = m_e c^2, \ \lambda_e = h/\gamma m_e v_e, f_e = m_e c^2/h, \ u_e = \lambda_e f_e = c^2/\gamma v_e, \ v_e \le c\right] \Rightarrow \ u_e >> c ... \\ & \left[hf_e = \tilde{E}_e = \gamma m_e c^2, \ \lambda_e = h/\gamma m_e v_e, f_e = \gamma m_e c^2/h, \ u_e = \lambda_e f_e = \left(c^2/v_e\right) > c\right] \Rightarrow v_e \le c \Rightarrow u_e \ge c , \end{aligned} \qquad (4.8\text{-}14)$$

We can find that there is not an agreement and mutual compatibility of (4.8-14) with basic **PWDC** relations (4.8-13), that are valid for any group and phase velocity of matter-waves. Consequently, we can draw the conclusion that a rest mass or rest energy not at all belongs to matter-waves energy. Total particle energy is also not equal to corresponding matter wave energy. ***What remains is that only kinetic or motional energy entirely belongs to, and/or presents matter wave energy***, as we can see in (4.8-12) and (4.8-13). Of course, a photon is an exception, because its total, wave and motional energy are mutually the same, since it has zero rest energy (or zero rest mass).

It is also obvious that probability and statistics still have no place in here elaborated analysis of Compton Effect. Of course, later we can show that innovated, ***deterministic electron matter wavefunction is related only to "motional electron-wave power" and to its kinetic energy*** (see more about here favored, natural wavefunctions in Chapters 4.0, 4.3 and 10.).

C) Let us now make a kind of mathematical experiment. We will address the electron and photon’s total energy, including its kinetic or motional energy and momentum, using relativistic momentum-energy 4-vectors. Since this is an almost elastic scattering (or we are approximately considering it as a “totally elastic impact”), the rest energy or rest mass of the system in states, 1 & 2, (before and after scattering) should be approximately the same.

$$\left\{\left[P_4^{(1)} = (\frac{\overrightarrow{hf_1}}{c}, \frac{mc^2 + hf_1 - W_1}{c})\right] (\Leftrightarrow) \left[P_4^{(2)} = (\frac{\overrightarrow{hf_2}}{c} + \vec{p}_e, \frac{\gamma mc^2 + hf_2 + W_2}{c})\right]\right\} \Rightarrow$$

$$\left\{\begin{array}{l} \left(\frac{\overrightarrow{hf_1}}{c}\right)^2 - \left(\frac{mc^2 + hf_1 - W_1}{c}\right)^2 = -\left(\frac{mc^2 - W_1}{c}\right)^2 \\ \left(\frac{\overrightarrow{hf_2}}{c} + \vec{p}_e\right)^2 - \left(\frac{\gamma mc^2 + hf_2 + W_2}{c}\right)^2 = -\left(\frac{mc^2 + W_2}{c}\right)^2 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \left(\overrightarrow{hf_1}\right)^2 - \left(mc^2 + hf_1 - W_1\right)^2 = -\left(mc^2 - W_1\right)^2 \\ \left(\overrightarrow{hf_2} + c\vec{p}_e\right)^2 - \left(\gamma mc^2 + hf_2 + W_2\right)^2 = -\left(mc^2 + W_2\right)^2 \end{array}\right\} \Rightarrow \quad (4.8\text{-}15)$$

$$\left\{\begin{array}{l} W_1 = mc^2, W_2 = -mc^2 \\ hf_1 = \gamma mc^2 + hf_2 + W_2 = (\gamma - 1)mc^2 + hf_2 = E_{ke} + hf_2 \end{array}\right\} \Rightarrow$$

$$\left\{\left[P_4^{(1)} = (\frac{\overrightarrow{hf_1}}{c}, \frac{mc^2 + hf_1 - mc^2}{c})\right] (\Leftrightarrow) \left[P_4^{(2)} = (\frac{\overrightarrow{hf_2}}{c} + \vec{p}_e, \frac{\gamma mc^2 + hf_2 - mc^2}{c})\right]\right\}$$

Here, W_1, W_2 should be relevant rest energies in the states 1 & 2, introduced to create mutually equivalent 4-vectors. As we can see later, ***equations of Compton Effect could also describe Photoelectric Effect***, since in case of ideal elastic collisions we have $hf_1 = (\gamma - 1)mc^2 + hf_2 = E_{ke} + hf_2$, but if incident photon is fully captured (or absorbed) by an atom, we will have $hf_2 \to 0$, and only an electron will be expulsed, what presents Photoelectric effect situation, which is much closer to a plastic collision case. Consequently, we will have new energy and moments balance (between involved particles, and matter waves), such as, $hf_1 = E_{ke} + hf_2 + W*$, where $W*$ is an internally captured, coupling, or absorbed energy (inside atoms), what is equivalent to M. Maric and A. Einstein explanation of the Photoelectric effect. On a similar way, we could extend the same analysis to explain a “pair of electron-positron creation” (and later annihilation) based on sufficiently high-energy incident photon interaction with an atom. ***Analyses elaborated in this chapter are selectively addressed, or conveniently decomposed, to show that typical moving-particles could be dualistically presented as equivalent matter-waves, and that typical waves have properties of moving-particles*** (and this way giving the theoretical and experimental support to Wave-Particles Duality concepts, in this book summarized as **PWDC**).

D) Let us now exploit ***classical (Minkowski-Einstein) 4-vector, relativistic invariance relations*** applied to an electron involved in the Compton Effect scattering, and we will again get the same results as in the case under A).

$$\left\{\begin{array}{l}\left[P_{4e}^{(1)}=(\vec{0},\frac{mc^2}{c})\right](\Leftrightarrow)\left[P_{4e}^{(2)}=(\vec{p}_e,\frac{\gamma mc^2}{c})\right],\\ \underline{\gamma m_e c^2=hf_1-hf_2+m_e c^2=E_{e-total}}\Big|_{v_e<<c}\cong m_e c^2+\frac{m_e v_e^2}{2},\\ \vec{p}_e=\vec{p}_2=\vec{p}_e=\tilde{p}_2=\gamma m\vec{v}_2=\frac{\overrightarrow{hf_1}}{c}-\frac{\overrightarrow{hf_2}}{c}=\underline{\gamma m_e\vec{v}_e}\Big|_{v_e<<c}\cong m_e\vec{v}_e=m\vec{v}_2\end{array}\right\}\Rightarrow$$

$$(\vec{p}_e)^2-\left(\frac{\gamma mc^2}{c}\right)^2=-\left(\frac{mc^2}{c}\right)^2\Rightarrow\left(\gamma mc^2\right)^2-\left(\vec{p}_e\right)^2c^2=m^2c^4\Rightarrow$$

$$(hf_1-hf_2+mc^2)^2-(\frac{hf_1}{c}\vec{e}_1-\frac{hf_2}{c}\vec{e}_2)^2c^2=m^2c^4\Rightarrow$$

$$h^2f_1f_2(1-\cos\theta)=h(f_1-f_2)mc^2=h\cdot\Delta f\cdot mc^2\Rightarrow$$

$$\frac{h}{mc}\frac{f_1f_2}{c}(1-\cos\theta)=\lambda_c\frac{f_1f_2}{c}(1-\cos\theta)=f_1-f_2=\Delta f\cong\frac{mv_e^2}{2h},$$

$$\lambda_1=c/f_1,\lambda_2=c/f_2,x=\frac{hf_1}{mc^2}=\frac{\lambda_c}{\lambda_1},$$

$$\frac{\lambda_2-\lambda_1}{c}=\frac{\Delta\lambda}{c}=\frac{1}{f_2}-\frac{1}{f_1}=\frac{f_1-f_2}{f_1f_2}=\frac{\Delta f}{f_1f_2}=\frac{h}{mc^2}(1-\cos\theta)=\frac{\lambda_c}{c}(1-\cos\theta),\qquad(4.8\text{-}16)$$

$$hf_2=\frac{hf_1}{1+x(1-\cos\theta)},\lambda_2=\lambda_1\left[1+x(1-\cos\theta)\right],\frac{f_1}{f_2}=\frac{\lambda_2}{\lambda_1}=1+x(1-\cos\theta),$$

and finally, we can again find the same wavelength and frequency differences (between incident and diffused photons), as before in (4.8-11),

$$\begin{array}{l}\Delta\lambda=\lambda_2-\lambda_1=\frac{h}{mc}(1-\cos\theta)=\lambda_c(1-\cos\theta)=c\frac{\Delta f}{f_1f_2},\\ \Delta f=f_1-f_2=\frac{f_1f_2}{c}\lambda_c(1-\cos\theta)=\frac{f_1f_2}{c}\Delta\lambda.\end{array}\qquad(4.8\text{-}17)$$

For instance, if an incident photon is totally reflected (back to its source, meaning, $\theta=\pi,\cos\theta=-1$), we can find theoretically maximal values for the same wavelength and frequency differences,

$$(\Delta\lambda)_{max.}=\frac{2h}{mc}=2\lambda_c\ ,\ (\Delta f)_{max.}=\frac{2hf_1f_2}{mc^2}=2\frac{\lambda_c}{c}f_1f_2=2\frac{f_1f_2}{f_c}\cong\frac{mv_{e-max.}^2}{2h}.\qquad(4.8\text{-}18)$$

We can again show that a moving electron, after an elastic impact with a photon, can equally be treated as a particle, or as an equivalent matter-wave packet (analogically, in the same way as summarized under A) and in “T.4.0. Photon – Particle Analogies" in chapter 4.1), as for example,

$$\left\{\begin{array}{l} E_{k2} = (\gamma-1)mc^2 = hf_1 - hf_2 = hf_1 \dfrac{x(1-\cos\theta)}{1+x(1-\cos\theta)} = hf_1 \dfrac{2x\cdot\cos^2\phi}{(1+x)^2 - x^2\cdot\cos^2\phi} = \\ = p_2u_2 = \dfrac{p_2v_2}{1+\sqrt{1-v_2^2/c^2}} = \left.\dfrac{m_2v_2^2}{(1+\sqrt{1-v_2^2/c^2})\sqrt{1-v_2^2/c^2}}\right|_{v_2<<c} \cong \dfrac{m_2v_2^2}{2} = \dfrac{m_2v_e^2}{2} = E_{ke} = \tilde{E}_e = hf_e \end{array}\right\} \Rightarrow$$

$$f_e = \frac{m_2v_2^2}{2h},\ \lambda_e = \frac{h}{p_e} = \frac{h}{m_2v_2},\ u_2 = u_e = \lambda_e f_e = \frac{v_2}{2} = \frac{v_e}{2},\ hf_e = \frac{m_2v_2^2}{2} = E_{k2} = E_{ke} = \tilde{E}_e = h(f_1 - f_2) = h\Delta f, \tag{4.8-19}$$

$$\left(\cos\theta = -1, \theta = \pi\right) \Rightarrow (E_{k2})_{max.} = hf_1 \frac{2x}{1+2x}.$$

We also know (from different experimental results) that photons and electrons are manifesting typically wave properties in cases of interference, superposition, wave scattering, and refractions, and here we simply presented how photons and electrons could be treated as particles and/or waves, supporting and proving the intrinsic wave-particle duality of matter in motion (based on **PVDC**).

[♣ Free thinking corner ---- (this presentation, below, is in process of evolving)

E) Usual analyses of the Compton Effect do not consider any ***wave energy or wave momentum being present in the transition zone of the collision process***. The objective of this example is to show that there is certain (hidden) phenomenology, which deals with involved fields and forces, here causally related to the wave energy and wave momentum, existing in a close temporal and spatial vicinity of a collision event. Let us first apply conservation laws (4.5), (4.8-3) and (4.8-5), to get all important energy members,

$$\begin{array}{l} E_{tot.} = hf_1 + mc^2 = E_{0C} + E_{kc} + E_{kr} = hf_2 + \gamma mc^2, \\ hf_1 = E_{kc} + E_{kr} = hf_2 + (\gamma-1)mc^2 = hf_2 + E_{k2}, \\ E_{0C} = m_c c^2,\ E_c = E_{0C} + E_{kc} = \gamma_c m_c c^2, \\ \tilde{p}_{f1} = \dfrac{hf_1}{c}\vec{e}_1 = \dfrac{hf_2}{c}\vec{e}_2 + \vec{p}_2,\ |\vec{e}_1| = |\vec{e}_2| = 1 \end{array}$$

$$\vec{v}_c = \frac{(\sum\vec{p})c^2}{\sum E} = \frac{(\frac{hf_1}{c}\vec{e}_1)c^2}{hf_1 + mc^2} = \frac{\frac{hf_1}{c}\vec{e}_1}{m + \frac{hf_1}{c^2}} = \frac{c\vec{e}_1}{1+\frac{mc^2}{hf_1}} = \frac{\frac{hf_1}{mc^2}}{1+\frac{hf_1}{mc^2}}c\vec{e}_1 = \frac{\vec{v}_2 + \frac{hf_2}{\gamma mc^2}c\vec{e}_2}{1+\frac{hf_2}{\gamma mc^2}} \Rightarrow$$

$$\Rightarrow \cos\alpha = \frac{\left[\dfrac{hf_1(1+\frac{hf_2}{\gamma mc^2})}{hf_1 + mc^2}\right]^2 - \left(\dfrac{hf_2}{\gamma mc^2}\right)^2 - (\frac{v_2}{c})^2}{2\dfrac{v_2}{c}\dfrac{hf_2}{\gamma mc^2}} \tag{4.8-20}$$

$$\gamma_c m_c = \frac{\sum E}{c^2} = \frac{hf_1 + mc^2}{c^2} = m + \frac{hf_1}{c^2} = \frac{hf_2 + \gamma mc^2}{c^2} = \gamma m + \frac{hf_2}{c^2},$$

$$m_c = m\sqrt{1 + 2\frac{hf_1}{mc^2}} = m\left[1 + \frac{hf_2}{mc^2}\sqrt{1 - v_2^2/c^2}\right]\sqrt{\frac{1 - v_c^2/c^2}{1 - v_2^2/c^2}}, \quad E_{oc} = m_c c^2$$

$$\gamma_c = \frac{1}{\sqrt{1 - \frac{v_c^2}{c^2}}} = \frac{1 + \frac{hf_1}{mc^2}}{\sqrt{1 + 2\frac{hf_1}{mc^2}}} = \frac{1+x}{\sqrt{1+2x}}, \gamma = \frac{1}{\sqrt{1 - \frac{v_2^2}{c^2}}}$$

$$\vec{p}_c = (\sum \vec{p}) = \frac{\sum E}{c^2}\vec{v}_c = (m + \frac{hf_1}{c^2})\vec{v}_c = \gamma_c m_c \vec{v}_c =$$
$$= \frac{\gamma mc^2 + hf_2}{c^2}\vec{v}_c = \gamma m(\vec{v}_2 + \frac{hf_2}{\gamma mc^2}c\vec{e}_2) = \vec{p}_2 + \frac{hf_2}{c}\vec{e}_2 = \frac{hf_1}{c}\vec{e}_1 = \tilde{p}_{f1},$$
$$p_2 = p_e = \frac{1}{c}\sqrt{E_{k2}(E_{k2} + 2mc^2)} = \frac{1}{c}\sqrt{E_{k2}(E_{k2} + \frac{2hf_1}{x})} =$$
$$= \frac{2hf_1}{c}\frac{(1+x)\cos\phi}{(1+x)^2 - x^2\cos^2\phi}, x = \frac{hf_1}{mc^2}. \tag{4.8-20}$$

$$E_{kc} = (\gamma_c - 1)m_c c^2 = hf_1 - E_{kr} = \frac{p_c v_c}{1 + \sqrt{1 - v_c^2/c^2}} = \frac{p_c^2}{\gamma_c m_c(1 + \sqrt{1 - v_c^2/c^2})} =$$

$$= hf_1\frac{\frac{hf_1}{mc^2}}{1 + \frac{hf_1}{mc^2} + \sqrt{1 + 2\frac{hf_1}{mc^2}}} = \left[1 + \frac{hf_1}{mc^2} - \sqrt{1 + 2\frac{hf_1}{mc^2}}\right]mc^2 = p_c u_c = hf_c =$$

$$= \frac{(\frac{hf_2}{c}\vec{e}_2 + \gamma m\vec{v}_2)^2}{(1 + \sqrt{1 - v_c^2/c^2})(\gamma m + \frac{hf_2}{c^2})} = (\frac{\gamma mc^2}{1 + \sqrt{1 - v_c^2/c^2}})\frac{(\frac{hf_2}{\gamma mc^2})^2 + \frac{v_2^2}{c^2} + 2\frac{v_2}{c}(\frac{hf_2}{\gamma mc^2})\cos\alpha}{1 + \frac{hf_2}{\gamma mc^2}}$$

$$E_{kc(1)} = E_{kc(2)} \Rightarrow$$
$$\left\{ m\sqrt{1 + 2\frac{hf_1}{mc^2}}(\frac{\frac{hf_1}{mc^2}}{1 + \frac{hf_1}{mc^2}}c\vec{e}_1)^2 \cong m\left[1 + \frac{hf_2}{mc^2}\sqrt{1 - v_2^2/c^2}\right]\sqrt{\frac{1 - v_c^2/c^2}{1 - v_2^2/c^2}}(\frac{\vec{v}_2 + \frac{hf_2}{\gamma mc^2}c\vec{e}_2}{1 + \frac{hf_2}{\gamma mc^2}})^2 \right\} \Leftrightarrow$$
$$\Leftrightarrow \left\{ \sqrt{1 + 2\frac{hf_1}{mc^2}}(\frac{\frac{hf_1}{mc^2}}{1 + \frac{hf_1}{mc^2}}c\vec{e}_1)^2 \cong \left[1 + \frac{hf_2}{mc^2}\sqrt{1 - v_2^2/c^2}\right]\sqrt{\frac{1 - v_c^2/c^2}{1 - v_2^2/c^2}}(\frac{\vec{v}_2 + \frac{hf_2}{\gamma mc^2}c\vec{e}_2}{1 + \frac{hf_2}{\gamma mc^2}})^2 \right\} \tag{4.8-21}$$

$$E_{kr} = hf_1 - E_{kc} = hf_2 + E_{k2} - E_{kc} = \frac{p_r v_r}{1+\sqrt{1-v_c{}^2/c^2}} = hf_r =$$

$$= \frac{p_r^2}{\gamma_c \mu_r (1+\sqrt{1-v_c{}^2/c^2})} = p_r u_r = \frac{m(\frac{hf_1}{c^2})}{m+\frac{hf_1}{c^2}}\left[\frac{(\vec{0}-\vec{c})^2}{1+\sqrt{1-c^2/c^2}}\right] = mc^2\frac{\frac{hf_1}{mc^2}}{1+\frac{hf_1}{mc^2}} =$$

$$= \frac{\gamma m(\frac{hf_2}{c^2})}{\gamma m+\frac{hf_2}{c^2}}\left[\frac{(\vec{v}_2-\vec{c})^2}{1+\sqrt{1-v_c^2/c^2}}\right] = (\frac{mc^2}{1+\sqrt{1-v_c^2/c^2}})\frac{\frac{hf_2}{mc^2}(1+\frac{v_2^2}{c^2}-2\frac{v_2}{c}\cos\alpha)}{1+\frac{hf_2}{\gamma mc^2}} =$$

$$= hf_2 + \frac{p_2 v_2}{1+\sqrt{1-v_2{}^2/c^2}} - (\frac{\gamma mc^2}{1+\sqrt{1-v_c^2/c^2}})\frac{(\frac{hf_2}{\gamma mc^2})^2+\frac{v_2^2}{c^2}+2\frac{v_2}{c}(\frac{hf_2}{\gamma mc^2})\cos\alpha}{1+\frac{hf_2}{\gamma mc^2}},$$

$$v_r^2 = v_{r1}^2 = (0-\vec{c})^2 = c^2,\ v_{r2}^2 = (\vec{v}_2-\vec{c})^2,$$

$$E_{kr(1)} = E_{kr(2)} \Rightarrow \left\{\frac{m\frac{hf_1}{c^2}}{m+\frac{hf_1}{c^2}}(0-c)^2 \cong \frac{\gamma m\frac{hf_2}{c^2}}{\gamma m+\frac{hf_2}{c^2}}\left[\frac{(\vec{v}_2-\vec{c})^2}{1+\sqrt{1-(\vec{v}_2-\vec{c})^2/c^2}}\right]\right\} \Leftrightarrow \qquad (4.8\text{-}22)$$

$$\Leftrightarrow \left\{\begin{array}{l} v_2 << c \Rightarrow \frac{f_1}{\left(1+\frac{hf_1}{mc^2}\right)} \cong \frac{f_2}{\left(1+\frac{hf_2}{mc^2}\right)}, \\ v_2\begin{pmatrix}\approx\\ \leq\end{pmatrix}c \Rightarrow \frac{f_1}{\left(1+\frac{hf_1}{mc^2}\right)} \cong \frac{f_2}{\left(1+\frac{hf_2}{\gamma mc^2}\right)}\left[\frac{(\vec{v}_2-\vec{c})^2}{2c^2}\right] \cong \frac{1}{2}f_2\frac{(\vec{c}-\vec{v}_2)^2}{c^2} \end{array}\right\}$$

$$\alpha = \theta+\phi = \angle(\vec{v}_2,\vec{e}_2),\ \gamma = (1-v_2^2/c^2)^{-0.5},\ \gamma_c = (1-v_c^2/c^2)^{-0.5}.$$

Now, we are in the position to find relevant wave elements of the electron after its elastic scattering with a photon, and ***we see that only the electron's kinetic energy presents its wave-energy (meaning that the electron rest mass or state of rest energy is not a part of matter-wave energy)***. We shall also find that when electron and incident photon get close enough, then their local Center of Mass System becomes a dominant place where de Broglie matter waves would be "players of greater importance" for the final products or results of an interaction.

$$E_{k2} = hf_1 - hf_2 = h(f_1-f_2) = \frac{p_2 v_2}{1+\sqrt{1-v_2^2/c^2}} = (\gamma_2-1)mc^2 = hf_e = p_2 u_2 =$$

$$= hf_1\frac{(hf_1/mc^2)(1-\cos\theta)}{1+(hf_1/mc^2)(1-\cos\theta)} = hf_1\frac{x(1-\cos\theta)}{1+x(1-\cos\theta)} = E_{ke} \Rightarrow$$

$$\lambda_e = \frac{h}{p_2} = \frac{v_2}{(1+\sqrt{1-v_2^2/c^2})(f_1 - f_2)} = \frac{\lambda_1\lambda_2}{\lambda_2 - \lambda_1}\left[\frac{v_2/c}{1+\sqrt{1-v_2^2/c^2}}\right],$$
$$f_e = \frac{E_{k2}}{h} = \frac{(\gamma_2 - 1)mc^2}{h} = \frac{p_2 v_2}{h(1+\sqrt{1-v_2^2/c^2})} = f_1 - f_2 = \Delta f, \tag{4.8-23}$$
$$u_2 = \lambda_e f_e = u_e = \frac{v_2}{1+\sqrt{1-v_2^2/c^2}} = \frac{E_{k2}}{p_2} =$$
$$= \frac{\lambda_1\lambda_2}{\lambda_2-\lambda_1}(f_1 - f_2)\left[\frac{v_2/c}{1+\sqrt{1-v_2^2/c^2}}\right] \cong \frac{\overline{\lambda}^2}{\Delta\lambda}\Delta f\left[\frac{v_2/c}{1+\sqrt{1-v_2^2/c^2}}\right] \Rightarrow \overline{\lambda}^2\frac{\Delta f}{\Delta\lambda} \cong c.$$

$$E_{kr} = hf_1 - E_{kc} = hf_1 - hf_c = hf_2 + E_{k2} - E_{kc} = hf_2 + hf_e - hf_c = hf_r = mc^2\frac{\frac{hf_1}{mc^2}}{1+\frac{hf_1}{mc^2}}$$

$$f_r = f_1 - f_c = f_2 + f_e - f_c = \left(\frac{mc^2}{h}\right)\frac{\frac{hf_1}{mc^2}}{1+\frac{hf_1}{mc^2}} = \frac{f_1}{1+\frac{hf_1}{mc^2}},$$

$$f_e = f_r + f_c - f_2 = \left(\frac{mc^2}{h}\right)\frac{\frac{hf_1}{mc^2}}{1+\frac{hf_1}{mc^2}} + f_c - f_2 = \frac{f_1}{1+\frac{hf_1}{mc^2}} + f_c - f_2 = f_1 - f_2, \tag{4.8-24}$$
$$(mc^2 >> hf_1) \Rightarrow f_r \cong f_1,\ (mc^2 << hf_1) \Rightarrow f_r \cong \frac{mc^2}{h}$$
$$E_{kc} = hf_c = hf_1 - E_{kr} = hf_1 - hf_r = \left[1+\frac{hf_1}{mc^2} - \sqrt{1+2\frac{hf_1}{mc^2}}\right]mc^2 = \left[1+x-\sqrt{1+2x}\right]mc^2,$$
$$f_c = f_1 - f_r = f_1 - \left(\frac{mc^2}{h}\right)\frac{\frac{hf_1}{mc^2}}{1+\frac{hf_1}{mc^2}} = \frac{mc^2}{h}\left[1+\frac{hf_1}{mc^2} - \sqrt{1+2\frac{hf_1}{mc^2}}\right] = f_1\frac{\frac{hf_1}{mc^2}}{1+\frac{hf_1}{mc^2}} = f_r\frac{hf_1}{mc^2},$$
$$(mc^2 >> hf_1) \Rightarrow f_c \cong f_1 \cong f_r\ ;\ (mc^2 << hf_1) \Rightarrow f_c \cong f_1 - \frac{mc^2}{h} \cong f_1 - f_r\ .$$

The virtual objects μ_r and m_c in the Center of Mass System, applying the same logic regarding motional energy of de Broglie matter waves, should have characteristic frequencies f_r and f_c. Since this is the case of an elastic collision, μ_r and m_c will eventually separate into a moving electron, γm, and a scattered photon, hf_2/c^2 (having characteristic frequencies f_e and f_2). Wavelengths, frequencies and phase and group velocities of such virtual objects (μ_r and m_c) vary (during a transitory phase of interaction), before they get stable and final frequency values f_e and f_2. Consequently, f_e and f_2 are causally related, or proportional to frequencies f_r and f_c (since what we know and calculate as f_e and f_2 are only their final values when interaction is completed). To continue the analysis of this situation, we can (mathematically) test several possibilities, such as:

$f_e = f_r$, $f_2 = f_c$, or $f_e = a \cdot f_r$, $f_2 = b \cdot f_c$, or $f_e = \alpha(f_r), f_2 = \beta(f_c)$,...

Practically, we assume that the Center of Mass (in a sufficiently close space-time vicinity of the impact) will become the “laboratory" where the electron's de Broglie matter wave frequency, f_e , and the frequency of the scattered photon, f_2, will be synthesized (from $\mathbf{f_r}$ and $\mathbf{f_c}$), generating the following results:

$$\{f_e = f_r ,\ f_2 = f_c\} \Rightarrow \left\{ \begin{matrix} \frac{f_c}{f_r} = \frac{hf_1}{mc^2} = \frac{f_c}{f_e} \\ \frac{f_e}{f_r} = 1 + \frac{f_c}{f_r} - \frac{f_2}{f_r} = \frac{f_1}{f_r} - \frac{f_2}{f_r} = 1 \end{matrix} \right\},$$

$$f_r = f_e = f_1 - f_2 = \frac{mc^2}{h}\left[\sqrt{1 + 2\frac{hf_1}{mc^2}} - 1\right] = \frac{\lambda_2 - \lambda_1}{\lambda_1\lambda_2}c = \frac{\lambda_c}{c}f_1 f_2 = \frac{hf_1 f_2}{mc^2},$$
$$f_c = f_2 = f_1 - f_r = f_1 - f_e = f_1 - \frac{mc^2}{h}\left[\sqrt{1 + 2\frac{hf_1}{mc^2}} - 1\right] = f_1(1 - \frac{hf_1}{mc^2}),$$

$$\lambda_c = \frac{h}{p_c} = \frac{c}{f_1} = \lambda_1 = \lambda_2 - \lambda_c(1 - \cos\theta),$$
$$u_c = f_c\lambda_c = c\left\{1 - \frac{mc^2}{hf_1}\left[\sqrt{1 + 2\frac{hf_1}{mc^2}} - 1\right]\right\} = c(1 - \frac{hf_2}{mc^2}),$$
$$\lambda_r = \frac{h}{p_r} = \lambda_1 + \frac{h}{mc} = \lambda_1 + \lambda_c = \lambda_c + \lambda_c ,\ (p_r = \frac{\frac{hf_1}{c}}{1 + \frac{hf_1}{mc^2}}),$$
$$u_r = f_r\lambda_r = \frac{mc^2}{h}\left[\sqrt{1 + 2\frac{hf_1}{mc^2}} - 1\right](\lambda_1 + \frac{h}{mc}) = c\left[\sqrt{1 + 2\frac{hf_1}{mc^2}} - 1\right](1 + \frac{mc^2}{hf_1}) =$$
$$= c\left[\sqrt{1 + 2x} - 1\right](1 + x) = c\frac{hf_2}{mc^2}(\frac{hf_1}{mc^2} - 1) = c\frac{hf_2}{mc^2}(x - 1),$$

$$\lambda_e = \frac{h}{p_2} = \frac{\lambda_1\lambda_2}{\lambda_2 - \lambda_1}\left[\frac{v_2 / c}{1 + \sqrt{1 - v_2^2 / c^2}}\right] = \frac{u_e}{c}(\frac{\lambda_1\lambda_2}{\lambda_2 - \lambda_1}),$$
$$u_e = u_2 = \lambda_e f_e = \frac{\lambda_1\lambda_2}{\lambda_2 - \lambda_1}\left[\frac{v_2 / c}{1 + \sqrt{1 - v_2^2 / c^2}}\right](f_1 - f_2) = \frac{v_2}{1 + \sqrt{1 - v_2^2 / c^2}} ,\qquad (4.8\text{-}25)$$

$$(f_r = f_e) \Rightarrow \cos\theta = \frac{(\frac{hf_1}{mc^2}+1)\left[\frac{hf_1}{mc^2} - \sqrt{1+2\frac{hf_1}{mc^2}}\right]+1}{\frac{hf_1}{mc^2}\left[\frac{hf_1}{mc^2} - \sqrt{1+2\frac{hf_1}{mc^2}}+1\right]} = \frac{(x+1)\left[x-\sqrt{1+2x}\right]+1}{x\left[x-\sqrt{1+2x}+1\right]} .$$

Continuing the same (mathematical) testing, by the elimination of unacceptable results (where one of them is given above), we should be able to find the most significant and exact relations between all frequencies, wavelengths, and velocities of interaction participants. ***The objective in the above-elaborated analysis is to show that mutually interacting objects create transitory (time and space dependent) variable and evolving phenomena, where dominant (interaction-decisive) frame is the local Center of Mass System.***

One of the remaining possibilities to analyze such situations is also to use differential forms of energy and momentum conservation laws, such as, $d\tilde{E}_i = c^2 d(\gamma\tilde{m}_i) = hdf_i = v_i d\tilde{p}_i = d(\tilde{p}_i u_i) = -dE_{ki} = -c^2 d(\gamma m_i) = -v_i dp_i = -d(p_i u_i)$, and in the process of integration we should be able to take care about specific boundary conditions (extending the same procedure to the effects of rotation, electromagnetic fields etc.).

*The most promising strategy in addressing similar problems is to understand that the incident photon changes its frequency (loses its initial energy), and the particle (or electron), which was in the state of rest, increasingly gets more of motional energy (in the transitional process when they approach each other). We can also conceptualize this situation as a kind of Doppler Effect, where the incident photon gradually reduces its frequency ($f_1 \to f_2$), or reduces its energy, and de Broglie electron's matter-wave gradually increases its frequency ($0 \to f_e$), in relation to the center of mass which has velocity v_c. Similar conclusions should also be valid for any other type of collision (see [**6**]).*

In fact, the principal message here is to show that every collision event (elastic or plastic) creates certain (dynamic and transient) field perturbation around collision participants, this way producing de Broglie matter waves. The energy of matter-waves is only a form of kinetic or motional energy composed of kinetic or other motional energies of interaction participants.

What we traditionally analyze as different collision types (found in all physics books) are mostly situations verifiable long before and long after the collision happens when we see and measure only steady or stationary states. For instance, the above analyzed Compton effect is traditionally explained based on energy and momentum conservation, considering only the initial situation long before the impact, and the situation long after the scattering happens (neglecting the transitory process between them), as for instance:

$$E_{tot.} = hf_1 + mc^2 = hf_2 + \gamma_2 mc^2 = hf_2 + E_{k2} + mc^2,$$
$$\frac{\mathbf{hf_2}}{\mathbf{c}}\vec{\mathbf{e}}_2 + \vec{\mathbf{p}}_2 = \frac{\mathbf{hf_1}}{\mathbf{c}}\vec{\mathbf{e}}_1 ,$$

and here an attempt is made to show that such a strategy is insufficient to fully describe Compton's and familiar interactions. ***In addition, we still did not consider***

involvement of any electromagnetic interaction here, but as we know, both photon and moving electrons have electromagnetic nature and properties. ♣]

The process which is opposite (or inverse) to Compton Effect is the **Continuous spectrum of x-rays** *(or photons) emission, caused by impacts of electrons (accelerated in the electrical field between two electrodes) with the anode as the target. The emission of x-ray photons starts when the electrons are abruptly stopped on the anode surface. If the final, impact-electron speed is non-relativistic,* $v << c$*, kinetic energy of such electron is* $E_{ke} = mv^2/2$*, and the maximal frequency of the emitted x-rays is found from the relation:* $\tilde{E} = hf_{max.} = mv^2/2 = E_{ke}$*, and in cases of relativistic electron velocities, we have* $hf_{max.} = (\gamma - 1)mc^2$ (both of them being experimentally confirmed as correct). *If we now consider electrons (before an impact happen) as matter waves, where the electron matter wave energy corresponds only to a kinetic or motional electron energy, without a rest-mass energy content, we will be able to find de Broglie, matter wave frequency of such electrons (just before their impact with the anode). With impact realization, electrons are fully stopped, and the energy content of their matter-waves is fully transformed and radiated in the form of X-ray photons (or into another form of waves), whose frequency corresponds to the matter wave frequency of electrons in the moment of the impact. This* ***equality of the frequencies of radiated X-ray photons and electron matter-waves (in the moment of impact) explains to us the essential nature of electron matter-waves (eliminating the possibility that the rest mass of electrons belongs to any matter-wave energy content).***

Let us now analyze the ***simplified case (of Compton's effect) that could also be placed between the Photoelectric and Compton Effect***. We can imagine that an incident photon is "fully absorbed" by its target, the free electron that was in the state of rest before the impact with a photon (and, of course, the electron will get certain kinetic energy after *the* impact). This situation will be presented, as before, graphically and with the table of input-output energy and momentum states, as follows:

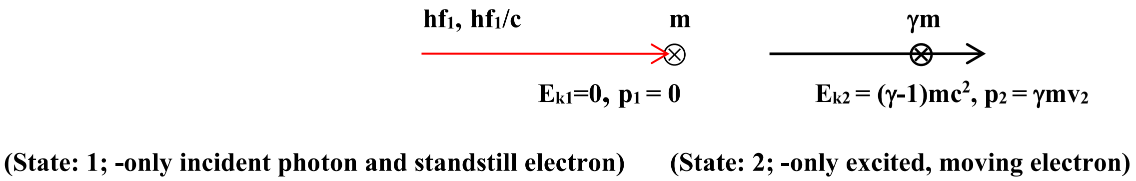


Fig.4.2.1 Direct Photon-Particle collision when the incident photon is fully absorbed by the electron. (Photon: hf_1, hf_1/c; Particle: mass m)

	States long before the collision (indexing: 1)	States just after the collision (indexing: 2)
Photon	$\tilde{E}_{f1} = hf_1, \tilde{p}_{f1} = hf_1/c$	$\tilde{E}_{f2} = 0, \tilde{p}_{f2} = 0$
Electron	$E_{k1} = 0, p_1 = 0,$ $v_1 = 0$	$E_{k2} = (\gamma - 1)mc^2 = p_2 u_2,$ $p_2 = \gamma m v_2 = p_e$

After the collision, the incident photon **(hf_1, hf_1/c)** disappears and its energy and momentum before impact are transformed into a moving particle (an electron) that was in a state of rest before the collision. When the particle (here an electron) was in a state of rest, we shall again assume that it did not have any wave energy or wave momentum (externally detectable). *The meaning of the particle-wave duality in this situation is that a particle (just) after the collision will get certain kinetic energy (*$E_{k2} = (\gamma-1)mc^2$, $p_2 = \gamma m v_2$*).* All particle and photon states before the collision will be marked using index "1" and after the collision using index "2", as already presented in the Fig.4.2.1 and in the table, above.

Let us first apply the energy and momentum conservation laws,

$$hf_1 + mc^2 = \gamma mc^2, hf_1 = (\gamma - 1)mc^2 = E_{k2},$$
$$\frac{hf_1}{c} = \gamma m v_2 = \gamma m v_e = p_2 = p_e$$

From the energy and momentum conservation laws we can find all other matter wave characteristics of the excited particle (excited electron) after the collision (of course, using the already known **PWDC** relations between group and phase velocity from (4.1) - (4.3)),

$$p_2 = \gamma m v_2 = \frac{hf_1}{c} = p_e, \lambda_2 = \frac{h}{p_2} = \frac{c}{f_1} = \frac{h}{(\gamma - 1)mc}, f_2 = f_e = f_1 = \frac{(\gamma - 1)mc^2}{h},$$

$$u_2 = \lambda_2 f_2 = \frac{v_2}{1 + \sqrt{1 - v_2^2/c^2}} = \frac{v_2}{1 + 1/\gamma} = c, \gamma = (1 - \frac{v_2^2}{c^2})^{-\frac{1}{2}} \Rightarrow v_2 = c,$$

$$E_{k2} = (\gamma - 1)mc^2 = p_2 u_2 = \frac{p_2 v_2}{1 + \sqrt{1 - v_2^2/c^2}} = \tilde{E}_2 = hf_2 = hf_1,$$

Obviously, it is not easy to imagine that any photon can immediately accelerate an electron from the state of rest to **c**, meaning that something is wrong in the above-analyzed example. Also, if we analyze the same case traditionally, when $hf_1 << mc^2$, we have $hf_1 \approx \frac{1}{2} m v_2^2, \frac{hf_1}{c} \approx m v_2, \gamma \approx 1 \Rightarrow v_2 \approx 2c$, which is an impossible result since the particle velocity reaches **2c**. ***The only logical conclusion is that we should consider that the interaction between the photon and the electron starts long before the physical impact (or before unification between them) happens.***

Let us now analyze the same situation in the Center of Mass System, in the time-space domain before the photon and the electron become the united object. We could also say that in this first phase of interaction, analyses of the elastic and plastic impact are identical (if we can say that we have two interacting objects). From the earlier analysis of the Compton Effect, we already know,

$$E_{kr} = hf_r = \frac{hf_1}{1+\frac{hf_1}{mc^2}} = hf_1 \frac{1}{1+x},\ E_{kc} = hf_c = hf_r x = E_{kr} x = hf_1 \frac{x}{1+x}\ ,\ x = \frac{hf_1}{mc^2}.$$

We also know that the energy E_{kr}, after a plastic impact materializes, is injected (absorbed) in the total-system mass, making the excited electron after the impact has a (temporarily) higher rest mass (higher than **m**), and a lower kinetic energy (lower than hf_1). Consequently, after gradual evolving of this plastic impact situation, we have only one object (a moving and excited electron that has temporarily increased its rest mass, **m*;** -The electron stays excited until it radiates again the initially "absorbed photon"),

$$\gamma_c m_c = \frac{\sum E}{c^2} = \frac{hf_1 + mc^2}{c^2} = m + \frac{hf_1}{c^2} = m(1+x),\ \{= \gamma m \Rightarrow \gamma = 1+x,\ ?!\},$$

$$m_c = m^* = m\sqrt{1+2\frac{hf_1}{mc^2}} = m\sqrt{1+2x} = m\sqrt{\frac{1-v_c^2/c^2}{1-v_2^2/c^2}} = m + \Delta m,\ E_{oc} = m_c c^2$$

$$\gamma_c = \frac{1}{\sqrt{1-\frac{v_c^2}{c^2}}} = \frac{1+\frac{hf_1}{mc^2}}{\sqrt{1+2\frac{hf_1}{mc^2}}} = \frac{1+x}{\sqrt{1+2x}},\ \gamma = \frac{1}{\sqrt{1-\frac{v_2^2}{c^2}}} = \sqrt{1+x^2}$$

$$\vec{p}_c = (\sum \vec{p}) = \frac{\sum E}{c^2}\vec{v}_c = (m + \frac{hf_1}{c^2})\vec{v}_c = \gamma_c m_c \vec{v}_c =$$

$$= \gamma m \vec{v}_c = \gamma m \vec{v}_2 = \vec{p}_2 = \frac{hf_1}{c}\vec{e}_1 = \tilde{p}_{f1},\ \lambda_2 = \frac{h}{p_2} = \lambda_e = \frac{c}{f_1},$$

$$v_c = v_2 = \frac{hf_1}{\gamma mc} = c\frac{x}{\gamma} = cx\sqrt{1-v_2^2/c^2} \Rightarrow v_2 = \frac{cx}{\sqrt{1+x^2}},$$

$$u_2 = \frac{v_2}{1+\sqrt{1-v_2^2/c^2}} = \frac{cx}{2} = \lambda_2 f_2 = c\frac{f_2}{f_1} = c\frac{f_e}{f_1},$$

$$E_{k2} = p_2 u_2 = \frac{hf_1}{c}\frac{cx}{2} = \frac{hf_1 x}{2} = hf_e \Rightarrow f_e = \frac{f_1 x}{2}$$

Now we can summarize the particle and wave properties of the excited electron, just after collision (before it radiates a photon).

$$v_2 = \frac{cx}{\sqrt{1+x^2}}, u_2 = \frac{v_2}{1+\sqrt{1-v_2^2/c^2}} = \frac{cx}{2} = \lambda_2 f_2 = \lambda_e f_e = c\frac{f_2}{f_1} = c\frac{f_e}{f_1},$$
$$E_{k2} = p_2 u_2 = \frac{hf_1}{c}\frac{cx}{2} = \frac{hf_1 x}{2} = hf_e \Rightarrow f_e = \frac{f_1 x}{2} = f_2$$
$$p_2 = mcx = \frac{hf_1}{c} = p_e, \lambda_e = \lambda_2 = \frac{h}{p_2} = \frac{c}{f_1},$$
$$x = \frac{hf_1}{mc^2}, \gamma = \frac{1}{\sqrt{1-\frac{v_2^2}{c^2}}} = \sqrt{1+x^2}.$$

For instance, let us first analyze the case (1°) when the incident photon has an extremely low energy, meaning that the electron after the collision can be treated as a non-relativistic particle.

1°

$$(hf_1 = \text{Low} << mc^2) \Rightarrow (v_2 << c, \gamma \approx 1, x << 1) \Rightarrow$$
$$v_2 = \frac{cx}{\sqrt{1+x^2}} \cong cx, u_2 = \frac{v_2}{1+\sqrt{1-v_2^2/c^2}} \cong \frac{cx}{2} = \frac{v_2}{2}$$
$$E_{k2} = p_2 u_2 = \frac{hf_1}{c}\frac{cx}{2} = \frac{hf_1 x}{2} = hf_e \Rightarrow f_e = \frac{f_1 x}{2} = f_2$$
$$p_2 = mcx = \frac{hf_1}{c}, \lambda_e = \lambda_2 = \frac{h}{p_2} = \frac{c}{f_1}.$$

The second case (2°) of interest is when the incident photon has a remarkably high energy and when the electron after the collision can be treated as a relativistic particle:

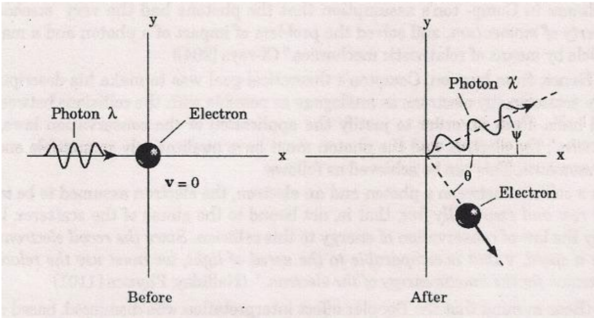
2°

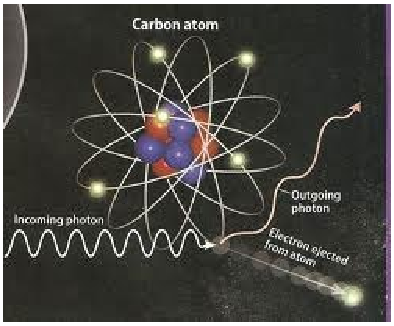
$$(hf_1 = \text{very high} >> mc^2) \Rightarrow (v_2 \approx c, \gamma \to \infty, x >> 1) \Rightarrow$$
$$v_2 = \frac{cx}{\sqrt{1+x^2}} \approx c, u_2 = \frac{v_2}{1+\sqrt{1-v_2^2/c^2}} \approx c,$$
$$E_{k2} = p_2 u_2 = \frac{hf_1}{c}\frac{cx}{2} = \frac{hf_1 x}{2} = hf_e \Rightarrow f_e = \frac{f_1 x}{2} = f_2$$
$$p_2 = mcx = \frac{hf_1}{c}, \lambda_e = \lambda_2 = \frac{h}{p_2} = \frac{c}{f_1}.$$

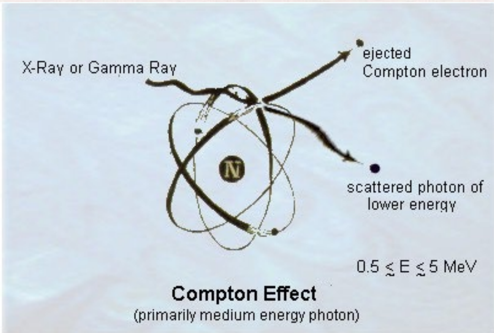
In both situations (non-relativistic 1°, and relativistic 2°), we can calculate all particles and wave characteristics of the excited electron (after collision), and the results look very realistic (or at least not directly contradictory to known conservation laws), contrary to the results of the traditional analysis of the same situation.

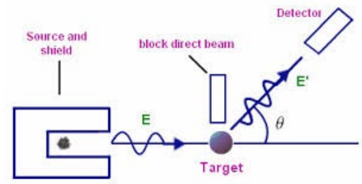
The situation under 2° is like conditions causing Cherenkov Effect: the accelerated (or excited) electron starts radiating photons behind (creating the back conus of its wave energy).

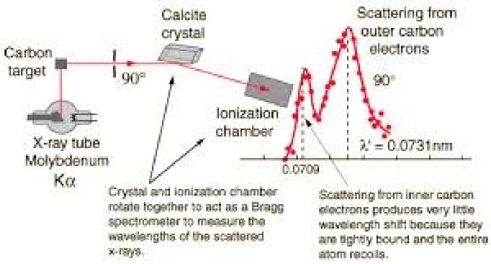
Compton Effect illustrations (found on the Internet)

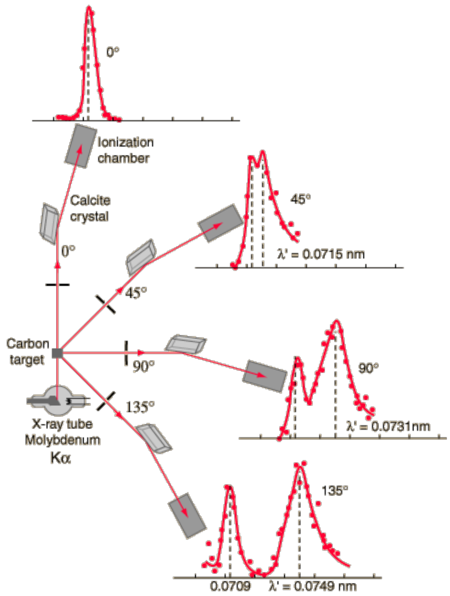




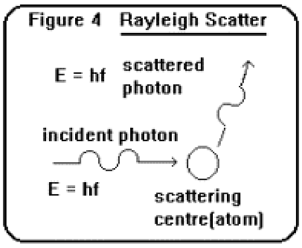


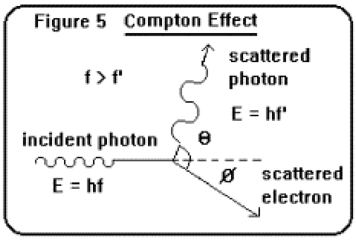


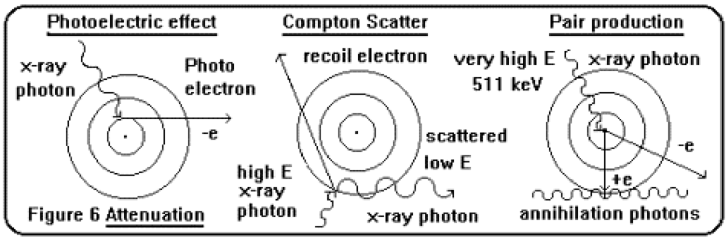


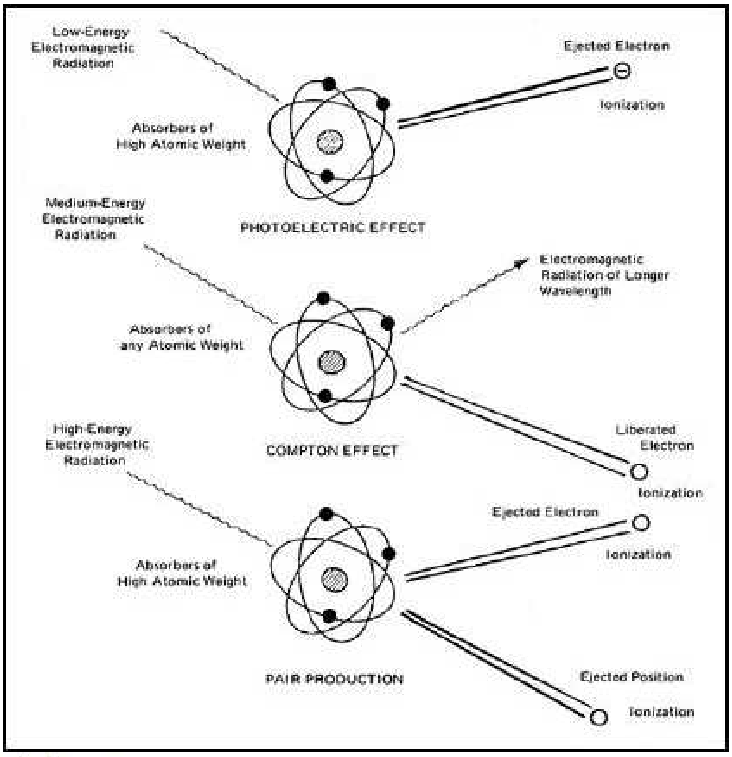


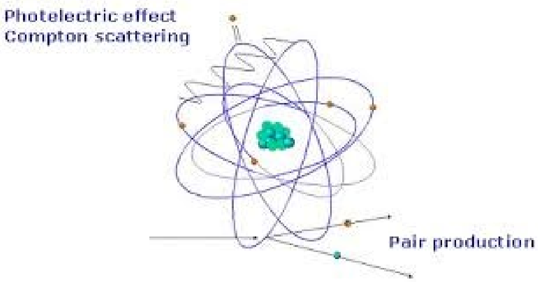
Compton Effect, Photoelectric Effect and Pair production pictures (from the Internet)



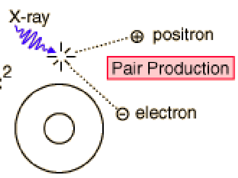


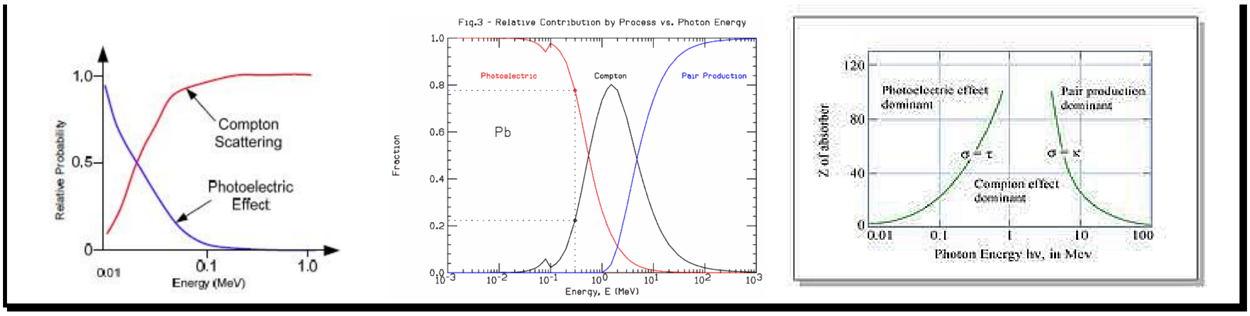






$$E_{threshold} = 2m_0c^2$$





[♣ 4.2.3. Example 4: Doppler Effect (*still a draft; -needs significant modifications*)

The Doppler Effect describes the frequency difference between the emitting source signal and received signal when the emitter and the receiver are mutually in relative motion.

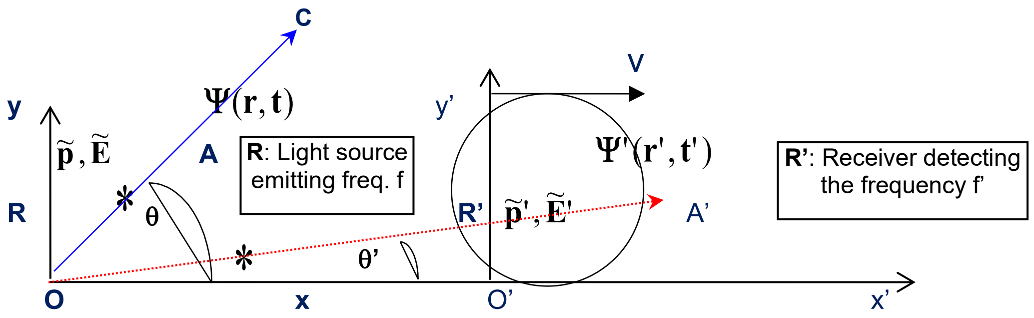
Let us imagine that the light source is in the referential system R: (Oxy), and signal receiver is in the system R': (O'x'y'), and that relative speed between them is v. In the center O of the system R is the light signal source, and in the center O' of the system R' is the receiver of the same light signal. O emits a monochromatic plane wave that has the frequency f, directed along **OA** (angle θ against Ox axis). Referential receiver system R' moves by uniform speed v relative to R, along their common axis Ox - O'x', and the received signal is detected by along **OA'** (angle θ' against O'x' axis).

Monochromatic plane-wave, in complex notation, in the system R can be characterized by its wave function $\overline{\Psi}(\mathbf{r},\mathbf{t})$:

$\overline{\Psi}(\mathrm{r},\mathbf{t}) = \mathbf{a}(\mathrm{r},\mathbf{t})\mathbf{e}^{-j(\omega t - kr)} = \mathbf{a}(\mathrm{r},\mathbf{t})\mathbf{e}^{j(k_x x + k_y y + k_z z - \omega t)}$, where $\mathbf{k}_x, \mathbf{k}_y$ i $\mathbf{k}_z$ are components of the wave vector $\vec{\mathbf{k}}$. In the 4-dimensional Minkowski space, photon wave vector $\overline{\mathbf{K}}_4$ and its radial position vector $\overline{\mathbf{R}}_4$ are known as, $\overline{\mathbf{K}}_4 = \overline{\mathbf{K}}_4(\mathbf{k}_x, \mathbf{k}_y, \mathbf{k}_z, \frac{\omega}{c}) = \overline{\mathbf{K}}_4(\vec{\mathbf{k}}, \frac{\omega}{c})$ and $\overline{\mathbf{R}}_4 = \overline{\mathbf{R}}_4(\mathbf{x}, \mathbf{y}, \mathbf{z}, \mathbf{ct}) = \overline{\mathbf{R}}_4(\vec{\mathbf{r}}, \mathbf{ct})$, making it possible to express the photon wave function using the product between $\overline{\mathbf{K}}_4$ and $\overline{\mathbf{R}}_4$:

$$\overline{\Psi}(\mathbf{r},\mathbf{t}) = \mathbf{a}(\mathbf{r},\mathbf{t})\mathbf{e}^{-j\overline{K}_4\overline{R}_4} = \mathbf{a}(\mathbf{r},\mathbf{t})\mathbf{e}^{j(\vec{k}\vec{r} - \omega t)} = \mathbf{a}(\mathbf{r},\mathbf{t})\mathbf{e}^{j(k_x x + k_y y + k_z z - \omega t)}$$

$$(\mathbf{j}^2 = -1,\ \mathbf{k} = \frac{2\pi}{\lambda},\ \omega = 2\pi\ \mathbf{f})$$



Doppler Effect Illustration

After applying Lorenz transformations on the above-described case, creating relations between R and R', we will get:

$$\overline{K}'_4 = \overline{K}'(k'_x, k'_y, k'_z, \frac{\omega'}{c}),$$

$$k'_x = \gamma\ (k_x - \beta\frac{\omega}{c}),\quad k'_y = k_y,\quad k'_z = k_z,\quad \frac{\omega'}{c} = \gamma\ (\frac{\omega}{c} - \beta\ k_x),\quad (k_{x,y,z} = \frac{2\pi}{h}\tilde{p}_{x,y,z}),$$

$$c = \lambda f = \frac{\omega}{k} = \frac{\omega'}{k'},\quad \tilde{E} = \frac{h}{2\pi}\omega,\quad \tilde{E}' = \frac{h}{2\pi}\omega',\quad \gamma = (1-\beta^2)^{-\frac{1}{2}},\ \beta = \frac{v}{c}\ .$$

$$\mathbf{k_x = k\cos\theta = \frac{2\pi}{c} f\cos\theta},$$

The Doppler frequency difference between light signals in R and R' can be found from,

$$f' = f\frac{\tilde{E}'}{\tilde{E}} = \gamma\ f(1-\beta\ \cos\theta),\quad \lambda' = \frac{c}{f'} = \lambda\frac{\tilde{p}}{\tilde{p}'} = \frac{\lambda}{\gamma\ (1-\beta\ \cos\theta)}\ .$$

This situation looks like adding (or reducing) certain frequency shift Δf to the source frequency $\mathbf{f}$, or adding (or reducing) a certain amount of motional (wave) energy $\Delta\tilde{E}$ to the source photon energy. We can imagine that between R and R' certain intermediary wave-coupling state materializes, realizing the mentioned energy difference. We can also associate the same wave coupling state to certain wave moment $\tilde{\mathbf{p}}^*$ and mass $\tilde{\mathbf{m}}^*$. Since the source and the receiver photon frequency, and relative speed between R and R' are known, we can calculate all characteristics of the wave coupling state (see the table with all results, below).

The message of the above given Doppler Effect analysis is that this is not only an observation-related phenomenon but much more, it is the case of real wave interactions and energy and momentum conservation rules. In addition, the same case can be analogically applied to any mass movement, explaining the nature of the particle-wave duality from a larger perspective than presently known (or saying differently, every relative motion between minimum 2 particles, or quasiparticles should create similar wave coupling state/s).

	Source Photon in R	Differential Wave Coupling State Between R' and R	Detected Photon in R'
Wave energy	$\tilde{E} = hf$	$\tilde{E}^* = \Delta\tilde{E} = \tilde{E}' - \tilde{E} = h(f' - f) = h\Delta f = hf^* =$ $= (\gamma - 1)\tilde{m}^* c^2 =$ $= hf[\gamma(1-\beta\cos\theta) - 1]$	$\tilde{E}' = hf'$
Moment	$\tilde{p} = \frac{hf}{c}$	$\vec{p}^* = \frac{\overrightarrow{hf'}}{c} - \frac{\overrightarrow{hf}}{c} = \gamma\ \tilde{m}^* \vec{v}$ $\tilde{p}^* = \gamma\ \tilde{m}^* v =$ $= \frac{hf}{c}\left[-\cos\theta \pm \sqrt{\gamma^2(1-\beta\cos\theta)^2 - \sin^2\theta}\right]$	$\tilde{p}' = \frac{hf'}{c}$
Frequency	f	$f^* = \Delta f = f' - f = \frac{\Delta\tilde{E}}{h} = \frac{\tilde{E}^*}{h} =$ $= f[\gamma(1-\beta\cos\theta) - 1]$	$f' = f' = f\frac{\tilde{E}'}{\tilde{E}} =$ $= \gamma f(1-\beta \cos\theta)$
Wavelength	$\lambda = \frac{c}{f}$	$\lambda^* = \frac{h}{\tilde{p}^*} =$ $= \frac{c}{f}\left[-\cos\theta \pm \sqrt{\gamma^2(1-\beta\cos\theta)^2 - \sin^2\theta}\right]$	$\lambda' = \frac{c}{f'} =$ $\lambda\frac{\tilde{p}}{\tilde{p}'} = \frac{\lambda}{\gamma\ (1-\beta\ \cos\theta)}$
Group Velocity	c	$v = \frac{\partial\tilde{E}^*}{\partial\tilde{p}^*}$	c
Phase Velocity	c	$u = \frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}} = \lambda^* f^* = \frac{\tilde{E}^*}{\tilde{p}^*}$	c
Effective Mass	$\tilde{m} = \frac{\tilde{E}}{c^2} = \frac{hf}{c^2}$	$\tilde{m}^* = \frac{\Delta\tilde{E}}{(\gamma-1)c^2} = \frac{h\Delta f}{(\gamma-1)c^2} = \frac{\tilde{p}^*}{\gamma v} =$ $= \frac{hf[\gamma(1-\beta\cos\theta) - 1]}{(\gamma-1)c^2} = \frac{hf^*}{(\gamma-1)c^2} =$ $= \frac{hf\left[-\cos\theta \pm \sqrt{\gamma^2(1-\beta\cos\theta)^2 - \sin^2\theta}\right]}{\gamma v c}$	$\tilde{m}' = \frac{\tilde{E}'}{c^2} = \frac{hf'}{c^2}$

The primary objective of the examples provided above was to demonstrate that **only** kinetic or motional energy contributes to de Broglie matter waves. This contrasts with the mainstream understanding in contemporary physics, where total energy, including rest mass energy, is typically included in the matter wave formulation.

It is important to note, however, that the mathematical analyses presented thus far have limitations. This approach only allows for relatively simple treatments of particle-wave duality. The complexity arises from the fact that we are dealing with both particle and field interactions, which necessitates the introduction of wave functions for a more complete description.

By establishing the wave function concept and formulating general wave equations, we can address these issues more comprehensively. However, even at the current level of analysis, there is already sufficient mathematical substance to qualitatively understand the concept of particle-wave duality as outlined in this paper, as well as its connection to inertial and reaction forces. ♣]

Download the last version here:
http://www.mastersonics.com/documents/revision_of_the_particle-wave_dualism.pdf

4.3. MATTER WAVES AND WAVE EQUATIONS

Wave motions, vibrations, and oscillations are ubiquitous in our universe. We have developed methods to create and detect them, describe them mathematically, and model various wave phenomena related to physics. These models connect general wave-motion concepts and equations with material properties, relevant geometries, and parameters of motion. Importantly, we understand that any temporal-spatial function or signal can be decomposed into simple harmonic sinusoidal components (or elementary waves). This decomposition is fundamental in mathematics and physics, producing measurable spectral characteristics and predictable interactions with matter. The principles and experimental facts behind this, based on "Fourier-Kotelnikov-Shannon-Nyquist-Whitaker" analysis and Denis Gabor's concepts of Complex Analytic Signals, are well documented (see [57] Michael Feldman, [109] Poularikas A. D., [110], and [111]).

Our universe also comprises various states of matter, such as particles, gases, plasmas, and fluids, which are always in relative motion. Through numerous experimental observations, we know that interacting particles and waves can combine to form new matter waves and/or particles in a variety of configurations. This means that masses, atoms, and different forms of matter are essentially compositions or superpositions of various matter wave states, including forms of electromechanical and electromagnetic resonators and oscillators. Waves can propagate both with and without particles, and particles synchronize and create wave-like motions when they move in large groups.

Moreover, it is evident that different wave formations and oscillations occur in multiple ways within and around atoms, molecules, and other particles. Consequently, mastering the conceptual, theoretical, and empirical aspects of wave motions and oscillations in relation to particles is a major focus of modern science and technology. By embracing the necessity and advantages of operating with wave functions, we delve into the foundations of modern quantum mechanics, and physics in general, since superposition and interference among matter states and matter waves (or wave functions) underpin everything in our universe. The concept of wavefunction modeling is elaborated in Chapters 4.0 and 10 of this book. This concept can later be mathematically transformed or normalized (losing its physical dimensions) to align with the non-dimensional, probabilistic wavefunction modeling in quantum theory.

All the force effects we experience are also linked to matter waves, especially evident in cases of stationary, static, or standing waves and resonant formations. For instance, what we perceive as an attractive force, such as gravity, may exist around resonant, nodal zones of *minimal oscillating amplitude* where matter and energy agglomerate within standing matter waves. In contrast, repulsive forces, which oppose gravitation, might exist around resonant antinode zones, where the oscillating amplitudes of cosmic standing matter waves are maximal. These standing-matter-waves and the associated attractive and repulsive forces can be electromagnetic, mechanical, electromechanical, or of another physical nature.

We often conceptually associate the four fundamental natural forces with certain field-charge entities, analogous to electric charges that generate electric fields and Coulomb forces. However, this analogy is less clear for gravitation, as gravitational charge is not merely mass. Natural forces are always detectable where there are non-zero gradients in energy and mass density. Even electric charges are likely manifestations of standing wave formations of electromagnetic energy-mass agglomerations, possibly akin to specifically structured photons, which implies they are never stationary. Weak nuclear forces are also manifestations of electromagnetic forces and fields related to specifically structured matter-waves. From this perspective, the concept of strong nuclear forces might need significant refinement.

Citation from [95], "A Student's Guide to Waves":

"A ***classical traveling wave is a self-sustaining disturbance of a medium, which moves through space transporting energy and momentum.***"

"What is required for a physical situation to be referred to as a wave is that ***its mathematical representation gives rise to a partial differential equation of a particular form, known as the wave equation***."

"[The essential feature of wave motion is that a ***condition of some kind transmitted from one place to another by means of a medium, but the medium itself is not transported.***"

"[A wave is] each of those ***rhythmic alternations of disturbance and recovery of configuration.***"

The most common defining characteristic is that ***a wave is a disturbance of some kind, that is, a change from the equilibrium (undisturbed) condition***. A string wave disturbs the position of segments of the string, a sound wave disturbs the ambient pressure, an electromagnetic wave disturbs the strengths of the electric and magnetic fields etc.

In propagating or traveling waves, the wave disturbance must move from place to place, carrying energy with it. But you should be aware that ***combinations of propagating waves can produce non-propagating disturbances, sue as those of a standing wave***.

In periodic waves, the wave disturbance repeats itself in time and space. So, if you stay in one location and wait long enough, you're sure to see the same disturbance as you've seen previously. And if you take an instantaneous snapshot of the wave, you'll be able to find different locations with the same disturbance. But ***combinations of periodic waves can add up to non-periodic disturbances such as a wave pulse***.

Finally, ***in harmonic waves, the shape of the wave is sinusoidal***, meaning that it takes the form of a sine or cosine function.

So, ***waves are disturbances that may or may notbe propagating, periodic and harmonic***.

In this book, we explore the existence of real, measurable, momentum-energy carrying matter-waves or wavefunctions, which include de Broglie waves. These wavefunctions here are presented as dimensional and deterministic entities, specifically as power-related wavefunctions (see equation 4.0.82 in Chapter 4.0 for an example).

This approach contrasts with assumptions in Orthodox Quantum Mechanics, where wavefunctions and de Broglie matter waves are treated exclusively as virtual mathematical objects, abstract, non-dimensional "probability waves" or probability distributions in wave form. Later in the book, we will explain why it has been convenient, though perhaps misleading, to associate a probabilistic nature with matter-wave wavefunctions in microphysics.

While probabilistic interpretation has proven mathematically effective within the frameworks of quantum theory (QT), Statistics, Probability Theory, and Signal Analysis, we argue that it is ontologically and conceptually insufficient, if not fundamentally flawed. The probability concept, though useful in mathematical modeling of sets with big number of similar participants, and in the structure of Orthodox Quantum Mechanics, falls short in providing a complete and unique understanding of microphysical reality. For instance, Probability, Statistics, and Parseval theorems, by accounting for all possible states of a given event, effectively represent laws of energy and momentum conservation in an average sense. This book will demonstrate why and how this artificially constructed QT works well within its limited scope.

The question of identifying the material carrier of de Broglie matter waves in modern physics remains unresolved, particularly within the frameworks of contemporary Relativity and Quantum Theory. In this book, we posit that in any wave motion, there must exist a carrier medium, spatial texture, or something else, either known, unknown, or beyond the reach of current technology. To deny such a material medium would defy common sense and contradict our entire understanding of natural and tangible wave motions, effectively legitimizing arbitrary assembled metaphysics or supporting "magic situations." Even an idealized vacuum state, after all, has measurable dielectric and magnetic permeability and susceptibility properties, which could be considered as attributes of a very fine fluid or aether.

Quantum Mechanics traditionally treats de Broglie matter waves and quantum interactions using the abstract concept of probability wavefunctions (without defined wave-phase function), considering these waves as non-dimensional, not instantly detectable in real-time, and virtual. In contrast, this book introduces a different perspective: the square of a wave function represents motional or kinetic energy, or temporal "power flow." This power-related wavefunction concept, which is dimensional and measurable, is introduced in Chapter 3.0 (see 3.5), further developed in Chapter 4.0 (see equations 4.0.1 to 4.0.5, including 4.0.11 on Generalized Wavefunctions and Unified Field Theory), and summarized in Chapter 10. In this framework, de Broglie matter-waves are treated mathematically as Complex Analytic Signal functions or Phasors, like any other wave phenomena in fields such as electromagnetism, acoustics, or fluid mechanics.

Moreover, we will show that, at a formal and mathematical level, there is no fundamental contradiction between the probabilistic wavefunction and the power-related, dimensional wavefunction. Both can be seen as mutually isomorphic, adhering to the same conservation laws in physics. When presented using the Analytic Signal model, the deterministic and power-related wavefunction accurately describes the space-time evolution of de Broglie matter waves or wave-packets within and around moving particles, consistent with the differential energy balance outlined in Chapter 4.0 (see 4.7) and further elaborated in Chapter 4.1.

Mathematically (to satisfy universally valid Parseval's theorem, related to signal energy equivalence and energy conservation between time and frequency domains of the same signal; -see Chapter 4.0; -equations (4.0.4), (4.0.5)), it is very convenient to consider the square of a wave function, $\Psi^2(t)$, being an active-power function $\mathbf{S(t)}$, or a matter-wave power (or wave "energy-current") since,

$$\{d\tilde{E} = hdf = dE_k = c^2 d(\gamma m) = vdp = d(pu) = -c^2 d\tilde{m} = -vd\tilde{p} = -d(\tilde{p}u)\} / dt$$

$$\Leftrightarrow \begin{Bmatrix} \Psi^2(t) = S(t) = \frac{d\tilde{E}}{dt} = h\frac{df}{dt} = c^2\frac{d(\gamma m)}{dt} = \frac{d(pu)}{dt} = v\frac{dp}{dt} = ... \quad (=)[W] \\ d\tilde{E} = \Psi^2(t)\cdot dt, \;\; dp = \frac{1}{v}d\tilde{E} = \frac{1}{v}\Psi^2(t)\cdot dt \end{Bmatrix},$$

$$\Rightarrow \begin{pmatrix} \tilde{E} = \int_{-\infty}^{+\infty}\Psi^2(t)dt = \int_{-\infty}^{+\infty}\hat{\Psi}^2(t)dt = \int_{-\infty}^{+\infty}\left|\frac{\overline{\Psi}(t)}{\sqrt{2}}\right|^2 dt = \int_{-\infty}^{+\infty}\left[\frac{a(t)}{\sqrt{2}}\right]^2 dt = \\ = \int_{-\infty}^{+\infty}\left|\frac{\overline{U}(\omega)}{\sqrt{2\pi}}\right|^2 d\omega = \int_{0}^{\infty}\left[\frac{A(\omega)}{\sqrt{\pi}}\right]^2 d\omega = \sum_{(i)}\tilde{E}_i = \sum_{(i)}\int_{-\infty}^{+\infty}\Psi_i^2(t)dt = \int_{-\infty}^{+\infty}\sum_{(i)}\left[\Psi_i^2(t)\right]dt = \\ = \sum_{(i)}\int_{-\infty}^{+\infty}\hat{\Psi}_i^2(t)dt = \int_{-\infty}^{+\infty}\sum_{(i)}\hat{\Psi}_i^2(t)dt = \sum_{(i)}\int_{-\infty}^{+\infty}\left[\frac{a_i(t)}{\sqrt{2}}\right]^2 dt = \int_{-\infty}^{+\infty}\sum_{(i)}\left[\frac{a_i(t)}{\sqrt{2}}\right]^2 dt = \\ = \sum_{(i)}\int_{-\infty}^{+\infty}\left|\frac{\overline{U}_i(\omega)}{\sqrt{2\pi}}\right|^2 d\omega = \int_{-\infty}^{+\infty}\sum_{(i)}\left|\frac{\overline{U}_i(\omega)}{\sqrt{2\pi}}\right|^2 dt = \sum_{(i)}\int_{0}^{\infty}\left[\frac{A(\omega)}{\sqrt{\pi}}\right]^2 d\omega = \int_{-\infty}^{+\infty}\sum_{(i)}\left[\frac{A(\omega)}{\sqrt{\pi}}\right]^2 d\omega \\ \Rightarrow \Psi^2(t) = \sum_{(i)}\left[\Psi_i^2(t)\right], \; \hat{\Psi}^2(t) = \sum_{(i)}\left[\hat{\Psi}_i^2(t)\right], \tilde{E} = \frac{1}{2}\int_{-\infty}^{+\infty}\left[\Psi^2(t) + \hat{\Psi}^2(t)\right]dt, \\ \Leftrightarrow \frac{1}{2}\{\Psi^2(t) + \hat{\Psi}^2(t)\} = \frac{dE}{dt} = \frac{dE_k}{dt} = \frac{d\tilde{E}}{dt} = \sum_{(i)}\frac{dE_i}{dt} = \sum_{(i)}\frac{dE_k}{dt} = \sum_{(i)}\frac{d\tilde{E}_i}{dt} = S(t). \end{pmatrix}, \quad (4.9\text{-}0)$$

where de Broglie or matter-wave energy is $\tilde{E} = \int S(t)dt = \int \Psi^2(t)dt = \tilde{p}u = hf = E_k$. All wave functions, Classical Wave equation, and *Schrödinger-like wave equations,* developed in Physics and later in this book, should be in compliance with (4.9-0), *and also fully merged with **P**article-**W**ave **D**uality **C**ode (or PWDC), first time formulated in Chapter 4.1, around equations (4.1)-(4.3), and later in Chapter 10, meaning that the Complex Analytic Signal wave function* $\overline{\Psi}(t)$ *created from* $\Psi(t)$ *will have all properties of de Broglie matter waves (such as matter-wave wavelength,* $\lambda = h/p = u/f$ *and relevant group and phase velocity* $v = u - \lambda du/d\lambda = -\lambda^2 df/d\lambda$*). For a wave function related to active power, it should be valid* $\left|\frac{\overline{\Psi}(t)}{\sqrt{2}}\right|^2 = \frac{1}{2}\{\Psi^2(t) + \hat{\Psi}^2(t)\} = \frac{d\tilde{E}}{dt} = \sum_{(i)}\frac{d\tilde{E}_i}{dt} = S(t)$ *(as in (4.9-0)).*

In this book, we explore the foundational links between wavefunctions, power, energy, and Parseval's theorem (or identity), which are both natural and mathematically valid. If we disregard these connections and treat the wavefunction purely as a probabilistic, non-dimensional concept without incorporating relevant phase functions, we risk creating conceptual errors. Alternatively, we may need to *introduce additional elements to probabilistic wavefunction processing to correct the initial artificial and weak conceptualization (what is already happening in QT).*

Orthodox Quantum Theory has effectively assembled a set of mathematical rules and assumptions that allow its probabilistic framework to serve as an "in-average" and isomorphic replacement for real, deterministic, and dimensional wavefunctions, while still satisfying the conservation laws of classical and deterministic physics. To create a meaningful and practically applicable theory, the founders of quantum mechanics postulated widely accepted ideas about wave-particle duality. They hybridized concepts from Probability, Statistics, Spectrum, and Signal Analysis with Particle–Wave Duality (as discussed further in Chapter 10 under "10.00 Deeper Meaning of PWDC").

Moreover, the classical second-order wave equation and its wavefunction were (in a somewhat intuitive and mathematically artificial manner) modified by Schrödinger to accommodate complex wave function solutions. This wavefunction and wave equation were then integrated with additional rules, postulates, and definitions to become probabilistically and statistically operational, ultimately aligning with conservation laws. After a series of mathematical refinements, this led to the current Orthodox Quantum Theory, which, despite being abstract and somewhat unnatural, works effectively within its self-defined terminology and boundaries. However, the future development of Quantum and Wave-Particle Duality Theory is likely to reveal that phenomena currently understood only through stochastic and probabilistic functions also possess a deterministic nature once appropriately clarified and modeled. While statistics and probability can always be applied in any natural or scientific field for "in-average" mathematical modeling, especially when dealing with large numbers of identical elements, this application is not exclusive to QT.

The primary objective of this book is to modify and re-establish the foundations of a more causal and less probabilistic Particle-Wave Duality, or more accurately, a Particle-Wave Unity theory. In line with the deterministic and tangible (dimensional, power-related) wavefunction introduced in Chapter 4.0, we will also utilize established mathematical tools from Spectrum and Signal Analysis, which are well-suited for handling wavefunctions and wave equations, regardless of the nature of the waves. In fact, the mathematical framework currently used in Orthodox Quantum Mechanics will largely remain applicable in any new matter-waves theory, serving as a valuable and instructive mathematical toolbox for future developments in wave equations and analyses. There is little need for significant changes in this regard.

We will demonstrate that the deterministic and tangible wavefunction, when modeled using the Analytic Signal approach, possesses a richer mathematical structure, with natural spatial, temporal, amplitude, phase, and spectral characteristics, compared to the artificial probability-based wavefunction in Orthodox QT (see Chapter 4.0 for more details).

The key message here is that the development of all wavefunctions and wave equations in modern physics, including those more general than currently known in quantum theory, does not require postulated Schrödinger's equation with the probabilistic assumptions of Orthodox QT. Instead, we only need to adhere to the conservation laws of physics, integrating them with the classical second-order partial *differential wave equation and the Particle-Wave Duality framework (or PWDC) described in this book (see equations 4.1 to 4.3 in Chapter 4.1 and 10.1 in Chapter 10). Ultimately, this must all be integrated into a more universally valid mathematical framework of Signal and Spectrum Analysis, based on Complex Analytic Signal modeling (via Hilbert transformation; see Chapters 4.0, 10, and later sections for more on Analytic Signals, including references [57], [58], and [59]).*

Despite the significant differences in how wavefunctions are treated in this book compared to Orthodox QT, it is easy to show that Schrödinger's wave equation, a cornerstone of microphysics, retains its form, properties, mathematical processing, and results, whether using a non-dimensional probability wavefunction or a "power-energy-carrying" dimensional wavefunction. Additionally, a modified version of Schrödinger's equation can be applied to cosmological scenarios, such as satellite and planetary motions, since it can be derived from the classical partial differential wave equation when the wavefunction is formulated as a Complex Analytic Signal function (see Chapter 2, equations 2.11.20, and Chapters 4.0 and 10 for more details).

While we may not yet have a complete understanding of the material carrier of de Broglie matter-waves (or the carrier of electromagnetic waves), this does not hinder our mathematical modeling of these wave phenomena. The material wave carrier is anyway implicitly considered. For now, we may set aside the question of the exact nature of the wavefunction and its material carriers, revisiting it later when these concepts become clearer. It is also natural and logical in physics to assume that a wave-propagation medium must exist.

It is noteworthy that many of the pioneers and experts in Orthodox QT have acknowledged that, while the theory works well, its underlying principles and origins are not fully understood, and the conceptual and ontological framework of modern QT remains incomplete. Orthodox QT is operational within its own self-defined, artificial scope of applied mathematical framework, but this framework is largely postulated and constructed to serve the needs of such Quantum Mechanics.

We also recognize that mathematics is the most effective language and tool for describing, explaining, and predicting phenomena in physics and nature when the mathematical conceptualization and modeling are well-founded on measurable, experimental facts and are smoothly integrated with the broader body of mathematics and physics.

In natural situations, where mathematical theory or modeling is not specifically constructed to address temporary or artificially created needs, conceptual, intellectual, philosophical, logical, deterministic, and realistic understanding of certain problems is clear and self-explanatory, free from internal contradictions. Naturally conceptualized theories are seamlessly integrated into the broader framework of physics and mathematics, making them intuitively clear and empirically verifiable without relying on artificial constructs or axiomatic assumptions.

When we construct, extend, and apply universally valid mathematics to physics, we should aim for clear, mutually connected conceptual pictures, a goal that Orthodox Quantum Theory has yet to achieve. This approach represents the natural, tangible *body of universally applicable mathematics. Unfortunately, contemporary Orthodox Quantum Theory, with its artificially constructed and postulated mathematical framework, partially remains disconnected from the broader, tangible body of physics and mathematics. This disconnection leads to a theory full of complexity, assumptions, dilemmas, and contradictions in relation to reality and the rest of physics.*

In this book, we establish a modeling approach for power- or energy-flow-related wavefunctions and wave equations that are smoothly connected with universally valid mathematics and physics, within the boundaries of well-known and universally applicable practices, such as those in mechanics, acoustics, and electromagnetism.

The same form of classical (second order, partial, differential) wave equation is established independently in many mutually different fields of Physics (much before Quantum Theory was formulated). ***It can be demonstrated that classical wave equation*** $\frac{1}{u^2}\frac{\partial^2\Psi}{\partial t^2}=\frac{\partial^2\Psi}{\partial r^2}(=)\nabla^2\Psi$ ***is the most important partial differential wave equation, which (with certain modifications) generally describes all kinds of matter waves****, such as sound waves, electromagnetic waves, fluid dynamics waves…, and which is in the background of all wave equations familiar to Schrödinger equation, as known in modern Physics and Quantum theory (QT). Variations of the same classical wave equation are also found in General Relativity theory. When operating with such wave equations (applied to Classical Physics), everything is tangible, clear, causal, explicable and without internal contradictions. Now, we will consider and demonstrate in three steps (see later: -1-, -2-, -3-), that mentioned classical, second order differential wave equation is the best starting platform for constructing other wave equations, including quantum-mechanical ones.* ***Schrödinger equation will simply surface to be the Classical Wave equation, where involved wave function*** $\overline{\Psi}$ ***is a Complex Analytic Signal function*** *as established in Chapter 4.0 (and merged with PWDC; -see more in (4.1) - (4.3) in Chapter 4.1, and (10.1) in Chapter 10.). This is additionally and directly supporting de Broglie-David Bohm QT interpretation; -regardless D. Bohm was also using postulated Schrödinger equation.*

At a young age, Schrödinger intuitively and perhaps serendipitously developed or assembled his famous equation. This groundbreaking work, which may not have been built on an extensive mathematical foundation, was likely inspired during his extracurricular activities at a hotel in the Austrian mountains, an idea highlighted by many modern science historians and devoted followers of the mainstream QT. However, after this fortunate postulation, Schrödinger himself many times expressed doubts about his creation.

Contemporary educational systems continue to celebrate and explain this non-traditional approach to the establishment of Schrödinger's equation. Today, we understand that it is essentially a complex differential wave equation smoothly derived from the classical wave equation, where the relevant wave function is modeled as a complex analytic signal (what also supports David Bohm, non-probabilistic QT interpretation).

In the following table (see below; -taken from ***[87]****, as an example) we can find several mutually analogical Classical Wave Equations, developed for different waving phenomena known in Physics. Starting from such wave equations, or from Classical Wave equations, we can easily develop several famous wave equations known in modern Quantum Theory.*

Propagation d'onde non dispersive. Cordes vibrantes 101

Tableau des célérités de phénomènes classiques non dispersifs

	Exemple de propagation non dispersive	Équation de d'Alembert	Onde	Célérité	Caractéristique du milieu	Problème type
Ondes transversales	Corde vibrante	$\frac{\partial^2 y}{\partial x^2} = \frac{\mu}{F}\frac{\partial^2 y}{\partial t^2}$	déplacement transversal $y(x,t)$	$c = \sqrt{\frac{F}{\mu}}$	F = tension de la corde μ = masse linéique de la corde	ch.2 n°2
	Ondes de Marée	$\frac{\partial^2 z}{\partial x^2} = \frac{1}{gh}\frac{\partial^2 z}{\partial t^2}$	déplacement de la surface de l'océan $z(x,t)$	$c = \sqrt{g \cdot h}$	h = profondeur de l'océan ($h \ll \lambda$) g = accélération de la pesanteur	ch.4 n°6
	Ondes de torsion dans un cylindre	$\frac{\partial^2 \theta}{\partial z^2} = \frac{\rho}{G}\frac{\partial^2 \theta}{\partial t^2}$	rotation d'un élément $\theta(z,t)$	$c = \sqrt{\frac{G}{\rho}}$	G = module de rigidité ρ = masse volumique	ch.2 n°16
	Ondes électromagnétiques dans le vide illimité	$\frac{\partial^2 B}{\partial x^2} = \sqrt{\varepsilon_0 \mu_0}\frac{\partial^2 B}{\partial t^2}$	champ électrique ou magnétique $B(x,t)$	$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$	ε_0 = permittivité du vide = $\frac{1}{36\pi \cdot 10^9}$ μ_0 = perméabilité du vide = $4\pi \cdot 10^{-7}$	ouvrage « ondes électromagnétique » (Dunod)
Ondes longitudinales	Ondes dans un ressort	$\frac{\partial^2 s}{\partial x^2} = \frac{\mu}{kL}\frac{\partial^2 s}{\partial t^2}$	déplacement longitudinal d'une spire du ressort $s(x,t)$	$c = \sqrt{\frac{kL}{\mu}}$	k = raideur du ressort L = longueur du ressort à vide μ = masse linéique du ressort	ch.2 n°17
	Ondes dans un barreau solide	$\frac{\partial^2 s}{\partial x^2} = \frac{\rho}{E}\frac{\partial^2 s}{\partial t^2}$	déplacement longitudinal de la tranche d'abscisse x $s(x,t)$	$c = \sqrt{\frac{E}{\rho}}$	E = module d'Young ρ = masse volumique	ch.2 n°15
	Ondes dans une ligne électrique	$\frac{\partial^2 i}{\partial x^2} = \Lambda \cdot \Gamma \frac{\partial^2 i}{\partial t^2}$	courant électrique $i(x,t)$	$c = \frac{1}{\sqrt{\Lambda \cdot \Gamma}}$	Λ = inductance linéique Γ = capacité linéique	ch.2 n°18

Classical differential wave equation is describing a wave that has (almost) the same wave-shape during its motion. If presented in one-dimension (traveling along the **x**, or **r**, or t axis), wave functions or solutions of relevant wave equation are composed of two similar waves (or wavelets), traveling with the same phase velocity, **u**, in mutually opposite directions (in the following elaborations marked as (+) and (-) directions).

- 1 -

The generalized wave function of such wave motions (or solution of Classical wave equation), applicable also to arbitrary function shapes can be formulated as $\Psi(t, r) = \Psi^{(+)}(r - ut) + \Psi^{(-)}(r + ut)$, where **u** is the *wave phase velocity*. It can be proven that for all of the particular wave functions such as $\Psi^{(+)}(r - ut)$, or $\Psi^{(-)}(r + ut)$, or

$\Psi^{(+)}(r-ut)+\Psi^{(-)}(r+ut)$, (when the wave propagates in linear media, without dispersion, with the same speed, being independent of wavelength, in both directions, and independent of amplitude), it is applicable the same Classical wave equation. **The most elementary solutions of such classical wave equations are also real, simple harmonic functions, since we know from Fourier and other Signal Analysis practices that arbitrary wave functions can be decomposed and presented as summations of elementary and simple harmonic wave components, such as,**

$$\Psi(r,t)=a\cdot\cos(kr-\omega t)+b\cdot\cos(kr+\omega t)=$$
$$=\Psi^{(+)}(r-ut)+\Psi^{(-)}(r+ut),\ u=\omega/k,\ (a,b)\ =\ \text{constants}.$$

Until here, we are still inside the framework of universally applicable mathematics about waves (well and naturally connected to Physics).

- 2 -

What is an even more important mathematical approach, which is easier and more productive, is that real **components of Classical wave equation can be formally extended and generalized with superposition of similar complex signal forms**, creating complex function such as,

$$\Psi(r,t)\to\overline{\Psi}(r,t)=a\cdot e^{j(kr-\omega t)}+b\cdot e^{j(kr+\omega t)},\ (a,b=\text{constants},\ j^2=-1),$$

what Schrödinger effectively, intuitively and subconsciously applied when he created his wave equation and this was the significant step that facilitated the creation of Quantum theory Wave Equation, as presently known (of course later merged with PWDC; -see about PWDC under (4.1) - (4.3) in chapter 4.1, and (10.1) in Chapter 10).

The transition from the realm of real numbers (or functions) to that of complex numbers (or functions) is a revolutionary step that requires significant mathematical and conceptual adjustments. These adjustments were not previously practiced if we aim to remain within the boundaries of universally applicable mathematics. Using complex functions instead of real functions is akin to extending our three- or four-dimensional understanding of the world to a higher number of dimensions. This shift opens imaginative and philosophical possibilities, especially when considering more advanced concepts like analytic or hypercomplex wavefunctions with multiple imaginary units for conceptualizing multidimensional worlds.

Schrödinger and other pioneers of Quantum Theory did what they could within the historical context of their scientific environment. They formulated contemporary Quantum Theory using partially ad-hoc mathematical postulates. While this framework is operational, it is not as naturally integrated with the broader body of physics and mathematics. Many of the founders and followers of this Quantum Theory were aware of this incoherence but accepted it because artificial construction still worked mathematically and produced satisfactory results within its defined boundaries.

To make sense of this, one must become familiar with a series of unnatural explanations, assumptions, definitions, and ad-hoc concepts, elements of Quantum Theory that were accepted by voting and consensus. These were seen as necessary mathematical tools to satisfy the laws of physics, at least "in average." Given that Quantum Theory has proven effective, one might ask, "Why not accept this original and well-functioning mathematical creation?"

While the founders and followers of Orthodox Quantum Theory accepted this platform, the author of this book proposes a shift in our intellectual creativity toward a more natural and robust mathematical modeling approach.

- 3 -
Obviously, the simple transition from real functions to complex functions is shown to be productive (regarding wave functions and wave equations). What is missing here (in contemporary Quantum theory) is to find a more natural, better connected, and smoother transition from real wavefunctions to complex wavefunctions (and to stay in the frames of universally applicable (natural) mathematics and conceptually clear Physics). In this book, we will simply consider that **solutions of any wave equation are best presentable as Complex, Analytic Signal functions**, created using Hilbert transform "**H**", as shown in (4.0.2) and (4.9), since such Analytic Signal functions are really, well, and naturally, presenting whole wave phenomenology known in Physics, as follows,

$$\Psi(r,t) \to \overline{\Psi}(r,t) = a(r,t)\cdot e^{j(kr-\omega t)} + b(r,t)\cdot e^{j(kr+\omega t)} = \Psi(r,t) + j\hat{\Psi}(r,t) =$$
$$= A(r,t)e^{j\varphi(r,t)} = A(r,t)\cos\varphi(r,t) + j\cdot A(r,t)\sin\varphi(r,t),$$
$$\Psi(r,t) = A(r,t)\cos\varphi(r,t), \hat{\Psi}(r,t) = A(r,t)\sin\varphi(r,t) = H\left[\Psi(r,t)\right].$$

We will see later that such approach (combined with **PWDC**, formulated in Chapter 4.1, around equations (4.1) - (4.3), and in Chapters 4.0 and 10.), can easily produce all important wave equations known in Quantum Theory (and Physics), on a more elegant, natural, simpler, clearer, and more elementary level (without introducing artificial assumptions), compared to contemporary Orthodox Quantum theory mathematical processing. Even entanglement effects (as known in microphysics) are explicable as (mathematical) connections between described, mutually coupled wavefunction-solutions of the same wave equation (for instance, like coupled photons, electrons, and other micro particles). Of course, the wave velocity (here phase velocity, $\mathbf{u} = \omega/\mathbf{k}$) will depend on the medium properties through which the wave is propagating. All other, more complex waveforms and wave packets are presentable as an integral or discrete superposition of elementary, simple harmonic waves, as shown in (4.0.1), including a superposition of elementary waves (4.0.8). In addition, synchronous propagation of two, (or many in-pairs) coupled wave components in mutually opposed directions should always be considered as generally valid for all wave motions (having a much deeper meaning in the world of Physics, than presently seen).

No matter how complex, brilliant, or imaginative the explanations, modeling and postulations in Quantum Mechanics books regarding wavefunctions and wave equations may be, these concepts can also be understood on a simpler and more deterministic level using Complex Analytic Signal wavefunction modeling, an approach that was not originally taken. Schrödinger and his contemporaries replaced and generalized classical wave equations and wavefunctions with simple complex functions, not with complex analytic signals. They then upgraded or postulated the missing mathematical structure based on their best intuitive estimations at the time.

Rather than seeking more natural and accurate mathematical models, proponents of Orthodox Quantum Theory continue to defend and reinforce this artificial and not entirely scientific approach, primarily because it looks powerful and works well mathematically. Many of them argue, "Why change something that works so well?" However, it is worth modifying and beneficial to rectify these "patchwork practices" in contemporary Quantum Theory, particularly in relation to Schrödinger's equation, uncertainty relations, and wavefunctions.

Let us now re-examine the foundations of wave equations and wavefunctions in physics using Complex Analytic Signals as wavefunctions, as discussed in Chapter 4.0. Since all natural and real-world matter waves, pulses, and signals can be decomposed into periodic, elementary sinusoidal waves using Fourier Analysis, we can start by analyzing the motion of these elementary waves in one spatial and one temporal dimension (x, t). This allows us to describe involved velocities, wavelengths, frequencies, signal energy, and power, while also addressing matter waves and wave-particle duality.

This approach will help us develop Schrödinger's and other familiar wave equations in a clear, elegant, and exact manner, without postulations, considering the basic elements and relations of particle-wave duality, which we summarize in this book as PWDC relations. By using Analytic Signal modeling for wavefunctions, we avoid the need to revisit the complicated and sometimes confusing historical interpretations of Schrödinger, Bohr, Heisenberg, P. Dirac, Louis de Broglie, and others when they were developing quantum wave mechanics. Here is the short, self-explanatory resume regarding wave motions of elementary sinusoidal waves:

$$\Psi(x,t) = A\sin(kx \mp \omega t) = A\sin\left[\frac{2\pi}{\lambda}(x \mp ut)\right] = A\sin\left[2\pi(\frac{x}{\lambda} \mp \frac{u}{\lambda}t)\right] = A\sin\left[2\pi(\frac{x}{\lambda} \mp f\cdot t)\right],\ \omega = 2\pi f,\ k = \frac{2\pi}{\lambda},$$

$$\psi(\omega t) = \underline{\psi(x,t)}\big|_{x=0} = \underline{\psi(kx,\omega t)}\big|_{x=0} = \psi(0,\omega t) = \psi\left(\frac{2\pi}{T}\cdot t\right),\ \text{sinusoidal wave only in a time domain } (x = 0)$$

$$\psi(kx) = \underline{\psi(x,t)}\big|_{t=0} = \underline{\psi(kx,\omega t)}\big|_{t=0} = \psi(kx,0) = \psi\left(\frac{2\pi}{\lambda}\cdot x\right),\ \text{sinusoidal wave only in a spatial domain } (t = 0)$$

$$\psi(kx - \omega t) = \psi(2\pi\frac{1}{\lambda}x - 2\pi\frac{1}{T}t) = \psi(2\pi\frac{1}{\lambda}\cdot x - 2\pi f\cdot t),\ \text{sinusoidal wave travels in a positive direction (right)}$$

$$\psi(kx + \omega t) = \psi(2\pi\frac{1}{\lambda}x + 2\pi\frac{1}{T}t) = \psi(2\pi\frac{1}{\lambda}\cdot x + 2\pi f\cdot t),\ \text{sinusoidal wave travels in a negative direction (left)}$$

$$\psi(kx,\omega t) = \psi(kx \mp \omega t) \Leftrightarrow \psi(kx + \omega t) + \psi(kx - \omega t),\ \text{sinusoidal waves travelling in a positive and negative directions (right \& left)} \Leftrightarrow \psi\left[k(x \mp \frac{\omega}{k}t)\right] = \psi\left[k(x \mp ut)\right] \Leftrightarrow \psi(x - ut) + \psi(x + ut).$$

A more general form of a wave function, $\boldsymbol{\Psi(x,t)}$ with an initial constant phase $\boldsymbol{\Phi}$, that shifts the same wave is, $\boldsymbol{\Psi(x,t) = A\sin(k\cdot x - \omega\cdot t + \Phi)}$.

The wave number **k** and the angular frequency $\boldsymbol{\omega = 2\pi f = 2\pi\frac{u}{\lambda}}$ are analogically defined as being directly dependent on involved spatial and temporal periodicities (or periods) such as wavelength $\boldsymbol{\lambda}$ and time period $\boldsymbol{T = \frac{1}{f}}$, meaning $\boldsymbol{k = \boxed{2\pi\cdot\frac{1}{\lambda}},\ \omega = 2\pi f = \boxed{2\pi\cdot\frac{1}{T}}}$.

The **wavefunction phase velocity u** is the velocity of a point on the wave $\boldsymbol{\Psi(x,t)}$ that has a constant phase (for example, its crest), and it is given by relations,

$$\mathbf{u = \lambda\cdot f = \frac{\lambda}{T} = \frac{\omega}{k} = \frac{\tilde{E}}{p}},$$

and on a similar way we define and develop **group wave-function velocity v** as the velocity of the envelope of a wave $\boldsymbol{\Psi(x,t)}$ (that has certain constant amplitude), and it is given by the following relations (see more in Chapters 4.0 and 4.1),

$$\mathbf{v = \frac{d\omega}{dk} = \frac{dE}{dp} = \frac{d\tilde{E}}{dp} = \frac{dE_k}{dp} = u - \lambda\frac{du}{d\lambda} = u + k\frac{du}{dk} = -\lambda^2\frac{df}{d\lambda}}.$$

Now, we can again, briefly repeat and summarize how mathematical approach to wave equations has been evolving from an ordinary classical wave equation (using real functions) towards using complex and hypercomplex functions.

-1-

$$\frac{1}{u^2}\frac{\partial^2\Psi}{\partial t^2}=\frac{\partial^2\Psi}{\partial r^2}(=)\nabla^2\Psi,\ u=\omega/k$$

Classical Wave Equation

This has been the basic and initial model for all other analogical extensions, of wave equations in Physics.

The wave function is a real function.

$$\begin{aligned}\Psi(r,t) &= a\cdot\cos\,(kr-\omega t)+b\cdot\cos\,(kr+\omega t)=\\ &= a\cdot\cos k(r-ut)+b\cdot\cos k(r+ut)=\\ &= \Psi^{(+)}(r-ut)+\Psi^{(-)}(r+ut),\ (a,b)\ = \text{constants}\end{aligned}$$

Solutions are also:

$$\Psi(r,t)=\Psi^{(+)}(r-ut)$$
$$\Psi(r,t)=\Psi^{(-)}(r+ut)$$
$$\Psi(r,t)=\Psi^{(+)}(r-ut)+\Psi^{(-)}(r+ut)$$

(including the convenient integral superposition of similar infinitesimal and elementary wavefunction components)

-2-

$$\frac{1}{u^2}\frac{\partial^2\overline{\Psi}}{\partial t^2}=\frac{\partial^2\overline{\Psi}}{\partial r^2}(=)\nabla^2\overline{\Psi},\ u=\omega/k$$

Complex Function Wave Equation

Schrödinger Equation and Wave Quantum Mechanics evolved from here (combined with certain ad hoc mathematical attachments, and with **PWDC**, as formulated in chapter 4.1, around equations (4.2)).

The wavefunction is a simple complex function.

$$\begin{aligned}\Psi(r,t)\rightarrow\overline{\Psi}(r,t) &= a\cdot e^{j(kr-\omega t)}+b\cdot e^{j(kr+\omega t)}=\\ &= a\cdot e^{jk(r-ut)}+b\cdot e^{jk(r+ut)}=\left|\overline{\Psi}(r,t)\right|e^{j\varphi(r,t)}=\\ &= \overline{\Psi}^{(+)}(r-ut)+\overline{\Psi}^{(-)}(r+ut),\ (a,b)=\text{constants}\end{aligned}$$

Solutions are also:

$$\overline{\Psi}(r,t)=\overline{\Psi}^{(+)}(r-ut)$$
$$\overline{\Psi}(r,t)=\overline{\Psi}^{(-)}(r+ut)$$
$$\overline{\Psi}(r,t)=\overline{\Psi}^{(+)}(r-ut)+\overline{\Psi}^{(-)}(r+ut)$$

(including the convenient integral superposition of similar infinitesimal and elementary wavefunction components)

-3-

$$\frac{1}{u^2}\frac{\partial^2\overline{\Psi}}{\partial t^2}=\frac{\partial^2\overline{\Psi}}{\partial r^2}(=)\nabla^2\overline{\Psi},\ u=\omega/k$$

Analytic Signal Wave Equation

In this book, considering wave functions as Complex, Analytic Signal Wave Functions, Schrödinger, and other wave equations of Wave Quantum Mechanics will be reestablished and generalized on a much simpler, natural, and more elementary way compared to contemporary Quantum theory situation (of course, combined with **PWDC**, formulated in chapter 4.1, around equations (4.2)).

The wave function is an Analytic Signal, complex function.

$$\begin{aligned}\Psi(r,t)\rightarrow\overline{\Psi}(r,t) &= a(r,t)\cdot e^{j(kr-\omega t)}+b(r,t)\cdot e^{j(kr+\omega t)}=\\ &= a(r,t)\cdot e^{jk(r-ut)}+b(r,t)\cdot e^{jk(r+ut)}=\left|\overline{\Psi}(r,t)\right|e^{j\varphi(r,t)}=\\ &= \overline{\Psi}^{(+)}(r-ut)+\overline{\Psi}^{(-)}(r+ut)=\Psi(r,t)+j\hat{\Psi}(r,t)=\\ &= \left[a(r,t)\cos(kr-\omega t)+b(r,t)\cos(kr+\omega t)\right]+\\ &+j\left[a(r,t)\sin(kr-\omega t)+b(r,t)\sin(kr+\omega t)\right]\end{aligned}$$

Solutions are also:

$$\overline{\Psi}(r,t)=\overline{\Psi}^{(+)}(r-ut)$$
$$\overline{\Psi}(r,t)=\overline{\Psi}^{(-)}(r+ut)$$
$$\overline{\Psi}(r,t)=\overline{\Psi}^{(+)}(r-ut)+\overline{\Psi}^{(-)}(r+ut)$$

(including the convenient integral superposition of similar infinitesimal and elementary wavefunction components; -see (4.10-12))

There is another (fourth) option for future explorations of wave functions by exploring that a much more general wavefunction ***will be a Hypercomplex, Analytic Signal, function***, as summarized below, under **-4-** (see more about Hyper-Complex Analytic Signal functions in Chapter 4.0, under equations (6.10) and in Chapter 10). Such approach has a potential to address a variety of micro-world entities appearing in impact and scatterings interactions (see also (4.3-0)-p,q,r,s… in Chapter 4.1, where Minkowski 4-vector of momentum-energy is extended towards hyper-complex space with three imaginary units).

<table>
<tr><td>

-4-

$$\frac{1}{u^2}\frac{\partial^2\overline{\Psi}}{\partial t^2}=\frac{\partial^2\overline{\Psi}}{\partial r^2}(=)\nabla^2\overline{\Psi},\ u=\omega/k,$$

$$\overline{\Psi}=\overline{\Psi}(r,t)=\overline{\Psi}[r(x,y,z),t]= \\ =\overline{\Psi}\{r[x(t),y(t),z(t)],t\}= \\ =\overline{\Psi}[x(t),y(t),z(t),t],$$

$$\frac{\partial}{\partial t}=\frac{d}{dt}-(v\cdot\vec{\nabla}),$$

$$\vec{\nabla}=(\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}),$$

$v(=)$ Frame velocity

Hyper Complex Analytic Signal Wave Equation

Will be left for future explorations.

(Again, such wave equation should be combined with **PWDC**, formulated in chapter 4.1, around equations (4.1) - (4.3)).

</td><td>

The wave function is an Analytic Signal, Hyper-complex function with 3 imaginary units.

$$\Psi(r,t)\rightarrow\overline{\Psi}(r,t)= \\ =a(r,t)\cdot e^{i_1(kr-\omega t)}+b(r,t)\cdot e^{i_1(kr+\omega t)}+ \\ +c(r,t)\cdot e^{i_2(kr-\omega t)}+d(r,t)\cdot e^{i_2(kr+\omega t)}+ \\ +e(r,t)\cdot e^{i_3(kr-\omega t)}+f(r,t)\cdot e^{i_3(kr+\omega t)}= \\ =a(r,t)\cdot e^{i_1k(r-ut)}+b(r,t)\cdot e^{i_1k(r+ut)}+ \\ +c(r,t)\cdot e^{i_2k(r-ut)}+d(r,t)\cdot e^{i_2k(r+ut)}+ \\ +e(r,t)\cdot e^{i_3k(r-ut)}+f(r,t)\cdot e^{i_3k(r+ut)}= \\ =\left|\overline{\Psi}(r,t)\right|e^{I\varphi(r,t)}= \\ =\overline{\Psi}^{(+)}(r-ut)+\overline{\Psi}^{(-)}(r+ut) \\ i_1^2=i_2^2=i_3^2=I^2=-1 \\ i_1i_2=i_3, i_2i_3=i_1, i_3i_1=i_2, i_2i_1=-i_3, i_3i_2=-i_1...$$

Solutions are also:

$$\overline{\Psi}(r,t)=\overline{\Psi}^{(+)}(r-ut) \\ \overline{\Psi}(r,t)=\overline{\Psi}^{(-)}(r+ut) \\ \overline{\Psi}(r,t)=\overline{\Psi}^{(+)}(r-ut)+\overline{\Psi}^{(-)}(r+ut)$$

(including a convenient integral superposition of similar infinitesimal and elementary wavefunction components; -see (4.10-12))

</td></tr>
</table>

In this book, we demonstrate that Schrödinger and other wave equations in Wave Quantum Mechanics can be derived directly from the Classical wave equation, with the wavefunction modeled as an Analytic Signal function (as outlined in Chapter 4.0). This approach reveals that nature, encompassing both macro and micro-physics, is seamlessly unified and that all wave equations are causally connected to the universally applicable Classical Wave Differential Equation. While Schrödinger constructed his equation using "divine inspiration" and intuitive assumptions, we achieve a similar, yet more general result through simple, smooth, and deterministic mathematical steps.

In Orthodox Quantum Theory, the wavefunction is defined probabilistically, without incorporating immediate phase information, which depends on spatial and temporal variables. To obtain meaningful spatial or spatial-temporal distributions or spectrum, this approach relies on probabilistic and statistical modeling, often requiring the identification of wave properties like superposition, diffraction, and interference through observation and testing. In contrast, by treating the wavefunction as a Complex Analytic Signal function, as we do in this book (especially in Chapter 4.0), the wavefunction inherently contains rich spatial and temporal phase and amplitude information, directly related to the principles of Particle-Wave Duality and Causality (PWDC), which are further explored in Chapters 4.1 and 10.

For example, when a solid body is composed of numerous atoms, molecules, or other small particles, these constituents naturally develop a certain level of internal coupling and synchronization, thanks to the wave-particle duality inherent in the particles involved. This perspective allows us to understand that every system of particles has a specific, joint phase function related to de Broglie matter-wave concept. Unfortunately, contemporary Quantum Theory, with its focus on probabilistic wavefunctions, tends to overlook these important phase-related properties.

Let us briefly (in three steps: **A**), **B**) and **C**)) summarize the starting points and results in the process of developing Schrödinger-like equations, **based on an Analytic Signal Wave function concept, as introduced in (4.9-0)**, almost apart (or independently) from any Quantum Theory background and assumptions:

A) By replacing (or extending) an arbitrary wave function $\Psi(t)$, (which could be applicable to represent any kind of waving process, wave-group, or motional particle, regardless if it would have probabilistic or deterministic nature), into its equivalent, temporal-spatial coordinates dependent complex form, $\Psi(t) \to \Psi(x,t) \to \overline{\Psi}(x,t)$, (considering that $\overline{\Psi}(x,t)$ will be treated as the **Complex Analytic Signal**, first time introduced by Dennis Gabor; -see [7] , [8], and Chapter 4.0.; -equations (4.0.1) - (4.0.5)), we will get:

$$\begin{aligned}
&\overline{\Psi}(x,t) = \Psi(x,t) + j\hat{\Psi}(x,t) = a(x,t)e^{j\varphi(x,t)} = \frac{1}{(2\pi)^2}\iint\limits_{(-\infty,+\infty)} U(\omega,k)e^{-j(\omega t-kx)}dkd\omega = \\
&= \frac{1}{\pi^2}\iint\limits_{(0,+\infty)} U(\omega,k)e^{-j(\omega t-kx)}dkd\omega = \frac{1}{\pi^2}\iint\limits_{(0,+\infty)} A(\omega,k)e^{-j(\omega t-kx+\Phi(\omega,k))}dkd\omega\ ,\ j^2 = -1, \qquad (4.9)\\
&U(\omega,k) = U_c(\omega,k) - j\,U_s(\omega,k) = \iint\limits_{(-\infty,+\infty)} \overline{\Psi}(x,t)\, e^{j(\omega t-kx)}dtdx = A(\omega,k)e^{-j\Phi(\omega,k)}\ ,
\end{aligned}$$

where $\hat{\Psi}(x,t)$ is the Hilbert transformation of $\Psi(x,t) = a(x,t)\cos\varphi(x,t)$, or $\hat{\Psi}(x,t) = H[\Psi(x,t)] = a(x,t)\sin\varphi(x,t)$.

More general wavefunction (instead of (4.9)) should be composed of at least two wave functions propagating in mutually opposite directions (see (4.10-12)), such as,

$$\begin{aligned}
&\Psi(x,t) \to \overline{\Psi}(x,t) = a(x,t)\cdot e^{j(kx-\omega t)} + b(x,t)\cdot e^{j(kx+\omega t)} = \\
&= a(x,t)\cdot e^{jk(x-ut)} + b(x,t)\cdot e^{jk(x+ut)} = \left|\overline{\Psi}(x,t)\right| e^{j\varphi(x,t)} = \\
&= \overline{\Psi}^{(+)}(x-ut) + \overline{\Psi}^{(-)}(x+ut) = \Psi(x,t) + j\hat{\Psi}(x,t) = \\
&= [a(x,t)\cos(kx-\omega t) + b(x,t)\cos(kx+\omega t)] + \\
&+ j[a(x,t)\sin(kx-\omega t) + b(x,t)\sin(kx+\omega t)],
\end{aligned}$$

what will not affect later results regarding the development of relevant differential wave equations, since general solutions of mentioned wave equations are always in agreement with $\overline{\Psi}(x,t) = a(x,t)\cdot e^{j(kx-\omega t)} + b(x,t)\cdot e^{j(kx+\omega t)}$, *or more correct in three dimensional spatial and one temporal coordinate, such wave function will be presentable as* $\overline{\Psi}(x,t) = a(x,y,z,t)\cdot e^{j(kr-\omega t)} + b(x,y,z,t)\cdot e^{j(kr+\omega t)}$, $r = r(x,y,z)$.

Solutions of Classical partial differential, second order wave equation are always presentable as two wavefunctions propagating in mutually opposite directions, or inwards and outwards in cases of more complicated spatial structures. This is kind of natural balance between always and coincidently present action and reaction forces, mutual inductions, mirror imaging effects, and it is closely related to respecting conservation laws and symmetries valid in Physics.

In fact, it will be demonstrated that Complex Analytic Signal (4.9) presents much more important (richer and more productive) generic framework for any wave function and wave equation formulation, than what we find in traditionally known Schrödinger and Quantum Mechanics wave equations practices. Starting from (4.9) we can easily develop all variants of Schrödinger, d'Alembert, Classical, and number of other familiar wave equations known in Physics.

Because of mathematical simplicity in developing Schrödinger equation, here we will consider $\overline{\Psi}(x,t)$ as a plane waves superposition (or a wave group, or wave packet) function of one spatial coordinate and a time, but we know that the more general case is $\overline{\Psi}(r,t)=\overline{\Psi}(x,y,z,t)$. We also know that a wave and particle nature of electromagnetic radiation (as photons), and motional particles with non-zero rest masses, can be very well, mutually, and analogically, compared using the concept of wave packets (based on analyzes and explanations of Compton and Photoelectric effects, Brag diffraction etc.). We will simply, and analogically consider here, based on de Broglie matter wave's hypothesis, that the same can be applied to all matter waves and motional particles, as follows:

B) By applying multiple derivations to (4.9), (see the similar procedure in **[5],** pages: 175-179), we will be able to get (4.9-1):

$$\left[\begin{array}{l}
\left\{\begin{array}{l}
\overline{\Psi}(x,t)=\Psi(x,t)+j\hat{\Psi}(x,t)=a(x,t)e^{j\varphi(x,t)}=\frac{1}{(2\pi)^2}\iint\limits_{(-\infty,+\infty)}U(\omega,k)e^{-j(\omega t-kx)}dkd\omega= \\
=\frac{1}{\pi^2}\iint\limits_{(0,+\infty)}U(\omega,k)e^{-j(\omega t-kx)}dkd\omega=\frac{1}{\pi^2}\iint\limits_{(0,+\infty)}A(\omega,k)e^{-j(\omega t-kx+\Phi(\omega,k))}dkd\omega= \\
=\iint\overline{\Psi}''dkd\omega=\iint\overline{\Psi}''dxdt=\Psi+jH[\Psi]=\Psi+j\hat{\Psi},\ \Psi=\mathrm{Re}\,\overline{\Psi},\hat{\Psi}=\mathrm{Im}\,\overline{\Psi}
\end{array}\right\}\Rightarrow \\
\Rightarrow\left\{\begin{array}{l}
\overline{\Psi}'_x=\frac{\partial\overline{\Psi}}{\partial x}=\frac{jk}{(2\pi)^2}U(\omega,k)e^{-j(\omega t-kx)}=jk\overline{\Psi},\ \overline{\Psi}''_x=\frac{\partial^2\overline{\Psi}}{\partial x^2}=jk\frac{\partial\overline{\Psi}}{\partial x}=-k^2\overline{\Psi}\Rightarrow\Delta\overline{\Psi}=-k^2\overline{\Psi}, \\
\overline{\Psi}'_t=\frac{\partial\overline{\Psi}}{\partial t}=\frac{-j\omega}{(2\pi)^2}U(\omega,k)e^{-j(\omega t-kx)}=-j\omega\overline{\Psi},\ \overline{\Psi}''_t=\frac{\partial^2\overline{\Psi}}{\partial t^2}=-j\omega\frac{\partial\overline{\Psi}}{\partial t}=-\omega^2\overline{\Psi}
\end{array}\right\}\Rightarrow \\
\Rightarrow\left[\overline{\Psi}=-\frac{1}{k^2}\frac{\partial^2\overline{\Psi}}{\partial x^2}=-\frac{1}{\omega^2}\frac{\partial^2\overline{\Psi}}{\partial t^2}\Leftrightarrow\frac{\partial^2\overline{\Psi}}{\partial x^2}-\frac{1}{\left(\frac{\omega}{k}\right)^2}\frac{\partial^2\overline{\Psi}}{\partial t^2}=0\Leftrightarrow\frac{\partial^2\overline{\Psi}}{\partial x^2}-\frac{1}{u^2}\frac{\partial^2\overline{\Psi}}{\partial t^2}=0\right]\Rightarrow \\
\Rightarrow\boxed{\nabla^2\overline{\Psi}-\frac{1}{u^2}\frac{\partial^2\overline{\Psi}}{\partial t^2}=\Delta\overline{\Psi}-\frac{1}{u^2}\frac{\partial^2\overline{\Psi}}{\partial t^2}=0\Leftrightarrow\left\{\begin{array}{l}\Delta\Psi-\frac{1}{u^2}\frac{\partial^2\Psi}{\partial t^2}=0\\ \Delta\hat{\Psi}-\frac{1}{u^2}\frac{\partial^2\hat{\Psi}}{\partial t^2}=0\end{array}\right\}}
\end{array}\right]\Rightarrow$$

$$\Rightarrow \begin{Bmatrix} \boxed{-\frac{\omega}{k^2}\Delta\overline{\Psi} = \omega\overline{\Psi} = \frac{-1}{\omega}\frac{\partial^2\overline{\Psi}}{\partial t^2} = -j\frac{\omega}{k}\nabla\overline{\Psi} = j\frac{\partial\overline{\Psi}}{\partial t}} \\ \Downarrow \\ -\frac{\omega}{k^2}\Delta\Psi = \omega\Psi = \frac{-1}{\omega}\frac{\partial^2\Psi}{\partial t^2} = \frac{\omega}{k}\nabla\hat{\Psi} = -\frac{\partial\hat{\Psi}}{\partial t} \\ -\frac{\omega}{k^2}\Delta\hat{\Psi} = \omega\hat{\Psi} = \frac{-1}{\omega}\frac{\partial^2\hat{\Psi}}{\partial t^2} = -\frac{\omega}{k}\nabla\Psi = \frac{\partial\Psi}{\partial t} \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} \boxed{\Delta\overline{\Psi} = \frac{k^2}{\omega^2}\frac{\partial^2\overline{\Psi}}{\partial t^2} = \frac{1}{u^2}\frac{\partial^2\overline{\Psi}}{\partial t^2} = -k^2\overline{\Psi} = jk\nabla\overline{\Psi} = -j\frac{k^2}{\omega}\frac{\partial\overline{\Psi}}{\partial t}} \\ \Downarrow \\ \Delta\Psi = \frac{k^2}{\omega^2}\frac{\partial^2\Psi}{\partial t^2} = \frac{1}{u^2}\frac{\partial^2\Psi}{\partial t^2} = -k^2\Psi = -k\nabla\hat{\Psi} = \frac{k^2}{\omega}\frac{\partial\hat{\Psi}}{\partial t} \\ \Delta\hat{\Psi} = \frac{k^2}{\omega^2}\frac{\partial^2\hat{\Psi}}{\partial t^2} = \frac{1}{u^2}\frac{\partial^2\hat{\Psi}}{\partial t^2} = -k^2\hat{\Psi} = k\nabla\Psi = -\frac{k^2}{\omega}\frac{\partial\Psi}{\partial t} \end{Bmatrix}.$$

(4.9-1)

Equivalent results to (4.9-1) can be, analogically and intuitively demonstrated, without the big need to "*invent*" and artificially attach any missing equation part, or to introduce imaginative and confusing discussions and concepts about waving probabilities and undulatory properties of involved statistical events like in Schrödinger wave equation historical development. Alternatively, if we simply start from the Classical Wave Equation, and analogically replace a real wave function $\Psi(x,t)$ with the corresponding Complex Analytic Signal wavefunction $\overline{\Psi}(x,t)$, and develop and combine relevant derivatives, we get again (4.9-1),

$$\begin{Bmatrix} \Psi(r,t) \to \overline{\Psi}(r,t) = a(r,t)\cdot e^{j(kr-\omega t)} + b(r,t)\cdot e^{j(kr+\omega t)} = \\ = a(r,t)\cdot e^{jk(r-ut)} + b(r,t)\cdot e^{jk(r+ut)} = \left|\overline{\Psi}(r,t)\right| e^{j\varphi(r,t)} = \\ = \overline{\Psi}^{(+)}(r-ut) + \overline{\Psi}^{(-)}(r+ut) = \Psi(r,t) + j\hat{\Psi}(r,t) = \\ = \left[a(r,t)\cos(kr-\omega t) + b(r,t)\cos(kr+\omega t)\right] + \\ + j\left[a(r,t)\sin(kr-\omega t) + b(r,t)\sin(kr+\omega t)\right] \end{Bmatrix} \to$$

$$\begin{bmatrix} \frac{1}{u^2}\frac{\partial^2\Psi}{\partial t^2} = \frac{\partial^2\Psi}{\partial x^2}(=)\nabla^2\Psi,\ u=\omega/k \\ \\ \Psi(x,t) \to \overline{\Psi}(x,t) = \\ = a\cdot e^{j(kx-\omega t)} + b\cdot e^{j(kx+\omega t)} = \\ = \left[a\cdot\cos(kx-\omega t) + b\cdot\cos(kx+\omega t)\right] + \\ + j\left[a\cdot\sin(kx-\omega t) + b\cdot\sin(kx+\omega t)\right] \end{bmatrix} \to \begin{bmatrix} \frac{1}{u^2}\frac{\partial^2\overline{\Psi}}{\partial t^2} = \frac{\partial^2\overline{\Psi}}{\partial x^2}(=)\nabla^2\overline{\Psi},\ u=\omega/k \\ \\ \overline{\Psi}(x,t) = \Psi(x,t) + j\hat{\Psi}(x,t) = \\ = a(x,t)\cdot e^{j(kx-\omega t)} + b(x,t)\cdot e^{j(kx+\omega t)} = \\ = \left[a(x,t)\cos(kx-\omega t) + b(x,t)\cos(kx+\omega t)\right] + \\ + j\left[a(x,t)\sin(kx-\omega t) + b(x,t)\sin(kx+\omega t)\right] \end{bmatrix} \Rightarrow$$

$$\Rightarrow \left[\text{eventually resulting in partial differential wave equations as in } (4.9\text{-}1)\right].$$

Or we can analogically start from the second order partial differential wave equation (of the real wavefunction), or from Classical Wave Equation, transform it into similar wave equation where wavefunction will be treated as the Complex Analytic Signal, and we will again get the same forms of Schrödinger wave equations (4.9-1), without any mathematical patchwork, assumptions, and postulations, as follows.

$$\left\{\begin{array}{l}\frac{\partial^2\Psi}{\partial x^2}-\frac{1}{u^2}\frac{\partial^2\Psi}{\partial t^2}=0\\ \Psi(x,t)\rightarrow\overline{\Psi}(x,t)=\\ =\overline{\Psi}^{(+)}(x-ut)+\overline{\Psi}^{(-)}(x+ut)\\ =a(x,t)\cdot e^{j(kx-\omega t)}+b(x,t)\cdot e^{j(kx+\omega t)}\\ x\rightarrow r(x,y,z)\end{array}\right\}\rightarrow\left\{\begin{array}{l}\frac{\partial^2\overline{\Psi}}{\partial r^2}-\frac{1}{u^2}\frac{\partial^2\overline{\Psi}}{\partial t^2}=\Delta\overline{\Psi}-\frac{1}{u^2}\frac{\partial^2\overline{\Psi}}{\partial t^2}=0\\ \overline{\Psi}(r,t)=\overline{\Psi}^{(+)}(r-ut)+\overline{\Psi}^{(-)}(r+ut)\\ =a(r,t)\cdot e^{j(kx-\omega t)}+b(r,t)\cdot e^{j(kx+\omega t)}\end{array}\right\}\Rightarrow$$

$$\Rightarrow\left\{\begin{array}{l}\Delta\Psi-\frac{1}{u^2}\frac{\partial^2\Psi}{\partial t^2}=0\\ \Psi(r,t)=\Psi^{(+)}(r-ut)+\Psi^{(-)}(r+ut)\\ \text{---}\\ \Delta\hat{\Psi}-\frac{1}{u^2}\frac{\partial^2\hat{\Psi}}{\partial t^2}=0\\ \hat{\Psi}(r,t)=\hat{\Psi}^{(+)}(r-ut)+\hat{\Psi}^{(-)}(r+ut)\end{array}\right\}\Leftrightarrow(4.9-1)\Rightarrow$$

$$\Rightarrow\left\{\begin{array}{l}\overline{\Psi}(r,t)=\overline{\Psi}^{(+)}(r-ut)+\overline{\Psi}^{(-)}(r+ut)=\Psi(r,t)+j\hat{\Psi}(r,t)=\\ =\left[\Psi^{(+)}(r-ut)+\Psi^{(-)}(r+ut)\right]+j\left[\hat{\Psi}^{(+)}(r-ut)+\hat{\Psi}^{(-)}(r+ut)\right]=\\ =\left[\Psi^{(+)}(r-ut)+j\hat{\Psi}^{(+)}(r-ut)\right]+\left[\Psi^{(-)}(r+ut)+j\hat{\Psi}^{(-)}(r+ut)\right]=\\ =a(r,t)\cdot e^{jk(r-ut)}+b(r,t)\cdot e^{jk(r+ut)}=\left|\overline{\Psi}(r,t)\right|e^{j\varphi(r,t)}\end{array}\right\}$$

This discussion is crucial to demonstrate what a natural, logical, smooth, and deterministic explanation of matter waves, wavefunctions, and the Schrödinger equation truly entails. Specifically, it shows how multiple opposing wave components, solutions to the Classical Wave Equations propagating in opposite directions, are synchronously generated and remain mutually coupled. This approach also addresses local, non-local, and other entanglement phenomena related to matter waves, photons, electrons, and atomic diffraction, superposition, and interference effects, without relying on "mathematical patchwork," probabilistic or statistical assumptions, or the arbitrary and simplistic "Bob and Alice" discussions often found in interpretations of Quantum Theory.

In this context, we place greater emphasis on the de Broglie–Bohm interpretation of Quantum Theory. However, it is important to note that both Orthodox Quantum Mechanics and the de Broglie-Bohm theory, despite their effectiveness, are artificial mathematical constructs. They are built on a foundation of postulated Schrödinger equation, with axioms and postulates that predominantly operate with abstract concepts, selected and merged under the principle of "whatever works and produces useful results." But de Broglie-Bohm theory with its pilot-wave function is very close or can be fully equivalent to Analytic Signal modelling of matter-waves, when Schrödinger equation can be simply and smoothly developed (without artificial patchwork assumptions and postulation).

C) Now, by implementing the following **P**article-**W**ave **D**uality **C**ode relations into (4.9) and (4.9-1), (as **PWDC** introduced in (4.1) - (4.3), and in Chapter 10),

$$\boxed{m^*=\gamma m, p=\gamma mv=m^*v=\tilde{p},\ \tilde{E}=hf=\hbar\omega=m^*vu=pu,\ u=\frac{\omega}{k}=\lambda f, v=\frac{d\omega}{dk}=u-\lambda\frac{du}{d\lambda}, \lambda=\frac{h}{p}}$$

we will be able to formulate several matter wave equations belonging to a family of Schrödinger-like equations, as follows (see such equations from (4.9-2) to (4.10-6)):

(1)

Let us start with pure matter waves (without rest masses), propagating in a free space without presence of potential-field or central forces, with potential energy, which is here $\mathbf{U_p = 0}$. Of course, the energy-momentum content of matter wave in question would have certain energy-mass equivalent (which is not a rest mass). Examples of such phenomena are electromagnetic waves and photons (propagating in a vacuum, far from any big masses agglomerations). In a few steps, from equations (4.9-1) we will get,

matter-waves equations in a free space, without rest mass

$$\begin{aligned}
&\frac{\hbar^2}{\tilde{\mathbf{m}}}\left(\frac{\mathbf{u}}{\mathbf{v}}\right)\Delta\overline{\Psi} + \tilde{\mathbf{E}}\,\overline{\Psi} = \mathbf{0},\ (\mathbf{U_p} = \mathbf{0}),\\
&\frac{\hbar^2}{\tilde{\mathbf{m}}}\left(\frac{\mathbf{u}}{\mathbf{v}}\right)\Delta\overline{\Psi} = -\mathbf{j}\hbar\frac{\partial\overline{\Psi}}{\partial\mathbf{t}} = -\tilde{\mathbf{E}}\,\overline{\Psi} = \frac{\hbar^2}{\tilde{\mathbf{E}}}\frac{\partial^2\overline{\Psi}}{\partial\mathbf{t}^2} = \mathbf{j}\hbar\mathbf{u}\nabla\overline{\Psi},\\
&\left(\frac{\tilde{\mathbf{E}}}{\hbar}\right)^2\cdot\overline{\Psi} + \frac{\partial^2\overline{\Psi}}{\partial\mathbf{t}^2} = \mathbf{0},\ \frac{\partial\overline{\Psi}}{\partial\mathbf{t}} + \mathbf{u}\nabla\overline{\Psi} = \mathbf{0},\\
&\Delta\overline{\Psi} - \frac{\mathbf{1}}{\mathbf{u}^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial\mathbf{t}^2} = \left(\frac{\tilde{\mathbf{E}}}{\hbar\mathbf{u}}\right)^2\overline{\Psi} + \frac{\mathbf{1}}{\mathbf{u}^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial\mathbf{t}^2} = \mathbf{j}\frac{\tilde{\mathbf{E}}}{\hbar\mathbf{u}^2}\frac{\partial\overline{\Psi}}{\partial\mathbf{t}} + \frac{\mathbf{1}}{\mathbf{u}^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial\mathbf{t}^2} = \mathbf{0}\,.
\end{aligned} \tag{4.9-2}$$

As the significant support to (4.9-1) and (4.9-2), we could draw relevant equivalent conclusions by comparing well-known examples of similar classical, electromagnetic wave equations dealing with electric, $\mathbf{E}$, and magnetic, $\mathbf{B}$, field-vectors,

$$\begin{aligned}
&\Delta\mathbf{E} - \varepsilon_0\mu_0\frac{\partial^2\mathbf{E}}{\partial\mathbf{t}^2} = \Delta\mathbf{E} - \frac{\mathbf{1}}{\mathbf{u}^2}\frac{\partial^2\mathbf{E}}{\partial\mathbf{t}^2} = \mathbf{0},\\
&\Delta\mathbf{B} - \varepsilon_0\mu_0\frac{\partial^2\mathbf{B}}{\partial\mathbf{t}^2} = \Delta\mathbf{B} - \frac{\mathbf{1}}{\mathbf{u}^2}\frac{\partial^2\mathbf{B}}{\partial\mathbf{t}^2} = \mathbf{0},\ \mathbf{u} = \frac{\mathbf{1}}{\sqrt{\varepsilon_0\mu_0}} = \mathbf{c}\,.
\end{aligned} \tag{4.9-2.1}$$

In reference [86], Victor Christianto, while in a rational and scientific frame of mind, elaborated on the correspondence, analogy, and mutual transformability between the classical wave equations of electromagnetic waves (4.9-2.1) and the quantum mechanical wave equations, including the Klein-Gordon and Schrödinger equations. He accomplished this by invoking and summarizing 'Ward & Volkmer's derivation of the Schrödinger equation from the Classical Wave equation,' effectively demonstrating how the Electromagnetic Classical Wave Equation can systematically evolve into the Schrödinger equation.

Additionally, reference [105], authored by Himanshu Chauhan, Swati Rawal, and R.K. Sinha, titled "Wave-Particle Duality Revitalized: Consequences, Applications, and Relativistic Quantum Mechanics", presents equivalent discussions on the evolution of Schrödinger-family differential wave equations towards the Classical wave equation, and vice-versa. These discussions are combined with a correct understanding of wave-particle duality foundations, referred to in this book as PWDC (with (4.1) - (4.3) and in Chapter 10).

The following resume (literature ref. under [86] and [39]) was summarized by Victor Christianto in his younger and still rational intellectual phase, before he started to work for J. C. He made his resume about Schrödinger wave equation based on publications and works of George Shpenkov (see literature references under [85], https://shpenkov.com/).

"A Review of Schrödinger Equation & Classical Wave Equation

Schrödinger equation

George Shpenkov points out that there are several weaknesses associated with (spherical solution of) Schrödinger's equation:

i. Its spherical solution is rarely discussed completely (especially in graduate or undergraduate quantum mechanics textbooks), perhaps because many physicists seem to feel obliged to hide from public that the spherical solution of Schrödinger's wave equation does not agree with any experiment.

ii. Schrödinger equation is able only to arrive at hydrogen energy levels, and it has to be modified and simplified for other atoms. For example, physicists are forced to use an approximate approach called Density Functional Theory (DFT) in order to deal with N-body system.1

iii. The introduction of variable wave number k in Schrödinger equation, depending on electron coordinates, and the omission of the azimuth part of the wave function, were erroneous [6]. Schrödinger's variable wave number should be questioned, because the potential function cannot influence the wave speed or consequently the wave number.

iv. Introduction of the potential function V in the wave equation, which results in dependence of the wave number k on the Coulomb potential, *generates divergences* that do not have physical justification. They are eliminated in an artificial way. [6, p.27]

v. Modern physics erroneously interprets the meaning of polar-azimuthal functions in Schrödinger's equation, ascribing these functions to atomic "*electron orbitals*". [1, p.5]

vi. Schrödinger arrived at a "correct" result of hydrogen energy levels using only a radial solution of his wave equation, with one major assumption: the two quantum numbers found in the solution of his wave equation were assumed to be the same with Bohr's quantum number.

vii. Quantum mechanics solutions, in their modern form, contradict reality because on the basis of these solutions, the existence of crystal substances-spaces is not possible. [6, p.26]

viii. Schrödinger's approach yields abstract phenomenological constructions, which do not reflect the real picture of the micro-world.[2]

ix. Schrödinger himself in his 1926 paper apparently wanted to interpret his wave equation in terms of vibration of string [3][4]. This is why he did not accept Born's statistical interpretation of his wave equation until he died. Einstein and de Broglie also did not accept the statistical interpretation of quantum mechanics.

x. The interpretation and the physical meaning of the Schrödinger's wave function was a problem for physicists, and it still remains so, although many researchers understand its conditional character [6]."

Of course, we could also make attempts to explore the evolution of classical electromagnetic (and other) wave equations like (4.9-2.1), when electromagnetic vectors and associated waves will take **Complex, or Hyper-Complex Analytic Signal forms**, eventually transforming (4.9-2.1) to wave equations as found in (4.9-2). See also familiar elaborations (3.7-1) and (3.7-2) from the third chapter of this book. *Nevertheless, regarding all wave equations, starting from (4.9) until (4.9-2.1), we should also consider as extremely relevant and unavoidable critical comments, elaborations, and proposals (found in [35]) from Thomas E. Phipps, Jr., about Maxwell equations, that can be equally applicable to all quantum theory wave equations.*

(2)

Next step will consider that matter waves phenomenology is also presentable (directly, indirectly, and analogically) as a motion of certain energy-momentum formation in an energy field of some bigger particle (which creates a potential field with energy U_p), still without rest mass involvement. For such situation, new, generalized Schrödinger-like equation, developed from (4.9-1), will evolve to its final form considering certain process of "wave-energy translation", as follows:

matter-waves equations in a negative potential energy field, without rest mass

$$\left[\tilde{E} \to \tilde{E}' = \tilde{E} - U_p,\ U_p \leq 0\right] \Rightarrow \left[\overline{\Psi} \to \overline{\Psi}',\ m^* \to m^*{}'...\right] \Rightarrow$$

$$-\frac{\hbar^2}{m^*{}'}(\frac{u}{v})'\Delta\overline{\Psi}' = j\hbar\frac{\partial\overline{\Psi}'}{\partial t} = (\tilde{E}-U_p)\overline{\Psi}' = \frac{-\hbar^2}{\tilde{E}-U_p}\frac{\partial^2\overline{\Psi}'}{\partial t^2} = -j\hbar u'\nabla\overline{\Psi}', \tag{4.9-3}$$

$$\left(\begin{array}{l} \Delta\overline{\Psi}' - \frac{1}{u'^2}\cdot\frac{\partial^2\overline{\Psi}'}{\partial t^2} = \left(\frac{\tilde{E}-U_p}{\hbar u'}\right)^2\overline{\Psi}' + \frac{1}{u'^2}\cdot\frac{\partial^2\overline{\Psi}'}{\partial t^2} = j\frac{\tilde{E}-U_p}{\hbar u'^2}\frac{\partial\overline{\Psi}'}{\partial t} + \frac{1}{u'^2}\cdot\frac{\partial^2\overline{\Psi}'}{\partial t^2} = 0, \\ \frac{\hbar^2}{m^*{}'}(\frac{u}{v})'\Delta\overline{\Psi}' + (\tilde{E}-U_p)\overline{\Psi} = 0 \end{array}\right)$$

Now, we can simply come back from (4.9-3) to usual wave functions notation, as,

$$\left[\overline{\Psi}' \to \overline{\Psi},\ m^*{}' \to m^*,\ \tilde{E}' = m^*{}'u'v' = \tilde{E} - U_p\right] \Rightarrow$$

$$\frac{\hbar^2}{m^*}(\frac{u}{v})\Delta\overline{\Psi} + (\tilde{E}-U_p)\overline{\Psi} = 0,$$

$$\frac{\hbar^2}{m^*}(\frac{u}{v})\Delta\overline{\Psi} - U_p\overline{\Psi} = -\tilde{E}\,\overline{\Psi} = -j\hbar\frac{\partial\overline{\Psi}}{\partial t} - U_p\overline{\Psi} = \frac{\hbar^2}{\tilde{E}-U_p}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2} - U_p\overline{\Psi}, \tag{4.10}$$

$$(\frac{\tilde{E}-U_p}{\hbar})^2\cdot\overline{\Psi} + \frac{\partial^2\overline{\Psi}}{\partial t^2} = 0\ ,\ \frac{\partial\overline{\Psi}}{\partial t} + u\nabla\overline{\Psi} = 0,$$

$$\Delta\overline{\Psi} - \frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2} = \left(\frac{\tilde{E}-U_p}{\hbar u}\right)^2\overline{\Psi} + \frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2} = j\frac{\tilde{E}-U_p}{\hbar u^2}\frac{\partial\overline{\Psi}}{\partial t} + \frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2} = 0.$$

The same equations from (4.10) can be transformed (or abbreviated) into operators' form:

$$\left(H = -\frac{\hbar^2}{m^*}(\frac{u}{v})\Delta + U_p\ (=)\ \text{Hamiltonian}\right) \Rightarrow \left\{H\overline{\Psi} = \tilde{E}\,\overline{\Psi} = j\hbar\frac{\partial}{\partial t}\overline{\Psi} + U_p\overline{\Psi} = ...\right\}$$

$$\Rightarrow \tilde{E} \Leftrightarrow H \Leftrightarrow j\hbar\frac{\partial}{\partial t} + U_p \Leftrightarrow -j\hbar u\nabla\ , \tag{4.11}$$

$$\left(\tilde{p}_i\overline{\Psi} = -j\hbar\nabla\overline{\Psi}\right) \Rightarrow \tilde{p}_i \Leftrightarrow -j\hbar\nabla \Leftrightarrow \frac{1}{u}(j\hbar\frac{\partial}{\partial t} + U_p) \Leftrightarrow \frac{1}{u}H.$$

In cases of non-relativistic velocities, after replacing relation between group and phase velocity with its approximate value $v \cong 2u$, *generalized Schrödinger's equation (4.10),* $\frac{\hbar^2}{m^*}(\frac{u}{v})\Delta\overline{\Psi} + (\tilde{E}-U_p)\overline{\Psi} = 0$ *turn into the traditionally known (original and non-relativistic) Schrödinger's wave equation,*

$$\left\{\begin{array}{c}\boxed{j\overline{\Psi}\frac{\partial\overline{\Psi}}{\partial t}=-\frac{\hbar^2}{2m^*}\Delta\overline{\Psi}+U_p\overline{\Psi}}=\tilde{E}\overline{\Psi}\\ \Updownarrow\\ \boxed{\begin{array}{l}\Psi\frac{\partial\hat{\Psi}}{\partial t}+\hat{\Psi}\frac{\partial\hat{\Psi}}{\partial t}=\frac{\hbar^2}{2m^*}\Delta\Psi-U_p\Psi=-\tilde{E}\Psi\\ \Psi\frac{\partial\Psi}{\partial t}+\hat{\Psi}\frac{\partial\Psi}{\partial t}=-\frac{\hbar^2}{2m^*}\Delta\hat{\Psi}+U_p\hat{\Psi}=\tilde{E}\hat{\Psi}\end{array}}\end{array}\right\}\Leftrightarrow\left\{\begin{array}{c}\boxed{\frac{\hbar^2}{2m^*}\Delta\overline{\Psi}+(\tilde{E}-U_p)\overline{\Psi}=0}\\ \Updownarrow\\ \boxed{\begin{array}{l}\frac{\hbar^2}{2m^*}\Delta(\Psi+j\hat{\Psi})+(\tilde{E}-U_p)(\Psi+j\hat{\Psi})=0\\ \frac{\hbar^2}{2m^*}\Delta\Psi+(\tilde{E}-U_p)\Psi=0\\ \frac{\hbar^2}{2m^*}\Delta\hat{\Psi}+(\tilde{E}-U_p)\hat{\Psi}=0\end{array}}\end{array}\right\}\Rightarrow$$

$$\Psi\frac{\partial(\Psi+\hat{\Psi})}{\partial t}+\hat{\Psi}\frac{\partial(\Psi+\hat{\Psi})}{\partial t}=\frac{\hbar^2}{2m^*}\Delta(\Psi-\hat{\Psi})-U_p(\Psi-\hat{\Psi})=-\tilde{E}(\Psi-\hat{\Psi})$$

$$\Psi\frac{\partial\Psi^{*+}}{\partial t}+\hat{\Psi}\frac{\partial\Psi^{*+}}{\partial t}=\frac{\hbar^2}{2m^*}\Delta\Psi^{*-}-U_p\Psi^{*-}=-\tilde{E}\Psi^{*-},\Psi^{*+}=\Psi+\hat{\Psi},\Psi^{*-}=\Psi-\hat{\Psi}$$

$$\frac{\hbar^2}{2m^*}\Delta(\Psi+\hat{\Psi})+(\tilde{E}-U_p)(\Psi+\hat{\Psi})=0\Leftrightarrow\frac{\hbar^2}{2m^*}\Delta\Psi^{*+}+(\tilde{E}-U_p)\Psi^{*+}=0, \qquad (4.12)$$

which (according to contemporary Quantum Theory literature) has the universal applicability in a micro world of physics, and has an almost divine origin, created in an extraordinary period of Schrödinger's intellectual illumination (when he was especially and multidisciplinary motivated). Fortunately, the same situation is much different from being divine, and it is much more deterministic, and very well explicable in quite simple terms (when treating wavefunction as an Analytic Signal, as established in Chapter 4.0). This is also an example of how obviously useful, but still incomplete equation, in a creative process, luckily well completed by trial-&-error attempts, being appropriately hybridized, and upgraded, eventually materialized as celebrated part of foundations of our modern physics (but all that divine and magical process could also be very simply and deterministically organized). In conclusion, based on everything elaborated until here, we can summarize the updated process of ***development of Schrödinger equation*** *as:*

1°
$$\boxed{\begin{array}{l}\Psi(x,t)\rightarrow\overline{\Psi}(x,t)=\Psi(x,t)+j\hat{\Psi}(x,t)=a(x,t)e^{j\varphi(x,t)}=\left|\overline{\Psi}(x,t)\right|e^{j\varphi(x,t)}\\ x\rightarrow x,y,z\ ,r=r(x,y,z),\ x\rightarrow r,\\ \Psi(r,t)=\mathrm{Re}\,\overline{\Psi}(r,t)=,\ \hat{\Psi}(r,t)=\mathrm{Im}\,\overline{\Psi}(r,t)=H\left[\Psi(r,t)\right],\\ \Psi(r,t)\rightarrow\overline{\Psi}(r,t)=\overline{\Psi}=\Psi(r,t)+j\hat{\Psi}(r,t)=a(r,t)e^{j\varphi(x,t)}=\left|\overline{\Psi}(r,t)\right|e^{j\varphi(x,t)}\end{array}}$$

Wavefunction is a Complex Analytic Signal function

$$\Downarrow$$

2°
$$\left(\Delta\Psi-\frac{1}{u^2}\frac{\partial^2\Psi}{\partial t^2}=0\right)\rightarrow\left(\Delta\overline{\Psi}-\frac{1}{u^2}\frac{\partial^2\overline{\Psi}}{\partial t^2}=0\right)$$

Clasical Wave Equation is replaced by Complex Analytic Wave Equation

$$\oplus$$

3°
$$m^*=\gamma m,p=\gamma mv=m^*v=\tilde{p},\ \tilde{E}=hf=\hbar\omega=m^*vu=pu,\ u=\frac{\omega}{k}=\lambda f,v=\frac{d\omega}{dk}=u-\lambda\frac{du}{d\lambda},\lambda=\frac{h}{p}$$

$$d\tilde{E}=hdf=dE_k=c^2d(\gamma m)=vdp=d(pu)=-c^2d\tilde{m}=-vd\tilde{p}=-d(\tilde{p}u)$$

$$\tilde{E}=\int_{-\infty}^{+\infty}\Psi^2(t)dt=\int_{-\infty}^{+\infty}\hat{\Psi}^2(t)dt=\int_{-\infty}^{+\infty}\left|\frac{\overline{\Psi}(t)}{\sqrt{2}}\right|^2dt=\int_{-\infty}^{+\infty}\left[\frac{a(t)}{\sqrt{2}}\right]^2dt=\frac{1}{2}\int_{-\infty}^{+\infty}\left[\Psi^2(t)+\hat{\Psi}^2(t)\right]dt=$$

$$=\frac{1}{2}\int_{-\infty}^{+\infty}\left\{\left[\Psi^+(t),\Psi^-(t)\right]^2+\left[\hat{\Psi}^+(t),\hat{\Psi}^-(t)\right]^2\right\}dt=\ ...$$

PWDC will be merged with Complex Analytic Wave Equation

$$\Downarrow$$

4° $\left(j\overline{\Psi}\frac{\partial\overline{\Psi}}{\partial t} = -\frac{\hbar^2}{2m^*}\Delta\overline{\Psi} + U_p\overline{\Psi} = \tilde{E}\overline{\Psi} \right) \Rightarrow \left(\frac{\hbar^2}{2m^*}\Delta\overline{\Psi} + (\tilde{E} - U_p)\overline{\Psi} = 0 \right)$

Schrödinger wave equation

$\Downarrow$

5° **Conclusions and consequences:**

- **Wavefunction will be modeled as the Complex Analytic Signal**
- **Hystorical development of Schrödinger equation will remain only as a non-optimal circumstantial reference**
- **Orthodox and other interpretations of QM can be revized, simplified and optimized**
- **Statistics and probability in relation to Wavefunction will lose their excusive ontological qualifications**

The above-presented forms of generalized Schrödinger's equation (4.9-2) and (4.10)) consider only motional wave energy as a wave-packet energy. This is the biggest difference between classical interpretation of Schrödinger's equation from contemporary quantum theory (which usually takes a particle with its rest-mass energy into account), and all wave-equations, (4.10) - (4.12), developed in this book.

We can also notice (and demonstrate) that Dirac's relativistic modification of Schrödinger's equation is (or should be) automatically included in equations (4.10) - (4.12), since the energy $\tilde{E}$ and ratio u/v are already treated as relativistic, velocity-dependent functions (in relation to Lorentz transformations; -see (4.1) - (4.3)).

In (4.10) we made wave-energy level-translation for the potential field energy, $\tilde{E} \to \tilde{E}' = \tilde{E} - U_p, U_p \neq 0$, and we got the following, familiar-looking (classical) Schrödinger's equation (since when $\mathbf{v << c \Rightarrow u/v = 1/2}$),

$$\frac{\hbar^2}{m^*}(\frac{u}{v})\Delta\overline{\Psi}+(\tilde{E} - \mathbf{U_p})\overline{\Psi} = 0 \Rightarrow \frac{\hbar^2}{2m^*}\Delta\overline{\Psi}+(\tilde{E} - \mathbf{U_p})\overline{\Psi} = 0.$$

"Schrödinger likely did not have knowledge of Analytic Signal functions and the Hilbert transformation when he, through intuition, productive trial and error, and brainstorming (perhaps even divine inspiration), developed his famous equation by adding the missing imaginary part. The concept of the Analytic Signal was theoretically established by Dennis Gabor much later. Nonetheless, Schrödinger intuitively and effectively created an equation that, whether intentionally or not, aligns with the Analytic Signal model. This equation has proven to be particularly useful and productive in Quantum Physics, gaining acceptance as an essential tool for describing the wave-particle duality of the microscopic world after numerous verifications.

In this chapter, we will demonstrate in a straightforward manner how the complex analytic signal-based Classical Wave Equation (4.9-1) logically and naturally generates Schrödinger's equation and other familiar wave equations in Quantum Theory. Unlike the early days of Quantum Theory, where divine inspiration, brilliant minds, ad hoc assumptions, and mathematical patchwork were required, this approach eliminates the need for such methods.

As a result, this book shows that micro-physics and early Quantum Theory concepts related to Particle-Wave Duality and Matter waves are more closely connected to Classical and Relativistic Physics (including Mechanics) than Orthodox Quantum Theory suggests, especially when the wavefunction and Schrödinger equation are treated differently. At the same time, we can appreciate the many mathematically adventurous, imaginative, and productive concepts developed in modern Quantum Theory, ideas that could never have emerged within the deterministic framework of Classical Physics and Mechanics.

However, it is important to note that modern Quantum Theory, including the Standard Model of Particle Physics, remains mathematically and conceptually incomplete. It is still somewhat artificial, overly complicated, and lacks the necessary mathematical foundations and realistic modeling to describe the physical world more naturally. For example, by treating the wavefunction as an Analytic Signal, as discussed in Chapters 4.0 and 10, we can make significant contributions to Wave Mechanics and Quantum Theory, leading to a more natural conceptualization of Particle-Wave Duality. The Standard Model of matter in our Universe should be updated with a more natural and robust mathematical framework or modeling."

(3)

Let us now create another *wave-energy level translation* in a potential field, to "capture" the total (relativistic) energy of a moving particle (which has its rest mass $\mathbf{m_0 = E_0 / c^2 = const.}$), as follows. $\tilde{\mathbf{E}} \to \tilde{\mathbf{E}}' = \tilde{\mathbf{E}} + \mathbf{E_0} - \mathbf{U_p}, \mathbf{U_p} \neq 0, \mathbf{E_0} = \mathbf{const.}$, (understanding that relevant energy equals the sum of motional energy, $E_k = \tilde{E}$, and rest energy $\mathbf{E_0}$, reduced for the energy of surrounding potential field U_p (since we associate negative values to potential energy), this way creating relevant Lagrangian), and let us apply such "*energy translation*" on the wave equations (4.9-2), as for instance,

matter-waves equations in a negative potential energy field, with rest mass involvement

$$\left[\tilde{\mathbf{E}} \to \tilde{\mathbf{E}}' = \tilde{\mathbf{E}} + \mathbf{E_0} - \mathbf{U_p} = \mathbf{E_{total}} - \mathbf{U_p}, (\mathbf{U_p} \leq 0, \mathbf{E_0} = \mathbf{const.})\right] \Rightarrow \left[\overline{\Psi} \to \overline{\Psi}', \mathbf{m}^* \to \mathbf{m}^{*}{'} ...\right] \Rightarrow$$

$$-\frac{\hbar^2}{\mathbf{m}^{*}{'}}(\frac{\mathbf{u}}{\mathbf{v}})'\Delta\overline{\Psi}' = \mathbf{j}\hbar\frac{\partial\overline{\Psi}'}{\partial \mathbf{t}} = (\tilde{\mathbf{E}} + \mathbf{E_0} - \mathbf{U_p})\overline{\Psi}' = \frac{-\hbar^2}{\tilde{\mathbf{E}} + \mathbf{E_0} - \mathbf{U_p}}\frac{\partial^2\overline{\Psi}'}{\partial \mathbf{t}^2} = -\mathbf{j}\hbar\mathbf{u}'\nabla\overline{\Psi}',$$

$$\begin{pmatrix} \Delta\overline{\Psi}' - \frac{1}{\mathbf{u}'^2}\cdot\frac{\partial^2\overline{\Psi}'}{\partial \mathbf{t}^2} = \left(\frac{\tilde{\mathbf{E}} + \mathbf{E_0} - \mathbf{U_p}}{\hbar\mathbf{u}'}\right)^2\overline{\Psi}' + \frac{1}{\mathbf{u}'^2}\cdot\frac{\partial^2\overline{\Psi}'}{\partial \mathbf{t}^2} = \mathbf{j}\frac{\tilde{\mathbf{E}} + \mathbf{E_0} - \mathbf{U_p}}{\hbar\mathbf{u}'^2}\frac{\partial\overline{\Psi}'}{\partial \mathbf{t}} + \frac{1}{\mathbf{u}'^2}\cdot\frac{\partial^2\overline{\Psi}'}{\partial \mathbf{t}^2} = \mathbf{0}, \\ \frac{\hbar^2}{\mathbf{m}^{*}{'}}(\frac{\mathbf{u}}{\mathbf{v}})'\Delta\overline{\Psi}' + (\tilde{\mathbf{E}} + \mathbf{E_0} - \mathbf{U_p})\overline{\Psi} = \mathbf{0} \end{pmatrix} \quad (4.9\text{-}4)$$

The next step is again to transform (4.9-4) to its basic energy-translated wave function,

$$\left[\overline{\Psi}' \rightarrow \overline{\Psi},\ \mathbf{m}^{*\prime} \rightarrow \mathbf{m}^{*},\ \tilde{E}' = \mathbf{m}^{*\prime}\mathbf{u}'\mathbf{v}' = \tilde{E} + E_0 - U_p = E_{total} - U_p\right] \Rightarrow$$

$$\frac{\hbar^2}{m^*}\left(\frac{u}{v}\right)\Delta\overline{\Psi} + (\tilde{E} + E_0 - U_p)\overline{\Psi} = 0$$

$$\frac{\hbar^2}{m^*}\left(\frac{u}{v}\right)\Delta\overline{\Psi} - U_p\overline{\Psi} = -(\tilde{E} + E_0)\overline{\Psi} = -j\hbar\frac{\partial\overline{\Psi}}{\partial t} - U_p\overline{\Psi} = \frac{\hbar^2}{\tilde{E} + E_0 - U_p}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2} - U_p\overline{\Psi} = \tag{4.10-1}$$

$$= j\hbar u\nabla\overline{\Psi} - U_p\overline{\Psi},\ \left(\frac{E_{total} - U_p}{\hbar}\right)^2\cdot\overline{\Psi} + \frac{\partial^2\overline{\Psi}}{\partial t^2} = 0,\ \frac{\partial\overline{\Psi}}{\partial t} + u\nabla\overline{\Psi} = 0$$

$$\Delta\overline{\Psi} - \frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2} = \left(\frac{\tilde{E} + E_0 - U_p}{\hbar u}\right)^2\overline{\Psi} + \frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2} = j\frac{\tilde{E} + E_0 - U_p}{\hbar u^2}\frac{\partial\overline{\Psi}}{\partial t} + \frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2} = 0.$$

Practically, now we can symbolically transform the same equations (4.10-1) into another operator's form, as follows:

$$\left(H = -\frac{\hbar^2}{m^*}\left(\frac{u}{v}\right)\Delta + U_p\ (=)\ \text{Hamiltonian}\right) \Rightarrow \left\{H\overline{\Psi} = E_{total}\overline{\Psi} = j\hbar\frac{\partial}{\partial t}\overline{\Psi} + U_p\overline{\Psi} = ...\right\}$$

$$\Rightarrow E_{total} \Leftrightarrow H \Leftrightarrow j\hbar\frac{\partial}{\partial t} + U_p \Leftrightarrow -j\hbar u\nabla, \tag{4.11-1}$$

$$\tilde{p}_i \Leftrightarrow -j\hbar\nabla - \frac{E_0 - U_p}{u} \Leftrightarrow \frac{1}{u}\left(j\hbar\frac{\partial}{\partial t} - E_0\right) \Leftrightarrow \frac{1}{u}(H - E_0 + U_p).$$

(4)

We can also make similar energy translation, $\tilde{E} \rightarrow \tilde{E}' = E_{total} = \tilde{E} + E_0$, for a particle with rest mass in a force-free space (without potential energy field, $U_p = 0$) and apply it to wave equations from (4.10-1), as follows,

matter-waves equations in a free space, with rest mass involvement

$$\left[\overline{\Psi}' \rightarrow \overline{\Psi},\ \mathbf{m}^{*\prime} \rightarrow \mathbf{m}^{*},\ \tilde{E}' = \mathbf{m}^{*\prime}\mathbf{u}'\mathbf{v}' = \tilde{E} + E_0 = E_{total},\ U_p = 0\right] \Rightarrow$$

$$\frac{\hbar^2}{m^*}\left(\frac{u}{v}\right)\Delta\overline{\Psi} + (\tilde{E} + E_0)\overline{\Psi} = \frac{\hbar^2}{m^*}\left(\frac{u}{v}\right)\Delta\overline{\Psi} + E_{total}\overline{\Psi} = 0,$$

$$\frac{\hbar^2}{m^*}\left(\frac{u}{v}\right)\Delta\overline{\Psi} = -(\tilde{E} + E_0)\overline{\Psi} = -j\hbar\frac{\partial\overline{\Psi}}{\partial t} = \frac{\hbar^2}{\tilde{E} + E_0}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2} = j\hbar u\nabla\overline{\Psi}, \tag{4.10-2}$$

$$\left(\frac{E_{total}}{\hbar}\right)^2\cdot\overline{\Psi} + \frac{\partial^2\overline{\Psi}}{\partial t^2} = 0,\ \frac{\partial\overline{\Psi}}{\partial t} + u\nabla\overline{\Psi} = 0,$$

$$\Delta\overline{\Psi} - \frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2} = \left(\frac{\tilde{E} + E_0}{\hbar u}\right)^2\overline{\Psi} + \frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2} = j\frac{\tilde{E} + E_0}{\hbar u^2}\cdot\frac{\partial\overline{\Psi}}{\partial t} + \frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2} = 0.$$

or, again, the same equation can be presented with an operators' form:

$$\left(H = -\frac{\hbar^2}{m^*}(\frac{u}{v})\Delta \text{ (=) Hamiltonian} \right) \Rightarrow \left(H\overline{\Psi} = E_{total}\ \overline{\Psi} = j\hbar\frac{\partial}{\partial t}\overline{\Psi} = -j\hbar u\nabla\overline{\Psi} \right)$$

$$\Rightarrow E_{total} \Leftrightarrow H \Leftrightarrow j\hbar\frac{\partial}{\partial t} \Leftrightarrow -j\hbar u\nabla\overline{\Psi}\ , \qquad (4.11\text{-}2)$$

$$\tilde{p}_i \Leftrightarrow -j\hbar\nabla - \frac{E_0}{u} \Leftrightarrow \frac{1}{u}(j\hbar\frac{\partial}{\partial t} - E_0) \Leftrightarrow \frac{1}{u}(H - E_0).$$

(5)

We can again exercise similar energy translation, $\tilde{\mathbf{E}} \to \tilde{\mathbf{E}}' = \mathbf{E}_{total} + \mathbf{U}_p = \tilde{\mathbf{E}} + \mathbf{E}_0 + \mathbf{U}_p$, for a particle with rest mass in a space with the positive potential energy field, $\mathbf{U}_p \geq \mathbf{0}$) and apply it to wave equations from (4.10-2),

matter-waves equations in a positive potential energy field, with rest mass involvement

$$\left[\overline{\Psi}' \to \overline{\Psi},\ \mathbf{m}^{*}{}' \to \mathbf{m}^*,\ \tilde{\mathbf{E}}' = \mathbf{m}^{*}{}'\mathbf{u}'\mathbf{v}' = \tilde{\mathbf{E}} + \mathbf{E}_0 + \mathbf{U}_p = \mathbf{E}_{total} + \mathbf{U}_p,\ \mathbf{U}_p \geq \mathbf{0}\right] \Rightarrow$$

$$\frac{\hbar^2}{\mathbf{m}^*}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + (\tilde{\mathbf{E}} + \mathbf{E}_0 + \mathbf{U}_p)\overline{\Psi} = \frac{\hbar^2}{\mathbf{m}^*}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + (\mathbf{E}_{total} + \mathbf{U}_p)\overline{\Psi} = \mathbf{0},$$

$$\frac{\hbar^2}{\mathbf{m}^*}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} = -(\tilde{\mathbf{E}} + \mathbf{E}_0 + \mathbf{U}_p)\overline{\Psi} = -\mathbf{j}\hbar\frac{\partial\overline{\Psi}}{\partial \mathbf{t}} = \frac{\hbar^2}{\tilde{\mathbf{E}} + \mathbf{E}_0 + \mathbf{U}_p}\cdot\frac{\partial^2\overline{\Psi}}{\partial \mathbf{t}^2} = \mathbf{j}\hbar\mathbf{u}\nabla\overline{\Psi},$$

$$(\frac{\mathbf{E}_{total} + \mathbf{U}_p}{\hbar})^2\cdot\overline{\Psi} + \frac{\partial^2\overline{\Psi}}{\partial \mathbf{t}^2} = \mathbf{0}\ , \frac{\partial\overline{\Psi}}{\partial \mathbf{t}} + \mathbf{u}\nabla\overline{\Psi} = \mathbf{0},$$

$$\Delta\overline{\Psi} - \frac{\mathbf{1}}{\mathbf{u}^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial \mathbf{t}^2} = \left(\frac{\tilde{\mathbf{E}} + \mathbf{E}_0 + \mathbf{U}_p}{\hbar\mathbf{u}}\right)^2\overline{\Psi} + \frac{\mathbf{1}}{\mathbf{u}^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial \mathbf{t}^2} = \mathbf{j}\frac{\tilde{\mathbf{E}} + \mathbf{E}_0 + \mathbf{U}_p}{\hbar\mathbf{u}^2}\cdot\frac{\partial\overline{\Psi}}{\partial \mathbf{t}} + \frac{\mathbf{1}}{\mathbf{u}^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial \mathbf{t}^2} = \mathbf{0}. \qquad (4.10\text{-}3)$$

or, again, the same equation can be presented with operators' form:

$$\left(H = -\frac{\hbar^2}{m^*}(\frac{u}{v})\Delta \text{ (=) Hamiltonian} \right) \Rightarrow \left(H\overline{\Psi} = (E_{total} + U_p)\overline{\Psi} = j\hbar\frac{\partial}{\partial t}\overline{\Psi} = -j\hbar u\nabla\overline{\Psi} \right)$$

$$\Rightarrow (E_{total} + U_p) \Leftrightarrow H \Leftrightarrow j\hbar\frac{\partial}{\partial t} \Leftrightarrow -j\hbar u\nabla\overline{\Psi}\ , \qquad (4.11\text{-}3)$$

$$\tilde{p}_i \Leftrightarrow -j\hbar\nabla - \frac{E_0}{u} \Leftrightarrow \frac{1}{u}(j\hbar\frac{\partial}{\partial t} - E_0) \Leftrightarrow \frac{1}{u}(H - E_0).$$

(6)

We can again make another energy translation, $\tilde{\mathbf{E}} \to \tilde{\mathbf{E}}' = \mathbf{E}_{total} - \mathbf{E}_0 = \tilde{\mathbf{E}} + \mathbf{U}_p$, for a particle with rest mass in a space with (a positive) potential energy field, $\mathbf{U}_p \neq \mathbf{0}$) and apply it to wave equations from (4.10-3),

matter-waves equations in a positive potential energy space, without rest mass involvement

$$\left[\overline{\Psi}' \to \overline{\Psi},\ \mathbf{m}^{*}{}' \to \mathbf{m}^{*},\ \tilde{\mathbf{E}}' = \mathbf{m}^{*}{}'\mathbf{u}'\mathbf{v}' = \tilde{\mathbf{E}} + \mathbf{U}_p, \mathbf{U}_p \geq 0\right] \Rightarrow$$

$$\frac{\hbar^2}{\mathbf{m}^*}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + (\tilde{\mathbf{E}} + \mathbf{U}_p)\overline{\Psi} = \frac{\hbar^2}{\mathbf{m}^*}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + (\tilde{\mathbf{E}} + \mathbf{U}_p)\overline{\Psi} = \mathbf{0},$$

$$\frac{\hbar^2}{\mathbf{m}^*}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} = -(\tilde{\mathbf{E}} + \mathbf{U}_p)\overline{\Psi} = -\mathbf{j}\hbar\frac{\partial\overline{\Psi}}{\partial \mathbf{t}} = \frac{\hbar^2}{\tilde{\mathbf{E}} + \mathbf{U}_p}\cdot\frac{\partial^2\overline{\Psi}}{\partial \mathbf{t}^2} = \mathbf{j}\hbar\mathbf{u}\nabla\overline{\Psi},$$

$$(\frac{\tilde{\mathbf{E}} + \mathbf{U}_p}{\hbar})^2\cdot\overline{\Psi} + \frac{\partial^2\overline{\Psi}}{\partial \mathbf{t}^2} = \mathbf{0}\ , \frac{\partial\overline{\Psi}}{\partial \mathbf{t}} + \mathbf{u}\nabla\overline{\Psi} = \mathbf{0},$$

$$\Delta\overline{\Psi} - \frac{\mathbf{1}}{\mathbf{u}^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial \mathbf{t}^2} = \left(\frac{\tilde{\mathbf{E}} + \mathbf{U}_p}{\hbar\mathbf{u}}\right)^2\overline{\Psi} + \frac{\mathbf{1}}{\mathbf{u}^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial \mathbf{t}^2} = \mathbf{j}\frac{\tilde{\mathbf{E}} + \mathbf{U}_p}{\hbar\mathbf{u}^2}\cdot\frac{\partial\overline{\Psi}}{\partial \mathbf{t}} + \frac{\mathbf{1}}{\mathbf{u}^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial \mathbf{t}^2} = \mathbf{0}. \qquad (4.10\text{-}4)$$

or, again, the same equation can be presented with the new operators' form:

$$\left(\mathrm{H} = -\frac{\hbar^2}{\mathrm{m}^*}(\frac{\mathrm{u}}{\mathrm{v}})\Delta\ (=)\ \text{Hamiltonian}\right) \Rightarrow \left(\mathrm{H}\overline{\Psi} = (\tilde{\mathrm{E}} + \mathrm{U}_p)\overline{\Psi} = \mathrm{j}\hbar\frac{\partial}{\partial \mathrm{t}}\overline{\Psi} = -\mathrm{j}\hbar\mathrm{u}\nabla\overline{\Psi}\right)$$

$$\Rightarrow (\tilde{\mathrm{E}} + \mathrm{U}_p) \Leftrightarrow \mathrm{H} \Leftrightarrow \mathrm{j}\hbar\frac{\partial}{\partial \mathrm{t}} \Leftrightarrow -\mathrm{j}\hbar\mathrm{u}\nabla\overline{\Psi}\ , \qquad (4.11\text{-}4)$$

$$\tilde{\mathrm{p}}_i \Leftrightarrow -\mathrm{j}\hbar\nabla - \frac{\mathrm{E}_0}{\mathrm{u}} \Leftrightarrow \frac{1}{\mathrm{u}}(\mathrm{j}\hbar\frac{\partial}{\partial \mathrm{t}} - \mathrm{E}_0) \Leftrightarrow \frac{1}{\mathrm{u}}(\mathrm{H} - \mathrm{E}_0).$$

Now we could summarize successive results regarding different energy-levels translations (what corresponds to different Lagrangian; -from (1) to 6)), as found in (4.9-2), (4.10), (4.10-1), (4.10-2), (4.10-3) and (4.10-4), and see that all of them produce mutually compatible and correct Schrödinger-like (or Dirac's) equations:

$$\begin{aligned}
&\frac{\hbar^2}{\mathbf{m}^*}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + (\tilde{\mathbf{E}} - \mathbf{U}_p)\overline{\Psi} = \mathbf{0}, (\mathbf{U}_p \neq \mathbf{0}),\\
&\frac{\hbar^2}{\mathbf{m}^*}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + \tilde{\mathbf{E}}\,\overline{\Psi} = \mathbf{0},\ (\mathbf{U}_p = \mathbf{0}),\\
&\frac{\hbar^2}{\mathbf{m}^*}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + (\tilde{\mathbf{E}} + \mathbf{U}_p)\overline{\Psi} = \frac{\hbar^2}{\mathbf{m}^*}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + (\tilde{\mathbf{E}} + \mathbf{U}_p)\overline{\Psi} = \mathbf{0},\\
&\frac{\hbar^2}{\mathbf{m}^*}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + (\tilde{\mathbf{E}} + \mathbf{E}_0 - \mathbf{U}_p)\overline{\Psi} = \mathbf{0},\\
&\frac{\hbar^2}{\mathbf{m}^*}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + \mathbf{E}_{total}\overline{\Psi} = \frac{\hbar^2}{\mathbf{m}^*}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + (\tilde{\mathbf{E}} + \mathbf{E}_0)\overline{\Psi} = \mathbf{0},\\
&\frac{\hbar^2}{\mathbf{m}^*}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + (\tilde{\mathbf{E}} + \mathbf{E}_0 + \mathbf{U}_p)\overline{\Psi} = \frac{\hbar^2}{\mathbf{m}^*}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + (\mathbf{E}_{total} + \mathbf{U}_p)\overline{\Psi} = \mathbf{0}.
\end{aligned} \qquad (4.10\text{-}5)$$

Equations like those in (4.10-5) should also be applicable, by analogy, to the electromagnetic wave vectors from (4.9-2.1). This is an area where we can explore mutual and extended levels of analogical predictions based on Maxwell's equations, which can be conveniently applied to matter-wave equations.

It is worth considering what Dirac's wave equation truly predicted and to what extent the prediction of matter-antimatter particles was solely based on Dirac's equation. While Dirac's prediction was indeed successful, it was partly influenced by already known experimental results that were established without and before Dirac's involvement. In other words, the success of Dirac's prediction was not entirely dependent on his equation, despite what Dirac and his followers later claimed as the strongest foundation for these predictions.

Unfortunately, subsequent generations of scientists in similar research fields have largely continued to uncritically and uncreatively repeat what Dirac and his early followers proposed during their initial phases of creative brainstorming and so-called divine inspiration.

We can also summarize previous results regarding different energy translations and corresponding operator forms of Schrödinger-like equations (found in (4.11), (4.11-1) - (4.11-4)), and see that all of them produce different operators,

$$\left\{\begin{array}{l}\tilde{\mathbf{E}} \Leftrightarrow \mathrm{H} \Leftrightarrow \mathbf{j}\hbar\frac{\partial}{\partial t}+\mathrm{U_p} \Leftrightarrow -\mathbf{j}\hbar u\nabla\ , \\ \tilde{\mathbf{p}}_i \Leftrightarrow -\mathbf{j}\hbar\nabla \Leftrightarrow \frac{1}{u}(\mathbf{j}\hbar\frac{\partial}{\partial t}+\mathbf{U_p}) \Leftrightarrow \frac{1}{u}\mathrm{H}\end{array}\right\}, \tag{4.11}$$

$$\left\{\begin{array}{l}\mathbf{E}_{total} \Leftrightarrow \mathrm{H} \Leftrightarrow \mathbf{j}\hbar\frac{\partial}{\partial t}+\mathrm{U_p} \Leftrightarrow -\mathbf{j}\hbar u\nabla\ , \\ \tilde{\mathbf{p}}_i \Leftrightarrow -\mathbf{j}\hbar\nabla - \frac{\mathrm{E_0}+\mathrm{U_p}}{u} \Leftrightarrow \frac{1}{u}(\mathbf{j}\hbar\frac{\partial}{\partial t}-\mathbf{E_0}) \Leftrightarrow \frac{1}{u}(\mathrm{H}-\mathrm{E_0}-\mathrm{U_p})\end{array}\right\}, \tag{4.11-1}$$

$$\left\{\begin{array}{l}\mathbf{E}_{total} \Leftrightarrow \mathrm{H} \Leftrightarrow \mathbf{j}\hbar\frac{\partial}{\partial t} \Leftrightarrow -\mathbf{j}\hbar u\nabla\ , \\ \tilde{\mathbf{p}}_i \Leftrightarrow -\mathbf{j}\hbar\nabla - \frac{\mathrm{E_0}+\mathrm{U_p}}{u} \Leftrightarrow \frac{1}{u}(\mathbf{j}\hbar\frac{\partial}{\partial t}-\mathrm{E_0}\ \mathbf{-U_p}) \Leftrightarrow \frac{1}{u}(\mathrm{H}-\mathrm{E_0}\ \mathbf{-U_p})\end{array}\right\}. \tag{4.11-2}$$

$$\left(\mathrm{H} = -\frac{\hbar^2}{m^*}(\frac{u}{v})\Delta \text{ (=) Hamiltonian}\right) \Rightarrow \left(\mathrm{H}\overline{\Psi} = (\mathrm{E}_{total} + \mathrm{U_p})\overline{\Psi} = \mathrm{j}\hbar\frac{\partial}{\partial t}\overline{\Psi} = -\mathrm{j}\hbar u\nabla\overline{\Psi}\right)$$

$$\Rightarrow (\mathrm{E}_{total} + \mathrm{U_p}) \Leftrightarrow \mathrm{H} \Leftrightarrow \mathrm{j}\hbar\frac{\partial}{\partial t} \Leftrightarrow -\mathrm{j}\hbar u\nabla\overline{\Psi}\ , \tag{4.11-3}$$

$$\tilde{p}_i \Leftrightarrow -\mathrm{j}\hbar\nabla - \frac{\mathrm{E_0}}{u} \Leftrightarrow \frac{1}{u}(\mathrm{j}\hbar\frac{\partial}{\partial t}-\mathrm{E_0}) \Leftrightarrow \frac{1}{u}(\mathrm{H}-\mathrm{E_0}).$$

$$\left(H=-\frac{\hbar^2}{m^*}(\frac{u}{v})\Delta \text{ (=) Hamiltonian}\right)\Rightarrow\left(H\overline{\Psi}=(\tilde{E}+U_p)\overline{\Psi}=j\hbar\frac{\partial}{\partial t}\overline{\Psi}=-j\hbar u\nabla\overline{\Psi}\right)$$

$$\Rightarrow(\tilde{E}+U_p)\Leftrightarrow H\Leftrightarrow j\hbar\frac{\partial}{\partial t}\Leftrightarrow -j\hbar u\nabla\overline{\Psi}\ , \qquad (4.11\text{-}4)$$

$$\tilde{p}_i\Leftrightarrow -j\hbar\nabla-\frac{E_0}{u}\Leftrightarrow\frac{1}{u}(j\hbar\frac{\partial}{\partial t}-E_0)\Leftrightarrow\frac{1}{u}(H-E_0).$$

The key takeaway from equations (4.10-3), (4.11), (4.11-1), and (4.11-2) is that some foundational elements of modern Quantum Mechanics and the particle-wave dualism concept should be reexamined, as proposed in this book. It's clear that different forms of Schrödinger's equations can be developed independently of Orthodox Quantum Theory, without relying on stochastic and probabilistic assumptions.

None of the equations in (4.10-5) justify treating total energy or total particle mass (including rest energy and rest mass) as equivalent to total particle matter-wave energy on a one-to-one basis. Only motional or kinetic energy generates de Broglie matter waves. Additionally, equations (4.11), (4.11-1) through (4.11-4) suggest that there are no universally applicable operators that can uniquely represent the entire family of Schrödinger equations. Each time we create an 'energy level shift or translation,' the operators also change their form or content.

Later in this book, it will be demonstrated that more universally valid operators for all Schrödinger-like equations can indeed be formulated, but in a form that differs from the oversimplified and rigid structure found in Orthodox Quantum Mechanics—see equations (4.22) through (4.28).

(7)

Anyway, in a certain unnecessarily complicated way, but with similar results to (4.10-3) - (4.10-5), the official Quantum Mechanics (regarding Schrödinger's equation), effectively generated results, as shown in (4.10-6). The particle energy has been treated, either as kinetic energy, as known in Classical Mechanics, or using the relativistic expression for the total particle energy (Dirac), including potential energy, in both cases, and by approximating **u/v = ½,** for **v<<c**; - see **[9]** and (4.10-3)),

$$-\frac{\hbar^2}{m^*}(\frac{u}{v})\Delta\overline{\Psi}+\mathbf{U_p}\,\overline{\Psi}=\frac{-\hbar^2}{\tilde{E}}\frac{\partial^2\overline{\Psi}}{\partial t^2}=j\hbar\frac{\partial\overline{\Psi}}{\partial t}=-j\hbar u\nabla\overline{\Psi}=\tilde{E}\,\overline{\Psi}\,,$$

$$\frac{\hbar^2}{m^*}(\frac{u}{v})\Delta\overline{\Psi}+(\tilde{E}-\mathbf{U_p})\,\overline{\Psi}=\left[\cong\frac{\hbar^2}{2m^*}\Delta\overline{\Psi}+(\tilde{E}-\mathbf{U_p})\,\overline{\Psi}\Big|_{v<<c}\right]=0\ ,$$

$$(\frac{\tilde{E}}{\hbar})^2\cdot\overline{\Psi}+\frac{\partial^2\overline{\Psi}}{\partial t^2}=0\ ,\frac{\partial\overline{\Psi}}{\partial t}+u\nabla\overline{\Psi}=0, \qquad (4.10\text{-}6)$$

$$-\frac{(\hbar u)^2}{\tilde{E}}\left[\Delta\overline{\Psi}-\frac{1}{u^2}\frac{\partial^2\overline{\Psi}}{\partial t^2}\right]+U_p\overline{\Psi}=0,$$

$$j\hbar\frac{\partial\overline{\Psi}}{\partial t}+\frac{\hbar^2}{\tilde{E}}\frac{\partial^2\overline{\Psi}}{\partial t^2}=0,-j\hbar u\nabla\overline{\Psi}+\frac{\hbar^2}{\tilde{E}}\frac{\partial^2\overline{\Psi}}{\partial t^2}=0,$$

$$\tilde{E}\,\overline{\Psi}+\frac{\hbar^2}{\tilde{E}}\frac{\partial^2\overline{\Psi}}{\partial t^2}=0,\ v\cong 2u.$$

From (4.10-6) we can extract the following operator forms of classical Schrödinger equation (most of them found in today's Quantum Mechanics):

$$\left(H = -\frac{\hbar^2}{m^*}(\frac{u}{v})\Delta + U_p \,(=)\text{ Hamiltonian} \right) \Rightarrow \left\{ H\bar{\Psi} = j\hbar\frac{\partial}{\partial t}\bar{\Psi} = -j\hbar u \nabla \right\}, \quad (4.11\text{-}3)$$
$$\tilde{E} \Leftrightarrow H \Leftrightarrow j\hbar\frac{\partial}{\partial t} \Leftrightarrow -j\hbar u\nabla \;,\; \tilde{p}_i \Leftrightarrow -j\hbar\nabla \Leftrightarrow j\frac{\hbar}{u}\frac{\partial}{\partial t} \Leftrightarrow \frac{1}{u}H, \frac{u}{v} \cong \frac{1}{2}.$$

However, we should not be overly captivated by simple, elementary, and non-essential operator formulations for expressing wave equations. It is important to avoid substituting real physics with trivial and unnecessarily complex mathematical theories that use operators with limited applicability.

The differences between the equations starting from (4.10-5) and those in (4.10-6), (4.11), (4.11-1), (4.11-2), and (4.11-3) are relatively minor, which may make it challenging to determine whether everything is in order. Historically, such seemingly small differences have led to significant challenges and debates in 20th-century physics, especially regarding the inclusion of stable rest mass or rest energy as part of matter-wave energy—a concept that is generally incorrect.

We are now able to clearly delineate the principal differences between the wave energy concepts applied in the historically known Schrödinger equation (4.10-6) and the generalized Schrödinger-like equations developed in this book (4.10-5).

Classical (contemporary and presently still official) particle-wave duality concept (applicable to equations (4.10-6)) considers rest mass as a static, constant, self-standing entity or parameter,

$$\left[\begin{array}{l} \tilde{E}_{classical} = E = hf = \hbar\omega \equiv \begin{cases} E_{total-Classic} = \dfrac{p^2}{2m} + U_p = E_k + U_p \;, \quad \text{or} \\ E_{total-Relativistic} = \sqrt{c^2p^2 + (E_0)^2} + U_p = E_k + E_0 + U_p \end{cases} \\ *!?\,\text{pay attention where the problem really is: } \left(\underline{(u \cong v/2)}\Big|_{v<<c}, (uv = c^2\,;\, v \le c) \Rightarrow (u \ge c; v << c) \right) *!? \end{array} \right]$$

Particle-wave duality in this book (applicable in equations (4.10-5)) in certain cases considers rest mass as potentially variable, being an evolving part or product of a specific interaction,

$$\left[\begin{array}{l} \tilde{\mathbf{E}}_{this-book} = \tilde{\mathbf{E}} = \mathbf{hf} = \hbar\omega = \tilde{\mathbf{p}}\mathbf{u} = (\textbf{particle} - \textbf{wave energy}) = \mathbf{E_k} = (\gamma - 1)\mathbf{mc^2} = \\ = \begin{cases} \underline{\mathbf{E}_{total-Classic} - \mathbf{U_p} = \dfrac{\mathbf{p^2}}{\mathbf{2m}} = \mathbf{pu} = \mathbf{E_k} = \dfrac{\mathbf{mv^2}}{\mathbf{2}}}\Big|_{v<<c} , \quad \textbf{or} \\ \mathbf{E}_{total-Relativistic} - \mathbf{E_0} - \mathbf{U_p} = \sqrt{\mathbf{c^2p^2 + (E_0)^2}} - \mathbf{E_0} = \mathbf{E_k} = \mathbf{pu} = \dfrac{\mathbf{pv}}{\mathbf{1 + \sqrt{1 - v^2/c^2}}} \end{cases} \\ \mathbf{d\tilde{E} = hdf = vd\tilde{p} = d(\tilde{p}u) = -dE_k = -vdp = -d(pu), u = \lambda f} = \dfrac{\mathbf{v}}{\mathbf{1 + \sqrt{1 - v^2/c^2}}} \\ \mathbf{v = u - \lambda(du/d\lambda) = -\lambda^2(df/d\lambda), \;\; (v/u) = 1 + \sqrt{1 - v^2/c^2} \;\; \Rightarrow \;\; 0 \le 2u \le \sqrt{uv} \le v \le c.} \end{array} \right] \quad (4.10\text{-}5)$$

Finally, after correlating various concepts, a sufficiently acceptable and operational mathematical model was established in the form of today's Orthodox Quantum Mechanics (see [9]). This framework represents a well-functioning, albeit somewhat artificial, mathematical theory that integrates common areas and similarities between particle and wave representations. It bridges or compensates for the differences highlighted in equations (4.10-5), often without recognizing them from the same perspective as elaborated in this book.

In many modern Quantum Mechanics texts, there are overly optimistic and affirmative statements about the achievements of contemporary Quantum Theory. These statements often aim to preempt criticism by emphasizing the successful results and applications derived from Schrödinger's equation within Orthodox Quantum Mechanics. Such claims frequently rely on pointing to correct experimental and theoretical backgrounds, asserting that numerous predictions and applications justify the ad hoc mathematical steps or models used. These models are not always systematically developed from foundational principles.

Orthodox Quantum Mechanics primarily addresses known experimental results and integrates these results into a coherent framework, creating an impression of comprehensive explanatory and predictive power. However, many of these supportive experimental results were discovered independently of Quantum Theory's theoretical guidance and were later explained as successes of probabilistic Quantum Theory. Over time, many predictions became apparent, often based on repeating experimental outcomes and adhering to well-known deterministic conservation laws of physics, irrespective of the specific mathematical options within Quantum Theory.

The reality concerning the Schrödinger equation is that it was initially formulated through an analogical generalization and hybridization of the Classical Wave Equation, related to the standing wave oscillations of a string. This classical wave equation, a specific form of the d'Alembert equation, was known in various fields of Classical Mechanics, Fluid Mechanics, Acoustics, and Maxwell's Electromagnetic Theory long before Schrödinger's work. Quantum Theory effectively repurposed these ordinary wave equations for the micro-world of matter-waves, where the displacement or amplitude of a string in standing wave oscillations was analogically transformed into a non-dimensional, probabilistic wave function.

Given that our universe is naturally unified, the phenomena of waves or matter-waves should share a fundamental mathematical structure. We have demonstrated that developing the Schrödinger equation required only the Complex Analytic Signal model and a hybridization of the Classical Wave Equation with the Particle-Wave Duality Concepts (PWDC) outlined in Chapters 4.1 and 10 of this book, an explanation not commonly found in standard Quantum Theory texts.

An important next step is to clarify the meaning of the complex Analytic Signal wavefunction (4.9), $\Psi(x,t)\rightarrow\overline{\Psi}(x,t)$, which is proposed here (as initially established in Chapter 4.0) as the most suitable model for all matter-waves and signals. This model differs from the conventional complex wavefunctions used in Quantum Mechanics.

In fact, we could also use only a real wave function, $\mathbf{\Psi(x,t)}$*, to represent de Broglie matter waves*. Because of number of mathematical conveniences, we transform a real wave function into its complex replacement $\Psi(x,t) \rightarrow \overline{\Psi}(x,t)$, known in mathematics as an Analytic Signal, where we can easily find and represent all signal characteristics (such as amplitude, phase, and frequency, both in time and frequency domains) that are equivalent (or similar) to any simple harmonic (sinusoidal) function $\Psi = a \cdot \sin\omega t$, as for instance (see chapter 4.0):

$$\overline{\Psi}(\mathbf{x},\mathbf{t}) = \Psi(\mathbf{x},\mathbf{t}) + \mathbf{j}\hat{\Psi}(\mathbf{x},\mathbf{t}) = \mathbf{a}(\mathbf{x},\mathbf{t})\mathbf{e}^{\mathbf{j}\varphi(\mathbf{x},\mathbf{t})} = \frac{\mathbf{1}}{\pi^2} \iint_{(\mathbf{0},+\infty)} \mathbf{A}(\omega,\mathbf{k})\mathbf{e}^{-\mathbf{j}(\omega\mathbf{t}-\mathbf{k}\mathbf{x}+\Phi(\omega,\mathbf{k}))}\mathbf{d}\mathbf{k}\mathbf{d}\omega,$$

$$\hat{\Psi}(x,t) = H\left[\Psi(x,t)\right], a(x,t) = \sqrt{\Psi^2 + \hat{\Psi}^2},$$

$$\varphi(x,t) = \text{arctang}\left[\hat{\Psi}(x,t)/\Psi(x,t)\right], \omega(t) = 2\pi f(t) = \partial\varphi/\partial t,\ \omega(x) = 2\pi f(x) = \partial\varphi/\partial x, \quad (4.10\text{-}6)$$

$$U(\omega,k) = U_c(\omega,k) - j\, U_s(\omega,k) = \iint_{(-\infty,+\infty)} \overline{\Psi}(x,t)\, e^{j(\omega t - kx)} dt dx = A(\omega,k)e^{-j\Phi(\omega,k)}.$$

Of course, more general wavefunction always has two coupled wave elements propagating in mutually opposite directions (or inwards and outwards), but mathematical processing, resulting with differential wave equations, and Analytic Signal structure will stay similar or the same as in (4.9-1) - (4.10-6),

$$\begin{aligned}&\Psi(r,t) \rightarrow \overline{\Psi}(r,t) = a(r,t)\cdot e^{j(kr-\omega t)} + b(r,t)\cdot e^{j(kr+\omega t)} = \\ &= a(r,t)\cdot e^{jk(r-ut)} + b(r,t)\cdot e^{jk(r+ut)} = \left|\overline{\Psi}(r,t)\right| e^{j\varphi(r,t)} = \\ &= \overline{\Psi}^{(+)}(r-ut) + \overline{\Psi}^{(-)}(r+ut).\end{aligned}$$

To fully understand all advantages of Analytic Signal forms, it is useful to refer to relevant chapters addressing Signal Analysis (see **[7]** and **[8]**). Here we can briefly say that Analytic Signal model (besides many other advantages) produces explicit forms of immediate and time-evolving amplitude, phase, and frequency functions of any signal (or arbitrary wave function), both in time and frequency domains (even in a joint time-frequency domain). This is not possible to have when using Orthodox Quantum Mechanics complex (and probabilistic) wave function (since phase functions have no use, or meaning in today's Quantum Mechanics, because "probability philosophy" takes care only about stochastic amplitude distributions and resulting mean effects of certain processes). *After operating with complex wave functions in the form of Analytic Signals, the final result can easily be transformed (back) into real wave function,* $\overline{\Psi}(\mathbf{x},\mathbf{t}) = \mathbf{a}(\mathbf{x},\mathbf{t})\mathbf{e}^{\mathbf{j}\varphi(\mathbf{x},\mathbf{t})} \rightarrow \Psi(\mathbf{x},\mathbf{t}) = \mathbf{a}(\mathbf{x},\mathbf{t})\cos\varphi(\mathbf{x},\mathbf{t})$*, as the solution relevant for wave functions and equations (being dimensionally the square root of relevant active power, and giving information regarding field distribution of certain matter-wave phenomena).* Contrary to Analytic Signal wavefunction (which could be either harmonic or an arbitrary function), in the contemporary Quantum Mechanics, the wavefunction is very much artificially formulated (from a very beginning and later; -see **[9]**) as a simple complex and harmonic function (and not as an Analytic Signal function). Later, results can again be transformed into real functions, finding their absolute values (also using specific complex operators for every specific case. In many aspects, comparing, (**a**) -typical Fourier signal analysis (applied on wave functions), (**b**) -Quantum Mechanics operations with wave functions, and (**c**) -Analytic Signal wavefunctions, we can notice many (mathematical) similarities between them. One should have passionate and profound attention to discover number of small details, to extract all finesses, differences, and advantages of the Analytic Signal model (see **[7]** and **[8]**). *When we come back to physics, we can distinguish two different, but mutually non-contradictory understandings of the wave function.*

a) In the Orthodox Quantum Theory, we say that probability (distribution) of finding the particle (including its rest mass and its total energy constituents) in a certain space-element is presented by the square of (the Schrödinger's) wavefunction. This is accepted axiomatically and by consensus of involved founders of Quantum theory.

b) And here (in this book), regarding the same moving particle, for its Analytic Signal Wavefunction, we consider the square of this wavefunction to be an active power (of the involved matter-wave phenomena, as first established in Chapter 4.0 of this book). Here, for a moving particle, we address spatial-temporal distribution of de Broglie, or Matter-Wave field that has amplitude, phase, group and phase velocity, power, and energy ... ($P = \Psi^2 = d\tilde{E}/dt = S$), (but not taking into an account the particle's rest mass, or rest energy). Of course, later, eventually we can also apply different averaging, smoothing, and statistical mathematical practices, and make comparisons with Orthodox Quantum Theory predictions.

*[♣ **COMMENTS & FREE-THINKING CORNER: There** are more mathematical possibilities (here only briefly mentioned) to be exploited in relation to Analytic Signal and wave equations, as for instance:*

1° Schrödinger equation (in this book) is created taking into account the complex analytic signal, wave function $\overline{\Psi}(x,t) = \Psi(x,t) + j\hat{\Psi}(x,t) = a(x,t)e^{j\varphi(x,t)}$*, and we can also formulate similar equations dependent only on* $\Psi(x,t)$ *or* $\hat{\Psi}(x,t)$*.*

2° The next interesting possibility would be to separate (or develop) wave equations dependent only on amplitude, $a(\mathbf{x},\mathbf{t})$ *and/or phase function,* $\boldsymbol{\varphi}(\mathbf{x},\mathbf{t})$ *to present all aspects of phase and group velocities, wave power and associated energy transfer.*

3° Since the wavefunction Ψ *and active power* $\mathbf{P}$ *are closely related we can also formulate all wave equations to be dependent only on active power function (*$P(t) = \Psi^2(t) = d\tilde{E}/dt$*). It is just a matter of intellectual rigidity and inertia that we are used treating wave functions as* $\Psi(t)$*, or in classical mechanics as certain amplitude function. Any wave motion is a form of energy transfer, or certain energy state with wave-motion attributes, or simply power in motion. Much more general case would be to establish (or transform) all wave equations to be related only to involved power* $P(t)$*, or that we start getting familiar with a new wavefunction which is only a power function (as first time established in Chapter 4.0 of this book). Creating new differential equations where only power function is used will be relatively easy, as for instance,*

$$\Psi^2(t) = P(t) \Rightarrow \frac{\partial P}{\partial t} = 2\Psi\frac{\partial \Psi}{\partial t} \Rightarrow \left\{ \begin{array}{l} \dfrac{\partial \Psi}{\partial t} = \dfrac{1}{2P^{0.5}}\dfrac{\partial P}{\partial t} \\ \dfrac{\partial^2 \Psi}{\partial t^2} = \dfrac{1}{2P^{0.5}}\dfrac{\partial^2 P}{\partial t^2} - \dfrac{1}{4P^{1.5}}(\dfrac{\partial P}{\partial t})^2 \\ .... \end{array} \right\}. \quad (4.10\text{-}7)$$

Later, by replacing $\frac{\partial\Psi}{\partial t}$ *and* $\frac{\partial^2\Psi}{\partial t^2}$ *(in all already known wave equations) with new members where only power* $P(t)$ *is figuring, we will get new set of (relatively simple) equivalent wave equations, without the need to use* $\Psi(t)$*. This is a small mathematical step, but consequences related to physics (and to the understanding of wave motions) would be significant.*

4° The Analytic Signal modeling of the wavefunction can easily be installed in the framework of the Fourier Integral Transform, which is based on summation of simple harmonic functions such as cosωt,

$$\Psi(t) = a(t)\cos\phi(t) = \int_{-\infty}^{\infty} U(\frac{\omega}{2\pi})e^{j2\pi ft}df = \int_{-\infty}^{\infty} U(\frac{\omega}{2\pi})\left\{\overline{H}\left[\cos 2\pi ft\right]\right\}df = F^{-1}\left[U(\frac{\omega}{2\pi})\right],$$

$$U(\frac{\omega}{2\pi}) = A(\frac{\omega}{2\pi})e^{j\Phi(\frac{\omega}{2\pi})} = \int_{-\infty}^{\infty}\Psi(t)e^{-j2\pi ft}dt = \int_{-\infty}^{\infty}\Psi(t)\left\{\overline{H}^*\left[\cos 2\pi ft\right]\right\}dt = F\left[\Psi(t)\right],\ \omega = 2\pi f, \tag{4.10-8}$$

where the meaning of symbols is:

F *(=) Direct Fourier transform,*

$\boldsymbol{F^{-1}}$ *(=) Inverse Fourier transform,*

$\overline{H} = 1 + jH$ *(=) Complex Hilbert transform,* $j^2 = -1$*,*

$\overline{H}^* = 1 - jH$ *(=) Conjugate complex Hilbert transform.*

$$\begin{aligned}
&\overline{H}\left[\cos\omega t\right] = e^{j\omega t},\quad H\left[\cos\omega t\right] = \sin\omega t,\\
&\overline{H}^*\left[\cos\omega t\right] = e^{-j\omega t},\quad H\left[\sin\omega t\right] = -\cos\omega t,\\
&e^{\pm j\omega t} = (1 \pm jH)\left[\cos\omega t\right],\\
&\overline{H}\left[\Psi(t)\right] = \overline{\Psi}(t) = \Psi(t) + jH\left[\Psi(t)\right] = \Psi(t) + j\hat{\Psi}(t),\\
&\overline{H}^*\left[\Psi(t)\right] = \overline{\Psi}^*(t) = \Psi(t) - jH\left[\Psi(t)\right] = \Psi(t) - j\hat{\Psi}(t).
\end{aligned} \tag{4.10-9}$$

The further generalization of the Fourier integral transformation can be realized by convenient replacement of its simple harmonic functions basis cosωt by some other (compatible) signal basis α(ω,t). Now, the general wave function (in the generalized framework of Fourier transform) can be represented as,

$$\Psi(t) = \int_{-\infty}^{\infty} U(\frac{\omega}{2\pi})\left\{\overline{H}\left[\alpha(\omega,t)\right]\right\}df = F^{-1}\left[U(\frac{\omega}{2\pi})\right],$$

$$U(\frac{\omega}{2\pi}) = \int_{-\infty}^{\infty}\Psi(t)\left\{\overline{H}^*\left[\alpha(\omega,t)\right]\right\}dt = F\left[\Psi(t)\right]. \tag{4.10-10}$$

Let us imagine that $\alpha(\mathbf{t})$ *presents the finite-energy, narrow-band, elementary signal that carries the energy of a single energy waveform. Now Planck's wave energy expression should correspond to,*

$$\int_{[t]}\left[\alpha(\mathbf{t})\right]^2 \mathbf{dt} = \frac{\mathbf{1}}{\mathbf{2\pi}}\int_{[0,\infty]}\left[B(\omega)\right]^2 \mathbf{d\omega} = \mathbf{h\,\overline{f}} = \mathbf{hf},\quad \mathbf{f} = \frac{\omega}{2\pi},\ \mathbf{h} = \mathbf{const.}. \tag{4.10-11}$$

5° Another possibility is to treat the wavefunction (4.9) as the composition of two (mutually coupled) waves, propagating in mutually opposed directions,

$$\begin{aligned}
&\overline{\Psi}(x,t) = \Psi^{+}(x,t) + j\hat{\Psi}^{+}(x,t) + \Psi^{-}(x,t) + j\hat{\Psi}^{-}(x,t) = \\
&= [\Psi^{+}(x,t) + \Psi^{-}(x,t)] + j[\hat{\Psi}^{+}(x,t) + \hat{\Psi}^{-}(x,t)] = \Psi(x,t) + j\hat{\Psi}(x,t) = a(x,t)e^{j\varphi(x,t)} = \\
&= \frac{1}{(2\pi)^2}\iint_{(-\infty,+\infty)} U^{+}(\omega,k)e^{-j(\omega t-kx)}dkd\omega + \frac{1}{(2\pi)^2}\iint_{(-\infty,+\infty)} U^{-}(\omega,k)e^{-j(\omega t+kx)}dkd\omega = \\
&= \frac{1}{\pi^2}\iint_{(0,+\infty)} U^{+}(\omega,k)e^{-j(\omega t-kx)}dkd\omega + \frac{1}{\pi^2}\iint_{(0,+\infty)} U^{-}(\omega,k)e^{-j(\omega t+kx)}dkd\omega = \\
&= \frac{1}{\pi^2}\iint_{(0,+\infty)} A^{+}(\omega,k)e^{-j(\omega t-kx+\Phi(\omega,k))}dkd\omega + \frac{1}{\pi^2}\iint_{(0,+\infty)} A^{-}(\omega,k)e^{-j(\omega t+kx+\Phi(\omega,k))}dkd\omega,\ j^2 = -1, \\
&U^{+/-}(\omega,k) = U_c^{+/-}(\omega,k) - j\,U_s^{+/-}(\omega,k) = A^{+/-}(\omega,k)e^{-j\Phi(\omega,k)}, \\
&a(x,t) = \sqrt{[\Psi^{+}(x,t) + \Psi^{-}(x,t)]^2 + [\hat{\Psi}^{+}(x,t) + \hat{\Psi}^{-}(x,t)]^2} = \sqrt{[\Psi(x,t)]^2 + [\hat{\Psi}(x,t)]^2}, \\
&\Psi(x,t) = \Psi^{+}(x,t) + \Psi^{-}(x,t),\ \hat{\Psi}(x,t) = \hat{\Psi}^{+}(x,t) + \hat{\Psi}^{-}(x,t), \\
&\tilde{E} = \int_{-\infty}^{+\infty}\Psi^2(t)dt = \int_{-\infty}^{+\infty}\hat{\Psi}^2(t)dt = \int_{-\infty}^{+\infty}\left|\frac{\overline{\Psi}(t)}{\sqrt{2}}\right|^2 dt = \int_{-\infty}^{+\infty}\left[\frac{a(t)}{\sqrt{2}}\right]^2 dt = \int_{-\infty}^{+\infty}\left|\frac{\overline{U}(\omega)}{\sqrt{2\pi}}\right|^2 d\omega = \int_{0}^{\infty}\left[\frac{A(\omega)}{\sqrt{\pi}}\right]^2 d\omega, \\
&\frac{1}{2}\left\{\Psi^2(t) + \hat{\Psi}^2(t)\right\} = \frac{dE}{dt} = \frac{dE_k}{dt} = \frac{d\tilde{E}}{dt} = \sum_{(i)}\frac{dE_i}{dt} = \sum_{(i)}\frac{dE_k}{dt} = \sum_{(i)}\frac{d\tilde{E}_i}{dt} = S(t).
\end{aligned} \tag{4.10-12}$$

From the signal energy (4.10-12), and wave equations (4.9-1) it is almost obvious that Ψ and its Hilbert couple $\hat{\Psi}$ are in a mutual coupling and synchronously communicating. This property presents **Quantum Entanglement** *because wave parts (or elements) of involved signals traveling in mutually opposite directions ($\Psi^{+}(x,t), \Psi^{-}(x,t)$, $\hat{\Psi}^{+}(x,t), \hat{\Psi}^{-}(x,t)$) are equally and coincidently present on both sides or directions of waves propagation (in the expressions for signal amplitude, power, and energy). We know for sure that Entanglement is an experimentally verifiable reality between specifically coupled particle-wave states like photons, electrons, protons, atoms, and small atomic clouds (see more about entanglement in [56] and in Chapter 10 of this book). Obviously, explanations based on probabilities and stochastics' grounds of Entanglement are in some way useful, but still not good enough explanation of the entanglement. For instance, there are publications, like in [36], with innovative thinking, proposals, arguments, and indications that all orbital and spinning moments (in micro and macro physics) could be mutually connected (universally, intrinsically, holistically, and synchronously, all-over our Universe), with mentioned entanglement couplings, or to have immediate communications (without delays).*

There are also numerous interesting possibilities to continue representing and analyzing wave functions using different signal basis functions (to be analyzed some other time).

For instance, originally, the Schrödinger wave equation was analogically established based on the standing-waves equation of an oscillating string. The string is a line that has limited (fixed) length *between two points of fixation. Then we imagine how we transform such an oscillating string into a similar, but circular (closed line) oscillating string to get self-closed and self-stabilized resonant states.*

One of the options for realizing such transformation is that we find the imaging or functional (bidirectional) transformation (or mapping) that is converting the line segment to the circle, and vice versa under the following initial conditions:

$$\left\{\begin{array}{l} y-y_1=k(x-x_1)\Leftrightarrow y=kx+b, \\ k=\dfrac{y-y_1}{x-x_1}=\dfrac{y_2-y_1}{x_2-x_1}, \\ b=y_1-kx_1=\dfrac{x_2y_1-x_1y_2}{x_2-x_1}, \\ x\in[x_1,x_2]\,, y\in[y_1,y_2]. \end{array}\right\} \underset{\text{transformation})}{\overset{\leftrightarrow}{(\text{bidirectional}}} \left\{\begin{array}{l} (x-x_0)^2+(y-y_0)^2=r^2, \\ 2\pi r=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}, \\ x_0=\dfrac{1}{2}(x_1+x_2),\ y_0=\dfrac{1}{2}(y_1+y_2), \\ r=r(x_0,y_0). \end{array}\right\}. \quad (4.10\text{-}13)$$

The same process can be additionally upgraded with the objective of finding another functional transformation that is converting the circle into a sphere:

$$\left\{\begin{array}{l} (x-x_0)^2+(y-y_0)^2=r^2, \\ 2\pi r=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}, \\ x_0=\dfrac{1}{2}(x_1+x_2),\ y_0=\dfrac{1}{2}(y_1+y_2), \\ r=r(x_0,y_0). \end{array}\right\} \leftrightarrow \left\{\begin{array}{l} (x-x_0)^2+(y-y_0)^2+(z-z_0)^2=r^2, \\ r=r(x_0,y_0,z_0). \end{array}\right\}. \quad (4.10\text{-}14)$$

Of course, for the sake of higher generality, we could extend the same task towards Line to Ellipse and Ellipse to Ellipsoid transformations (in both directions).

The topology or geometry platform in the background of here conceptualized links between linear, planar, and spatial objects, could belong to Möbius transformations and Riemann Sphere concepts, where such transformations are something obvious and natural.

After establishing the above-mentioned imaging (or transformations) we will be able to make the equivalency and correspondence (in both directions) between standing waves equations on a <u>line segment (or string)</u> and standing waves on a <u>corresponding circle and ellipse</u>, as well as to extend such standing waves on a <u>Sphere and Ellipsoid</u>. ♣]

4.3.1. Wave Energy and Mean Values

Up to present we only demonstrated applicability and compatibility of Planck's wave energy relation (of certain narrow-band wave packet), $\tilde{E}_i = hf_i$, with Energy and Momentum conservation laws, as well as with de Broglie matter wavelength, $\lambda_i = h/p_i$, without precisely showing what really makes those relations correct. The answer to the simplest question of how and why only one specific frequency (multiplied by Planck's constant) can represent the motional energy of a quantized wave group (or what means that frequency) should be found. The first intuitive and logical starting point is the mean frequency, $\bar{f}$, of the corresponding narrow-banded matter-wave group calculated concerning its energy (since wave group, or wave packet, or de Broglie matter wave is composed of an infinity of elementary simple-harmonic waves, covering certain, not too wide, frequency interval: $0 \le f_{min.} \le \bar{f} \le f_{max.} <<< \infty$).

$$\tilde{E} = \int_{-\infty}^{+\infty} P(t)dt = \int_{-\infty}^{+\infty} \Psi^2(t)dt = \int_{-\infty}^{+\infty} \hat{\Psi}^2(t)dt = \frac{1}{2}\int_{-\infty}^{+\infty} \left|\overline{\Psi}(t)\right|^2 dt = \frac{1}{2}\int_{-\infty}^{+\infty} a^2(t)dt = \tilde{p}u = h\overline{f}\,. \qquad (4.13)$$

As known from Signal and Spectrum analysis, time, and frequency domains of a wave function (4.9) can be connected using Parseval's theorem, thus the energy of de Broglie matter wave can also be presented as,

$$\tilde{E} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |U(\omega)|^2 d\omega = \frac{1}{\pi}\int_{0}^{\infty} [A(\omega)]^2 d\omega = \tilde{p}u = h\overline{f}\,. \qquad (4.14)$$

Now (by definition) we can find the mean frequency as,

$$\overline{f} = \frac{\frac{1}{\pi}\int_{0}^{\infty} f\cdot[A(\omega)]^2 d\omega}{\frac{1}{\pi}\int_{0}^{\infty} [A(\omega)]^2 d\omega} = \frac{\frac{1}{\pi}\int_{0}^{\infty} f\cdot[A(\omega)]^2 d\omega}{\tilde{E}}, \qquad (4.15)$$

and replace it with the wave energy expression, (4.14),

$$\tilde{E} = \frac{1}{\pi}\int_{0}^{\infty} [A(\omega)]^2 d\omega = h\overline{f} = h\frac{\frac{1}{\pi}\int_{0}^{\infty} f\cdot[A(\omega)]^2 d\omega}{\tilde{E}} \Rightarrow \tilde{E}^2 = \frac{h}{\pi}\int_{0}^{\infty} f\cdot[A(\omega)]^2 d\omega\,. \qquad (4.16)$$

Using one of the most general formulas valid for all definite integrals (and applying it to (4.16)), we can prove that the wave energy (of a de Broglie, narrow-band wave group) should be equal to the product between Planck's constant and mean frequency of the wave group in question, as follows:

$$\left\{\begin{array}{l} \int_{a}^{b} f(x)\cdot g(x)dx = f(c)\int_{a}^{b} g(x)dx\,, a < c < b,\ g(x) \geq 0, \\ f(x) \text{ and } g(x) - \text{continuous in } [a \leq x \leq b], \\ f(x) = f\,, g(x) = [A(\omega)]^2 > 0, x = \omega \in (0, \infty) \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left[\tilde{E}^2 = \frac{h}{\pi}\int_{0}^{\infty} f\cdot[A(\omega)]^2 d\omega = \frac{h}{\pi}\cdot\overline{f}\cdot\int_{0}^{\infty} [A(\omega)]^2 d\omega = h\overline{f}\cdot\tilde{E}\right] \Rightarrow \tilde{E} = h\overline{f}\,. \qquad (4.17)$$

If Planck's energy of certain wave group, effectively and essentially deals with mean frequency (of that narrow-band wave group), the same should be valid for de Broglie wavelength, as well as for its phase and group velocities. All the mentioned items should be treated as mean values describing the motion of an effective center of inertia, or center of gravity of that wave group). Consequently, we do not need to mark them, as was the case with a mean frequency (since we know that all of them should anyway be mean values $\overline{f} = f$, $\overline{\lambda} = \lambda$, $\overline{u} = u$, $\overline{v} = v$). *The next important conclusion is that* ***all forms of wave equations, (4.10-3), effectively deal only with mean values (regarding energy, momentum, frequency, wavelength, velocities...).***

For instance, group and phase velocity can also be found as mean values in the following way:

$$\overline{v}_g = \overline{v} = \frac{\frac{1}{\pi}\int_0^\infty v\cdot[A(\omega)]^2 d\omega}{\frac{1}{\pi}\int_0^\infty [A(\omega)]^2 d\omega} = \frac{\frac{1}{\pi}\int_0^\infty \frac{d\omega}{dk}\cdot[A(\omega)]^2 d\omega}{\frac{1}{\pi}\int_0^\infty [A(\omega)]^2 d\omega} = \frac{\frac{1}{\pi}\int_0^\infty \frac{d\omega}{dk}\cdot[A(\omega)]^2 d\omega}{\tilde{E}} = \frac{d\tilde{E}}{d\tilde{p}} \Rightarrow \quad (4.17\text{-}2)$$

$$\Rightarrow \tilde{E}\frac{d\tilde{E}}{d\tilde{p}} = \frac{1}{\pi}\int_0^\infty \frac{d\omega}{dk}\cdot[A(\omega)]^2 d\omega = \frac{1}{\pi}\int_0^\infty \frac{d\tilde{E}}{d\tilde{p}}\cdot[A(\omega)]^2 d\omega \Rightarrow \tilde{E} = \frac{\frac{1}{\pi}\int_0^\infty \frac{d\tilde{E}}{d\tilde{p}}\cdot[A(\omega)]^2 d\omega}{\frac{d\tilde{E}}{d\tilde{p}}} = h\overline{f},$$

$$\overline{v}_f = \overline{u} = \frac{\frac{1}{\pi}\int_0^\infty u\cdot[A(\omega)]^2 d\omega}{\frac{1}{\pi}\int_0^\infty [A(\omega)]^2 d\omega} = \frac{\frac{1}{\pi}\int_0^\infty \frac{\omega}{k}\cdot[A(\omega)]^2 d\omega}{\frac{1}{\pi}\int_0^\infty [A(\omega)]^2 d\omega} = \frac{\frac{1}{\pi}\int_0^\infty \frac{\omega}{k}\cdot[A(\omega)]^2 d\omega}{\tilde{E}} = \frac{\tilde{E}}{\tilde{p}} \Rightarrow \quad (4.17\text{-}1)$$

$$\Rightarrow \tilde{E}\frac{\tilde{E}}{\tilde{p}} = \frac{1}{\pi}\int_0^\infty \frac{\omega}{k}\cdot[A(\omega)]^2 d\omega,\ \tilde{E}\frac{\omega}{k} = \frac{1}{\pi}\int_0^\infty \frac{\omega}{k}\cdot[A(\omega)]^2 d\omega \Rightarrow \tilde{E} = h\overline{f}.$$

*[♣ **COMMENTS & FREE-THINKING CORNER:** We could additionally test the Planck's radiation law, regarding narrow-band photon energy* $\mathbf{E} = \mathbf{hf} = \frac{\mathbf{h}}{2\pi}\omega$. *It is well proven that a photon has the wave energy equal to the product between Planck's constant h and photon's frequency f. Photon is a wave phenomenon, and it should be presentable using certain time-domain wave function* $\psi(\mathbf{t}) = \mathbf{a}(\mathbf{t})\cos\varphi(\mathbf{t})$, *expressed in the form of an Analytic signal. Since the Analytic signal gives the chance to extract immediate signal amplitude* $\mathbf{a}(\mathbf{t})$, *phase* $\varphi(\mathbf{t})$, *and frequency* $\omega(\mathbf{t}) = \frac{\partial\varphi(\mathbf{t})}{\partial \mathbf{t}} = 2\pi\mathbf{f}(\mathbf{t})$, *let us explore the meaning of Planck's energy when: instead of constant photon frequency (valid for a single photon), we take the mean wave-group frequency,* $\Omega = \langle\omega(\mathbf{t})\rangle$, *of the time-domain wave function* $\psi(\mathbf{t})$.

$$\mathbf{E} = \mathbf{hf} = \frac{\mathbf{h}}{2\pi}\omega$$

$$\mathbf{E} = \int_{[T]} \psi^2(\mathbf{t})d\mathbf{t} = \int_{[T]} [\mathbf{a}(\mathbf{t})\cos\varphi(\mathbf{t})]^2 d\mathbf{t} = \int_{[T]} \mathbf{a}^2(\mathbf{t})d\mathbf{t}\ (=)\ \mathbf{hf} = \frac{\mathbf{h}}{2\pi}\langle\omega\rangle$$

$$\psi(\mathbf{t}) = \mathbf{a}(\mathbf{t})\cos\varphi(\mathbf{t}) = -\mathbf{H}[\hat{\psi}(\mathbf{t})],\ \hat{\psi}(\mathbf{t}) = \mathbf{a}(\mathbf{t})\sin\varphi(\mathbf{t}) = \mathbf{H}[\psi(\mathbf{t})]$$

$$\mathbf{a}(\mathbf{t}) = \sqrt{\psi^2(\mathbf{t}) + \hat{\psi}^2(\mathbf{t})}$$

$$\varphi(\mathbf{t}) = \mathbf{arctg}\frac{\hat{\psi}(\mathbf{t})}{\psi(\mathbf{t})}$$

$$\omega(\mathbf{t}) = \frac{\partial\varphi(\mathbf{t})}{\partial \mathbf{t}} = 2\pi\mathbf{f}(\mathbf{t}) \Rightarrow \quad \Omega = \langle\omega(\mathbf{t})\rangle = \frac{1}{\mathbf{T}}\int_{[T]} \omega(\mathbf{t})d\mathbf{t}\ (=)\ \frac{\frac{1}{\mathbf{T}}\int_{[T]} \mathbf{a}^2(\mathbf{t})\omega(\mathbf{t})d\mathbf{t}}{\int_{[T]} \mathbf{a}^2(\mathbf{t})d\mathbf{t}}$$

$$E = hf = \frac{h}{2\pi}\omega \Rightarrow E = \frac{h}{2\pi}\Omega \Rightarrow$$

$$E = \frac{h}{2\pi}\frac{1}{T}\frac{\int_{[T]} a^2(t)\omega(t)dt}{\int_{[T]} a^2(t)dt} (=) \frac{h}{2\pi}\frac{1}{T}\int_{[T]} \omega(t)dt = \int_{[T]} a^2(t)dt \Rightarrow$$

$$\Rightarrow \frac{\left[\int_{[T]} a^2(t)dt\right]^2}{\frac{1}{T}\int_{[T]} a^2(t)\omega(t)dt} = \frac{h}{2\pi} = \text{Const.}, \quad \text{or} \quad \frac{\int_{[T]} a^2(t)dt}{\frac{1}{T}\int_{[T]} \omega(t)dt} = \frac{h}{2\pi} = \text{Const.}$$

Depending on how we calculate the mean frequency, we should be able to prove the above given relations, or to find a family of wave functions which describe photon in a time domain, or in any case, to see how universal Planck's energy law could evolve regarding the energy of arbitrary wave functions. ♣]

4.3.2. Inertial and Reaction Forces

Now we can formulate the starting platform for establishing unified force/fields theory (valid for interactions between different motional states of particles, quasi-particles, and similar matter-waves objects). De Broglie waves address waiving or oscillating field structure inside and around interacting objects, this way naturally manifesting specific forces and field formations between them. If de Broglie, or matter-wave function $\Psi(r,t)$ represents an active power, we should be able to determine a force and field distribution in its space of definition, based on the idea first introduced in the second chapter of this book (see equations from (2.5.1) to (2.9) and later (4.3)), regarding the unity of linear and rotational motions:

$$\begin{aligned} &dE = c^2 d(\gamma m) = mc^2 d\gamma = vdp + \omega dL = \Psi^2 dt, \quad p = \gamma mv, \quad \gamma = (1 - v^2/c^2)^{-0.5} \\ &\Psi^2 = \frac{dE}{dt} = v\frac{dp}{dt} + \omega\frac{dL}{dt} = \frac{dx}{dt}\frac{dp}{dt} + \frac{d\alpha}{dt}\frac{dL}{dt} = vF + \omega\tau = mc^2\frac{d\gamma}{dt} = \text{Power} = [W] \end{aligned} \tag{2.5.1}$$

The objective behind (2.5.1) is to establish the concept that any particle (regarding its total energy content) is composed of linear motion components $\mathbf{vdp}$ and spinning motion components, ωdL, and that both of them are intrinsically involved in creating total particle energy ($dE = mc^2 d\gamma = vdp + \omega dL$). The relation between linear and rotational motion components (related to the same moving particle) could be "analogically visualized" as a relation between current and voltage components inside of a closed, oscillating R-L-C circuit (where electric and magnetic field mutually communicate by currents and voltages through inductive and capacitive elements). The wave function behind such modeling should have two wave components (one from linear motion and the other from spinning),

$$\begin{aligned} &\Psi^2 = \frac{d\tilde{E}}{dt} = v\frac{dp}{dt} + \omega\frac{dL}{dt} = vF + \omega\tau = -v\frac{d\tilde{p}}{dt} - \omega\frac{d\tilde{L}}{dt} = -v\tilde{F} - \omega\tilde{\tau} = \\ &= vF_{linear} + \omega F_{angular} = \Psi^2_{linear} + \Psi^2_{angular} = mc^2\frac{d\gamma}{dt} \\ &\left.\begin{cases} \Psi^2_{linear} = vF_{linear} \Leftrightarrow F_{linear} = \dfrac{dp}{dt} = -\dfrac{d\tilde{p}}{dt} = \dfrac{1}{v}\Psi^2_{linear} \\ \Psi^2_{angular} = \omega F_{angular} \Leftrightarrow F_{angular} = \tau = \dfrac{dL}{dt} = -\dfrac{d\tilde{L}}{dt} = \dfrac{1}{\omega}\Psi^2_{angular} \end{cases}\right\} \Rightarrow F_q = \frac{1}{V_q}\Psi^2_q \\ &v = \frac{d\tilde{E}}{d\tilde{p}} = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} = -\frac{\lambda^2}{2\pi}\frac{d\omega}{d\lambda}, \; u = \frac{\tilde{E}}{\tilde{p}} = \lambda f = \lambda\frac{\omega}{2\pi}, \; \tilde{E} = hf, \end{aligned} \tag{4.18}$$

where **v** and **u** are group and phase velocities of de Broglie wave packet. Practically, for any specific situation, it would be necessary to find the wave function, $\Psi(r,t)$, by solving relevant Schrödinger's equation (4.10) - (4.12), and then to determine the linear and radial force-field structure (see also some attempts to generalize force laws formulated by (4.30), (4.31) and (4.37)).

Nuclear (and other known or still undiscovered) forces, because of specific force field distributions, should also be presentable like (4.18) or (4.37).

In addition to (4.18), we could evoke the ideas of the philosopher R. Boskovic, who was the first to explain the (need of) existence of certain alternating force-field (with attractive and repulsive forces) in a narrow zone of impact between two objects (see [6]).

If the above-mentioned concept proves logical, or at least introducing fruitful brainstorming ideas, the road to a unified field theory will be largely paved (see also (4.26), (4.29), (4.30), (4.31), (5.15) and (5.16) to get an idea how the particle-wave duality, universal force law, and Schrödinger-like equations can additionally be upgraded).

[♣ COMMENTS & FREE-THINKING CORNER:

R. Boskovic, of South-Slavic origin with a Serbian surname from his father and Italian heritage from his mother, introduced the concept of universal natural forces nearly three centuries before Quantum Theory was established. This occurred during a historical period when the Balkan region, predominantly Slavic and Serbian, had not yet fully defined its future national identities, like Croats, Bosnians, Slovenians, and Macedonians.

Later, ideological forces, still active in the region, sought to reshape the identities of the Balkan population. These forces manipulated geographical, religious, and national identities, creating new national distinctions while magnifying regional identities. Despite these changes, the Serbian language remained a unifying factor, resisting transformation.

The Balkans, a region rich in historical and cultural intersections, flourished during the declining Roman Empire. This area was a melting pot of Byzantine, Roman Catholic, Slavic, and European entities and influences.

During the Roman Empire, over fifteen emperors came from the Balkan region, which overlaps with areas historically inhabited by Serbs and other Slavic groups. This trend of assimilating Slavic populations continued under the Ottoman Empire, where many military and administrative leaders were Serbs. Notably, Serbian was one of the two official languages of the Ottoman Empire for almost five centuries, highlighting its significance in administration across the Balkans. Croatian and Bosnian languages, emerging later, were not used in the administration of the Ottoman Empire.

Interestingly, early Christianity, before the divisions into Orthodox and Catholic branches, was established in Serbia by Roman Emperor Constantine, born to a Serbian mother. However, in modern times, a newly formed Catholic version of Christianity led to the conversion of some Serbs to Croats, driven by Germanic and Vatican-supported efforts that involved violence against those who resisted conversion.

In addition to historical and religious aspects, the Balkans have contributed significantly to science and technology. Prominent figures such as Nikola Tesla, Milutin Milankovic, Mihajlo Pupin, Mileva Maric (Einstein's first wife), and Pavle Savic have made impactful contributions to the modern world. Tesla's inventions have been foundational to modern industry and technology. However, it is tragic that many members of Tesla's family were killed by Croats during World War II, a consequence of extremist actions by some factions within Croatia.

It is important to clarify that these actions were perpetrated by certain radical groups and do not reflect the views of the entire Croatian population. Many Croats continue to support historical transitions and actions, often backed by Western influences and the Vatican.

The author also acknowledges the significant contributions of Asian, Arab, and Indian scholars whose innovations have been borrowed or copied by European and Western innovators. While these examples may not directly pertain to Wave-Particle Duality, they are crucial for challenging our understanding of history and promoting objectivity and inclusivity in the study of scientific and cultural advancements.

It is especially important to notice that (based on (4.2) and (4.18)) the particle-wave duality concept is extended to any situation where motional energy (regardless of its origin) is involved. This indicates that motional energy is intrinsically coupled with de Broglie matter waves, creating inertia-like reaction forces in the form of waves (where reaction and inertial waving forces can have gravitational, mechanical, rotational, electromagnetic... or some other nature). The most significant relation, found in (4.2), leading to such conclusion is the connection between wave and kinetic energy, $d\tilde{E} = h \cdot df = v d\tilde{p} = h \cdot v df_s = d(\tilde{p}u) = dE_k = vdp = d(pu)$, *which is in agreement with de Broglie* *wavelength* $\lambda = h/\tilde{p}$, *and with relations expressing group and phase velocities* $v = d\omega/dk = dE/dp$, $u = \omega/k = \tilde{E}/\tilde{p}$, $v = u - \lambda du/d\lambda$. *Using a similar conclusion process (based on analogies), we could also develop the following particle-wave duality relations* $\omega d\tilde{L} = -\omega dL$, $ud\tilde{q}_{electric} = -udq_{electric}$, $id\tilde{q}_{magnetic} = -\mathbf{idq_{magnetic}}$, *(where* $\tilde{q}_{electric}$, $q_{electric}$ *and* $\tilde{q}_{magnetic}$, $q_{magnetic}$ *are electric and magnetic charges). From the point of view of cosmology and "expanding universe" we should also conclude that what we see as an expansion (driving force, or positive energy: characterized by* dE_k, dp, dL,... *) should always be balanced with something what we, most probably, do not see (named here as reaction energy or inertial forces,* $-\mathbf{d\tilde{E}}, -\mathbf{d\tilde{p}}, -\mathbf{d\tilde{L}}$,... *). It looks that here we are dealing with forces described by Newton action-reaction, or inertia law (extended to rotation, electromagnetism etc.). In (4.2) we also find that after integration of differential relations connecting particle and wave aspect of motion, we can get very useful finite differences relations, such as:* $\Delta E_k = \Delta\tilde{E}$, $\Delta p = -\Delta\tilde{p}$, $\Delta L = -\Delta\tilde{L}$, $\Delta q = -\Delta\tilde{q}$, $\Delta\dot{p} = -\Delta\dot{\tilde{p}}$, $\Delta\dot{L} = -\Delta\dot{\tilde{L}}$, $\Delta\dot{q} = -\Delta\dot{\tilde{q}}$,... *(see the end of chapter 5 of this paper, where advantages of using Central Differences are presented). Modern physics also addresses the same problem, non-systematically and often implicitly (when analyzing origins of Inertia), using the terminology of "transient mass fluctuations, electromagnetic radiation reaction forces, induction laws, inertial reaction forces" etc.*

This book initially started by establishing the wide analogy platform between different physical entities. For instance (see T.1.1 to T.1.6 and T.3.1 to T.3.3), there is a multilevel analogy between speed and voltage (or potential difference), and between electric charge and linear and angular momentum. This could be directly and imaginatively applied (in fact tested) in the equation (4.2) that connects group and phase speed $v_g = v$, $v_p = u$ *(producing that group velocity,* $v_g = v$, *can be analog to "**group voltage**" (=)* u_g, *or* $v_g = v \Leftrightarrow u_g$, *and phase velocity,* $v_p = u$, *analog to "**phase voltage**" (=)* u_p*", or* $v_p = u \Leftrightarrow u_p$,

$$\left\{\begin{array}{l}(v = v_{group} = v_g \Leftrightarrow u_g),\ (u = v_{phase} = v_p \Leftrightarrow u_p), (q \Leftrightarrow p, \tilde{q} \Leftrightarrow \tilde{p}), \\ v_g = v_p - \lambda\dfrac{dv_p}{d\lambda} = v_p + \tilde{p}\dfrac{dv_p}{d\tilde{p}} = \dfrac{d\tilde{E}}{d\tilde{p}},\ v_p = \dfrac{\tilde{E}}{\tilde{p}}\end{array}\right\} \Rightarrow$$

$$\Rightarrow\ u_g = u_p - \lambda'\frac{du_p}{d\lambda'} = u_p + \tilde{q}\frac{du_p}{d\tilde{q}} = \frac{d\tilde{E}}{d\tilde{q}}, u_p = \frac{\tilde{E}}{\tilde{q}}. \qquad (4.19)$$

*Using analogies (see T.1.8, Generic Symmetries and Analogies of the Laws of Physics) in the similar way as in (4.19) we can (hypothetically) extend "**group phase**" concept to magnetic field and rotational motion (connecting "**magnetic group-voltage**" and "**magnetic phase-voltage**" as well as "**angular group**" and "**angular phase**" velocity) etc., (see also (3.4), (3.5), T.3.1 - T.3.3 for understanding the meaning of magnetic voltages and currents). It is another question (still not answered here) to prove if the "**group phase**" concept (4.19), based only on analogies, is universally applicable, and how to integrate it into positive knowledge of today's relevant physics (see also (5.15) and (5.16)). One of the possibilities that should be analyzed is to connect "**group phase**" concept to Retarded Lorentz Potentials (known in Maxwell Electromagnetic Theory). ♣]*

4.3.3. Probability and Conservation Laws

It is evident that many wave equations, both in Quantum Mechanics and beyond (e.g., (4.10) through (4.12)), can be developed without necessarily attributing a probabilistic nature to wave functions. Importantly, modifying the interpretation of the wave function from a probabilistic to a natural, dimensional "active power" function does not compromise the flexibility and positive results of Quantum Mechanics. By simply normalizing the "active power" wave function (4.9), we can maintain its dimensionless quality and utilize it similarly to current Quantum Mechanics practices, but with a richer and more naturally suited mathematical framework derived from Signal Analysis and Analytic Signal concepts.

The wave function concept presented in this book extends broadly to various temporal-spatial signals, including voltages, currents, velocities, forces, and moments (see Chapter 4.0 and (4.0.82)). In electrical sciences, we already have a well-developed and practical concept of complex Phasors. We understand how currents and voltages are phase-shifted or transformed by different electrical loads (or impedances), and how we define and analyze various forms of power, such as active, reactive, apparent, and complex. We also analyze coherence and correlation between different signals or Phasors.

There are established and natural analogies between electrical and mechanical systems, which should be applied to mechanics and physics (see the first chapter of this book). By integrating the concepts and methodologies of electrical Phasors into the realm of mechanics and Wave-Particle Duality, we can greatly enhance our understanding of interactions in these fields. Despite this potential, current approaches remain limited by insufficiently clear and incomplete probabilistic strategies. Nevertheless, Probability and Statistics will continue to play a significant role in the analysis and modeling of mass data and associated processes across all scientific and practical fields.

In this book, we model de Broglie matter waves as phenomena that unify linear motion with rotation (see (4.3) - (4.4), Fig. 4.1, and Chapter 10). The generalized Schrödinger equations (4.10) through (4.12) and (4.10-3) provide additional support for the hypothesis presented here: fields and forces associated with rotation, in conjunction with linear motions, offer a more comprehensive explanation of Wave-Particle Duality.

To understand the profound background regarding how Orthodox Quantum Mechanics successfully established a wave function as a probability function, let us start from natural conservation laws as the strongest platform in physics. Most conservation laws in physics are covered by the law of energy conservation, and several laws of different vector-values conservations (such as moments conservation laws). Since a total energy input, $\mathbf{E}_{\mathbf{inp.}}$, of one isolated system (passing through certain transformation) will always stay equal to its total energy output, $\mathbf{E}_{\mathbf{outp.}}$, and since the same is valid for all other important vector properties of that system, $\vec{\mathbf{A}}_{\mathbf{inp.}} = \vec{\mathbf{A}}_{\mathbf{outp.}}$ (its moments, for instance), we can easily formulate the following generalized and normalized forms of such conservation laws:

$$\left\{\begin{array}{l} E_{tot.} = E_{inp.} = \sum_{(i)} E_i = E_{outp.} = \sum_{(j)} E_j \\ \vec{A}_{tot.} = \vec{A}_{inp.} = \sum_{(i)} \vec{A}_i = \vec{A}_{outp.} = \sum_{(j)} \vec{A}_j = \left|\vec{A}_{tot.}\right| \cdot \vec{a}_0 = A \cdot \vec{a}_0 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \begin{cases} 1 = \frac{E_{inp.}}{E_{tot.}} = \sum_{(i)} \frac{E_i}{E_{tot.}} = \frac{E_{outp.}}{E_{tot.}} = \sum_{(j)} \frac{E_j}{E_{tot.}} \\ 1 = \frac{\left|\vec{A}_{inp.}\right|}{A} = \left|\sum_{(i)} \frac{\vec{A}_i}{A}\right| = \frac{\left|\vec{A}_{outp.}\right|}{A} = \left|\sum_{(j)} \frac{\vec{A}_j}{A}\right| = \left|\vec{a}_0\right| \end{cases} \Leftrightarrow \begin{cases} 1 = \sum_{(i)} e_i = \sum_{(j)} e_j \\ 1 = \left|\sum_{(i)} \vec{a}_i\right| = \left|\sum_{(j)} \vec{a}_j\right| \end{cases}. \tag{4.20}$$

For situations where average motional or wave energy $\langle \tilde{E} \rangle$ of certain system is stable (meaning conserved) we can also apply similar normalization like in (4.20), for instance using motional or wave energy relations from (4.9-0) and (4.13),

$$\begin{cases} \langle \tilde{E} \rangle = \int_{-\infty}^{+\infty} P(t)dt = \int_{-\infty}^{+\infty} \Psi^2(t)dt = \int_{-\infty}^{+\infty} \hat{\Psi}^2(t)dt = \frac{1}{2}\int_{-\infty}^{+\infty} \left|\overline{\Psi}(t)\right|^2 dt = \frac{1}{2}\int_{-\infty}^{+\infty} a^2(t)dt = \int_{-\infty}^{+\infty} d\tilde{E} \\ d\tilde{E} = hdf = dE_k = c^2 d(\gamma m) = vdp = d(pu) \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} \frac{\tilde{E}}{\langle \tilde{E} \rangle} = \frac{1}{\langle \tilde{E} \rangle}\int_{-\infty}^{+\infty} P(t)dt = \frac{1}{\langle \tilde{E} \rangle}\int_{-\infty}^{+\infty} d\tilde{E} = \\ = \frac{1}{\langle \tilde{E} \rangle}\int_{-\infty}^{+\infty} \Psi^2(t)dt = \frac{1}{\langle \tilde{E} \rangle}\int_{-\infty}^{+\infty} \hat{\Psi}^2(t)dt = \frac{1}{2\langle \tilde{E} \rangle}\int_{-\infty}^{+\infty} \left|\overline{\Psi}(t)\right|^2 dt = \frac{1}{2\langle \tilde{E} \rangle}\int_{-\infty}^{+\infty} a^2(t)dt = 1 \end{cases} \Leftrightarrow \tag{4.20-1}$$

$$\left\{ \frac{\tilde{E}}{\langle \tilde{E} \rangle} = \int_{-\infty}^{+\infty} \frac{P(t)}{\langle \tilde{E} \rangle}dt = \int_{-\infty}^{+\infty} \frac{d\tilde{E}}{\langle \tilde{E} \rangle} = \int_{-\infty}^{+\infty} \frac{\Psi^2(t)}{\langle \tilde{E} \rangle}dt = \int_{-\infty}^{+\infty} \frac{\hat{\Psi}^2(t)}{\langle \tilde{E} \rangle}dt = \int_{-\infty}^{+\infty} \frac{\left|\overline{\Psi}(t)\right|^2}{2\langle \tilde{E} \rangle}dt = \int_{-\infty}^{+\infty} \frac{a^2(t)}{2\langle \tilde{E} \rangle}dt = 1 \right\} \Leftrightarrow$$

$$\Leftrightarrow \int_{-\infty}^{+\infty} de = 1,\ de = \frac{P(t)}{\langle \tilde{E} \rangle}dt = \frac{d\tilde{E}}{\langle \tilde{E} \rangle} = \frac{\Psi^2(t)}{\langle \tilde{E} \rangle}dt = \frac{\hat{\Psi}^2(t)}{\langle \tilde{E} \rangle}dt = \frac{\left|\overline{\Psi}(t)\right|^2}{2\langle \tilde{E} \rangle}dt = \frac{a^2(t)}{2\langle \tilde{E} \rangle}dt$$

Based on (4.20-1) we can conclude that the square of a normalized wavefunction $\psi^2(t)$ (originating from power function, as established in Chapter 4.0) will be,

$$\psi^2(t) = \frac{P(t)}{\langle \tilde{E} \rangle} = \frac{\Psi^2(t)}{\langle \tilde{E} \rangle} = \frac{\hat{\Psi}^2(t)}{\langle \tilde{E} \rangle} = \frac{\left|\overline{\Psi}(t)\right|^2}{2\langle \tilde{E} \rangle} = \frac{a^2(t)}{2\langle \tilde{E} \rangle} \Rightarrow \int_{-\infty}^{+\infty} \psi^2(t)dt = 1. \tag{4.20-2}$$

Similar and more tangible superposition and normalization should also be applicable on a 4-vector of momentum-energy, from Relativity theory, as for instance (just to give some brainstorming ideas to start with),

$$\left\{\begin{array}{l} P_4(\vec{P},\frac{E}{c}) = (\sum_{(i)}\vec{p}_i, \frac{\sum_{(i)}E_i}{c}) \Rightarrow \left(\sum_{(i)}\vec{p}_i\right)^2 - \frac{\left(\sum_{(i)}E_i\right)^2}{c^2} = -\frac{-\left(\sum_{(i)}E_{0i}\right)^2}{c^2} \\ E = E_{tot.} = E_{inp.} = \sum_{(i)}E_i = E_{outp.} = \sum_{(j)}E_j \\ \vec{P} = \vec{P}_{tot.} = \vec{P}_{inp.} = \sum_{(i)}\vec{p}_i = \vec{P}_{outp.} = \sum_{(j)}\vec{p}_j = \left|\vec{P}_{tot.}\right|\cdot\vec{a}_0 = P\cdot\vec{a}_0,\ P = \left|\vec{P}\right| \\ (\vec{p}_i, \frac{E_i}{c}) \Rightarrow \vec{p}_i^2 - \frac{E_i^2}{c^2} = -\frac{E_{0i}^2}{c^2},\ \vec{P}^2 - \frac{E^2}{c^2} = -\frac{E_0^2}{c^2} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} E^2 = E_0^2 + c^2\vec{P}^2 \\ E_i^2 = E_{0i}^2 + c^2\vec{p}_i^2 \\ \left(\sum_{(i)}E_i\right)^2 = \left(\sum_{(i)}E_{0i}\right)^2 + c^2\left(\sum_{(i)}\vec{p}_i\right)^2 \\ \sum_{(i)}E_i^2 = \sum_{(i)}E_{0i}^2 + c^2\sum_{(i)}\vec{p}_i^2 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left\{\left[\left(\sum_{(i)}E_i\right)^2 - \sum_{(i)}E_i^2\right] = \left[\left(\sum_{(i)}E_{0i}\right)^2 - \sum_{(i)}E_{0i}^2\right] + c^2\left[\left(\sum_{(i)}\vec{p}_i\right)^2 - \sum_{(i)}\vec{p}_i^2\right]\right\} \Rightarrow \sum_{(i\neq j)}E_iE_j = \sum_{(i\neq j)}E_{0i}E_{0j} + c^2\sum_{(i\neq j)}\vec{p}_i\vec{p}_j$$

$$\Rightarrow \left\{\begin{array}{l} \frac{\left(\sum_{(i)}E_i\right)^2}{E^2} = \frac{\left(\sum_{(i)}E_{0i}\right)^2}{E^2} + \frac{c^2}{E^2}\left(\sum_{(i)}\vec{p}_i\right)^2 = 1 \\ \frac{\sum_{(i)}E_i^2}{E^2} = \frac{\sum_{(i)}E_{0i}^2}{E^2} + \frac{c^2}{E^2}\sum_{(i)}\vec{p}_i^2 = 1 \ldots (?!) \end{array}\right\} \Leftrightarrow \left\{\begin{array}{l} \left(\sum_{(i)}\frac{E_i}{E}\right)^2 = \left(\sum_{(i)}\frac{E_{0i}}{E}\right)^2 + \left(\sum_{(i)}\frac{c\vec{p}_i}{E}\right)^2 = 1 \\ \sum_{(i)}\frac{E_i^2}{E^2} = \sum_{(i)}\frac{E_i^2}{E^2} + \sum_{(i)}\frac{c^2\vec{p}_i^2}{E^2} = 1 \ldots (?!) \end{array}\right\}.$$

(4.20-3)

Or, if we exploit similar 4-vector relations (4.0.5-4) from chapter 4.0, we will be able to get a normalized sum of all energy-momentum members of certain system as,

$$\left\{\begin{array}{l} (p, \frac{E}{c}) \Rightarrow p^2 - \frac{E^2}{c^2} = -\frac{E_0^2}{c^2} \Leftrightarrow E^2 = E_0^2 + c^2p^2 = E_0^2 + E_p^2 \Rightarrow E = \left|\overline{E}\right| = \gamma mc^2 \\ E = \left|\overline{E}\right| = \gamma mc^2, E_0 = mc^2, E_p = cp = \gamma mcv,\ \gamma = (1 - \frac{v^2}{c^2})^{-0.5}, E_k = E - E_0 = (\gamma - 1)E_0 = \frac{\gamma - 1}{\gamma}E \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left(\begin{array}{l} \overline{E} = \sqrt{E_0^2 + E_p^2}\cdot e^{j\cdot arctg(\alpha)} = \left|\overline{E}\right|\cdot e^{j\cdot arctg(\alpha)} = E\cdot e^{j\cdot\theta} \\ \overline{E}_k = \frac{\gamma - 1}{\gamma}\overline{E} = E_k\cdot e^{j\cdot arctg(\alpha)} = E_k\cdot e^{j\cdot\theta} = \tilde{E}\cdot e^{j\cdot\theta} \end{array}\right) \Rightarrow$$

$$\Rightarrow \left\{\begin{array}{l} \overline{E}_{tot.} = \sum_{(i)}\overline{E}_i = \sum_{(i)}\sqrt{E_{0i}^2 + E_{pi}^2}\cdot e^{j\cdot arctg(\alpha_i)} = \sqrt{\sum_{(i)}(E_{0i}^2 + E_{pi}^2)}\cdot e^{j\cdot\theta_\Sigma} = \left|\overline{E}_{tot.}\right|\cdot e^{j\cdot\theta_\Sigma} = E_{tot.}\cdot e^{j\cdot\theta_\Sigma}, \\ \left|\overline{E}_{tot.}\right|^2 = E_{tot.}^2 = \left|\sum_{(i)}\overline{E}_i\right|^2 = \sum_{(i)}(E_{0i}^2 + E_{pi}^2),\ \left|\sum_{(i)}\overline{E}_{ki}\right| = \sqrt{\sum_{(i)}\overline{E}_{ki}^2} \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left\{1 = \frac{E_{tot.}^2}{E_{tot.}^2} = \left|\sum_{(i)}\frac{\overline{E}_i}{E_{tot.}}\right|^2 = \sum_{(i)}(\frac{E_{0i}^2 + E_{pi}^2}{E_{tot.}^2}) = \sum_{(i)}e_i^2\right\}.$$

(4.20-4)

Founders of Quantum Mechanics succeeded to model the wave function, that can mimic, represent, or (analogically, and in-average, or approximately) replace and generalize equations of normalized energy and vectors conservation laws, (4.20), (4.20-1), (4.20-3), and (2.20-4) of a system in certain transformation, $\left(\sum_{(i)}e_i, \sum_{(j)}e_j\right) = \frac{E_{i,j}}{E_{tot.}}$, $\sum_{(i)}e_i^2 = \sum_{(j)}e_j^2 = 1$, $\left(\left|\sum_{(i)}\vec{a}_i\right|, \left|\sum_{(j)}\vec{a}_j\right|\right) = \left|\frac{\vec{A}_{i,j}}{A}\right|$, by convenient finite or infinite summations (for instance using Fourier Integral Transformation, and framework of Signal Analysis, Probability theory and Statistics). This practically replaces finite (or discrete) summation elements $\sum_{(i)} e_i^2, \sum_{(j)} e_j^2, \left|\sum_{(i)} \vec{a}_i\right|^2, \left|\sum_{(j)} \vec{a}_j\right|^2$ by an equivalent integral summation form of certain wave function that could have a finite or infinite number of elements (which we can modify and fit to represent spectral and/or probability components of such wave function, such as $\int_{-\infty}^{+\infty} \psi^2(t)dt = 1$). When dealing with a sufficiently large number of elements in a system, whether these elements are mathematically constructed through abstractions, approximations, generalizations, or by applying integral transformations and Signal Analysis decomposition methods, it becomes feasible to apply or approximate laws of Probability and Statistics to the system. This approach allows us to effectively use Probability and Statistics on elementary signal-function bases of the system, provided that necessary conditions for mathematical fitting and integration are met. This can yield applicable and reasonably accurate results.

Quantum Mechanics, through consensus and theoretical development, has effectively formulated or modeled the wave function as a probability-based function $|\psi|^2$. This approach involves adopting specific rules for replacement, which are analogous to the concepts of energy conservation, averaging, and normalization formalism, as presented in equations (4.20) to (4.20-4), as follows,

$$\left\{\sum_{(i)} e_i^2 = \sum_{(j)} e_j^2 = 1\right\} \Leftrightarrow \left\{\begin{array}{l} \int |\psi|^2 dq = 1 \text{ , } dq = dx_1 \cdot dx_2 \cdot ... \cdot dx_n \\ \psi = \psi(x_1, x_2, ...x_n) = \sum_m \alpha_m \psi_m \text{ , } \alpha_m = \text{const.} \end{array}\right\} \tag{4.21}$$

Understanding matter-wave duality and its associated energy components supports this approach. Since interacting particles and waves are surrounded and coupled with various energy-momentum entities, we can extend these concepts to spatial and temporal distributions of relevant states. Probability and statistics provide a useful mathematical framework for approximating and applying energy-momentum conservation principles. The sum of probabilities in a system, always equal to 1, is analogous to the sum of normalized energy components in the same system.

The distinction between dimensional power-related and non-dimensional probabilistic wave functions become less clear when dealing with a large or infinite number of particle-wave components. In practice, the sum of probabilities approximates the sum of normalized energies of wave packets involved in the process. To develop a robust theoretical framework, it is sufficient to unify signal analysis and probability theory concepts, particularly by applying Parseval's theorems and identities for discrete and infinite summations or wave superposition, as seen in contemporary Quantum Theory.

This methodology is not only applicable to Quantum Theory but also to any system with many similar or identical participants. Another strong support for this approach comes from the fact that temporal-spatial dependent wave functions or signals, including power functions, motional energy functions, currents, voltages, forces, velocities, and moments, can be precisely decomposed into simple harmonic components or similar elementary wave groups, such as Gaussian-Gabor signals. This decomposition can be analyzed using discrete and integral forms of Fourier and Analytic Signal analysis, supported by sampling theories like Kotelnikov-Shannon and Whittaker-Nyquist (see more in [57, 58, and 59]).

When decomposing a wave function into its elementary waveforms, these wave elements remain real matter-waves that can be experimentally detected or verified. Solutions to Classical or Schrödinger wave equations involve sets of wave function components propagating in various directions, suggesting that wave functions could be present everywhere with certain probabilities. However, probability is not the unique or essential property of any wave function. Probability modeling of wave functions is merely a useful approximation to represent power-related, dimensional wave functions.

In developing Orthodox Quantum Theory, the process involved establishing a mathematical framework of normalizations, transformations, and definitions to support and defend the theory (see (4.20), (4.20.1) - (4.20-4) and (4.21)). For example, the statement representing the probability distribution of an energy state can be viewed as a mathematical equivalent to a probability function $|\psi|^2$, aligning with fundamental conservation laws and spectral and signal analysis principles.

The probability of a multi-component event obeys the law of total probability conservation (the sum of all individual probabilities equals one), which can be considered an equivalent formulation. The normalization of wave functions in relation to conservation laws will be further discussed later in this chapter.

Modern Quantum Theory sought to align its wave functions with physical laws, including Hamiltonian and Lagrangian mechanics, by unifying and generalizing existing conservation laws and incorporating statistical, probabilistic, and signal analysis concepts. When necessary, new mathematical rules and bridges were invented to maintain coherence within the theory.

Amalie Noether's theorem (formulated in 1905) significantly contributed to this process. The theorem states:

- For every continuous symmetry of the laws of physics, there must be a corresponding conservation law.

- For every conservation law, there must be continuous symmetry.

This theorem provided a logical basis for completing the Quantum Theory framework by aligning it with recognized symmetries and conservation laws. Although Noether's theorem was fully appreciated later, it played a crucial role in finalizing and generalizing Orthodox Quantum Theory.

The popularization and simplification of Quantum Theory have unfortunately led to significant conceptual confusion and mystification. Many adherents have accepted these simplified explanations as the definitive and unchallengeable truth. Despite its practical success, the fundamental principles of Quantum Theory are clear and straightforward, rooted in a complex mapping of basic conservation laws.

This confusion is partly due to the unintentional misinterpretations and oversimplifications introduced by those popularizing Quantum Theory. As a result, many well-meaning followers have accepted these interpretations as absolute and divine truths. While it is true that Quantum Theory has been remarkably successful, with its effectiveness often celebrated by its proponents, the statement that "nobody understands why it works so well" underscores the need for a clearer understanding.

It is essential to rectify this situation. As illustrated in equations (4.20) and (4.20.1) - (4.20-4), the foundation of Quantum Theory is quite tangible, clear, and simple. The theory continues to incorporate, albeit in a modified and generalized form, a complex functional mapping and reformulation of basic conservation laws, which has been made more complicated than necessary.

The author of this book believes that the principles of Signal and Spectrum Analysis provide a sufficient mathematical foundation for formulating most of the rules, theorems, definitions, and models in modern Quantum Mechanics, provided they are properly integrated with the concepts from Particle-Wave Duality Conservation (PWDC) and other conservation laws (see Chapter 10 for further details). Probability and Statistics serve as a complementary framework that refines and formalizes Orthodox Quantum Mechanics, often resembling a summation of probabilities as shown in equation (4.21).

Probability and Statistics are especially useful in contexts involving many similar or identical items, which is common in the microcosm of matter and various physics-related systems. Given that many wave equations, such as those in (4.10-3), primarily operate with mean values (e.g., energy, momentum, frequency, wavelength, velocities; -see (4.17)), transitioning to Quantum Mechanics using Probability Theory and Statistics involves only minor theoretical and modeling adjustments.

However, the use of Probability and Statistics is particularly convenient when we disregard immediate spatial-temporal phase information, as well as the intrinsic synchronizations, couplings, and entanglement effects between Analytic Signal pairs $\psi(t)$ and $\hat{\psi}(t)$. It is important to recognize that while operating with mean values is beneficial, it is not sufficient to claim that Microphysics exclusively deals with items manageable by Probability and Statistics alone.

One of the problems in (4.21) to be solved is in the fact that normalized energy members $\sum_{(i)} \mathbf{e}_i = \sum_{(j)} \mathbf{e}_j = \mathbf{1}$, $\sum_{(i)} e_i^2 = \sum_{(j)} e_j^2 = 1$ present summations of state of rest (static, or constant) energy members, $\mathbf{e}_{oi}$ and $\mathbf{e}_{0j}$, and their remaining motional energies, $\mathbf{e}_{mi}$ and $\mathbf{e}_{mj} \Rightarrow$ $\sum_{(i)} \mathbf{e}_i = \sum_{(i)} (\mathbf{e}_{oi} + \mathbf{e}_{mi}) = \sum_{(j)} \mathbf{e}_j = \sum_{(j)} (\mathbf{e}_{oj} + \mathbf{e}_{mj}) = \mathbf{1}$.

Quantum Mechanics pretends to represent all such energy members (static = rest energy, and dynamic = motional energy members) in the same way (using wave functions, as wave groups or wave packets). ***In this book, we support the platform that proper wavefunction can represent only motional (dynamic)***

energy (or power), and that states of rest and motional energy members should be separately treated. Also, after normalizing vector components, (4.21); $\left|\sum_{(i)} \vec{a}_i\right| = \left|\sum_{(j)} \vec{a}_j\right| = 1$, we additionally sacrifice their motional, or immediate, time-space evolving phase information (losing the chance to deal with the concept of active and reactive power and energy, and to analyze optimal active power transfer, like in electronics). To find a solution to such and similar problems, Quantum Mechanics simply introduced different (and mathematically manageable) artificial complex functions, normalization or renormalization rules, Feynman diagrams, Operators' algebra etc., without modifying the foundations of Orthodox Quantum Mechanics, making this situation more complicated and "conceptually fogy" (but still mathematically operational and applicable to Physics).

*[♣ **COMMENTS & FREE-THINKING CORNER:***

The official history of modern Quantum Theory is not presented as it is here, nor does it follow the mathematical approach outlined in equations (4.20) - (4.21). According to the author, the success of Quantum Theory can be attributed to the effective integration of conservation laws of physics with the mathematical principles of Signal Analysis, Probability, and Statistics, combined with the effects of rotation and linear motion, which are framed here as Particle-Wave Duality Conservation (PWDC).

The probability framework in Quantum Mechanics is useful but not fundamentally essential or superior. It serves as a practical tool for modeling, aligning with the normalized and dimensionless wave functions that respect conservation laws (at least on average). This methodology is straightforward when applicable and logical. However, it's crucial to recognize that the probabilistic nature of certain phenomena does not mean that reality is intrinsically probabilistic. We might discover that de Broglie matter waves and electromagnetic waves involve hidden parameters or undiscovered aspects of reality. It is also possible that we are interacting with unknown oscillatory media or undetectable fluids.

Despite our inability to directly observe such media, our mathematical models can accurately predict the results of particle-wave interactions within them. From an observational standpoint, it may appear that we are dealing with probabilities. For example, Schrödinger's equation provides reliable predictions in micro-world physics, confirming that our mathematical models for matter wave phenomenology are effective, even though our conceptual understanding remains incomplete.

To address this gap, one could imagine an invisible, fluid-like medium where wave propagation, perturbations, and other phenomena occur. Particles moving within such media would generate de Broglie waves, like how a stone thrown into water creates ripples.

The terms "Quantum," "Quant," "Quantization," and "Quantum Mechanics" have become fashionable over the past century, leading to a form of intellectual inertia and emotional attachment to these concepts. These terms describe fixed, quantifiable units of measurement, which are fundamental to understanding matter and energy states. Quantization essentially refers to counting these discrete units.

In this book, we find that "quantum" typically corresponds to minimal, elementary, or resonant wave formations, often related to half-wavelength domains. These elementary wave components are the building blocks of matter and energy-momentum states, which are countable using integers. This quantization is a property of stable material structures, where energy-momentum exchanges occur in these discrete units.

It is important to remember that terms associated with quantum theory describe basic, countable phenomena, rather than invoking mystical or artificial concepts. Modern science and technology must continually clarify and justify new findings to avoid creating theories that are unnecessarily complex or unnatural.

A significant concern is the tendency for established mainstream science to resist change, like the historical persistence of the Ptolemaic geocentric model due to ideological and dogmatic reasons. Just as the geocentric model was maintained despite contrary evidence, there is a risk of modern physics and Quantum Theory becoming entrenched in outdated or flawed paradigms. We must strive to avoid repeating such historical mistakes and ensure that our understanding of Quantum Theory evolves naturally and accurately. ♣]

4.3.4. Matter Waves and Unified Field Theory

By simple algebraic transformations of different forms of Schrödinger equations (4.9-2), (4.10), (4.10-1) - (4.10-5), valid for different levels of energy translation, such as,

$$\left\{\begin{array}{l}
\Delta\overline{\Psi}-\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=\left(\frac{\tilde{E}}{\hbar u}\right)^2\overline{\Psi}+\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=j\frac{\tilde{E}}{\hbar u^2}\frac{\partial\overline{\Psi}}{\partial t}+\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=0.\\
\Delta\overline{\Psi}-\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=\left(\frac{\tilde{E}-U_p}{\hbar u}\right)^2\overline{\Psi}+\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=j\frac{\tilde{E}-U_p}{\hbar u^2}\frac{\partial\overline{\Psi}}{\partial t}+\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=0.\\
\Delta\overline{\Psi}-\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=\left(\frac{\tilde{E}+E_0-U_p}{\hbar u}\right)^2\overline{\Psi}+\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=j\frac{\tilde{E}+E_0-U_p}{\hbar u^2}\frac{\partial\overline{\Psi}}{\partial t}+\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=0.\\
\Delta\overline{\Psi}-\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=\left(\frac{\tilde{E}+E_0}{\hbar u}\right)^2\overline{\Psi}+\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=j\frac{\tilde{E}+E_0}{\hbar u^2}\cdot\frac{\partial\overline{\Psi}}{\partial t}+\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=0.\\
\Delta\overline{\Psi}-\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=\left(\frac{\tilde{E}+E_0+U_p}{\hbar u}\right)^2\overline{\Psi}+\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=j\frac{\tilde{E}+E_0+U_p}{\hbar u^2}\cdot\frac{\partial\overline{\Psi}}{\partial t}+\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=0.\\
\Delta\overline{\Psi}-\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=\left(\frac{\tilde{E}+U_p}{\hbar u}\right)^2\overline{\Psi}+\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=j\frac{\tilde{E}+U_p}{\hbar u^2}\cdot\frac{\partial\overline{\Psi}}{\partial t}+\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=0.
\end{array}\right\}\Rightarrow \quad (4.22)$$

$$\Rightarrow \boxed{\begin{array}{l}\Delta\overline{\Psi}-\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=\left(\frac{L}{\hbar u}\right)^2\overline{\Psi}+\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=j\frac{L}{\hbar u^2}\frac{\partial\overline{\Psi}}{\partial t}+\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=0,\\ L\in\left[(\tilde{E}),\text{ or }(\tilde{E}-U_p),\text{ or }(\tilde{E}+E_0-U_p),\text{ or }(\tilde{E}+E_0),\text{ or }(\tilde{E}+E_0+U_p),\text{ or }(\tilde{E}+U_p)...\right]\end{array}},$$

we can easily obtain generalized (or universal) wave equation with different Lagrangian, or (for instance) a wave equation that has the same form as Classical or electromagnetic waves equation $\Delta\Psi=\Delta(\mathcal{E},\mathcal{H})=\frac{1}{c^2}\frac{\partial^2(\mathcal{E},\mathcal{H})}{\partial t^2}$, {$(\mathcal{E},\mathcal{H})$ = (*electric and magnetic fields*)}.

From (4.22), especially from $\Delta\overline{\Psi}=\frac{1}{u^2}\frac{\partial^2\overline{\Psi}}{\partial t^2} \Leftrightarrow \Delta(\mathcal{E},\mathcal{H})=\frac{1}{c^2}\frac{\partial^2(\mathcal{E},\mathcal{H})}{\partial t^2}$, *it is obvious that electromagnetic waves (in a free space, U_p = 0) are just one of measurable (and non-probabilistic) manifestations of de Broglie matter waves (when stable rest mass does not exist* $\Rightarrow \tilde{E}=E_{total}=E_k$*).* See also familiar elaborations (3.7-1) and (3.7-2) from the third chapter of this book.

There are also forms of (4.22) where rest mass should be involved, and where we can recognize other forms of matter waves. We also see that the famous Schrödinger equation *is nothing else but just another analogically modified form of well-known (classical, D'Alembert) wave equation, valid in electromagnetic theory, mechanics, acoustics, fluid dynamics, simply formulated in the form of a Complex, Analytic Signal... Going backwards to some of the earlier chapters of this book, we shall be able to defend initial hypothetical statements (of this book) that New Theory of Gravitation could be constructed following analogy with Faraday-Maxwell Electromagnetic Theory (of course, first by upgrading both to become more generally valid and more compatible for unification).*

The way to establish a Unified Field Theory (suggested in this paper) will go back to the presentation of analogies found at the beginning of this paper. To give an idea how to relate Classical and Schrödinger equations (4.22) to Gravitation (and to any other field), we should remember that the square of the wave function in this book presents the active power function $\Psi^2(t,r) = P(t,r) = d\tilde{E}/dt = -dE_k/dt$ (as originally established in Chapter 4.0). Since we are already used to express power functions as products between corresponding current and voltage (in electro technique), or force and velocity (in mechanics) etc. (*see the first chapter of this book dealing with analogies*), this will directly enable us to develop new forms of wave equations (valid for Gravitation and other fields), formally like Schrödinger and Classical wave equations (4.22), but dealing with velocities, forces, currents, voltages ..., as for instance,

$$\Psi^2(t,r) = P(t,r) = \frac{d\tilde{E}}{dt} (=) \text{Active Power } (=)$$
$$(=)\begin{Bmatrix} i(t)\cdot u(t) & (=) & [\text{Current} \cdot \text{Voltage}], \text{ or} \\ f(t)\cdot v(t) & (=) & [\text{Force} \cdot \text{Velocity}], \text{ or} \\ \tau(t)\cdot\omega(t) & (=) & [\text{Orb.} - \text{moment} \cdot \text{Angular velocity}], \text{or} \\ (\vec{E} \times \vec{H})\cdot\vec{S} & (=) & [\overrightarrow{\text{Pointyng Vector}}] \cdot \overrightarrow{\text{Surface}} \\ ----- & (=) & ---------- \\ s_1(t)\cdot s_2(t) & (=) & [(\text{signal}-1)\cdot(\text{signal}-2)] \end{Bmatrix} \quad (4.23)$$

(see also (4.0.82) in Chapter 4.0).

*[♣ **COMMENTS & FREE-THINKING CORNER:** We can also profit from the well-developed methodology (in electronics) related to an optimal active power transfer, to real, imaginary, and complex power, to real and complex impedances etc., applying by analogy, models, structures, and conclusions (already developed in Electronics), to Classical Mechanics, Gravitation, Quantum Mechanics...*

Here we try to connect arbitrary Power Function (product between current and voltage, or product between force and velocity, or product between any other relevant, mutually conjugate functions creating power) to a Wavefunction as it is known in Quantum Mechanics. Energetically analyzed, any wave propagation in time and frequency domain can be mutually (space-time-frequency) correlated using Parseval's theorem. Consequently, the immediate (time domain) Power signal can be presented as the square of the wave function $\Psi^2(t)$. Analysis of the optimal power transfer can be extended to any wave propagation field (and to arbitrarily shaped signals). Thus, we profit enormously (in booth, directions) after unifying traditional concepts of Active, Reactive and Apparent power with the $\Psi^2(t)$ wave function mathematics, based on Analytic Signal methodology.

In Quantum Mechanics the wave function $\Psi^2(t)$ *is conveniently modeled as a probability function, but effectively it behaves like normalized and dimensionless Power function, and here it will be closely related to Active Power or power delivered to a load expressed in Watts as its units:*

$$\Psi^2(t) = P(t) = S(t)\cos\theta(t) = \frac{1}{2}(ui + \hat{u}\hat{i}) = Q(t)\cdot \cot an\,\theta(t)\ (=)[W]\ .$$

The power reflected from a load, or Reactive Power, can be formulated as:

$$Q(t) = S(t)\sin\theta(t) = \frac{1}{2}(u\hat{i} - \hat{u}i) = \Psi^2(t)\cdot\tan\theta(t) = P(t)\cdot\tan\theta(t)\,(=)\ [VAR]$$

Electric Power and Energy transfer analysis (especially for arbitrary voltage and current signal forms) can be related to Wave function analysis if we establish the Wavefunction (or more precisely, the square of the wavefunction) in the following way (see more in Chapter 4.0):

$$P(t) = \Psi^2(t) = [a(t)\cos\varphi(t)]^2 = \text{Wave function}\ ,\ t\in[T],$$
$$\Psi(t) = a(t)\cos\varphi(t),\ \hat{\Psi}(t) = a(t)\sin\varphi(t),$$
$$\overline{\Psi}(t) = \Psi(t) + j\hat{\Psi}(t) = \Psi(t) + jH[\Psi(t)] = a(t)e^{j\varphi(t)} = \frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty} U(\omega)e^{-j\omega t}d\omega =$$
$$= \frac{1}{\pi^2}\int_{0}^{+\infty} U(\omega)e^{-j\omega t}d\omega = \frac{1}{\pi}\int_{(0,+\infty)} A(\omega)e^{-j\omega t}d\omega,$$
$$U(\omega) = U_c(\omega) - j\,U_s(\omega) = \int_{(-\infty,+\infty)} \overline{\Psi}(t)\ e^{j\omega t}dt = A(\omega)e^{-j\Phi(\omega)}\ ,$$
$$U_c(\omega) = A(\omega)\cos\Phi(\omega),\ U_s(\omega) = A(\omega)\sin\Phi(\omega),$$
$$a(t) = \sqrt{\Psi(t)^2 + \hat{\Psi}(t)^2}\ ,\quad \varphi(t) = \operatorname{arctg}\frac{\hat{\Psi}(t)}{\Psi(t)}\ ,$$
$$A^2(\omega) = U^2{}_c(\omega) + U^2{}_s(\omega),\quad \Phi(\omega) = \operatorname{arctg}\frac{U_s(\omega)}{U_c(\omega)}\ ,$$
$$T\cdot\langle P(t)\rangle = \int_{-\infty}^{+\infty} P(t)dt = \int_{-\infty}^{+\infty}\Psi^2(t)dt = \int_{-\infty}^{+\infty}\hat{\Psi}^2(t)dt = \frac{1}{2}\int_{-\infty}^{+\infty}\left|\overline{\Psi}(t)\right|^2 dt =$$
$$= \frac{1}{2}\int_{-\infty}^{+\infty}\left|\Psi(t) + j\hat{\Psi}(t)\right|^2 dt = T\cdot\langle\hat{P}(t)\rangle = \int_{-\infty}^{+\infty}\hat{P}(t)dt =$$
$$= \frac{1}{2}\int_{-\infty}^{+\infty} a^2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty}|U(\omega)|^2 d\omega = \frac{1}{\pi}\int_{0}^{\infty}[A(\omega)]^2 d\omega,$$

As we can see, every single wavefunction has at least two wave components (since to create power function it is essential to make the product of two relevant, mutually conjugate signals, like current and voltage, velocity and force, or some other equally important couple of signals: see (4.18)):

$$\Psi^2(t) = P(t) = S(t)\cos\theta(t) = \frac{1}{2}(ui + \hat{u}\hat{i}) = \Psi_1{}^2(t) + \Psi_2{}^2(t),\ \Psi_1^2 = \frac{ui}{\sqrt{2}},\ \Psi_2^2 = \frac{\hat{u}\hat{i}}{\sqrt{2}},$$

and it shouldn't be too big success to causally explain quantum mechanical diffraction, superposition and interference effects, when a "single wave object" and/or a single particle (like an electron, or photon) passes the plate with (at least) two small, diffraction holes, because in reality (in this situation) there isn't a single object (there are always minimum 2 mutually conjugate wave elements and their mixed products, somehow energetically coupled with their environment, extending the number of interaction participants). What looks to us like strange quantum interaction, or interference of a single *wave or particle object with itself, in fact, presents an interaction of at least 2 wave entities with some other, third object (* $\Psi^2(t) = \Psi_1^2(t) + \Psi_2^2(t)$ *). Somehow, Nature always creates complementary and conjugate couples of important elements (signals, particles, energy states…) belonging to every kind of matter motions. We can also say that every object (or energy state) in our universe has its non-separable and conjugate image (defined by Analytic Signal concept). Consequently, the quantum mechanical wave function and wave energy should represent only a motional energy (or power) composed of a minimum of two mutually coupled wave functions (* $\Psi^2(\mathbf{t}) = \Psi_1^2(\mathbf{t}) + \Psi_2^2(\mathbf{t})$ *).*

Here applied mathematics, regarding wave functions $\Psi^2(\mathbf{t}) = \Psi_1^2(\mathbf{t}) + \Psi_2^2(\mathbf{t})$*, after making appropriate normalizations and generalizations, would start (only superficially and approximately) looking like applying Probability Theory laws, like in the contemporary Quantum Theory. Consequently, modern Quantum Theory could also be treated as the generalized mathematical modeling of the micro world phenomenology, by conveniently unifying all conservation laws of physics in a joint, dimensionless, mutually well-correlated theoretical platform, creating a new mathematical theory that is different by appearance, but isomorphic to remaining Physics. Also, a kind of generalized analogy with Norton and Thevenin's theorems (known in Electric Circuit Theory) should also exist (conveniently formulated) in all other fields of Physics and Quantum Theory, since the cause or source of certain action produces a certain effect, and vice versa and such events are always mutually coupled. ♣]*

4.3.5. Wave Function and Euler-Lagrange-Hamilton Formalism

One of the biggest achievements of Classical Mechanics is Euler-Lagrange-Hamilton formalism derived from Calculus of Variations. In this methodology, we usually formulate the most appropriate Lagrange function, or **Lagrangian (=) L**, and apply Euler-Lagrange equations on it, to find all relevant elements of certain complex motion. Without going too far in discussing Euler-Lagrange formalism, just analyzing different forms of Schrödinger wave equations (4.22), we can conclude that **Lagrangian** can also have different (floating or variable energy level) forms, as for instance,

$$\left[\begin{array}{l}
\frac{\hbar^2}{m^*}(\frac{u}{v})\Delta\overline{\Psi}+(\tilde{E}-U_p)\overline{\Psi}=0,(U_p\neq 0)\Rightarrow\left\{\underline{L=\tilde{E}-U_p}=-\frac{\hbar^2}{m^*}(\frac{u}{v})\frac{\Delta\overline{\Psi}}{\overline{\Psi}}\Leftrightarrow E_k-U_p,\frac{1}{c^2}\leq(\frac{\tilde{E}}{L})\frac{1}{u^2}<\infty\right\} \\
\frac{\hbar^2}{m^*}(\frac{u}{v})\Delta\overline{\Psi}+\tilde{E}\overline{\Psi}=0,\ (U_p=0)\Rightarrow\left\{\underline{L=\tilde{E}}=-\frac{\hbar^2}{m^*}(\frac{u}{v})\frac{\Delta\overline{\Psi}}{\overline{\Psi}}\Leftrightarrow E_k,\frac{1}{c^2}\leq\frac{1}{u^2}<\infty\right\}, \\
\frac{\hbar^2}{m^*}(\frac{u}{v})\Delta\overline{\Psi}+(\tilde{E}+U_p)\overline{\Psi}=0\Rightarrow\left\{\underline{L=\tilde{E}+U_p}=-\frac{\hbar^2}{m^*}(\frac{u}{v})\frac{\Delta\overline{\Psi}}{\overline{\Psi}}\Leftrightarrow E_k+U_p,\frac{1}{c^2}\leq(\frac{\tilde{E}}{L})\frac{1}{u^2}<\infty\right\} \\
\frac{\hbar^2}{m^*}(\frac{u}{v})\Delta\overline{\Psi}+(\tilde{E}+E_0-U_p)\overline{\Psi}=0\Rightarrow\left\{\underline{L=E_{total}-U_p}=-\frac{\hbar^2}{m^*}(\frac{u}{v})\frac{\Delta\overline{\Psi}}{\overline{\Psi}},\frac{1}{c^2}\leq(\frac{\tilde{E}}{L})\frac{1}{u^2}<\infty\right\}, \\
\frac{\hbar^2}{m^*}(\frac{u}{v})\Delta\overline{\Psi}+E_{total}\overline{\Psi}=\frac{\hbar^2}{m^*}(\frac{u}{v})\Delta\overline{\Psi}+(\tilde{E}+E_0)\overline{\Psi}=0\Rightarrow\left\{\underline{L=E_{total}}=-\frac{\hbar^2}{m^*}(\frac{u}{v})\frac{\Delta\overline{\Psi}}{\overline{\Psi}},\frac{1}{c^2}\leq(\frac{\tilde{E}}{L})\frac{1}{u^2}<\infty\right\}, \\
\frac{\hbar^2}{m^*}(\frac{u}{v})\Delta\overline{\Psi}+(\tilde{E}+E_0+U_p)\overline{\Psi}=\frac{\hbar^2}{m^*}(\frac{u}{v})\Delta\overline{\Psi}+(E_{total}+U_p)\overline{\Psi}=0\Rightarrow \\
\Rightarrow\left\{\underline{L=\tilde{E}+E_0+U_p}=-\frac{\hbar^2}{m^*}(\frac{u}{v})\frac{\Delta\overline{\Psi}}{\overline{\Psi}}\Leftrightarrow E_k+E_0+U_p,\frac{1}{c^2}\leq(\frac{\tilde{E}}{L})\frac{1}{u^2}<\infty\right\}.
\end{array}\right]\Rightarrow$$

$$\Rightarrow\boxed{\begin{array}{l}\Delta\overline{\Psi}-\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=\left(\frac{L}{\hbar u}\right)^2\overline{\Psi}+\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=j\frac{L}{\hbar u^2}\frac{\partial\overline{\Psi}}{\partial t}+\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=0, \\ L\in\left[\tilde{E}\text{ or }\tilde{E}-U_p\text{ or }\tilde{E}+E_0-U_p\text{ or }\tilde{E}+E_0\text{ or }\tilde{E}+E_0+U_p\text{ or }\tilde{E}+U_p...\right]\end{array}}. \qquad (4.24)$$

[♣ __COMMENTS & FREE-THINKING CORNER:__ Based on (4.24) and (4.10-3) we can formulate the unique and even more general forms of relativistic Schrödinger (or Dirac's) wave equations, which replace all previously formulated wave equations,

$$\left\{\begin{array}{l}
\Delta\overline{\Psi}-\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=\left(\frac{L}{\hbar u}\right)^2\overline{\Psi}+\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=j\frac{L}{\hbar u^2}\frac{\partial\overline{\Psi}}{\partial t}+\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=0, \\
\frac{\hbar^2}{m^*}(\frac{u}{v})\Delta\overline{\Psi}+L\overline{\Psi}=0\ ;\ \Delta\overline{\Psi}=\frac{1}{u^2}\frac{\partial^2\overline{\Psi}}{\partial t^2}, \\
(\frac{L}{\hbar})^2\overline{\Psi}+\frac{\partial^2\overline{\Psi}}{\partial t^2}=0,\ \frac{L}{\hbar}\overline{\Psi}=j\frac{\partial\overline{\Psi}}{\partial t}, \\
L\in\left[\tilde{E}\text{ or }\tilde{E}-U_p\text{ or }\tilde{E}+E_0-U_p\text{ or }\tilde{E}+E_0\text{ or }\tilde{E}+E_0+U_p\text{ or }\tilde{E}+U_p...\right] \\
S=\int_{t_1}^{t_2}L(q_i,\dot{q}_i...,t)dt=\text{extremum},
\end{array}\right\}\Rightarrow$$

$$\left(H=-\frac{\hbar^2}{m^*}(\frac{u}{v})\Delta\ (=)\ \text{Hamiltonian}\right)\Rightarrow\left(H\overline{\Psi}=L\overline{\Psi}=j\hbar\frac{\partial}{\partial t}\overline{\Psi}=-j\hbar u\nabla\overline{\Psi}\right)\Rightarrow$$

$$\Rightarrow L\Leftrightarrow H\Leftrightarrow j\hbar\frac{\partial}{\partial t}\Leftrightarrow -j\hbar u\nabla\overline{\Psi},\ \tilde{E}-U_p\leq L<\infty,\tilde{E}\Leftrightarrow E_k,\overline{\Psi}=\overline{\Psi}(t,r),j^2=-1, \qquad (4.25)$$

where Lagrangian L should be considered as the floating (and variable: $\mathbf{L} = \mathbf{E}_{tot.} - \mathbf{U}_p$, $0 \leq \tilde{\mathbf{E}} \leq \mathbf{E}_{tot.}, \mathbf{U}_p \geq 0$) energy level, and $\tilde{\mathbf{E}} - \mathbf{U}_p$ as its lowest or minimal energy level. For instance, in the case of electromagnetic waves in a free space ($\mathbf{U}_p = 0$), floating energy level (Lagrangian) is equal to the wave energy $\mathbf{L} = \tilde{\mathbf{E}}$. This is (most probably) related to the fact that photons do not have a rest mass (but when strong γ photon penetrates sufficiently close to an atom, it can transform its energy-momentum state into an electron-positron pair, because there is a certain potential field involved in the reaction). By performing an energy translation on the scale (or axis) of a floating energy level $\tilde{\mathbf{E}} - \mathbf{U}_p \leq \mathbf{L} < \infty$, we should be able "to materialize" all elementary particles and other energy forms of our universe. It should also be highlighted that all of the wave equations in (4.25) are mutually compatible or describe the same wave phenomena. The solutions of such equations are essentially dependent on the solutions of Euler-Lagrange equations applied on the relevant Lagrangian L. It is also interesting to notice that in (4.25) the factor $(\frac{\tilde{E}}{L})\frac{1}{u^2}$ should determine a kind of wave (phase) velocity. If we just limit this speed to be $\frac{1}{c^2} \leq (\frac{\tilde{E}}{L})\frac{1}{u^2} < \infty$, we shall get $(\frac{\tilde{\mathbf{E}}}{\mathbf{L}}) \geq (\frac{\mathbf{u}}{\mathbf{c}})^2$ and $-\mathbf{L} \leq \frac{\tilde{\mathbf{E}} - \mathbf{L}}{1 - \frac{\mathbf{u}^2}{\mathbf{c}^2}} \leq \frac{\mathbf{U}_p}{1 - \frac{\mathbf{u}^2}{\mathbf{c}^2}}$.

If we imagine that $(\frac{\tilde{\mathbf{E}}}{\mathbf{L}})\frac{1}{\mathbf{u}^2} = \frac{1}{\mathbf{v}^2}$ determines the group speed of the same wave group (because there is not a big choice of possible velocities of a wave packet, since $\mathbf{u}$ is already its phase speed), we get $(\frac{u}{v})^2 = (\frac{\tilde{E}}{L})$, and since from (4.2) we already know the relation between group and phase velocity, we shall easily obtain the following relations: $\{0 \leq (\frac{\mathbf{u}}{\mathbf{c}})^2 \leq (\frac{u}{v})^2 = (\frac{\tilde{\mathbf{E}}}{\mathbf{L}}) = \left\{\frac{1}{2} \leq 1 / \left[1 + \sqrt{1 - \frac{v^2}{c^2}}\right]^2 \leq 1\right\}\}$. This mathematical exercise (regarding group and phase velocity) at present, is brainstorming to initiate the search for the real meaning of the velocity involved in $(\frac{\tilde{\mathbf{E}}}{\mathbf{L}})\frac{\mathbf{1}}{\mathbf{u}^2} = \frac{\mathbf{1}}{\mathbf{v}^2}$).

We can also use the following expressions (from (4.1) - (4.3) and T.4.1.):

$$\left\{\begin{array}{l} L = \frac{\tilde{p}c^2}{u} = \tilde{E}(\frac{c}{u})^2 = hf(\frac{c}{u})^2 = E_{tot.} - U_p = \sqrt{c^2p^2 + (E_0)^2} = \frac{pv}{1 + \sqrt{1 - v^2/c^2}} + E_0 = pu + E_0, \\ E_{tot.} = \sqrt{c^2p^2 + (E_0)^2} + U_p = E_k + E_0 + U_p = pu + E_0 + U_p = \frac{pv}{1 + \sqrt{1 - v^2/c^2}} + E_0 + U_p, \\ \tilde{p} = (\frac{u}{c})\sqrt{p^2 + (\frac{E_0}{c})^2} = \frac{\tilde{E}}{u} = mu\sqrt{(\frac{\gamma v}{c})^2 + 1} = \frac{mv}{(1 + \sqrt{1 - v^2/c^2})\sqrt{1 - v^2/c^2}} = \frac{mu}{\sqrt{1 - v^2/c^2}} \\ \tilde{p} = \frac{\tilde{E}}{u} = p = \gamma mv = \frac{E_k}{u}, \ E_0 = mc^2 \end{array}\right\}. \quad (4.25\text{-}2)$$

Of course, if once somebody finds and proves that **SRT** is wrong in certain of its segments, we should be ready for new refinements of wave equations (and this is, most probably, already the case based on recent publications from Thomas E. Phipps, Jr.; -see literature under [35]).

From (4.25) we can find Lagrangian **L** and apply Euler-Lagrange-Hamilton equations on it, considering all possible coordinates and relevant field parameters of certain wave function (4.23), as for instance:

$$\begin{aligned} &\overline{\Psi} = \overline{\Psi}(q_i, \dot{q}_i, ..., t),\ S = \int_{t_1}^{t_2} L(q_i, \dot{q}_i, ..., t)dt = \text{extremum},\ H\overline{\Psi} = 0\,, \\ &k = \frac{2\pi}{\lambda} = \frac{2\pi}{h}\tilde{p},\ \tilde{E}\text{-}U_p \le L < \infty\,,\ L \in \left\{ (\tilde{E} - U_p), ...\ \tilde{E}, ...\ (E_{total} - U_p), ...\ E_{total} ... \right\}, \\ &\tilde{E} = \int_{-\infty}^{+\infty} \Psi^2(\omega t - kx)dt = \int_{-\infty}^{+\infty} \left|\overline{\Psi}(\omega t - kx)\right|^2 dt \Rightarrow \\ &\frac{d\tilde{E}}{dt} = 2(\omega - k\frac{dx}{dt})\int_{-\infty}^{+\infty} \Psi(\omega t - kx)dt, \frac{\partial \tilde{E}}{\partial x} = -2k\int_{-\infty}^{+\infty} \Psi(\omega t - kx)dt \Rightarrow \\ &\frac{d\tilde{E}}{dt} = \frac{\omega - k\frac{dx}{dt}}{-k}\cdot\frac{\partial \tilde{E}}{\partial x} = (v - u)\cdot\frac{\partial \tilde{E}}{\partial x} \Rightarrow \frac{d\tilde{p}}{dt} = (1 - \frac{u}{v})\cdot\frac{\partial \tilde{E}}{\partial x} = \frac{\sqrt{1 - v^2/c^2}}{1 + \sqrt{1 - v^2/c^2}}\cdot\frac{\partial \tilde{E}}{\partial x}\,. \end{aligned} \tag{4.26}$$

In simpler situations when Lagrangian presents a function of certain coordinates and their first derivatives, $L = L(q_i, \dot{q}_i, t)$, Euler-Lagrange equations will generate,

$$\left. \begin{cases} L = L(q_i, \dot{q}_i, t), \overline{\Psi} = \overline{\Psi}(q_i, \dot{q}_i ..., t)\ ,\ \tilde{E} = \int \left\|\overline{\Psi}\right\|^2 \cdot dt\,, \\ S = \int_{t_1}^{t_2} L(q_i, \dot{q}_i, t)dt = \text{extremum},\ H\overline{\Psi} = 0, \end{cases} \right\} \Rightarrow \tag{4.27}$$
$$\Rightarrow \frac{d}{dt}\left[\frac{\partial L(q_i, \dot{q}_i, t)}{\partial \dot{q}_i}\right] - \frac{\partial L(q_i, \dot{q}_i, t)}{\partial q_i} = 0.$$

In all other situations when Lagrangian presents more complex function, Euler-Lagrange equations will be,

$$\left. \begin{cases} L = L(q_i, \dot{q}_i, \ddot{q}_i, \dddot{q}_i, ..., q_i^{(n)}, t), \overline{\Psi} = \overline{\Psi}(q_i, \dot{q}_i, \ddot{q}_i, \dddot{q}_i, ..., q_i^{(n)}, t)\ ,\ \tilde{E} = \int \left\|\overline{\Psi}\right\|^2 \cdot dt\,, \\ S = \int_{t_1}^{t_2} L(q_i, \dot{q}_i, \ddot{q}_i, \dddot{q}_i, ..., q_i^{(n)}, t)dt = \text{extremum},\ H\overline{\Psi} = 0 \end{cases} \right\} \Rightarrow \tag{4.28}$$
$$\Rightarrow \frac{\partial \int L dt}{\partial q_i} - \frac{d}{dt}\left[\frac{\partial \int L dt}{\partial \dot{q}_i}\right] + \frac{d^2}{dt^2}\left[\frac{\partial \int L dt}{\partial \ddot{q}_i}\right] - ... + (-1)^n \cdot \frac{d^n}{dt^n}\left[\frac{\partial \int L dt}{\partial q_i^{(n)}}\right] = 0.$$

Orthodox Quantum Mechanics has built much of its mathematical framework on the foundations of Euler-Lagrange and Hamiltonian mechanics, largely adhering to the constraints of Classical Mechanics. The Euler-Lagrange equations were extended to Hamiltonian mechanics, introducing Poisson brackets and leading to the formulation of the Evolution Equation, Schrödinger's Equation, and Operator Mechanics. Although the elegant symmetries and forms of Euler-Lagrange and Hamiltonian equations, developed over a century ago to describe Classical Newtonian

Mechanics, still influence foundational physics, their applicability is often limited to non-relativistic contexts (v << c).

It is time to advance and update this robust framework, which is based on the Calculus of Variations, to a more generalized level of physics. This involves moving beyond the constraints of Classical, Relativistic, and Quantum Mechanics, which were established through a process of incremental adjustments and patchwork. By incorporating the relativistic form of the Lagrangian, we propose extending the applicability of the Euler-Lagrange equations (as outlined in equations (4.26) - (4.28)). This extension could lead to the development of a new theory that would update and potentially replace the traditional Euler-Lagrange and Hamiltonian mathematics.

In equations (4.25) - (4.28), we do not need to limit our treatment of motional and wave energy and Lagrangian functions to the Classical Mechanics perspective. By incorporating relativistic-compatible expressions for energy, we can extend the applicability of the Euler-Lagrange concept beyond its traditional constraints. While Classical Mechanics provides attractive and symmetrical mathematical forms when using its formulas for mass, momentum, and energy, this should not confine the Euler-Lagrange concept to these limitations. As a universally valid result derived from Hamilton's Principle and the Calculus of Variations, the Euler-Lagrange framework will remain relevant even if physics develops a theory that supersedes Einstein's Relativity Theory, if mass, momentum, and energy are described differently within the new theoretical framework.
This revision aims to clarify the proposed advancements and adjustments to the Euler-Lagrange and Hamiltonian frameworks, emphasizing the need for a more generalized approach to modern physics. ♣]

Equations (4.25) - (4.28) illustrate the connections between Classical, Relativistic, and Quantum Mechanics, and the Particle-Wave Duality Theory presented in this book. These equations are universally applicable to various aspects of particle-wave duality, including Gravitation, Electromagnetism, Linear and Rotational motions, and coupled Action-Reaction forces. For further elaboration, see T4.1, T5.2, (4.29) - (4.31), Uncertainty Relations (5.15), and (5.16), which complement this discussion. Summarizing the results from equations (4.25) - (4.28), we can formulate:

*1. **Particle-Wave Duality Code (PWDC)**: Defined by relations compatible with Relativity Theory, as found in (4.1), (4.2), and (4.3).*
*2. **General Wavefunction**: Initially established in Chapter 4.0, represented as an Analytic Signal (4.9) and described by the Schrödinger equation.*
*3. **Dynamic Energy and de Broglie Waves**: Only motional (dynamic or field) energy creates de Broglie matter waves, respecting Energy and Momentum conservation laws (as stated in the second part of (4.10-5)).*
*4. **Calculus-of-Variation Principles**: These principles, derived from Euler-Lagrange and Hamilton mechanics, frame the concepts above (from 1 to 3).*

Since Orthodox Quantum Mechanics (OQM) remains the predominant and officially accepted theory of the micro world, it is useful to highlight the key differences and similarities between the particle-wave duality concept presented here and that of

Orthodox Quantum Mechanics (or Schrödinger's wave mechanics). Here is a comparative analysis based on the above points:

*1. **Particle-Wave Duality Code (PWDC)** in OQM: Orthodox Quantum Mechanics integrates PWDC with its probabilistic wave function and Schrödinger's equation. However, it does not fully utilize the options found in (4.1) - (4.3) and remains constrained by Classical Mechanics in terms of particle speed and energy. A notable drawback of OQM is its imprecise treatment of phase and group velocities and the omission of spatial-temporal phase information. Additionally, OQM does not address coupled Action-Reaction forces or intrinsic rotational effects.*

*2. **Wavefunction and Schrödinger Equation** in OQM: The wavefunction and Schrödinger equation in OQM are somewhat artificially constructed. Unlike the general model representing wavefunctions as Analytic Signals (4.9), OQM's wavefunction does not incorporate immediate temporal-spatial-frequency or signal phase information. Although the OQM wave function is eventually accurate and operational, it lacks the comprehensive phase information that would provide a complete representation.*

*3. **Energy and Rest Mass in OQM**: In OQM, the wavefunction encompasses a particle's kinetic and rest energy, and rest mass states (summarized by the first part of (4.10-5)). While this approach is generally effective, it falls short in analyzing relativistic particles and loses constant terms (e.g., rest mass and energy) during differentiation.*

*4. **Conservation Laws in OQM**: OQM employs a probabilistic framework that indirectly respects Energy and Momentum conservation laws through dimensionless probability functions. These functions are derived from the normalization of energy and momentum conservation equations. Although this approach is compatible with conservation laws and integrates aspects of Statistics and Probability theory with Signal and Spectrum Analysis, it operates in averaged statistical terms rather than reflecting the immediate signal phase of de Broglie waves.*

*5. **Euler-Lagrange and Hamilton Mechanics in OQM**: Euler-Lagrange and Hamiltonian mechanics cannot be directly applied within the dimensionless, probabilistic framework of OQM. However, OQM indirectly incorporates much of the mathematical apparatus from non-relativistic Euler-Lagrange and Hamiltonian theory, accepting the constraints and conservation laws of Classical Mechanics.*

In conclusion, while Orthodox Quantum Mechanics has proven effective in many respects, it relies on a framework that only partially aligns with the more general principles and broader applicability proposed here.

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*[♣ **COMMENTS & FREE-THINKING CORNER:***

The author of this book views the probabilistic philosophy of Quantum Mechanics, including its probabilistic wave function and numerous assumptions, as just one of several acceptable mathematical modeling approaches. While it is not necessarily the best or most general, it has gained *significant support due to its average compatibility with Energy and Momentum conservation laws, as well as its alignment with established mathematics from Signal and Spectrum Analysis and Probability Theory.*

The probabilistic approach in Quantum Mechanics has benefited from its mathematical elegance and its ability to fit well with established physics, thanks to the foundational work of de Broglie, Heisenberg, and Schrödinger on particle-wave duality. This has led to the development of Orthodox Quantum Mechanics, which, despite its long-standing success, represents a major conceptual and philosophical obstacle to future advances in the field.

The current framework of Orthodox Quantum Mechanics may be hindering progress by masking or obscuring the path to new, more advanced concepts in Physics. As such, there is a need for a comprehensive refinement of modern physics, particularly Orthodox Quantum Mechanics. This paper suggests that a more significant advancement would involve creating a Unified and Open Architecture Field Theory—one that integrates all known natural forces and allows for the incorporation of yet-to-be-discovered forces. Such a theory might resemble a form of Superstring Theory. ♣]

4.3.6. Extensions of Wave Function and Universal Field Theory

It is especially important to notice that the wavefunction presented in this book is different from Quantum Mechanic's wavefunction. Generally valid in this book is that we can associate the wave function with any form of motional-energy, power (or particularly to linear motion, to Rotation, to motion in Electric field and to motion in the Magnetic field, or to any of their combinations, as established in Chapter 4.0). If there are other forms of motional energy presently unknown or not mentioned, for each one of them it should exist one associated wave function. However, what looks probable regarding a plurality of motional energy components is that the real motion energy nature could have its profound origins only in some of the different manifestations of an electromagnetic field.

To develop a more general (and Relativity compatible) forms of wave and Euler-Lagrange equations we shall use the expression for kinetic energy in a differential form, that is (mathematically) the same in Classical and Relativistic Mechanics (instead of using the expression for non-relativistic kinetic energy), as for instance:

$$dE_k = vdp = d\tilde{E} = hdf ,$$

since in Classical Mechanics for kinetic energy we have,

$$\{E_k = mv^2/2 = pv/2, p = mv, m = \text{const.}\} \Rightarrow \underline{dE_k} = mvdv = vd(mv) = \underline{vdp} ,$$

and we can get the same result in Relativistic Mechanics,

$$\left\{E_k = \frac{\gamma mv^2}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{pv}{1+\sqrt{1-\frac{v^2}{c^2}}} = (\gamma-1)mc^2, \gamma = 1/\sqrt{1-\frac{v^2}{c^2}}, p = \gamma mv\right\} \Rightarrow \underline{dE_k = vdp}.$$

It is also obvious that the differential of kinetic energy, from the point of view of Classical Mechanics, can be found as $dE_k = pdv \ (= vdp)$, but we eliminate this possibility since in Relativistic Mechanics we can get only $dE_k = vdp$. This way we indirectly implement compatibility of any form of kinetic energy (and wave functions developed based on motional energy) with Lorentz transformations and Euler-Lagrange equations.

In a situation where particle only performs the rotation, its kinetic (rotational and relativistic) energy, by analogy with the above-given example (see T.3.1-T.3.3 and T.4.3.1), can be expressed as a function of its angular momentum and angular speed, $dE_k = \omega d(J\omega) = \omega dL$.

The square of the wave function in this paper is just the active (motional) power, and using analogies developed before (see T.1.8, Generic Symmetries, and Analogies of the Laws of Physics), and meaning and content of the wave function as first time established in Chapter 4.0, we can summarize several forms of possible wavefunctions, presented in the following table:

T.4.3.1

Linear motion	$dE_k = vdp = -\Psi^2 \cdot dt,$ $\Psi^2 = v \cdot \tilde{F}$	**v = velocity,** $\tilde{F}$ **= force**	**p = momentum,** $dp = -d\tilde{p}$, $\tilde{F} = d\tilde{p}/dt$
Rotation	$dE_{kr} = \omega dL = -\Psi^2 \cdot dt,$ $\Psi^2 = \omega \cdot \tilde{\tau}$	ω **= angular velocity,** $\tilde{\tau}$ **= torque**	**L = angular momentum,** $dL = -d\tilde{L}$, $\tilde{\tau} = d\tilde{L}/dt$
Electric field	$dE_{ke} = udq_e = -\Psi^2 \cdot dt,$ $\Psi^2 = u \cdot \tilde{i} = i_{mag.} \cdot \tilde{u}_{mag.}$	**u = electric voltage,** $\tilde{i}$ **= electric current**	q_e **= electric charge,** $dq_e = -d\tilde{q}_e$, $\tilde{i} = d\tilde{q}_e / dt$
Magnetic field	$dE_{km} = id\Phi = -\Psi^2 \cdot dt,$ $\Psi^2 = u_{mag.} \cdot \tilde{i}_{mag.} = i_{el.} \cdot \tilde{u}_{el.}$	$u_{mag.}$ **= magn. voltage** $\tilde{i}_{mag.}$ **= magn. current**	Φ **= magnetic flux,** $d\Phi = -d\tilde{\Phi}$, $\tilde{i}_{mag.} = d\tilde{\Phi}/dt$

where: $(u = u_{el.}, \tilde{u} = \tilde{u}_{el.}) \equiv (i_{mag.}, \tilde{i}_{mag.}); (i = i_{el.}, \tilde{i} = \tilde{i}_{el.}) \equiv (u_{mag.}, \tilde{u}_{mag.});$

$q = q_{el.} = \Phi_{el.}, \tilde{q} = \tilde{q}_{el.} = \tilde{\Phi}_{el.}, q_{mag.} = \Phi_{mag.} = \Phi, \tilde{q}_{mag.} = \tilde{\Phi}_{mag.} = \tilde{\Phi}.$

In a case when all the above energy elements are present in the same motional situation, we shall have:

$$\begin{aligned} dE_{k-total} &= \sum_i dE_k(q_i, \dot{q}_i, \ddot{q}_i, \dddot{q}_i, ..., q_i^{(n)}, t) = vdp + \omega dL + udq_e + id\Phi + ... = \\ &= -vd\tilde{p} - \omega d\tilde{L} - ud\tilde{q}_e - id\tilde{F} - ... = \Psi^2_{total}dt = \sum_i \Psi^2(q_i, \dot{q}_i, \ddot{q}_i, \dddot{q}_i, ..., q_i^{(n)}, t)dt = \\ &= \sum_i \Psi^2(q_i, v, \omega, u, i, p, L, q_e, \Phi, ..., t)dt = d\tilde{E}_{total}. \end{aligned} \tag{4.29}$$

We could again apply the Euler-Lagrange formalism, (4.26) - (4.28) on (4.29) and analyze much more complex field situations, this way approaching the fields' unification objective from a more general platform.

For instance, we could now generalize the meaning of (multi-component, linear) force, extending by analogy the force expression (4.18), (respecting also (4.19) and (4.26) - (4.29)), to the following form:

$$\begin{aligned} &\tilde{F}(t,r) = \frac{1}{v}\Psi^2(t,r) \Rightarrow \tilde{F}_i(q_i, \dot{q}_i, \ddot{q}_i, \dddot{q}_i, ..., q_i^{(n)}, t) = \frac{1}{\dot{q}_i}\Psi^2(q_i, \dot{q}_i, \ddot{q}_i, \dddot{q}_i, ..., q_i^{(n)}, t), \\ &\tilde{F} = \sum_{(i)} \alpha_i \tilde{F}_i = \sum_{(i)} \frac{\alpha_i}{\dot{q}_i}\Psi_i^2 = \sum_{(i)} \frac{\alpha_i}{\dot{q}_i} \frac{d\tilde{E}(q_i, \dot{q}_i, \ddot{q}_i, \dddot{q}_i, ..., q_i^{(n)}, t)}{dt} = \\ &= \sum_{(i)} \alpha_i \frac{E_i}{q_i} = \sum_{(i)} \alpha_i \frac{E_{ki}}{q_i}, \quad \alpha_i \in \{Const...\} \forall_i, \end{aligned} \tag{4.30}$$

$$\tilde{E} = -k^2 L \frac{\overline{\Psi}}{\Delta\overline{\Psi}} = Lu^2 \Delta\overline{\Psi} / \frac{\partial^2 \overline{\Psi}}{\partial t^2} = j\frac{\hbar k^2}{\Delta\overline{\Psi}} \frac{\partial \overline{\Psi}}{\partial t} = j\hbar u^2 \frac{\Delta\overline{\Psi}}{\overline{\Psi}} \frac{\partial \overline{\Psi}}{\partial t}.$$

It looks that generalized force law (4.30) represents only dynamical (transitory, or time-dependent) forces, but we can easily see that this is not correct, since every component of motional (or kinetic) energy E_{ki} plus certain constant energy $\mathbf{E_{0i}}$ can create the total energy $E_{total-i}$,

$$\begin{aligned} & E_{total-i}=E_{ki}+E_{0i}\,,\; E_{0i}=\text{Constant.}\Rightarrow \frac{dE_{ki}}{dt}=\frac{dE_{total-i}}{dt}\Rightarrow \\ & \Rightarrow \tilde{\mathbf{F}}=\sum_{(i)}\alpha_i\tilde{\mathbf{F}}_i=\sum_{(i)}\alpha_i\frac{\dot{\tilde{E}}_i}{\dot{q}_i}=\sum_{(i)}\alpha_i\frac{\dot{E}_{ki}}{\dot{q}_i}=\sum_{(i)}\alpha_i\frac{\dot{E}_{total-i}}{\dot{q}_i}\,,\alpha_i=\text{const.}. \end{aligned} \quad (4.31)$$

The generalized force law (4.31) should be considered (presently) more as an example for discussion in the direction of creating a more practical force formula, since the force is a vector. Constants $\boldsymbol{\alpha}_i$ should take care of dimensional agreements and generalized coordinates $\dot{\mathbf{q}}_i$ should belong to the set of generalized velocities, $q_i \in \{v_i, \omega_i, ...\}$.

The most important "secret" (hidden) in the force law (4.31) is that certain motional (time-dependent and stationary) energy and field components are somehow trapped (blocked or permanently captured) inside every constant energy level $\mathbf{E_{0i}}$. *For instance, every magnet (electromagnet or permanent magnet) is created as the result of a certain current flow, but in the case of permanent magnets, we do not see such current flow. This is because it is permanently captured inside small, atomic-size magnet domains, created by circulation of electrons around their cores. Consequently, the ultimate structure of matter (or of our universe) is that we have frozen (permanently captured, stationary and stable) energy states (or levels)* $\mathbf{E_{0i}}$, *and free energy levels* $\mathbf{E_{ki}}$. *Whenever we can penetrate (experimentally or theoretically) into an internal structure of such constant energy states* $\mathbf{E_{0i}}$, *we find different motional energy states, and sometimes other relatively stable energy states, or particles. Again, it is obvious that the most important forces creating such (internal) energy structure should be the forces in some close relation to the rotation. Now it becomes clearer that the real sources of Gravity should be hidden in a certain more sophisticated entity than in only what is related to a particle rest mass.*

Almost a perfect symmetry between electric and magnetic voltages and currents has been established as the result of presentations based on analogies (see T.3.1-T.3.3 and T.4.3.1). Of course, this symmetry exists as an achievement of Maxwell-Faraday Electromagnetic Theory, and here it becomes evident just because of the way it is presented. *It is also clear (after reading the entire contents of this book) that presently we do not have the same level of symmetry regarding Linear motion and Rotation, and that the missing link (between them) should be related to de Broglie matter waves. Therefore, every rectilinear motion should have some associated effects of rotation and waving (and vice versa, since every form of rotation is also a source of harmonic oscillations). Consequently, a new theory must be established to cover all aspects of here presented analogies and particle-wave duality phenomenology.*

4.3.7. Matter-Wave 4-Vectors in Minkowski Space and Elements for New Topology

The principal idea here is that when creating a New Topology Basis, we should consider only coordinates, motional elements and/or degrees of freedom which are really (and always) contributing to the total energy of a certain system (see also (5.16)). Based on (4.30) and (4.31) we can extract the elements for a New (Universal Field) Topology if we determine the resulting velocity $\mathbf{V}_{\Sigma}$ *and its path element* $\mathbf{dX}_{\Sigma}$ *caused by complex (multi-component) force* $\mathbf{F}_{\Sigma}$*, or by complex momentum* $\mathbf{P}_{\Sigma}$*, as for instance:*

$$\mathbf{F}_{\Sigma} = \sum_{(i)} \alpha_i \frac{\dot{E}_{ki}}{\dot{q}_i} = \sum_{(i)} \alpha_i \frac{\dot{E}_{total\text{-}i}}{\dot{q}_i} = \\ = \sum_{(i)} \frac{\alpha_i}{\dot{q}_i} \frac{dE(q_i, \dot{q}_i, \ddot{q}_i, \dddot{q}_i, ..., q_i^{(n)}, t)}{dt} = \frac{dP_{\Sigma}}{dt} = \dot{P}_{\Sigma} = \frac{1}{V_{\Sigma}} \frac{dE_{\Sigma}}{dt} \Rightarrow$$

$$\tilde{\mathbf{F}}_{\Sigma} = -\mathbf{F}_{\Sigma} = -\sum_{(i)} \alpha_i \frac{\dot{E}_{ki}}{\dot{q}_i} = -\sum_{(i)} \alpha_i \frac{\dot{E}_{total\text{-}i}}{\dot{q}_i} = \sum_{(i)} \alpha_i \frac{\dot{\tilde{E}}_i}{\dot{q}_i} = \\ = -\sum_{(i)} \frac{\alpha_i}{\dot{q}_i} \frac{dE(q_i, \dot{q}_i, \ddot{q}_i, \dddot{q}_i, ..., q_i^{(n)}, t)}{dt} = -\frac{dP_{\Sigma}}{dt} = -\dot{P}_{\Sigma} = \dot{\tilde{P}}_{\Sigma} \Rightarrow$$

$$V_{\Sigma} = \frac{dE_{\Sigma}}{dP_{\Sigma}} = \frac{d\left[\sum_{(i)} E_i\right]}{dP_{\Sigma}} = \frac{\dot{E}_{\Sigma}}{\mathbf{F}_{\Sigma}} = \frac{\sum_{(i)} \dot{E}_i}{\mathbf{F}_{\Sigma}} = \frac{dX_{\Sigma}}{dt}, \tag{4.32}$$

$$dP_{\Sigma} = \sum_{(i)} \frac{\alpha_i}{\dot{q}_i} dE(q_i, \dot{q}_i, \ddot{q}_i, \dddot{q}_i, ..., q_i^{(n)}, t),$$

$$E_i = E(q_i, \dot{q}_i, \ddot{q}_i, \dddot{q}_i, ..., q_i^{(n)}, t),$$

$$dX_{\Sigma} = V_{\Sigma} dt = \frac{\sum_{(i)} dE_i}{dP_{\Sigma}} dt = \frac{\sum_{(i)} \dot{E}_i}{F_{\Sigma}} dt = \frac{\sum_{(i)} dE_i}{F_{\Sigma}} = \frac{\sum_{(i)} dE_i}{\sum_{(i)} \frac{\alpha_i}{\dot{q}_i} dE_i} dt = \frac{\sum_{(i)} dE_i}{\sum_{(i)} \alpha_i \frac{dE_i}{dq_i}}.$$

With (4.32) only the principal idea and starting platform for creating New Topology is formulated (but to finalize this task will require much more effort; -see also (5.15) - (5.17)).

In contemporary physics (especially in Relativity Theory) we use (under Lorentz transformations) covariant forms of 4-vectors in Minkowski space, as the most elegant and most general synthesis of Energy and Momentum conservation laws. For instance, covariant 4-vectors of particle velocity (here we can also say group velocity when we are using wave packet or wave group model) and momentum are known as:

$$\overline{V}_4 = \overline{V}[\gamma\vec{v}, \gamma c], \overline{V}_4^2 = \text{invariant}, \ \overline{P}_4 = m\overline{V}_4 = \overline{P}\left[\vec{p} = \gamma m\vec{v}, \frac{E}{c} = \gamma mc\right] =$$
$$= \overline{P}(\vec{p}, \frac{E}{c}) = \overline{P}\left[\vec{p}, (\frac{E_0}{c} + \frac{\tilde{E}}{c})\right], \overline{P}_4^2 = \text{invariant}., \tilde{E} = E_k = (\gamma - 1)mc^2 = pu, \ E_0 = mc^2. \quad (4.33)$$

The wave energy and wave momentum can be connected in a similar way using differential forms of 4-vector wave momentum,

$$\overline{P}_4 = \overline{P}(\vec{p}, \frac{E}{c}) = \overline{P}\left[\vec{p}, (\frac{E_0}{c} + \frac{\tilde{E}}{c})\right] = \overline{P}_{4,0}(0, \frac{E_0}{c}) + \tilde{P}_4(\vec{p}, \frac{\tilde{E}}{c}), \vec{p} = \tilde{p},$$
$$d\overline{P}_4 = d\tilde{P}_4 = d\tilde{P}(\tilde{p}, \frac{\tilde{E}}{c}) = \tilde{P}(d\tilde{p}, \frac{d\tilde{E}}{c}) = \tilde{P}(\frac{d\tilde{E}}{v}, \frac{d\tilde{E}}{c}) = \tilde{P}(\frac{\Psi^2}{v}dt, \frac{\Psi^2}{c}dt) = \frac{1}{v}\overline{\Psi}_4^2 dt \Rightarrow$$
$$\overline{\Psi}_4^2 = \frac{v d\tilde{P}_4}{dt} = \overline{\Psi}_4^2(\frac{v d\tilde{p}}{dt}, \frac{v}{c}\frac{d\tilde{E}}{dt}) = \overline{\Psi}_4^2(\frac{d\tilde{E}}{dt}, \frac{v}{c}\frac{d\tilde{E}}{dt}) = \overline{\Psi}_4^2(\Psi^2, \frac{v}{c}\Psi^2) \Rightarrow \quad (4.34)$$
$$\overline{P}_4 = \overline{P}_4(\int \frac{1}{v}\overline{\Psi}_4^2 dt) = \overline{P}_4(\int \frac{1}{v}\Psi^2 dt, \frac{1}{c}\int \Psi^2 dt) = \overline{P}(\vec{p}, \frac{E}{c}),$$
$$\overline{P}_4^2 = (\int \frac{1}{v}\Psi^2 dt)^2 - (\frac{1}{c}\int \Psi^2 dt)^2 = \text{invariant} \ (= \ mc^2),$$
$$d\tilde{E} = hdf = v d\tilde{p} = d(\tilde{p}u) = dE_k = dE = vdp = d(pu) = \Psi^2 dt.$$

A 4-wave vector and 4-phase velocity vector could also be formulated as,

$$\overline{V}_4 = \overline{V}(\gamma v, \gamma c) = \overline{V}(\frac{v}{\sqrt{1 - \frac{v^2}{c^2}}}, \frac{c}{\sqrt{1 - \frac{v^2}{c^2}}}) = \text{ 4-group velocity} \Rightarrow$$
$$\Rightarrow \overline{U}_4 = \overline{U}(\frac{u}{\sqrt{1 - \frac{u^2}{c^2}}}, \frac{c}{\sqrt{1 - \frac{u^2}{c^2}}}) = \text{ 4-phase velocity},$$
$$\overline{K}_4 = \frac{2\pi}{h}\tilde{P}_4 = \overline{K}(\vec{k}, \frac{\omega}{c}) \ (= \text{ 4-wave vector}), \quad (4.35)$$
$$\overline{V}_4^2 = \text{inv.}, \ \overline{U}_4^2 = \text{inv.}, \ \overline{K}_4^2 = \text{inv.}, \ \overline{K}_4\overline{U}_4 = \text{inv.} = 0,$$
$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{h}\tilde{p}, \omega = 2\pi f, \ u = \lambda f = \frac{\omega}{k} = \frac{\tilde{E}}{\tilde{p}},$$
$$v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} = \frac{d\omega}{dk} = \frac{d\tilde{E}}{d\tilde{p}}.$$

From (4.34) we can also develop the force 4-vectors which are Lorentz-covariant in Minkowski space,

$$\tilde{F}_4 = \gamma\frac{d\tilde{P}_4}{dt} = \left[\gamma\frac{d\tilde{p}}{dt}, \frac{\gamma}{c}\frac{d\tilde{E}}{dt}\right] = \left[\gamma\tilde{F}, \gamma\frac{\vec{v}}{c}\tilde{F}\right] =$$
$$= -\gamma\frac{d\overline{P}_4}{dt} = -\overline{F}_4 = -\left[\gamma\frac{d\vec{p}}{dt}, \frac{\gamma}{c}\frac{dE}{dt}\right] = -\left[\gamma\vec{F}, \gamma\frac{\vec{v}}{c}\vec{F}\right], \tag{4.36}$$
$$\tilde{F}_4^{\ 2} = \overline{F}_4^{\ 2} = \text{inv.}, \tilde{F}_4 \cdot \overline{V}_4 = -\overline{F}_4 \cdot \overline{V}_4 = \tilde{F}'_4 \cdot \overline{V}'_4 = -\overline{F}'_4 \cdot \overline{V}'_4 = 0.$$

All binary or two-states interactions can be treated in the most general way by applying (4.33) - (4.36). (4.37) also express Newton law of action and reaction (regarding inertial forces), $\overline{F}_4 = -\tilde{F}_4$. To expose a more general view of force law (4.36), we should treat such force as being multi-component (complex) force, as presented in (4.32), modeling it towards creating Lorentz-covariant 4-vectors in the Minkowski space.

Implicitly, from 4-vectors found in (4.34) and (4.36) we can also determine what should mean Lorentz-covariant wave function Ψ^2 in the Minkowski space. This way, we have a chance to establish higher compatibility between Quantum Mechanics and Relativity Theory. ***The full power and advantages of Minkowski 4-vectors will be exposed if we properly and creatively unite the Analytic Signal concept and 4-vectors, what is resulting in relevant complex (Analytic Signal) Phasors, analog to Phasors in electrical sciences (see more in Chapters 4.0 and 10).***

*[♣ **COMMENTS & FREE-THINKING CORNER** (only brainstorming ideas based on analogies):*
Using analogies (see T.3.1-T.3.3) we could "invent" possible (and hypothetical) directions of further extensions of 4-space vectors (in Minkowski space) towards rotation, electromagnetism etc. Let us reformulate the 4-vector of velocity (4.33) to become a direct consequence of 4-vector of momentum,

$$\overline{V}_4 = \frac{1}{m}\overline{P}_4 = (\frac{\vec{p}}{m}, \frac{E}{mc}) = (\frac{\vec{p}}{m}, \frac{pc}{mv}) = (\frac{\gamma m\vec{v}}{m}, \frac{\gamma mc^2}{mc}) = \overline{V}(\gamma\vec{v}, \gamma c) \Rightarrow$$
$$\Rightarrow V^2(\gamma\vec{v}, \gamma c) = \text{invariant}. \tag{4.38}$$

In terms of rotation (based on analogies), instead of linear speed **v***, we have angular speed* ω*, and instead of speed* c *we should have maximal angular speed* ω_c*. By replacing all values from (4.38) with their analogies in rotational motion we will get:*

$$\left\{\begin{array}{l} \overline{\Omega}_4 = \frac{1}{\mathbf{J}}\overline{L}_4 = \overline{\Omega}(\gamma\vec{\omega}, \gamma\omega_c), \vec{L} = \gamma\mathbf{J}\vec{\omega} \\ \gamma = (1 - v^2/c^2)^{-1/2} = (1 - \omega^2/\omega_c^{\ 2})^{-1/2} \end{array}\right\} \Rightarrow \overline{\Omega}^2(\gamma\vec{\omega}, \gamma\omega_c) = \text{inv.}, \tag{4.39}$$

In terms of the magnetic field (using conclusions based only on analogies), instead of **v** *we have electric current (or "magnetic voltage")* $i_{el.}$*. Instead of speed* c *we would have a certain maximal electric current* $i_{el\text{-}c.}$*. By replacing all values from (4.38) with their analogies in the magnetic field we would formally or analogically get:*

$$\overline{I}_{el.-4} = \overline{I}_{el.}(\gamma i_{el.}, \gamma i_{el-c.}) = \overline{I}_{el.}(\gamma i_{el.}, \gamma i_{el.} \cdot \frac{v}{c}),\ i_{el-c.} = i_{el.} \cdot \frac{v}{c},$$
$$(\overline{I}_{el.-4})^2 = -(i_{el-c.})^2,\ \gamma = (1 - v^2/c^2)^{-1/2} = \left[1 - (i_{el.}/i_{el-c.})^2\right]^{-1/2}. \tag{4.40}$$

*In terms of the electric field (using analogies), instead of **v** we have electric voltage (or "magnetic current") $u_{el.}$, and instead of speed **c** we would have maximal electric voltage $u_{el-c.}$. By replacing all values from (4.38) with their analogies in the electric field we will get:*

$$\overline{U}_{el.-4} = \overline{U}_{el.}(\gamma u_{el.}, \gamma u_{el-c.}) = \overline{U}_{el.}(\gamma u_{el.}, \gamma u_{el.} \cdot \frac{v}{c}),\ u_{el-c.} = u_{el.} \cdot \frac{v}{c},$$
$$(\overline{U}_{el.-4})^2 = -(u_{el-c.})^2,\ \gamma = (1 - v^2/c^2)^{-1/2} = \left[1 - (u_{el.}/u_{el-c.})^2\right]^{-1/2}. \tag{4.41}$$

We could continue similar mathematical experiments (creating analogies of possible 4-space vectors) towards many other values found in T.3.1-T.3.3. The principal question, whether something like that really produces useful, logical, and correct results (or results that later can be transformed into more realistic formulas), could be analyzed some other time.

Another important conceptual difference between our usual modeling regarding mechanical movements and electromagnetic family of phenomena (where currents and voltages are involved) is that in most of the situations regarding electromagnetic phenomenology we know or understand that electric circuits are by their nature kind of fully closed circuits (Kirchoff's and Ohm's laws etc.). Regarding (mechanical) particles motions we are often talking about (somehow free hanging) linear and/or rotational motions without defining what should be their closed mechanical circuits. Both electrical and mechanical phenomena, models and motions should have similar, closed circuits nature. This is certainly in relation to Lorentz transformations and 4-vector rules, as presently formulated, and a reason for extending Lorentz and 4-vectors framework, towards unified models that would take care about the unity of linear and rotational motions (what is basically proposed here).

In the first three chapters of this book, we already established analogies between different "field charges" or values equivalent to linear and orbital moments. Let us try "analogically" to extend all such values to 4-vectors in Minkowski space. By analogy with the linear momentum 4-vector (see (2.5.1-7)) we would be able to formulate a similar relation for an orbital moment, as for instance,

$$\overline{P}_4 = m\overline{V}_4 = \overline{P}(\vec{p}, \frac{E}{c}) = \overline{P}(\gamma m\vec{v}, \gamma mc) \Leftrightarrow \overline{L}_4 = \overline{L}(\vec{L}, \frac{E}{\omega_c}) = J\overline{\Omega}_4 = (\vec{L}, \frac{\gamma J\omega_c^2}{\omega_c}) \Rightarrow$$
$$\Rightarrow \left\{ \overline{P}_4^2 = \overline{P}^2(\vec{p}, \frac{E}{c}) = \vec{p}^2 - \frac{E^2}{c^2} = \vec{p}'^2 - \frac{E'^2}{c^2} = \vec{p}''^2 - \frac{E''^2}{c^2} = \text{invariant} = -\frac{E_0^2}{c^2} = -M^2c^2, \right\} \Rightarrow$$
$$\Rightarrow \left\{ \overline{L}_4^2 = \overline{L}^2(\vec{L}, \frac{E}{\omega_c}) = \vec{L}^2 - \frac{E^2}{\omega_c^2} = \vec{L}'^2 - \frac{E'^2}{\omega_c^2} = \vec{L}''^2 - \frac{E''^2}{\omega_c^2} = \text{invariant} = -\frac{E_0^2}{\omega_c^2} = -J^2\omega_c^2 \right\},$$
$$E^2 = E_0^2 + \vec{p}^2c^2 = E_0^2 + \vec{L}^2\omega_c^2,\ E_0 = mc^2 + E_s = Mc^2 = J\omega_c^2,$$
$$\vec{p} = \gamma M\vec{v},\ \vec{L} = \gamma J\vec{\omega} = \vec{p} \times \vec{R},$$
$$dE = dE_k = vdp = \omega dL,\ pdv = Ld\omega,\ vdp + pdv = \omega dL + Ld\omega,$$
$$E_k = E - E_0 = (\gamma - 1)Mc^2 = \frac{pv}{1 + \sqrt{1 - \frac{v^2}{c^2}}} = (\gamma - 1)J\omega_c^2 = \frac{L\omega}{1 + \sqrt{1 - \frac{v^2}{c^2}}},$$
$$\vec{v} = \vec{v}_t = \vec{\omega}R,\ pv = L\omega,\ pc = L\omega_c,\ \frac{c}{\omega_c} = \frac{v}{\omega} = \sqrt{\frac{J}{M}},\ \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{\omega^2}{\omega_c^2}}}. \tag{4.33-1}$$

The same particle could have linear, orbital, and electromagnetic moments (at the same time), and we would need to find a way to address such complex moments reality, for instance, by proposing united 4-vectors and scalar fields formulation (see more of similar ideas in Chapter 10). Based on analogies, we could propose the following table, T.4.3.2, just to stimulate creative curiosity.

T.4.3.2	**[Q] = CHARGES / MOMENTS**	**Meaning**	**4-vectors: Moment / Force** (just ideas, proposals)
Gravitation & Linear Motion	**p = mv**	**Linear moment**	$\overline{P}_4 = \overline{P}(\vec{p}, \frac{E}{c}) = m\overline{V}_4,$ $\overline{F}_4 = \gamma\frac{d\overline{P}_4}{dt} = (\gamma\vec{F}, \frac{\gamma}{c}\frac{dE}{dt}) = (\gamma\vec{F}, \gamma\vec{F}\cdot\frac{\vec{v}}{c})$
Spinning	$\mathbf{L = J\omega}$	**Orbital moment**	$\overline{L}_4 = \overline{L}(\vec{L}, \frac{E}{\omega_c}) = J\overline{\Omega}_4,$ $\overline{\tau}_4 = \gamma\frac{d\overline{L}_4}{dt} = (\gamma\vec{\tau}, \frac{\gamma}{\omega_c}\frac{dE}{dt}) =$ $= (\gamma\vec{\tau}, \frac{\gamma}{\omega_c}\vec{\tau}\cdot\vec{\omega}) = (\gamma\vec{\tau}, \gamma\vec{\tau}\cdot\frac{\vec{v}_t}{c})$
Electric Field	$\mathbf{\Phi_{el} = q_{el.}}$	**Electric charge**	**? ! (to think about)** $\overline{Q}_{el.-4} = \int \overline{I}_{el.-4}dt = \overline{Q}(q_{el.}, q_{el.-c}) =$ $= \overline{Q}(\int\gamma i_{el.}dt, \int\gamma i_{el.}\cdot\frac{v}{c}dt),$ $\overline{I}_{el.-4} = \overline{I}_{el.}(\gamma i_{el.}, \gamma i_{el.}\cdot\frac{v}{c})$
Magnetic Field	$\mathbf{\Phi_{mag} = q_{mag.}}$	**Magnetic flux**	**? ! (to think about)** $\overline{\Phi}_{4-mag.} = \int \overline{U}_{el.-4}dt = \overline{\Phi}(q_{mag.}, q_{mag-c}) =$ $= \overline{\Phi}(\int\gamma u_{el.}dt, \int\gamma u_{el.}\cdot\frac{v}{c}dt),$ $\overline{U}_{el.-4} = \overline{U}_{el.}(\gamma u_{el.}, \gamma u_{el.}\cdot\frac{v}{c})$

Obviously, for scalar values of electric charge and magnetic flux (in T.4.3.2) it would not be analogically applicable to place them directly in a format of the 4-vectors of Minkowski-space (as here presented), but just for the purpose of brainstorming (and maybe in relation to some future redefinition of such entities, which would give them more of vectors meanings), here is initiated the first step. ♣]

4.3.8. Mass, Particle-Wave Duality and Real Sources of Gravitation

By applying different analogies all over this paper we see that mass and gravitation somehow "avoid" being simply presentable, following similar patterns of other natural forces and their sources (or charges). It looks (based upon here established analogies in earlier chapters) that real sources of gravity (between particles with non-zero rest masses) should be mutually coupled, relevant linear and orbital moments, also coupled with other electrodynamic parameters such as involved electric charges and magnetic fluxes. This is contrary to the common opinion that pure, static or standstill rest masses are proper gravity charges, or primary sources of Gravitation (based on a direct analogy with Coulomb laws for electric and magnetic charges. See also Chapter 2: GRAVITATION, starting from equations (2.3) to (2.4-3)).

To understand better what mass means, let us summarize the most important expressions concerning the energy of a particle (moving in a free space):

$$\begin{aligned} &E = E_{total} = E_0 + E_k = \gamma mc^2 = \sqrt{E_0^2 + p^2c^2} = m_{total}c^2 = m_t c^2, \\ &E_k = E - E_0 = (\gamma - 1)mc^2 = E(1 \pm \sqrt{1 - \frac{p^2c^2}{E^2}}) = m_{motional}c^2 = m_m c^2, \\ &E_0 = mc^2 = m_0 c^2, \end{aligned} \qquad (4.41\text{-}1)$$

The most general understanding of the mass concept is to present mass just as another form of *"particle energy storage (devised by the constant c^2)"*, where $\mathbf{m_t}$ is the total mass, $\mathbf{m_m}$ is the motional mass and $\mathbf{m} = \mathbf{m_o}$ is the initial rest mass:

$$\begin{aligned} &m_t = \frac{E}{c^2} = \gamma m = m^* = m + m_m = m + \tilde{m} = \frac{1}{c^2}\sqrt{E_0^2 + p^2c^2} = \frac{E_0}{c^2}\sqrt{1 - \frac{p^2c^2}{E_0^2}}, \\ &m_m = \tilde{m} = \frac{E_k}{c^2} = (\gamma - 1)m = m_t - m = \frac{E}{c^2}(1 \pm \sqrt{1 - \frac{p^2c^2}{E^2}}), \\ &m = m_0 = \frac{E_0}{c^2}. \end{aligned} \qquad (4.41\text{-}2)$$

This way, the same mass concept can be extended to all constituents of our universe, such as elementary particles and quasi-particles that do not have a rest mass (like photons), as well as to energy of fields and waves around particles. We could also describe the mass as a "*space-time-spread*" entity. Here, minimal internal mass content captured by internal particle space (by a particle geometry, or by its mechanical or solid boundaries) equals $\mathbf{m}$, and external mass content, spread in the space around the particle, equals $(\gamma - 1)\mathbf{m}$, what effectively (for moving particle) makes the total (relativistic) particle mass equal $m_t = \gamma m = m^* = m + (\gamma - 1)m = m + \tilde{m}$. This kind of conceptualization is directly linked to particle-wave duality of matter, as imaginatively described in Chapter 10. under "*10.1 Hypercomplex Analytic Signal functions and interpretation of energy-momentum 4-vectors in relation to matter-*

waves and particle-wave duality". Of course, here are only introduced ideas and indicative directions how to understand mass, energy, and wave-particle duality.

Let us now transform relations for energy and mass into relations applicable to photons,

$$\tilde{E} = E_p = E = E_{total} = E_0 + E_k = E_k = \tilde{p}c = m_t c^2 = \tilde{m}c^2,\ \tilde{m} = m_t = \frac{\tilde{E}}{c^2} = \frac{\tilde{p}}{c} = m_p,$$
$$\tilde{E} = E_p = E_k = E - E_0 = E = E(1 \pm \sqrt{1 - \frac{p^2c^2}{E^2}}) = m_m c^2 = \tilde{m}c^2, m_m = \tilde{m} = \frac{\tilde{E}}{c^2} = \frac{\tilde{p}}{c}, \qquad (4.41\text{-}3)$$
$$E_0 = 0,\ \ m = m_0 = m_{p0} = \frac{E_0}{c^2} = 0.$$

Now, it is possible to introduce a new understanding of a stable rest mass as the form of motional (wave) energy, conveniently stabilized and (for instance) captured and packed into a big number of more elementary vortex or rotating formations of self-sustaining, self-closed stationary and standing waves (see similar and complementary conceptualization in Chapter 2., under "*2.3. How to account Rotation concerning Gravitation*").

All stable-mass particles also have natural, external electromagnetic and matter-waves field couplings, and different energy-mass-momentum communications with a surrounding Universe.

If the total *internal wave energy content* of a certain particle is, $\mathbf{E}_{k-int.} = \tilde{\mathbf{E}} = \tilde{\mathbf{E}}_{int} = \tilde{\mathbf{m}}_{int.}\mathbf{c}^2$, then the particle rest mass should be $\mathbf{m}_0 = \tilde{\mathbf{m}}_{int.} = \frac{\tilde{\mathbf{E}}_{int}}{\mathbf{c}^2}$. This way, we can equally treat "internal rest mass", and "externally spread (moving) mass", since externally spread mass is already defined as an equivalent to a kinetic particle energy (devised by $\mathbf{c}^2$). The equality of mass treatments is again valid, since in both cases (when we calculate mass), we are talking about kinetic or motional (or wave) energy, but:

1° For a rest mass we are considering or using as relevant only an internal wave-formation, or internal motional energy content, and

2° For a mass equivalent corresponding to an "external" particle motion we will consider its "external" particle kinetic energy $\mathbf{m}_m = \frac{\mathbf{E}_k}{\mathbf{c}^2} = (\gamma - 1)\mathbf{m}$.

Of course, this situation (regarding the mass concept) becomes much more general and better unified if we treat the particle kinetic energy also as a form of wave energy (what presents the proper meaning of the **P**article-**W**ave **D**uality in this book). In case of an elementary particle, we will have,

$$
\begin{aligned}
&m_0 = \tilde{m}_{int.} = \frac{\tilde{E}_{int}}{c^2} = \frac{h\langle f_{int.}\rangle}{c^2}, \\
&m_m = m_{ext.} = \frac{E_k}{c^2} = (\gamma-1)m = \frac{\tilde{E}_{ext.}}{c^2} = \frac{h\langle f_{ext.}\rangle}{c^2} \\
&E_k = (\gamma-1)mc^2 = \tilde{E}_{ext.} = \tilde{E} = h\langle f_{ext.}\rangle = hf \\
&m_t = \tilde{m}_{int.} + m_{ext.} = \frac{E}{c^2} = \frac{\tilde{E}_{int} + \tilde{E}_{ext.}}{c^2} = \frac{E_0 + E_k}{c^2} = \gamma m = m^*
\end{aligned}
\tag{4.41-4}
$$

Here used terminology “internal and external” motional (or wave) energy is maybe not the best choice of names, but here it serves the need for simplified and faster (or an indicative) explanation of the fact that what we consider as particles (with our instruments), “externally” only looks like as particles, but “internally” such entities have their proper wave nature. Also, similar wave nature should exist "externally", as a motional energy of particles (being fully connected or coupled with internal wave structure of a relevant particle in motion). Rotation, spinning, and wave packing is what makes the difference between particles with rest masses and freely propagating waves.

Finally, the mass concept (as here introduced) tells us that in the background of what makes our universe should only be a certain kind of kinetic, motional or wave energy, “**assembled in different packing, stabilized or free propagating, and/or resonant mass-energy-momentum formats**”.

Particularly interesting consequences of previous assumptions and analyses are extending **the PWDC** concept to the internal macro-particles (or elementary mass) domains, meaning that:

A) A single particle has its "external" kinetic or waves energy content, $\mathbf{E_k = (\gamma-1)mc^2 = \tilde{E}_{ext.} = h\langle f_{ext.}\rangle = \tilde{E} = hf}$ *, and*

B) The same particle can be at the same time "internally excited" on many ways (for instance, by heating, or passing electrical currents and/or mechanical vibrations through it), this way getting an elevated content of internal wave energy, $\tilde{E}_{int.} = \mathbf{hf}_{int.}$*. Consequently, the internally excited particle would become able to radiate the surplus of its "internal" wave energy (when conditions will be satisfied), what really happens in many well-known experimental situations. Sometimes, the surplus of internally excited wave states is so high that a macroparticle becomes a spontaneous radiator of real (smaller) particles, such as electrons,* α*-particles etc., besides radiating photons (all of that leading to the conclusion that differences between real particles that have stable rest masses, and pure waves without rest mass content, is not in their essential nature, but only in their appearance, or in a way of “**energy packing**”).*

Until here, we presented only a kind of simplified and approximate picture about real masses. It is already empirically known that relativistic mass descriptions have some not completely explained nature, exceptions, and deviations. For instance, by heating certain mass we have both negative and positive contributions of the total mass (see more in [101]), but Relativity Theory is confirming only the positive contribution of added heat energy to a total mass (see more in Chapter 9 of this book).

Quote (The Negative Temperature Dependence of Gravity is a Reality, Professor Alexander L. Dmitriev and Sophia A. Bulgakova, [101]): "Temperature dependence of force of gravitation - one of the fundamental problems of physics. The negative temperature dependence of the weight of bodies is confirmed by laboratory experiments and like Faraday phenomenon in electrodynamics is a consequence of natural "conservatism" of a physical system, its tendency to preserve a stable condition. Realization of experimental research of the influence of the temperature of bodies on their gravitational interaction is timely and, undoubtedly, will promote the progress of development of physics of gravitation and its applications".

………………………………

Mass is a property influenced by the internal arrangement, formatting, and spatial configuration of the fields and binding energy of its constituents. This means that stable, static, and constant mass exists only conditionally, within specific conditions. For example, mass is composed of atoms, which in turn are made up of neutrons, protons, and electrons. Typically, the total mass of an atom is less than the sum of the masses of its individual neutrons, protons, and electrons when considered independently. This difference is known as the atomic or nuclear mass defect.

However, there are exceptions where stable atoms exhibit a mass defect with an opposite sign. In these cases, the rest and stable mass content of certain particles exists conditionally, influenced by specific internal fields and wave configurations. This suggests that the real boundaries of mass, energy, and momentum, along with associated electromagnetic phenomena, are broader than what is considered strictly mechanical, geometric, or static. This understanding has implications for our interpretation of Uncertainty Relations.

It is important to recognize that discussions about various forms of energy, mass, and matter waves should be grounded in a relatively simple conceptual framework, as illustrated in Figure 4.1.2 and related to energy conservation (equation 4.5-3). More advanced and sophisticated concepts related to mass-energy states should be rooted in this basic framework. Chapter 10, titled "Particles and Self-Closed Standing Matter Waves," addresses similar issues and emphasizes that mass likely has a Complex (or Hypercomplex) nature. This is well-integrated with matter waves and wave-particle duality properties, as discussed around equations (10.1.1) to (10.1.5).

*[♣ **COMMENTS & FREE-THINKING CORNER: Let** us now try to show how linear and orbital moments could assist the creation of rest masses. We can analyze a stable and neutral particle (consisting of one or many atoms, or a big number of different elementary entities) in the state of relative rest, which has a mass m. We can also say that such particle consists of different molecules, atoms, electrons, nuclear and sub-nuclear particles, and fields, all of them moving, circulating, spinning, or oscillating inside the "particle shell". Externally, it appears that a particle with rest mass is stable, electrically neutral, and in the state of rest. Consequently, total linear and orbital (internal) particle moments, $\vec{P}_{int.-total}$,$\vec{L}_{int.-total}$ (looking externally, outside of the "particle shell"), should be close to zero, or should have negligibly small values (if we make a vector sum of all internal particle moments), as for instance,*

$$\vec{P}_{int.-total} = \sum_{(i)} \vec{p}_i = \sum_{(i)} m_i \vec{v}_i \rightarrow 0 \text{ , } \vec{L}_{int.-total} = \sum_{(i)} \vec{L}_i = \sum_{(i)} J_i \vec{\omega}_i \rightarrow 0 \text{ .} \qquad (4.42)$$

For the particle in the state of rest we could also estimate its center-of-mass linear speed, and center-of-inertia angular speed, $\vec{v}_c$, $\vec{\omega}_c$ that should again be negligibly small or close to zero values (when the particle is globally in its rest state),

$$\vec{v}_c = \frac{\vec{P}_{\text{int.-total}}}{\sum_{(i)} m_i} = \frac{\sum_{(i)} m_i \vec{v}_i}{\sum_{(i)} m_i} \to 0 \ , \ \vec{\omega}_c = \frac{\vec{L}_{\text{int.-total}}}{\sum_{(i)} \mathbf{J}_i} = \frac{\sum_{(i)} \mathbf{J}_i \vec{\omega}_i}{\sum_{(i)} \mathbf{J}_i} \to 0 \ . \tag{4.43}$$

Now, combining (4.42) and (4.43) we can find the total mass and total moment of inertia of the particle in the state of rest as the result of an internal superposition and mutual interferences of all its internal, linear and orbital moments,

$$m = \sum_{(i)} m_i = \frac{\left|\vec{P}_{\text{int.-total}}\right|}{\left|\vec{v}_c\right|} = \frac{\left|\sum_{(i)} m_i \vec{v}_i\right|}{\left|\vec{v}_c\right|} = \frac{E_0}{c^2} = \frac{1}{c^2}\int_{[P]} \vec{v} d\vec{p} = \text{const.} \ ,$$

$$\mathbf{J} = \sum_{(i)} \mathbf{J}_i = \frac{\left|\vec{L}_{\text{int.-total}}\right|}{\left|\vec{\omega}_c\right|} = \frac{\left|\sum_{(i)} \mathbf{J}_i \vec{\omega}_i\right|}{\left|\vec{\omega}_c\right|} = \sum_{(i)} m_i r_i^2 = \frac{E_0}{\omega_c^2} = \frac{1}{\omega_c^2}\int_{[L]} \vec{\omega} d\vec{L} = Const. \tag{4.44}$$

Effectively, in (4.44) we have divisions between two values; both negligibly small, but the results of such divisions are constant and realistically high numbers, meaning that matter or mass should always be in some motional state.

Just for the purpose of creating certain quantifiable and simple mathematical forms (at least dimensionally correct), we could "invent" the following indicative relations (that would be later, most probably modified, but presently are good enough to show that stable particle in a state of rest is intrinsically composed of permanently moving and rotating internal wave-energy states):

$$E_o = J\omega_c^2 = mc^2 = \int_{[P\text{-int.}]} \vec{v} d\vec{p} = \int_{[L\text{-int.}]} \vec{\omega} d\vec{L} = hf_c \Rightarrow \omega_c = c\sqrt{\frac{m}{J}} = \frac{c}{r^*} = 2\pi f_c \Rightarrow$$
$$\Rightarrow f_c = \frac{c}{2\pi}\sqrt{\frac{m}{J}} , \ c = \lambda_c f_c \Rightarrow \lambda_c = 2\pi r^* . \tag{4.44-1}$$

If we go back to the Newton law of gravitation between two masses (see (2.3), (2.4) - (2.4-3), in Chapter 2., we can conclude that there we explicitly deal with masses, but implicitly also with their linear and orbital moments, like in (4.44), and with many other dynamic properties of internal (and external) mass constituents. There are some other, more profound (macrocosmic and microcosmic) consequences of such active mass modeling, especially if in (4.44) we apply Minkowski space 4-vectors, (4.32) - (4.37), in order to establish a more complex active mass modeling, as speculated in Chapter 10., under "10.1 Hypercomplex Analytic Signal functions and interpretation of energy-momentum 4-vectors in relation to matter-waves and particle-wave duality" (see also (2.3) - (2.4-3), (4.5-1) and (5.15) - (5.17)).

Under certain conditions, we know that rest mass can be created combining energy states (matter waves) that do not have their own rest masses, as for instance,

$$m = m_0 = \frac{1}{c^2}\sqrt{E_t^2 - P_t^2 c^2} \quad , \quad E_t = \sum_{(i)} E_{t-i} \quad , \quad \vec{P}_t = \sum_{(i)} \vec{P}_{t-i} \quad . \tag{4.44-2}$$

By analogy (presently without claiming that such a result would be correct) we could create a similar relation for a corresponding static moment of inertia,

$$J = J_0 = \frac{1}{\omega_c^2}\sqrt{E_t^2 - L_t^2\omega_c^2} \ , \ E_t = \sum_{(i)} E_{t-i} \ , \ \vec{L}_t = \vec{r}^* \times \vec{P}_t = \sum_{(i)} \vec{L}_{t-i} \ , (\left|\vec{L}_{t-i}\right| = \frac{c^2}{r^* \omega_c^2}) . \tag{4.44-3}$$

By combining (4.44-2) and (4.44-3) we get,

$$mc^2 + P_t^2c^2 = J\omega_c^2 + L_t^2\omega_c^2 = E_t^2 = \gamma\, mc^2 \quad . \qquad (4.44\text{-}4)$$

Most probably that combining (4.44-4) and (4.33-1) we would be able to find more appropriate relations that show that linear and rotational motion elements are well synchronized and united (or coupled like mutually conjugate functions, producing what we consider as a wave-particle duality).

Like in (4.44), using analogies given in T.3.1-T.3.3, $\mathbf{m} \leftrightarrow \mathbf{C},\ \mathbf{J} \leftrightarrow \mathbf{L}$, *we could later create new formulations of electric capacitance and magnetic inductance. This situation can also be compared to an atom structure: Externally, far-enough, in a space around an atom, we could measure that atom is electrically neutral and in the state of relative rest, although inside the atom structure everything is waving and moving (and there is no electromagnetic neutrality there). The next step is to conclude that the total energy of active (or moving) atom constituents, conveniently integrated into a stable and (externally) neutral atom, creates atom-mass (where rest mass can be presented using Einstein relation* $E_{tot.} = mc^2$*). It is also clear that very particular dynamic "gearing and fitting" between atom constituents should be realized to create an atom (and this gearing and resonant fitting we are presently explaining as a quantum or discretized nature of matter; -see also familiar Wilson-Sommerfeld rules, (5.4.1)).*

The same situation could also be analyzed using generalized Schrödinger equation, and equations of Relativistic Electrodynamics (producing more complex mathematical picture, than here presented), but important qualitative, phenomenological, and conceptual aspects of particle-wave duality and action-reaction forces would not be clearly and simply identified as here presented.

We could also conclude that de Broglie matter waves (belonging to moving particles) should be very much present inside an atom structure, complementing the above-mentioned dynamic gearing and fitting among atom constituents. A similar concept can be extended to any other stable particle. The same statement differently formulated is that sources of de Broglie matter waves should be found inside an internal structure of particles, in the form of standing wave formation of internal particle structure (externally manifesting as orbital moments and spin attributes). When a particle changes its state of motion, intrinsic, internal wave formation (previously being in the state of stationary and standing waves structures) is becoming an active, de Broglie matter wave, producing externally measurable consequences, as for instance: Photoelectric and Compton Effect, particles diffraction…

Every wave motion has its equivalent mass that can be found (combining (4.17-1) and (4.44)) as,

$$m^* = \frac{\tilde{E}}{uv} = \frac{\left[\int_0^\infty [A(\omega)]^2 d\omega\right]^3}{\pi\left[\int_0^\infty \frac{\omega}{k}\cdot[A(\omega)]^2 d\omega\right]\left[\int_0^\infty \frac{d\omega}{dk}\cdot[A(\omega)]^2 d\omega\right]}$$

, or in case of electromagnetic waves,

$$m^* = \frac{\tilde{E}}{uv} = \frac{\tilde{E}}{c^2} \qquad (4.45)$$

Intuitively, we see that a stable particle (which has stable rest mass) should have certain stationary waving structure (internally organized and balanced), and we also know that in many interactions' stable particles manifest particle-wave duality properties. Moreover, particles could disintegrate into pure wave-energy constituents. It is also known that a convenient superposition of pure wave elements could produce a stable particle (electron-positron creation, for instance). Consequently, the general case of a stable particle should be that its internal constituents are composed of wave-mass elements in the form of (4.45). The proper internal and dynamic "gearing and fitting" of all particle constituents should produce a stable particle, which has stable rest mass (found by applying the rules of Relativity Theory: -using known connections between total energy, rest energy and particle momentum). Of course, there are many intermediary and mixed particle-wave states and objects, which sometimes behave more as particles or as waves.

*Obviously, the particle-wave duality concept favored in this book creates sufficiently clear frontier between stable particles (which have constant rest mass in their center of mass reference system), and wave or particle-wave phenomena that belong only to different states of motion. Contrary to this position, we also know that internal structure of a stable particle is composed of wave and particle-wave constituents, which are properly "geared and fitted" (looking only externally) producing a stable particle (**based on standing waves, resonant field structures**). What is missing in such conceptual picture of particle-wave duality is to explain conditions when certain dynamic combination (superposition) of waves transforms into a stable particle; -for instance, when wave mass, given by (4.45), will create a stable particle (**by closing an open and relatively free propagating waveform into itself, internally structured as standing waves, or self-stabilized field in resonance**). The cornerstones and frames for wave-to-particle transformation should be found in the following ideas:*

1. ***A stable particle will be created when a wave-mass (4.45) is transformed into a constant (externally measurable and stable) rest mass,*** $\mathbf{m} \to \mathbf{m_0}$***, that is time independent and localized in certain limited space (satisfying also (4.44)).***
2. ***A stable particle should have (in its Center of Mass System) non-zero rest mass.***
3. ***The center-of-mass velocity of all internal wave-particle constituents (in its Laboratory System) should be equal zero.***
4. ***Rest mass (created from a wave group as kind of superposition and interaction between involved wave groups) can be formulated based on Minkowski-space 4-vector relation between total energy and momentum, as:*** $m = m_0 = (\sqrt{E^2_{tot.} - P^2_{tot.}c^2})/c^2$ ***...*** *Here we can make another conceptual step regarding wave-packing nature of stable particles that have non-zero rest masses. Let us again start with the general relation between mass, total energy and total momentum presented as:*

$$mc^2 = E_0 = \sqrt{E^2_{tot.} - P^2_{tot.}c^2} = \sqrt{E^2 - p^2c^2} = \text{const.} \Rightarrow$$
$$\Rightarrow E_0^2 = E^2 - p^2c^2 = (E - pc)\cdot(E + pc) = \gamma mc(c - v)(c + v) = \text{Const.}$$

In other words, we could state that a rest mass $\mathbf{m}$ *is created as superposition (cross-correlation, or interaction) between two wave groups propagating in mutually opposed directions, that have corresponding time-space domain functions* $\overline{\psi}_1(t,x) = \overline{\psi}(\omega t - kx)$ *and* $\overline{\psi}_2(t,x) = \overline{\psi}(\omega t + kx)$ *.*

Now relations between mutually corresponding time and frequency domains can be conceptually presented (carrying an over-simplified brainstorming message) as,

$$E_0^2 (\Leftrightarrow) \begin{Bmatrix} E_0 \cdot E_0 \\ \overline{\psi}(t); \overline{\psi}(t) \end{Bmatrix} (\Leftrightarrow) \begin{Bmatrix} (E - pc)\cdot(E + pc) \\ \overline{\psi}_1(t,x);\ \overline{\psi}_2(t,x) \end{Bmatrix} (\Leftrightarrow) \begin{Bmatrix} \text{Energy} \\ \text{time - space} \end{Bmatrix}$$
$$E = E(\omega, k)\ ,\ p = \frac{h}{2\pi}k = \frac{h}{\lambda}$$
$$\overline{\psi}_1(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} U_1(\omega)e^{j\omega t}d\omega = \frac{1}{2\pi}\int_{-\infty}^{+\infty} U(E - pc)e^{j\omega t}d\omega = a(t)e^{j\Phi(E-pc)}$$
$$\overline{\psi}_2(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} U_2(\omega)e^{j\omega t}d\omega = \frac{1}{2\pi}\int_{-\infty}^{+\infty} U(E + pc)e^{j\omega t}d\omega = a(t)e^{j\Phi(E+pc)} \quad (4.46)$$
$$\overline{\psi}(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} U_1 \cdot U_2 e^{j\omega t}d\omega = \frac{1}{2\pi}\int_{-\infty}^{+\infty} U(E, pc)e^{j\omega t}d\omega = a(t)e^{j\Phi(E,pc)} =$$
$$= \int_{-\infty}^{+\infty} \psi_1(t)\cdot\psi_2(t-\tau)e^{j\omega t}d\omega = \int_{-\infty}^{+\infty} \psi_1(t-\tau)\cdot\psi_2(t)e^{j\omega t}d\omega.$$

It is evident that a stable rest mass can be conceptualized as the space-time cross-section (or convolution) of mutually phase-shifted wave functions $\psi_1(t,x)$ *and* $\psi_2(t,x)$*. Through this process, the common space-time wave content is "trapped" into a stable wave packet format. In other words, we are proposing a model where waves of matter can form stable particles. Using a similar approach, this framework could later be extended to describe how particles might transform back into waves.*

Continuing this analysis may reveal additional conditions for wave-to-particle transformations, likely tied to solutions of Schrödinger-type wave equations. Orthodox Quantum Mechanics does not strictly differentiate between waves and particles, treating all particles and particle-wave entities as waves (including those with rest mass) and assigning a wave function to every "quantum entity." While this approach works well within the Quantum Theory framework, it overlooks a distinction: free, unbounded waves propagate differently compared to spatially localized standing waves, where the wave structure is self-contained and forms resonant patterns.

This difference is crucial for the particle-wave duality discussed here and differs from the conventional view in Quantum Theory, which incorporates rest mass and rest energy into the free wave energy. In many practical scenarios, this distinction remains unnoticed or irrelevant because, when creating differential wave equations like Schrödinger's, constant mass and energy terms are eliminated (since their derivatives are zero).

The main goal of this discussion is to present an intuitive and conceptually clear interpretation of the true nature of mass. This also implies that our current understanding of Gravitation in physics may need a reevaluation, as the concept of mass itself should be significantly updated. While this concept of mass requires further refinement, the groundwork presented here offers a new perspective.

To advance this understanding, we should systematically analyze the differentiation between mass and energy in relation to particles or wave groups in various states of motion. A starting point would be to categorize all possible distinct states of motion. For instance, Table T.4.3.3 lists, and Fig. 4.3.1 illustrate different states of motion, representing the diverse combinations of linear and rotational dynamics. Eventually, we can define at least 16 distinct energy-momentum-mass states (labeled 1.0.1 to 4.3.2 in T.4.3.3) that describe the complementary nature of linear motion and rotation.

For a deeper exploration of this topic, refer to Chapter 10, titled "Particles and Self-Closed Standing Matter Waves," which is specifically dedicated to these issues.

T.4.3.3. The chart of possible particle states (only a brainstorming)

<table>
<tr><th>Different motional states of a stable particle</th><th>1. Standstill state</th><th>2. Linear Motion</th><th>3. Rotational Motion</th><th>4. Combined Linear Motion and Rotation</th></tr>
<tr><td rowspan="3">From the point of view of different observers (see 1. and 2., below):</td><td rowspan="3">1.0. No external motion. The particle is in the state of rest, looking from the outside space.</td><td rowspan="3">2.0. A particle as a whole is only in linear motion relative to a certain external system of reference.</td><td>3.1. A particle as a whole is only rotating around a certain point, relative to the certain external system of reference (or center, which is in external space, not captured by the particle). No linear motion.</td><td>4.1. A particle as a whole is in linear motion and at the same time rotating relative to a certain external system of reference (or center, which is in external space, not captured by the particle).</td></tr>
<tr><td>3.2. A particle as a whole is only rotating around itself (or around its center of gravity, or around one of its axes). No linear motion.</td><td>4.2. A particle as a whole is in linear motion and at the same time rotating around itself, relative to a certain (moving) system of reference that is inside the domain captured by the particle.</td></tr>
<tr><td>3.3. A particle as a whole is rotating around itself (or around its center of gravity, or around one of its axes), and at the same time also rotating around a certain point which is in external space, not captured by the particle (performing a multi-component rotational motion). No linear motion.</td><td>4.3. A particle as a whole is in linear motion and at the same time rotating relative to a certain external system of reference, and also rotating around itself, relative to a certain (moving) system of reference that is inside the domain captured by the particle.</td></tr>
<tr><td>Comments:</td><td>Only internal particle constituents or matter waves are in complex motion (united rotation and linear motion inside the particle structure; -nothing of that being visible externally).</td><td colspan="3">No externally visible rotation exists; -internal particle structure and internal matter-wave motions are not visible externally.</td></tr>
<tr><td rowspan="3">1. The (virtual) observer who is placed inside the particle structure</td><td rowspan="3">1.0.1.</td><td rowspan="3">2.0.1.</td><td>3.1.1.</td><td>4.1.1.</td></tr>
<tr><td>3.2.1.</td><td>4.2.1.</td></tr>
<tr><td>3.3.1.</td><td>4.3.1.</td></tr>
<tr><td rowspan="3">2. An observer who is placed in the external particle space (external, independent system of reference, not captured by the particle domain).</td><td rowspan="3">1.0.2.</td><td rowspan="3">2.0.2.</td><td>3.1.2.</td><td>4.1.2.</td></tr>
<tr><td>3.2.2.</td><td>4.2.2.</td></tr>
<tr><td>3.3.2.</td><td>4.3.2.</td></tr>
</table>

The same energy state (particle, wave packet…) could have many mutually coupled levels of its (internally and externally), energetically atomized structure, where each level would have its own linear and a rotational couple of motional components, symbolically visualized on the Fig.4.3.1 with 4 of such levels (see also chapter 6., MULTIDIMENSIONALITY, where an attempt is made to formulate similar concepts mathematically).

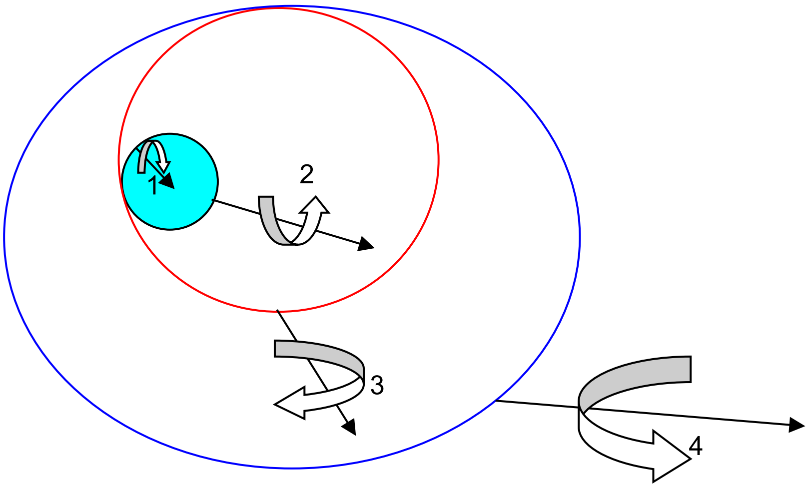


Fig.4.3.1. Symbolic visualization of multilevel linear-rotational motional couples ♣*]*

5. UNCERTAINTY RELATIONS AND ELEMENTARY MATTER DOMAINS

In this chapter we address universally valid **Uncertainty** and **Certainty** relations, created from the point of view of mathematics or signal, spectral analysis, applicable (with necessary adjustments) on all matter-waves and "energy-moments-states" in micro and macro domains of physics. The main foundations and stepstones of such Uncertainty relations are:

1. **Finite Durations and Signal Analysis**: Every real signal, wavefunction, wave-group, particle, or "energy-momentum" state in Physics has finite spatial, temporal, and spectral durations. Fourier and Complex Analytic signal processing reveal that these wavefunctions can be decomposed, composed, or reconstructed using elementary harmonic or sinusoidal signals. Particularly relevant for modeling moving particles are compact or finite wave-groups or wave-packets, which are formed by the superposition of a narrow band of harmonic signals. Mathematical idealizations from Signal Analysis shows that if a wave-packet has an infinitesimally short temporal duration, its spectral domain will have an infinite frequency range, and vice versa. Similarly, infinitesimally small spatial displacements produce infinite linear moments (and vice versa). However, in the real physical world, signal durations are always finite in all domains, due to the limited energy of real signals. Parseval's theorem states that the energy in mutually conjugated domains is the same, reflecting the principle or law of energy conservation. In this book, we emphasize the total durations of signals across all domains, rather than statistical signal measures like standard deviations. Here, we approach the concept of wavefunctions and their modeling (applicable to both the micro and macro realms) primarily from a deterministic standpoint, as elaborated in Chapters 4.0 and 10. However, when defendable and applicable, these wavefunctions can be normalized and/or transformed into the isomorphic, non-dimensional, probabilistic-like framework of Quantum Theory.

2. **Gaussian Signals and Stability**: From Signal and Spectral Analysis, we know that signals with Gaussian or bell-curve envelopes (in both their original and spectral domains) have finite temporal, spatial, and spectral durations in all conjugate domains. This natural stability and compactness condition ensures that stable particles, elementary particles, and other stable energy-momentum states are well-localized, finite in energy, non-dispersive, and well-defined across all relevant domains. In such cases, **Uncertainty Relations** can be seen as **Certainty Relations**. Electromagnetic photons fall into this category of finite temporal, spatial and frequency domains.

3. **Understanding Uncertainty Relations**: In contemporary physics uncertainty relations trace their origins to Heisenberg's invented or almost postulated formulation, to describe quantum statistical phenomena in microphysics. Uncertainty Principle can also be smoothly and causally developed through mathematics. Quantum Theory approach to Uncertainty, however, sometimes creates the misconception that uncertainty relations are exclusive to Heisenberg's postulated principle, and unique to the microworld. This misinterpretation may be partly due to Heisenberg's very young age, good intuition and intelligence at the time, which, despite his limited mathematical training, led him to profound, lucky-brainstorming insights. Another common misunderstanding is the confusion between the physical dimensions of particles or solid-state objects and the temporal-spatial-spectral dimensions and characteristics of their wave functions or wave packets. Real wave packets representing moving

particles are dynamic oscillatory states, with interdependent spatial, temporal, and spectral properties. These properties are coupled and synchronized, necessitating a holistic, rather than isolated, analysis. Phenomena such as single-photon or electron diffraction and interference effects vividly illustrate the effects of these matter-waves coupled dynamics.

4. **Universal Mathematical Foundations**: We will begin by discussing the generally applicable, universally valid mathematical foundations of Uncertainty or Inequality Relations, which concern the absolute or total values of mutually conjugate domain durations both in micro and macro universe. By starting here, we can develop a comprehensive understanding of all other Physics-related Uncertainty Relations within our Universe (see the introduction to Uncertainty Relations in Chapter 4.0, and later in this chapter, under Sections 5.1 and 5.5).

5. **Quantum Theory and Uncertainty:** It's important to note that original Heisenberg Uncertainty Relations in Quantum Theory (QT), as initially formulated, differ slightly from those found in later contemporary QT publications, and from here elaborated only mathematically defined uncertainties. Over time, emphasis has shifted towards stochastic interpretations within Orthodox QT, rather than focusing on the more universally valid and stable mathematical concept of Uncertainty Relations based on relations between total domain durations. In mathematics, Uncertainty Relations originally pertain to the absolute durations of signals in space, time and frequency domains. When applied to matter-wave packets or wave-groups, these relations naturally extend to Physics, without relying on Heisenberg's brainstorming and intuitive assumptions, or the assumed statistical aspects of contemporary QT. A detailed explanation of how wave-packet energy is represented can be found in Chapters 4.0 and 9, particularly around equations 9.8 to 9.11.

6. **Statistical and Signal-Spectrum Analysis:** Using knowledge from Statistics, Probability Theory, and Signal-Spectrum Analysis, we can also develop Uncertainty Relations or inequalities concerning average and mean values, standard deviations, and signal spectral distributions, whenever the mathematical conditions for statistical processing are met. These principles are applicable to both the micro and macro-Universe. The exclusive association (or postulation) of Heisenberg's Uncertainty Principle with the microworld of QT is not supported by any fundamental necessity. Heisenberg's formulation was a well-educated guess, most probably based on already known mathematical uncertainty relations, but it remains incomplete and somewhat misleading. As QT evolves, Heisenberg's contribution will likely be seen as an important, though limited, step in the development of Uncertainty relations, much like the historical development and postulation of original Schrödinger's equation will be replaced by its mathematical, exact and smooth, deterministic and clear development (see more in Chapter 4.3).

7. **Wavefunction Modeling and Signal Theory:** Finally, we should recognize that the most general modeling of wavefunctions is based on the Complex Analytic Signal model (see Chapter 4.0). By leveraging results from modern Informatics, Signal Analysis, and Telecommunications theory, particularly the “Kotelnikov-Shannon-Nyquist-Whittaker” sampling and reconstruction theorems, we can better understand the nature of Uncertainty Relations (and signal quantizing) between different signal domain durations. These concepts, which have broader implications than both

mathematical and QT Uncertainty Relations, were not fully understood or formulated during Heisenberg's time. In this book, we extend Uncertainty Relations to encompass similar relations between Shannon-Nyquist sampling intervals or frequencies, sufficient for the total reconstruction of analog signals.

Citation from: Mark John Fernee. Works at University of Queensland May 3: Quora

How does the Fourier transform tell us the uncertainty principle?

"The Fourier transform deals with conjugate variables, such as time and frequency, or position and momentum. So, let's start with the underlying principle, and that is the superposition of sine waves can be used to build any waveform. Now consider a sine wave, it has no beginning or end, but it has a perfectly defined frequency. Now consider a spike that is zero everywhere except at a single point. How many sine waves would we need to construct such a spike? It turns out that we'd need infinitely many covering all frequencies. Now the spike has a perfectly defined time or position (depending on your axis) but a completely undefined frequency or wave-vector (i.e., momentum). So, we have two extremes: A perfect sinusoid that has an undefined position in time or space; or a spike of perfectly defined time and space, but undefined frequency and momentum. This is purely a mathematical property that arises because we wanted to understand the superposition of sine waves. Because there are two extremes, the question arises about what would be the optimum midpoint where both time and frequency, or position and momentum, were sufficiently well-defined? Clearly, there must be some uncertainty in both measurements at such a point, as perfect certainty can only exist in one property if the other is completely undefined. The minimum product of the uncertainties is the basis of the Heisenberg uncertainty principle".

Citation: **Heisenberg Uncertainty Principle,** David Kahana, physicist unhinged May 16

Did a scientist find a loophole in Heisenberg's uncertainty principle?

https://www.quora.com/Did-a-scientist-find-a-loophole-in-Heisenbergs-uncertainty-principle

"First, let's drop the vague term Heisenberg's Uncertainty Principle - since Heisenberg never actually proved an uncertainty principle for position and momentum, and he certainly didn't prove one for time and energy, but rather he strongly argued and suggested that such a relation should exist for both pairs of variables, proceeding heuristically since he understood that position and momentum were a canonically conjugate pair of "matrices" in the terminology that Born and Jordan made up for Heisenberg's fundamental theoretical objects. Note very carefully they were not simply finite dimensional matrices such as we are all familiar with or at least imagine we are familiar with. It's a common human failing or hope that because you can understand the simplest possible sentences in a language, you also can understand the most general sentences, but in fact you can't. It was Born and Jordan who first formalized Heisenberg's breakthrough paper of July 1925 into what became known as matrix mechanics. There then followed a very intense argument between Schrödinger and supporters of his wave mechanics, and Heisenberg and the supporters of his matrix mechanics. After all, jobs were at stake, reputations had to be made. Schrödinger argued, after Max Born once again intervened and gave Schrödinger the proper interpretation of Schrödinger's own wavefunction, that wave mechanics and matrix mechanics were equivalent. Heisenberg argued that wave mechanics was "crap". Heisenberg insisted that discontinuous quantum jumps were absolutely basic and that any continuous and visualizable theory such as Schrödinger's was abhorrent and must be beside the point.

But there existed the Born rule in addition to Schrödinger's deterministic evolution equation for the wavefunction, without which any quantum theory was incomplete, and the measurement process was then largely a mystery. The measurement problem is still in my view a black hole from which very little has so far emerged from those who travelled inside the event horizon of that sub-area of theoretical physics and reside and work there, or maybe even it is an area of philosophy. Heisenberg and Schrödinger argued endlessly and violently in Copenhagen, with Niels Bohr I am sure, smoking his pipe and enjoying the whole thing immensely, listening to the youngsters and no doubt intervening to hold forth endlessly on some point or another at least once a day - and very little light was actually generated for all of that heat.

Meanwhile, PAM Dirac was looking at both approaches, and he finally settled the matter, they were indeed exactly equivalent formulations of the same theory. But this all took time to come out. So, the history is very confusing.

Start with Heisenberg's first paper on the idea of an uncertainty principle: I am not saying Heisenberg doesn't deserve credit for it, but I am saying that his original argument was not convincing.

Heisenberg argued that because position and momentum matrices obeyed a canonical commutation relation of the form: [X, P] = −i1, in his theory as formulated by Born and Jordan, the disturbance that any simultaneous measurement of such canonically conjugate variables that was attempted, would be defeated by the disturbance inherent in the measurement effect. The measuring device he said, would disturb the measurement inevitably since something must interact with the particle that was observed (he considered the case of a single massive particle), and that would always result in an inherent irreducible uncertainty in the measurement of both position and momentum.

*Heisenberg **did not state** in his original work on the subject of any actual inequality, but merely argued that due to the existence of this commutator, the uncertainty in any simultaneous measurement of both position and momentum for a single* *particle must be on the order of ℏ, that is - $\Delta x \Delta p \sim \hbar$ (sic). The position momentum uncertainty relation, let's be quite clear, is stated as an inequality, and it was only proved sometime later, after Heisenberg first published his thinking on the matter. Opinions differ on who first proved the inequality. You can take your choice, I would go with Kennard, since he was first into print, but Hermann Weyl also proved it, and he did so within the same year, I think. So, who knows which one was actually first? It takes time to write up a result …*

However, let this be said - if this inequality is false, then quantum mechanics is false, or else John von Neumann's axiomatization of quantum mechanics is incorrect. Because the position momentum uncertainty relation: $\Delta x \Delta p \geq \hbar 2$

is a trivial corollary of the axioms - it follows because the Cauchy-Schwartz inequality is valid in a Hilbert space and all the quantum states for a single particle, which is all that Heisenberg was considering, lies in a Hilbert space.

So, inequality is automatically satisfied by any quantum state.

Notice the factor of 2 that appears in inequality relative to what Heisenberg heuristically argues for in his original work on the uncertainty principle. So, we can, first of all, violate "Heisenberg's order of magnitude argument" without violating the actual position momentum uncertainty principle, which is a weaker uncertainty, by a factor 2, than what Heisenberg argues for.

I suggest that while it is very interesting to demonstrate "entanglement", meaning to show nothing more than that a highly correlated quantum state for a two or more particle system exists, in this case what one might by a small stretch of the imagination call a pair of macroscopic systems even if at very low temperature, and to do this for the coupled oscillations of two such systems is a very, very nice achievement, and may well have implications for better measurement devices for various purposes - or at least we can hope so, there is actually no violation of quantum mechanics implied here at all, much less a violation of the uncertainty principle as I stated it above.

On reading the arxiv preprints of these works no such claim is made, and "entangled states", I actually hate the word entangled since entangled states are just correlated many particle quantum states in which a conservation law generally exists and which entail stronger correlations than are possible classically, that's all they are, are also quantum states, they don't violate quantum mechanics, and so, they are subject to all the uncertainty principles that quantum mechanics implies about quantum states.

Now as to whether this violates "the Heisenberg Uncertainty Principle" and whether that matters to you or not, I leave it up to you to decide. I hope I have made it clear it doesn't matter to me because "the Heisenberg Uncertainty Principle" is not actually a well-defined notion, but instead is a phrase which is tossed around all the time by people who half the time don't understand what Heisenberg argued for and what he actually proved and did accomplish, and who are promoting quantum woo half the time. What Heisenberg did accomplish was the singlehanded CREATION of matrix mechanics even if he didn't formulate it completely on his own and he made the first SUGGESTION - and it's a brilliant suggestion at that - that an uncertainty relation should exist. isn't that enough?!"

The historical path that led Heisenberg to the uncertainty principle did not involve Fourier signal analysis. Instead, Heisenberg pioneered the linear algebra approach to Quantum Mechanics, where his famous uncertainty principle is derived from the commutation properties of matrices. At the time, Heisenberg formulated his intuitive and largely accurate assumptions about uncertainty, but he did not provide rigorous mathematical proof (like nobody involved in Orthodox QT, until present). Regardless of the historical context behind the original formulation of Heisenberg's Uncertainty Relations, the only logically consistent, conceptually clear, and mathematically robust explanation for universally applicable Uncertainty Relations comes from Fourier Signal Analysis. This approach, grounded in mathematics rather than Quantum Theory assumptions, deals with absolute or total signal (or wavefunction) durations in mutually conjugate domains, such as the original and spectral domains. Quantum Mechanics, in its stochastic framework, interpreted these relations between durations in conjugate domains as statistical averages between involved standard deviations. While this interpretation looks valid, it has its limitations.

In this book, we will explore all these aspects of uncertainty relations, presenting them in various ways. The underlying natural and mathematical principles should be consistent across different scales, whether in the micro or macro world of physics. Understanding what Heisenberg originally had in mind and how he developed his ideas about uncertainty is less critical than grasping the universally valid mathematical approach to uncertainty, rooted in Fourier and Complex Analytic Signal analysis. Beyond the mutually conjugated wavefunction or signal domains, we recognize that signals can be represented in temporal and/or spatial domains. In physics, real signals are always defined in a combined spatial-temporal domain. We also know that the spatial and temporal intervals and their corresponding spectral functions of the same object, matter state, or wave-packet are interlinked and mutually proportional, as seen in Relativity theory. These signal dimensions (or durations) also depend on the signal propagation velocities, and only the effective spatial and temporal dimensions of a particular matter state should be considered, rather than being simply (and often incorrectly) mixed or substituted.

Since Uncertainty Relations address signal (or wave-packet) durations in their Original and Spectral domains, we should first review the established conservation laws and symmetries between these domains. These concepts were initially introduced in the first chapter of this paper, concerning electromechanical analogies, and are summarized in Table T.5.1.

In 1905, a mathematician named Amalie Nether proved the following theorem (regarding universal laws of Symmetries):

-For every continuous symmetry of the laws of physics, there must exist a conservation law.
-For every conservation law, there should exist continuous symmetry.
Since Uncertainty Relations are addressing signal (or wave-packet) durations in their Original and Spectral, spatial, temporal and frequency domains, let us first summarize the already known conservation laws and symmetries between Original and Spectral domains with T.5.1 (in this book initially introduced in the first chapter, regarding electromechanical Analogies).

T.5.1. Symmetries of the Laws of Physics

Original Domains $\leftrightarrow$	$\leftrightarrow$ **Spectral Domains**
Time = t	**Energy =** $\tilde{E}$ **(or frequency =** f**, or mass =** m**)**
Time Translational Symmetry	**Law of Energy Conservation**
Displacement = $x = S\dot{p} = S\tilde{F}, \ (\tilde{F} = force)$	**Momentum =** $\tilde{p} = \tilde{m}\dot{x} = \tilde{m}v = p$
Space Translational Symmetry	**Law of Momentum Conservation**
Angle = $\alpha = S_R\dot{\mathbb{L}} = S_R\tau$	**Angular momentum =** $\mathbb{L} = \mathbb{J}\dot{\alpha} = \mathbb{J}\omega$
Rotational Symmetry	**Law of Angular Momentum Conservation**
Electric Charge = $q_{el.} = \Phi_{el.} = C\dot{q}_{mag.} = Ci_{mag.}$	**Magn. Charge =** $q_{mag.} = \Phi_{mag.} = L\dot{q}_{el.} = Li_{el.}$
Law of Total Electric Charge Conservation	**The Electric Charge-reversal Symmetry**
The Magnetic Charge-reversal Symmetry	**"Total Magnetic Charge" Conservation**

(Mono-magnetic charge does not exist as a free and self-standing, natural entity)

Apparently (as we can see from T.5.1), couples of the conjugate, mutually Original and Spectral domains, specified using simple analogies, are also in formal agreement with the symmetries and conservation laws of our universe (see T.1.8 from the first chapter of this book). There are many mathematical forms of uncertainty relations between mutually coupled or conjugate and absolute domains-durations, or interval-lengths (in connection with entities presented in table T.5.1), belonging to a specific signal, wavefunction, or any motional energy state, and related to spectral, or *Fourier transformation domains* of that signal. Let us start with the simplest, already known (most generally valid, both for micro and macro world of Physics) mathematical forms of Uncertainty relations that can be applied to any waveform, or wave packet, as often found in Signal (or Spectrum) Analysis, Telecommunications Theory and in certain earlier works regarding QT (see (5.5)). We will consider within this analysis, that there is a wave function or wave packet mathematical model, which is by its definition or formation equivalent (only in a couple of most important kinematic aspects) to certain moving particle, or to a specific motional "energy-moments" state. The wave-energy of the (narrow band) wave-packet in question ($\tilde{E} = hf$ or $\tilde{E} = Hf$, $[h, H]$ (=) constants) should be equal to the total motional (or kinetic) energy of its *equivalent particle couple*. The wave linear momentum of the wave-packet will be equal to the particle momentum, and the relevant *group wave-velocity* will be equal to the particle velocity (see more in Chapters 4.1 and 10).

Let us now consider only the ***absolute or total wave-packet durations (or lengths)*** *in all its domains* (in time, space and frequency, as it is already introduced in Chapter 4.0 – "Wave functions, wave velocities and uncertainty relations", starting from the equations under (4.0.55) and (5.5)). Then, "***non-statistical***" uncertainty relations, addressing a world of microphysics, molecules, atoms, and subatomic entities, are given as:

$$\Delta x \cdot \Delta p = \Delta t \cdot \Delta E = h \cdot \Delta t \cdot \Delta f \geq h/2 \Leftrightarrow \Delta x \cdot \Delta \tilde{p} = \Delta t \cdot \Delta \tilde{E} = h \cdot \Delta t \cdot \Delta f \geq h/2, \quad (5.1)$$

where meaning of the symbols is the same as introduced all over this book ($\mathbf{x}$ -displacement, $\mathbf{p} = \tilde{\mathbf{p}}$ -particle and/or wave momentum, $\mathbf{E_k} = \tilde{\mathbf{E}}$, $\Delta\mathbf{E} = \Delta\mathbf{E_k}$, -particle and/or wave-packet motional energy, $\mathbf{f}$ -mean frequency, $\mathbf{t}$ -time and $\mathbf{h}$ -Planck's constant, see expressions (4.1) - (4.3)). Of course, similar domains relation can be formulated for macro-world objects since there are Planck analog constants $H_i >>> h$ applicable in a world of solar systems and galaxies (see more in Chapters 2 and 10).

Since here we are still addressing an integral wave packet or associated motional particle (which should be mutually and only kinematically equivalent on a described way), it is clear that *in (5.1) momentum and energy values are the total momentum and total motional energy amounts, regardless of using delta-difference symbols, effectively meaning* $\Delta p = \Delta\tilde{p} = \tilde{p} = p$ *and* $\Delta\tilde{E} = \Delta E = \tilde{E} = E_k = c^2 \Delta m$. To avoid repeating lengthy introductions and explanations regarding relations (5.1), it is very much recommendable first to review most of the necessary background (related to wave packets and introductory aspects of uncertainty relations) from Chapter 4.0; - "Wave functions wave velocities and uncertainty relations".

Here is also appropriate to mention that modern QT modified a little bit the original and unambiguous mathematical meaning of Uncertainty Relations (5.1) by considering delta differences (Δ), or essential and total signal durations, as statistical, standard-deviation intervals, because this has been more appropriate to support probabilistic and statistical thinking, modeling and conceptual environment forged by the Orthodox QT establishment. *The position of the author of this book is that it should exist only one basic, fundamental set of*

Uncertainty (and Certainty) Relations or inequalities (universally applicable in Mathematics and Physics), which is ontologically purely mathematical, and all other kinds of Uncertainty relations surfacing in Quantum Physics should be equivalent or identical to general mathematical foundations of Uncertainty (of course, properly assigned to physics-related items).

The principal objective in this book is to establish a much broader conceptual framework regarding universally valid Uncertainty Relations than presently found in Physics (condensing and simplifying, as much as possible, associated mathematical complexity). *What we can overwhelmingly find in Microworld Physics literature is that in many ways, all Uncertainty relations are intrinsically linked to Planck's constant* h. *This is traditional, and still contemporary platform, but we can also find that the Planck's constant applicable in the world of atoms, photons, and elementary particles, has its macro-world equivalent in a much bigger constant* $H >> h$, *or it has series of such constants* (H_i, $i = 1,2,3\ldots$)$>> h$, *which are relevant for macro systems like planets, solar systems and galaxies with intrinsic periodicities related structures (see Chapter 2; -equations (2.11.12) - (2.11.21), and formulations about PWDC in Chapter 10). The fact is that traditional, micro-world Planck's constant* h *cannot be the unique and exclusive attribute of our macro-Universe, since macro-world states are synthetized or superimposed micro-world states. This fact brings a new light on future analyses and presentations regarding generalized Uncertainty relations (since we will be able to create at least two sets of micro and macro worlds Uncertainty relations, each of them with its own Planck-like constants, most probably being in some way deterministically connected). To comply with simplicity and continuity with already existing knowledge about Uncertainty relations in Physics, in this chapter, we will (only initially) pay attention to the micro-world environment, where Planck constant,* h, *has its significant place. Then, we will realize that most of the uncertainty relations developed for micro-world events are analogically applicable as similar relations valid for a macro-world of planetary systems, stars, and galaxies (using new and much bigger constant* H, *or number of such constants, as shown in Chapter 2., but this time devoted to planetary and other periodical motions within our Universe). Signal analysis (or mathematics) does not make any difference between signals from micro or macro world situations; -it is universally valid and applicable on every physics-related, mathematically formulated wave function.*

When using electromechanical analogies (established in the earlier chapters of this book), it is possible to address and additionally extend all Uncertainty Relations (5.1). Taking data from T.1.2, T.3.1, T.3.2, T.3.3, and T.5.1, let us analogically create T.5.2, producing the results that will **dimensionally** give a mutually analog and complementing energy-time product $\Delta t \Delta E$ for situations regarding electromagnetic fields and linear and rotational motions (always applied on the same object or wave-group). Here, we will consider only total amounts, or total (non-statistical) domains durations of relatively isolated and compact, self-standing energy states, such as a motional particle, or an equivalent wave-packet.

T.5.2. Analogies between electromagnetic fields and mechanical motions

Electro-Magnetic Field	Linear Motion	Rotation
$u_{el.} = i_{mag.} = -\frac{d\Phi_{mag.}}{dt} = -\frac{dq_{mag.}}{dt} \downarrow$ (=) El.volt.(=) Mag.current	$v = \frac{dx}{dt}$ (=) Velocity $\downarrow$	$\omega = \frac{d\alpha}{dt}$ (=)Angular Velocity $\downarrow$
$i_{el.} = u_{mag.} = +\frac{d\Phi_{el.}}{dt} = +\frac{dq_{el.}}{dt}$ (=) El.current (=)Mag. voltage	$F = \frac{dp}{dt}$ (=) Force	$\tau = \frac{dL}{dt}$ (=) Torque

$P = u_{el.} i_{el.} = i_{mag.} u_{mag.} = \frac{dE}{dt} = -\frac{dq_{mag.}}{dt} \cdot \frac{dq_{el.}}{dt} (=) Power (=) \Psi^2$	$P = vF = \frac{dx}{dt} \cdot \frac{dp}{dt} = \frac{dE}{dt}$ $(=)$ Power $(=)$ Ψ^2	$P = \omega\tau = \frac{d\alpha}{dt} \cdot \frac{dL}{dt} = \frac{dE}{dt}$ $(=)$ Power $(=)$ Ψ^2
$\Phi_{mag.} = q_{mag.}$ (=) **Magn. Flux / Charge**	x (=) Displacement	α (=) Angle
$\Phi_{el.} = q_{el.}$ (=) **El. Flux / Charge**	**p (=) Momentum**	**L (=) Angular Momentum**
ΔE = PΔt (=) Energy	**ΔE = PΔt (=) Energy**	**ΔE = PΔt (=) Energy**
$\Delta\Phi_{mag.}\Delta q_{el.} = \Delta q_{mag.}\Delta\Phi_{el.} = P(\Delta t)^2 = \Delta E\, \Delta t$	$\Delta x \Delta p = P(\Delta t)^2 = \Delta E\, \Delta t$	$\Delta\alpha\Delta L = P(\Delta t)^2 = \Delta E\, \Delta t$

Combining (5.1) and (4.2) with relations found in the bottom line of T.5.2, and with analogies summarized in the first chapter of this book, it would be possible (taking a complete and finite, minimal stable-size, or energy amount of the wave-packet of certain kind) to extended uncertainty relations (5.1) for two more members, as follows,

$$\begin{aligned}
&\Delta q_{mag.} \cdot \Delta q_{el.} = \Delta\alpha \cdot \Delta L = h \cdot \Delta t \cdot \Delta f = \Delta x \cdot \Delta p = \Delta t \cdot \Delta\tilde{E} = c^2 \Delta t \cdot \Delta m = \Delta s_1 \cdot \Delta s_2 \geq h/2, \\
&\Delta E = \frac{\Delta q_{mag.} \cdot \Delta q_{el.}}{\Delta t} = \frac{\Delta\alpha \cdot \Delta L}{\Delta t} = \frac{\Delta x \cdot \Delta p}{\Delta t} = \overline{v} \cdot \Delta p = h\Delta f = c^2 \Delta m = P\Delta t = \Delta\tilde{E} \geq \frac{h}{2\Delta t}, i_{el.} = \frac{\Delta q_{el.}}{\Delta t}, \quad (5.2) \\
&\Delta m = \frac{\Delta E}{c^2} = \frac{\Delta q_{mag.} \cdot \Delta q_{el.}}{c^2 \Delta t} = \frac{\Delta q_{mag.}}{c^2} i_{el.} = \frac{\Delta\alpha \cdot \Delta L}{c^2 \Delta t} = \frac{\Delta x \cdot \Delta p}{c^2 \Delta t} = \frac{\overline{v} \cdot \Delta p}{c^2} = \frac{h\Delta f}{c^2} = \frac{P\Delta t}{c^2} \geq \frac{h}{2c^2 \Delta t}.
\end{aligned}$$

With (5.2) we are again addressing the same “energy-moment” entity, object, or matter-wave state, meaning that such state should have (on some way) mutually coupled and dependent linear and angular moments, linear and angular displacements, and associated electromagnetic-moments complexity. Later, we will see that in addition to uncertainty relations (or inequalities), such mutually connected properties (of linear, angular, and electromagnetic energy and related motions) should also comply with “Kotelnikov-Shannon-Nyquist-Whitaker” theorems regarding signals’ sampling and reconstructing (concerning elementary sampling intervals and total signal durations in different, mutually conjugate signal definition domains). Important background here is the universal tendency of matter constituents to be mutually synchronized (or to evolve towards mutual spectral or resonant synchronizations).

The above-mentioned mathematical options seem a bit oversimplified, but we will see that the relations (5.2) and their consequences are correct. For instance, when we take $\mathbf{\Delta q_{el.}} = \mathbf{e}$, as an elementary (stable and minimal possible) electric charge of an electron, we will get $(\Delta q_{el.} = \Delta\Phi_{el.})_{min.} = e \Rightarrow \Delta\Phi_{mag.} = \Delta q_{mag.} \geq h/2e$. It is already known (in a micro world physics) that $\mathbf{h/2e}$ presents an elementary charge of a magnetic flux, being an elementary magnetic charge, $(\Delta q_{mag.})_{min.} = h/2e$ (of course, natural, free-standing, isolated and static magnetic monopole does not exist, but certain kind of (mathematically equivalent and effective) one-side closed electric current-loop should be in the background of such coupled magnetic charges. Since magnetic flux is anyway related to a certain magnetic dipole or closed electric current loop (with two magnetic poles), electric charge, or delta-interval in (5.2), should also be one part of the corresponding electric dipole with positive and negative electric charges. Similarly, we can also support the resonant-like quantization of orbiting motions ($\mathbf{\Delta\alpha, \Delta L}$) and connect them to spin (and gyromagnetic) properties of elementary particles (or possibly find other couples of conjugate variables, $\mathbf{s_1}$, $\mathbf{s_2}$ satisfying similar relations). To simplify further elaborations of Uncertainty inequalities (or relations), in (5.2) is introduced formal and symbolic generalization of all mutually conjugate “delta segments” with $\Delta s_1 \cdot \Delta s_2 \geq h/2$, when the following Uncertainty inequalities are equally applicable (see [62]),

$$\left\{\begin{array}{l} \Delta s_1 \cdot \Delta s_2 = (\Delta s_1)_{min.} \cdot (\Delta s_2)_{max.} = (\Delta s_1)_{max.} \cdot (\Delta s_2)_{min.} \geq h/2 \\ \dfrac{(\Delta s_2)_{min.}}{(\Delta s_1)_{min.}} = \dfrac{(\Delta s_2)_{max.}}{(\Delta s_1)_{max.}} = \dfrac{\Delta s_2}{\Delta s_1} \end{array}\right\} \Rightarrow$$
$$(\Delta s_1 \cdot \Delta s_2)_{min.} = h/2, \; (\Delta s_1 \cdot \Delta s_2)_{max.} > h/2,$$
$$\left(\text{or } (\Delta s_1 \cdot \Delta s_2)_{max.} = n \cdot \frac{h}{2}, \; n = 2,3,4,... \;\; ?! \right). \tag{5.2.1}$$

What is significant here is that both particles and waves are treated as being kinematically mutually equivalent, while applying the same Uncertainty Relations (and the same PWDC relations, as described in Chapters 4.1 and 10.). The only difference is that real (or ordinary) particles (with non-zero rest masses) should be considered as “frozen” or well-packed, self-stabilized and self-closed standing waves structures (this way creating non-zero rest masses). Here, we have one challenging situation related to how we understand and specify the total durations or lengths of certain (solid body) real particle and its kinematic properties (in mutually conjugate original and spectral domains), compared to similar total durations, lengths and/or dimensions of the kinematically and mathematically equivalent wave group. Real (solid and rigid) particle dimensions or absolute, total lengths and durations are not directly and fully equal to similar geometric properties of the corresponding and kinematically equivalent wave group, meaning that certain compromising and realistic, mathematically correct solutions should be established here, regarding absolute particle durations in all domains. ***Furthermore, it looks logical to conclude that in cases of stable elementary particles from the world of micro-physics, meaning of UNCERTAINTY is transformed into a CERTAINTY, where an inequality sign, “≥ or ≤” is transformed into an equality sign, “=”.*** Obviously, (based on (5.2) and (5.2.1)), we can conclude (maybe at this time still a bit prematurely) that metrics and energy-formatting of nature, regarding its elementary parts (such as atoms and elementary particles), have come to certain, conditionally non-divisible (and minimal), stabilized, atomized or discretized units, or building blocks. This way Nature realizes optimal matter packing (or formatting) with certain minimal and finite elementary domain intervals. Such elementary and minimal matter building blocks should satisfy the following “resonant gearing, fitting, packing, or *elementary-certainty-relations*”,

$$\left(\Delta q_{mag.} \cdot \Delta q_{el.}\right)_{min.} = \left(\Delta\alpha \cdot \Delta L\right)_{min.} = h \cdot \left(\Delta t \cdot \Delta f\right)_{min.} = \left(\Delta x \cdot \Delta p\right)_{min.} =$$
$$= \left(\Delta t \cdot \Delta\tilde{E}\right)_{min.} = c^2\left(\Delta t \cdot \Delta m\right)_{min.} = \left(\Delta s_1 \cdot \Delta s_2\right)_{min.} = h/2, \; (\Delta x)_{min.} = \frac{\lambda}{2} = \frac{h}{2p},$$
$$\left\{\begin{array}{l} \tilde{E} = \tilde{E}(f), \; f = \omega/2\pi\left(\approx \dfrac{1}{\Delta t}\right), \;\; p = p(k), \; k = 2\pi/\lambda = \dfrac{2\pi}{h} p\left(\approx \dfrac{\pi}{\Delta x}\right), \\ v = \dfrac{\Delta\tilde{E}}{\Delta p} = \dfrac{\Delta E}{\Delta p} = \dfrac{\Delta\omega}{\Delta k} = \dfrac{\Delta x}{\Delta t} = \dfrac{\Delta s_2}{\Delta s_1} = \overline{v} \leq c \; (\cong \text{constant}) \end{array}\right\}, \tag{5.3}$$

where $(\Delta s_1)_{min.}$ and $(\Delta s_2)_{min.}$ symbolize all other, minimal, and elementary, quantifiable, finite interval lengths, which are also mutually conjugate variables.

Analogically concluding (or just hypothetically exploring), for macrocosmic systems, like stable solar systems and galaxies, we need to apply macrocosmic "Planck-analog" constant H, and we will have similar ***CERTAINTY*** *relations applicable on relevant macro participants,*

$$\left(\Delta q_{mag.}\cdot\Delta q_{el.}\right)_{min.}=\left(\Delta\alpha\cdot\Delta L\right)_{min.}=h\cdot\left(\Delta t\cdot\Delta f\right)_{min.}=\left(\Delta x\cdot\Delta p\right)_{min.}=$$

$$=\left(\Delta t\cdot\Delta\tilde{E}\right)_{min.}=c^2\left(\Delta t\cdot\Delta m\right)_{min.}=\left(\Delta s_1\cdot\Delta s_2\right)_{min.}=H/2,\ (\Delta x)_{min.}=\frac{\lambda}{2}=\frac{H}{2p}.$$

Here, in (5.2) and (5.3), we are dealing with mutually proportional and coupled, finite, absolute, spatial, frequency and time durations of matter-wave packets (in different domains of their existence). From mathematics (or signal analysis), we know that if a time domain duration of certain signal is very short, its spectral function duration is very long or tends to become infinite, and vice versa (valid equally for spatial distances and spatial frequencies; -here valid for Δx and Δp). ***Anyway, Nature, or our Universe, is on some convenient and natural way always creating durations of both time, and frequency domains, of relatively stable particles and wave-packets, to be finite and energy limited (like in cases of Gaussian and Gabor, bell-curve-envelope wavelets, or wave-packets, since Gaussian waveforms are well defined and finite, both in a temporal and frequency domain).***

We could also draw more far-reaching conclusions, based on relations (5.2). For instance, if we would like to realize "mass-radiator", or "mass-emitter", practically meaning to create and/or radiate a mass surplus as an equivalent to specific dynamic, and/or oscillatory motional "energy-momentum" situation, we just need to address (creatively and innovatively) the following relations, developed from (5.2),

$$\Delta m=\frac{\Delta x\cdot\Delta p}{c^2\Delta t}=\boxed{F\frac{\Delta x}{c^2}}=\frac{\overline{v}\cdot\Delta p}{c^2}=\frac{\Delta\alpha\cdot\Delta L}{c^2\Delta t}=\boxed{\tau\frac{\Delta\alpha}{c^2}}=\frac{\overline{\omega}\cdot\Delta L}{c^2}=\frac{h\Delta f}{c^2}=\frac{P\Delta t}{c^2}=\frac{\Delta E}{c^2}=$$

$$=\frac{\Delta q_{mag.}\cdot\Delta q_{el.}}{c^2\Delta t}=\frac{\Delta q_{mag.}}{c^2}i_{el.}\geq\frac{constant}{\Delta t},\ \boxed{\Delta m=\frac{\Delta x}{c^2}F=\frac{\Delta\alpha}{c^2}\tau\rightarrow F=\frac{\Delta m}{\Delta x}c^2=\frac{\Delta\alpha}{\Delta x}\tau}$$

Mass defect or surplus Δm (or this can also be a radiated kinetic energy, matter waves, Tesla's Radiant energy, or some mass evaporation) will be directly proportional to involved linear and angular displacements, $\Delta x, \Delta\alpha$, or linear and angular forces F, τ, created by certain oscillatory process, $\Delta m=\frac{\Delta x}{c^2}F=\frac{\Delta\alpha}{c^2}\tau$, ($\tau(=)\text{torque}$). Alternatively, the same situation and conclusions can be extended to pulsed and/or oscillatory, linear, and angular motions, electromagnetic excitations, etc. (see more in Chapter 10.). For instance, if we apply any of mentioned (external) excitation on certain crystalline mass, such mass starts radiating certain "energy-moments" matter-wave states (what is the background of "magic power of crystals").

Gravitation (including antigravitation) and flying objects thrust related technologies (or forces) could be developed starting from such relations. The simplified understanding of natural forces can also be generalized based on "Uncertainty and Certainty Relations", as for example,

$$\Delta x \cdot \Delta p = \Delta t \cdot \Delta \tilde{E} = \Delta\alpha \cdot \Delta L = h \cdot \Delta t \cdot \Delta f = c^2 \Delta t \cdot \Delta m = \Delta q_{mag.} \cdot \Delta q_{el.} = \Delta s_1 \cdot \Delta s_2 \ (= h/2, \text{ or } H/2) \Rightarrow$$

$$\Rightarrow F\ (=)\ \frac{\Delta p}{\Delta t} = \frac{\Delta E}{\Delta x} = \frac{\Delta \tilde{E}}{\Delta x} = \frac{h \cdot \Delta f}{\Delta x} = \frac{c^2 \cdot \Delta m}{\Delta x} = \frac{\Delta\alpha}{\Delta t} \cdot \frac{\Delta L}{\Delta x} = \frac{\omega \cdot \Delta L}{\Delta x} = \frac{v \cdot \Delta p}{\Delta x} = \frac{\Delta q_{mag.} \cdot \Delta q_{el.}}{\Delta x \cdot \Delta t} = \frac{i_{el.} \cdot \Delta q_{mag.}}{\Delta x} \Rightarrow$$

$$\Rightarrow F = grad(E) = \nabla E = \nabla(\omega \cdot \Delta L) = \nabla(v \cdot \Delta p) = \nabla(i_{el.} \cdot \Delta q_{mag.}) = \dots ,$$

meaning (at least dimensionally and qualitatively) that every spatial, energy-related gradient (or mass agglomeration) is producing effects we identify as different forces. Alternatively, we could also creatively say that broader understanding of Uncertainty relations is directly influencing or explaining effects of natural forces (see more in Chapter 10. under (10.2-2.2) and "10.02 MEANING OF NATURAL FORCES"). Obviously, that contemporary understanding of nuclear forces should also significantly evolve from a present state of oversimplified and unclear, or too simple "labelling statements", and from indirect, fuzzy and naive verbal descriptions without tangible mathematical modeling (saying that such forces should exist on some way, what is a trivial statement, and not an explanation), towards much more mathematically sophisticated, resonant, nodal-zones related force-effects within matter-wave states with specific by standing-waves stabilized structures (within an atom-nucleus zone). See more about generalized natural forces in Chapter 10, under 10.02.

The intrinsic structure of our universe is most-probably organized in a way that all relevant, mutually coupled, or conjugate signal durations are finite in all domains (explicable while using reasonable approximations, and convenient mathematical modeling such as Gaussian-Gabor and bell-curves shaped Analytic signals). Mutual domains proportionality and coupling is closely related to relevant signal or matter waves speed of propagation, $v = \frac{\Delta \tilde{E}}{\Delta p} = \frac{\Delta E}{\Delta p} = \frac{\Delta\omega}{\Delta k} = \frac{\Delta x}{\Delta t} = \frac{\Delta s_2}{\Delta s_1} = \bar{v} \leq c\ (\cong \text{constant})$; -see relation (4.0.66) from Chapter 4.0. *The equality sign in (5.3) is also mathematically defendable in cases when relevant signals or wave-packets (or their wave functions) are presentable in the form of Gaussian-envelope window functions (in all of mutually conjugate domains) because the Gaussian-envelope function is optimally concentrated (finite or limited) in its joint time-frequency domain.* This is indirectly telling us how we should make modeling of wave functions, or wave packets including photons, that have a relevance in physics (or to be useful for creating or representing moving particles).

Since all energy states, masses, particles, and waves in our universe are always in different states of mutually related or relative motions, we can define the unique energy-propagation speed of such energy states, or matter waves, which is naturally connecting all matter waves durations in relevant "space-time-frequency" domains as in (5.3), $v = \frac{\Delta \tilde{E}}{\Delta p} = \frac{\Delta E}{\Delta p} = \frac{\Delta\omega}{\Delta k} = \frac{\Delta x}{\Delta t}$.
Depending on the properties of specific (non-dispersive) media, where matter waves are being observed or analyzed, we know that relevant wave-energy propagation-speed has its relatively stable value. For instance, sound-waves velocities in different gas, liquid or solid states are mutually different. Electromagnetic waves velocities in different materials are also propagation-media parameters-dependent. All of them are also relatively stable and limited if propagation media parameters are stable … This is at the same time the very important condition describing the stability and integrity of a specific energy state (also presenting stability condition of all micro and macro particles in our universe, since

$$v = \frac{\Delta \tilde{E}}{\Delta p} = \frac{\Delta E}{\Delta p} = \frac{\Delta\omega}{\Delta k} = \frac{\Delta x}{\Delta t} = const. \Rightarrow \quad \Delta \tilde{E} = \Delta E = const. \cdot \Delta p, \quad \Delta\omega = const. \cdot \Delta k, \quad \Delta x = const. \cdot \Delta t$$

…). *In other words, time and space (or spatial and temporal lengths) are mutually directly proportional, where the constant of proportionality has dimensions of certain speed (see more in Chapter 10).*

………………………………

Here is convenient place to clarify the meaning of associated forces and velocities that are linked to Uncertainty relations. Let us take mathematical Uncertainty relations between total or absolute durations of certain wave-packet (or a moving particle), as given in (5.1).

$$\Delta x \cdot \Delta p = \Delta t \cdot \Delta E = h \cdot \Delta t \cdot \Delta f \geq h/2 \Leftrightarrow \Delta x \cdot \Delta \tilde{p} = \Delta t \cdot \Delta \tilde{E} = h \cdot \Delta t \cdot \Delta f \geq h/2 . \qquad (5.1)$$

Let us now rename relevant delta (Δ) values, since here we are talking about absolute, total, and finite domain durations, as follows,

$$\left(\Delta x = X,\ \Delta p = P,\ \Delta t = T,\ \Delta E = \Delta \tilde{E} = E_k = \tilde{E}\right) \Leftrightarrow X \cdot P = T \cdot \tilde{E} \geq h/2 .$$

Since here we are addressing certain moving and stable object, particle, or an equivalent wave packet, we could say (assuming that wave phenomena or wave-packet in question is not dispersive), that initial absolute amounts $X, P, T, \tilde{E}$ will, after certain minor time interval δt, evolve as follows,

$$X \cdot P = T \cdot \tilde{E} \geq h/2 \Rightarrow \left\{ \begin{array}{l} (X+\delta x)\cdot(P+\delta p) = (T+\delta t)\cdot(\tilde{E}+\delta\tilde{E}) \geq h/2, \\ X \cdot P = T \cdot \tilde{E} \geq h/2, \\ \delta x \cdot \delta p = \delta t \cdot \delta\tilde{E} \geq h/2,\ v \cdot \delta p = \delta E = d\tilde{E} = dE = dE_k = vdp,\ v = \delta x/\delta t = dx/dt \end{array} \right\} \Rightarrow$$

$$(X+\delta x)\cdot(P+\delta p) = (T+\delta t)\cdot(\tilde{E}+\delta\tilde{E}) =$$

$$= \boxed{X \cdot P} + X \cdot \delta p + \delta x \cdot P + \boxed{\delta x \cdot \delta p} = \boxed{T \cdot \tilde{E}} + T \cdot \delta\tilde{E} + \delta t \cdot \tilde{E} + \boxed{\delta t \cdot \delta\tilde{E}} \Rightarrow$$

$$\Rightarrow \left\{X \cdot \delta p + \delta x \cdot P = T \cdot \delta\tilde{E} + \delta t \cdot \tilde{E}\right\} \Rightarrow$$

$$\Rightarrow \left\{ \begin{array}{l} \text{Force} = F\ (=)\ \dfrac{\delta p}{\delta t} = \dfrac{1}{X}\left[\tilde{E} + T \cdot \left(\dfrac{\delta\tilde{E}}{\delta t}\right) - P \cdot v\right] \\ \text{Force} = F\ (=)\ \dfrac{\delta\tilde{E}}{\delta x} = \dfrac{1}{T}\left[P + X \cdot \left(\dfrac{\delta p}{\delta x}\right) - \dfrac{\tilde{E}}{v}\right] \end{array} \right\} \Rightarrow$$

$$\Rightarrow \tilde{E} = P \cdot v - T \cdot \left(\frac{\delta\tilde{E}}{\delta t}\right) + \frac{X}{T}\left[P + X \cdot \left(\frac{\delta p}{\delta x}\right) - \frac{\tilde{E}}{v}\right] = \int v \cdot dp .$$

$$\Rightarrow X = T \cdot v + \frac{\tilde{E}}{F} - v\frac{P}{v(\delta p/\delta x)} = T \cdot v + \frac{1}{F}\left(\tilde{E} - v \cdot P\right) = \int_{(T)} vdt$$

In conclusion, we can say that the meaning of force is related to spatial and temporal energy distribution, or to relevant (mass-energy) gradients, and to absolute durations (size, lengths, or dimensions) of associated wave-packets in all their (original and spectral) domains, also implicating that Uncertainty relations and associated Forces phenomenology are mutually related.

The next conclusion is that vibrational or oscillatory phenomenology creates forces, since in force formulations we have members like, $\frac{\delta p}{\delta t} = \frac{dp}{dt}, \frac{\delta\tilde{E}}{\delta t} = \frac{d\tilde{E}}{dt}$, *and we already know that spatial and temporal durations are mutually related. Here, we also see that any solid body or stable, moving particle with well-defined geometry has different effective sizes, when analyzed as an equivalent matter-wave packet.*

Relativity theory concepts about Gravitation, related to spatial-temporal geometry, deformations, and transformations, could have certain grounds in "Uncertainty or Certainty" relations and mass-energy gradients, as initiated here (see more about natural forces in Chapters 2., and 10.). Especially interesting or challenging situations we can exercise if $(\delta x, \Delta x)$ will be oscillatory modulated as $a(x,t) \cdot \sin(\omega^* t \pm k^* x) \cdot [\delta x, dx, \Delta x]$, *and such simple harmonic, external excitations, we could apply on crystalline structures, minerals and stones, expecting to produce or radiate different matter-waves, or also (with certain additional dynamic complexity) to create anti-gravitation effects (see more in Chapter 10. under (5.3-3) and 5.2)).*

The message here is again that dominant and primary Uncertainty relations are mathematical relations (between absolute durations of mutually conjugate domains), and that Heisenberg Uncertainty relations (including statistically formulated uncertainty relations) should be strictly and entirely developed only from basic, generally applicable, and always valid mathematical Uncertainty relations between absolute and total signal durations. Later formulated statistical makeup of Uncertainty in Physics could also be appropriately updated.

In fact, no one field theory, like Maxwell electromagnetic theory, and A. Einstein Relativistic theories will be completed, more universally valid, and better mutually connected, if questions regarding satisfying Uncertainty relations (and Parseval's theorems) are not appropriately addressed there.

We should not forget that statistical processing of sets with sufficiently big number of similar or identical items, when applied <u>in all natural and other sciences</u> is especially useful, practical, very correct, universal, and powerful mathematical tool for mass data modeling, mainstream or averaged curve fittings, events spreading or distributions estimations, trends determination etc. We should not misunderstand the non-doubtful and universally applicable power of mathematical theories like Statistics and Probability and wrongly consider them as being presentable as fundamental ontological basis or origin of certain specific disciplines of Natural Sciences (like promoted, and "by consensus accepted" in Orthodox Quantum community). Powerful, sophisticated, and particularly good mathematical tools (like Statistics and probability) are only useful tools (for good artists or operators, when applicable) and should not be treated as foundations and stepstones of specific branches of Physics, because to apply such tools, we first need to conceptualize, model, build, or establish relevant data).

Mathematics was initially advancing by learning from Nature, explaining, measuring, and mastering surfaces and spaces, this way formulating its laws, gradually becoming the best language, calculating and analytic toolbox, and logic of our Universe. Now, we see that contemporary mathematics has been advancing enormously, almost independently, generalizing what it was initially learned from Nature, and we now have chances to make deductive "mining of mathematical treasures", going backwards to basic principles, or foundations of Physics. For instance, we find that Uncertainty relations (and Parseval's theorems) should always be satisfied or respected in Physics and in all signal definition domains of certain "energy-moments" state (essentially presenting forms of energy conservation laws). We also find that other mathematically formulated concepts and conclusions are universally applicable, and we need to implement such universal mathematical foundations, critically and creatively, within theories of contemporary Physics. For example, Signal Analysis based on "Kotelnikov-Nyquist-Shannon-Whittaker" sampling and signals reconstruction techniques, and Complex Analytic Signals, established by Dennis Gabor, are universally applicable to all kinds of signals and wavefunctions in mathematics and physics.

If we consider that all elementary, (relatively stable) matter building blocks are self-closed standing-waves formations, we can transform (5.3) into an equivalent matter-wave half-wavelength $\frac{\lambda}{2}$ as,

$$\left\{\lambda = h/p = h/\Delta p,\ \Delta p = \frac{\Delta\tilde{E}}{(\frac{\Delta x}{\Delta t})} = \frac{\Delta\tilde{E}}{\overline{v}} = \frac{c^2\Delta m}{\overline{v}},\ \overline{v} \leq c\right\} \Rightarrow$$

$$\frac{\lambda}{2} = \frac{h}{2\Delta p} \leq \frac{\Delta q_{mag.}\cdot\Delta q_{el.}}{\Delta\tilde{E}}\cdot\overline{v} = \frac{\Delta\alpha\cdot\Delta L}{\Delta\tilde{E}}\cdot\overline{v} = \frac{h\cdot\Delta t\cdot\Delta f}{\Delta\tilde{E}}\cdot\overline{v} = \frac{\Delta s_1\Delta s_2}{\Delta\tilde{E}}\overline{v} = \frac{\Delta x\cdot\Delta p}{\Delta\tilde{E}}\cdot\overline{v} = \overline{v}\cdot\Delta t = \Delta x \Rightarrow \quad (5.4)$$

$$\frac{\lambda_{min.}}{2} = \left(\frac{\Delta q_{mag.}\cdot\Delta q_{el.}}{\Delta\tilde{E}}\cdot\overline{v}\right)_{min.} = \left(\frac{\Delta\alpha\cdot\Delta L}{\Delta\tilde{E}}\cdot\overline{v}\right)_{min.} = \left(\frac{h\cdot\Delta t\cdot\Delta f}{\Delta\tilde{E}}\cdot\overline{v}\right)_{min.} =$$

$$= \left(\frac{\Delta s_1\Delta s_2}{\Delta\tilde{E}}\overline{v}\right)_{min.} = \left(\frac{\Delta x\cdot\Delta p}{\Delta\tilde{E}}\cdot\overline{v}\right)_{min.} = (\overline{v}\cdot\Delta t)_{min.} = (\Delta x)_{min.} = \left(\frac{h}{2\Delta p}\right)_{min.} = \frac{h}{2mc},$$

what presents kind of resonant states and standing-waves relations between each couple of mutually conjugate variables (like acoustic resonators, antenna dipoles, etc.). Here we should also notice that $\frac{\lambda_{min.}}{2} = \frac{\lambda_c}{2} = \frac{h}{2mc}$ is the Compton wavelength $\lambda_c = h/mc$ (just to initiate an indicative and challenging brainstorming).

Intentionally choosing the matter-wave half-wavelength $\lambda/2$, as the most crucial elementary and resonant energy-formatting and packing unit (or size) of anything that is quantifiable in our universe, (see (5.2) - (5.4)), we are merely stating that quantization and standing-waves relations are phenomenological and conceptual synonyms (regarding elementary matter-domains and different matter packing or formatting). Clear manifestations of such quantizing events are present only in cases of self-closed, atomized domains, and energy exchanges between them, where the smallest space-domain unit is equal to a half-wavelength $\lambda/2$, or to certain of familiar (analogical and more applicable) space-periodicity related values from T.5.3 and T.5.4. Bottom line explanation of the quantum nature of our Universe is that all stable energy-moments-states (for instance masses) are composed of quasi-resonant, standing-matter waves structures. Such states are also mutually communicating and interacting by exchanging similar (finite, discretized and countable) packets or quanta of "energy-moments matter-wave states". Quantizing of matter has two phenomenological aspects, such as, Energy-communications between matter-states are not continuous but more like sending or exchanging small, finite, or discrete packets of energy, and if we structurally associate some integers with such energy quanta, this is closely related to standing-waves wavelengths counting. The real and mathematically valid theory about matter quantizing is much closer to rules of "Kotelnikov-Nyquist-Shannon-Whitaker" sampling, signal analysis and signals synthesis (or reconstruction), than to what modern Quantum theory is promoting regarding quantization. Of course, associated Euler-Lagrange-Hamilton equations and familiar principles and concepts (taken from analytical Mechanics) are additionally arranging realistic situations in the same field. More of the common supporting arguments regarding standing-waves matter structure can be found in the Appendix, Chapter 10. PARTICLES AND SELF-CLOSED STANDING MATTER WAVES.

5.0.1. Meaning of Time and Certainty Relations

There are lot of polemic and philosophical, mathematical, perceptual, emotional, tangible measurements based, and other elaborations about the meaning of time, and still, we do not have the final and completed picture, description or definition about time as one of fundamental dimensions of our universe. Let us state several properties of the time or temporal dimension,

1. For spatial dimensions like (x, y, z) we know what zero length, finite length ΔL, and positive and negative or infinity length meanings and values are. From Relativity theory we know that spatial and temporal dimensions, lengths or intervals are mutually dependent (defined using Lorentz transformations, … $\Delta L = c\Delta t$). This has been experimentally verified and theoretically described. In space, we measure the distance ΔL between two points, but we do not define what a dot (or point) is. We only intuitively and axiomatically describe the point. The same applies to time: we measure the distance between two time points (dashes, or moments) and do not know what these fixed time points are, but we do know and measure what the time interval Δt is between two time points (or moments). The answer to what time is we can compare to an answer to the question of what space is, but in both cases, we only measure the corresponding distances or intervals, and the rest are descriptive, undefined, basic and initial concepts, axiomatically accepted as they are. All of that is linked to, or the consequence of CPT Symmetry.

2. Anyway, for us, time-vector, time-arrow, time tendency, or natural time property is that time is defined from certain zero-time spot towards positive infinity, or future. Negative values of time in physics are often only mathematical interpretations, or situations addressing past events, but "real energy-moments" states or objects cannot travel in negative time direction, except mathematically when time-symmetry is in a compliance with CPT and universal, natural conservation laws.

3. All natural dimensions of our universe are or should be mutually perpendicular, or mutually phase-shifted for $\pi/2$. This condition of mutual dimensional orthogonality or perpendicularity secures that all of dimensions (x, y, z, t) describe non-overlapping, non-interacting, independent "energy-moments" states, regarding spacetime dimensionality. For spatial dimensions (x, y, z) mentioned orthogonality is easily understandable, or visually clear, since (x, y, z) are mutually orthogonal or mutually phase shifted for 90° (or $\pi/2$). To make temporal dimension t phase shifted to (x, y, z) for $\pi/2$, Minkowski created his complex spatial-temporal, "mass-energy-moment" 4-vectors, where temporal dimension t is on the ict imaginary axis ($i^2 = -1$), and this way A. Einstein Special Relativity theory was mathematically formalized on a very simple, correct and elegant way, producing (mathematically and experimentally) verifiable results in cases of all "energy-moments" states and events regarding particles, photons and other quantized wave-groups scattering and impacts interactions, meaning that ict imaginary time axis has been well defined or selected. Consecutively, if (x, y, z) are real, visible, tangible dimensions, then temporal dimension t is on some mathematical way the pure imaginary dimension, which can take positive and negative imaginary values. While (x, y, z) are mutually orthogonal and phase shifted for $+\pi/2$ or $-\pi/2$. Positions like +x and -x are mutually phase-shifted for $+\pi$ or $-\pi$. The same is valid for ±y, ±z, and ±ict. Here we could also conceptualize that r = r(x, y, z), and $\Psi = \Psi(r, t)$, being a wavefunction or signal that will be presented in a form of the Complex Analytic Signal or Phasor, such as $\overline{\Psi}(r,t) = \Psi(r,t) + i \cdot \hat{\Psi}(r,t),\ i^2 = -1,\ \hat{\Psi}(r,t) = H\left[\Psi(r,t)\right]$.

What is interesting here, regarding real and imaginary parts of an Analytic Signal is that they are mutually phase-shifted for $\pi/2$, this way immediately creating conditions of mutual orthogonality, what is also the way of dimensional extension, being similar as applied in Relativity theory. In other words, spatial and temporal dimensions should be phase-shifted for $\pi/2$, or coordinate system (r, ict) is also mutually orthogonal, meaning phase-shifted, between **r** and **t** for $\pi/2$, (r = r(x, y, z)). This is enriching our conceptual understanding of time flow, from certain initial, zero-time-spot towards infinity (see more about Analytic Signal modeling in Chapter 4.0). If temporal and spatial lengths or intervals are mutually linked and dependent (as in Relativity theory), and space has three dimensions such as (x, y, z), it is analogically and intuitively logical that time also could have three temporal axes (t_x, t_y, t_z), $(dr)^2 = (dx)^2 + (dy)^2 + (dz)^2$, $(dt)^2 = (dt_x)^2 + (dt_y)^2 + (dt_z)^2$, $(dr)^2 + (Icdt)^2 = (ds)^2, I^2 = -1$, and in this case we will define the Analytic Signal as a hypercomplex function with three imaginary units $(i, j, k)^2 = (-1, -1, -1)$, this way opening horizon for an extended meaning of CPT Symmetry and Multidimensionality of spacetime conceptualization (see more of this in Chapter 6).

4. In time and space, everything that constitutes our Universe occurs. This involves energetic and other dynamic parameters, such as mechanical linear and angular moments, and accompanying electromagnetic, electrical, and magnetic moments. In this process, we adhere to the laws of conservation of energy, linear and angular or rotational moments. All movements in our Universe are mutually relative. Stable states that are in some way uniform, calm, and unchanging are called inertial states in mechanics, and there are analogous states in electromagnetism. When spatial-temporal changes occur in these uniform, stable states, various mechanical and electromagnetic forces arise. This means that the dimensions of space and time, or spatial and temporal intervals, are directly dependent on various manifestations of energy, power, and different dynamic, mechanical, and electromagnetic moments and forces that occur. This also means that the essence of time cannot be explained only through our intuitive, perceptual, philosophical, and ordinary human language, because time and space have more components or parameters upon which they depend. Space and time create a kind of dimensional box in which everything that is part of our universe happens. Our senses are not sufficient to fully define or describe time and space based on what we see and feel (we are still not that perfect). Our linguistic capabilities do not have equivalent terms, meanings or expressions for everything that needs to describe space and time. Language of Mathematics and Physics is required. Therefore, systems, concepts, states and processes that are multidimensional or dependent on several parameters, including specific geometry, cannot be described in just a few sentences, or in one highly simplified linguistic way.

5. Classical, second-order, partial, differential wave equation (including Schrödinger equation as its derivative), which is describing all matter-waves in micro and macro world of Physics, always has solutions as at least two wave groups (or also 4, 6, 8, …wave groups). Such solutions are its spatial-temporal or (x, y, z, ict)-dependent solutions, always being (in-pairs) mutually opposed, or propagating in mutually opposite directions (inwards and outwards), meaning being mutually phase shifted (in the Minkowski space) for $+\pi$ or $-\pi$. We often neglect negative-direction, or inwards-directed solutions of wave equations, because in most cases we do not have an easy and simple visualization, but mathematically, experimentally and as 4-vectors in the Minkowski space of Relativity theory, such solutions comply with all natural,

conservation laws, and CPT Symmetry, meaning that all mentioned temporal and spatial directions should be naturally possible, acceptable and realistic.

6. Fourier integral transformations, related to signal analysis or decomposition in a time domain decompose certain complicated or arbitrary signal-form into number of simple-harmonic or sinusoidal elementary waves, where time t is taking values from its negative infinity to positive infinity (or from $-\infty$ to $+\infty$). Real time, or present situation is this initial arbitrary signal, but Fourier analysis shows that such or any temporal reality is in fact a superposition of all elementary, simple-harmonic wave forms belonging to negative and positive (or past and future) infinity on time axis. That means that any wavefunction or signal in real present-time is in some way composed or superimposed by its negative and positive time values, canceling everything on both infinity sides, and leaving only the initial wavefunction in its present state. To negative time values, we associate everything what was in the past, and to positive time values we associate the future situations of the same wavefunction or signal. Only the real-time initial signal is always in its present state or real time. All of that is again part of the CPT Symmetry.

7. Action equal to reaction and different "currents-voltages-charges induction laws," are universal laws of our universe, meaning that we always have and use positive and negative, realistic temporal and spatial values or directions of wavefunctions and "energy-moments" states propagation (including inwards and outwards directions in spherical geometry situations).

8. Mirror symmetrical mathematical imaging and mapping situations or states in Physics have mutually inverted, or opposed ± signs, and state values, or meanings. We already perform (in electromagnetic theory) useful and meaningful calculations based on virtual interactions between an original and its mirror state. In other words, positive and negative times can be synchronously used while respecting natural conservation laws and CPT Symmetry.

9. There are processes that change in a similar way to the time we experience. For example, the entropy of the universe seems to us only to increase in a positive time direction, or it always increases, and intuitively we can compare this to time that only progresses from the present to the future. But this is an incomplete and oversimplified, analogous entropy interpretation, because we have already explained that space and time depend on more parameters (energies, moments, forces, matter waves, specific geometry of involved participants, etc.). Also, old or Classical Thermodynamics, the second law of Thermodynamics, and meaning of Entropy, should be significantly updated taking into consideration the total motional energy of certain system (including electromagnetic and other matter waves), and not only thermal and/or mechanical motions of involved particles.

10. All of this about dimensionality is best stated through our international system of units in physics. The world of these standardized and harmonized units of measurement are basic concepts or starting points that are sometimes not completely defined ontologically or essentially (though they have certain descriptions, explanations and mutual relations). Everything else that happens, or is being measured in our universe are intervals, differentials, integrals, moments, energies, interferences, and superpositions of these basic items, concepts, or better said, relations of relevant intervals. Time is just one of the basic concepts that is always used and still not essentially defined, but we easily do define and measure time intervals.

11. Anyway, we still do not completely know or understand the philosophical, perceptual, emotional and ontological meaning of time, but logically and naturally, we know that all such meanings should also comply with mathematical and experimental meaning of time (especially regarding time intervals). To obtain such latest and total harmony between different time aspects we need to differentiate between essential and ontological meaning of time, and what time, used in our engineering applications and mathematical calculations pragmatically means. We are practically measuring or operating with time intervals, or time segments Δt (not with time). This Δt situations should not be mixed with essential, ontological and philosophical meaning of time. It is very logical and understanding that here we can exploit Certainty Relations (5.3), which are giving relations between all elementary domains and mass-energy-moments found in (5.3), to extract and define Δt, or effective, real-time intervals, as for example,

$$\begin{bmatrix} \left(\Delta q_{mag.} \cdot \Delta q_{el.}\right)_{min.} = \left(\Delta\alpha \cdot \Delta L\right)_{min.} = h \cdot \left(\Delta t \cdot \Delta f\right)_{min.} = \left(\Delta x \cdot \Delta p\right)_{min.} = \\ = \left(\Delta t \cdot \Delta\tilde{E}\right)_{min.} = c^2\left(\Delta t \cdot \Delta m\right)_{min.} = h/2,\ \Delta\tilde{E} = h\Delta f = c^2\Delta m = \Delta E \end{bmatrix} \Rightarrow \tag{5.4-0}$$

$$\Rightarrow \boxed{\Delta t = \frac{\Delta x \cdot \Delta p}{\Delta\tilde{E}} = \frac{\Delta\alpha \cdot \Delta L}{\Delta\tilde{E}} = \frac{\Delta q_{mag.} \cdot \Delta q_{el.}}{\Delta\tilde{E}} = \frac{h}{2\Delta\tilde{E}}}$$

As we can see, all elementary or minimal, discrete, temporal, spatial, mechanical and electromagnetic Δ intervals, including relevant energy, mass, and frequency intervals, (of a finite, and stable Gaussian signal of matter-wave packet) are mutually linked and dependent. This is the reason why visual, intuitive, emotional, perceptual, and philosophical descriptions are not producing generally satisfactory, very useful and ontological time-meaning explanations. In other words, present science-philosophers should be more multidisciplinary educated, creative, good mathematicians and physicians to be able to properly elaborate the meaning of time. Ordinary human-language is not enough for meaning of time description.

5.0.2. Matter waves wavelengths and elementary domains

The Schrödinger equation is among the most famous equations closely related to matter-waves and QT. We should not forget that it can be smoothly developed and formulated by an analogical transformation of the Classical wave equation of standing waves oscillations of a string, where an integer number of half-wavelength segments is a natural stability condition (and where a relevant wave function is presented as a Complex Analytic Signal). Of course, the meaning of matter wavelength should be analogically extended to all other aspects of spatially self-structured waves and fields, such as to rotational or angular motions and to corresponding electromagnetic values (as we can find in T.5.3 and T.5.4). By continuing developing the same concept, we can find that Euler-Lagrange-Hamilton formalism presents another framework to express a tendency to optimal and dynamic energy packing and synchronization of interacting matter in motion.

Let us now address the (resonant) periodicity of motional states, fields and wave packets using the framework of analogies (established in the first and third chapter of this book) and Wilson-Sommerfeld action integrals (see [9]), known from early days of

Quantum Mechanics. By “playing with analogies”, based on data from T.3.1 and T.3.3, (Chapter 3), combining them with “*elementary particle metrics”* (5.3) and (5.4), we could introduce even broader meaning of (new) de Broglie-like wavelengths. Such wavelengths can also be considered as periodical and quantifiable, resonant matter wave intervals, being discrete matter building blocks, or elementary “Certainty Intervals” (as already elaborated in this chapter, in Chapter 4.1, by T.4.2, and in Chapter 10). For instance, we will be able to formulate analogically “*de Broglie-like, matter waves angular wavelength*”, $\vec{\theta} = \frac{h}{L}$ and “*de Broglie-like electromagnetic charges and wavelengths*”, $\vec{\Phi}_{mag.} = \frac{h}{\Phi_{el.}} = \frac{h}{q}$, $\vec{\Phi}_{el.} = q = \frac{h}{\Phi_{mag.}}$, as given in the T.5.3.

T.5.3 Analogical Parallelism between Different Aspects of Matter Waves

Waves Periodicity of Fields and Motional States	[DISPLACEMENTS] $[X] = [q_{mag.}, q_{el.}, x, \alpha]$	[CHARGES] $[Q] = [q_{el.}, q_{mag.}, p, L]$	De Broglie, or Periodicity Wave Intervals $[\vec{X}][Q] = [h]$
Electric Field (and total electric charge conservation)	$\Phi_{mag} = L_{mag} i_{el} = q_{mag}$	$[C_{el.} u_{el.}^1]$ (=) $[q_{el.} u_{el.}^0]$ (=) $[q_{el.}]$ (=) $[\Phi_{el}]$	$\vec{\Phi}_{mag.} = \frac{h}{\Phi_{el.}} = \frac{h}{q} = \lambda_e = \frac{h}{q_{el.}} = q_{mag.}$ (new)
Magnetic Field (and total, or dynamic magnetic charge conservation)	$\Phi_{el.} = L_{el.} i_{mag.} = q_{el.}$	$[C_{mag.} u_{mag.}^1]$ (=) $[q_{mag.} u_{mag.}^0]$ (=) $[q_{mag.}]$ (=) $[\Phi_{mag}]$	$\vec{\Phi}_{el.} = q = \frac{h}{\Phi_{mag.}} = \lambda_m = \frac{h}{q_{mag.}} = q_{el.}$ (new)
Gravitation & Linear Motion (and linear momentum conservation)	$x = Sf$	$[mv^1]$ (=) $[pv^0]$ (=) $[p]$	$\vec{\lambda} = \frac{h}{p} = \lambda$ (already known)
Rotation (and angular momentum conservation)	$\alpha = S_R M$	$[J\omega^1]$ (=) $[L\omega^0]$ (=) $[L]$	$\vec{\theta} = \frac{h}{L}$ (new)

(Periodicity – here invented, unifying formulation, $q_{mag.} = \Phi_{mag.}$ ***is not a free and independent magnetic charge)***

The idea here is to show that micro-world quantization is not too far from standing matter waves, resonant energy packing (of self-closed or self-stabilized oscillatory formations), since the general meaning of quantization is that elementary and stable matter domains (elementary particles) are composed or structured from elementary matter-waves intervals such as $\lambda = \frac{h}{p}$, $\vec{T} = \frac{h}{\vec{E}}$, $\vec{\theta} = \frac{h}{L}$, $\vec{\Phi}_{el.} = \frac{h}{\Phi_{mag.}}$, $\vec{\Phi}_{mag.} = \frac{h}{\Phi_{el.}}$, $\vec{s}_{1,2} = \frac{h}{s_{2,1}}$, multiplied by integers (see later (5.2.1) and (5.4.1)). The conceptual grounds of Strings Theory are not too far from what is elaborated here (see *Appendix, Chapter 10. PARTICLES AND SELF-CLOSED STANDING MATTER WAVES*).

In fact, what T.5.3, equations (5.2) - (5.4), and Wilson-Sommerfeld action integrals really describe (and predict) should be a kind of universal (resonant) matter waves periodicity, or kind of fields, waves and charges, energy, and space-related atomization (based on standing waves formations, where half-wavelength, $\lambda/2$, is the minimal building-block size), when describing finite and relatively stable matter states. This way we can formulate "fitting and gearing rules", expressing some of the fundamental Symmetries of our universe (or simply saying, we are expressing "**optimal matter waves and energy formatting and matter packing rules**"). Here we can also formulate an extension of de Broglie particle-wave hypothesis. *Since Particle-Wave Duality, periodicity, waving, and resonant quantization is associated to all linear motions (already known in the form of* $\lambda = h/p$, $\tilde{E} = hf$ *), the same or analogical idea (under similar standing-waves conditions, such as,*
$\lambda = \frac{h}{p},\ \overleftrightarrow{T} = \frac{h}{\tilde{E}},\ \overleftrightarrow{\theta} = \frac{h}{\mathbf{L}},\ \overleftrightarrow{\Phi}_{el.} = \frac{h}{\Phi_{mag.}},\ \overleftrightarrow{\Phi}_{mag.} = \frac{h}{\Phi_{el.}}, \vec{s}_{1,2} = \frac{h}{s_{2,1}}$*)*
should apply to rotational motions, to electromagnetic phenomena, to any kind of motional energy, and to all other fields and motions in Nature (including the macro world of astronomy). Also, all such manifestations should exist coincidently being mutually well synchronized (often united, coupled inside of the same object, or belonging to the same energy state). These principles are fundamentally rooted in translational and rotational symmetry, which underlie the conservation laws of linear and angular momentum, as well as energy conservation. All stable, linear, uniform, and inertial motions are relative motions; -this concept of relative motion also extends to stable rotational or orbital motions, which are accelerated by nature.

The wave-particle duality of matter serves as a "theoretical, conceptual and tangible bridge" between linear motions, characterized by spatial translational symmetry, and rotational motions, characterized by spatial rotational symmetry. Additionally, linear mechanical motions of masses can be analogically and phenomenologically associated with the linear motions of electrically charged particles in an electric field. Similarly, rotational motions of masses can be analogically linked to the rotational (spinning and helical) motions of electrically charged particles in a magnetic field.

By analogy with (5.2), and using the newly formulated "*de Broglie-like matter wavelengths*" from T.5.3, we can express the following, extended matter-waves periodicity relations:

$$\left\{ \begin{array}{l} 2\Delta q_{mag.} \cdot \Delta q_{el.} = 2\Delta\alpha \cdot \Delta L = 2h \cdot \Delta t \cdot \Delta f = 2\Delta x \cdot \Delta p = 2\Delta t \cdot \Delta\tilde{E} = 2c^2 \Delta m \cdot \Delta t = 2\Delta s_1 \cdot \Delta s_2 \geq h, \\ \lambda = \frac{h}{p},\ \overleftrightarrow{T} = \frac{h}{\tilde{E}} = \frac{h}{\Delta\tilde{E}} = \frac{h}{\Delta E},\ \overleftrightarrow{\theta} = \frac{h}{\mathbf{L}} = \frac{h}{\Delta\tilde{\mathbf{L}}} = \frac{h}{\Delta\mathbf{L}}, \frac{\lambda}{\overleftrightarrow{\theta}} = \frac{\mathbf{L}}{p}, p = \Delta p, \mathbf{L} = \Delta\mathbf{L}\ \ldots, \\ \overleftrightarrow{\Phi}_{el.} = \frac{h}{\Phi_{mag.}} = \frac{h}{\Delta\Phi_{mag.}},\ \overleftrightarrow{\Phi}_{mag.} = \frac{h}{\Phi_{el.}} = \frac{h}{\Delta\Phi_{el.}}, \vec{s}_{1,2} = \frac{h}{s_{2,1}} = \frac{h}{\Delta s_{2,1}},\ \frac{\overleftrightarrow{\Phi}_{el.}}{\overleftrightarrow{\Phi}_{mag.}} = \frac{\Phi_{el.}}{\Phi_{mag.}}, i_{el.} = \frac{\Delta q_{el.}}{\Delta t} \\ h = \lambda \cdot p = \overleftrightarrow{T} \cdot \tilde{E} = \overleftrightarrow{\theta} \cdot \mathbf{L} = \overleftrightarrow{\Phi}_{el.} \cdot \Phi_{mag.} = \overleftrightarrow{\Phi}_{mag.} \cdot \Phi_{el.} . \end{array} \right\} \Rightarrow$$

$$\left.\begin{array}{l}\lambda \leq 2\frac{\Delta s_1 \Delta s_2}{\Delta \tilde{E}}\cdot\overline{v} = 2\frac{\Delta q_{mag.}\cdot\Delta q_{el.}}{\Delta \tilde{E}}\cdot\overline{v} = 2\frac{\Delta\alpha\cdot\Delta L}{\Delta \tilde{E}}\cdot\overline{v} = 2\frac{h\cdot\Delta t\cdot\Delta f}{\Delta \tilde{E}}\cdot\overline{v} = 2\frac{\Delta x\cdot\Delta p}{\Delta \tilde{E}}\cdot\overline{v} = 2\overline{v}\cdot\Delta t = 2\Delta x,\\ \ddot{T} \leq 2\frac{\Delta s_1\cdot\Delta s_2}{\Delta \tilde{E}} = 2\frac{\Delta q_{mag.}\cdot\Delta q_{el.}}{\Delta \tilde{E}} = 2\frac{\Delta\alpha\cdot\Delta L}{\Delta \tilde{E}} = 2\frac{h\cdot\Delta t\cdot\Delta f}{\Delta \tilde{E}} = 2\frac{\Delta x\cdot\Delta p}{\Delta \tilde{E}} = 2\frac{c^2\Delta m\cdot\Delta t}{\Delta \tilde{E}} = 2\Delta t,\\ \ddot{\theta} \leq 2\frac{\Delta s_1 \Delta s_2}{\Delta \tilde{L}} = 2\frac{\Delta q_{mag.}\cdot\Delta q_{el.}}{\Delta \tilde{L}} = 2\frac{\Delta\alpha\cdot\Delta \tilde{L}}{\Delta \tilde{L}} = 2\frac{h\cdot\Delta t\cdot\Delta f}{\Delta \tilde{L}} = 2\frac{\Delta x\cdot\Delta p}{\Delta \tilde{L}} = 2\frac{\Delta t\cdot\Delta \tilde{E}}{\Delta \tilde{L}} = 2\overline{\omega}\cdot\Delta t = 2\Delta\alpha\ ,\\ \ddot{\Phi}_{el.} \leq 2\frac{\Delta s_1\cdot\Delta s_2}{\Delta\Phi_{mag.}} = 2\frac{\Delta q_{mag.}\cdot\Delta q_{el.}}{\Delta\Phi_{mag.}} = 2\frac{\Delta\alpha\cdot\Delta L}{\Delta\Phi_{mag.}} = 2\frac{h\cdot\Delta t\cdot\Delta f}{\Delta\Phi_{mag.}} = 2\frac{\Delta x\cdot\Delta p}{\Delta\Phi_{mag.}} = 2\frac{\Delta t\cdot\Delta \tilde{E}}{\Delta\Phi_{mag.}} = 2\frac{\Delta \tilde{E}\cdot\Delta t}{\Delta\Phi_{mag.}} = 2\Delta q_{el.},\\ \ddot{\Phi}_{mag.} \leq 2\frac{\Delta s_1\cdot\Delta s_2}{\Delta\Phi_{el.}} = 2\frac{\Delta q_{mag.}\cdot\Delta q_{el.}}{\Delta\Phi_{el.}} = 2\frac{\Delta\alpha\cdot\Delta L}{\Delta\Phi_{el.}} = 2\frac{h\cdot\Delta t\cdot\Delta f}{\Delta\Phi_{el.}} = 2\frac{\Delta x\cdot\Delta p}{\Delta\Phi_{el.}} = 2\frac{\Delta t\cdot\Delta \tilde{E}}{\Delta\Phi_{el.}} = 2\frac{\Delta \tilde{E}\cdot\Delta t}{\Delta\Phi_{el.}} = 2\Delta q_{mag.}.\end{array}\right\}\Rightarrow$$

$$\Rightarrow\left\{\begin{array}{l}\lambda_{min.} = \left(\frac{h}{p}\right)_{min.} = 2(\Delta x)_{min.} = \frac{h}{mc},\\ \ddot{T}_{min.} = \left(\frac{h}{\tilde{E}}\right)_{min.} = 2(\Delta t)_{min.} = \left(\frac{h}{c^2\Delta m}\right)_{min} = \frac{h}{mc^2},\\ \ddot{\theta}_{min.} = \left(\frac{h}{L}\right)_{min.} = 2(\Delta\alpha)_{min.} = \frac{2\pi}{n},\ n = 1,2,3,...,\\ \left(\ddot{\Phi}_{el.}\right)_{min.} = \left(\frac{h}{\Phi_{mag.}}\right)_{min.} = 2(\Delta q_{el.})_{min.} = 2e,\\ \left(\ddot{\Phi}_{mag.}\right)_{min.} = \left(\frac{h}{\Phi_{el.}}\right)_{min.} = 2(\Delta q_{mag.})_{min.} = \frac{h}{2e}.\end{array}\right\}\Leftrightarrow\left\{\begin{array}{l}\frac{\lambda_{min.}}{2} = \frac{1}{2}\left(\frac{h}{p}\right)_{min.} = (\Delta x)_{min.} = \frac{h}{2mc},\\ \frac{\ddot{T}_{min.}}{2} = \frac{1}{2}\left(\frac{h}{\tilde{E}}\right)_{min.} = (\Delta t)_{min.} = \left(\frac{h}{2c^2\Delta m}\right)_{min} = \frac{h}{2mc^2},\\ \ddot{\theta}_{min.} = \left(\frac{h}{L}\right)_{min.} = 2(\Delta\alpha)_{min.} = \frac{2\pi}{n},\ n = 1,2,3,...,\\ \frac{\left(\ddot{\Phi}_{el.}\right)_{min.}}{2} = \frac{1}{2}\left(\frac{h}{\Phi_{mag.}}\right)_{min.} = (\Delta q_{el.})_{min.} = e,\\ \left(\ddot{\Phi}_{mag.}\right)_{min.} = \left(\frac{h}{\Phi_{el.}}\right)_{min.} = 2(\Delta q_{mag.})_{min.} = \frac{h}{2e}.\end{array}\right\}$$

Magnetic flux quantum (From Wikipedia, the free encyclopedia); *-Motivating to think about and make associations and initiate brainstorming… Pay attention on $\Phi_0 = h/(2e) = 2.067\,833\,636\times10{-}15$ Wb.*

*The **magnetic flux quantum** Φ_0 is the quantum of magnetic flux passing through a superconductor. The phenomenon of flux quantization was discovered B. S. Deaver and W. M. Fairbank[2] and, independently, by R. Doll and M. Nabauer,[3] in 1961. The quantization of magnetic flux is closely related to the Little-Parks effect but was predicted earlier by Fritz London in 1948 using a phenomenological model.*

*The inverse of the flux quantum, $1/\Phi_0$, is called the **Josephson constant** and is denoted K_J. It is the constant of proportionality of the Josephson Effect, relating the potential difference across a Josephson junction to the frequency of the irradiation. The Josephson Effect is very widely used to provide a standard for high precision measurements of potential difference, which (since 1990) have been related to a fixed, "conventional" value of the Josephson constant, denoted K_{J-90}.*

Values (CODATA 2006)[1]
Units
$\Phi_0 = 2.067\,833\,667(52)\times10^{-15}$ Wb
$K_J = 483\,597.891(12)\times10^9$ Hz/V
$K_{J-90} = 483\,597.9\times10^9$ Hz/V

***Magnetic flux quantum** Φ_0 is a property of a supercurrent (superconducting electrical current) that the magnetic flux passing through an area bounded by such a current is quantized. The quantum of magnetic flux is a physical constant, as it is independent of the underlying material as long as it is a superconductor. Its value is $\Phi_0 = h/(2e) = 2.067\,833\,636\times10^{-15}$ Wb.*

If the area under consideration consists entirely of superconducting material, the magnetic flux through it will be zero, for supercurrents always flow in such a way as to expel magnetic fields from the interior of a superconductor, a phenomenon known as the Meissner effect. A non-zero magnetic flux may be obtained by embedding a ring of superconducting material in a standard (non-superconducting) medium. There are no supercurrents present at the center of the ring, so magnetic fields can pass through. However, the supercurrents at the boundary will arrange themselves so that the total magnetic flux through the ring is quantized in units of Φ_0. This is the idea behind SQUIDs, which are the most accurate type of magnetometer available.

A similar effect occurs when a type II superconductor is placed in a magnetic field. At sufficiently high field strengths, some of the magnetic field may penetrate the superconductor in the form of thin threads of material that have turned normal. These threads, which are sometimes called ***fluxons*** *because they carry magnetic flux, are in fact the central regions ("cores") of vortices in the supercurrent. Each fluxon carries an integer number of magnetic flux quanta.*

Quantization of magnetic flux is determined by a unified geometric theory [4] of electromagnetism and gravitation. The elementary electric charge is simultaneously determined by the theory. Furthermore, these charge and flux quanta directly give the fractions which appear in the fractional quantum Hall effect.

Measuring the magnetic flux

The magnetic flux quantum may be measured with high precision by exploiting the Josephson effect. In fact, when coupled with the measurement of the von Klitzing constant $R_K = h/e^2$, this provides the most precise values of Planck's constant h obtained to date. This is remarkable since h is generally associated with the behavior of microscopically small systems, whereas the quantization of magnetic flux in a superconductor and the quantum Hall effect are both collective phenomena associated with thermodynamically large numbers of particles.

Wilson-Sommerfeld action integrals (see [9] and (5.4.1)), related to any stable, periodical or wave motion on a self-closed stationary orbit, applied over one period of a motion, present the kind of general quantifying rule (for all closed standing waves, which are energy carrying structures) what was successfully used in supporting N. Bohr's Planetary Atom Model. By analogical extension of Wilson-Sommerfeld action integrals to all "**CHARGE (=) Q**" elements found in T.5.3 and (5.2.1), we can formulate the following quantifying expressions (again between mutually conjugate variables) that are also in agreement with "Periodicity relations, or de Broglie Wave Intervals" from T.5.3, presenting important metrics of different elementary energy-momentum states:

<u>Metrics of Elementary Particles:</u>

$$\left\{\begin{array}{l} 2\left|\Delta q_{mag.}\cdot\Delta q_{el.}\right|_{min.} = 2\left|\Delta\alpha\cdot\Delta L\right|_{min.} = 2h\cdot\left|\Delta t\cdot\Delta f\right|_{min} = 2\left|\Delta x\cdot\Delta p\right|_{min.} = 2\left|\Delta t\cdot\Delta\tilde{E}\right|_{min.} = \\ = 2c^2\left|\Delta t\cdot\Delta m\right|_{min.} = 2\left|\Delta s_1\cdot\Delta s_2\right|_{min.} = h, \\ \left\{\lambda = \frac{h}{p} = \overleftrightarrow{\lambda},\ \tilde{E} = hf = h\frac{1}{\overleftrightarrow{T}} \Leftrightarrow \overleftrightarrow{T} = \frac{h}{\tilde{E}}\right\} \Rightarrow \left\{\overleftrightarrow{\theta} = \frac{h}{L},\ \overleftrightarrow{\Phi}_{el.} = \frac{h}{\Phi_{mag.}},\ \overleftrightarrow{\Phi}_{mag.} = \frac{h}{\Phi_{el.}}, \overleftrightarrow{s}_1 = \frac{h}{s_2}\right\}, \\ [X] = \left[q_{mag.}, q_{el.}, x, \alpha\right],\ [Q] = \left[q_{el.}, q_{mag.}, p, L\right],\ \Phi_{el.} = q_{el.},\ \Phi_{mag.} = q_{mag.}. \end{array}\right\} \Leftrightarrow$$

$$\Leftrightarrow \begin{Bmatrix} \vec{\Phi}_{mag.} \cdot \Phi_{el.} = \vec{\Phi}_{el.} \cdot \Phi_{mag.} = \vec{\lambda} \cdot p = \vec{\theta} \cdot L = \vec{T} \cdot \tilde{E} = ... = \vec{s}_1 \cdot s_2 = h, \\ (\vec{\Phi}_{mag.} \cdot \Phi_{el.})_1 + (\vec{\lambda} \cdot p)_2 + (\vec{\theta} \cdot L)_3 + (\vec{T} \cdot \tilde{E})_4 + ... + (\vec{s}_1 \cdot s_2)_m = mh, \ m = 1,2,3... \end{Bmatrix} \Rightarrow$$

$$\Rightarrow [\vec{X}][Q] = [h] \Rightarrow$$

Wilson-Sommerfeld action integrals

$$\begin{Bmatrix} \oint_{C_n} p_\lambda d\lambda = n_\lambda h, \ \oint_{C_n} L_\theta d\theta = n_\theta h, \\ (n_\lambda, n_\theta) = \text{integers } (=1,2,3,...) \end{Bmatrix} \wedge \begin{Bmatrix} \oint_{C_n} \Phi_{el.} d\Phi_{magn.} = n_{el.} h, \ \oint_{C_n} \Phi_{mag.} d\Phi_{el.} = n_{mag.} h, \\ \oint_{C_n} \tilde{E}_n dt = nh, \ (n_{el.}, n_{mag.}, n) = \text{integers} \end{Bmatrix} \Rightarrow$$

$$\Rightarrow \begin{cases} \oint_{C_n} [X] d[Q]^T = \oint_{C_n} [Q] d[X]^T = \oint_{C_n} \tilde{E} dt = n \cdot [X] \cdot [Q]^T = [n] \cdot [Q] \cdot [X]^T = [n]h, \ n = 1,2,3,... \\ [X]^T = [X(t)]^T = \begin{bmatrix} q_{mag.} \\ q_{el.} \\ x \\ \alpha \end{bmatrix}, [Q]^T = [Q(t)]^T = \begin{bmatrix} q_{el.} \\ q_{mag.} \\ p \\ L \end{bmatrix}, [n] = \begin{bmatrix} n_{el.} \\ n_{mag.} \\ n_\lambda \\ n_\theta \end{bmatrix} \end{cases} \quad (5.4.1)$$

What we could additionally conclude from T.5.3 and Wilson-Sommerfeld action integrals (5.4.1) is that Uncertainty relations (recognized by using the symbol "≤") in the form (5.1) - (5.2), have certain mathematical power only for cases of relatively unbounded, open-space and sufficiently freely propagating and spatial-temporal evolving wave motions. On the contrary, stable, and self-closed spatial structures, (5.3) - (5.4), like stationary and resonant energy states, and standing waves formations inside atoms and elementary particles (where rest masses could be involved), effectively and "in average" alter this form of Uncertainty, *making it much more certain than uncertain. Something remarkably similar can be shown valid for closed-paths periodical motions of planetary or solar systems like elaborated in the second chapter (except that Planck-like constant will be something else, H >>h). In other words, instead of "≤", or "≥" in cases related to localized and stable energy states, like atoms and certain elementary particles are, we should have an equality sign "=", since only an integer number of half or full wavelengths of any relevant matter-wave entity that creates standing waves there (as in (5.2.1)), could exist in such structures with number of periodicities (see T.5.4).* ***The minimal and basic set of such elementary building entities of our universe is captured by (5.4.1).*** *Quantization in Physics or in our Universe is related only to such self-closed, stabilized standing-waves states or systems (but there are also many other transient and not quantized, evolving states of matter). Quantum World and Quantum Physics are dealing with optimal packing, interactions, and energy-momentum exchanges between here described elementary and resonantly self-closed building blocks of matter. Later in this chapter (see (5.14-1)), we will formulate more general conditions for signal discretization or atomization, explaining the meaning of elementary matter-waves domains as being energy finite Gaussian-Gabor signals, optimally concentrated in all mutually conjugate domains. See more supporting information in the Appendix of this book, Chapter 10. PARTICLES AND SELF-CLOSED STANDING MATTER WAVES.*

In contemporary physics and electromagnetism, the mathematical descriptions of the unity and interactions between electric and magnetic charges and fields are more refined than the analogous relationships between linear and rotational motions in mechanics. To achieve the same level of elegance, simplicity, and symmetry seen in the interplay between electric and magnetic fields and charges, the essential definitions and parameters of linear and rotational motions (as outlined in T.5.4) may need to be reformulated or slightly modified.

It is also worth revisiting the often-overlooked works of the early pioneers of electromagnetism, particularly those who preceded Maxwell. For instance, Ampère's force law between electric-current elements and Wilhelm Eduard Weber's force law *between electric charges demonstrated a conceptual continuity between electromagnetism and Newtonian mechanics, a continuity that seems diminished in today's emphasis on Maxwell's field equations, Lorentz transformations, relativity, and orthodox quantum theory.*

By approaching these historical concepts with open-minded and intellectually flexible thinking, and by remodeling and updating the ideas of Ampère, Weber, and others (such as Faraday, Lenz, Biot, and Savart), we could establish a stronger unity between mechanics and electrodynamics than is currently practiced. This approach would also allow us to identify and address the weaknesses in relations between Maxwell's equations, Lorentz transformations, relativity, and quantum theory.

To extend mentioned analogies and symmetry between electromagnetic and mechanical motions, let us summarize the already introduced concept of wavelength analogies and symmetries between mechanical and electromagnetic characteristics of matter waves (T.4.2 and T.5.3), by creating table T.5.4. The terminology in T.5.4 is in some cases slightly modified (compared to what we could find in Physics literature) to additionally expose previously mentioned analogies.

T.5.4. Wavelength analogies in different frameworks

Matter Wave Analogies	Linear Motion	Rotation	Electric Field	Magnetic Field
Characteristic Charge	Linear Momentum $\mathbf{p}$	Orbital Momentum $\mathbf{L} = \mathbf{pR}$	Electric Charge $\mathbf{q_e = q}$	"Magnetic Charge" $\mathbf{q_m = \Phi}$
Matter Wave *Periodicity*	*Linear Path Periodicity* $\boldsymbol{\lambda = \frac{h}{p}}$ (Linear motion Wavelength)	*Angular Motion Periodicity* $\theta = \frac{h}{L}$ (Angular motion Wavelength)	*"Electric Periodicity"* $\lambda_e = \frac{h}{q_e} = q_m$	*"Magnetic Periodicity"* $\lambda_m = \frac{h}{q_m} = q_e$
Standing Waves on a circular self-closed path	$n\lambda = 2\pi R$ $p = n\frac{h}{2\pi}\cdot\frac{1}{R}$	$n\theta = 2\pi$ $L = n\frac{h}{2\pi}$	$\lambda_e\lambda_m = q_e q_m = h$	
	$\theta L = \lambda p = h$, $\theta = \frac{\lambda}{R} = \frac{2\pi}{n}$			

(Periodicity – here adopted the term for unifying all de Broglie wavelengths, $\mathbf{q_m = \Phi}$ is not a free and independent, self-standing magnetic charge)?

The next unifying, though still challenging, step in this process is to demonstrate the intrinsic coupling between magnetic phenomena and rotation, as well as between familiar electric events and linear motion. This can be explored both macroscopically and microscopically, starting with the well-known relationships between orbital and magnetic moments in atomic structures, such as those involving electrons and protons.

Understanding the structures of elementary particles, based on concepts outlined in sections (5.2.1) and (5.4.1), as well as the wavelength analogies in T.5.3 and T.5.4, can provide a clear and straightforward picture of the fundamental structure of our *universe. An analogous concept regarding the structural stability of standing waves (referenced in equations (2.11.10) to (2.11.20) in the second chapter) can also be applied to the solar and galactic systems within our universe, grounded in generalized orbital and spin conservation laws. For planetary and some galactic systems, we might consider an analogous macro-world Planck-like constant, denoted as H, which is significantly larger than the micro-world Planck constant h. This macro constant would be applicable whenever we encounter stable orbiting and spinning planetary and galactic systems with periodic motion elements, potentially hosting standing-wave formations that conserve total angular momentum.*

In other words, the successful application of Planck's constant h, or a specific macrocosmic constant H, to model processes or describe certain interactions directly indicates that we are dealing with spatially closed and resonant objects, particles, fields, and wave formations where matter-waves create standing waves, much like those supported by Wilson-Sommerfeld integrals. To better understand why and how the same Uncertainty Relations apply to both micro and macro systems, we should recall that the real mass-energy-momentum boundaries of a macro-object differ significantly from the strict spatial or geometric limits of its static (or rest) mass. Planck's constant h is primarily associated with photons, interatomic entities with numerous periodicities, and the corresponding (essentially electromagnetic) matter-waves. For macroparticles, solar systems, and astronomical configurations, we should analogically apply different and much larger Planck-like constants (see the second chapter on Gravitation). This approach helps explain why Uncertainty Relations can be extended to the macro masses.

Stable, self-contained standing matter-wave systems also represent inertial states, suggesting that the Newtonian definition of inertia should be appropriately extended. Multilevel and structurally complex standing-wave quantization, as presented in T.5.4 and sections (5.2.1) to (5.4.1), can be analogically compared to Finite Element Analysis results when searching for modal and natural resonant (or standing wave) states in solid bodies. Macroparticles and solid objects typically exhibit numerous resonant frequencies and harmonics, representing various standing-wave states, and possess frequency zones devoid of resonant properties. However, as we delve deeper into smaller spatial matter domains, down to the atomic level and within atoms, we once again encounter self-stabilized, self-contained, standing-wave, resonant or oscillatory structures with intrinsic periodicities.

....

*We usually think about time domain vibrations as something what has temporal periodicity and produces vibrational or wave energy effects (like audio signals and music). On the same way (or analogically), different objects or static, **geometry related solid shapes with spatial periodicities (like crystalline structures), when on some way agitated, can produce dynamic effects**, affecting surrounding masses (by radiating waves with linear and angular moments; -see more in Chapter 10.).* To demonstrate such unusual behaviors of geometry-related periodicities, let us start with a certain finite-energy wave packet, which has the following wave function:

$$\begin{bmatrix} \Psi(t) \to A(\omega) \\ \Psi(x) \to A(k) \end{bmatrix} \Rightarrow \begin{bmatrix} \Psi(x,t) \to A(k,\omega), \\ \varphi(=)\text{phase}(=)\,\omega t \pm kx \end{bmatrix} \Rightarrow \overline{\Psi}(x,t) = \left|\overline{\Psi}\right| \cdot e^{I\varphi}$$

Extended, de Broglie, or matter-wave properties and wave-particle-duality relations of such wave-packet are (see more in Chapter 10):

$$\left.\begin{array}{l} \omega=\omega_t=2\pi f_t=2\pi f=k_t=\left|\frac{\partial\varphi}{\partial t}\right|=\frac{2\pi}{T}=2\pi\frac{\tilde{E}}{h}, \\ k=k_x=\omega_x=2\pi f_x=\left|\frac{\partial\varphi}{\partial x}\right|=\frac{2\pi}{\lambda}=2\pi\frac{p}{h} \\ k_\alpha=\omega_\alpha=2\pi f_\alpha=\left|\frac{\partial\varphi}{\partial\alpha}\right|=\frac{2\pi}{\theta}=2\pi\frac{L}{h} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} u=\frac{\omega}{k}=\frac{f_t}{f_x}=\frac{\lambda}{T}=\lambda f=\frac{\tilde{E}}{p}=\frac{\partial x}{\partial t}(=) \\ \text{phase velocity} \\ v=\frac{d\omega}{dk}=u-\lambda\frac{du}{d\lambda}=-\lambda^2\frac{df}{d\lambda}=\frac{d\tilde{E}}{dp}=\frac{dx}{dt}(=) \\ \text{group velocity} \end{array}\right\}$$

Spatial, temporal, and angular periodicity or periods of certain energy-moments characterized structure, and their mutual relations are (see more in Chapter 10, under PWDC):

$$\begin{bmatrix} \lambda=\frac{h}{p}=\frac{2\pi}{k_x} \\ p=\frac{h}{2\pi}k_x=hf_x=\frac{h}{\lambda} \\ \Delta p=\frac{h}{2\pi}\Delta k=h\Delta f_x=\frac{h}{2\Delta x} \\ (\Delta x\cdot\Delta p)_{min.}=h/2 \end{bmatrix}, \begin{bmatrix} T=\frac{h}{\tilde{E}}=\frac{2\pi}{k_t} \\ \tilde{E}=\frac{h}{2\pi}\omega_t=hf_t=\frac{h}{T} \\ \Delta\tilde{E}=\frac{h}{2\pi}\Delta\omega=h\Delta f_t=\frac{h}{2\Delta t} \\ (\Delta t\cdot\Delta\tilde{E})_{min.}=h/2 \end{bmatrix}, \begin{bmatrix} \theta=\frac{h}{L}=\frac{2\pi}{k_\alpha} \\ L=\frac{h}{2\pi}k_\alpha=hf_\alpha=\frac{h}{\theta} \\ \Delta L=\frac{h}{2\pi}\Delta k_\alpha=h\Delta f_\alpha=\frac{h}{2\Delta\alpha} \\ (\Delta\alpha\cdot\Delta L)_{min.}=h/2 \end{bmatrix}$$

Certainty Relations between relevant, dimensional durations or Gaussian wave packet lengths (when wavefunction is well localized and limited in all its domains) are:

$$\begin{aligned} &(\Delta t\cdot\Delta\tilde{E})_{min.}=h\cdot\left(\Delta t\cdot\frac{\Delta\omega_t}{2\pi}\right)_{min.}=h\cdot(\Delta t\cdot\Delta f_t)_{min.}=(\Delta x\cdot\Delta p)_{min.}=h\left(\Delta x\cdot\frac{\Delta k_x}{2\pi}\right)_{min.}=h(\Delta x\cdot\Delta f_x)_{min.}= \\ &=(\Delta\alpha\cdot\Delta L)_{min.}=h\left(\Delta\alpha\cdot\frac{\Delta k_\alpha}{2\pi}\right)_{min.}=h(\Delta\alpha\cdot\Delta f_\alpha)_{min.}=(\Delta q_{mag.}\cdot\Delta q_{el.})_{min.}=c^2(\Delta t\cdot\Delta m)_{min.}=h/2 \\ &(\Delta t\cdot\Delta\omega_t)_{min.}=(\Delta x\cdot\Delta k_x)_{min.}=(\Delta\alpha\cdot\Delta k_\alpha)_{min.}=\pi,\ (\Delta t\cdot\Delta f_t)_{min.}=(\Delta x\cdot\Delta f_x)_{min.}=(\Delta\alpha\cdot\Delta f_\alpha)_{min.}=1/2 \end{aligned}$$

Consequently, spatial, and temporal periodicities of certain energy-moments state or object are (potentially and still hypothetically) producing mutually related dynamic and motional effects, such as:

$$\left\{\begin{bmatrix} \begin{pmatrix} \text{spatial periodicity} \\ \text{linear cristalline structures} \end{pmatrix} \\ f_x=\frac{1}{\lambda}=\frac{p}{h}=\frac{k_x}{2\pi}=\frac{\omega_x}{2\pi} \\ \text{generating linear momentum} \\ p=hf_x, \Delta p=h\Delta f_x \end{bmatrix}, \begin{bmatrix} \begin{pmatrix} \text{temporal periodicity} \\ \text{oscillatory circuits} \end{pmatrix} \\ f_t=\frac{1}{T}=\frac{\tilde{E}}{h}=\frac{k_t}{2\pi}=\frac{\omega_t}{2\pi} \\ \text{generating wave energy} \\ \tilde{E}=hf_t, \Delta\tilde{E}=h\Delta f_t \end{bmatrix}, \begin{bmatrix} \begin{pmatrix} \text{angular periodicity} \\ \text{angular cristalline structures} \end{pmatrix} \\ f_\alpha=\frac{1}{\theta}=\frac{L}{h}=\frac{k_\alpha}{2\pi}=\frac{\omega_\alpha}{2\pi} \\ \text{generating angular momentum} \\ L=hf_\alpha, \Delta L=h\Delta f_\alpha \end{bmatrix}\right\}$$

If our Universe is globally and structurally oscillating, rotating and performing number of relative motions, generating and receiving electromagnetic, radiant and heat energy (as Nikola Tesla conceptualized within his Dynamic Gravity theory), then stable and solid structures, for instance like big solid masses of granite and other crystalline materials in different geometric shapes (or like big pyramids, rocky mountains and enormous granite sarcophaguses), could be affected by mentioned (radiant energy) perturbations, start resonating on different modal and natural and Schuman resonant frequencies and producing specific radiations, streaming of fluids, acoustic and electromagnetic waves emissions, explicable based on Certainty and Uncertainty relations between different durations or lengths of relevant original and spectral domains of mentioned objects. Big masses are simply presenting resonant boxes, big receivers and emitters of matter waves (producing and receiving radiant energy, dominantly in a very low frequency range). *Also, (judging by geometry or shape) other complex-geometric objects could serve as resonators and specific frequency, amplitude and phase modulators of surrounding and natural matter waves, this way explaining unusual or strange electromagnetic and acoustic effects measurable inside and around crystalline structures considered as big stone containers, boxes, sarcophaguses, pyramids, and pyramidal Rocky Mountains.*

For instance, a certain big mass composed of hard stones, granite and minerals has countless small crystals (on a molecular level). Such crystals (presumably oscillating because of different environmental and thermal excitations) have certain spatial periodicity with a mean spatial frequency, $f_x = \frac{1}{\lambda} = \frac{p}{h} = \frac{k_x}{2\pi} = \frac{\omega_x}{2\pi}$. Every (excited and oscillating) spatial periodicity structure is also generating (oscillating) linear moments, $p = hf_x, \Delta p = h\Delta f_x$, and such linear moments should produce certain streaming and wave-like manifestations within surrounding fluids, masses and on electromagnetically charged particles. Such interesting and unexpected effects are measurable and already detected inside and around big pyramids. Based on spontaneous linear moments generating, and based on surrounding spatial periodicity, internal crystalline and atomic resonators could be exited and start producing resonant oscillations with certain mean frequency of temporal periodicity $f_t = \frac{1}{T} = \frac{\tilde{E}}{h} = \frac{k_t}{2\pi} = \frac{\omega_t}{2\pi}$, this way creating matter waves, having mean energy $\tilde{E} = hf_t, \Delta\tilde{E} = h\Delta f_t$. If the same pyramid (or stone object) internally has another type of spatial and torsional or angular crystalline structure, this could generate an oscillating angular periodicity $f_\alpha = \frac{1}{\theta} = \frac{L}{h} = \frac{k_\alpha}{2\pi} = \frac{\omega_\alpha}{2\pi}$ that would radiate spinning and spiraling matter-waves and particles with angular moments $L = hf_\alpha, \Delta L = h\Delta f_\alpha$.

....

As we can find in [36], Anthony D. Osborne, & N. Vivian Pope, effectively and analogically (could be also unintentionally), made a significant extension of Wilson-Sommerfeld standing-waves concept (5.4.1) to the macro-world of planets, stars, and galaxies, but apparently, here we are starting to realize that there is a lot more to do around similar temporal and spatial periodicity-related concepts. Before addressing the Universe and galaxies, let us first analyze the stability conditions of micro-world of atoms and elementary particles, as follows.

A) Because we already know that all elementary particles (and quasiparticles) have certain intrinsic and quantized spin and/or orbital moments (in units $\frac{h}{2\pi} = \hbar$), this clearly tells us that kind of stable, periodical rotation and/or spinning is present there. Every elementary particle exists as a measurable entity (meaning that it can be localized with a certain precision in specific time and space). *The spatial domain of such elementary particle-wave object is hosting an effective spinning-mass* $\boldsymbol{m}$ *(or rotating matter wave with an energy content of* $\boldsymbol{mc^2}$*), which could be geometrically conceptualized as a thin and narrow toroid or ring, having spatially or radially distributed mass* $\boldsymbol{m}$*. The condition, which makes such conceptualization defendable, is that when the average radius of rotation* $\boldsymbol{R}$ *is much bigger than any other rotating particle or rotating ring dimensions, a moment of inertia of such object is in all cases approximately equal to* $J = mR^2$. Angular periodicity belonging to such an object, that is analog to (here invented) angular matter wavelength (see T.5.4), should be $\theta = \frac{h}{L}$. Since we are describing a spatially localized and relatively stable object on the way that only integer number of such angular periodicity sectors captures the entire particle domain, $n\theta = 2\pi = n\frac{h}{L}$, the orbital moment of the intrinsic rotation in question would be

$$L = n\frac{h}{2\pi} = n\hbar, \ \mathbf{n = 1, 2, 3} \ldots$$

B) Now, the idea about self-closed, orbiting circular zone (of an elementary particle structure) is already getting stronger legitimacy, and we could (mathematically) imagine the same rotating object (or an elementary particle) as being an equivalent rotating mass or wave group that revolves around its center of mass. Such rotating mass on its closed-line orbit performs a kind of linear motion, which should be presentable as a matter wave that has its de Broglie matter wavelength equal to $\lambda = \frac{h}{p}$.
Since we are describing the same rotating object as before, only an integer number of such wavelengths should cower the circular zone perimeter (like in Wilson-Sommerfeld standing-waves concept (5.4.1)),

$$n\lambda = n\frac{h}{p} = 2\pi R \Leftrightarrow p = n\frac{h}{2\pi}\cdot\frac{1}{R} = n\hbar\cdot\frac{1}{R},\ \mathbf{L} = pR = \mathbf{J}\omega = \mathbf{J}\frac{v}{R}\ . \quad (5.4.2)$$

The "rotating motional energy" associated to an orbital moment $\mathbf{L} = \mathbf{n}\hbar$, should be equal to a particle linear-motion energy (on a self-closed circular path, where an equivalent mass content moves with its tangential velocity $v = \omega R$), which is associated to linear particle momentum $p = n\hbar\cdot\frac{1}{R}$. Here, we are describing two mathematical aspects of the same motion, belonging to the same, stable and spatially localized object. Such motional energy equivalence means that the following identities are automatically satisfied (when for linear and orbital moments we take quantized values, $\mathbf{p} = \mathbf{n}\hbar\cdot\frac{\mathbf{1}}{\mathbf{R}}$, $\mathbf{L} = \mathbf{n}\hbar$):

$$\left\{pv = \mathbf{L}\omega, \frac{mv^2}{2} = \frac{\mathbf{J}\omega^2}{2}, vdp = \omega d\mathbf{L}, \frac{dp}{p} = \frac{dL}{\mathbf{L}}\right\} \text{ and } \left\{p = n\hbar\cdot\frac{1}{R}\ ,\ \mathbf{L} = n\hbar\right\} \quad (5.4.3)$$
$$\Leftrightarrow \{n\hbar\cdot\frac{v}{R} = n\ \hbar\omega,\ v = \omega R\ , \theta\mathbf{L} = \lambda p = h,\ \theta = \frac{\lambda}{R} = \frac{2\pi}{n}\},$$

what is obviously and indicatively correct.

C) The conceptual picture regarding the basic structure of elementary particles, as just presented, is oversimplified, but being clear and elegant. Most probably mentioned elementary and rotating or spinning matter-wave domains are internally composed of electromagnetic waves in a specific form of stationery and standing waves structures, since most elementary particles have their magnetic moments, and most of them are in some ways being sensitive to external electric and magnetic fields. What is characteristic of many elementary particles is that they have stable gyromagnetic ratios, meaning that both magnetic and orbital moments in question are causally and strongly coupled, being mutually proportional and coincidently present (effectively having the same origin). Here, we also come closer to understanding how nature creates elementary matter domains with rest masses, using (electromagnetic) waves and fields as a primary building substance. Also, spin numbers of bosons and fermions could be addressed inside the same picture given here, since involved quantizing integers regarding linear and orbital motion-periodicity can be mutually different, as for instance, $p = n\hbar\cdot\frac{1}{R}$, $\mathbf{L} = m\cdot\hbar$, $n, m \in [1,2,3...]$. From the nature of here described self-closed circular zone (captured by an elementary particle), it is clear that in some cases integer $\mathbf{m} = \mathbf{n}$, and in other examples could be m = **n/2**, or m = **n/2k,** $n,m,k \in [1,2,3...]$ this way respecting $n\hbar \cdot \frac{v}{R} = m\hbar\omega,\ v = \frac{m}{n}\omega R = \omega R$ ... (see also Chapter 4.1, Fig.4.1, and equations under (4.3)).

In any case, specific and relatively stable elementary domain, or its equivalent wave-packet (or corresponding moving particle) will always have its minimal and elementary building blocks (as standing waves formations) mutually related as found in (5.4),

$$2\left|\Delta q_{mag.} \cdot \Delta q_{el.}\right|_{min.} = 2\left|\Delta\alpha \cdot \Delta L\right|_{min.} = 2h \cdot \left|\Delta t \cdot \Delta f\right|_{min} = 2\left|\Delta x \cdot \Delta p\right|_{min.} = 2\left|\Delta t \cdot \Delta\tilde{E}\right|_{min.} =$$
$$= 2c^2\left|\Delta t \cdot \Delta m\right|_{min.} = 2\left|\Delta s_1 \cdot \Delta s_2\right|_{min.} = h.$$

D) Also, based on (5.2), and later elaborations (from (5.2.1) until (5.4.1)) it is clear that everything that could be relatively stable, space-time isolated, or qualified as an energy-finite particle, or an equivalent wave packet, should have in its structure coupled elements of linear and rotational motions ($\Delta x, \Delta p$ and $\Delta\alpha, \Delta L$). This is also equivalent to coupling of mutually dependent electrical and magnetic charges ($\Delta q_{mag.}, \Delta q_{el.}$), equal to the coupling of (associated) elementary spatial and temporal blocs ($\Delta x, \Delta t$), and causally related to the meaning of energy, linear and angular forces, mass, and time, since,

$$\Delta E = \frac{\Delta q_{mag.} \cdot \Delta q_{el.}}{\Delta t} = \frac{\Delta\alpha \cdot \Delta L}{\Delta t} = \frac{\Delta x \cdot \Delta p}{\Delta t} = \overline{v} \cdot \Delta p = F\Delta x = \tau\Delta\alpha = h\Delta f = c^2\Delta m = P\Delta t \geq \frac{h}{2\Delta t}.$$

Sometimes, like in the case of Gravitation, we do not see explicitly where the vital place of electric and magnetic charges is, but it should indeed exist (within an internal structure of interacting participants or masses). Regarding Gravitation, presently we are wrongly considering masses as the only relevant gravitational charges (or sources), but based on (5.2) – (5.2.2), we see that every mass is composed of coupled electric and magnetic charges, dipoles, multipoles, or combined linear and spinning motional states, including appearance of liner and angular (or torsional) forces F and τ (see also (5.4-0) in this chapter, and (2.4-4.6) – (2.4-4.8), (2.4-4.11) in Chapter 2, about gravitation),

$$\Delta m = \frac{\Delta q_{mag.} \cdot \Delta q_{el.}}{c^2\Delta t} = \frac{\Delta q_{mag.}}{c^2} i_{el.} = \frac{\Delta\alpha \cdot \Delta L}{c^2\Delta t} = \frac{\Delta x \cdot \Delta p}{c^2\Delta t} = \boxed{\tau\frac{\Delta\alpha}{c^2} = F\frac{\Delta x}{c^2}} =$$
$$= \frac{\overline{v} \cdot \Delta p}{c^2} = \frac{H\Delta f}{c^2} = \frac{P\Delta t}{c^2} = \frac{\Delta E}{c^2} \geq \frac{H}{2c^2\Delta t}. \quad (5.2.2)$$

Consequently, we are close to the conclusion that in the background of Gravitation we should experience an electromagnetic attraction, because even electrically and magnetically neutral, moving macro masses (when being in a mutual vicinity) have a tendency for creating internal electric and magnetic dipoles or polarizations, and linear and angular force elements (because of having linear and angular accelerated motions). Here it is beneficial to exploit an analogical mass-energy conceptualization, if we treat a photon in a usual and well-proven way, as known in cases of analyses of Compton effect, Photoelectric effect, electron-positron creation, or annihilation, etc. (see in more details in

Chapter 4.1. under "4.1.1.1. Photons and Particle-Wave Dualism"). Photon is known to have an equivalent mass that is equal to $m_{ph.} = \frac{\Delta E}{c^2} = \frac{\tilde{E}}{c^2} = \frac{hf}{c^2}$, and we also know that big masses are making gravitational bending of photon beams (meaning attracting photons). It is also known for a photon to be an electromagnetic wave with alternating and mutually coupled (also mutually orthogonal) electric and magnetic field vectors, what could be coupling between specific electric and magnetic dynamic charges, $m_{ph.} = \frac{\Delta q_{mag.} \cdot \Delta q_{el.}}{c^2 \Delta t}$. It is also commonly accepted that we can associate an orbital or spinning moment with a photon equal to $L_{ph.} = \frac{h}{2\pi} \Rightarrow m_{ph.} = \frac{\Delta\alpha \cdot \Delta L}{c^2 \Delta t} = \frac{\omega \cdot L_{ph.}}{c^2} = \frac{hf}{c^2}$. As we can see, all equivalent qualifications and quantifications of a photon are again confirming (by analogy) that every finite and space-time localized mass or particle, or energy state, should be in its natural structure kind of coupling of mutually-conjugate states, like we see in (5.2) – (5.2.2).

Another essential meaning of Uncertainty-relations presented here is that we are also this way defining mutual relations, sizes, limits, and dimensions of elementary matter-building blocks, and that specific time interval (or temporal duration) is involved in all of them. For instance, for a photon, relevant time interval, period, or duration will be,

$$\left(m_{ph.} = \frac{\Delta q_{mag.} \cdot \Delta q_{el.}}{c^2 \Delta t} = \frac{\Delta\alpha \cdot \Delta L}{c^2 \Delta t} \right) \Rightarrow \Delta t = \frac{\Delta q_{mag.} \cdot \Delta q_{el.}}{m_{ph.} c^2} = \frac{\Delta\alpha \cdot \Delta L}{m_{ph.} c^2} = \frac{\Delta q_{mag.} \cdot \Delta q_{el.}}{\tilde{E}_{ph.}} = \frac{\Delta\alpha \cdot \Delta L}{\tilde{E}_{ph.}}. \tag{5.2.3}$$

If we like to get an idea what could happen with such elementary matter blocks in motion, we can simply apply (on (5.2.2)) the known relation between a rest mass and its equivalent mass in motion, as for instance,

$$\gamma \Delta m = \frac{\Delta q_{mag.} \cdot \Delta q_{el.}}{c^2 (\frac{\Delta t}{\gamma})} = \frac{\Delta\alpha \cdot \Delta L}{c^2 (\frac{\Delta t}{\gamma})} = \frac{\Delta x \cdot \Delta p}{c^2 (\frac{\Delta t}{\gamma})} = \gamma \frac{\overline{v} \cdot \Delta p}{c^2} = \frac{h(\gamma \Delta f)}{c^2} = \gamma \frac{\Delta E}{c^2} \geq \frac{h}{2c^2 (\frac{\Delta t}{\gamma})}, \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}. \tag{5.2.4}$$

The conclusion from (5.2.4) is that specific dimensions (sizes, or intervals) of elementary building blocks of matter should also be (mutually) dependent involving Lorentz factor $\gamma = 1/\sqrt{1 - \frac{v^2}{c^2}}$. For instance, if motional (elementary domain) mass is equal to $\gamma \Delta m$, the direct consequence of such mass evolution will be that relevant elementary time domain will evolve to $\frac{\Delta t}{\gamma}$, to keep the validity of (5.2.2). Something similar we should be able to conclude related to Δx and Δp, or to $\Delta\alpha$ and ΔL …

Of course, similar results (regarding actual and proper time) are known in Relativity Theory, but here we are introducing the possibility to extend, correct and revise such insights and practices in a simpler, straightforward, and more generally valid way.

E) We typically discuss matter waves in the context of the de Broglie wavelength, Planck and Einstein's energy of an elementary wave packet, and the relationships between group and phase velocity, as well as time-space dependent, oscillatory signals. When considering time-domain wavefunctions, our intuitive perception is that waves are moving, oscillating entities with specific temporal-spatial periodicity and dynamics. To better understand the time and frequency domains of wavefunctions, we use spectral analysis techniques, such as Fourier integral transformation, analytic signal methods, the Hilbert transform, and the Kotelnikov-Shannon-Nyquist-Whittaker sampling and reconstruction methods. These techniques allow us to generate spectral distributions that depend on amplitude, phase, power, energy, time, and frequency, as initially discussed in Chapter 4.0.

In our everyday experience, time-domain functions appear as something dynamic, alive, or active. In contrast, frequency or spectral domain functions, which are associated with these time-domain functions, seem static and relatively stable. Spectral functions can also be derived from spatially dependent functions, such as those related to standing waves, crystalline structures, and diversely shaped objects. For example, when examining a thin plate with one or several vertical slits, like those used in diffraction experiments to visualize light or particle diffraction, we can use Fourier transformation to produce spectral images of these spatial-domain functions. These images yield amplitude distributions that resemble waveforms, as further explored in Chapter 10 under “10.00 DEEPER MEANING OF PWDC.”

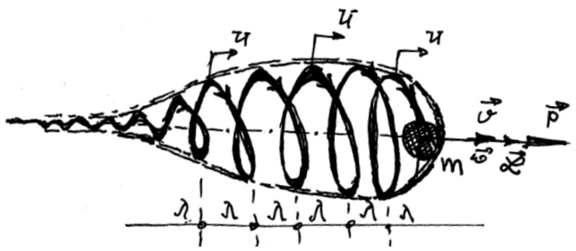
Our tendency not to associate such geometrical and spectral situations with matter waves stems from our biological nature and habitual ways of thinking. However, this is where Uncertainty (or "Certainty") relations, such as those expressed in equations (5.3) and (5.4), provide additional insight into the plausible reality of matter waves. In summary, matter waves (and other types of waves) are not exclusively tied to time-domain wave functions. Alongside time-domain functions, we also have space-domain wave functions, and these are interdependent, as governed by relations (5.3) and (5.4).

$$\left(\Delta q_{mag.} \cdot \Delta q_{el.}\right)_{min.} = \left(\Delta\alpha \cdot \Delta L\right)_{min.} = h \cdot \left(\Delta t \cdot \Delta f\right)_{min.} = \left(\Delta x \cdot \Delta p\right)_{min.} =$$
$$= \left(\Delta t \cdot \Delta\tilde{E}\right)_{min.} = c^2 \left(\Delta t \cdot \Delta m\right)_{min.} = \left(\Delta s_1 \cdot \Delta s_2\right)_{min.} = h/2, \; (\Delta x)_{min.} = \frac{\lambda}{2} = \frac{h}{2p}.$$

This is producing that energy-related spectral function will be temporal-frequency dependent, $\tilde{E} = \tilde{E}(f), \; f = \omega / 2\pi \left(\approx \frac{1}{\Delta t} \right)$, that momentum will be spatial-frequency dependent, $p = p(k), \; k = 2\pi / \lambda = \frac{2\pi}{h} p \left(\approx \frac{\pi}{\Delta x} \right)$, and that the signal or group velocity (of energy propagation) will be space-time-frequency dependent, $v = \frac{\Delta\tilde{E}}{\Delta p} = \frac{\Delta E}{\Delta p} = \frac{\Delta\omega}{\Delta k} = \frac{\Delta x}{\Delta t} = \overline{v}.$

[♣ COMMENTS & FREE-THINKING CORNER:

WHAT UNCERTAINTY RELATIONS REALLY PRESENT*: Let us create an oversimplified model for the visual and intuitive understanding of the relation between real and stable particle (or constant mass) size and its matter-wave size in its spatial, temporal and frequency domains (see the picture below). We could realistically assume that mass in question is (geometrically or spatially) smaller than space (or volume) captured by its* *kinematically equivalent matter-wave packet. We can also consider that if the particle is stable, the spatial length* Δx *of its matter-wave equivalent should be on some way* *<u>standing waves</u> structured and stabilized, meaning that* $\Delta x = n \cdot \frac{\lambda}{2},\ n = 1, 2, 3....$ *Now, we assume that mass in question, m, has specific speed v and linear moment,* $p = mv$*, and that it started to move from the state of relative rest, meaning that mass gained the linear momentum of,* $\Delta p = p - p_0 = mv,\ (p_0 = 0, p = mv)$.



Now we can create the self-explanatory Uncertainty Relations product, without involving any probability or statistics-related thinking,

$$\Delta x \cdot \Delta p = n \cdot \frac{1}{2}\lambda \cdot mv = n \cdot \frac{1}{2}\frac{H}{mv} \cdot mv = n \cdot \frac{H}{2} \geq \frac{H}{2},\ n \geq 1$$

$$\left(\tilde{E} = E_k = Hf = \frac{pv}{1 + \sqrt{1 - \frac{v^2}{c^2}}},\ \lambda = \frac{H}{p} = \frac{H}{mv},\ u = \lambda v, v = u - \lambda \frac{du}{d\lambda} \right).$$

Let us observe mass m in linear motion from its initial position (when x = 0) arriving after certain time Δt *to its new location, passing the length interval equal to* Δx *, which is precisely equal to the effective total length of the same matter wave packet* Δx *that belongs to mass m.*

Under such conditions, we could say that a particle with mass m has an average linear speed equal to, $v = \frac{\Delta x}{\Delta t}$*, where* Δt *is its temporal duration. We also know that, both in classical and relativistic mechanics, the differential of kinetic (or total) energy is equal to* $\left(d\tilde{E} = dE_k = vdp = Hdf\right) \Leftrightarrow \left(\Delta\tilde{E} = \Delta E_k = v\Delta p = H\Delta f\right)$*, meaning that the same mass m has its total motional energy content equal to* $\Delta\tilde{E} = \Delta E_k$ *, and it captures certain frequency interval equal to* Δf*. We already know, or we can again verify here, that the product between motional energy content and corresponding temporal duration (of the same matter wave packet) is,*

$$\Delta E \cdot \Delta t = v\Delta p \cdot \Delta t = \frac{\Delta x}{\Delta t}\Delta p \cdot \Delta t = \Delta x \cdot \Delta p \geq \frac{H}{2},\ (\Delta E = \Delta\tilde{E} = \Delta E_k = v\Delta p = H\Delta f).$$

Now, we can just rewrite the same relations as,

$$\frac{\Delta x \cdot \Delta p}{\Delta t \cdot \Delta E} = v\frac{\Delta p}{\Delta E} = v\frac{\Delta p}{H\Delta f} = 1.$$

In cases when our matter-wave packet is a photon, we will have, $v\frac{\Delta p}{\Delta E} \to c\frac{\Delta p}{\Delta E} = c\frac{\Delta\tilde{p}}{\Delta\tilde{E}}$,

$$\left(c\frac{\Delta p}{\Delta E} = c\frac{\Delta p}{h\Delta f} = 1\right) \Leftrightarrow \left(c\frac{dp}{dE} = c\frac{dp}{hdf} = 1\right).$$

Also, we can extend the same relationship using relativistic energy form $dE = c^2 dm$ *, getting already well-known relations,*

$$\left(v\frac{dp}{dE} = v\frac{dp}{hdf} = v\frac{dp}{c^2 dm} = 1 \right) \Leftrightarrow \left(dE = c^2 dm = vdp = hdf \right).$$

Now, we can realize what the real and essential meaning of Uncertainty relations is explained entirely without probabilities and statistics related items. We operated only with total signal (or wave packet) durations, or lengths in different spectral and original domains, and we assumed that the wave packet in question creates standing waves where $\Delta x = n \cdot \frac{\lambda}{2}, \; n = 1, 2, 3...$ *Similar situations are extendable on the same way to standing waves formatting of* $\Delta E, \Delta p, \Delta f$ *.*

By conceptualizing in this way, we can see that Uncertainty Relations are equally applicable to both the micro and macro worlds of physics, without any inherent limitations. This means that statistical interpretations are not necessarily required, either ontologically or mathematically. However, the statistical approach remains valid and useful whenever the appropriate conditions for its application are met.

In classical or macro-world mechanics, we often have the false impression that we can fully and precisely define all mass parameters and properties, such as position, momentum, and kinetic energy, without considering Uncertainty Relations. This misconception arises because spatial macro mass geometry, position, volume, and domain are not addressed in the same way as they are in micro-world physics.

To apply Uncertainty Relations in macro-mechanics, we need to consider the relevant spatial-temporal dimensions, matter waves (or wavefunctions), and kinematic parameters in a manner like their treatment in microphysics. For more details, see Chapters 4.0, 4.3, and 10. ♣]

The real background of diffraction and interference patterns observed through single or multiple vertical slits in experiments with photons, electrons, and other microparticles should not be mysterious. The patterns created by vertical slits, defined by their spatial geometry, have specific spatial-frequency-dependent spectral distributions. These distributions channels and guide the diffracting particles, as the entire diffracting system, both before and after the diffraction, can be considered at least a two-body coupled system where temporal and spatial dimensions are interconnected. While probability and distribution explanations of matter waves are useful and applicable in statistical contexts, they are not always entirely accurate or universally valid as ontological grounds.

Additionally, we should consider how to apply Uncertainty Relations to hypothetical events such as the "Big Bang," which is theorized to have occurred within an extremely short time interval, implying an exceptionally large spectral domain. Similarly, hypothetical concepts such as "Dark Matter," "Dark Mass," and "Dark Energy" should adhere to Uncertainty Relations (5.2.2) - (5.2.4). We should avoid inventing virtual entities or labels to unknown entities, merely to reconcile with Relativity theory without satisfying well-established mathematical principles.

*[♣ **COMMENTS & FREE-THINKING CORNER**:*

Quantizing and Kotelnikov-Shannon-Whittaker-Nyquist Sampling Theorem

Here we will show mathematically that signal (or wave function) quantizing and standing waves structural packing are mutually related and important when creating particles and other matter structures. (see more in chapter 10. of this book).

Let us take a time and frequency band-limited, duration-limited, energy finite, signal $\psi(t)$, *where* T *(=) total time duration of the signal* $\psi(t)$, *and* $F = F_{max.}$ *(=) total frequency spectrum duration (or width) of* $\psi(t)$ *in its frequency domain* $A(\omega)$. *If we cannot say that* $\psi(t)$ *is time and frequency durations limited in both domains (in* $\psi(t)$ *and in* $A(\omega)$*), we could just say (for instance) that **T** and **F** are signal durations where 99% or 99.99%... of the signal energy is captured (to be sure that approximately we took almost the whole object or wave packet). Of course, the meaning of signal energy here is closely related to the application of Parseval's theorem (the same energy should be considered both in time and frequency domains). From Uncertainty relations (between total time and frequency durations **T** and **F** of the same signal) we also know that it is generally valid* $TF \geq \frac{1}{2}$.

A band-limited signal $\psi(t)$ *with bandwidth* $\Omega_{max} = 2\pi F_{max.}$ *is entirely determined by the countable set of samples* $\psi(n \cdot \Delta t)$ *of the signal* $\psi(t)$ *if the time sampling interval satisfies* $\Delta t \leq \pi/\Omega_{max}$. *Furthermore,* $\psi(t)$ *may be obtained from these values using the following sampling and reconstruction relations:*

$$\psi(t) = \sum_{n=-\infty}^{\infty} \psi(n \cdot \Delta t) \cdot \frac{\sin\left(\frac{\omega_s (t - n\Delta t)}{2}\right)}{\frac{\omega_s (t - n\Delta t)}{2}} = \sum_{k=-\infty}^{\infty} \psi(n \cdot \Delta t) \cdot \frac{\sin\left(\omega^* t - \omega_n \Delta t\right)}{\left(\omega^* t - \omega_n \Delta t\right)}$$

$$\omega_s = 2\pi f_s = \frac{4\pi}{\Delta t} = 2\omega^* > 2\Omega_{max} = 4\pi F,\ f_s > 2F,$$

$$\Delta t \leq \frac{1}{2F} = \frac{\pi}{\Omega_{max}},\ \omega^* = \frac{\omega_s}{2},\ \omega_n = n\frac{\omega_s}{2}, F = \frac{\Omega_{max}}{2\pi} = F_{max.}, n = 1,2,3...$$

If we now start (backwards) from a known spectral function $A(\omega)$, *(*$\psi(t) \to A(\omega)$*), we could again analogically apply the same sampling theorem*

$$\left(t \to \omega,\ \Delta t \to \Delta\omega,\ \omega_s \to t_s,\ \omega^* \to t^*, F \to T\right) \Rightarrow$$

$$t_s \frac{\omega}{2} = t^*\omega,\ t_s > 2T, t_n = n\frac{t_s}{2},\ \Delta\omega = 2\pi\Delta f \leq \frac{\pi}{T},\ \Delta f \leq \frac{1}{2T}, n = 1,2,3...$$

$$A(\omega) = \sum_{n=-\infty}^{\infty} A(n \cdot \Delta\omega) \cdot \frac{\sin\left(\frac{t_s (\omega - n\Delta\omega)}{2}\right)}{\frac{t_s (\omega - n\Delta\omega)}{2}} = \sum_{n=-\infty}^{\infty} A(n \cdot \Delta\omega) \cdot \frac{\sin\left(t^*\omega - t_n \Delta\omega\right)}{\left(t^*\omega - t_n \Delta\omega\right)}.$$

As we can see, for energy finite signals (or wave functions) the same signal can be discretized or quantized (on similar ways, using sinc functions $\frac{\sin\varphi(x,t)}{\varphi(x,t)}$*) both in its time and frequency domain. Taking into account known uncertainty relation (between total* *signal durations in a time and frequency domain) and known or necessary sampling intervals, we can create the following, extended uncertainty relations between minimal or sufficient sampling intervals,*

$$\left\{TF \geq \frac{1}{2}, (\Delta t)_{max.} = \Delta t \leq \frac{1}{2F}, (\Delta f)_{max.} = \Delta f \leq \frac{1}{2T}\right\} \Rightarrow$$
$$\left\{0 < \Delta t \cdot \Delta f \leq \frac{1}{2} \leq TF \leq \frac{1}{4\Delta t \cdot \Delta f}\right\} \Rightarrow (\Delta t \cdot \Delta f)_{min.} = \frac{1}{2}.$$

An optimal signal discretization (or quantization) implies that minimal sampling signal intervals or durations in all signal domains may also define wavefunction frequency and wavelength. Quantum Theory Uncertainty relations are also related to sampling intervals, but this is now a much clearer and more deterministic concept than Uncertainty presented in contemporary Quantum Theory.

Now we can generalize wave function, meaning to make it presentable as an analytic complex signal or sinc-function,

$$\psi(t) = \sum_{n=-\infty}^{\infty} \psi(n \cdot \Delta t) \cdot \frac{\sin(\omega^* t - \omega_n \Delta t)}{(\omega^* t - \omega_n \Delta t)} = a(t) \cdot \cos \varphi(t),$$
$$\hat{\psi}(t) = H[\psi(t)] = a(t) \cdot \sin \varphi(t) = \sum_{n=-\infty}^{\infty} \hat{\psi}(n \cdot \Delta t) \cdot \frac{\sin(\omega^* t - \omega_n \Delta t)}{(\omega^* t - \omega_n \Delta t)},$$
$$\overline{\Psi}(t) = \psi(t) + j\hat{\psi}(t) = a(t) \cdot e^{j\varphi(t)},\ a(t) = |\psi(t)| = |\hat{\psi}(t)| = |\overline{\Psi}(t)| = \sqrt{\psi^2(t) + \hat{\psi}^2(t)},\ j^2 = -1,$$

We can again apply Whittaker-Nyquist-Kotelnikov-Shannon Sampling Theorem and realize wave function quantizing or discretization as,

$$\overline{\Psi}(t) = \sum_{n=-\infty}^{\infty} \overline{\Psi}(n \cdot \Delta t) \cdot \frac{\sin(\omega^* t - \omega_n \Delta t)}{(\omega^* t - \omega_n \Delta t)} = e^{j\varphi(t)} \cdot \sum_{n=-\infty}^{\infty} a(n \cdot \Delta t) \cdot \frac{\sin(\omega^* t - \omega_n \Delta t)}{(\omega^* t - \omega_n \Delta t)}$$

In cases of planar unidirectional elementary waves (propagating in a positive x-direction), the wave function could be something as,

$$\overline{\Psi}(t) \Rightarrow \overline{\Psi}(\omega t - kx) = \sum_{n=-\infty}^{\infty} \overline{\Psi}(n \cdot \Delta t) \cdot \frac{\sin(\omega^* t - \omega_n \Delta t - kx)}{(\omega^* t - \omega_n \Delta t - kx)} =$$
$$= e^{j(\omega t - kx)} \cdot \sum_{n=-\infty}^{\infty} a(n \cdot \Delta t) \cdot \frac{\sin(\omega^* t - \omega_n \Delta t - kx)}{(\omega^* t - \omega_n \Delta t - kx)} = e^{j(\omega t - 2n\pi - kx)} \cdot \sum_{n=-\infty}^{\infty} a(n \cdot \Delta t) \cdot \frac{\sin(\omega^* t - \omega_n \Delta t - kx)}{(\omega^* t - \omega_n \Delta t - kx)}$$

intended to produce synchronized, uniform, and linear superposition of elementary (or quantized) waves, with the unique group and phase velocity, as follows, and finally, we will have,

$$(\omega t - 2n\pi - kx = \omega^* t - \omega_n \Delta t - kx) \Rightarrow \omega t - 2n\pi = \omega^* t - \omega_n \Delta t \Rightarrow$$
$$\omega = \omega^*,\ 2n\pi = \omega_n \Delta t = n\frac{\omega_s}{2}\Delta t \Leftrightarrow 4\pi = \omega_s \Delta t,\ \omega_s = \frac{4\pi}{\Delta t}$$

F) As we know (empirically) every wave phenomenon (or matter wave, wave-packet, photon, quasi-particle …) in a certain environment with fixed and stable properties has its relatively stable and fixed energy propagation speed (or group velocity), which will be denoted here as $v = v_c = const.$. In all such cases, the same matter wave would also have its phase velocity $u = \lambda f$. It is also possible to verify the inequality relation between any group and phase velocity (valid for any matter wave propagation) as $0 \leq u = \lambda f < 2u \leq \sqrt{uv} \leq v \leq v_c \leq c$. Consequently, if the speed of energy-propagation of specific matter wave is stable and constant $v = v_c = const.$, under different motional conditions relative to a particular observer, its phase velocity would be variable (dependent on the relative speed between the observer and matter wave generator). Analyzing matter waves this way, we will also be able to address *Doppler Effect relations*.

Let us start with the (always-valid) relation between any group and phase velocity, making the following chain of conclusions (under $v = \overline{v} = v_c = const.$),

$$\left\{\begin{array}{l} v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} = u + p\frac{du}{dp} = \frac{d\omega}{dk} = \frac{d\tilde{E}}{dp} = h\frac{df}{dp} = \frac{2u}{1+\frac{uv}{c^2}} = (1+\frac{1}{\gamma})u \ , \\ \lambda = \frac{h}{p} = \frac{h}{\gamma mv} = \frac{h}{mc}\cdot\frac{c}{\gamma v} = \lambda_0\cdot\left(\frac{c}{\gamma v}\right), \ \lambda_0 = \frac{h}{mc} = const., \gamma = 1/\sqrt{1-\frac{v^2}{c^2}}, \\ f = \frac{1}{\Delta t} = \frac{E_k}{h} = \frac{mc^2}{h}(\gamma-1) = f_0\cdot(\gamma-1) = \frac{1}{\Delta t_0}\cdot(\gamma-1), \ f_0 = \frac{mc^2}{h} = Const. \Rightarrow \Delta t = \frac{\Delta t_0}{(\gamma-1)}, \\ u = \lambda f = \lambda_0 f_0\cdot\frac{\frac{\gamma v}{c}}{1+\gamma} = \frac{\gamma}{1+\gamma}\cdot v = \frac{\omega}{k} = \frac{\tilde{E}}{p} = \frac{hf}{p} = \frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{E_k}{p}, \ \ \lambda_0 f_0 = u_0 = c \\ \overline{v} = \frac{\Delta\tilde{E}}{\Delta p} = \frac{\Delta E}{\Delta p} = \frac{\Delta\omega}{\Delta k} = \frac{\Delta x}{\Delta t}, \ \ \Rightarrow 0 \leq u < 2u \leq \sqrt{uv} \leq v \leq c. \end{array}\right\} \Rightarrow$$

$$v = \overline{v} = v_c = const. \Rightarrow$$

$$\Rightarrow \left\{\begin{array}{l} v = v_c = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} = u + p\frac{du}{dp} = \frac{d\omega}{dk} = \frac{d\tilde{E}}{dp} = h\frac{df}{dp} \Rightarrow \\ v_c d\lambda = ud\lambda - \lambda du \Rightarrow v_c\frac{d\lambda}{\lambda^2} = d(\frac{u}{\lambda}), \\ v_c\frac{d\lambda}{\lambda^2} = -df \Rightarrow \frac{v_c}{\lambda} = f + F_0 \Leftrightarrow \lambda = \frac{v_c}{f+F_0}, \ u = v_c\frac{f}{f+F_0}, \ F_0 = CONST. \\ v_c dk = d\omega \Rightarrow v_c k = \omega + \omega_0 \Leftrightarrow k = \frac{2\pi}{\lambda} = \frac{\omega+\omega_0}{v_c} \Leftrightarrow \lambda = \frac{v_c}{f+F_0}, \\ v_c dp = udp + pdu = d\tilde{E} = hdf \Rightarrow v_c dp = d(up) = d\tilde{E} = hdf \Rightarrow \\ \Rightarrow v_c p = up + const. = \tilde{E} + E_0 = hf + hF_0 \Leftrightarrow (v_c - u)p = const. \Leftrightarrow \\ \Leftrightarrow p = \frac{const.}{v_c - u} = h\frac{f+F_0}{v_c} = \frac{\tilde{E}+E_0}{v_c} = \frac{h}{\lambda} \Rightarrow \lambda = \frac{h}{const.}(v_c - u) = \frac{v_c}{f+F_0} = \frac{h}{p} \end{array}\right\} \Rightarrow$$

$$\Rightarrow \begin{cases} \lambda = \frac{h}{p} = \frac{v_c}{f+F_0} = \frac{v_c/f}{1+\frac{F_0}{f}} = \lambda_0 \cdot \frac{c}{\gamma v} \leq \frac{v_c}{f},\ \lambda_0 = \frac{h}{mc} = \text{const.} \\ f = \frac{v_c}{\lambda} - F_0 = f_0 \cdot (\gamma - 1),\ f_0 = \frac{mc^2}{h} = \text{Const.}, \\ u = \lambda f = \frac{\tilde{E}}{p} = v_c \frac{f}{f+F_0} = v_c \frac{1}{1+\frac{F_0}{f}} = \frac{\gamma}{1+\gamma} \cdot v = \frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}} \leq v_c, \\ \tilde{E} = hf = \frac{v_c p}{1+\frac{F_0}{f}} = v_c p + E_0 = v_c p - hF_0,\ p = \frac{h}{v_c}(f+F_0) \geq \frac{hf}{v_c},\ hf + hF_0 = v_c p, \\ E_0 = v_c p(\frac{1}{1+\frac{F_0}{f}} - 1) = -v_c p(\frac{\frac{F_0}{f}}{1+\frac{F_0}{f}}) = -v_c p \frac{F_0}{f+F_0} = -hF_0\ . \end{cases}$$

The relations established until here should be in the background of Doppler Effect situations.

G) *The wave-mechanics picture of elementary particles (as described above and in the second chapter with equations from (2.5.1) to (2.11)), can additionally be upgraded and generalized. For instance, if we consider that some matter form (initially without a rest mass) aggregates as a spinning object, this should be certain kind of motional and wave energy form, mathematically presentable as a wave packet, serving as a mathematical equivalent to certain rotating particle, or as certain toroidal form of rotating spatially distributed mass. Here, we are guided by the idea that rotation that creates closed toroidal form should be the mechanism of initial rest mass aggregation (since we already know that elementary particles always have their magnetic and mechanical, orbital and/or spinning moments). Here we are attempting to show that the very first, original elementary matter substance belongs to certain superposition of waveforms and fields (most probably composed of electromagnetic photons, fields, and waves). Such initial or primary "matter or mass substance", when being in a specific motion (with elements of rotation, and when creating standing waves on a closed path) produces stabilized particles with rest masses. Let us consider that rotating form in question does not make any (externally detectable) linear motion regarding the Laboratory System, or we can say that its center of mass linear velocity equals zero,* $v_c = 0$ *, and that its rotational or spinning speed* ω *is constant. Tangential (group) velocity here is* $v = \omega R$ *. If we now express the mentioned wave rotation as a linear motion of a certain equivalent particle along its circular path, we will have the following differential form of its motional energy:*

$$dE_{motional} = dE = d\tilde{E} = dE_{linear-motion} = dE_{rotational-motion} =$$

$$= \underline{vdp} = \omega R dp = \underline{\omega d\mathbf{L}} = c^2 dm = hdf\ ,\ d\mathbf{L} = Rdp = n(\frac{h}{2\pi})\frac{dp}{p}, \tag{5.4.4}$$

$$\begin{Bmatrix} mv^2 = \mathbf{J}\omega^2,\ \mathbf{J} = mR^2, v = \omega R,\ 2\pi R = n \cdot \lambda,\ \lambda = h/p, u = \lambda f \\ \mathbf{L} = \mathbf{J}\omega,\ \theta = h/\mathbf{L}, q \cdot \theta = 2\pi = qh/\mathbf{L}\ ,\ (n,q) = 1,2,3,... \end{Bmatrix}$$

Again, to underline, the condition which makes such conceptualization logical (like in (5.4.4)) is that when the average radius of rotation of specific particle R is much larger than any other dimension and radius of such equivalent rotating ring, or torus, moment of inertia of such thin walls object is in all cases approximately equal to $\mathbf{J} = mR^2$*.*

Now, we could imagine that initial rotating energy content ($\omega d\mathbf{L} = vdp = c^2 dm$ *) starts making additional linear motion related to its Laboratory System (* $v_c dp$ *), having a specific non-zero* <u>*center of mass velocity*</u> $\underline{v_c \neq 0}$ *. Here we are implicitly neglecting what is happening inside the rotating energy* *form, considering it (by observing externally) to be a particle that has certain rest mass content* $m = \tilde{m}$ *. The new motional energy picture, analog to (5.4.4), will be,*

$$\begin{aligned}
&dE = d\tilde{E} = v_c dp + \omega dL = c^2 d(\gamma m) \\
&\begin{Bmatrix} p = \gamma m v_c ,\ \gamma = (1 - v_c^2 / c^2)^{-0.5}, \\ v_c = u - \lambda \frac{du}{d\lambda},\ \lambda = \frac{h}{p},\ u = \lambda f = \lambda \frac{\omega}{2\pi} \end{Bmatrix} \\
&E_{motional} = \tilde{E} = E_{linear\text{-}motion} = E_{rotational-motion} = pu = \tilde{m}uv = \tilde{m}c^2 = hf
\end{aligned} \quad (5.4.5)$$

Also, assuming that every linear motion is only a particular case (or approximation) of specific curvilinear or rotational motion (with a sufficiently large radius of rotation), we could again make an equivalency between two aspects of such motion, as in (5.4.4),

$$\begin{aligned}
&dE = d\tilde{E} = v_c dp + \omega dL = \omega_c dL + mc^2 = c^2 d(\gamma m) \\
&dE = d\tilde{E} = v_c dp + \omega_c dL^* = c^2 d(\gamma m) \\
&v_c dp = \omega_c dL,\ \omega dL = mc^2
\end{aligned}$$

(See also Chapter 4.1, Fig.4.1 and equations under (4.3))

H) *In cases of creation of real elementary particles, we should have some rotational, pure wave motion (with zero rest mass) that creates self-closed standing waves, and this way certain wave-packet becomes a stable elementary particle (like an electron). Such self-closed wave object observed externally looks much more as a particle that has its rest mass (being a carrier of its total energy). When the same particle is internally observed, there is no rest mass, and we will have the following total energy associated with such an object,*

$$\begin{aligned}
&E_{total} = E_{motional} = \tilde{E} = E_{linear\text{-}motion} = E_{rotational-motion} = pu = \tilde{m}uv = \tilde{m}c^2 = hf \\
&E_{total}^2 = E_0^2 + p^2c^2 = p^2c^2 = (\tilde{m}c^2)^2 \Leftrightarrow pc = \tilde{m}c^2 \Leftrightarrow p = \tilde{m}c = \tilde{m}v \Leftrightarrow v = c = u
\end{aligned} \quad (5.4.6)$$

From (5.4.6), we see that the wave packet in question rotates with group (or tangential) velocity that is constant and equal to c (meaning that internal content of such object are photons). Now, from the equivalency $pv = L\omega$ *, replacing group velocity with c, we get,*

$$\begin{aligned}
&\left(pv = L\omega,\ v = c = \omega_c R,\ L = n\frac{h}{2\pi} \right) \Leftrightarrow \\
&pc = L\omega_c = n\frac{h}{2\pi}\omega_c = nhf_c = \tilde{m}c^2 = E_{total} = E_{motional} = hf, \\
&L = \frac{pc}{\omega_c} = n\frac{h}{2\pi},\ f = nf_c,\ \omega_c = 2\pi f_c = const.\ c = Const., \\
&f = nf_c = \frac{\tilde{m}c^2}{h},\ p = \tilde{m}c = \frac{hf}{c} = \frac{nhf_c}{c},\ n = 1,2,3,...
\end{aligned} \quad (5.4.7)$$

We can find similar challenging concepts regarding the structure of elementary particles in many publications coming from Bergman, David L. and Lucas, Jr., Charles W. and their collaborators (see mentioned literature under [16 to 20]. [22], [25], [83], [88]).

What is explained here is that specific pure wave state, without any rest mass content (observed internally), under certain conditions transforms itself into a state that externally starts behaving as having rest mass. The fundamental laws governing behaviors and appearance of such matter state are as usual: Law of Total Energy Conservation, Law of Total Linear Momentum Conservation, and Law of Total Angular Momentum Conservation. Let us imagine that the matter-wave state in question is a set of spatially distributed matter-waves, which is passing certain transformation. The following conservation laws can describe such a locally isolated system,

$$\begin{aligned} &E_{total} = E = \sum_{(i)} E_i = \sum_{(j)} E_j = \gamma mc^2, \frac{\partial E}{\partial t} = 0, \\ &\vec{P}_{total} = \vec{P} = \sum_{(i)} \vec{p}_i = \sum_{(j)} \vec{p}_j = \gamma m\vec{v}_c, \frac{\partial P}{\partial t} = 0, \\ &\vec{L}_{total} = \vec{L} = \sum_{(i)} \vec{L}_i = \sum_{(j)} \vec{L}_j = J\vec{\omega}_c = \gamma mR^2\vec{\omega}_c, \frac{\partial L}{\partial t} = 0, \end{aligned} \quad (5.4.8)$$

Also, total energy conservation should satisfy the following relations,

$$\begin{aligned} &\left\{ \begin{array}{l} E_{total}^2 = E_0^2 + P^2c^2 = E^2, \\ dE = \omega_c dL + v_c dP = c^2 d(\gamma m) \end{array} \right\} \Rightarrow \\ &E = \int dE = \int \omega_c dL + \int v_c dP = \gamma mc^2, \\ &\int \omega_c dL = m_0c^2 + L^2\omega_c^2 = mc^2, m_0 = const. \\ &\int v_c dP = (\gamma - 1)mc^2 \\ &\gamma = (1 - v_c^2/c^2)^{-0.5} \end{aligned} \quad (5.4.9)$$

l) *Until here, we did not make any differentiation between the mechanical or angular rotation* $\omega_m = \omega_{gm}$ *and wave angular speed* $\omega = 2\pi f$ *, since we had only circular wave motion (without rest mass).*

Let us imagine that specific particle (which has non-zero rest mass) rotates, having tangential velocity $v = v_g = \omega_m R = \omega_{gm} R$ *, where the radius of rotation is R = const. and* $\omega_m = \omega_{gm}$ *is the mechanical, angular particle velocity (presenting number of full rotations per second), and let us find all particle and matter wave parameters associated to such movement. Practically, the same concept of a wave packet, which has its group and phase velocity, in cases or rotational motions should be analogically extended to the rotating wave packet that has a group and phase angular velocity, (using indexing: m (=) mechanical, f (=) phase, g (=) group), as for instance,*

$$\begin{aligned} &\mathbf{v = v_g = R\omega_{gm} = \frac{d\omega}{dk} (=)\ group\ wave\ velocity (=) particle,\ linear\ velocity,} \\ &\mathbf{\omega_{gm} = \frac{v}{R} = 2\pi f_{gm} (=) group\ angular\ velocity\ or\ frequency (=) particle\ angular\ velocity,} \\ &\mathbf{u = v_f = R\omega = \frac{\omega}{k} = \lambda f (=)\ phase,\ wave\ velocity,\ Rk = 1,\ \lambda = 2\pi R,} \\ &\mathbf{\omega = \omega_f = \frac{u}{R} = 2\pi f (=) angular\ wave\ frequency,\quad \frac{\omega_{gm}}{\omega} = \frac{f_{gm}}{f} = \frac{v}{u},} \\ &\mathbf{k = \frac{2\pi}{\lambda} = \frac{2\pi}{h} p (=)\ wave\ vector,\ \tilde{E} = hf,} \\ &\left\{ \begin{array}{l} \left[v = u - \lambda \frac{du}{d\lambda} = u + k\frac{du}{dk} = \frac{d\omega}{dk} \right] \cdot \frac{1}{R} \Leftrightarrow \\ \frac{v}{R} = \frac{u}{R} - \lambda \frac{d(\frac{u}{R})}{d\lambda} = \frac{u}{R} + k\frac{d(\frac{u}{R})}{dk} = \frac{1}{R}\frac{d\omega}{dk} \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} \omega_{gm} = \omega - \lambda \frac{d\omega}{d\lambda} = \omega + k\frac{d\omega}{dk} = \frac{1}{R}\frac{d\omega}{dk} \\ \omega = \frac{\omega}{Rk} = 2\pi f \end{array} \right\} \end{aligned} \quad (5.4.10)$$

$$(0 \le v \le c) \Rightarrow 1 \le \frac{\omega_{gm}}{\omega} = \frac{f_{gm}}{f} = \frac{v}{u} \le 2$$

$$(v << c) \Rightarrow v = 2u \Rightarrow \frac{\omega_{gm}}{\omega} = \frac{f_{gm}}{f} = 2$$ *(See also (4.3), chapter 4.1, Matter Waves)*

$$(v \cong c) \Rightarrow (v = u) \cong c \Rightarrow \frac{\omega_{gm}}{\omega} = \frac{f_{gm}}{f} = 1$$

Matter waves associated with any particle in motion are practically defined by PWDC relations (see equations (4.2) - (4.3), Chapter 4.1, and Chapter 10, "10.00 DEEPER MEANING OF PWDC"). Particle mechanical, rotating and spinning properties are different compared to familiar frequency related, matter-wave properties. We should make a difference between mechanical rotation (number of mechanical revolutions about certain center), and orbital matter-waves frequency of associated matter waves. In cases of inter-atomic circular motions, where standing matter waves are an intrinsic structural property, conditions for creating standing waves are:

$$\mathbf{2\pi R = n\lambda \Rightarrow Rk = n = 1, 2, 3... \Rightarrow \omega_{gm} = \frac{1}{R}\frac{d\omega}{dk}, \frac{\omega_{gm}}{\omega} = \frac{v}{u} = \frac{k}{\omega}\frac{d\omega}{dk}} \tag{5.4.11}$$

The differentiation between mechanical and matter wave angular (frequency-related) parameters has often been overlooked. Instead of distinguishing frequency-related matter-wave parameters from traditional mechanical rotation parameters, some ad hoc and exotic postulates have been introduced to correct or support theories regarding elementary particles and atomic structures. These additional principles have been accepted in Orthodox Quantum Mechanics to address complex issues such as the gyromagnetic ratio, spin attributes, correspondence principle, and the relationships between orbital and magnetic moments.

The example discussed in (5.4.11) should be generalized and aligned with Wilson-Sommerfeld rules (see (5.4.1)). In other words, quantization is not universally applicable to all motions, fields, waves, and charges without specific structural conditions. Resonant and standing-wave conditions are typically necessary for quantization, occurring when stable, elementary matter domains interact and exchange moments and energy in various ways. The nature and configuration of these matter domains, including their fitting, synchronization, and internal and external packing, are closely related to standing waves and resonant structures.

Attempts to apply the quantization concepts of Quantum Theory universally to all fields and waves can be misleading and inadequately defined. Often, what we observe in particle-wave duality is the application of signal analysis and synthesis principles, like those in spectrum analysis and digital signal processing techniques (e.g., Shannon–Kotelnikov-Nyquist-Whittaker). The precise mathematical quantization of signals or wave groups is frequently neglected in favor of a more generalized quantization approach, as promoted by contemporary Quantum Theory.

True quantization, where discrete intervals and integers play a significant role, reflects the energy packing within stable objects when space is limited. Energy quantization can also manifest during interactions between atoms and elementary particles, but not all energy exchanges or formats in the universe are quantifiable by integers or take the form of standing waves. For example, transient, progressive, and space-time evolving wave motions in particle interactions may not adhere to clear and straightforward energy quantization concepts.

This book proposes that a significant advancement would occur if certain outdated dogmas and ad hoc postulates of Quantum Theory started evolving in the directions suggested here. Researchers should seriously consider new ideas that diverge from Orthodox Quantum Theory. Although Quantum Theory has proven effective, mathematically, our goal is to transform its inexplicable, abstract, or postulated aspects into concepts that are more tangible and better integrated with the broader framework of tangible Physics. ♣**]**

5.1. Uncertainty in Physics and Mathematics

In contemporary physics, the Uncertainty Principle is usually presented mostly in connection with Heisenberg's Uncertainty Relations. It is also almost exclusively linked to (micro world) Planck's constant $\mathbf{h}$. For real, correct, and full understanding of many forms of Uncertainty Relations, it is, for the time being, better to forget that Heisenberg made any invention regarding Uncertainty, since what we know and have from mathematics is already enough, and much more generally applicable.

In other words, we should know (or learn) that Uncertainty is not "married" almost exclusively with Statistical interpretation of Quantum Theory and with Heisenberg assumed Uncertainty, and that it primarily belongs to mathematics (meaning it is universally applicable, presenting mutually conjugate absolute domain-lengths relations, valid both in micro-world of atoms and elementary particles, and in a macro-world of cosmic formations). Even Planck's constant $\mathbf{h}$, which has its significant place in all micro-world events (regarding atoms, photons, and elementary particles), has its (analogical) macro-world equivalent in a new, much bigger constant $\mathbf{H} >> \mathbf{h}$ (see chapter 2; -equations (2.11.12) - (2.11.21)). It is also the current case that Uncertainty, as presented in contemporary Physics (mostly in Quantum Theory), is often used as a supporting background for many oversimplifications, mystifications, and justifications of the number of logical, conceptual, and methodological uncertainties in Physics. *The here-upgraded concept of Uncertainties will show that this is a much more productive and more diversified concept compared to one presented and applied in contemporary QT and physics.*

A much more general approach to universally valid Uncertainty relations (in connection to quantum manifestations of all energy formats) should start from a band-limited, energy-finite wave function $\Psi(t)$. Here we shall treat the square of the wave function as a power: $\Psi^2(\mathbf{t}) = \mathbf{Power} = \mathbf{dE}/\mathbf{dt}$, but later, we could also transform, normalize, and treat $\mathbf{\Psi(t)}$ as a dimensionless function without influencing the results of the analysis that follows.

Let us designate with $\mathbf{T}$ and $F = \Omega / 2\pi$ the absolute and finite time and frequency durations of a wave function $\Psi(t)$ and its spectral function $\mathbf{A(f)}$, as we find in (4.9), where ($t \in [\mathbf{T}]$, $0 < \mathbf{T} < \infty$, $-\infty < \mathbf{t} < \infty$), and ($\mathbf{f} \in [\mathbf{F}]$, $0 < \mathbf{F} < \infty$, $0 < \mathbf{f} < \infty$, $2\pi F = \Omega, 2\pi f = \omega$). We can also say that $\mathbf{T}$ and $\mathbf{F}$ are absolute (or total) time and frequency lengths of $\Psi(t)$ and its spectrum $\mathbf{A}(f)$).

Here we are considering the wave function, $\Psi(t)$ as an Analytical Signal form (first time introduced by Dennis Gabor, in connection with Hilbert transform, see **[7]** and **[8]**), as well as a finite, energy-limited function, both in its time or frequency domain. It is an advantage of Analytical Signals that they cover only natural domains of real-time and frequency: $-\infty < \mathbf{t} < \infty$, $\mathbf{0} \leq \mathbf{f} < \infty$. This is opposite to the traditional Signal Analysis (Fourier analysis), where frequency can take negative values, but in all other aspects Analytic Signals produces the same or equivalent results, as in the case of Fourier Signal Analysis, including producing additional dynamic, spectral signal properties, time-frequency dependent, which the Fourier analysis is not able to generate. Before we start developing concepts of Uncertainties, it would be highly recommendable to go to Chapter 4.0 of this book (Wave functions wave velocities and uncertainty relations) in order not to repeat already presented Uncertainties background.

Spectrum Analysis shows (without any doubts, and any applicability limits to the micro or macro world) general validity of the following Uncertainty relation (which is equivalent to (5.1)):

$$\mathrm{TF} > \frac{1}{2},\ \mathrm{T}\Omega > \pi\,. \tag{5.5}$$

This is the most general, "*master uncertainty framework*" that can later evolve towards Heisenberg micro-world Uncertainty where Planck constant **h** is the most relevant. Analogically, in cases of Uncertainty relations for periodical, stable planetary-systems motions, another Planck-like constant **H>>h** is relevant (see more in Chapter 2.). There is nothing here exclusively related to standard deviations and statistical meaning of signal durations. T and F are absolute and total signal durations. Of course, there are logical and clear situations (when we deal with a significant number of identical or similar members, events, or participants) where absolute signal durations could be, very practically, conveniently, and approximately, or statistically replaced with relevant standard deviations (but not with so much exclusivity and ontological weight as ambitiously postulated in the contemporary Quantum Theory).

If specific transformation (5.6) happens to a wave function $\Psi(t)$, changing its time and frequency lengths, **T** and **F**, for the amounts **Δt** and **Δf**, the same signal transformation will automatically influence all other space and energy-related parameters of **Ψ(x, t)** to change, producing similar Uncertainty relations, as given in (5.1). Then, an effective physical signal length **L** and signal wave energy $\tilde{\mathbf{E}}$ will also change, as for instance (the following results are taken from Chapter 4.0):

$$\left\{\begin{array}{l} \mathrm{T} \to \mathrm{T} \pm \Delta t > 0, \mathrm{F} \to \mathrm{F} \pm \Delta f > 0, \mathrm{L} \to \mathrm{L} \pm \Delta x > 0, \mathrm{K} \to \mathrm{K} \pm \Delta k > 0 \\ \Delta\tilde{\mathrm{E}} = h\ \Delta f = \overline{v}\Delta p = \tilde{\mathrm{F}}\Delta x, \tilde{\mathrm{F}} = \frac{\Delta p}{\Delta t} = \text{force}, \\ 0 < \delta\, t \cdot \delta f = \delta\, x \cdot \frac{\delta k}{2\pi} = \delta\, x \cdot \delta f_x < \frac{1}{2} < \mathrm{F} \cdot \mathrm{T} = \frac{1}{2\pi} \cdot \mathrm{K} \cdot \mathrm{L} = \mathrm{F}_x \cdot \mathrm{L} \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left\{\begin{array}{l} \tilde{\mathrm{E}} \to \tilde{\mathrm{E}} \pm \Delta\tilde{\mathrm{E}}, \\ \overline{v} = \frac{\Delta x}{\Delta t} = \frac{\Delta\tilde{\mathrm{E}}}{\Delta p} = \frac{\Delta\omega}{\Delta k} = \frac{\delta\, x}{\delta\, t} = \frac{\delta\omega}{\delta k} = \frac{\delta\tilde{\mathrm{E}}}{\delta p} \end{array}\right\}. \tag{5.6}$$

In cases when time and frequency changes are either positive or negative, we have:

$$T\cdot F>\frac{1}{2}\Leftrightarrow\left\{\begin{array}{l}(T+\Delta t)\cdot(F+\Delta f)>\frac{1}{2}\\(T-\Delta t)\cdot(F-\Delta f)>\frac{1}{2}\end{array}\right\}\Rightarrow\left\{\begin{array}{l}\left[T^2-(\Delta t)^2\right]\cdot\left[F^2-(\Delta f)^2\right]>\frac{1}{4}\\T\cdot F+\Delta t\cdot\Delta f>\frac{1}{2}\\T\cdot\Delta f+\Delta t\cdot F>0\end{array}\right\}\Leftrightarrow$$

$$\Leftrightarrow\left\{\begin{array}{l}1-(\frac{\Delta t}{T})^2-(\frac{\Delta f}{F})^2+(\frac{\Delta t}{T})^2\cdot(\frac{\Delta f}{F})^2>\frac{1}{4}\\1+(\frac{\Delta t}{T})\cdot(\frac{\Delta f}{F})>\frac{1}{2T\cdot F},T\cdot F>\frac{1}{2}\\(\frac{\Delta t}{T})+(\frac{\Delta f}{F})>0\end{array}\right\}. \quad (5.7)$$

Let us additionally conceptualize the same idea (about sudden signal duration changes) using the moving particle Energy-Momentum 4-vector from the Minkowski-space of Relativity Theory, by (mathematically) introducing mutually coupled changes of total system energy and belonging total momentum, applying discrete, central differentiations method,

$$\begin{array}{l}\overline{P}_4=\overline{P}\left[\vec{p}=\gamma m\vec{v},\frac{E}{c}=\gamma mc\right],\overline{P}^2=\vec{p}^2-\frac{E^2}{c^2}=-\frac{E_0^2}{c^2},E_0=mc^2,E=\gamma E_0\Rightarrow\\\vec{p}^2c^2+E_0^2=E^2,(p\to p\pm\Delta p)\Leftrightarrow(E\to E\pm\Delta E)\Rightarrow\end{array} \quad (5.7.1\text{-}1)$$

$$\begin{array}{l}\left\{\begin{array}{l}(p+\Delta p)^2c^2+E_0^2=(E+\Delta E)^2\\(p-\Delta p)^2c^2+E_0^2=(E-\Delta E)^2\\\overline{v}=\frac{\Delta x}{\Delta t}=\frac{\delta x}{\delta t}=\frac{\delta\omega}{\delta k}=\frac{\Delta\omega}{\Delta k}\end{array}\right\}\Leftrightarrow\left\{c^2\cdot p\ \Delta p=E\ \Delta E\Leftrightarrow\frac{\Delta E}{\Delta p}=c^2\frac{p}{E}=\overline{v}\right\}\Rightarrow\\\Rightarrow\frac{\Delta x}{\Delta t}=\frac{\delta x}{\delta t}=\frac{\Delta E}{\Delta p}=c^2\frac{p}{E}=\overline{v}=h\frac{\Delta f}{\Delta p}=\frac{\Delta\tilde{E}}{\Delta p}=\frac{\Delta\omega}{\Delta k}=\frac{\delta\omega}{\delta k}\leq c\ .\end{array} \quad (5.7.1\text{-}2)$$

What we can see from (5.7.1-2) is that sudden energy-momentum changes in a specific system (a moving particle, here) are causally related to its average center-of-mass velocity, which is at the same time equal to the system average group velocity. This now gives a new and more tangible meaning to Uncertainty Relations that should be well analyzed before we make other conclusions and associated statistical makeup. Before, we found that signal domain proportionality only dimensionally and quantitatively indicated that this could be the signal group velocity (or particle velocity), and now we can safely confirm that this is the real case.

The idea here is to show that so-called Uncertainty Relations are causally related to a velocity of matter waves propagation. The meaning of that is that at the same time when a specific motional object or signal experiences a sudden change of its energy-related parameters, matter-waves are automatically created, and this produces results captured by Uncertainty Relations (indirectly saying that there is no such Uncertainty in its old and traditional meaning).

For instance, we can express the average group velocity $\bar{v}$ associated with the transformations (5.7.1-2) as:

$$\left\{u=\frac{\omega}{k}, \omega=ku\right\} \Rightarrow \Delta\omega=(k+\frac{1}{2}\Delta k)(u+\frac{1}{2}\Delta u)-(k-\frac{1}{2}\Delta k)(u-\frac{1}{2}\Delta u)=$$
$$=k\Delta u+u\Delta k \Leftrightarrow \bar{v}=\frac{\Delta\omega}{\Delta k}=u+k\frac{\Delta u}{\Delta k} \Leftrightarrow \quad (5.7.1\text{-}3)$$
$$\Leftrightarrow \left\{v=u+k\frac{du}{dk}=\frac{d\omega}{dk}=\frac{d\tilde{E}}{dp}=\frac{dx}{dt}=\text{immediate group velocity}\right\}.$$

Such group velocity (found using finite differences) is fully analog to its differential form where infinitesimal signal changes are involved. By merging average group velocity with Uncertainty Relations, we will again see that they are mutually comparable,

$$\left\{\begin{array}{l} \bar{v}=u+k\frac{\Delta u}{\Delta k}=\frac{\Delta\omega}{\Delta k}=\frac{\Delta\tilde{E}}{\Delta p}=\frac{\Delta x}{\Delta t}=\text{average group velocity} \\ \text{and} \\ \left|\Delta x\ \Delta p\right|=\left|\Delta t\ \Delta\tilde{E}\right|=h\left|\Delta t\ \Delta f\right|>h/2, \Delta\tilde{E}=h\Delta f, \\ 0<\delta t\cdot\delta f=\delta x\cdot\delta f_x<\frac{1}{2}\leq F\cdot T=F_x\cdot L\leq\frac{1}{4\cdot\delta t\cdot\delta f}=\frac{1}{4\cdot\delta x\cdot\delta f_x} \end{array}\right\} \Rightarrow$$
$$\Rightarrow \bar{v}=\frac{\Delta x}{\Delta t}=\frac{\Delta E}{\Delta p}=h\frac{\Delta f}{\Delta p}=\frac{\Delta\tilde{E}}{\Delta p}=\frac{\Delta\omega}{\Delta k}=u+k\frac{\Delta u}{\Delta k}=\frac{\delta x}{\delta t}=\frac{\delta\omega}{\delta k}=\frac{dx}{dt}=\frac{d\omega}{dk}. \quad (5.7.1\text{-}4)$$

A very interesting fact regarding the average group velocity "$\bar{v}$" in (5.7.1-3) and (5.7.1-4), which could pass unnoticed, is that the full analogy between the expression for the average group velocity "$\bar{v}$" and the expression for the immediate group velocity "**v**" is not made as an approximation, automatically by formal and simple replacement of infinitesimal difference "d" with discrete, delta difference "Δ". The real development of the average group velocity (as given here) is based on applying practices of symmetrical central differences to basic definitions of group and phase velocity, and it is fully correct. This shows that there is a deterministic connection between physics of continuum and physics of discrete or finite steps (to support better this statement it would be necessary to devote certain time to learning about properties of central, symmetrical differences).

*[♣ **COMMENTS & FREE-THINKING CORNER**:*

Since we know relations between group and phase velocity (of certain specific wave packet) in connection with signal wavelength and frequency, as given in (4.2), we can find the absolute motional parameter frames where all the signal parameters, caused by transformation (5.6), should be expected, for instance:

$$\left\{ v = u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda} = \frac{dE}{dp} = \frac{dx}{dt}, \lambda = \frac{h}{p}, dE = hdf = c^2 d(\gamma m), p = \gamma m v \right\} \Rightarrow$$

$$\overline{v} = \frac{\Delta x}{\Delta t} = \frac{\Delta E}{\Delta p} = \frac{1}{\lambda_{max.} - \lambda_{min.}} \int_{\lambda_{min.}}^{\lambda_{max.}} (-\lambda^2 \frac{df}{d\lambda}) d\lambda = \frac{-1}{\Delta\lambda} \int_{f_{max.}}^{f_{min.}} \lambda^2 df = \frac{-h}{\Delta\lambda} \int_{f_{max.}}^{f_{min.}} \frac{1}{p^2} d(hf) =$$

$$= \frac{-h}{\Delta\lambda} \int_{[\Delta E]} \frac{1}{p^2} dE = \frac{-h}{\Delta\lambda} \int_{[\Delta v]} \frac{c^2}{(\gamma m v)^2} d(\gamma m) = \frac{-hc^2}{m\Delta\lambda} \int_{[\Delta v]} \frac{d\gamma}{\gamma^2 v^2} = \frac{-h}{m\Delta\lambda} \int_{[\Delta v]} \frac{dv}{v\sqrt{1 - v^2/c^2}} =$$

$$= \frac{h}{2m\Delta\lambda} \left[\frac{1 - \sqrt{1 - v^2/c^2}}{1 + \sqrt{1 - v^2/c^2}} \right]_{V_{MIN.}}^{V_{MAX.}} =$$

$$= \frac{v_{min.} v_{max.}}{2\left[v_{max.} \sqrt{1 - v_{min.}{}^2/c^2} - v_{min.} \sqrt{1 - v_{max.}{}^2/c^2} \right]} \left[\frac{1 - \sqrt{1 - v^2/c^2}}{1 + \sqrt{1 - v^2/c^2}} \right]_{V_{MIN.}}^{V_{MAX.}} \leq$$

$$\leq \frac{v_{min.} v_{max.}}{2\left[v_{max.} \sqrt{1 - v_{min.}{}^2/c^2} - v_{min.} \sqrt{1 - v_{max.}{}^2/c^2} \right]} = \frac{h}{2m\Delta\lambda},$$

$$\Delta\lambda = \lambda_{max.} - \lambda_{min.} = \frac{h}{\gamma(v_{min.}) m v_{min.}} - \frac{h}{\gamma(v_{max.}) m v_{max.}} =$$

$$= \frac{h}{m} \left[\frac{v_{max.} \sqrt{1 - v_{min.}{}^2/c^2} - v_{min.} \sqrt{1 - v_{max.}{}^2/c^2}}{v_{min.} v_{max.}} \right] \leq \frac{h}{2m\overline{v}},$$

$$\lambda_{max.} = \frac{h}{m} \left[\frac{v_{max.} \sqrt{1 - v_{min.}{}^2/c^2}}{v_{min.} v_{max.}} \right], \lambda_{min.} = \frac{h}{m} \left[\frac{v_{min.} \sqrt{1 - v_{max.}{}^2/c^2}}{v_{min.} v_{max.}} \right],$$

$$\overline{\lambda} = \frac{h}{\overline{p}} = \frac{h}{m\overline{v}} \sqrt{1 - \overline{v}^2/c^2} \leq \frac{h}{m\overline{v}}, \frac{\Delta\lambda}{\overline{\lambda}} \leq \frac{1}{2},$$

$$\lambda = \frac{h}{p} \Rightarrow p\Delta\lambda + \lambda\Delta p + \Delta p\Delta\lambda = 0, \ (1 + \frac{\Delta p}{p})(1 + \frac{\Delta\lambda}{\lambda}) = 1, \ \frac{\Delta\lambda}{\lambda} = -\frac{\frac{\Delta p}{p}}{1 + \frac{\Delta p}{p}}.$$

$$u_{min} = \lambda_{min.} f_{min.} = \frac{v_{min.}}{1 + \sqrt{1 - v_{min.}{}^2/c^2}}, \ u_{max.} = \lambda_{max.} f_{max.} = \frac{v_{max.}}{1 + \sqrt{1 - v_{max.}{}^2/c^2}},$$

$$\overline{u} = \overline{\lambda}\,\overline{f} = \frac{h\overline{f}}{m\overline{v}} \sqrt{1 - \overline{v}^2/c^2} = \frac{\overline{v}}{1 + \sqrt{1 - \overline{v}^2/c^2}},$$

$$f_{max.} = \frac{1}{\lambda_{max.}} \left[\frac{v_{max.}}{1 + \sqrt{1 - v_{max}^2/c^2}} \right] = \frac{m}{h} \left[\frac{v_{min.} v_{max.}}{(1 + \sqrt{1 - v_{max.}{}^2/c^2}) \sqrt{1 - v_{min}^2/c^2}} \right],$$

$$f_{min.} = \frac{1}{\lambda_{min.}} \left[\frac{v_{min.}}{1 + \sqrt{1 - v_{min}^2/c^2}} \right] = \frac{m}{h} \left[\frac{v_{min.} v_{max.}}{(1 + \sqrt{1 - v_{min}^2/c^2}) \sqrt{1 - v_{max}^2/c^2}} \right],$$

$$\overline{f} = \frac{m\overline{v}^2}{h(1 + \sqrt{1 - \overline{v}^2/c^2})\sqrt{1 - \overline{v}^2/c^2}},$$

$$\Delta f = f_{max.} - f_{min.} \leq \frac{v_{max.}}{\lambda_{max.}} - \frac{v_{min.}}{2\lambda_{min.}} = \frac{m(v_{min.} v_{max.})}{h} (1 - \frac{1}{2\sqrt{1 - v_{max}^2/c^2}}),$$

$$\overline{E}_k = \tilde{E} = \frac{\overline{p}\overline{v}}{1+\sqrt{1-\overline{v}^2/c^2}} = \frac{m\overline{v}^2}{(1+\sqrt{1-\overline{v}^2/c^2})\sqrt{1-\overline{v}^2/c^2}} = h\overline{f}, \quad m = \frac{E_{total}}{c^2},$$

$$v <<< c \Rightarrow$$

$$\overline{v} \cong \frac{v_{min.}v_{max.}}{2(v_{max.}-v_{min.})}, \frac{\Delta v}{\overline{v}} \cong 2\frac{(v_{max.}-v_{min.})^2}{v_{min.}v_{max.}},$$

$$\overline{u} = \overline{\lambda}\overline{f} \cong \frac{1}{2}\overline{v} \cong \frac{v_{min.}v_{max.}}{4(v_{max.}-v_{min.})},$$

$$\Delta\lambda \cong \frac{h}{m}\left[\frac{v_{max.}-v_{min.}}{v_{min.}v_{max.}}\right], \overline{\lambda} \cong \frac{h}{m\overline{v}} \cong \frac{2h}{m}\left[\frac{v_{max.}-v_{min.}}{v_{min.}v_{max.}}\right] \cong 2\Delta\lambda, \frac{\Delta\lambda}{\overline{\lambda}} \cong \frac{1}{2}, \tag{5.7.2}$$

$$\Delta f \cong \frac{m(v_{min.}v_{max.})}{2h}, \overline{f} \cong \frac{m}{2h}\overline{v}^2 \cong \frac{m}{2h}\left[\frac{v_{min.}v_{max.}}{2(v_{max.}-v_{min.})}\right]^2, \frac{\Delta f}{\overline{f}} \cong 4\frac{(v_{max.}-v_{min.})^2}{v_{min.}v_{max.}},$$

$$\overline{E}_k = \tilde{E} = \frac{m}{2}\left[\frac{v_{min.}v_{max.}}{2(v_{max.}-v_{min.})}\right]^2 = \frac{E_{total}}{2c^2}\left[\frac{v_{min.}v_{max.}}{2(v_{max.}-v_{min.})}\right]^2, \quad m = \frac{E_{total}}{c^2}.$$

Particle-wave duality and Uncertainty relations will become even more interesting research subjects if we express the signal length variation from (5.6), in connection to signal wavelength variation as $\Delta x \cong const. \times \Delta\lambda$, or if the total signal length presents the integer multiple of the average signal half-wavelength, $L = n\frac{\overline{\lambda}}{2}$, $n = 1,2,3...$

Results and relations from equation (5.7.2) should be applicable to explain the spectral nature of blackbody radiation and Planck's law. Regardless of the specific implications of these results in relation to signal transformation (as discussed in (5.6) and (5.7)), a key takeaway for understanding Uncertainty Relations is the importance of clarity about what we are discussing.

From a purely mathematical standpoint, Uncertainty Relations are straightforward: they describe the absolute relationship between time and frequency domain durations of relevant signals. When considering a signal or wave function $\Psi(x,t)$ that represents a real object in motion, we must apply Uncertainty Relations accurately, considering relevant factors such as velocity, momentum, and energy.

A significant source of confusion and misunderstanding in Physics arises from the arbitrary methods used to connect the mathematical aspects of Uncertainty (related to wavefunction domain durations) with the dimensions of real, geometry-related particles. While mathematical Uncertainty Relations, when correctly applied, are clear and comprehensible, the Uncertainties in microphysics as presented in contemporary Quantum Theory remain challenging and require further elaboration.

By examining equations (5.6) and (5.7.2) in the context of earlier equations (5.1) through (5.5), we can better understand how Uncertainty Relations in physics might need to be reformulated and generalized within the framework provided by (5.6) and (5.7.2). This approach will offer a more comprehensive understanding of real and generally valid Uncertainty Relations and the quantum nature of physics.

For example, the relations and mathematical expressions from (5.6) to (5.7.2) should apply to any macro-object in the universe. Hypothetically, if we use (5.6) and (5.7) to describe the total size and boundaries of the universe, approximating it geometrically as an "expanding balloon", and given our understanding of its growth based on Hubble's Law and astronomical observations, we could make significant predictions and draw conclusions by applying relevant Uncertainty Relations correctly. This could lead us to question and reassess some of the foundational principles currently accepted in modern physics. ♣*]*

Understanding Uncertainty relations in physics (presently still on a mathematical level) is also related to our choice of signal duration intervals. Until here, we have been using (or talking about) real, absolute, or total signal lengths. Now we will once more extend already established Uncertainty Relations of absolute signal duration intervals, taking into consideration corresponding *signal standard deviation intervals*.

Since Orthodox Quantum Mechanics mostly deals with statistical distributions and probabilities, relevant signal lengths are represented by signal variance intervals, which are statistics or standard deviations of certain variables around their mean values. Consequently, mathematical expressions of basic Uncertainty Relations, when using variance intervals or statistical deviations, present another aspect of Uncertainty Relations (not mentioned before, but exclusively used in today's Orthodox Quantum Mechanics theory), and here, such approach will be integrated into a chain of all other, already known Uncertainty Relations. The statistics' concept of variance is used to measure the signal's energy spreading in time and frequency domains. For instance, for an energy-finite wavefunction, we can define the following variances (see **[7]**, pages: 29-37, **[8]**, pages: 273-277 and **[79],** pages: 57-60):

$$\begin{gathered}(\sigma_t)^2 = \Delta^2 t = \frac{1}{\tilde{E}}\int_{-\infty}^{+\infty}(t-\langle t\rangle)^2\left|\overline{\Psi}(t)\right|^2 dt = \int_{-\infty}^{+\infty} t^2\frac{\left|\overline{\Psi}(t)\right|^2}{\tilde{E}}dt - \langle t\rangle^2 < T^2, \\ (\sigma_\omega)^2 = \Delta^2\omega = \frac{1}{\pi\tilde{E}}\int_{0}^{+\infty}(\omega-\langle\omega\rangle)^2\left|A(\omega)\right|^2 d\omega = \frac{1}{\pi}\int_{0}^{+\infty}\omega^2\frac{\left|A(\omega)\right|^2}{\tilde{E}}d\omega - \langle\omega\rangle^2 < (2\pi F)^2, \\ \omega = 2\pi f,\ \sigma_\omega = 2\pi\sigma_f,\ \tilde{E} = \left\|\overline{\Psi}(t)\right\|^2 = \int_{-\infty}^{+\infty}\left|\overline{\Psi}(t)\right|^2 dt = \frac{1}{\pi}\int_{0}^{+\infty}\left|A(\omega)\right|^2 d\omega,\end{gathered} \tag{5.8}$$

where mean time and mean frequency should be found as:

$$\langle t\rangle = \frac{1}{\tilde{E}}\int_{-\infty}^{+\infty} t\left|\overline{\Psi}(t)\right|^2 dt,\ \langle\omega\rangle = \frac{1}{\pi\tilde{E}}\int_{0}^{+\infty}\omega\left|A(\omega)\right|^2 d\omega = 2\pi\langle f\rangle = 2\pi\overline{f}\,. \tag{5.9}$$

If two functions, $\mathbf{\Psi(t)}$ *and* $A(\omega)$*, form a Fourier-integral pair, then they cannot both be of short duration. This is supported by the scaling theorem,*

$$\Psi(at) \leftrightarrow \frac{1}{|a|}A(\frac{\omega}{a})\,, \tag{5.10}$$

where ***"a"*** *is a real constant. The above claim, (5.10), also known as the* ***Uncertainty Principle****, can be given various interpretations, depending on the meaning of the term "duration".*

Using the time and frequency variances, (5.8), as the significant signal duration intervals, found for an energy-finite wave function $\Psi(t)$, it is possible to prove the validity of the following Uncertainty Principle (see **[7]** and **[8]**):

$$\begin{gathered}\text{If } \sqrt{t}\Psi(t)\to 0 \text{ for } |t|\to\infty, \\ \text{then:} \\ 2\pi\mathbf{TF} = \mathbf{T\Omega} > \mathbf{TF} > \sigma_t\sigma_\omega = 2\pi\sigma_t\sigma_f = \sqrt{(\Delta^2 t)(\Delta^2\omega)} = 2\pi\sqrt{(\Delta^2 t)(\Delta^2 f)} \geq \frac{1}{2}\,, \\ \mathbf{TF} = \frac{\mathbf{T\Omega}}{2\pi} > \frac{\mathbf{TF}}{2\pi} > \frac{\sigma_t\sigma_\omega}{2\pi} = \sigma_t\sigma_f = \frac{\sqrt{(\Delta^2 t)(\Delta^2\omega)}}{2\pi} = \sqrt{(\Delta^2 t)(\Delta^2 f)} \geq \frac{1}{4\pi}\end{gathered} \tag{5.11}$$

In the variance relations (5.11) we consider as apparent that absolute (or total) time and frequency durations, **T** and **F,** can never be shorter than time and frequency variances, σ_t and σ_f (and usually, they should be much larger than σ_t and σ_f). It is also clear that statistical, (5.11), and Quantum Mechanic's aspect of Uncertainty should be fully integrated (meaning correctly estimated) within absolute durations or interval values Uncertainty Relations, (5.7) - (5.7.2), as for instance,

$$\begin{aligned} &0 < \delta t \cdot \delta f = \delta x \cdot \delta f_x < \frac{1}{2} \le \sigma_t \cdot \sigma_f = \sigma_x \cdot \sigma_{f\text{-}x} < F \cdot T = F_x \cdot L \le \frac{1}{4 \cdot \delta t \cdot \delta f} = \frac{1}{4 \cdot \delta x \cdot \delta f_x}, \\ &0 < \delta t \cdot \delta \tilde{E} = \delta x \cdot \delta p < \frac{h}{2} \le 2\pi\sigma_t \cdot \sigma_{\tilde{E}} = \sigma_x \cdot \sigma_p < \tilde{E} \cdot T = P \cdot L \le \frac{h}{4 \cdot \delta t \cdot \delta f} = \frac{h}{4 \cdot \delta x \cdot \delta f_x}. \end{aligned} \quad (5.12)$$

In cases where normal, Gaussian or bell-curve amplitudes-envelopes distributions are applicable for certain wave packets (what means that 99% signal energy is captured by six times of standard deviation length, $6\sigma_{t,f,x,p,E}$) it should also be valid,

$$\begin{aligned} &\frac{1}{2} \le \sigma_t \cdot \sigma_f = \sigma_x \cdot \sigma_{f\text{-}x} < 36 \cdot \sigma_t \cdot \sigma_f = 36 \cdot \sigma_x \cdot \sigma_{f\text{-}x} < F \cdot T = F_x \cdot L, \\ &\frac{h}{2} \le 2\pi\sigma_t \cdot \sigma_{\tilde{E}} = \sigma_x \cdot \sigma_p < 36 \cdot 2\pi\sigma_t \cdot \sigma_{\tilde{E}} = 36 \cdot \sigma_x \cdot \sigma_p < \tilde{E} \cdot T = P \cdot L. \end{aligned} \quad (5.12\text{-}1)$$

In other words, (5.12) extends (and explains) the meaning of Uncertainty relations (in connection with (5.7) and (5.11)) and presents a kind of universal discretization of a wave function. To be sure that here elaborated Uncertainty Relations are correctly applied, when *related to certain wave-packet integrity and stability*, group velocity of such wave packet should satisfy the extended relation, $v = \frac{\delta x}{\delta t} = \frac{\sigma_x}{\sigma_t} = \frac{\sigma_f}{\sigma_{f\text{-}x}} = \frac{L}{T} = \frac{F}{F_x} \le c$ (see also equations (4.0.72) and (4.0.76) from Chapter 4.0, supporting the same form of group velocity).

In Mathematics, modern Telecommunications Theory and Digital Signal Processing practices we can find many methods and formulas for discretized signal representations, meaning that time-continuous signals, or wavefunctions, could be adequately represented (errorless, without residuals) if we just implement sufficiently short *time increments sampling* (of a particular continuous signal), and create discrete series of such signal samples. For instance, if a continuous wave function, $\Psi(t)$ is frequency band-limited (at the same time it should also be an energy-finite function), by applying Kotelnikov-Shannon-Nyquist-Whittaker Sampling theorem, we can present a function $\Psi(t)$, concerning its sample values $\Psi(n \cdot \delta t)$, for instance as superposition of sinc-functions (see **[8]**),

$$\begin{aligned} &\Psi(t) = a(t)\cos\varphi(t) = \sum_{n=-\infty}^{+\infty} \Psi(n \cdot \delta t) \frac{\sin\ \Omega(t - n \cdot \delta t)}{\Omega(t - n \cdot \delta t)} = \\ &= \sum_{n=-\infty}^{+\infty} a(n \cdot \delta t) \frac{\sin\Omega\ (t - n \cdot \delta t)}{\Omega(t - n \cdot \delta t)} \cos\varphi(n \cdot \delta t), \Psi(n \cdot \delta t) = a(n \cdot \delta t)\cos\varphi(n \cdot \delta t), \\ &\delta t \le \frac{\pi}{\Omega} = \frac{1}{2F}, \end{aligned} \quad (5.12\text{-}2)$$

where Ω is the highest frequency in the spectrum of $\Psi(t)$, and we could consider that Ω is the total frequency duration of the signal $\Psi(t)$.

If in the relation (5.7) we take $\Delta t = \delta t$, where δt is the sampling time interval taken from (5.12), then it will be $\Delta f = \delta f$, where δf should be the frequency-sampling interval when applying the Kotelnikov-Shannon-Whittaker-Nyquist sampling theorem on $A(f) = A(\omega/2\pi)$. It is also important to notice that δt and δf in (5.6) and (5.7), can take positive and negative values, but in the sampling process, they take only positive values. This way we can express the function $A(f)$ concerning its sample values $A(n \cdot \delta f)$, such as,

$$\mathbf{A(f)} = \sum_{n=-\infty}^{+\infty} \mathbf{A(n \cdot \delta f)} \frac{\sin 2\pi T(f - n \cdot \delta f)}{2\pi T(f - n \cdot \delta f)}, \ \delta f \leq \frac{1}{2T} \Leftrightarrow T \cdot \delta f \leq \frac{1}{2},$$

$$(T + \delta t)(F + \delta f) > \frac{1}{2}, \ \frac{1}{2} < TF \leq \frac{1}{4\delta t \cdot \delta f}, \ \delta t \cdot \delta f \leq \frac{1}{2}.$$

$$2\delta t \cdot \delta f \leq 4\delta t \cdot \delta f \cdot TF \leq 1. \quad (5.13)$$

Now, from (5.12) and (5.13) we can get even more general Uncertainty and energy expressions than before, such as,

$$|\delta t| \cdot |\delta f| \leq \left(T \cdot |\delta f| \approx |\delta t| \cdot F\right) \leq \frac{1}{2} \leq \sigma_t \sigma_\omega = 2\pi\sigma_t\sigma_f < TF \leq \frac{1}{4\delta t \cdot \delta f} < 2\pi TF,$$

$$\frac{|\delta t| \cdot |\delta f|}{TF} \leq \left(\frac{|\delta f|}{F} \approx \frac{|\delta t|}{T}\right) \leq \frac{1}{2TF} \leq \frac{\sigma_t\sigma_\omega}{TF} = \frac{2\pi\sigma_t\sigma_f}{TF} \leq \frac{1}{4\delta t \cdot \delta f \cdot TF} < 2\pi,$$

$$|\delta t| < \sigma_t < T, \ |\delta f| < \sigma_f < F, \ \frac{|\delta t|}{T} \approx \frac{|\delta f|}{F},$$

$$\tilde{E} = \int_{-\infty}^{+\infty} \Psi(t)^2 dt = \frac{1}{\pi}\int_{0}^{+\infty} |A(\omega)|^2 d\omega = \delta t \cdot \sum_{n=-\infty}^{+\infty} |\Psi(n \cdot \delta t)|^2 = \delta f \cdot \sum_{n=-\infty}^{+\infty} |A(n \cdot \delta f)|^2 =$$

$$= \int_{-\infty}^{+\infty} \hat{\Psi}^2(t)dt = \frac{1}{2}\int_{-\infty}^{+\infty} |\overline{\Psi}(t)|^2 dt = \frac{1}{2}\int_{-\infty}^{+\infty} \Psi(t)\Psi^*(t)dt = \frac{1}{2}\int_{-\infty}^{+\infty} a^2(t)dt = \tilde{p}u = h\overline{f}, \quad (5.14)$$

$$\overline{f} = \frac{\overline{\omega}}{2\pi} = \frac{1}{\tilde{E}}\int_{-\infty}^{+\infty} f(t)|\overline{\Psi}(t)|^2 dt = \frac{1}{\tilde{E}}\int_{-\infty}^{+\infty} \frac{\partial\varphi}{\partial t}|\overline{\Psi}(t)|^2 dt = \frac{1}{2\pi^2\tilde{E}}\int_{0}^{+\infty} \omega|A(\omega)|^2 d\omega =$$

$$= \sqrt{\frac{1}{2\pi^2 h}\int_{0}^{+\infty} \omega|A(\omega)|^2 d\omega} = \sqrt{\frac{1}{h}\int_{-\infty}^{+\infty} f(t)|\overline{\Psi}(t)|^2 dt} = \sqrt{\frac{1}{h}\int_{-\infty}^{+\infty} \frac{\partial\varphi}{\partial t}|\overline{\Psi}(t)|^2 dt} \ . \quad (5.14)$$

The next possibility for analogical extension of uncertainty and energy relations (5.14), could be to introduce similar relations between linear and orbital mechanical moments and electromagnetic charges, in a similar way as (5.3) was created.

As we know from Signal Analysis, it should be clear that if a specific signal (or wave) has a limited or very short duration in its time domain, then its duration in its frequency domain is very large, and vice versa (see (5.10)). Contrary to such general (mathematical) knowledge, here (in (5.14)), we are talking about energy finite and/or limited-duration signals in both (time and frequency) domains. This is mathematically not entirely correct, but under reasonable approximations (considering, for instance, 99% of the signal energy in both domains, and especially in cases of Gaussian-envelope signals) could be practically satisfied. Nature anyway (intrinsically) implements or produces an effective "signal filtering and time-frequency shrinking", during signal creation and propagation (also performing some signal modulations, shaping and attenuation) what eventually makes signal duration, and its energy content will be limited and well localized, both in its time and frequency domain.

To illustrate what a finite and limited duration (of an elementary) wave function means in time and frequency domains, let us imagine that we can find (or calculate) an equivalent, averaged wave function, which will replace real (arbitrary shaped) wave-packet function, $\Psi(\mathbf{x},\mathbf{t})$ *. We will place this new (mathematically useful) wave function into a rectangular-shape-amplitude borders (in both, time, and frequency, this way creating some equivalent "rectangular frames", or molds). We shall also request that this elementary, finite-energy wavefunction (or narrow-band wave-packet* $\Psi(\mathbf{x},\mathbf{t})$ *) has energy equal to one energy quant* $\tilde{\mathbf{E}} = \mathbf{hf}$*. For instance, the original signal amplitude in a time domain,* $\mathbf{a(t)}/\sqrt{2}$*, will be replaced by its effective and constant amplitude,* $\overline{\mathbf{a}}/\sqrt{\mathbf{2}}$*. The real signal duration,* $\mathbf{T}$*, in a time domain, will be replaced by an effective signal duration,* $\overline{\mathbf{T}}$*. The real signal amplitude in a frequency domain,* $A(\omega)/\sqrt{\pi}$*, will be replaced by its effective and constant amplitude,* $\overline{\mathbf{A}}/\sqrt{\pi}$*. The real signal duration,* $\mathbf{F}$*, in a frequency domain will be replaced by effective signal duration,* $\overline{\mathbf{F}}$*. Also, the signal mean (central, or carrier) frequency,* $\mathbf{f}$*, will be replaced by its effective central frequency* $\bar{\mathbf{f}}$*, (4.15), placed in the middle (like in a center of gravity) point of the interval* $\overline{\mathbf{F}}$*. Instead of rectangular signal frames or molds (as a method for signal averaging), we could also say that both time and frequency domain waveforms of the same wave function would be like Gaussian pulses (or Gaussian window functions) because the Gaussian function is optimally concentrated, finite, and limited in its joint time-frequency domain.*

Now, based on general wave-energy expressions found in (5.14), all relevant parameters of the one wave packet, approximated as a rectangular shape elementary wave-packet, or signal, can be presented as:

$$\tilde{E} = \int_{[t]} [\Psi(t)]^2 dt = \frac{\overline{a}^2}{2}\overline{T} = \frac{\overline{A}^2}{\pi} 2\pi\overline{F} = 2\overline{A}^2\overline{F} = h\overline{f} = \overline{pu}\,,\ \overline{TF} = \frac{1}{2},$$

$$\overline{T} = \frac{\sqrt{2}\overline{A}}{\overline{a}} = \left(\frac{2\overline{A}}{\overline{a}}\right)^2 \overline{F} = \frac{1}{2\overline{F}}\,, \overline{F} = (\frac{\overline{a}}{2\overline{A}})^2\overline{T} = \frac{\overline{a}}{2\sqrt{2}\overline{A}} = \frac{1}{2\overline{T}},$$

$$\overline{f} = \frac{\overline{\omega}}{2\pi} = \frac{1}{2\pi^2\tilde{E}}\int_0^{+\infty} \omega|A(\omega)|^2 d\omega = \sqrt{\frac{1}{2\pi^2 h}\int_0^{+\infty} \omega|A(\omega)|^2 d\omega}\,,$$

$$\overline{\lambda} = \frac{h}{\overline{p}},\ \overline{u} = \overline{\lambda}\,\overline{f} = \left\langle\frac{\omega}{k}\right\rangle = \left\langle\frac{\tilde{E}}{p}\right\rangle,\ \overline{v} = \left\langle\frac{\Delta\omega}{\Delta k}\right\rangle = \left\langle\frac{d\tilde{E}}{dp}\right\rangle, \tag{5.14-1}$$

$$h = 2\overline{A}^2 \cdot \frac{\overline{F}}{\overline{f}} = 2\overline{A}^2 \cdot \frac{\Delta f}{\overline{f}} = 6.62606876 \text{ x } 10^{-34}\text{Js},\ \Delta f = \frac{h\overline{f}}{2\overline{A}^2}$$

$$\frac{\overline{F}}{\overline{f}} = \frac{1}{2}\left(\frac{h}{\overline{A}}\right)^2 = \frac{1}{2\overline{T}\,\overline{f}},$$

$$\Psi(x,t) = a(x,t)\frac{\sin(\underline{\Delta\omega}\ t - \underline{\Delta k}\, x)}{(\underline{\Delta\omega}\ t - \underline{\Delta k}\, x)}\cos(\omega t - kx)\ (\Leftrightarrow) \text{wave packet}.$$

As we can see from (5.14-1), if the matter-wave packet has a higher mean frequency $\overline{f}$, its frequency width or duration $\overline{F}$ is shorter. Consequently, for low and exceptionally low mean frequency matter-wave packets (like in phenomenology related to gravitation, planetary and galactic systems), frequency duration $\overline{F}$ of corresponding wave-packets is exceptionally long.

With (5.14-1) we are again formulating conditions for signal discretization or defining meaning of elementary matter-waves domains (as being energy finite, optimally concentrated, Gaussian signals in all mutually conjugate domains), as addressed all over this chapter, since conditions and relations in (5.14-1), (5.2.1), (5.3) and (5.4.1) are mutually comparable and equivalent.

Another far-reaching aspect of uncertainty-relations (and signals quantifying) could be developed if instead of relatively stable, mean-frequency found in (5.14) and (5.9), we use immediate, time variable frequency **f(t)**, from the Analytical Signal model,

$$\omega(t) = \partial\varphi/\partial t = 2\pi f(t),$$
$$(\overline{\Psi}(x,t) = \Psi(x,t) + j\hat{\Psi}(x,t) = a(x,t)e^{j\varphi(x,t)}, \hat{\Psi} = H[\Psi]), \tag{5.14-2}$$

what is left to be analyzed another time.

Regarding here-presented aspects of Uncertainty relations we should make a difference between Macro-Uncertainty Relations valid between total interval lengths of mutually coupled or conjugate spectral domains (expressed in absolute conjugate-variables intervals), and Micro-Uncertainty Relations between minimal signal sampling segments, when signal is "atomized" (or sampled) by means of Digital Signal Processing. Consequently, we should be able to formulate the complete set of United Micro and Macro Uncertainty Relations, as well as merge them with the extended meaning of "de Broglie periodicity intervals" presented in

T.5.4). The contemporary Heisenberg concept of Uncertainty Relations in Physics is still slightly different when compared with the Mathematical Uncertainty and here-described objectives and results (and sometimes presents a more confusing and uncertain, than clear view regarding that problematic).

5.2. Uncertainty Relations, Fields, and Transformation Domains

In modern physics (starting from Heisenberg) we often find oversimplified (and sometimes mystified) comments about Uncertainty relations without proper explanation of what kind of entities, values, phenomena, categories, signals, dimensions, domains, and tangible-physics items, such Uncertainty relates (see for instance T.5.1 and T.5.2). Most of microphysics literature is presenting Uncertainty relations almost exclusively as something typically valid for micro-world of elementary particles and other quantum entities, what is not correct, because general time-frequency domain, signal analysis does not assume any (a priory) size limitations on the "time-space-frequency" and "object-shape-size" represented by certain wave function.

Several crucial situations are explaining the roots and background of Uncertainty relations. From mathematics, there is not 1:1 or point-to-point mapping, imaging, or correspondence between specific function in its Original and its Spectral domain (valid in both directions, whatever we take as Original or Spectral domain). This property first found and described in mathematics dealing with Signal Analysis, Spectrum Analysis, Fourier transformations, etc., is generally valid and applicable without limitations to all wavefunctions from micro and macro physics. From Physics point of view, the most significant achievement was the formulation of generally valid conservation laws (and associated universal principles) such as: Energy conservation, Momentum conservation, etc., found as consequences of space-time uniformity and isotropy (in isolated inertial systems). Euler-Lagrange-Hamilton Theory, as well as Quantum Mechanics differently and correctly contributed to the conclusion that time and energy domains are Original and Spectral domain to each other. Max Planck and Mileva and Albert Einstein also formulated the significant relation for photon as narrow-band wave-packet energy, $\tilde{\mathbf{E}} = \mathbf{hf}$, this way connecting energy and frequency (because this way doing, fully correct explanations of Photoelectric and Compton effects confirmed such concept). Einstein showed that there is a direct equivalence or proportionality between mass end energy as $\mathbf{E} = \mathbf{mc^2}$, and that the same relation extends to any form of energy, to particles, quasiparticles, fields, and waves. Luis de Broglie (indirectly and probably unintentionally) discovered another *(Original Domain)-to-(Spectral Domain) couple*, by correctly formulating his matter-wavelength $\lambda = h / p$, this way connecting position (distance, length, or spatial dimension) with linear momentum, leading to $p = hf_s = \hbar\mathbf{k}$ (where $\mathbf{f_s}$ is the spatial frequency).

Physics found that "Nature obeys Fourier Spectrum Analysis", or that predictions of Spectrum Analysis are confirmable by experimental evidence (this way precisely connecting time and frequency, and position-momentum domains). It is obvious that Physics deals with two (mutually coupled and conjugate) worlds: the world of (our perceptible) Original domains, and the world of corresponding Spectral domains. Using such concept as a guiding idea (combined with already developed analogies from earlier chapters of this book, as a predictive platform for generating new physics related concepts), we can again formulate several essential Original-to-Spectral, mutually conjugate domain couples, which should be the building blocks of our Universe, as for instance (see T.1.6, T.3.1, T.3.2, T.3.3 and T.5.1):

T. 5.5. Mutually conjugate variables

Original Domains $\leftrightarrow$	$\leftrightarrow$ Spectral Domains
Time = t	Energy = $\tilde{E}$, (and/or frequency = $f = \tilde{E}/h$)
Displacement = $x = S\dot{\tilde{p}} = S\tilde{F}$, ($\tilde{F}$ = force)	Momentum = $\tilde{p} = \tilde{m}\dot{x} = \tilde{m}v$
Angle = $\alpha = S_R\dot{L} = S_R\tau$	Angular momentum = $L = J\dot{\alpha} = J\omega$
Electric charge = $q_{el.} = \Phi_{el.} = C\dot{q}_{mag.} = Ci_{mag.}$	Magn. charge = $q_{mag.} = \Phi_{mag.} = L\dot{q}_{el.} = Li_{el.}$

Since there is no chance to make 1:1, or point-to-point mapping, imaging, or correspondence between Original and Spectral domain points, there should exist certain intervals-relation between coupled domains, describing the amount of mutual interval matching or mismatching, named in physics the Uncertainty relations, (see (5.2), (5.3), (5.7) and (5.14)). Also, in physics we do not have any strong platform to say which domain is an Original, and which one is its Spectral domain (since both could coincidently exist, be equally important, mathematically, and experimentally verifiable). For instance, Quantum Mechanics (in connection with traditional Schrödinger's equation) formulates and exploits (at least) two of (bi-directional) Original to Spectral domain transformations (or associations), also found in T.1.1 and T.5.5, such as: (time)-(energy), given by $t \leftrightarrow f$, and (position)-(momentum), $\tilde{p} \leftrightarrow -j\hbar\frac{\partial}{\partial x}$. By analogy (see **T.5.5**), we could also imagine (or propose) to introduce two more associations (concerning rotation and electromagnetic field), for instance: (angle)-(angular momentum), $L \leftrightarrow -j\hbar\frac{\partial}{\partial \alpha}$, and (electric charge)-(magnetic charge), $q_{mag.} \leftrightarrow -j\hbar\frac{\partial}{\partial q_{el.}}$. Here it is convenient to mention that independent, separate, and self-standing unipolar magnetic charges naturally do not exist (except as mutually coupled parts of magnetic dipoles).

Here we are in a strong position to explain the most interesting conceptual platform of this book predicting that the field of Gravitation should have its *complementing field* (at present still hypothetical), caused by mass rotation, in the same way as Electric and Magnetic fields are mutually dependent and complementary fields (see also (4.30) and (4.31)). As we can see from the analogies given in T.5.5, as well as from $q_{mag.} \leftrightarrow -j\hbar\frac{\partial}{\partial q_{el.}}$, we should be able to find (analogically) the couple of _Gravitation-Rotational_, mutually conjugate charges, $q_{gravitat.}, q_{rotat.}$ (analog to electric and magnetic charges, $q_{el.}, q_{mag.}$), that will mutually respect similar relation as one valid between electric and magnetic charges. For instance, such charges will be an Original and Spectral domain to each other or satisfy the mapping and operator relation: $q_{rotat.} \leftrightarrow -j\hbar\frac{\partial}{\partial q_{gravitat.}}$).

Direct and inverse Fourier transforms (of certain wave function), as relations between different charges (or between Original and Spectral domain-couples) should also satisfy Uncertainty Relations (5.2) and (5.5), and can be symbolically and analogically (on a simplified and intuitive way), generally presentable as:

$$F\left\{\Psi\left(\|S\|\right)\right\} = U\left(\|Q\|\right) = \int_{-\infty}^{+\infty} \Psi\left(\|S\|\right)\cdot e^{-j\|Q\|\cdot\|S\|} d\|S\| ,$$

$$F^{-1}\left\{U\left(\|Q\|\right)\right\} = \Psi\left(\|S\|\right) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} U\left(\|Q\|\right)\cdot e^{-j\|Q\|\cdot\|S\|} d\|Q\| ,$$

$$\left\{\begin{array}{l} \|S\| = \begin{pmatrix} t \\ x \\ \alpha \\ q_{el.} \\ q_{gravitat.} \end{pmatrix} = \begin{pmatrix} \text{time} \\ \text{position} \\ \text{angle} \\ \text{el.chrg.} \\ \text{grv.chrg.} \end{pmatrix}, \|Q\| = \begin{pmatrix} f;(\tilde{E} = hf) \\ \tilde{p};(\lambda = h/\tilde{p}) \\ L \\ q_{mag.} \\ q_{rotat.} \end{pmatrix} = \begin{pmatrix} \text{frequency} \\ \text{momentum} \\ \text{ang.momnt.} \\ \text{mag.chrg.} \\ \text{rotat.charg.} \end{pmatrix}, \\ \|\Delta S\|\cdot\|\Delta Q\|^T = \|\Delta Q\|\cdot\|\Delta S\|^T \geq \frac{h}{2} \end{array}\right\} \Leftrightarrow$$

$$\Leftrightarrow \begin{pmatrix} h\cdot(\Delta t\cdot\Delta f) \\ \Delta x\cdot\Delta p \\ \Delta\alpha\cdot\Delta L \\ \Delta q_{el.}\cdot\Delta q_{mag.} \\ \Delta q_{gravitat.}\cdot\Delta q_{rotat.} \end{pmatrix} \geq \frac{h}{2}. \tag{5.15}$$

Delta intervals in relations (5.15) should be considered as total, absolute signal lengths or durations in both mutually conjugated domains (not as standard deviations and statistical items).

It is interesting to notice that in the case of electric and magnetic charges (and fields) it is possible to have both, Original and Spectral domain equally and coincidentally present in the same, real-time (and in the same space), and that both create (electric and magnetic) fields around them. Is something like that possible for any other conjugate couple of Original-Spectral domains (5.15), is the question to answer? What should be the Original-Spectral domain couple that is most relevant for gravitation ($\Delta q_{gravitat.}, \Delta q_{rotat.}$), is also the question to answer. Anyway, it would be difficult and illogical to show that **(t, x, α) (=) (time, length, angle),** from (5.15), are sources of some presently known or unknown fields. Also, we must find the answer about what should be the essential, minimal, and complete sets of elements of Original and Spectral, mutually conjugate domains ($\|\mathbf{S}\|, \|\mathbf{Q}\|$) relevant for the description of our universe.

It is almost needless to say that Uncertainty relations like (5.2), (5.3) and (5.14) apply to all the values found in (5.15), T.5.1, T5.2, and T.5.5, belonging to vectors of Original and Spectral domains ($\|\mathbf{S}\|, \|\mathbf{Q}\|$). This time we can create another generalization of uncertainty-relations (5.2) and (5.15), based on (4.32), such as:

$$\left\{\begin{array}{l} dP_{\Sigma} = \sum_{(i)} \frac{\alpha_i}{\dot{q}_i} dE(q_i, \dot{q}_i, \ddot{q}_i, \dddot{q}_i, ..., q_i^{(n)}, t) \Rightarrow \Delta P_{\Sigma} = \sum_{(i)} \frac{\alpha_i}{\dot{q}_i} \Delta E_i \\ E_i = E(q_i, \dot{q}_i, \ddot{q}_i, \dddot{q}_i, ..., q_i^{(n)}, t), \\ dX_{\Sigma} = V_{\Sigma} dt = \frac{\sum_{(i)} dE_i}{dP_{\Sigma}} dt = \frac{\sum_{(i)} \dot{E}_i}{F_{\Sigma}} dt = \frac{\sum_{(i)} dE_i}{F_{\Sigma}} \Rightarrow \Delta X_{\Sigma} = \frac{\sum_{(i)} \Delta E_i}{F_{\Sigma}} \\ \mathcal{F}\left\{\Psi\left(\|X_{\Sigma}\|\right)\right\} = U\left(\|P_{\Sigma}\|\right) = \int_{-\infty}^{+\infty} \Psi\left(\|X_{\Sigma}\|\right) \cdot e^{-j\|P_{\Sigma}\| \cdot \|X_{\Sigma}\|} d\|X_{\Sigma}\|, \\ \mathcal{F}^{-1}\left\{U\left(\|P_{\Sigma}\|\right)\right\} = \Psi\left(\|X_{\Sigma}\|\right) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} U\left(\|P_{\Sigma}\|\right) \cdot e^{-j\|P_{\Sigma}\| \cdot \|X_{\Sigma}\|} d\|P_{\Sigma}\| \end{array}\right\} \Rightarrow$$

$$\|\Delta X_{\Sigma}\| \cdot \|\Delta P_{\Sigma}\| = \left| \frac{\sum_{(i)} \Delta E_i}{F_{\Sigma}} \cdot \sum_{(i)} \frac{\alpha_i}{\dot{q}_i} \Delta E_i \right| \geq \frac{h}{2}. \qquad (5.16)$$

We could also adjust all components of vectors $\|\mathbf{S}\|, \|\mathbf{Q}\|$ and $\|\mathbf{X}_{\Sigma}\|, \|\mathbf{P}_{\Sigma}\|$ from (5.15) and (5.16) to be Lorentz-covariant in Minkowski space (for instance to create 4-space vectors $\|\overline{\mathbf{S}}_4\|, \|\overline{\mathbf{Q}}_4\|$ and $\|\overline{\mathbf{X}}_{\Sigma-4}\|, \|\overline{\mathbf{P}}_{\Sigma-4}\|$, similar to (4.33) - (4.37)). Since the product between any couple of Lorentz-covariant vectors (in the Minkowski space) always presents an (inertial coordinate system) invariant, it is possible to show that uncertainty relations, or products made of such 4-vectors (in fact made between mutually related, conjugate, absolute, total duration intervals of such 4-vectors),

$$\|\Delta\overline{S}_4\| \cdot \|\Delta\overline{Q}_4\| = \|\Delta\overline{X}_{\Sigma 4}\| \cdot \|\Delta\overline{P}_{\Sigma 4}\| = \underline{\text{Constant } (= \text{invariant})} \geq \frac{h}{2}, \qquad (5.17)$$

are no more uncertain relations (being equally applicable to the world of microparticles, or planetary systems and galaxies, considering that Planck $\mathbf{h}$-constant will be replaced by another $\mathbf{H}$-constant valid for planetary systems). Also, when creating a New Topology and New Metrics of our universe, we should find a way to incorporate and merge results and predictions from T.5.2, (4.32), (4.37), (5.15), (5.16) and (5.17) in it.

As an example, let us explore (5.17) as the case of uncertainty relations between absolute amounts or durations of energy $\tilde{E}$, time t, momentum p and signal length or position x. Until present, we know such uncertainty relations (without considering 4-vectors), as $\Delta t \cdot \Delta\tilde{E} = h \cdot (\Delta t \cdot \Delta f) = \Delta x \cdot \Delta p \geq \frac{h}{2}$. Let us now consider that momentum $\mathbf{p}$ is replaced by the corresponding 4-vector $\overline{P}_4 = (p, \frac{E}{c})$, and that the same Uncertainty Relation will be applicable between corresponding 4-vectors,

$$\left[p \to \overline{P}_4 = (p, \frac{E}{c}) \right] \Rightarrow \left[\Delta p \to \Delta\overline{P}_4 = (\Delta p, \frac{\Delta E}{c}) \right],$$
$$\left[\Delta p \cdot \Delta x \geq \frac{h}{2} \right] \Rightarrow \left[\left| \Delta\overline{P}_4 \cdot \Delta\overline{X}_4 \right| \geq \frac{h}{2} \right] \Leftrightarrow \left| (\Delta\overline{P}_4)^2 \cdot (\Delta\overline{X}_4)^2 \right| \geq \left(\frac{h}{2}\right)^2, \tag{5.18}$$
$$(\Delta\overline{P}_4)^2 = (\Delta p)^2 - \left(\frac{\Delta E}{c}\right)^2 = -\left(\frac{E_0}{c}\right)^2 = \text{invariant}.$$

There are at least two different options for how to continue from (5.18). We can first consider that Δx is an ordinary, total signal length (in three-dimensional space), equal to $|\Delta x| = |\Delta\overline{X}_4|$, and get,

$$\left| (\Delta\overline{P}_4)^2 \cdot (\Delta\overline{X}_4)^2 \right| = \left| (\Delta\overline{P}_4)^2 \cdot (\Delta x)^2 \right| = \left\| \left[(\Delta p)^2 - \left(\frac{\Delta E}{c}\right)^2 \right] \cdot (\Delta x)^2 \right\| = \left| -\left(\frac{E_0}{c}\right)^2 \cdot (\Delta x)^2 \right| \geq \left(\frac{h}{2}\right)^2 \Rightarrow$$
$$\Rightarrow \left[(\Delta p)^2 \cdot (\Delta x)^2 - \left(\frac{\Delta E}{c}\right)^2 \cdot (\Delta x)^2 = \left(\frac{E_0}{c}\right)^2 \cdot (\Delta x)^2 \geq \left(\frac{h}{2}\right)^2 \right] \Rightarrow \tag{5.19}$$
$$\Rightarrow \Delta x = \frac{hc}{2\sqrt{c^2(\Delta p)^2 - (\Delta E)^2}} \geq \frac{hc}{2E_0} = \frac{hc}{2mc^2} = \frac{1}{2}\frac{h}{mc} = \frac{\lambda_c}{2}.$$

Another option is to consider that Δx can analogically be replaced by its Relativistic theory 4-vector, $\Delta\overline{X}_4 = (\Delta x, c\Delta t)$ and this will produce,

$$\left[x \to \overline{X}_4 = (x, ct) \right] \Rightarrow \left[\Delta x \to \Delta\overline{X}_4 = (\Delta x, c\Delta t) \right], (\Delta\overline{X}_4)^2 = (\Delta x)^2 - c^2(\Delta t)^2$$
$$\left| (\Delta\overline{P}_4)^2 \cdot (\Delta\overline{X}_4)^2 \right| = \left\| \left[(\Delta p)^2 - \left(\frac{\Delta E}{c}\right)^2 \right] \cdot \left[(\Delta x)^2 - c^2(\Delta t)^2 \right]^2 \right\| = \left| -\left(\frac{E_0}{c}\right)^2 \cdot \left[(\Delta x)^2 - c^2(\Delta t)^2 \right] \right| =$$
$$= E_0^2 \cdot \left| (\Delta t)^2 - \frac{(\Delta x)^2}{c^2} \right| \geq \left(\frac{h}{2}\right)^2 \Leftrightarrow \left| c^2(\Delta t)^2 - (\Delta x)^2 \right| \geq \left(\frac{hc}{2E_0}\right)^2 \Leftrightarrow \tag{5.20}$$
$$\Leftrightarrow \Delta x \geq \sqrt{\left| (\Delta x)^2 - c^2(\Delta t)^2 \right|} \geq \frac{hc}{2E_0} = \frac{hc}{2mc^2} = \frac{1}{2}\frac{h}{mc} = \frac{\lambda_c}{2}.$$

Anyway, in both situations regarding relativistic interval length, we can see that minimal signal (or wave packet) length is equal to the half of Compton wavelength, $(\Delta x)_{minimal} = \frac{hc}{2E_0} = \frac{1}{2}\frac{h}{mc} = \frac{\lambda_c}{2}$. This is the same indicative conclusion as found earlier in (5.4) and (5.2.1), meaning that minimal length of certain signal (that represents moving particle) cannot be shorter than belonging Compton, resonant half-wavelength, concerning standing waves, atomized or discretized signal packing. In other words, Uncertainty Relations are defining the elementary building blocks of matter, or frontiers between a micro and macro world of Physics.

From (5.19) and (5.20) we could have some doubts regarding 4-vector associated to the total signal duration, but principal conclusions regarding the significance of Compton wavelength and standing waves formatting of elementary matter domains will still stay valid. The definition of 4-dimensional relativistic space-time interval (as we find it in Relativity theory, and in this book) is maybe still speculative and controversial concept (probably not completely proven), and most probably that it can (and should) be modified and upgraded starting from 4-vector of relativistic velocity (which can be considered as correctly defined), as for instance,

$$\begin{aligned}
&\overline{V}_4 = (\gamma v, \gamma c) = \frac{d}{dt}\overline{X}_4 \Rightarrow d\overline{X}_4 = \overline{V}_4 dt = (\gamma v \cdot dt, \gamma c \cdot dt) \Rightarrow \\
&\Rightarrow (\Delta S)^2 = (\Delta\overline{X}_4)^2 = \left(\int_{[\Delta T]} \gamma v \cdot dt\right)^2 - \left(c\int_{[\Delta T]} \gamma \cdot dt\right)^2, \; v = v(x,y,z,t), \; \gamma = 1/\sqrt{1-v^2/c^2}, \\
&\left|(\Delta\overline{P}_4)^2 \cdot (\Delta\overline{X}_4)^2\right| = \left\|\left[(\Delta p)^2 - \left(\frac{\Delta E}{c}\right)^2\right] \cdot \left[\left(\int_{[\Delta T]} \gamma v \cdot dt\right)^2 - \left(c\int_{[\Delta T]} \gamma \cdot dt\right)^2\right]\right\| = \\
&= \left|-\left(\frac{E_0}{c}\right)^2 \cdot \left[\left(\int_{[\Delta T]} \gamma v \cdot dt\right)^2 - \left(c\int_{[\Delta T]} \gamma \cdot dt\right)^2\right]\right| = \left|\left(\frac{E_0}{c}\right)^2 \cdot \left[\left(c\int_{[\Delta T]} \gamma \cdot dt\right)^2 - \left(\int_{[\Delta T]} \gamma v \cdot dt\right)^2\right]\right| = \\
&= E_0^2 \cdot \left|\left(\int_{[\Delta T]} \gamma \cdot dt\right)^2 - \frac{1}{c^2}\left(\int_{[\Delta T]} \gamma v \cdot dt\right)^2\right| = \left(E_0 \int_{[\Delta T]} \gamma \cdot dt\right)^2 \geq \left(\frac{h}{2}\right)^2 \Leftrightarrow \\
&\Leftrightarrow \Delta S = \sqrt{\left|\left(\int_{[\Delta T]} \gamma c \cdot dt\right)^2 - \left(\int_{[\Delta T]} \gamma v \cdot dt\right)^2\right|} = c\int_{[\Delta T]} \gamma \cdot dt \geq \frac{hc}{2E_0} = \frac{h}{2mc} = \frac{\lambda_c}{2}.
\end{aligned} \tag{5.21}$$

ΔT in (5.21) is the total signal (or wave packet) duration in its time domain.

In Relativity Theory the same situation regarding challenging relations between 4-vectors of space-time and velocity are addressed by considering the proper time t_0, which is replacing observer time, $t = \gamma t_0$, but (based on (5.19) – (5.21)) there is still a space to ask if the picture about correct foundations of space-time 4-vector is completed.

Similarly, we can address Uncertainty Relations from (5.15), (5.16) and (5.17) replacing normal vectors (from the three-dimensional space) with similar 4-vectors in the Minkowski space. ***Next decisive and essential step will be to unite Minkowski 4-vectors and Analytic Signal concepts (see more in Chapter 10).***

[♣ COMMENTS & FREE-THINKING CORNER:

By extending the concept of coupling between Original and Spectral domains, we can reorganize and generalize the analogy and symmetry tables established in the first chapter of this book. This approach also allows us to generalize Schrödinger-like equations (such as (4.25)) to apply across all domains of Physics, including Quantum Mechanics, Gravitation, and Maxwell's Electromagnetic Theory.

For example, in this book, the square of the wave function represents power. Power can be expressed in various forms, such as the product of velocity and force, voltage and current, or angular velocity and torque (as shown in T.5.2). In some cases, we may encounter the sum of these power components (see also (4.30) and (4.31)). By applying the generalized Schrödinger equation (4.25) to such wave functions, we can develop new wave equations where currents, voltages, velocities, forces, and torques are explicitly incorporated. This approach aligns with Maxwell's electromagnetic theory, which predates Schrödinger's formulation.

When the wave function is considered solely as a normalized probability distribution in the four-dimensional space-time domain, diversifying Schrödinger-like equations becomes challenging. Each new attribute, such as current, voltage, force, momentum, or torque, requires formulating an appropriate operator. The founders of Orthodox Quantum Mechanics focused primarily on modeling the wave function within a probability framework. Subsequent adherents have largely maintained this focus, but while this framework remains effective, it should not be regarded as the only or final approach.

Modern Statistical Electrodynamics has introduced similar, though often implicitly formulated, challenges to Orthodox Quantum Mechanics, suggesting the need for re-evaluation of its unique and irreplaceable status.

Regarding Gravitation, different moments (mass, linear momentum, and angular momentum, ($\mathbf{m}$, $\mathbf{p}$, and $\mathbf{L}$)) are significant, but they do not fully address the concept of Gravity-Rotational charges as discussed here. This indicates a potential need for modifications and upgrades to current Gravitation Theory, particularly considering predictions based on electromechanical analogies. This raises an essential question: What the real sources of Gravity and Inertia are if static mass alone is insufficient to describe these phenomena (as suggested by (4.30) - (4.32) and (5.15) and (5.16))?

The answer likely lies in understanding interactions, such as those described by force expressions (2.1), (2.2), and (2.4). This issue may be further clarified within a multidimensional space or coupling environment (see [10] and [11]). It is also crucial to recognize that mass always creates a gravitational field or space deformation around itself, as stated in Newton's Law of Universal Gravitation and Einstein's General Relativity. This book proposes, based on analogies and insights, how to explore more general sources of gravitation that could be analogous or symmetrical to electromagnetic field sources. ♣]

5.3. Central Differences, Uncertainty and Continuum

There is another platform closely related to signals quantizing, Uncertainty relations, errors estimation, and to the possibility of creating different analogies, which is a direct consequence of mathematics of finite differences, and whose significance has not been exposed and exploited enough in physics. In table T.5.2, the replacement of infinitesimal differences with similar finite differences ($dt = \Delta t$, $dE = \Delta E$…) was obvious and directly made, just following the idea to create dimensional analogies. It will be good to pay attention to the background platform that makes such replacements possible and to know when this is entirely correct (see [82]).

If finite differences (of Δ - types) <u>belong to the class of central or symmetrical differences</u>, then, applying them to a large group of continual functions (often used in mathematical physics), we will obtain the same (or in some cases almost the same) results as in differential analysis with infinitesimal differences. This situation gives us a chance, whenever something like that is applicable, to transform many differential equations of mathematical physics, almost directly (by replacing $d (=) \Delta$, $dy \to \Delta y$, $dx \to \Delta x$...), into simpler algebraic, "<u>quantified, discrete and finite" equations</u>, respecting the rules of central and finite differences. In many cases the same method also replaces higher levels of infinitesimal derivatives $\mathbf{d^n}$ with their simple analog and finite, differential $\mathbf{\Delta^n}$ **-** operators.

The central difference of the function $F(x)$ can be defined as $\Delta F(x) = F(x + \alpha\ \Delta x) - F(x - (1- \alpha)\Delta x)$, where $0 \leq \alpha \leq 1$. If $\alpha = 1/2$, then $\Delta F(x)$ presents central and symmetrical difference. There are cases of functions and differential equations where the first derivation $dF(x)/dx$, is identical (or almost identical) to $\Delta F(x)/\Delta x$, where $\Delta F(x)$ is the central and symmetrical difference, ***without applying $\Delta x \to 0$ (meaning that we can consider Δx = Const.).*** Based on the previously mentioned specifics of central differences, we can try to explain the relations between the Physics of Continuum and Quantum Physics, and provide the part of the answer why, where/when and how nature made quantization of its elementary entities.

The mathematical models of reversible, continual, smooth, and deterministic processes in physics (related to their differential equations) seem to belong to the family of functions where differential, central-symmetrical ***Δ^n-operators*** *can replace infinitesimal derivatives d^n. This conditional (or somewhat hypothetical) statement should be taken as a starting platform for a new research task.*

For instance, as can be found in chapter 4.0 (equations (4.0.73) to (4.0.76)), we can apply the symmetrical central differences method to determine mutual relations between relevant energy-momentum domains' variations. This is, in a few elementary steps, creating an almost completed picture of Uncertainty Relations, and at the same time reinforcing foundations of Particle-Wave Duality as follows,

$$\left\{\begin{array}{l}\overline{P}_4=\overline{P}\left[\vec{p}=\gamma m\vec{v},\frac{E}{c}=\gamma mc\right],\\ \overline{P}^2=\vec{p}^2-\frac{E^2}{c^2}=-\frac{E_0^2}{c^2},\\ E_0=mc^2,\ \ E=\gamma E_0,\\ \vec{p}^2c^2+E_0^2=E^2,\\ p\to p\pm\Delta p,\\ E\to E\pm\Delta E\end{array}\right\}\Rightarrow\left\{\begin{array}{l}(p+\Delta p)^2c^2+E_0^2=(E+\Delta E)^2\\ (p-\Delta p)^2c^2+E_0^2=(E-\Delta E)^2\\ \overline{v}=\frac{\Delta x}{\Delta t}=\frac{\delta x}{\delta t}=\frac{\delta\omega}{\delta k}=\frac{\Delta\omega}{\Delta k}\end{array}\right\}\Leftrightarrow\left\{c^2\cdot p\,\Delta p=E\,\Delta E\Leftrightarrow\frac{\Delta E}{\Delta p}=c^2\frac{p}{E}=\overline{v}\right\}\Rightarrow$$

$$\Rightarrow\frac{\Delta x}{\Delta t}=\frac{\delta x}{\delta t}=\frac{\Delta E}{\Delta p}=c^2\frac{p}{E}=\overline{v}=h\frac{\Delta f}{\Delta p}=\frac{\Delta\tilde{E}}{\Delta p}=\frac{\Delta\omega}{\Delta k}=\frac{\delta\omega}{\delta k}\leq c.$$

(4.0.73), (4.0.74)

$$\left\{\begin{array}{l}u=\frac{\omega}{k},\ \omega=ku,\\ k\to k\pm\Delta k,\\ u\to u\pm\Delta u\end{array}\right\}\Rightarrow\Delta\omega=(k+\frac{1}{2}\Delta k)(u+\frac{1}{2}\Delta u)-(k-\frac{1}{2}\Delta k)(u-\frac{1}{2}\Delta u)=$$

(4.0.75)

$$=k\Delta u+u\Delta k\Leftrightarrow\left\{\begin{array}{l}\overline{v}=\frac{\Delta\omega}{\Delta k}=u+k\frac{\Delta u}{\Delta k}=\frac{\Delta\tilde{E}}{\Delta p}=\frac{\Delta x}{\Delta t}\\ =\text{average group velocity}\end{array}\right\}\Leftrightarrow\left\{\begin{array}{l}v=u+k\frac{du}{dk}=\frac{d\omega}{dk}=\frac{d\tilde{E}}{dp}=\frac{dx}{dt}\\ =\text{immediate group velocity}\end{array}\right\}.$$

$$\left\{\begin{array}{l}\overline{v}=u+k\frac{\Delta u}{\Delta k}=\frac{\Delta\omega}{\Delta k}=\frac{\Delta\tilde{E}}{\Delta p}=\frac{\Delta x}{\Delta t},\\ |\Delta x\,\Delta p|=|\Delta t\,\Delta\tilde{E}|=h|\Delta t\,\Delta f|>h/2,\ \ \Delta\tilde{E}=h\Delta f,\\ 0<\delta t\cdot\delta f=\delta\,x\cdot\delta f_x<\frac{1}{2}\leq F\cdot T=F_x\cdot L\leq\frac{1}{4\cdot\delta t\cdot\delta f}=\frac{1}{4\cdot\delta x\cdot\delta f_x}\end{array}\right\}\Rightarrow$$

$$\Rightarrow\overline{v}=\frac{\Delta x}{\Delta t}=\frac{\Delta E}{\Delta p}=h\frac{\Delta f}{\Delta p}=\frac{\Delta\tilde{E}}{\Delta p}=\frac{\Delta\omega}{\Delta k}=u+k\frac{\Delta u}{\Delta k}=\frac{\delta x}{\delta t}=\frac{\delta\omega}{\delta k}=\frac{dx}{dt}=\frac{d\omega}{dk}.\quad(4.0.76)$$

The key takeaway from this review of Uncertainty Relations is that there is a singular fundamental Uncertainty Principle that is universally applicable across Mathematics, Quantum Mechanics, and all Physics.

It is evident that the current treatment of Uncertainty Relations in Physics is still incomplete and requires further updates. The particle-wave duality theory and Schrödinger's wave equation need to be revised to encompass all aspects of matter waves, including thermodynamics, gravity, rotational dynamics of particles and fields, and electromagnetic theory (as initiated in sections (5.15) to (5.21)). Our goal is to establish universally valid Uncertainty Relations that address signal or wave-packet lengths and durations in relevant space, time, and frequency domains. Specifically, we aim to develop:

A) Relations between absolute signal duration values across different domains.
B) Relations between statistically derived mean-duration values and their associated standard deviations in all domains.
C) Relations considering optimal signal sampling intervals in accordance with Nyquist-Kotelnikov-Shannon-Whittaker sampling rules.

These relations must be integrated and compared with modern methods and practices in "Error Analysis" and "Central Differences" calculations (see [96] for more details).

In conclusion, this review provides new insights into the fundamental understanding of Uncertainty and Certainty Relations. Future work should focus on formulating a more comprehensive and general concept of Uncertainty that extends beyond the current practices in Physics.

[♣ COMMENTS & FREE-THINKING CORNER: Examples, ... in process... only a reminder

Analogies, if well and correctly established and applied, are the best possible platform for understanding and connecting different fields of physics. Knowing relevant analogies, we can (almost) start from every distinct physics theory and (try to) extend its most significant forms/laws to other fields.

*For instance, let us apply **central differences** on the simplest form of D. Bernoulli's fluid flow equation (valid for tube segment with negligible viscosity, laminar fluid flow):*

$$\frac{1}{2}\rho v^2 + \rho g z + p = \text{const.} = p_0 \qquad (5.22)$$

*where $\rho = \frac{\mathbf{m}}{\mathbf{V}}$ -is fluid density, **v** -is fluid speed, g -is gravitational constant, p -is pressure in the fluid segment, and z -is a vertical coordinate/position of the analyzed fluid segment. Apparently, (5.22) can be transformed as follows (**applying central, Δ differences, introduced above**):*

$$\begin{aligned}
&\frac{\frac{1}{2}mv^2}{V} + \frac{mgz}{V} + p \;=\; \frac{E_k}{V} + \frac{E_p}{V} + p = \frac{\sum_{(i)}(E_{ki} + E_{pi})}{V} + p \;=\; \text{const.} = p_0 \Rightarrow \\
&\Rightarrow \sum_{(i)}(E_{ki} + E_{pi}) + pV = p_0 V \Leftrightarrow \sum_{(i)} E_i + pV = p_0 V \Leftrightarrow \sum_{(i)} E_i + (p - p_0)V = 0 \\
&\Rightarrow \sum_{(i)} \Delta E_i + \;(p - p_0)(\Delta V) + (\Delta p)V + \Delta p \Delta V = \sum_{(i)} \Delta E_i + \Delta(pV) = \\
&= \sum_{(i)} \Delta E_i + \Delta(\frac{F}{S} S \cdot \delta x) = \sum_{(i)} \Delta E_i + \Delta(F \cdot \delta x) = 0 \;, E_i = E_{ki} + E_{pi} .
\end{aligned} \qquad (5.23)$$

where V is the volume of the analyzed fluid/tube segment and, $\mathbf{E_k}$ and $\mathbf{E_p}$ are corresponding kinetic and potential energy of the fluid segment. The last part of (5.23) can be taken as the starting point for creating different analogous forms, where the meaning of the fluid and energy could be extended to something else, for instance to a fluid of electrons, a fluid of photons, phonons … Indeed, we cannot say that (5.23) is fully and immediately applicable to other fields phenomena in physics (before implementing necessary adjustments to it), but we can be sure that the creative process (based on analogies) has already started (even if (5.23), in some later step will be significantly upgraded). ♣]

Download the last version here:
http://www.mastersonics.com/documents/revision_of_the_particle-wave_dualism.pdf

6. DIFFERENT POSSIBILITIES FOR MATHEMATICAL FOUNDATIONS OF A MULTIDIMENSIONAL UNIVERSE

Mathematical Physics has long been developing various theoretical and conceptual frameworks to explain how our universe could be multidimensional or contain multiple, non-interacting, independent worlds, each with its unique combination of dimensions. While practical, directly testable confirmations of extended multidimensionality are still lacking, we possess robust mathematical tools, indications, and concepts suggesting that some form of multidimensionality could exist in our Universe.

Perceptual Reality vs. Theoretical Multidimensionality

Our perceptual and empirical reality indicates that we live in a four-dimensional world (three spatial dimensions and one temporal dimension), where these dimensions are mutually related and orthogonal. Currently, we can only visualize and detect this four-dimensional world. Empirically, we do not know what higher dimensions (such as the fifth, sixth, etc.) or other independent worlds with different combinations of dimensions might be like. However, mathematics presents numerous possibilities for these higher dimensions.

Detecting Higher Dimensions

If higher and/or different dimensions exist, they could produce detectable effects on certain boundary matter-states in our perceptual and experimentally accessible four-dimensional world. Extreme perturbations from other multidimensional worlds might influence measurable parameters within our four-dimensional world, affecting motional, electromagnetic, and other matter states. For instance, frequency, phase, and amplitude of specific matter-waves could be additionally excited or modulated by occurrences in higher dimensions, resulting in new frequency harmonics, amplitude ripples, phase shifts, and different signal modulations.

Additionally, we might detect inexplicable energy deficits, surpluses, spikes, or fluctuations in matter and vacuum states around us, and variations in dielectric and magnetic permeability, potentially affecting the frequency and wavelengths of photons and other matter-waves appearing in our four-dimensional world and hypothetically assume some of such events as signals from other multidimensional worlds.

Strong and impulsive, high-energy discharges, explosions, implosions and by impacts produced effects might produce extradimensional or multidimensional perturbations, affecting “moments-energy-states” within our four-dimensional world. Something like that was experimented by Nikola Tesla regarding impulsive, high-energy, DC, resonant AC, and capacitors discharges, when he noticed the appearance of unusual radiant energy.

Hypothetical Proposals for Multidimensional Effects

A) Black Holes as Multidimensional Interfaces:

Black holes could serve as observable candidates for multidimensional effects. Conceptualized as energy and mass agglomerations acting as "gravitational-force matter-sinks," black holes might also be "electromagnetic force or energy sinks." Given energy and momentum conservation laws, every vortex-sink (or black hole) should have an energy input and an energy output. Our understanding of black holes might still be incomplete, neglecting electromagnetic aspects and their potential connections to higher-dimensional worlds.

B) Casimir Effect:

The Casimir effect might manifest cross-frontier communications between different multidimensional worlds. It could be a simple manifestation of natural, wideband electromagnetic noise reception, involving mutual coupling and resonant synchronization between overlapping resonant states of parallel metal plates.

C) Atomic Interactions:

Atoms could be structures where electron clouds bidirectionally exchange photons with other atoms and with the atom's nucleus. This creates stable, internally and externally balanced resonant structures. Atomic perturbations, such as fission or fusion, might send signals to other multidimensional worlds, with transient and reflected "echo matter states" indicating the existence of such worlds.

D) Statistical Fluctuations:

Hidden matter-states variables, captured through statistical or probabilistic modeling, might indicate interactions with a surrounding multidimensional environment. Zero-point energy fluctuations (ZPE) could serve as examples, with selective signal extraction potentially revealing signals from other multidimensional worlds.

E) Mathematical Constructs:

Mathematically created dimensions or worlds, without experimental backing, are theoretically possible, but not yet sufficiently realistic assumptions. Mathematics, grounded in tangible physics, remains the best framework for understanding multidimensional aspects of the Universe. Well-constructed mathematical models addressing physics may one day reveal insights into still unknown multidimensional realities.

F) Complex and Hypercomplex Numbers:

Consistent mathematical concepts, such as Imaginary, Complex, and Hypercomplex numbers, functions, signals, and Analytic Signal Phasors, provide a solid foundation for modeling multidimensional spaces and waves in Physics. These models, particularly those evolving from Complex Analytic Signals to Hypercomplex Analytic Signals, offer structured ways to understand possible spatial or dimensional extensions, much like the Minkowski space in Special Relativity theory.

Conclusion

By creatively proposing and imagining various hypothetical options, we might find effects or imprints of other multidimensional and multiverse worlds. This exploration bridges our current understanding with the potential of discovering a more complex, multidimensional reality.

G) The easiest way of simple analogical thinking related to **understanding multidimensional spaces** is to create cross-sections or projections of multidimensional structures towards lower levels of dimensionality. For instance, let us project a 3-dimensional object to a 2-dimensional object. Any cross section or projection of a 3-dimensional sphere $z = a = \text{const.} \Rightarrow x^2 + y^2 + z^2 = r^2$, will be a circle, or ellipse such as $x^2 + y^2 = r^2 - a^2 = r_1^2$ in a 2-dimensional plane. Analogically extending the same example, any cross section or projection of a 4-dimensional sphere $(x_1^2 + x_2^2 + x_3^2 + x_4^2 = r_2^2)$ will become a 3-

dimensional sphere, ($x_4 = a = const., \; x_1^2 + x_2^2 + x_3^2 = x^2 + y^2 + z^2 = r_2^2 - a^2 = r^2$) etc. Anyway, in our explorations of Nature (and in any other aspect of life), we are often applying analogical, inductive, and deductive conclusions, combined with interpolations, extrapolations and “cross-sections or projections”, inwards and outwards.

The simplest, analogically created basis of an extended, multidimensional universe (while staying in the theoretical frames of Riemannian metrics and 4-vector or n-vector rules of Minkowski-Space of the Theory of Relativity) is:

$$\{\text{Space \&Time}\} \Leftrightarrow \begin{Bmatrix} (x_1,x_2,...,x_n), t \\ (x_1,x_2,...,x_n),(t_1,t_2,...,t_n) \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} (r,t),\, r = r(x_1,x_2,...,x_n) \\ t = t(t_1,t_2,...,t_n) \end{Bmatrix} \Leftrightarrow (r_n,t). \quad (6.1)$$

There are many possibilities how to conceptualize and shape multidimensional or multi-coordinate basis (6.1), and here we will touch only a couple of them, that are sufficiently imaginative and challenging (mostly serving like brainstorming and indicative options). We can assume, as the first step towards multidimensionality, that every spatial dimension or coordinate from the multidimensional basis (6.1) could have its real and imaginary part, whatever that should mean,

$$\begin{Bmatrix} [r(x,t) = (x_1,x_2,...,x_n,t) \Leftrightarrow (r,t)] \to \overline{r}(r,t) = \overline{r}(\overline{x}_i,...) \\ \overline{x}_i = x_{i(real)} + j\,x_{i(imaginary)},\; i = 1,2,3,...,n;\; j^2 = -1 \end{Bmatrix}. \quad (6.2)$$

A similar concept has already been legitimized in the Minkowski space of Relativity Theory, where time is represented on the imaginary axis. This approach facilitates the unification of space and time, extending three-dimensional spatial structures into four-dimensional space-time formations. In practical terms, an additional dimension (time) is mathematically integrated with the three tangible spatial coordinates by introducing an orthogonal imaginary axis with a single imaginary unit (denoted as j^2 = -1).

Using the Minkowski-Einstein framework of complex energy-momentum four-vectors, we can now analyze, solve, and predict interactions between particles, wave groups (such as photons), and other energy-momentum states in physics. This framework provides a clear example of how the time dimension can be understood and mathematically conceptualized by placing it on an axis of imaginary units. This effectively means that all temporal and spatial dimensions are mutually orthogonal, and phase shifted.

Given that spatial and temporal domains are proportionally related and coupled, it follows that any space-time framework should have an equal number of spatial and temporal dimensions. Extending this idea further, we could place any new (yet undiscovered) dimensions on an imaginary axis or apply this concept to more complex systems, such as quaternions and hypercomplex functions (or phasors). We could even formulate hypercomplex Minkowski n-vectors, thereby mathematically creating new multidimensional structures.

For more details, see Chapter 10, particularly Section 10.1, "Hypercomplex Analytic Signal Functions and Interpretation of Energy-Momentum Four-Vectors in Relation to Matter-Waves and Particle-Wave Duality."

1. Let us now introduce pragmatic convention (compatible with (6.1) and (6.2)) by saying that all measurable or detectable spatial dimensions (x, y, z), of our world ($(\overline{x}_1 = x_{1(real)} = x_1 = x), (\overline{x}_2 = x_{2(real)} = x_2 = y), (\overline{x}_3 = x_{3(real)} = x_3 = z)$) have only real parts from (6.2), and all other, like time, and still unknown and hidden spatial dimensions would have only imaginary parts from (6.2). **One of the brainstorming options here is to treat other dimensions in the same way as time is treated in the Minkowski space of the Theory of Relativity.** Thus, multidimensional basis (6.2) can be modified as,

$$\begin{aligned}
&(\overline{x}_1, \overline{x}_2, \overline{x}_3, \overline{x}_4, ..., \overline{x}_n, t) \Leftrightarrow (\underleftrightarrow{x_{1(real)}, x_{2(real)}, x_{3(real)}}, \underleftrightarrow{x_{4(imag.)}, x_{5(imag.)}, ..., x_{n(imag.)}}, t) \Leftrightarrow \\
&\Leftrightarrow (\underleftrightarrow{x_{1(real)}, x_{2(real)}, x_{3(real)}}, \underleftrightarrow{jx_{4(imag.)}, jx_{5(imag.)}, ..., jx_{n(imag.)}}, jct) \Leftrightarrow \\
&\Leftrightarrow \left[(x_{1(real)}, x_{2(real)}, x_{3(real)}), j(x_{4(imag.)}, x_{5(imag.)}, ..., x_{n(imag.)}, ct) \right] \Leftrightarrow \\
&\Leftrightarrow \left[(x_{1(real)}, x_{2(real)}, x_{3(real)}), jct^* \right] \Leftrightarrow \left[(x_1, x_2, x_3), jct^* \right] \Leftrightarrow \left[(x, y, z), jct^* \right], \\
&ct^* = (x_{4(imag.)}, x_{5(imag.)}, ..., x_{n(imag.)}, ct).
\end{aligned} \tag{6.3}$$

In fact, in (6.3) all higher-level dimensions are associated with an extended meaning of an effective time dimension $t^* = (\frac{x_{4(imag.)}}{c}, \frac{x_{5(imag.)}}{c}, ..., \frac{x_{n(imag.)}}{c}, t)$. Let us now (based on (6.3)), try to express the multidimensional space-time interval, as hypothetically being "*coordinates-invariant*", using the analogy with space-time interval from the Relativity theory, as,

$$\begin{aligned}
&(\Delta S)^2 = (\Delta x_1)^2 + (\Delta x_2)^2 + (\Delta x_3)^2 - c^2 (\Delta t^*)^2 = \\
&= (\Delta x_1)^2 + (\Delta x_2)^2 + (\Delta x_3)^2 - c^2 (\Delta t)^2 - (\Delta x_4)^2 - (\Delta x_5)^2 - ... - (\Delta x_n)^2 .
\end{aligned} \tag{6.4}$$

Alternatively, we can express the same interval in its infinitesimal form as,

$$\begin{aligned}
&(dS)^2 = (dx_1)^2 + (dx_2)^2 + (dx_3)^2 - c^2 (dt^*)^2 = \\
&= (dx_1)^2 + (dx_2)^2 + (dx_3)^2 - c^2 (dt)^2 - (dx_4)^2 - (dx_5)^2 - ... - (dx_n)^2 .
\end{aligned} \tag{6.4-1}$$

Given that our four-dimensional universe remains the primary measurable reality we can empirically assess, there are at least two possible interpretations (labeled as 1.a and 1.b) for understanding the space-time interval equation (6.4) in a way that aligns with both our experience and contemporary Relativity Theory.

Currently, we lack definitive empirical evidence that confirms the relativistic space-time interval is always invariant across all coordinate systems. While the concept is widely accepted, it remains a highly probable yet partially speculative and still-debated idea (see [49]: *Universal Invariance: A Novel View of Relativistic Physics*, T. E. Phipps, Jr.). A modified definition of the space-time interval will be introduced later in equation (6.17-1).

With this context in mind, let us consider the following conceptual scenarios as potential frameworks for establishing the mathematical foundations of multidimensional universes based on varying interpretations of space-time intervals.

1. a) One of options related to (6.3) is that higher dimensions of our or other universes (if any) always present extremely short or negligible time-space intervals,

$$(\Delta x_4)^2 + (\Delta x_5)^2 + ... + (\Delta x_n)^2 \cong 0 . \quad (6.5)$$

or,

1. b) That higher dimensions somehow contribute to our real time-scale modification, or to creation of certain temporal or phase shift, $t^* \cong t + \tau$,

$$\begin{aligned} c^2(\Delta t^*)^2 &= c^2(\Delta t)^2 + (\Delta x_4)^2 + (\Delta x_5)^2 + ... + (\Delta x_n)^2, \\ (\Delta t^*)^2 &= (\Delta t)^2 + \frac{(\Delta x_4)^2}{c^2} + \frac{(\Delta x_5)^2}{c^2} + ... + \frac{(\Delta x_n)^2}{c^2} = [\Delta(t+\tau)]^2 . \end{aligned} \quad (6.6)$$

Anyway, both options, (6.5) and (6.6) effectively produce the well-known space-time interval (from the Relativity Theory),

$$(\Delta S)^2 = (\Delta x_1)^2 + (\Delta x_2)^2 + (\Delta x_3)^2 - c^2(\Delta t^*)^2 . \quad (6.7)$$

In (6.6), all spatial dimensions are presented by their real parts, and only the effective time dimension (related to other higher, for us still hidden dimensions) is associated to an imaginary coordinate axis. Here, (6.6), presents the effective time-shift where invisible (hidden or higher) dimensions create mentioned time shifts,

$$(\Delta t^*)^2 = (\Delta t)^2 + \frac{(\Delta x_4)^2}{c^2} + \frac{(\Delta x_5)^2}{c^2} + ... + \frac{(\Delta x_n)^2}{c^2} = [\Delta(t+\tau)]^2 .$$

This concept offers an intriguing avenue for exploration (even for science fiction speculations about time travel) because it suggests the potential parallel existence of multiple worlds, each sharing some dimensions while differing in others. These worlds could be phase-shifted along the time scale or spatial-temporal coordinates, varying across different fundamental and spectral intervals. Due to this phase-shifting, such worlds would remain invisible, untouchable, and nearly non-interacting with one another—like mutually orthogonal functions in mathematics.

If we inhabit one of these phase-shifted worlds, the "residual presence" of other parallel worlds might form a conceptual "ether" or a spatial matrix. This matrix could act as a carrier medium for the propagation of matter waves in our universe (e.g., electromagnetic waves, gravitational waves, etc.). Interestingly, we already know that such a "spatial matrix" has measurable and significant electromagnetic properties, characterized by the dielectric permittivity and magnetic permeability of vacuum, ε_0 , μ_0. These constants are interconnected and give rise to the universal speed of light in a vacuum, $c = 1/\sqrt{\varepsilon_0\mu_0}$).

If, for some reason, constants ε_0 and μ_0 fluctuate, the speed of photons in this environment (at least when averaged) could still appear constant. This would imply that while the frequency and wavelength of photons might vary, the product $\varepsilon_0\mu_0 = \varepsilon_1\mu_1 = ... = \varepsilon_x\mu_x = 1/c^2$, $\varepsilon_x\mu_x = 1/c^2 = 1/(\lambda_x f_x)^2$ remains the same.

Generally applicable relation between group and phase velocity (u and v), of all matter-waves is also applicable to photons, since we have,

$$\begin{bmatrix} v = u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda} \\ u = \lambda f = \lambda_x f_x = c = 1/\sqrt{\varepsilon_x \mu_x} = \text{constant} \end{bmatrix} \Rightarrow v = c = u = \text{constant}.$$

Practically, we assume or have indicative arguments that photons have the same group and phase velocity (v and u), which is equal to universal constant c = u = v = constant (equal to the speed of light in a vacuum). What can change or fluctuate here (in cases of perturbations) are relevant wavelength λ_x and frequency f_x, or dielectric and magnetic constants ε_x and μ_x. Here we can directly establish proportionality between the temporal Δt and spatial Δs durations or intervals, such as, $v = c = u = ds/dt \Rightarrow ds = c \cdot dt \Rightarrow \Delta s = c \cdot \Delta t$, without any need to refer to big inventions and postulates from Relativity theory.

The speed of light, denoted as **constant c**, plays a central role in many fundamental aspects of physics, such as the relationships between energy, momentum, mass, time, space, and even the fine structure constant. A particularly intriguing fact is that **photons** exhibit only transverse oscillations, where the electric and magnetic field vectors oscillate perpendicular to one another.

By drawing **analogies** between wave motions in different states of matter, as gaseous, liquid, and solid, we can observe that in extremely hard, rigid materials (which are nearly non-elastic), **transverse waves** dominate, while **longitudinal waves** are almost nonexistent. Extending this analogy, if we consider the vacuum or the medium that carries photons (whether it be the concept of **ether** or a "spatial matrix"), this medium might be equivalent to an infinitely strong and rigid material (for electromagnetic waves). In this view, the vacuum would have properties akin to a material with an extremely high **Young's modulus**, a measure of rigidity, allowing photons to propagate as transverse waves, much like how they behave in this idealized environment.

Additionally, it is worth considering the possibility that **higher spatial dimensions**, which are still not detectable by our current sensors and instruments, might play a role in this "exotic," rigid medium that supports photon propagation. While this is still a **preliminary hypothesis** and an oversimplified brainstorming exercise, it may indicate that we still must learn about the true nature of electromagnetic wave propagation.

Interestingly, similar phenomena can be observed in mechanical systems: it is possible to generate fully **transverse acoustic or mechanical vibrations**, oscillating orthogonally to the direction of propagation, that travel through solid materials like metals (wires or tubes). These vibrations, much like photons in fiber optic cables, can propagate over exceedingly long distances with minimal **attenuation**, especially when compared to longitudinal waves.

2. The concepts discussed above could be made compatible with the formalism of **Minkowski space** in relativity theory. In such cases, we can employ **ordinary complex numbers** or **analytic signal functions** (which use a single imaginary unit $\mathbf{j^2 = -1}$) as a convenient way to unify the spatial and temporal domains by acting as a **time-axis marker**. This approach facilitates rich mathematical processing, particularly in relation to **conservation laws** in physics. For example, it aids in conceptualizing **de Broglie matter waves** and in the mathematical treatment of **impact** and **scattering interactions**.

As an advanced step, we can extend this framework to introduce a new, **Minkowski-equivalent multidimensional space**, using **quaternions** or **hypercomplex functions**, starting from the same foundation as earlier models but incorporating at least three imaginary units. (For further details on **Hypercomplex Analytic functions**, refer to Chapter 4.0, section "4.0.2.6. Hyper-complex Analytic Signal", and Chapter 10, section "10.1 Hypercomplex Analytic Signal functions and interpretation of energy-momentum 4-vectors in relation to matter-waves and particle-wave duality.")

To provide an indicative example of how a **hypercomplex-model** might be applied, consider the case of **quarks** in contemporary physics. Quarks are mathematically defined and fundamental to particle physics, yet they are not directly detectable. Instead, they are "masked" and exist in sets of three elementary particles, along with corresponding sets of **antiparticles** and their **color** variants. Each quark also possesses other attributes, leading to a complex structure of 36 quarks currently recognized, with the possibility of more in the future.

This brainstorming exercise aims to explore how **hypercomplex wave functions**, with a potentially infinite number of imaginary unit triplets, could be used to construct an **innovative multidimensional model**. This model would allow for a more organized classification of wavefunctions, with the goal of representing the structures of elementary particles more accurately.

In hypercomplex functions, the basic imaginary unit **I** is expanded into a set of three more elementary units: **i, j, k**. We could preliminarily explore how to mathematically formulate such a **multidimensional, extended Minkowski space** as a potential framework for future developments, as follows,

$$\left\{(\overline{x}_1,\overline{x}_2,...,\overline{x}_n,t) \Leftrightarrow (\overline{r},t) \Leftrightarrow \overline{r}(t),\ \overline{x}_i = \overline{x}_i(t)\right\} \Rightarrow \left\{\overline{z},\ t\right\} \Leftrightarrow \overline{z}(t) \Rightarrow$$

$$\Rightarrow \left\{\begin{array}{l} (\overline{x}_1,\overline{x}_2,\overline{x}_3,\ jct) \to (\overline{x}_1,\overline{x}_2,\overline{x}_3,\ Ict) \Rightarrow \left\{\overline{x}_1,\ \overline{x}_2,\ \overline{x}_3,\ c(it_i + jt_j + kt_k)\right\} \\ \left[\begin{array}{l} Ict\ =\ c(it_i +\ jt_j +\ kt_k), \\ (t^2 = t_i^{\ 2} +\ t_j^{\ 2} +\ t_k^{\ 2},\ \ t_i t_j k - t_i t_k j + t_j t_k i = 0,\ ??!!) \\ \text{or,}\ \ Ict\ = ic_i t_i +\ jc_j t_j +\ kc_k t_k), \\ (c^2 t^2 = c_i^2 t_i^{\ 2} +\ c_j^2 t_j^{\ 2} +\ c_k^2 t_k^{\ 2},\ \ c_i c_j t_i t_j k - c_i c_k t_i t_k j + c_j c_k t_j t_k i = 0,\ ??!!) \end{array}\right], \\ I^2 = i^2 = j^2 = k^2 = -1, \\ i\cdot j = k,\ \ j\cdot k = i,\ \ k\cdot i = j,\ j\cdot i = -k,\ \ k\cdot j = -i,\ \ i\cdot k = -j, \\ e^{I\varphi} = \cos\varphi + I\cdot\sin\varphi \\ e^{i\cdot\varphi_i} = \cos\varphi_i + i\cdot\sin\varphi_i\ , e^{j\cdot\varphi_j} = \cos\varphi_j + j\cdot\sin\varphi_j\ ,\ e^{k\cdot\varphi_k} = \cos\varphi_k + k\cdot\sin\varphi_k \end{array}\right\} \Rightarrow$$

$$\overline{z} = |\overline{z}|e^{I\varphi} = |\overline{z}|e^{i\varphi_i + j\varphi_j + k\varphi_k} = |\overline{z}|e^{i\varphi_i}e^{j\varphi_j}e^{k\varphi_k} = |\overline{z}|(\cos\varphi_i + i\cdot\sin\varphi_i)(\cos\varphi_j + j\cdot\sin\varphi_j)(\cos\varphi_k + k\cdot\sin\varphi_k) =$$
$$= r + a_i\cdot i + a_j\cdot j + a_k\cdot k = r + I\cdot A = \overline{z}_i + \overline{z}_j + \overline{z}_k = |\overline{z}_i|e^{i\varphi_i} + |\overline{z}_j|e^{j\varphi_j} + |\overline{z}_k|e^{k\varphi_k}, |\overline{z}|^2 = r^2 + A^2,$$
$$r = |\overline{z}|\cos\varphi,\ A = |\overline{z}|\sin\varphi,\ I\varphi = i\varphi_i + j\varphi_j + k\varphi_k,$$

$$\Rightarrow \left\{ \begin{array}{l} A\cdot I = a_i\cdot i + a_j\cdot j + a_k\cdot k = Ae^{I(\frac{\pi}{2}+2m\pi)} = I\cdot|\overline{z}|\sin\varphi = \\ = a_i e^{i(\frac{\pi}{2}+2n\pi)} + a_j e^{j(\frac{\pi}{2}+2p\pi)} + a_k e^{k(\frac{\pi}{2}+2q\pi)},\ m, n, p, q = 1,2,3,... \\ I = \frac{a_i}{A}\cdot i + \frac{a_j}{A}\cdot j + \frac{a_k}{A}\cdot k = e^{I(\frac{\pi}{2}+2m\pi)} = I\cdot\frac{|\overline{z}|}{A}\sin\varphi, \\ \varphi\cdot I = \varphi_i\cdot i + \varphi_j\cdot j + \varphi_k\cdot k, \\ I = \frac{\varphi_i}{\varphi}\cdot i + \frac{\varphi_j}{\varphi}\cdot j + \frac{\varphi_k}{\varphi}\cdot k = \frac{a_1}{A}\cdot i + \frac{a_2}{A}\cdot j + \frac{a_3}{A}\cdot k = e^{I(\frac{\pi}{2}+2m\pi)}, \\ \frac{\varphi_i}{\varphi} = \frac{a_i}{A}, \frac{\varphi_j}{\varphi} = \frac{a_j}{A}, \frac{\varphi_k}{\varphi} = \frac{a_k}{A},\ (A^2 = a_i^2 + a_j^2 + a_{3k}^2,\ \varphi^2 = \varphi_i^2 + \varphi_j^2 + \varphi_k^2\ ??!!) \end{array} \right\} \Rightarrow$$

$$\Rightarrow \left\{ \begin{array}{l} \cos\varphi = \quad \cos\varphi_i\cos\varphi_j\cos\varphi_k - \sin\varphi_i\sin\varphi_j\sin\varphi_k, \\ \sin\varphi = \left[\begin{array}{l} \frac{\varphi}{\varphi_i}(\cos\varphi_i\sin\varphi_j\sin\varphi_k + \sin\varphi_i\cos\varphi_j\cos\varphi_k), \\ \frac{\varphi}{\varphi_j}(\cos\varphi_i\sin\varphi_j\cos\varphi_k - \sin\varphi_i\cos\varphi_j\sin\varphi_k), \\ \frac{\varphi}{\varphi_k}(\text{sins}\,\varphi_i\sin\varphi_j\cos\varphi_k + \cos\varphi_i\cos\varphi_j\sin\varphi_k) \end{array} \right] \end{array} \right\}. \qquad (6.8)$$

What could be particularly interesting in the hypercomplex coordinate basis (6.8), is the possibility to unite linear and rotational (or torsional) aspects of certain motion within the same mathematical concept. Anyway, most elementary particles, quasiparticles, and matter waves have (apart from their linear or translational motion parameters) also rotational or angular attributes, like spin and orbital moments (see chapter 4.1; "4.1.2. De Broglie Matter Waves and Hidden Rotation"). For instance, torsional and spinning matter-wave parameters mentioned could be introduced in the following way (just to start a creative thinking about such modeling):

$$\overline{z} = |\overline{z}|e^{I\varphi} = |\overline{z}|e^{I(\omega t \mp kr)},\ \omega = \frac{\partial\varphi}{\partial t},\ k = \frac{\partial\varphi}{\partial x},$$
$$\overline{x}_n = |\overline{x}_n|e^{i\varphi_n} = |\overline{x}_n|e^{i(\omega_n t \mp k_n x_n)}, \omega_n = \frac{\partial\varphi_n}{\partial t}, k_n = \frac{\partial\varphi_n}{\partial x}, n = 1,2,3. \qquad (6.8\text{-}1)$$

When relevant wave functions are conveniently presented in a hypercomplex space, described by (6.8), we would have a lot of additional freedom regarding mathematically rich modeling of motions, interactions, and analyzing energy-momentum states. For instance, using couples of mutually complex-conjugate functions, and algebraic sign variation in front of different imaginary units (+ or – sign), we will be able to present, or only mark and separate different energy-momentum states of particles, waves, antiparticles, etc., as for example,

$$\overline{z}^* = r - a_1 \cdot i - a_2 \cdot j - a_3 \cdot k = r - I \cdot A = |\overline{z}| e^{-I\varphi} = \overline{z}_i^* + \overline{z}_j^* + \overline{z}_k^* = \\ = |\overline{z}_i| e^{-i\varphi_i} + |\overline{z}_j| e^{-j\varphi_j} + |\overline{z}_k| e^{-k\varphi_k}, |\overline{z}^*|^2 = r^2 + A^2 = |\overline{z}|^2 = \overline{z} \cdot \overline{z}^*, \tag{6.9}$$

whatever shows appropriate in a certain case of interest.

3. In order to reach a higher level of unity and applicability of the above-mentioned options **1.** and **2.** (for instance, in relation to the quantum mechanical wave function), already introduced complex functions (figuring in (6.2) - (6.9)), can also be treated as multidimensional **Hypercomplex Analytic Signal wave functions or Phasors** (see: eq. (4.9), **[7]** and **[8]**), and be presented in some of the following ways:

3. a) Using hypercomplex representation (6.8) and (6.9), we can transform the multidimensional basis (6.2) or (6.3), and express the ***Hypercomplex, Analytic Signal wave function,*** as follows,

$$(\overline{r}, t) \Leftrightarrow (\underleftrightarrow{x_{1(real)}, x_{2(real)}, x_{3(real)}}, \underleftrightarrow{x_{4(imag)}, x_{5(imag)}, \dots, x_{n(imag)}}, t) \Rightarrow$$

$$\Rightarrow (\underleftrightarrow{x_{1(real)}, x_{2(real)}, x_{3(real)}}, \underleftrightarrow{Ix_{4(imag)}, Ix_{5(imag)}, \dots, Ix_{n(imag)}, Ict}) \Leftrightarrow$$

$$\Leftrightarrow \left\{ r(\underleftrightarrow{x_{1(real)}, x_{2(real)}, x_{3(real)}}), Ict^* \right\} \Leftrightarrow (r, Ict^*), \; I^2 = -1$$

$$r = r(x_1, x_2, x_3), \; t^* = t^*(t, \frac{x_4}{c}, \frac{x_5}{c}, \dots, \frac{x_n}{c}).$$

$$\overline{\Psi}(\mathbf{r}, \mathbf{t}) = \Psi(\mathbf{r}, \mathbf{t}) + \mathbf{I} \cdot \mathbf{H}[\Psi(\mathbf{r}, \mathbf{t})] = \Psi(\mathbf{r}, \mathbf{t}) + \mathbf{I} \cdot \hat{\Psi}(\mathbf{r}, \mathbf{t}) =$$

$$= \overline{\Psi}_i + \overline{\Psi}_j + \overline{\Psi}_k = |\overline{\Psi}(\mathbf{r}, \mathbf{t})| \cdot \mathbf{e}^{\mathbf{I} \cdot \varphi(\mathbf{r}, \mathbf{t})}, \; \overline{\Psi}_{i,j,k} = \Psi_{i,j,k} + \begin{bmatrix} \mathbf{i} \\ \mathbf{j} \\ \mathbf{k} \end{bmatrix} \cdot \hat{\Psi}_{i,j,k} = |\overline{\Psi}_{i,j,k}| \cdot \mathbf{e}^{\begin{bmatrix} i \\ j \\ k \end{bmatrix} \cdot \varphi_{i,j,k}}, \tag{6.10}$$

$$|\overline{\Psi}(r,t)|^2 = [\Psi(r,t)]^2 + [\hat{\Psi}(r,t)]^2 = \Psi^2 + \hat{\Psi}^2 = |\overline{\Psi}|^2, \; \varphi(r,t) = \mathrm{arctg} \frac{\hat{\Psi}(r,t)}{\Psi(r,t)} = \varphi,$$

$$|\overline{\Psi}_{i,j,k}|^2 = [\Psi_{i,j,k}]^2 + [\hat{\Psi}_{i,j,k}]^2, \; \varphi_{i,j,k} = \mathrm{arctg} \frac{\hat{\Psi}_{i,j,k}}{\Psi_{i,j,k}}, \omega_{i,j,k} = \frac{\partial \varphi_{i,j,k}}{\partial t}$$

$$\left|\overline{\Psi}\right|^2=\left|\overline{\Psi}_i\right|^2+\left|\overline{\Psi}_j\right|^2+\left|\overline{\Psi}_k\right|^2=\Psi^2+\hat{\Psi}^2=\hat{\Psi}_i^2+\hat{\Psi}_j^2+\hat{\Psi}_k^2+\Psi_i^2+\Psi_j^2+\Psi_k^2,$$
$$\Psi^2=\hat{\Psi}_i^2+\hat{\Psi}_j^2+\hat{\Psi}_k^2,\ \ \hat{\Psi}^2=\Psi_i^2+\Psi_j^2+\Psi_k^2,$$
$$\Psi=\left|\overline{\Psi}\right|\cdot\cos\varphi=\Psi_i+\Psi_j+\Psi_k,\ \hat{\Psi}=\left|\overline{\Psi}\right|\cdot\sin\varphi=H[\Psi]=\hat{\Psi}_i+\hat{\Psi}_j+\hat{\Psi}_k,$$
$$H\left[\hat{\Psi}\right]=-\Psi=\hat{\Psi}+i\cdot\hat{\Psi}_i+j\cdot\hat{\Psi}_j+k\cdot\hat{\Psi}_\kappa,$$
$$\Psi_{i,j,k}=\left|\overline{\Psi}_{i,j,k}\right|\cdot\cos\varphi_{i,j,k},\hat{\Psi}_{i,j,k}=H\left[\Psi_{i,j,k}\right]=\left|\overline{\Psi}_{i,j,k}\right|\cdot\sin\varphi_{i,j,k},$$
$$H[\Psi_1\Psi_2]\ =\ (\Psi_1\Psi_2)\frac{\Psi_1\hat{\Psi}_2+\hat{\Psi}_1\Psi_2}{\Psi_1\Psi_2-\hat{\Psi}_1\hat{\Psi}_2},\ H\left[\frac{\Psi_2}{\Psi_1}\right]\ =\ (\frac{\Psi_2}{\Psi_1})\frac{\Psi_1\hat{\Psi}_2-\hat{\Psi}_1\Psi_2}{\Psi_1\Psi_2+\hat{\Psi}_1\hat{\Psi}_2}\ ,$$

$$\begin{aligned}
&I\cdot\hat{\Psi}(r,t)=i\cdot\hat{\Psi}_i+j\cdot\hat{\Psi}_j+k\cdot\hat{\Psi}_\kappa=e^{I(\frac{\pi}{2}+2m\pi)}\cdot\hat{\Psi}(r,t),\\
&I\cdot\varphi(r,t)=i\cdot\varphi_i+j\cdot\varphi_j+k\cdot\varphi_k=e^{I(\frac{\pi}{2}+2m\pi)}\cdot\varphi(r,t),\\
&I=i\cdot\frac{\varphi_i}{\varphi}+j\cdot\frac{\varphi_j}{\varphi}+k\cdot\frac{\varphi_\kappa}{\varphi}=i\cdot\frac{\hat{\Psi}_i}{\hat{\Psi}}+j\cdot\frac{\hat{\Psi}_j}{\hat{\Psi}}+k\cdot\frac{\hat{\Psi}_\kappa}{\hat{\Psi}}=e^{I(\frac{\pi}{2}+2m\pi)},\\
&\frac{\varphi_i}{\varphi}=\frac{\hat{\Psi}_i}{\hat{\Psi}},\frac{\varphi_j}{\varphi}=\frac{\hat{\Psi}_j}{\hat{\Psi}},\frac{\varphi_k}{\varphi}=\frac{\hat{\Psi}_k}{\hat{\Psi}},\\
&\frac{\varphi_{i,j,k}}{\varphi}=\frac{\hat{\Psi}_{i,j,k}}{\hat{\Psi}}=\frac{H\left[\Psi_{i,j,k}\right]}{H[\Psi]}=\frac{\operatorname{arctg}\frac{\hat{\Psi}_{i,j,k}}{\Psi}}{\operatorname{arctg}\frac{\hat{\Psi}}{\Psi}},
\end{aligned}\qquad (6.10)$$

$$\left[\begin{array}{l}\text{?!... to be verified}\\ \left(\begin{array}{l}\Psi=\Psi_i+\Psi_j+\Psi_k,\Psi^2=\hat{\Psi}_i^2+\hat{\Psi}_j^2+\hat{\Psi}_k^2,\hat{\Psi}_i\hat{\Psi}_j+\hat{\Psi}_j\hat{\Psi}_k+\hat{\Psi}_i\hat{\Psi}_k=0,\\ \hat{\Psi}=\Psi_i+\Psi_j+\Psi_k,\hat{\Psi}^2=\Psi_i^2+\Psi_j^2+\Psi_k^2,\Psi_i\Psi_j+\Psi_j\Psi_k+\Psi_i\Psi_k=0,\\ (\varphi=\varphi_i+\varphi_j+\varphi_k,\varphi^2=\varphi_i^2+\varphi_j^2+\varphi_k^2,\varphi_i\varphi_j+\varphi_j\varphi_k+\varphi_i\varphi_k=0)\end{array}\right)\end{array}\right].$$

What we see from (6.10) is that important wave function elements (such as amplitude, phase, frequency, etc.) cannot be found if we do not take into account both, an original wave function Ψ, and its Hilbert couple $\hat{\Psi}$, meaning that in reality, both of such wave functions should coincidently exist (see Chapter 4.0 for more information about Analytic Signals).

The *hypercomplex analytic signal wave function*, which has an arbitrary number of imaginary units (higher than 3), could be expressed in a similar way, as the following summation:

$$\overline{\Psi}(t)=\Psi(t)+I\,\hat{\Psi}(t)=a_0(t)e^{I\,\varphi_0(t)}=a_0(t)\left[\cos\varphi_0(t)+I\ \sin\varphi_0(t)\right]=$$
$$=a_0(t)e^{\sum_{(k)}i_k\varphi_k(t)}=\sum_{(k)}a_k(t)e^{i_k\varphi_k(t)}=\sum_{(k)}\overline{\Psi}_k(t)\ ,$$

or, as the multiplication,

$$\Psi(t)=\frac{a_n(t)}{2^{n+1}}\prod_{k=0}^{n}(e^{I\varphi_k(t)}+e^{-I\varphi_k(t)}),\hat{\Psi}(t)=\frac{a_n(t)}{(2i)^{n+1}}\prod_{k=0}^{n}(e^{I\varphi_k(t)}-e^{-I\varphi_k(t)}),$$
$$\overline{\Psi}(t)=\Psi(t)+I\hat{\Psi}(t)=\frac{a_n(t)}{2^{n+1}}\left\{\prod_{k=0}^{n}(e^{I\varphi_k(t)}+e^{-I\varphi_k(t)})+\frac{1}{(i)^n}\prod_{k=0}^{n}(e^{I\varphi_k(t)}-e^{-I\varphi_k(t)})\right\}, \quad (6.10\text{-}1)$$

$$\begin{bmatrix} \overline{\Psi}_k(t)=a_k(t)e^{i_k\varphi_k(t)}=\Psi_k(t)+i_k\hat{\Psi}_k(t), \\ \cos\varphi_k=\frac{1}{2}(e^{I\varphi_k(t)}+e^{-I\varphi_k(t)}),\ \sin\varphi_k=\frac{1}{2i}(e^{I\varphi_k(t)}-e^{-I\varphi_k(t)}), \\ \varphi_k(t)=\mathrm{arctg}\frac{\hat{\Psi}_k(t)}{\Psi_k(t)},\ \varphi_0^2(t)=\sum_{(k)}\varphi_k^2(t),\quad \omega_k(t)=\frac{\partial\varphi_k(t)}{\partial t}=2\pi f_k(t), \\ a_k^2(t)=a_{k-1}^2(t)+\hat{a}_{k-1}^2(t)=\Psi_{k+1}^2(t)=\Psi_k^2(t)+\hat{\Psi}_k^2(t), \\ a_0^2(t)=\left|\overline{\Psi}(t)\right|^2=\Psi^2(t)+\hat{\Psi}^2(t)=\sum_{(k)}a_k^2(t)+2\sum_{(i\neq j)}\Psi_i(t)\Psi_j(t),\ \forall i,j,k\in[1,n] \end{bmatrix}.$$

To present structure of different elementary particles, quasiparticles, wave-packets etc., it will be conceivable to establish certain well-operating practice how to use analytic, hypercomplex wave functions and phasors for an appropriate and natural mathematical modeling with a deep meaning in Physics. Later, for instance, we may conclude that what we have been trying to isolate or define as a single elementary particle, or detectable entity, presents ***a structured world*** that has its unbounded, internally, and externally expanding structures and dimensions. Such structured body of an analytic hypercomplex wavefunction, having multiplicative energy levels (characterized by integers as, n = 1, 2, 3…) is recognizable from the expression:

$$\overline{\Psi}(t)=\Psi(t)+I\hat{\Psi}(t)=\frac{a_n(t)}{2^{n+1}}\left\{\prod_{k=0}^{n}(e^{I\varphi_k(t)}+e^{-I\varphi_k(t)})+\frac{1}{(i)^n}\prod_{k=0}^{n}(e^{I\varphi_k(t)}-e^{-I\varphi_k(t)})\right\},$$

where wave components going inwards and outwards (in mutually opposite directions) are marked by different + and - signs in the exponent (such as $\mathbf{e^{I\varphi_k(t)}, e^{-I\varphi_k(t)}}$). This is probably the main reason why our attempts to find and classify all elementary particles within dimensions of our universe will never end. By going into higher or deeper energy levels (regarding particle impacts and relevant products in collider accelerators), we are just opening a countless number of new matter-waves combinations of **mutually coupled**, amplitudes modulating, inwards and outwards expanding matter-waves of dynamically dependent and dualistic wave-particle structures (existing both temporally and spatially; - see more in Chapter 4.0). A much better strategy would be to find the right logical and conceptual mathematical modelling structure of wave functions regarding how to understand and present them within naturally developing and evolving hypercomplex analytic wavefunctions and Phasors.

Here, we could also intuitively address elementary particles creation (when a particle with a stable rest mass is created). Certain multidimensional wave function (matter-wave) can be presented as an amplitude-modulated product of many cosine functions, each of them having different temporal and spatial frequencies

$\omega_n(t) = \frac{\partial \varphi_n(t)}{\partial t} = 2\pi f_n(t)$ and $k_n(x) = \frac{\partial \varphi_n(t)}{\partial x} = 2\pi f_n(x)$. Such frequency harmonics in a proper environment could create self-closed, stable structures with standing waves, which would externally manifest as different stable particles.

It is worth noting that certain hypercomplex functions, such as those incorporating three imaginary units, can often be decomposed into a sum of three ordinary complex functions, each defined by a single imaginary unit. Additionally, some hypercomplex functions can also be represented as a product of three simpler complex functions, where each function similarly uses only one distinct imaginary unit.

This decomposition approach allows for a separate treatment of triadic or trinary structures, such as those found in quark models. In such cases, each simple complex function operates along a unique, mutually orthogonal imaginary axis, providing a mathematical framework to explore more complex spatial structures and symmetries. This framework offers a deeper understanding when compared to simpler dyadic or binary cases, which often involve straightforward oppositions like positive and negative states, matter and antimatter, or basic mirror symmetries.

Through this perspective, we can open a new window for analyzing multidimensional structures, thereby extending our comprehension beyond traditional binary symmetries.

Remarks: *Hypercomplex-Quaternions and Analytic signal wave function modeling can assume that signal phase function is also presentable as a Hypercomplex function, having its real and imaginary parts, such as,*

$$\overline{\Psi}(r,t) = \overline{\Psi}(x,y,z,t) = \Psi(r,t) + I \cdot H[\Psi(r,t)] = \Psi(r,t) + I \cdot \hat{\Psi}(r,t) =$$
$$= \Psi(r,t) + i \cdot \hat{\Psi}_x(r,t) + j \cdot \hat{\Psi}_y(r,t) + k \cdot \hat{\Psi}_z(r,t) =$$
$$= \overline{\Psi}_i + \overline{\Psi}_j + \overline{\Psi}_k = \left|\overline{\Psi}(r,t)\right| \cdot e^{\overline{\varphi}(r,t)},$$
$$\overline{\varphi}(r,t) = \varphi_R(r,t) + I\varphi_I(r,t) = \varphi_R(r,t) + i \cdot \varphi_i + j \cdot \varphi_j + k \cdot \varphi_k = \left|\overline{\varphi}(r,t)\right| e^{\overline{\varphi}(r,t)},$$
$$I \cdot \varphi_I(r,t) = i \cdot \varphi_i + j \cdot \varphi_j + k \cdot \varphi_k, \; I^2 = i^2 = j^2 = k^2 = -1.$$

This will give us even larger mathematical modeling freedom.

3. b) Following a quite simple analogy, based on the knowledge that our universe has three spatial and one temporal dimension (x_1, x_2, x_3, t), this situation has been easily generalized (in modern Physics) creating multidimensional spaces analogically extending the basis of the three spatial dimensions into n spatial dimensions $(x_1, x_2, x_3, \ldots, x_n, t)$. This is the simplest choice, and only one of numerous possibilities to imagine what could be a good starting platform to analyze hypothetical and multidimensional universes. Most probably, we still do not know what the actual choice of the Universe is, (we could only think, or believe that a basis $(x_1, x_2, x_3, \ldots, x_n, t)$ is the most realistic multidimensional platform for a future exploration regarding multidimensionality). We have seen that every spatial-temporal coordinate of our universe (x_1, x_2, x_3, t) is also presentable as having its own real and imaginary component $[(x_{1r}, x_{1i}), (x_{2r}, x_{2i}), (x_{3r}, x_{3i}), t]$. It is also imaginable that a time component, **t**, has its real and imaginary components, $\left[(x_{1r},x_{1i}), (x_{2r},x_{2i}), (x_{3r},x_{3i}), (t_r,t_i)\right]$ **thus creating modified Minkowski space using hypercomplex 4-vectors,** or we can introduce analogical n-vectors (as already mentioned), etc. From Hypercomplex Minkowski 4-vectors, or analogical n-vectors, we can develop corresponding Complex and Hypercomplex Analytic Signal Phasors, and profit from number of advantages of such Phasors in relation to Wave-Particle Duality (see more in Chapter 10.).

The most interesting situation (coming from Hypercomplex Analytic Signal functions) is that every single imaginary unit $\mathbf{i}, \mathbf{j}, \mathbf{k}$ from (6.8) - (6.10) can additionally split into three new imaginary units, creating new levels of endlessly growing (higher dimensional, or sub-dimensional) triplets, or something like fractals, such as,

$$
\begin{aligned}
& i \rightarrow (i_{11}, i_{12}, i_{13})..., \; j \rightarrow (j_{11}, j_{12}, j_{13})..., \; k \rightarrow (k_{11}, k_{12}, k_{13})... \\
& i_{mn} \rightarrow (i_{mn1}, i_{mn2}, i_{mn3})..., \; j_{mn} \rightarrow (j_{mn1}, j_{mn2}, j_{mn3})..., \; k_{mn} \rightarrow (k_{mn1}, k_{mn2}, k_{mn3})... \\
& \overline{\Psi}(r, t) = \overline{\Psi}(x, y, z, t) = \Psi(r, t) + I \cdot H\left[\Psi(r, t)\right] = \Psi(r, t) + I \cdot \hat{\Psi}(r, t), \\
& \Psi(r, t) = \Psi_x(r, t) + \Psi_y(r, t) + \Psi_z(r, t), \; \hat{\Psi}_{x,y,z}(r, t) = H\left[\Psi_{x,y,z}(r, t)\right], \\
& I \cdot \hat{\Psi}(r, t) = i \cdot \hat{\Psi}_x(r, t) + j \cdot \hat{\Psi}_y(r, t) + k \cdot \hat{\Psi}_z(r, t) =
\end{aligned}
$$

$$
= \begin{bmatrix}
i_{11} \cdot \hat{\Psi}_{x1}(r, t) + i_{12} \cdot \hat{\Psi}_{x2}(r, t) + i_{13} \cdot \hat{\Psi}_{x3}(r, t) \\
+ \\
j_{11} \cdot \hat{\Psi}_{y1}(r, t) + j_{12} \cdot \hat{\Psi}_{y2}(r, t) + j_{13} \cdot \hat{\Psi}_{y3}(r, t) \\
+ \\
k_{11} \cdot \hat{\Psi}_{z1}(r, t) + k_{12} \cdot \hat{\Psi}_{z2}(r, t) + k_{13} \cdot \hat{\Psi}_{z3}(r, t)
\end{bmatrix}. \qquad (6.11)
$$

Following the same patterns of brainstorming, we could imagine the basis of multidimensional universes extending each of the spatial coordinates into a new set of three elements (associating different, hypercomplex imaginary units to every triplet of spatial coordinates), as for instance:

$$
\mathbf{(x_1, x_2, x_3, t) \Rightarrow \left[(x_{11}, x_{21}, x_{23}), (x_{12}, x_{22}, x_{32}), (x_{13}, x_{23}, x_{33}), \ldots (x_{1n}, x_{2n}, x_{3n}), t)\right]}. \quad (6.12)
$$

Hypercomplex numbers and functions can be formulated in a more general way than in (6.8) and (6.9), using $\mathbf{n}$ elementary complex units (instead of three), and this way we can exercise new and more complex multi-dimensional coordinate systems...

*"**Experimenting**" with algebraic sign (**+** and/or **–**), with different combinations of hypercomplex imaginary units (**i, j, k**), and with couples of complex and complex-conjugate wave functions and their phase-shifted Hilbert couples mentioned in 3.a) and 3.b), we can try to address various **symmetry related structures** known in Physics.*

3. c) Until present, the time dimension **t** or **Ict** was implicitly considered as a homogenous and isotropic, single and independent variable. Mathematically, we can show that the time dimension and absolute speed **c** could be treated as anisotropic, coordinates-dependent, physical (and vector) values, **at least in one of three different ways**, as follows,

$$I \cdot ct = \begin{Bmatrix} (i_x ct + i_y ct + i_z ct), \text{ or} \\ (i_x c_x t + i_y c_y t + i_z c_z t), \text{ or} \\ (i_x ct_x + i_y ct_y + i_z ct_z), \text{ or} \\ (i_x c_x t_x + i_y c_y t_y + i_z c_z t_z) \end{Bmatrix} \begin{pmatrix} \Rightarrow \\ (=) \end{pmatrix} I \cdot \begin{Bmatrix} c, t, & \text{or} \\ (c_x + c_y + c_z), t & \text{or} \\ c, (t_x + t_y + t_z), & \text{or} \\ (c_x, c_y, c_z), (t_x, t_y, t_z) & \end{Bmatrix}$$

$$I^2 = i_x^2 = i_y^2 = i_z^2 = -1, \ (i_x = i, i_y = j, i_z = k, x_1 = x, x_2 = y, x_3 = z, \quad (6.13)$$

$$c^2 = c_x^2 + c_y^2 + c_z^2, \ t^2 = t_x^2 + t_y^2 + t_z^2,$$

while keeping the relativistic space-time interval $\Delta\mathbf{S}$, and absolute speed and time amplitudes ($\mathbf{c}$, $\mathbf{t}$), unchanged,

$$(\Delta \mathbf{S})^2 = (\Delta \mathbf{x_1})^2 + (\Delta \mathbf{x_2})^2 + (\Delta \mathbf{x_3})^2 - \mathbf{c}^2 (\Delta \mathbf{t})^2 = \mathbf{const.}.$$

In other words, there is a possibility that the constant and maximal, absolute speed $\mathbf{c}$ has variable and unlimited, mutually coordinate-dependent, velocity vector components $(\mathbf{c}_x, \mathbf{c}_y, \mathbf{c}_z)$, and that something similar is valid for the relevant and associated time dimension components $(\mathbf{t}_x, \mathbf{t}_y, \mathbf{t}_z)$. Maybe, some of the speed components, hypothetically higher than $\mathbf{c}$, could be mathematically (still hypothetically) related to entanglement effects in relation to "Bell's interconnections theorem", and to David Bohm's concept of particle-wave phenomenology of pilot waves. Another option radiating from similar ideas is that any Multidimensional Universe should have an equal number of mutually coupled, spatial and temporal components or dimensions, since in Relativity theory time and space are mutually linked and interdependent (and this should be a generally extendable situation).

In Chapter 4.1 of this book, we can find all relations between the group and the phase velocity applicable to any wave motion that has sinusoidal wave components and harmonic nature of its spectrum (see eq. (4.2)).

$$\begin{Bmatrix} v = u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda} = u + p \frac{du}{dp} = \frac{d\omega}{dk} = \frac{d\tilde{E}}{dp} = h \frac{df}{dp} = \frac{df}{df_s} = \frac{2u}{1 + \frac{uv}{c^2}}, \\ u = \lambda f = \frac{\omega}{k} = \frac{\tilde{E}}{p} = \frac{hf}{p} = \frac{f}{f_s} = \frac{v}{1 + \sqrt{1 - \frac{v^2}{c^2}}} = \frac{E_k}{p} \Rightarrow \\ \Rightarrow \quad 0 \le 2u \le \sqrt{uv} \le v \le c, \\ d\tilde{E} = hdf = mc^2 d\gamma, \quad \frac{df}{f} = (\frac{dv}{v}) \cdot \frac{1 + \sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{v^2}{c^2}} \Rightarrow \frac{\Delta f}{\overline{f}} = (\frac{\Delta v}{\overline{v}}) \cdot \frac{1 + \sqrt{1 - \frac{\overline{v}^2}{c^2}}}{1 - \frac{\overline{v}^2}{c^2}} \end{Bmatrix} \quad (4.2)$$

Relations (4.2) are also applicable to photons, electromagnetic waves, and speed of light, where the speed of light should correspond to the group velocity in (4.2). When an electromagnetic wave propagates in a vacuum and free space, all three velocities will become mutually equal, $\mathbf{v = u = c \cong 300000\ km/s}$, and here should lie a part of the explanation what the meaning of the universal constant $\mathbf{c}$ is. Another challenging question is if we would be able to send part of the energy or meaningful information at speeds higher than $\mathbf{c}$.

3. d) Whatever mathematical basis we choose to describe a multidimensional universe, it must be compatible with the energy-related aspects of dimensionality as defined in previous equations, such as (4.32), (5.15), (5.16), and (5.17). Complex and hypercomplex numbers, along with their associated functions, typically offer a suitable and natural mathematical framework, enabling more elegant and concise formulations of multidimensional physical realities.

However, a fundamental question remains: on what basis do we select (or impose) a specific dimensional structure, such as $\mathbf{(x_1, x_2, x_3, t)} \Rightarrow \mathbf{(x_1, x_2, x_3, \ldots, x_n, t)}$, or $(x_1,x_2,x_3...,x_n\ ,\ t_1,t_2,t_3...,t_n)$, to define an n-dimensional universe? Do we genuinely know Nature's preference regarding the dimensionality of the universe? The intent here is not to criticize the existing mathematical models of n-dimensionality, which may very well be correct, but rather to explore alternative options and assess which of these frameworks aligns more closely with the actual structure of our multidimensional universe.

This line of inquiry remains highly speculative and hypothetical. One notable approach not yet discussed is Superstring Theory, which introduces the concept of compactified spatial dimensions. Nevertheless, the author of this paper holds the view that hypercomplex analytic signals and n-dimensional phasors offer a more promising and natural mathematical foundation for investigating multidimensional reality.

All the above-introduced options regarding the mathematical basic frames of a multidimensional universe can be generalized by modifying the multi-dimensional basis, given by (6.1), introducing hypercomplex spatial coordinates $(\overline{\mathbf{x}}_1, \overline{\mathbf{x}}_2, \ldots, \overline{\mathbf{x}}_n)$, where the time dimension is not explicitly present (but couples of spatial and temporal components always exist inside), such as:

$$\{\textbf{(SPACE) \& TIME}\} \Leftrightarrow \{(x_1,x_2,...,x_n) + t\} \Leftrightarrow (r,t) \Rightarrow (\overline{x}_1,\overline{x}_2,...,\overline{x}_n)\ . \qquad (6.14)$$

In (6.14) the time dimension is created only from imaginary parts of generalized hypercomplex space coordinates $(\overline{\mathbf{x}}_1, \overline{\mathbf{x}}_2, \ldots, \overline{\mathbf{x}}_n)$, or n-vectors in the following way:

$$\begin{aligned}
&\overline{x}_i = x_{i(real)} + I\, x_{i(imaginary)} = x_i e^{I\theta_{xi}},\ \ x_i = \sqrt{x^2_{\ i(real)} + x^2_{\ i(imaginary)}}\ , \\
&\theta_{xi} = \arctan \frac{x_{i(imaginary)}}{x_{i(real)}},\ i = 1,2,3,...,n;\ I^2 = -1, \\
&(\overline{x}_1,\overline{x}_2,...,\overline{x}_n) \Leftrightarrow \{(x_{1(real)}, x_{2(real)}, ...x_{n(real)}), (x_{1(imaginary)}, x_{2(imaginary)}, ...x_{n(imaginary)})\} \\
&\Leftrightarrow (r,t),\ r = r(x_{1(real)}, x_{2(real)}, ...x_{n(real)}),\ \ t\ =\ t(x_{1(imaginary)}, x_{2(imaginary)}, ...x_{n(imaginary)})
\end{aligned} \qquad (6.15)$$

There are many mathematical possibilities for how to treat and analyze (6.15), operating with only one, three or more hypercomplex imaginary units. Practically, a kind of generalized multidimensional (hypercomplex) Minkowski n-space of the Relativity Theory is immediately recognizable in (6.15) and (6.16), when we introduce the following n-vector notation:

$$(\overline{x}_1,\overline{x}_2,...,\overline{x}_n) \Leftrightarrow \{(x_{1(real)},x_{2(real)},...x_{n(real)}),(x_{1(imaginary)},x_{2(imaginary)},...x_{n(imaginary)})\}$$
$$\Leftrightarrow (r,ct),\ r = r(x_{1(real)},x_{2(real)},...x_{n(real)}),\ t = \frac{1}{c}(x_{1(imaginary)},x_{2(imaginary)},...x_{n(imaginary)}) \qquad (6.16)$$

The 4-dimensional relativistic space-time interval based on (6.16), unified with Uncertainty relations from Chapter 5, could be expressed as:

$$(\Delta S)^2 = (\Delta x_{1(real)})^2 + (\Delta x_{2(real)})^2 + (\Delta x_{3(real)})^2 + (\delta s)^2 - c^2(\Delta t)^2,$$
$$(\delta s)^2 = (\Delta x_{4(real)})^2 + (\Delta x_{5(real)})^2 + ... + (\Delta x_{n(real)})^2 \geq \left[\frac{h}{2\Delta p}\right]^2 \approx 0,$$
$$c^2(\Delta t)^2 = (\Delta x_{1(imaginary)})^2 + (\Delta x_{2(imaginary)})^2 + ... + (\Delta x_{n(imaginary)})^2, \qquad (6.17)$$
$$\delta s \cdot \Delta p = \Delta t \cdot \Delta\tilde{E} \geq \frac{h}{2}.$$

The definition of 4-dimensional relativistic space-time interval (as we find it presently in Relativity theory and in this paper; -see Chapter 5.; -equations (5.18) – (5.21) and Chapter 10) is still a somewhat speculative and challenging concept. It is not experimentally or undoubtedly proven, and most probably that it would be modified using 4-vector of relativistic velocity (which can be considered as correctly defined), as for instance,

$$\overline{V}_4 = V(\gamma v, \gamma c) = \frac{d}{dt}\overline{X}_4 \Rightarrow d\overline{X}_4 = \overline{V}_4 dt = X(\gamma v \cdot dt, \gamma c \cdot dt) \Rightarrow$$
$$\Rightarrow (\Delta S)^2 = \left(\int_{[\Delta T]} \gamma v \cdot dt\right)^2 - \left(c\int_{[\Delta T]} \gamma \cdot dt\right)^2,\ v = v(x,y,z,t) = v(x_1,x_2,x_3,t), \qquad (6.17\text{-}1)$$
$$t = t(x_4,x_5,x_6,...x_n),\ \gamma = 1/\sqrt{1-v^2/c^2}.$$

In future analyses of **multidimensional-space coordinate systems**, it will be essential to determine which spatial coordinate-frame (such as the one proposed in equations (6.1) – (6.17-1), or similar models like in [49]) is the most natural and realistic for effective applications in physics.

In this section, we have speculated on various **multidimensional spacetime frameworks**, drawing on analogies with **Riemannian** and **Minkowski space** from relativity theory. These explorations examine new possibilities when **ordinary complex functions** are extended into the realm of **analytic hypercomplex functions**. While the current **Minkowski 4-vector concept** used in relativity theory has been intellectually stimulating and has yielded particularly strong results in particle interaction studies, it is likely that its interpretation and practical applications will evolve into something more generalized, potentially resembling **Hypercomplex Analytic Phasors**. (For more on this, see Chapter 10, section "10.1 Hypercomplex Analytic Signal functions and interpretation of energy-momentum 4-vectors in relation to matter-waves and particle-wave duality.")

Similarly, **Quantum Mechanical wave functions** could be reformulated using **hypercomplex analytic signal functions**, creating new possibilities for integrating **Relativity** and **Quantum Theory** (if such integration is even necessary, given that both fields are expected to undergo significant evolution). The potential superiority of **multidimensional hypercomplex analytic signal functions** should become apparent to open-minded, visionary individuals when considering the overwhelming mathematical and wave-modeling advantages of the **ordinary Analytic Signal**, which is based on the **Hilbert Transform**. This model has proven far superior to any other signal analysis method established before the introduction of the **Analytic Signal** (see more in Chapters 4.0, 4.3, and 10).

It is unfortunate that **Quantum Theory**, during its early development, did not establish the **wave function** as an **Analytic Signal** or **Phasor**. This omission likely occurred because the concept of the Analytic Signal was introduced only after **Quantum Theory** had already canonized and overly glorified its own artificial methods for wave function mathematical processing. By that time, quantum theorists viewed the existing framework as functioning well enough, making it seem unnecessary or impractical to restart from scratch, even if the **Analytic Signal** approach could offer advantages and improvements.

Moreover, the sheer volume of published references, recognitions, and prizes that Quantum Theory had accumulated made any attempt to overhaul the established framework politically and scientifically difficult to defend. The author of this book, however, believes that **Quantum Theory** would perform equally well, if not better, if its **wave function** framework were rebuilt using **Hypercomplex Analytic Signals and Phasors**. (See Chapter 4.0 for more on this modeling approach.)

[♣ COMMENTS & FREE-THINKING CORNER: **STILL IN PROCESS ⇒**

For our Universe, we usually say that it has three spatial dimensions and conveniently, we add a time dimension. Three spatial dimensions are just spatial coordinates, having the same physical nature (being some distances measured in the same way using the same units). How such ***identical nature coordinates***, could be separate or independent dimensions? Here is the Quote from the books ***Lone Pine Writings by Eric Dollard***:

"It is customary to consider space boundaries as a CUBIC, or third degree set of coordinates. The three coordinates are length, width, and height, taken from a corner of the cube. Think of a sugar cube, the sugar is the space, and the corners define the boundaries. These three coordinates, length, width, and height are WRONGLY known as the three dimensions of our space. This is a major mind virus, and it is hard to erase it. There is only one dimension of space, SPACE, a metrical dimension. Any number of coordinates in any number of geometries can serve to define the boundaries of said space. The use of the cubic three is habitual. The dimension of space is considered to exist in degrees or powers of a unit space dimension, here centimeters, l (lowercase L). So, we can say cubic centimeters, or square centimeters, etc."

6.1. Hypercomplex, In-depth Analysis of Wavefunctions

Starting from the signal representation in the form of an analytical signal (**1.1**) and (**1.2**) from the chapter 4.0, (which reminds to a simple harmonic form of a signal), we can continue the same procedure of factorization unlimitedly, representing every new amplitude function a(t), by a new form of analytical signal. Thus, it will be:

$$\Psi(t) = \frac{1}{\pi}\int_0^{\infty}[U_c(\omega)\cos\omega t + U_s(\omega)\sin\omega t]d\omega = \frac{1}{\pi}\int_0^{\infty}[A(\omega)\cos(\omega t + \Phi(\omega))]d\omega =$$
$$= a(t)\cos\varphi(t) = \Psi_0(t) = a_0(t)\cos\varphi_0(t) =$$
$$= a_1(t)\cos\varphi_1(t)\cos\varphi_0(t) = \Psi_1(t)\cos\varphi_0(t) =$$
$$= a_2(t)\cos\varphi_2(t)\cos\varphi_1(t)\cos\varphi_0(t) = \Psi_2(t)\cos\varphi_1(t)\cos\varphi_0(t) =$$

....

$$= a_n(t)\prod_{i=0}^{n}\cos\varphi_i(t) = \Psi_n(t)\prod_{i=0}^{n-1}\cos\varphi_i(t) = a_n(t)\cdot\cos\varphi_0(t)\prod_{i=1}^{n}\cos\varphi_i(t),$$
$$\left[a(t) = a_0(t) = a_n(t)\prod_{i=1}^{n}\cos\varphi_i(t),\ \cos\varphi(t) = \cos\varphi_0(t)\right]. \quad (6.18)$$

Previously obtained, factorized signal form reminds us of a multiple amplitude modulation of the signal, where every following level represents an amplitude function of the previous level, so in that sense, we can talk about the in-depth structure of the signal, whose structural levels may be presented by following functions:

$$\Psi_1(t) = a_0(t) = a_1(t)\cos\varphi_1(t) = \left|\overline{\Psi}(t)\right| = \left|\overline{\Psi}_0(t)\right| = \frac{\Psi_0(t)}{\cos\varphi_0(t)},$$

$$\Psi_2(t) = a_1(t) = a_2(t)\cos\varphi_2(t) = \left|\overline{a}_0(t)\right| = \left|\overline{\Psi}_1(t)\right| = \frac{\Psi_0(t)}{\cos\varphi_1(t)\cos\varphi_0(t)},$$

.... (6.19)

$$\Psi_n(t) = a_{n-1}(t) = a_n(t)\cos\varphi_n(t) = \left|\overline{a}_{n-2}(t)\right| = \left|\overline{\Psi}_{n-1}(t)\right| = \frac{\Psi_0(t)}{\prod_{i=0}^{n-1}\cos\varphi_i(t)}.$$

From (3.15) and (3.16) one may notice that the following relations remain:

$$\Psi_{k+1}(t) = a_k(t) = a_{k+1}(t)\cos\varphi_{k+1}(t) = a_n(t)\prod_{i=k+1}^{n}\cos\varphi_i(t),\ k < n,$$

$$\prod_{i=1}^{n}\Psi_i(t) = \prod_{i=1}^{n}a_n(t)\cdot\prod_{i=1}^{n}\cos\varphi_i(t)\ ,\quad \frac{\Psi_k(t)}{\Psi_{k+1}(t)} = \cos\varphi_k\ ,\ \prod_{i=0}^{n}\cos\varphi_i(t) = \frac{\prod_{i=0}^{n}\Psi_i(t)}{\prod_{i=1}^{n+1}\Psi_i(t)}\ , \quad (6.20)$$

$$\frac{a_1}{a_0}\cos\varphi_1 = \frac{a_2}{a_1}\cos\varphi_2 = ... = \frac{a_n}{a_{n-1}}\cos\varphi_n = 1.$$

If we briefly address signal velocities, we can conclude that the phase velocity of the basic signal $\Psi(t)$ (as well as the common phase velocity of its components) is determined only by the $\cos\phi_0$. In addition, the group velocity of the signal $\Psi(t) = \Psi_0(t) = a_0(t)\cos\phi_0(t)$ propagation is determined only as the velocity of its amplitude function $a_0(t)$. By considering the factorization of the signal, one may conclude that there are series of partial group velocities, which by appropriate addition (like the relativistic determination of the velocity of the inertia center) must form the resulting (central, group) velocity of the amplitude member $a_0(t)$.

*A problem that is interesting to be solved mathematically is the case of factorized signal presentation (6.18), where we could fix several $\cos\varphi_i(t)$ members in advance, giving them specific (or arbitrary) values for phase functions, regardless of if such values could be found in the initially factorized signal form. Let us take the example where only three of such cosine-phase members (indexed with **A**, **B**, and **C**) are fixed by our intervention,*

$\Psi(t) = a_n(t)\cdot\prod_{i=0}^{n}\cos\varphi_i(t) = a_n(t)\cdot\cos\varphi_A(t)\cdot\cos\varphi_B(t)\cdot\cos\varphi_C(t)\cdot\prod_{i=3}^{n}\cos\varphi_i^*(t)$ *The question here is if we could still find the right product of remaining cosine members* $\prod_{i=3}^{n}\cos\varphi_i^*(t)$ *that will fit exactly to represent the same signal (without any error). Why and where could be such factorization interesting? Let us imagine that we intend to present the process of disintegration of certain microparticle, initially presentable by* $\Psi(t)$. *Based on experimental data, we may know (in advance) the resulting amplitude function* $a(t) = a_0(t) = a_n(t)\prod_{i=1}^{n}\cos\varphi_i(t)$ *of the same particle before its disintegration. In addition, knowing the experimental results of particle disintegration, we could find that three new particles (for the needs of this example) are clearly detectable (particles indexed with **A**, **B**, and **C**), and that possibly we have certain signal residuals, or energy dissipation, producing some other elements, not easy to detect. Hypothetically, we will attempt to present all such residual elements using the factorization* $\prod_{i=3}^{n}\cos\varphi_i^*(t)$. *If here described scenario can be mathematically modeled to describe real situations from microphysics, this will pave a way to start organizing elementary particles in a much better way than presently known.*

Every structural level of the signal from (6.19) can be associated with correspondent complex, analytical signal, by the same procedure as given in (4.02),

$$\{\Psi_1(t)\} \to \overline{\Psi}_1(t) = a_0(t) + i_1\hat{a}_0(t) = a_1(t)e^{i_1\varphi_1(t)},$$

$$\{\Psi_2(t)\} \to \overline{\Psi}_2(t) = a_1(t) + i_2\hat{a}_1(t) = a_2(t)e^{i_2\varphi_2(t)}, \ldots\ldots\ldots\ldots\ldots\ldots$$

$$\{\Psi_n(t)\} \to \overline{\Psi}_n(t) = a_n(t) + i_n\hat{a}_n(t) = a_n(t)e^{i_n\varphi_n(t)},$$

$$\{\Psi_n(t)\} \to \overline{\Psi}(t) = \Psi(t) + I\hat{\Psi}(t) = a_0(t)e^{I\varphi_0(t)}, \qquad (6.21)$$

where $\hat{\Psi}_{k+1}(t) = \hat{a}_k(t) = H[a_k(t)]$.

One may notice that in (6.19), an indexed imaginary unit is introduced, which for now should be understood, and interpreted, as an already known (common) imaginary unit:

$$I^2 = i^2 = j^2 = i_1^{\ 2} = i_2^{\ 2} = i_3^{\ 2} = \ldots. = i_n^{\ 2} = -1. \qquad (6.22)$$

However, later such indexed imaginary units will serve for evolving from a simple complex space to a hyper-complex space with many imaginary units.

The specificity of penetrating into the structural depth of the signal (or into the wave function) becomes obvious if we determine energies of the structural levels of the signal (in the same way as in (4.04)):

$$\tilde{E}_0 = \int_{-\infty}^{+\infty}\Psi^2(t)dt = \int_{-\infty}^{\infty}a_0^2(t)\cos^2\varphi_0(t)\,dt = \frac{1}{2}\int_{-\infty}^{+\infty}a_0^2(t)\,dt,$$

$$\tilde{E}_1 = \int_{-\infty}^{+\infty} \Psi_1^2(t)dt = \int_{-\infty}^{\infty} a_0^2(t)\,dt = \frac{1}{2}\int_{-\infty}^{+\infty} a_1^2(t)\,dt = 2\tilde{E}_0 \ ,$$

$$\tilde{E}_2 = \int_{-\infty}^{+\infty} \Psi_2^2(t)dt = \int_{-\infty}^{\infty} a_1^2(t)\,dt = \frac{1}{2}\int_{-\infty}^{+\infty} a_2^2(t)\,dt = 4\tilde{E}_0 \ , \qquad (6.23)$$

....

$$\tilde{E}_n = \int_{-\infty}^{+\infty} \Psi_n^2(t)dt = \int_{-\infty}^{\infty} a_{n-1}^2(t)\,dt = \frac{1}{2}\int_{-\infty}^{+\infty} a_n^2(t)\,dt = 2^n\tilde{E}_0 = 2\tilde{E}_{n-1}, n = 0,1,2...$$

$$(\tilde{E}_0 = (\gamma-1)m_0c^2 = \frac{\tilde{E}_n}{2^n} \Rightarrow m_0 = \frac{\tilde{E}_0}{(\gamma-1)c^2} = \frac{\tilde{E}_n}{2^n(\gamma-1)c^2}, \gamma = (1-v^2/c^2)^{-0.5}).$$

It is obvious that penetrating deeper structural levels of the signal "**n**" demands consumption of more and more energy for factor 2^n, to isolate and analyze those levels (in a way like the analysis of nuclear and elementary particles).

Let us go back now to the structural levels of the signal in the form of (6.19). We will call the basic level of the signal in its complex form a hypercomplex analytical function of the signal, and we will represent it as in (4.02), apart from that, the complex unit "j" will be replaced by the hypercomplex, imaginary unit "I":

$$\overline{\Psi}(t) = \Psi(t) + I\hat{\Psi}(t) = a_0(t)e^{I\varphi_0(t)} = a_0(t)\left[\cos\varphi_0(t) + I\sin\varphi_0(t)\right], \qquad (6.24)$$

where the hypercomplex, imaginary unit "**I**" is defined as a linear combination over the base of elementary hypercomplex units "$i_1, i_2, i_3, ..., i_n$", as:

$$I\varphi_0(t) = i_1\varphi_1(t) + i_2\varphi_2(t) + ... + i_n\varphi_n(t) = \sum_{k=1}^{n} i_k\varphi_k(t),$$

$$I^2 = i_1^2 = i_2^2 = i_3^2 = ... = i_n^2 = -1,$$

$$i_j i_k = 0, \forall j \neq k \Rightarrow \varphi_0^2(t) = \sum_{k=1}^{n} \varphi_k^2(t), \qquad (6.25)$$

$$e^{i_k\varphi_k(t)} = \cos\varphi_k(t) + i_k \sin\varphi_k(t), \forall k \in [1, n].$$

According to the introduction of the hypercomplex wave function $\overline{\Psi}(t)$ in a previous way, it is possible to show the validity of the following relations:

$$\overline{\Psi}(t) = \Psi(t) + I\hat{\Psi}(t) = a_0(t)e^{I\varphi_0(t)} = a_0(t)\left[\cos\varphi_0(t) + I\sin\varphi_0(t)\right] =$$
$$= a_0(t)e^{\sum_{(k)} i_k\varphi_k(t)} = \sum_{(k)} a_k(t)e^{i_k\varphi_k(t)} = \sum_{(k)} \overline{\Psi}_k(t), \qquad (6.26)$$

where:

$$\overline{\Psi}_k(t) = a_k(t)e^{i_k\varphi_k(t)} = \Psi_k(t) + i_k\hat{\Psi}_k(t) \ ,$$

$$\varphi_k(t) = \text{arctg}\frac{\hat{\Psi}_k(t)}{\Psi_k(t)}, \ \varphi_0^2(t) = \sum_{(k)} \varphi_k^2(t), \qquad (6.27)$$

$$a_k^2(t) = a_{k-1}^2(t) + \hat{a}_{k-1}^2(t) = \Psi_{k+1}^2(t) = \Psi_k^2(t) + \hat{\Psi}_k^2(t),$$

$$a_0^2(t) = \left|\overline{\Psi}(t)\right|^2 = \Psi^2(t) + \hat{\Psi}^2(t) = \sum_{(k)} a_k^2(t) + 2\sum_{(i\neq j)} \Psi_i(t)\Psi_j(t), \ \forall i, j, k \in [1, n].$$

Based on the structural analysis of the signal, introducing its structural levels through (6.19), it is obvious that we are in a position that, if we know the function of the basic signal level $\Psi(t)$, we can determine its remaining (depth) levels of a higher order:

$$\Psi(t) = \Psi_0(t) \rightarrow \Psi_1(t) \rightarrow \Psi_2(t) \ldots \rightarrow \Psi_n(t), \quad (6.28)$$

and also go back from any level $\Psi_k(t)$, $0 < k \leq n$ (using inverse transformations) to the basic level of the signal $\Psi_0(t)$,

$$\Psi_k(t) \rightarrow \Psi_{k-1}(t) \rightarrow \Psi_{k-2}(t) \rightarrow \ldots \rightarrow \Psi_0(t) \cdot \quad (6.29)$$

If we continue the procedure of inverse transformations from (6.29) in the same direction (backward), we will be able to determine sub-structural signal levels (or its history),

$\Psi_k(t) \rightarrow \Psi_{k-1}(t) \rightarrow \Psi_{k-2}(t) \rightarrow \ldots$	$\rightarrow \Psi_0(t) \rightarrow$	$\rightarrow \Psi_{-1}(t) \rightarrow \Psi_{-2}(t) \rightarrow \ldots \rightarrow \Psi_{-n}(t)$
depth levels of the signal	basic level	Sub-structural levels
(future of the signal, $\tilde{E}_k$)	(present state $\tilde{E}_0$)	(past of the signal, $\tilde{E}_{-k}$)

One can show that eigen-energies of the substructural signal levels (or its prehistory, i.e., energy spent to form that signal) will be, similarly to the formula for the energies of the structural (depth) levels (6.23), equal to

$$\tilde{E}_{-n} = 2^{-n}\tilde{E}_0 \ , n = 0, 1, 2, \ldots \quad (6.30)$$

One may immediately notice that sum of all energies of the substructural or past signal levels is equal to the sum of a convergent series (converging towards $\tilde{E}_0$),

$$\sum_{k=1}^{n} \tilde{E}_{-k} = \sum_{k=1}^{n} 2^{-k}\tilde{E}_0 \leq \tilde{E}_0, \quad (n \rightarrow \infty) , \quad (6.31)$$

which obviously represents the form of the Law of energy conservation, while the series of the energies of the higher or deeper structural signal levels $(n \rightarrow +\infty)$ is divergent, i.e.,

$$\sum_{k=1}^{n} \tilde{E}_k = \sum_{k=1}^{n} 2^{k}\tilde{E}_0 \rightarrow \infty, \quad (n \rightarrow \infty) \cdot \quad (6.32)$$

As equation (6.32) indicates, delving deeper into the structure of a given signal (or matter) requires progressively greater amounts of energy, an endeavor that mirrors the ongoing quest in modern particle accelerators. We can draw a parallel between this structural analysis, which relies on signal factorization, and the classical Fourier analysis, where signals are expressed as sums of orthogonal functions.

In this context, Fourier analysis can be viewed as a type of "surface-level" signal decomposition, focusing on a single structural level. Conversely, factorization goes beyond this, serving as a "spatial" or "volumetric" structural-decomposition method, delving into the internal layers and deeper structures of the signal. We might think of it as a form of "signal mining."

This factorized approach suggests that, with suitable mathematical modifications, a similar methodology could be applied to analyze the internal structure of matter, such as atoms and subatomic particles. Additionally, this model could have significant applications in telecommunications, data processing, and the analysis of various signals that carry information about complex processes.

ANALYTIC SIGNAL FORMS

$$\overline{H}\left[\Psi(t)\right] = \overline{\Psi}(t) = \Psi(t) + I\cdot\hat{\Psi}(t) = a(t)\cdot e^{I\cdot\varphi(t)} \quad , \quad \overline{H} = 1 + I\cdot H,\ I^2 = -1$$

AS SUPERPOSITION	AS MULTIPLICATION
$\overline{\Psi}(t) = \sum_{(k)} \overline{\Psi}_k(t) = \sum_{(k)} \Psi_k(t) + I\sum_{(k)} \hat{\Psi}_k(t)$	$\overline{\Psi}(t) = \Psi_n(t)\prod_{(i=0)}^{n-1}\cos\varphi_i(t) + I\Psi_n(t)\prod_{(i=0)}^{n-1}\sin\varphi_i(t)$

RELATIONS BETWEEN ADDITIVE AND MULTIPLICATIVE ELEMENTS

$$\Psi(t) = \sum_{(k)} \Psi_k(t) = \Psi_n(t)\prod_{(i=0)}^{n-1}\cos\varphi_i(t) = a_0(t)\cos\varphi_0(t) = a(t)\cos\varphi(t) = -H\left[\hat{\Psi}(t)\right]$$

$$\hat{\Psi}(t) = \sum_{(k)} \hat{\Psi}_k(t) = \Psi_n(t)\prod_{(i=0)}^{n-1}\sin\varphi_i(t) = a_0(t)\sin\varphi_0(t) = a(t)\sin\varphi(t) = H\left[\Psi(t)\right]$$

$$\overline{\Psi}_k(t) = \Psi_k(t) + i_k\hat{\Psi}_k(t) = a_k(t)e^{i_k\varphi_k(t)}\ ,\Psi_k(t) = -H\left[\hat{\Psi}_k(t)\right],\ \hat{\Psi}_k(t) = H\left[\Psi_k(t)\right]$$

$$\overline{\Psi}(t) = \Psi(t) + I\hat{\Psi}(t) = \frac{a_n(t)}{2^{n+1}}\left\{\prod_{k=0}^{n}(e^{I\varphi_k(t)} + e^{-I\varphi_k(t)}) + \frac{1}{(i)^n}\prod_{k=0}^{n}(e^{I\varphi_k(t)} - e^{-I\varphi_k(t)})\right\}$$

Signal Amplitude

$$a(t) = a_0(t) = \left|\overline{\Psi}(t)\right| = \sqrt{\Psi^2(t) + \hat{\Psi}^2(t)} = a_n(t)\prod_{(i=1)}^{n}\cos\varphi_i(t) = \Psi_{n+1}(t)\prod_{(i=1)}^{n}\cos\varphi_i(t)$$

$$a_k(t) = \Psi_{k+1}(t) = \left|\overline{\Psi}_k\right| = \sqrt{\Psi_k^2(t) + \hat{\Psi}_k^2(t)} = a_{k+1}(t)\cos\varphi_{k+1}(t) = a_n(t)\prod_{(i=k+1)}^{n}\cos\varphi_i(t),\ k < n$$

$$a_{n-1}(t) = \Psi_n(t) = \left|\overline{\Psi}_{n-1}(t)\right| = \sqrt{\Psi_{n-1}^2(t) + \hat{\Psi}_{n-1}^2(t)} = a_n(t)\cos\varphi_n(t) = \frac{\Psi_0(t)}{\prod_{(i=0)}^{n-1}\cos\varphi_i(t)}$$

Signal Phase

$$\varphi_0(t) = \varphi(t) = \operatorname{arctg}\frac{\hat{\Psi}(t)}{\Psi(t)} = \sqrt{\sum_{(k)}\varphi_k^2(t)},\ \varphi_k(t) = \operatorname{arctg}\frac{\hat{\Psi}_k(t)}{\Psi_k(t)}$$

$$I^2 = i_1^2 = i_2^2 = ... = i_n^2 = -1\ ,\ i_j i_k = 0, \forall\, j \neq k\ \ \text{(hypercomplex imaginary units)}$$

$$I\varphi_0(t) = I\varphi(t) = i_1\varphi_1(t) + i_2\varphi_2(t) + ... + i_n\varphi_n(t) = \sum_{k=1}^{n} i_k\varphi_k(t)$$

$$e^{i_k\varphi_k(t)} = \cos\varphi_k(t) + i_k\sin\varphi_k(t),\ \ \varphi_0^2(t) = \sum_{k=1}^{n}\varphi_k^2(t)$$

Signal instant frequency

$$\omega_i(t) = 2\pi f_i(t) = \frac{\partial\varphi_i(t)}{\partial t},\ \ i = 0,1,2,...k,...n$$

The parallelism between Time and Frequency Domains	Analytic Signal: **Time Domain**	Analytic Signal: **Frequency Domain**
Complex Signal	$\overline{\Psi}(t) = a(t)e^{j\varphi(t)}$ $= \Psi(t) + j\hat{\Psi}(t)$ $= \int_{(0,+\infty)} \overline{U}(\omega)e^{-j\omega t}d\omega$ $= \int_{(0,+\infty)} A(\omega)e^{-j(\omega t+\Phi(\omega))}d\omega$	$\overline{U}(\omega) = A(\omega)e^{-j\Phi(\omega)}$ $= U_c(\omega) - j\,U_s(\omega)$ $= \int_{[t]} \overline{\Psi}(t)\,e^{j\omega t}dt$ $= \int_{[t]} a(t)e^{j(\omega t+\varphi(t))}dt$
Real and imaginary signal components	$\psi(t) = a(t)\cos\varphi(t)$ $= -H[\hat{\psi}(t)],$ $\hat{\psi}(t) = a(t)\sin\varphi(t)$ $= H[\psi(t)]$	$U_c(\omega) = A(\omega)\cos\Phi(\omega)$ $U_s(\omega) = A(\omega)\sin\Phi(\omega)$
Signal Amplitude	$a(t) = \sqrt{\psi^2(t) + \hat{\psi}^2(t)}$	$A(\omega) = \sqrt{U_c^{\,2}(\omega) + U_s^{\,2}(\omega)}$
Instant Phase	$\varphi(t) = \mathrm{arctg}\dfrac{\hat{\psi}(t)}{\psi(t)}$	$\Phi(\omega) = \arctan\dfrac{U_s(\omega)}{U_c(\omega)}$
Instant Frequency	$\omega(t) = \dfrac{\partial\varphi(t)}{\partial t}$	$\tau(\omega) = \dfrac{\partial\Phi(\omega)}{\partial\omega}$
Signal Energy	$E = \int_{[t]} \lvert\overline{\psi}(t)\rvert^2 dt$ $= \int_{[t]} [a(t)]^2 dt$	$E = \dfrac{1}{2\pi}\int_{-\infty}^{+\infty} \lvert\overline{U}(\omega)\rvert^2 d\omega$ $= \dfrac{1}{\pi}\int_0^{\infty} [A(\omega)]^2 d\omega$
Central Frequency	$\Omega = \dfrac{\int_{[t]} \omega(t)\cdot a^2(t)dt}{\int_{[t]} a^2(t)dt}$	$\Omega = \dfrac{\int_0^{\infty} \omega\cdot[A(\omega)]^2 d\omega}{\int_0^{\infty} [A(\omega)]^2 d\omega}$
"Central Time"	$T = \dfrac{\int_{[t]} t\cdot[a(t)]^2 dt}{\int_{[t]} [a(t)]^2 dt}$	$T = \dfrac{\int_{[\omega]} \tau(\omega)\cdot[A(\omega)]^2 d\omega}{\int_{[\omega]} [A(\omega)]^2 d\omega}$
Standard Deviation	$\sigma_\Omega^2 = \dfrac{1}{\Delta t}\int_{[t]} \lvert\omega(t) - \Omega\rvert^2 dt$	$\sigma_T^2 = \dfrac{1}{\Delta\omega}\int_{[\omega]} \lvert\tau(\omega) - T\rvert^2 d\omega$
Uncertainty Relations	$\omega(t)\cdot\tau(\omega) \cong \Delta\omega\cdot\Delta\tau(\omega) \cong \sigma_\Omega\cdot\sigma_T \cong \Omega\cdot T \cong \Delta\omega\cdot\Delta t \cong \pi\,(!?)$	

Energy-momentum vectors and multidimensional Hypercomplex universe

The hypercomplex wave function is a much richer concept regarding the usual complex functions, and on the following example, we can show some more advantages of such representations of the wave function. Let us start with the previously defined hypercomplex wave function (6.24) and let us represent it in one of the possible (modified) forms:

$$\overline{\Psi}(t) = \Psi(t) + I\,\hat{\Psi}(t) = a_0(t)e^{I\,\varphi_0(t)} = a_0(t)\left[\cos\varphi_0(t) + I\ \sin\varphi_0(t)\right] =$$
$$= a_{01}(t)e^{I_1\varphi_{01}(t)} + a_{02}(t)e^{I_2\varphi_{02}(t)} + a_{03}(t)e^{I_3\varphi_{03}(t)} =$$

$$= \Psi_1(t) + \Psi_2(t) + \Psi_3(t) + i_1\,\hat{\Psi}_1(t) + i_2\,\hat{\Psi}_2(t) + i_3\,\hat{\Psi}_3(t) =$$
$$= \left[\Psi_{11}(t) + \Psi_{12}(t) + \Psi_{13}(t)\right] + \left[\Psi_{21}(t) + \Psi_{22}(t) + \Psi_{23}(t)\right] + \left[\Psi_{31}(t) + \Psi_{32}(t) + \Psi_{33}(t)\right] +$$
$$+\left[i_{11}\,\hat{\Psi}_{11}(t) + i_{12}\,\hat{\Psi}_{12}(t) + i_{13}\,\hat{\Psi}_{13}(t)\right] + \left[i_{21}\,\hat{\Psi}_{21}(t) + i_{22}\,\hat{\Psi}_{22}(t) + i_{23}\,\hat{\Psi}_{23}(t)\right] +$$
$$+\left[i_{31}\,\hat{\Psi}_{31}(t) + i_{32}\,\hat{\Psi}_{32}(t) + i_{33}\,\hat{\Psi}_{33}(t)\right] =$$
$$= \left[\Psi_{11}(t) + i_{11}\,\hat{\Psi}_{11}(t)\right] + \left[\Psi_{12}(t) + i_{12}\,\hat{\Psi}_{12}(t)\right] + \left[\Psi_{13}(t) + i_{13}\,\hat{\Psi}_{13}(t)\right] +$$
$$+\left[\Psi_{21}(t) + i_{21}\,\hat{\Psi}_{21}(t)\right] + \left[\Psi_{22}(t) + i_{22}\,\hat{\Psi}_{22}(t)\right] + \left[\Psi_{23}(t) + i_{23}\,\hat{\Psi}_{23}(t)\right] +$$
$$+\left[\Psi_{31}(t) + i_{31}\,\hat{\Psi}_{31}(t)\right] + \left[\Psi_{32}(t) + i_{32}\,\hat{\Psi}_{32}(t)\right] + \left[\Psi_{33}(t) + i_{33}\,\hat{\Psi}_{33}(t)\right] =$$
$$= a_{01}(t)e^{I_1\varphi_{01}(t)} + a_{02}(t)e^{I_2\varphi_{02}(t)} + a_{03}(t)e^{I_3\varphi_{03}(t)} = \ \dots\ ,$$

so that following relations stand:

$$\Psi(t) = \Psi_1(t) + \Psi_2(t) + \Psi_3(t),$$
$$I\,\hat{\Psi}(t) = i_1\,\hat{\Psi}_1(t) + i_2\,\hat{\Psi}_2(t) + i_3\,\hat{\Psi}_3(t)\ ,$$
$$\Psi_1(t) = \left[\Psi_{11}(t) + \Psi_{12}(t) + \Psi_{13}(t)\right],$$
$$\Psi_2(t) = \left[\Psi_{21}(t) + \Psi_{22}(t) + \Psi_{23}(t)\right],$$
$$\Psi_3(t) = \left[\Psi_{31}(t) + \Psi_{32}(t) + \Psi_{33}(t)\right],$$

$$i_1\,\hat{\Psi}_1(t) = \left[i_{11}\,\hat{\Psi}_{11}(t) + i_{12}\,\hat{\Psi}_{12}(t) + i_{13}\,\hat{\Psi}_{13}(t)\right],$$
$$i_2\,\hat{\Psi}_1(t) = \left[i_{21}\,\hat{\Psi}_{21}(t) + i_{22}\,\hat{\Psi}_{22}(t) + i_{23}\,\hat{\Psi}_{23}(t)\right],$$
$$i_3\,\hat{\Psi}_3(t) = \left[i_{31}\,\hat{\Psi}_{31}(t) + i_{32}\,\hat{\Psi}_{32}(t) + i_{33}\,\hat{\Psi}_{33}(t)\right].$$

The previous model of the wave function becomes a more interesting one, with the following introduction of the rule of three-dimensional (as vectors) orthogonality of the imaginary axes:

$$i_1 \times i_2 = i_3\ ,\quad i_2 \times i_3 = i_1\ ,\quad i_3 \times i_1 = i_2\ ,$$
$$i_2 \times i_1 = -i_3\ ,\quad i_3 \times i_2 = -i_1\ ,\quad i_1 \times i_3 = -i_2\ ,$$
$$i_{1n} \times i_{2n} = i_{3n}\ ,\ i_{2n} \times i_{3n} = i_{1n}\ ,\ i_{3n} \times i_{1n} = i_{2n}\ ,$$
$$i_{2n} \times i_{1n} = -i_{3n}\ ,\ i_{3n} \times i_{2n} = -i_{1n}\ ,\ i_{1n} \times i_{3n} = -i_{2n}\ ,\quad n \in [1,2,3]\ .$$

....

By continuing the previous approach, we can develop various forms of signal analysis and synthesis, further breaking down the hypercomplex wave function into distinct levels. These levels could potentially be used to model specific matter-state structures found in the realm of elementary and subatomic particles or even to contribute to the formulation of a universal field theory.

The use of multilevel hypercomplex structuring, employing multiple sets of “imaginary triplets of units”, opens the possibility of describing a type of multidimensional space that differs significantly from the conventional multidimensional frameworks currently in use. To make this concept of a "Hypercomplex Universe" more practical and applicable to physics, we could draw an analogy with the 4-vectors used in the Minkowski space of Relativity Theory.

By extending the traditional imaginary unit into triplets of hypercomplex imaginary units, we can formulate equivalent hypercomplex functions or hypercomplex 4-vectors. For instance, we can define a **hypercomplex momentum** vector by following a similar structure to that of the linear momentum vector. This approach is detailed in Chapter 4.1, paragraph 4.1.2.1 ("Hypercomplex Functions Interpretation"), and further elaborated in Chapter 10 ("Hypercomplex Analytic Signal Functions and Interpretation of Energy-Momentum 4-Vectors in Relation to Matter-Waves and Particle-Wave Duality").

$$\left\{\begin{array}{l} \vec{P}_4 = \left(\vec{p}, \frac{E}{c}\right) \Leftrightarrow \left(\vec{p}, I\frac{E}{c}\right) = \left(\vec{p}_i, i\frac{E_i}{c_i}\right) + \left(\vec{p}_j, j\frac{E_j}{c_j}\right) + \left(\vec{p}_k, k\frac{E_k}{c_k}\right) \\ \vec{P}_4 = \left(\vec{p}, \frac{E}{c}\right) = \text{invariant} \Rightarrow \vec{p}^2 - (\frac{E}{c})^2 = -(\frac{E_0}{c^2})^2 \Leftrightarrow E_0^2 + p^2c^2 = E^2 = (E_0 + E_k)^2 \\ \vec{p} = \vec{p}_i + \vec{p}_j + \vec{p}_k, E = E_0 + E_k, E_i = E_{0i} + E_{ki}, E_j = E_{0j} + E_{kj}, E_k = E_{0k} + E_{kk} \\ I^2 = i^2 = j^2 = k^2 = -1, ij = k,\ jk = i, ki = j, ji = -k, kj = -i, ik = -j \end{array}\right\} \Rightarrow \quad (4.3\text{-}0)\text{-s}$$

$$\left\{\begin{array}{l} I\frac{E}{c} = i\frac{E_i}{c_i} + j\frac{E_j}{c_j} + k\frac{E_k}{c_k}, \\ \left(\frac{E}{c}\right)^2 = \left(\frac{E_i}{c_i}\right)^2 + \left(\frac{E_j}{c_j}\right)^2 + \left(\frac{E_k}{c_k}\right)^2 \\ \left(\frac{E_0}{c}\right)^2 = \left(\frac{E_{0i}}{c_i}\right)^2 + \left(\frac{E_{0j}}{c_j}\right)^2 + \left(\frac{E_{0k}}{c_k}\right)^2 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} p^2 = \frac{E^2 - E_0^2}{c^2} = \frac{E_i^2 - E_{0i}^2}{c_i^2} + \frac{E_j^2 - E_{0j}^2}{c_j^2} + \frac{E_k^2 - E_{0k}^2}{c_k^2} = \\ = \vec{p}_i^2 + \vec{p}_j^2 + \vec{p}_k^2 \end{array}\right\}$$

In equation (4.3-0)-s, constants c_i, c_j, c_k with the dimensions of speed, such as the universal speed constant $c \cong 3 \cdot 10^8 m/s$, could be equal to $c = c_i = c_j = c_k$ (though the underlying mathematics allows for other possibilities). The energies indexed with 0 and k in (4.3-0)-s effectively pave the way for explaining the creation of numerous products during impact reactions: the "0" index corresponds to particles with rest mass, while the k-index represents kinetic energy states.

This extended energy-momentum framework can later be integrated into a more universal complex and hypercomplex analytic signal representation of wave functions, which eventually leads to the well-known wave equations of Quantum Theory. It also forms the basis for new concepts related to a multidimensional universe (see Chapter 4.3 and Chapter 6, equations (6.10) – (6.13)).

The presence of three distinct imaginary units in (4.3-0)-s intuitively suggests the existence of energy triplets, such as three quarks or three anti-quarks, hinting at the possibility of a more general and precise concept of supersymmetry in microphysics. This approach could significantly modify, enrich, and simplify the Standard Model by providing a deeper and more unified understanding of particle interactions. ♣]

CONCLUSION

This book offers a fresh perspective on particle-wave duality, gravitation, and electromagnetic theory, primarily by exploring electromechanical analogies alongside Continuous Symmetries. The main objective is to revisit the concepts of particle-wave duality and gravitation through a unified system of electromechanical analogies, as introduced in Chapter 1. These analogies (T.1.3 – T.1.8) are suggested as foundational steps towards developing a Unified Physics or Unified Field Theory.

The book introduces a consistent, intuitive, and analogy-based framework for creating Universal and Unified Physics, with the following key contributions:

1. Mathematical Analogies: *These are derived from similarities in relevant mathematical formulas and differential equations across various physical phenomena, underscoring the interconnectedness of seemingly distinct phenomena.*

2. Geometric and Topological Analogies: *These arise from the shared geometry or topology of equivalent mechanical and electromagnetic oscillatory and resonant circuits.*

3. Energy-Momentum Analogies: *Established by comparing energy-momentum equations and conservation laws across different physical contexts.*

4. Historical Context: *Recognizing that even Maxwell's Electromagnetic theory was initially and analogically grounded in Fluid Dynamics, as discussed in Chapter 3.*

5. Wave Phenomena: *The book posits that all waves, oscillations, and resonances in physics are manifestations of matter waves, best modeled using the Complex Analytic Signal framework, independent of probabilistic concepts (Chapter 4.0).*

6. Electromagnetic Universe: *It critiques the contemporary focus on a mechanical, electromagnetically neutral perspective, arguing that a comprehensive understanding of the universe must integrate the electromagnetic dimension, as mechanical, electromagnetic, and wave-particle duality aspects are inseparable.*

7. Natural Mathematics: *The book advocates for grounding physics in fundamental, experimentally verifiable mathematics, avoiding reliance on artificial and Game-theory constructs (Chapter 10).*

This approach leads to universally applicable structures in physics, ensuring clarity and preventing paradoxes often associated with artificially constructed theories, such as those in Game Theory.

The book proposes that the traditional concept of "Particle-Wave Duality" in physics should evolve into "Particle-Wave Unity" (here summarized under PWDC = Particle Wave Duality Code). It demonstrates that key wave equations and uncertainty relations in orthodox quantum theory can be logically derived without relying on probabilistic and statistical foundations, simply by adopting different mathematical and conceptual premises (Chapters 4.0, 4.1, 4.2, 4.3, 8, 9, and 10).

The concept of Wave-Particle Duality is applicable to both micro and macro phenomena, particularly in systems with intrinsic periodicities that can host stable, standing-wave structures. It serves as a bridge between linear motions with translational symmetry and rotational motions with rotational symmetry, united by universally valid conservation laws. The book also draws analogies between the linear and rotational motions of masses and electrically charged particles, emphasizing the interconnectedness with wave-particle duality and spatial and temporal dimensions.

In contemporary physics, theories describing gravitational and cosmological phenomena often overlook the electromagnetic aspect of the universe. This book argues that the electromagnetic universe is the dominant reality, with the mechanical and geometrical universe merely projections of this electromagnetic reality.

Key contributions and innovative ideas presented in the book include:

*1. **Gravitation and Matter-Wave States**: A new perspective on gravitation is presented, viewing it as related to resonant, standing matter-wave states of vibrating masses with coupled linear and angular moments, challenging the notion that static masses are the sole source of gravitation (Chapters 2, 8, and 10). Objects in states of stable, uniform linear and/or angular inertial motions effectively keep both inertial states as synchronously and coincidently, mutually coupled, thanks to intrinsically associated matter-waves phenomenology. Both particle and wave characteristics coexist simultaneously, being in the same time coupled with familiar electromagnetic moments. Gravitation could be a consequence emanating from phenomenology and laws of Electromagnetism and Mechanics, and not a self-standing, Fundamental Natural force.*

*2. **Nature of Forces**: It suggests that all fundamental forces, including gravitational and nuclear forces, are related to spatial and temporal energy and moments gradients. Also, resonant effects creating standing matter-waves (including electromagnetic vibrations), inside and between atoms and other masses manifest attractive and repulsive forces in nodal and antinodal zones, what could serve as an explanation for vibration-related nuclear forces (Chapters 5 and 10). Practically, the effects of forces and currents manifest when inertial states of matter in motion are being changed (during acceleration, or in oscillatory motions).*

***3. Nature of Matter:** Matter is described as self-organizing, standing matter-wave formations with electromagnetic and mechanical complexity, complementing and updating the de Broglie concept of matter waves. The book aligns these ideas with String Theory, suggesting that self-closed and stabilized standing waves are the building blocks or skeleton of matter (Chapter 10).*

***4. Uncertainty and Certainty Relations: The Uncertainty Principle is extended to apply to both micro and macro** worlds, including **Certainty Relations** valid for stable matter formations, and offering a new understanding of elementary particles and multidimensional structures of matter (Chapters 5 and 10).*

The book concludes that the universe is predominantly composed of electromagnetic fields, waves, and electromagnetically charged states, with other forms of matter also being condensed electromagnetic energy states. The ideas presented aim to unify our understanding of natural forces, black holes, dark matter, and other cosmic phenomena, suggesting that contemporary explanations are incomplete and in need of fundamental revision (Chapter 10).

Overall, this book challenges modern quantum and relativistic theories with a more deterministic and intuitive framework. It advocates adopting more natural mathematical models to better describe the motions, waves, and oscillations in our universe, moving closer to a Unified Field Theory.

The book extends the concept of Wave-Particle Duality, applicable to both the micro and macro realms, and explores a multi-level system of physics and mathematics-related analogies. These analogies provide a foundation for new hypothetical predictions regarding de Broglie matter-waves phenomenology (or particle-wave duality), offering insights into gravitation, quantum theory, and potential upgrades to Maxwell-Faraday Electromagnetic Theory.

The book explores the hypothetical phenomenology of the unity of masses and fields, examining the coupling between electromagnetic and mechanical motions, both linear and rotational. It also suggests that the field of gravitation could have a complementary field, much like the mutually coupled electric and magnetic fields are. Despite being composed of electromagnetic entities like atoms, electrons, protons, and neutrons (all of which have spinning, orbital, and/or electromagnetic moments or dipoles), the hypothetical complementary field of gravitation has not yet been fully understood or modeled.

The book criticizes the common misconception that electric charges are static parameters, arguing that they naturally have dynamic nature, like mechanical moments. This implies a continuous radiant electromagnetic energy flow between electric charges, as speculated by Nikola Tesla. It suggests a specific coupling and unity between linear and torsional aspects of any mechanical motion, akin to the intrinsic coupling between electric and magnetic fields, where the foundation of wave-particle duality lies.

In essence, the field of gravitation, together with its hypothetical torsional or spinning field complements, could be a manifestation of the electromagnetic field. It is also suggested that gravitational force could be closely related to the formation of standing matter waves in a spatially and temporally resonating universe, acting towards nodal or mass-agglomerating zones of such standing waves, like acoustic or ultrasonic levitation in fluids.

This book offers a novel perspective on the probabilistic and ontological foundations of contemporary Orthodox Quantum Theory (QT), arguing that QT can be more realistic and deterministic with more appropriate mathematical modeling (based on Complex Analytic Signal).

The text explains why Orthodox QT works well mathematically but only within its self-defined content, boundaries, and an artificially constructed mathematical environment. Orthodox QT lacks a clear conceptual structure and sufficient causal grounding. Through innovative mathematical modeling and upgraded wave-particle duality concepts, this book formulates more general forms of Schrödinger and Dirac-like matter-wave equations without relying on the artificial assumptions of modern probabilistic QT, using only elementary mathematics.

The generalization of the Schrödinger equation is achieved by employing powerful mathematical modeling of physics-related wavefunctions in the form of Complex Analytic Signal functions or Phasors, created based on the Hilbert transformation. This approach respects specific and universally applicable relations between wave-packet, group, and phase velocity. Additionally, the book argues that rest energy and rest mass should not be considered part of the wave-packet model, implying that de Broglie's particle-wave phenomenology should only relate to states of motional energy.

The book also provides a clear explanation of the stochastic and probabilistic quantum mechanical wavefunction, stripping away much of its probabilistic or statistical decor. It reveals that the probabilistic behaviors in modern QT arise from "in-average" mathematical modeling of particle-wave duality using Probability, Statistics, and Signal Analysis, effectively satisfying basic conservation laws of Physics <u>on average</u>.

Orthodox QT respects energy and momentum conservation laws in an isomorphic manner, but it does not account for instantaneous, real-time-space dependent signal phase due to the nature and limitations of its mathematical modeling and the presumed stochastic ontological nature of the phenomena. This oversight produces "magic effects" in diffraction experiments with isolated particles, yielding results akin to those with waves in fluids. Briefly summarizing, this book suggests significant upgrade of mathematical and conceptual foundations of QT, reducing the need for exclusive and dominant ontological and probabilistic assumptions.

The proposed new natural fields unification platform is initially based on exploiting unified multi-level analogies and basic continuous symmetries between different natural couples of mutually original and spectral, or conjugate domains, such as time-frequency, momentum-position, electric-magnetic charges, and angle-orbital momentum. This is followed by appropriate conceptual and mathematical modeling, upgrading, and integration into Physics.

In this book, uncertainty relations are generalized and treated as the matching (or mismatching) absolute interval relations between original and spectral signal durations, as well as between their elementary parts, showing how signals are being synthesized. This concept is extended far beyond its current application in Orthodox Quantum Theory (QT), as it is applied to both the micro and macro universe.

Consequently, the particle-wave duality concept, presented here more as particle-wave unity, is extended to any situation involving motional or time-dependent energy flow, regardless of its origin. In this framework, any change in motional (or driving) energy is inherently coupled with associated de Broglie matter waves, creating inertia-like waving-effects. These inertial effects could manifest gravitational, mechanical, electromagnetic, or another nature, depending on the interaction-participants. This concept is also valid for variations in linear, spinning, or orbital moments. In fact, all forms of ordinary, mechanical, and electromagnetic oscillations and waves known in physics are considered de Broglie matter-waves.

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Le Locle, Switzerland, 2025.

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- Wireless Giant of the Pacific - 381 pages

  This is a monumental book authored and compiled by Eric Dollard. It discusses his understanding of the Alexanderson antenna system, which operated with longitudinal electrostatic waves instead of transverse electromagnetic waves. That means it is a communication system that has instantaneous propagation with no time delay, thereby defeating Einsteinian physics.

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APPENDIX:

8. BOHR's ATOM MODEL, GRAVITATION AND PARTICLE-WAVE DUALISM

(Draft. Still in preparation)

Here is a short resume about what we know and practice regarding N. Bohr's atom model.

Citation from: https://en.wikipedia.org/wiki/Atom,
"In 1913 the physicist Niels Bohr proposed a model in which the electrons of an atom were assumed to orbit the nucleus but could only do so in a finite set of orbits, and could jump between these orbits only in discrete changes of energy corresponding to absorption or radiation of a photon.[15] This quantization was used to explain why the electrons' orbits are stable (given that normally, charges in acceleration, including circular motion, lose kinetic energy which is emitted as electromagnetic radiation, see *synchrotron radiation*) and why elements absorb and emit electromagnetic radiation in discrete spectra.[16]

Later in the same year, Henry Moseley provided additional experimental evidence in favor of Niels Bohr's theory. These results refined Ernest Rutherford's and Antonius Van den Broek's model, which proposed that the atom contains in its nucleus a number of positive nuclear charges that is equal to its (atomic) number in the periodic table. Until these experiments, the atomic number was not known to be a physical and experimental quantity. That it is equal to the atomic nuclear charge remains the accepted atomic model today.[17]

Chemical bonds between atoms were now explained, by Gilbert Newton Lewis in 1916, as the interactions between their constituent electrons.[18] As the chemical properties of the elements were known to largely repeat themselves according to the periodic law,[19] in 1919 the American chemist Irving Langmuir suggested that this could be explained if the electrons in an atom were connected or clustered in some manner. Groups of electrons were thought to occupy a set of electron shells about the nucleus.[20]..."

The foundation of this chapter was established by the author in 1975 and has since been enriched with numerous comments and additions. While a comprehensive synthesis and internal harmonization of this chapter have yet to be completed, the core message remains timely and original. **This chapter serves as an extension or addition to Chapter 2 on Gravitation, given the analogy between Bohr's atom model and planetary or solar systems, showing that the internal electromagnetic field of atom structures with associated and coupled electromagnetic and mechanical moments are the real source of gravitation.**

Bohr's atom model also offers valuable analogical insights into Gravitation, providing an intuitive and complementary perspective to the theories of I. Newton and A. Einstein. Additionally, Nikola Tesla's Dynamic Gravity Theory aligns closely with Rudjer Boskovic's Natural Force concepts and relates conceptually to the extended N. Bohr hydrogen atom model discussed here (see [97] and [117]). In summary, this book advocates the concept that all atoms, masses, and other stable "mass-energy-moment" states in our Universe communicate synchronously and bi-directionally through the exchange of radiative energy in the form of matter-waves, charged particles, and photons. This energy exchange, primarily electromagnetic, is hypothesized to produce gravitational effects, as Nikola Tesla's Dynamic Gravity theory suggests—though his theory was never fully published, fragments are available online (e.g., http://teslaresearch.jimdo.com/dynamic-theory-of-gravity/).

N. Tesla's and R. Boskovic's ideas on gravitation, Universal Natural Force, and Radiant energy imply that all atoms in the Universe communicate electromagnetically, both externally and internally, creating closed-circuit flows of energy, mass, momentum, currents, and voltages. These ideas resonate with Newton's laws of action-reaction, electromagnetic induction, quantum entanglement, and the "in-pairs" solutions of classical wave equations, where oppositely propagating waves are created synchronously. More on these concepts and analogies can be found in Chapter 1, Chapter 4.1 (e.g., Fig. 4.1.6, An Illustration of the Closed-Circuit Energy Flow), and Chapter 4.2 (under (4.8-1) - (4.8-4)).

Bohr's original atom model describes energy exchanges between electrons' stationary states through the emission and absorption of photons. Here extended Bohr atom model speculates on similar energy exchanges between electron states and the atomic nucleus, within the nucleus itself, and between atoms and external space. Since electrons and protons attract each other electromagnetically, changes in one group immediately affect the other, embodying the universal principle of "action equals reaction." In this view, atoms function as communication hubs, connected and active both internally and externally, forming dynamically stabilized sets of resonators with standing-wave force-field structures on both micro and macro scales. This complex electromagnetic force field creates stationary, spatially closed resonant states with self-contained standing-matter-wave structures, such as electron orbits within atoms. This field structure extends infinitely inwards and outwards, analogous to the resonant and quantized states of electrons (implicating that our macro universe is dominantly electric or electromagnetic fields and currents structured system, where effects of gravitation are only consequences or secondary manifestations of such electric universe).

Furthermore, these bidirectional standing matter-wave communications among atoms and masses echo N. Tesla's ideas on Radiant energy and the forces generating gravitation. This phenomenon is also related to M. Planck's and A. Einstein's Black Body Radiation (see [100], M. Sonnleitner, M. Ritsch-Marte, and H. Ritsch, "Attractive Optical Forces from Blackbody Radiation," and references [114], [115], [116]). Here, it is speculated that cosmic microwave background radiation, traditionally considered a remnant of the Big Bang, could instead be ordinary electromagnetic radiation resulting from energy exchanges within and between atomic nuclei, electron clouds, and surrounding atoms. This radiation might be a "*frequency-shifted mirror image*" of M. Planck's Blackbody radiation from electron states, as explored in Chapter 9 on Blackbody Radiation.

The hydrogen atom, as the simplest atom, serves as an ideal mini-laboratory for examining the fundamentals of particle-wave dualism and gravitation. Despite its obsolescence and limitations, Bohr's hydrogen atom model provides accurate predictions related to light spectra, offering insights into particle-wave duality in an elementary and intuitively understandable way. This analysis aims to reassess the validity of particle-wave dualism at its simplest level, using Bohr's hydrogen atom model to identify and validate the Particle Wave Duality Code (PWDC), as elaborated in Chapters 4.1 and 10. The following discussion will explore the roots of modern particle-wave duality concepts found in Bohr's model, a connection that has been overlooked both during its inception and in contemporary times. By uncovering the true sources of particle-wave duality, we can support the further development of these ideas. This analysis will address essential questions regarding the understanding of particle-wave dualism, as derived from Bohr's model, which remains sufficiently relevant despite being overshadowed by modern Quantum Physics and alternative atom models such as those by M. Kanarev, C. Lucas, and David L. Bergman (see [44], [16] - [22]).

While the limitations of Bohr's atom model have been well-documented in comparison with Quantum Mechanics modeling, where problems are addressed using probabilistic wave functions and Schrödinger's equation, this discussion does not aim to challenge that established understanding. Instead, the goal is to demonstrate, through an analysis (based on analogies) of Bohr's hydrogen atom model , that the fundamentals of Particle Wave Duality can also be encapsulated in the principles classified as PWDC in this book (see about PWDC in Chapters 4.1 and 10).

1. Matter waves, or de Broglie waves, are manifestations of all states involving "motional mass-energy-moments." It's important to note that rest-energy and rest particle mass do not contribute to the de Broglie matter-wave or the corresponding wave group (associated with a moving particle).

   Inertial motions, such as orbital motions within atoms and planetary systems, include stationary, stable, periodic, non-forced, natural, orbital, and rotating (continuous and smooth) inertial motions. Oscillatory motions also fall under this category since rotating and orbiting motions are often sources of oscillations or can be mutually transformed. These orbital motions generate stationary, standing, stable, self-contained matter-waves.

   Conversely, only non-inertial changes involving sudden acceleration—of motional energy within a "mass-energy-moments" entity directly produce unbounded progressive de Broglie or matter waves and associated forces. All waves and oscillations known in physics naturally belong to the same family of matter-waves and should be modelled using a consistent mathematical framework.

2. De Broglie wavelength $\lambda = \frac{h}{p}$ and Planck-Einstein energy of the narrow-band wave packet $\tilde{E} = hf \, (= \tilde{m}c^2)$ are intrinsic and mutually compatible mathematical elements or relations between the group "$\mathbf{v}$" and phase velocity "$\mathbf{u}$" of de Broglie matter-waves, $v = u - \lambda \frac{du}{d\lambda} = \frac{d\tilde{E}}{dp} = \frac{\tilde{E}}{p} - \frac{h}{p} d(\frac{\tilde{E}}{p})/d(\frac{h}{p})$ (also being mathematically presentable as wave groups, or wave packets). The group velocity is at the same time equal to the velocity of the particle that is represented by the same wave group. The phase velocity (of the wave group) is never higher than its group velocity and assumed to be never higher than the speed of light. See (4.0.73) - (4.0.76) from chapter 4.0.

3. The **P**article **W**ave **D**uality of matter (**PWD**) is the consequence of dynamic and intrinsic coupling between linear and angular motions and their surrounding fields'

manifestations in any of two or multi-body systems. **PWD** is also in a close relation to all mutually coupled “action-reaction” and inertial forces between interacting and moving, or oscillating objects (regardless of the involved forces and fields nature present in two-body or multiple-body systems), while respecting all conservation laws of Physics (especially energy and moments conservation laws). Here, we can again summarize the most relevant relations applicable to any energy-finite and narrow-band wave group or to its dualistic motional-particle equivalent, as already classified under **PWDC** (see more about **PWDC** in Chapters 4.1 and 10.):

$$\lambda = \frac{h}{\tilde{p}} = \frac{h}{p}, \tilde{E} = E_k = hf = \hbar\omega = \tilde{m}vu = pu = h\frac{\omega}{2\pi} = \frac{h}{\tau} = (\gamma - 1)mc^2, k = \frac{2\pi}{\lambda} = \frac{2\pi}{h}p,$$
$$\omega = \frac{2\pi}{T} = \frac{2\pi}{h}\tilde{E}, \quad u = \lambda f = \frac{\tilde{E}}{p} = \frac{\omega}{k}, v = v_g = u - \lambda\frac{du}{d\lambda} = \frac{d\tilde{E}}{dp} = \frac{d\omega}{dk}, \tilde{m} = \gamma m, p = \gamma mv = \tilde{m}v = \tilde{p}.$$

Now, we will try to extract, identify, and prove the above-formulated elements of the **PWDC**, in relation to N. Bohr atom model.

A short summary of Bohr’s atom model (for assumed non-relativistic velocities of electrons, $v << c$) is included in two of N. Bohr’s postulates and in one hypothesis about the electrons motion in an atom-field.

Let it be (for the Bohr’s planetary atom model):
m – mass of an electron, **e** – charge of an electron, **n** – main quantum number, **h** – Planck’s constant, **v** - orbital velocity of an electron, $\omega_m = 2\pi f_m$ - angular revolving frequency related to an electron rotation around its nucleus, **r** – radius of the electron orbit, $p = mv$ – linear, orbital momentum, **Z** – number of protons in an atom nucleus, ε – energy level (or wave energy) of the stationary electron orbit, $\mathbf{f_{12}}$ – frequency of the emitted or absorbed photon, and $\mathbf{f_s}$ – eigenvalue frequency of the stationary electron wave (in the orbit denoted with principal quantum number **n = 1, 2, 3,…**).

Bohr initially conceptualized or assumed that (in a hydrogen atom) an electron (as a particle) is quite mechanically rotating around its nucleus (a proton), meaning being in a natural, not-forced inertial and orbital motion. To impose certain stability to such planetary atom model, Bohr introduced the postulate of allowed, stationary and stable electron atom orbits (by direct analogy with gravitation-related planetary or solar systems). To be stable, stationary electron orbits must satisfy the condition of standing waves, or quantization of the involved orbital momentum (as planets rotating around a sun do), meaning that such mechanical rotation of an electron is respecting:

$$pr = mvr = n\frac{h}{2\pi}, \quad n = 1, 2, 3, ... \tag{8.1}$$

as well as respecting all other conservation laws of physics.

The second Bohr postulate is related to the conditions when an electron may emit (or absorb) the quantum of electromagnetic emission (a photon), during the change of the energy level of the electron between its two stationary orbits:

$$hf_{12} = \varepsilon_2 - \varepsilon_1 = \Delta\varepsilon = h\Delta f, \quad \varepsilon_2 > \varepsilon_1 \quad . \tag{8.2}$$

Of course, something similar (or equivalent) is valid when a photon is either leaving or entering the atom space captured by electron orbits (meaning that certain electrons will reduce or increase the energy level of its stationary orbit). Such energy transitions and exchanges can be considered (or better to say approximated) as discrete, sudden, and fast changes.

Finally, Bohr assumes that electron orbits around the nucleus of the (hydrogen) atom are circular, and he sets the condition of the dynamic balance of the attractive and repulsive electrostatic forces between the electron (that revolves) and the static atom nucleus (very much mathematically analog to gravitational planetary systems situation):

$$\frac{1}{4\pi\varepsilon_0}\frac{e(eZ)}{r^2} = \frac{mv^2}{r} . \tag{8.3}$$

From the previous postulates and conditions (8.1), (8.2), and (8.3), one may get the elements that characterize the circular motion of the electrons around the atom nucleus, such as:

-radius of an electron orbit,

$$r = r_n = \frac{n^2 h^2 \varepsilon_0}{\pi m e^2 Z} = \frac{v_n}{\omega_m}, \left(\frac{1}{n^2} = \frac{h^2 \varepsilon_0}{\pi m e^2 Z} \cdot \frac{1}{r_n}\right), n = 1,2,3,... \tag{8.4}$$

-orbital velocity of the electron,

$$v = v_n = \omega_m r_n = 2\pi f_m r_n = \frac{nh}{2\pi m r} = \frac{Ze^2}{2nh\varepsilon_0}, n = 1,2,3,..., \tag{8.5}$$

-the kinetic energy of the electron,

$$E_k = \frac{1}{2}mv^2 = \frac{Ze^2}{8\pi\varepsilon_0 r_n}, n = 1,2,3,..., \tag{8.6}$$

-and the potential (electrostatic) energy of the electron,

$$U = \int_{\infty}^{r} F dr = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r} = -2E_k = -mv^2 \quad ,(F_\infty = 0) . \tag{8.7}$$

The total orbital energy of the electron (apart from its rest energy) in the interatomic field of a hydrogen atom is equal to the sum of its kinetic and potential energy:

$$\varepsilon_B = E_k + U = -E_k = -\frac{Ze^2}{8\pi\varepsilon_0 r} = -\frac{mZ^2e^4}{8n^2h^2{\varepsilon_0}^2} = -\frac{1}{2}mv^2, \ n = 1,2,3,... \ . \tag{8.8}$$

Applying the second Bohr's postulate (8.2) onto the (8.8), one may find the energy of the electromagnetic emission of the atom, at the transition of its electron between two (internal) stationary energy levels $\varepsilon_2 > \varepsilon_1, n_2 > n_1$,

$$hf_{12} = \varepsilon_2 - \varepsilon_1 = \Delta\varepsilon_B = \frac{mZ^2e^4}{8h^2\varepsilon_0^2}(\frac{1}{n_1^2} - \frac{1}{n_2^2}) = \frac{Ze^2}{8\pi\varepsilon_0}(\frac{1}{r_{n1}} - \frac{1}{r_{n2}}) = \Delta E_k \,. \quad (8.9)$$

The indexing with **"B"** in the previous (and following) expressions is introduced to emphatically indicate that certain values directly and exclusively emanate from the originally established Bohr's atom model (because later, there will be other indexed symbols), due to the later comparison of Bohr's results with other available results.

An electron and an atom nucleus also revolve (as a two-body system) about their common center of mass. Therefore, the mass of the nucleus **M** should enter the equations of assumed orbital, electron motions. Sommerfeld has shown that equation (8.9) still stands if **m** is replaced by the so-called reduced mass μ (contributing with fine correction of (8.9); -see [9] and [92]),

$$\left\{\begin{array}{l} m \rightarrow m_r = \mu = \frac{mM}{m+M} \cong m \;,\; \frac{M}{m} = 1836.13 \;, \\ (m \;=\; m_e = \text{electron mass},\; M \;=\; m_p = \text{nucleus mass}) \end{array}\right\} \Rightarrow$$
$$\Rightarrow hf_{12} = \varepsilon_2 - \varepsilon_1 = \Delta\varepsilon_B = \frac{\mu Z^2e^4}{8h^2\varepsilon_0^2}(\frac{1}{n_1^2} - \frac{1}{n_2^2}) \,. \quad (8.9\text{-}1)$$

Since the nucleus mass (or proton) is much greater than the electron mass, if we analyze the same system (nucleus-electron) in the Laboratory and in the center-of-mass system, **we can easily realize that the electron's stationary (or orbital) wave carries practically only the motional energy of the revolving electron**. Consequently, we can easily and with high certainty find that this stationary wave energy (in the case of non-relativistic electron velocities: see (8.1) - (8.9-1)) will be expressed as,

$$\varepsilon = \varepsilon_s = hf_s = \frac{1}{2}\mu v^2 = \frac{1}{2}\mathbf{J}_e\omega_m^2 = \frac{1}{2}\mathbf{L}_e\omega_m = \frac{\mu Z^2e^4}{8h^2\varepsilon_0^2}\cdot\frac{1}{n^2} \cong \frac{1}{2}mv^2 \,,\; \mathbf{J}_e = \mu r^2 = \mathbf{J}_m \,,$$
$$\Leftrightarrow v = \frac{Ze^2}{2h\varepsilon_0}\cdot\frac{1}{n} \,,\; \mathbf{L}_e = \mathbf{J}_e\omega_m = \mathbf{L}_m = \mu r^2\omega_m = \frac{1}{\omega_m}\cdot\frac{\mu Z^2e^4}{4h^2\varepsilon_0^2}\cdot\frac{1}{n^2} = \frac{r}{v}\cdot\frac{\mu Z^2e^4}{4h^2\varepsilon_0^2}\cdot\frac{1}{n^2} = n\frac{h}{2\pi}. \quad (8.9\text{-}2)$$

In (8.9-2) we are applying indexing " m " to underline that here we are talking about almost an ordinary ***mechanical*** rotation (in the frames of the initial Bohr's atom model). Since an electron is assumed rotating, there is another index " e " addressing electron (because later we will upgrade such atom model and introduce different indexing for another level of spinning, or rotation which will be differently associated to the same electron). Then, it will be shown and proven that (8.9-1) and (8.9-2) are fully correct (presenting a small and fine, but conceptually important correction of the original Bohr spectral formula (8.9)), and that it contains very essential information regarding the **PWDC** (see also (8.25) - (8.30)).

Originally, Bohr introduced the concept of an electron's particle-like inertial revolution, both in mass and electric charge, around the atomic nucleus. This model, to a certain extent, began to yield verifiable and especially useful results, particularly in the measurement of light spectra, which are often observed in gaseous states. The spectral lines experimentally found in the emission and absorption spectra of hydrogen atoms, as well as those of many heavier and more complex atoms, are in perfect agreement with theoretical predictions described by equations (8.9) and (8.9-1). This confirms the applicability of Bohr's atomic model, within its assumed limitations.

However, it is important to note that Bohr's model provides limited insight into the structure and nature of the atom, as well as the underlying phenomenology associated with particle-wave dualism (PWD). This is because the model is based on two postulates and one hypothesis that is not entirely realistic or self-standing. Despite this, equation (8.9) has enabled excellent theoretical predictions of the experimentally obtained spectral distributions of light radiation and absorption.

One may therefore conclude that a more careful and comprehensive analysis of Bohr's model could potentially reveal a more fundamental, physics-related essence that has not yet been fully exposed. Since the model produces very accurate spectral measurement results, this deeper understanding could further explain Bohr's postulates and shed new light on the nature of matter and the essence of particle-wave dualism. In this paper, we refer to this still-unexposed physical essence as the Particle Wave Duality Concept (PWDC). The more fundamental and analogically extended picture suggested here is that all natural, unforced orbital, periodic, and inertial motions, including planetary motions, may potentially host standing matter-wave formations.

Retrospectively, Bohr's atomic model does not explicitly account for the wave-like nature of electrons. Nevertheless, the postulate (8.2) implies the emission or absorption of electromagnetic wave quanta during transitions of electrons between stationary orbits, which implicitly suggests that standing-wave properties are related to the orbital motion of electrons even before energy changes occur (8.9). However, we currently lack strong arguments to claim that every photon is a specific quantum of its electromagnetic energy, $\tilde{E} = h \cdot f$ as this is only applicable in cases of finite-energy and narrow frequency band (Gaussian) electromagnetic waves (photons or wave groups).

In the framework of Bohr's atomic model, the moment of inertia *of* a rotating electron—initially considered as a particle is $J = mr^2$. This moment of inertia remains the same if we consider the electron mass as rotating while being distributed around a thin-walled equivalent hollow torus that covers the same space as the electron's orbit or path and has the same total mass m. Within this context, we have the freedom to conceptualize the electron as a helical matter-wave on a toroidal coil structure. This idea is further explored in the second chapter of this book, specifically in sections 2.3.2 ("Rotation and Stable Rest-Mass Creation"), T.2.4, T.2.5, and T.2.6, as well as in equations (2.11.2) and (2.11.3). Additional relevant details can be found in chapter 4.1, particularly in Figures 4.1.2 to 4.1.4 and section T.4.3.

This approach aligns with similar, significant, and original ideas introduced by M. Kanarev, as well as by C. Lucas and David L. Bergman. Notably, the electron (and atom) models proposed by Lucas, Bergman, and Kanarev appear more realistic and advanced compared to Bohr's original atom model and other contemporary Quantum Theory concepts. Despite the differences, these models yield largely identical or similar results that are experimentally verifiable through spectroscopic measurements, assumed electron energy levels, relevant frequencies, and orbital and magnetic moments. For a comparable concept of the electron's magnetic field, one can refer to the Henry Augustus Rowland effect around a rotating conductor, as presented by Jean de Climont. An excellent model of the helical electron, compatible with the helical matter waves conceptualized in this book, is detailed in Oliver Consa's work titled "g-factor and Helical Solenoidal Electron Model" [108].

Historically, since N. Bohr's model was the first sufficiently successful atom model, we will attempt to draw as many relevant conclusions as possible from it in the context of PWDC. Subsequently, we will explore opportunities to unify, update, and integrate the most successful aspects of the models proposed by Bohr, Kanarev, Bergman, and Lucas [120].

To provide an intellectual and conceptual analogy, consider the Ptolemaic geocentric planetary system. Despite being fundamentally erroneous, this model produced surprisingly accurate mathematical predictions of planetary motions, verifiable by astronomical observations. The success of this model was due to the introduction of additional assumptions, such as the gearbox-like rotations and synchronized natural periodic motions of planets and moons. These assumptions allowed the model to compensate for its initial conceptual errors by introducing new, self-correcting orbiting elements based on the intrinsic, natural periodicities of planetary motions within solar systems.

A similar analogy might be applicable when comparing Bohr's model with those of Bergman, Lucas, Kanarev, Climont, and Consa. All these models produce a significant portion of identical and correct results related to electromagnetic spectral measurements, thanks to the mutually coupled intrinsic and structural periodicities and synchronized motions of models participants. Thus, the atom's structure can be formally explained in conceptually different ways, such as by relating dynamically stable mechanical systems to presentations in both laboratory and center-of-mass reference frames.

Most of the atom models already mentioned produce mathematically similar and experimentally verifiable electromagnetic emissions and absorptions, spectral results. This consistency suggests that differences between the energy levels of electrons and photon transitions are essentially the same. This concept is explored further in Chapter 10.

A similar principle applies to the Quantum Theory model of the hydrogen atom, which is based on solving the Schrödinger equation. Due to intrinsic, mutually related and synchronized temporal and spatial periodicities, and the presence of structural standing electron waves within real atoms, the Quantum Theory model operates successfully, although it could be considered somewhat analogous to erroneous Ptolemaic construction.

Ironically, while N. Bohr’s original atom model has limited relevance when explaining real atomic structure, it is more applicable to the innovative modeling of solar or planetary systems, as discussed in Chapter 2 (“2.3.3. Macro-Cosmological Matter-Waves and Gravitation").

In general, systems with inherent structural periodicities, such as planets, atoms, or particles, exhibiting circular, spinning, and synchronized orbital and periodic motions can often be approximated by simplified mathematical models, even if these models are somewhat artificial, like Ptolemaic model of our planetary system. These models can be transformed into one another through deterministic mappings and spatial transformations, yet only one will be conceptually and physically correct. Nonetheless, mathematically, these models may be equivalent and predict measurable results with sufficient accuracy.

Given the stable, synchronized periodicity relationships in these systems, it is possible to map and transform between "Ptolemaic models" and the relevant energy-momentum states of organized matter-wave structures like atoms. This is like how Ptolemaic models falsely placed Earth at the center of the universe while still providing sufficiently accurate predictions.

While Bohr's planetary atom model may not be the best representation of an atom's real spatial structure, it remains useful because it is causally connected to other, more accurate atom models through strong periodicity relationships and deterministic functional mapping and transformations. Consequently, we can derive relevant conclusions within the framework of Bohr’s model that also apply to other atom models.

This reasoning extends to other competitive and more accurate atom models, such as those introduced by M. Kanarev, Charles Lucas, David Bergman, and Oliver Consa. However, Quantum Theory atom modelling remains outside the scope of the concepts used here.

In physics, quantization is related to discrete energy amounts exchanges between quantum or resonant systems, and mathematically related to numerical indexing, quantifications, integer counting, and the arithmetic relationships between energy-momentum states, stationary and standing resonant states of matter, and intrinsic periodicities in structured “energy-moments” systems. It is important to distinguish between temporal and spatial quantizing, as matter state periodicities can be temporal, spatial, and spectral.

Finally, we must recognize that humans can make significant conceptual errors. Sometimes, these errors are masked by self-correcting mathematical models and assumptions that function correctly due to embedded natural periodicities. Like the long-standing acceptance of the geocentric planetary system, we may still be holding onto outdated models or theories in modern physics without fully acknowledging their limitations.

Let us start with an intuitive, simplified geometric concept of the interpretation of **stationary** or stable and inertial-motion electron orbits, (which we directly connect with the assumed existence of stable orbital electron waves, limiting ourselves, for now, only on circular orbits and stable, inertial motions). We can say that the stationary electron orbit is the orbit to which the stationary (standing) electron wave belongs, such that the mean perimeter of that orbit is equal to the integer number of the wavelengths of the stationary electron wave, i.e.,

$$2\pi r = n\lambda_s ,\ n = 1,2,3,\ldots . \qquad (8.10)$$

Indexing with "$_s$" should remind us that we are talking about stationary electron orbits and stationary energy states.

Comparing the condition of "*stationarity*" (8.10) and Bohr's postulate (8.1), one may notice a direct and indisputable analogy from which it takes only one step to the posing of the de Broglie's wave hypothesis (which, surprisingly, was formulated almost 10 years later),

$$2\pi r = n\frac{h}{mv} = n\frac{h}{p} = n\lambda_s , \qquad (8.11)$$

i.e., the wavelength of an orbital electron wave equals:

$$\lambda_s = \frac{h}{p} . \qquad (8.12)$$

Conditions of such particle-wave duality and "*stationarity*", (8.10) – (8.12), are later generalized by Sommerfeld (1915, about 10 years before Luis de Broglie eventually realized that here we should have electron or matter waves), extending such standing-waves concept to any orbit-shape and any degree of freedom of its internal (spatial, linear and orbital) moments "p_k" and coordinates "q_k" -related energy states, as for instance,

$$\oint p_k dq_k = n_k h,\ n_k \in [1, 2, 3, ...] . \qquad (8.11\text{-}1)$$

All here-mentioned conceptual upgrading of N. Bohr atom model contributed (later) to smaller results improvements related to spectral formula (8.9-1), showing new evolutional directions for advanced atoms modeling, and additionally supporting here-underlined statements about **PWDC**. Of course, ideas about standing electron (or electromagnetic) waves, (8.10) – (8.12) and (8.11-1) are also conceptually supporting and explaining Bohr's postulates, establishing spatially and temporally synchronized and stabilized atom structure.

The quantization of electron momentum and the quantum nature of atomic structure are often misunderstood. The key to describing the stability of atomic structures lies in the spatial and structural formatting of resonant states involving standing, self-contained inertial motions of matter waves. In these structures, the presence of integer numbers of half-wavelengths is both natural and necessary, as it is the only possible way to structure and count these states. Bohr-Sommerfeld quantization is an alternative but equivalent method for describing these spatial standing-wave formations.

Since the introduction of N. Bohr's atom model, Quantum Theory has extensively explored various aspects of the quantum nature of matter and atoms. This exploration is a direct consequence of how standing waves are organized, with integer numbers indexing the energy and moments of wave formations (more on this in Chapter 10).

Real Quantization in physics is more closely related to the synthesis and decomposition of signals or waveforms, as described by the "Kotelnikov-Shannon-Nyquist-Whitaker" signal analysis and synthesis. It's important to note that quantization can be linked to both the temporal and spatial periodicities of matter states (see Chapter 10 for further discussion).

It seems clear that electron, on its stationary orbit inside an atom (whatever that means), is presentable as a kind of mixed or dual entity (having mutually equivalent and commuting properties of a particle and a wave, where waves are dominant, since inside atoms we cannot really imagine or defend electrons as particles). To literal recognition of such dual object, in this book, we will use the following wave expressions and symbols for the wavelength and wave energy of (de Broglie) matter waves: $\tilde{E} = hf$, $\lambda = \frac{h}{\tilde{p}}$, $(E_k \to \tilde{E},\ p \to \tilde{p})$. In order to support the use of wave symbols, $\tilde{E}$, $\tilde{p}$, we can remember that energy $E_f = \tilde{E}_f$, mass $m_f = \tilde{m}_f$, and wavelength λ, of an electromagnetic energy wave-packet or photon, can be given by analogical relations (see T.4.0. Photon – Particle Analogies from Chapter 4.1),

$$\tilde{E}_f = hf = (m_f - m_{f0})c^2 = m_f c^2 = \tilde{m}_f c^2 = E_{kf},\ \tilde{p}_f = \frac{hf}{c} = p_f,\ \ \tilde{m}_f = m_f = \frac{hf}{c^2},\ \ \lambda_f = \frac{h}{\tilde{p}_f},$$

moreover, all of them are effectively (and particle-wave analogically) proven valid and applicable in analyses of Photoelectric, Compton and other familiar impacting and scattering effects (without explicit usage of here proposed wave symbolic).

Results, (8.11) - (8.12), and (8.9-2) imply that Bohr's atom model and de Broglie's matter-wave hypothesis have become mutually complementary with mathematical elements that simultaneously support and prove the correctness of both concepts.

Until here, for the assumed electron wave we may say that we know only its wavelength. Of course, we know that the difference between the eigenenergies of two stationary electron waves, (8.9) very precisely predicts emissive or absorptive atom spectral lines, but we still do not know the absolute amounts of the energies of stationary electron waves ($\varepsilon = \varepsilon_s = hf_s$), which take a part in that difference. Yet, formally, we may write Bohr's postulate (8.2) in a form that takes into consideration (still unknown) orbital stationary frequencies of that wave, so it will be,

$$hf_{12} = h(f_{s2} - f_{s1}) = \varepsilon_2 - \varepsilon_1 = \Delta\varepsilon = \Delta\varepsilon_B\ . \qquad (8.13)$$

In addition, we will try to determine (in absolute values) all elements of the stationary electron wave, such as its phase and group velocity, frequency, wavelength, and energy. Of course, the term stationary electron wave is quite justified by the fact that atoms can absorb or emit an electromagnetic wave group (like a photon), i.e., it is likely that appropriate interferences and superposition of the two familiar wave phenomena will happen. Thereupon a possibility intuitively obtruded is that an electron in orbital revolving around the atom nucleus possesses, besides particle attributes, and some wave attributes (like a photon). Naturally, an electron is no longer a particle localized in some point, but some (helically guided) standing wave structure of distributed "electromagnetic mass" and distributed charge of an electron across the perimeter of the corresponding stationary orbit (which we will here call de Broglie's matter wave). *See on the Internet somewhat similar electron magnetic field concept regarding Henry Augustus Rowland effect around rotating conductor, presented by Jean de Climont, [117].*

Until here we did not care about relativistic mass-energy velocity dependencies since the first objective was to recognize and prove the most important relations belonging to the **P**article **W**ave **D**uality **C**ode (= **PWDC**), $\tilde{E} = hf$, $\lambda = \frac{h}{\tilde{p}}$. There are some other elements of **PWDC** that will be mentioned later (More about **PWDC** foundations can be found in Chapter 4.1 and Chapter 10.).

To identify the orbital eigenfrequency f_s of the electron wave (before any energy jumps, emissions, or photons absorptions), it is necessary to assume the nature of that wave, i.e., what energy content the electron stationary wave exists on.

For example, we may express the total energy of an electron in orbital motion as,

$$\varepsilon_t = mc^2, \quad (m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma m_0, \; m_0 = \text{const.}), \qquad (8.14)$$

while its total motion energy is,

$$\varepsilon_k = (m - m_0)c^2 = (\Delta m)c^2 = \frac{mv^2}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{pv}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{\mathbf{L}_m \omega_m}{1+\sqrt{1-\frac{v^2}{c^2}}}, \qquad (8.15)$$
$$p = mv = \gamma m_0 v, \mathbf{L}_m = \mathbf{J}_m \omega_m = mr^2 \omega_m = 2\pi m r^2 f_m = mvr, pv = \mathbf{L}_m \omega_m .$$

We may notice at once that energy ε_B from (8.8), which we consider as the motional energy of the electron in the electric field of the atom, must be included in some amount in (8.14) and/or (8.15). In addition, we will notice that we may now represent Bohr's postulate (8.2) by the difference of two energy levels (which are in direct and mutually translated dependence, i.e., which mutually differ in some constant) in several ways, using energy expressions (8.8), (8.13), (8.14), or (8.15):

$$hf_{12} = h(f_{s2} - f_{s1}) = \varepsilon_2 - \varepsilon_1 = \Delta\varepsilon = \Delta\varepsilon_B = \Delta\varepsilon_t = \Delta\varepsilon_k . \qquad (8.16)$$

In expressions (8.14)-(8.16) and later on, electron's non-relativistic kinetic or total energy E_k, E, is simply transformed into its relativistic motional or total energy $\varepsilon_k, \varepsilon$, implicitly indicating that relativistic motional and/or total energy could (or should) have a complex electromagnetic nature, containing mixed particle and wave manifestations of electron's energy.

If we are not sure which one of the previous energies ($\varepsilon, \varepsilon_B, \varepsilon_t, \varepsilon_k$) from (8.16) is in an absolute amount equal to the energy of the stationary electron wave $\varepsilon = \varepsilon_s = hf_s$, then we could not determine what the frequency of the stationary electron wave is. The thing we know initially (or may determine) is the set of all possible assumptions (where some of them figure only temporarily, fictionally, and dimensionally as frequencies) among which at least one will represent the required, realistic stationary, eigenfrequency f_s. Later, by elimination, we will find the real eigenfrequency (in its absolute amount) that satisfies the selection criterion (particularly by satisfying the criterion of group and phase velocity connection, and other elements of **PWDC**).

Now (in process of searching for real, orbital eigenfrequency f_s) we may quote the following "candidate" frequencies (connected to the energy states of electrons on stationary orbits):

- the frequency of the mechanical rotation of the electron as a particle, around the atom nucleus, which is found in (8.4), (8.5) and (8.15),

$$f_m = \frac{\omega_m}{2\pi} = f_B' = \frac{v}{2\pi r} = \frac{mZ^2e^4}{4n^3h^3\varepsilon_0^2} = \frac{2\varepsilon_B}{nh} = \frac{2}{n}f_B \quad , \tag{8.17}$$

- the frequency that is dimensionally found in (8.8),

$$f_B = f_B'' = \frac{\varepsilon_B}{h} = \frac{mZ^2e^4}{8n^2h^3\varepsilon_0^2} = \frac{n}{2}f_B' = \frac{n}{2}f_m , \tag{8.18}$$

- the frequency that is obtained based on the total energy of the electron (8.14),

$$f_t = \frac{\varepsilon_t}{h} = \frac{mc^2}{h} , \tag{8.19}$$

- and the frequency that is obtained from the motional energy (8.15),

$$f_k = \frac{\varepsilon_k}{h} = \frac{(\Delta m)c^2}{h} . \tag{8.20}$$

Considering all previous frequencies from (8.17) to (8.20), the second Bohr's postulate (8.16) formally becomes:

$$\begin{aligned} hf_{12} &= h(f_{s2} - f_{s1}) = \varepsilon_2 - \varepsilon_1 = \Delta\varepsilon = \Delta\varepsilon_B = \Delta\varepsilon_t = \Delta\varepsilon_k = \\ &= \frac{1}{2}h(n_2 f_{B2}' - n_1 f_{B1}') = h(f_{B2} - f_{B1}) = h\Delta f_B = \\ &= h(f_{t2} - f_{t1}) = h\Delta f_t = \\ &= h(f_{k2} - f_{k1}) = h\Delta f_k \ . \end{aligned} \tag{8.21}$$

As one may see, in (8.21) several different frequencies (that are translated, mutually and linearly) figure here, which leads to equality of their differences.

Now one may determine the phase velocity of a stationary electron wave, as a product of its corresponding wavelength, (8.12), and some of the frequencies from (8.17) to (8.20). Thus, it is possible to determine several phase velocities, from which only one (real and exact) will be required to show which electron wave (orbital) frequency (from (8.21)) is the most relevant:

$$u_m = \lambda_s f_m = \lambda_s f_B{}' = u_B{}' = \frac{Ze^2}{2n^2 h\varepsilon_0} \quad ,$$
$$u_B = \lambda_s f_B = \lambda_s f_B{}'' = u_B{}'' = \frac{n}{2} u_B{}' = \frac{n}{2} u_m = \frac{Ze^2}{4n^2 h\varepsilon_0} \quad , \tag{8.22}$$
$$u_t = \lambda_s f_t = \frac{c^2}{v}$$
$$u_k = \lambda_s f_k = c\sqrt{\frac{m - m_0}{m + m_0}} .$$

Phase velocities from (8.22) are mutually very different, but the physical essence of the electron orbital wave should have (or defend) only one of them. To establish the needed selection criterion, let us start with the fact that the orbital velocity of an electron (8.5) is equal to the group velocity of the stationary electron wave associated with it. Then, we will search (among possibilities from (8.22)) for the phase velocity of a stationary electron wave as the phase velocity that satisfies the general equation of the group and phase velocity relation of (any) wave group,

$$v = u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda} = \frac{d\tilde{E}}{dp} = \frac{\tilde{E}}{p} - \frac{h}{p} d(\frac{\tilde{E}}{p})/d(\frac{h}{p}) . \tag{8.23}$$

The equation (8.23) is obtained from definitional expressions for group and phase velocity of the wave group, which are generally valid for any form of harmonic wave motions (See more in (4.0.73) - (4.0.76) from Chapter 4.0),

$$v = \frac{d\omega}{dk} = \frac{d\tilde{E}}{d\tilde{p}} , \; u = \frac{\omega}{k} = \frac{\tilde{E}}{\tilde{p}} = \lambda f \; , \; \omega = 2\pi f \; , \; k = \frac{2\pi}{\omega} . \tag{8.24}$$

Therefore, it is obvious that equation (8.23) may become the key arbiter in the explanation of the energy essence of the electron wave (i.e., in finding its real phase velocity and orbital frequency).

In fact, the relations (8.23) and (8.24) also belong to **PWDC** (**P**article **W**ave **D**uality **C**ode), including $\tilde{E} = hf$, $\lambda = \frac{h}{\tilde{p}}$, too.

One may check and prove that equations and relations (8.23) - (8.24) describe a real stationary electron wave that has:

-phase velocity (see (8.22)),

$$u_s = u_k = \lambda_s f_k = \lambda_s f_s = c\sqrt{\frac{m-m_0}{m+m_0}} = c\sqrt{\frac{\gamma-1}{\gamma+1}} = \frac{v}{1+\sqrt{1-(\frac{v}{c})^2}},\ m=\gamma m_0, v = v_s,$$

-orbital eigenfrequency,

$$f_s = f_k = \frac{\varepsilon_k}{h} = \frac{(\Delta m)c^2}{h} = n\frac{f_m}{2}(1+\frac{u^2}{c^2}) = n\frac{mZ^2e^4}{8n^3h^3{\varepsilon_0}^2}(1+\frac{u^2}{c^2}),$$

-energy,

(8.25)

$$\varepsilon_s = hf_s = \varepsilon_k = (\Delta m)c^2 = muv\ ,$$

-and wavelength,

$$\lambda_s = \frac{h}{p} = \frac{h}{c\sqrt{m^2 - {m_0}^2}} = \frac{h}{m_0 c\sqrt{\gamma^2-1}}.$$

Due to obtaining greater evidence in checking previous solutions, considering the cases when the orbital velocity of an electron is relatively small, $v << c$, previous expressions from (8.25), after adequate approximations, are reduced to the results known from the original Bohr's atom model. In addition, one also obtains some new values (that Bohr's model does not generate) and which are typical for the wave concept of electron motion. Therefore, it is:

- the phase velocity of the electron wave,

$$u_s = u_k = \lambda_s f_k = \lambda_s f_s = \frac{\omega}{k} = c\sqrt{\frac{m-m_0}{m+m_0}} = \left.\frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}}\right|_{v<<c} \approx \tag{8.26}$$

$$\approx \frac{1}{2}v = \frac{1}{2}v_s = u_B = \frac{n}{2}u_m = \frac{Ze^2}{4nh\varepsilon_0},$$

-the group velocity of the electron wave (see (4.0.17), (4.0.26), and (4.0.28)),

$$v = \frac{d\omega}{dk} = \left.\frac{2u_s}{1+\frac{{u_s}^2}{c^2}} = v_s\right|_{v<<c} \approx 2u_s = \frac{Ze^2}{2nh\varepsilon_0} \tag{8.27}$$

-the frequency of the orbital electron wave (see (4.0.31) and (4.0.39)),

$$f_s = f_k = \frac{\varepsilon_k}{h} = \frac{(\Delta m)c^2}{h} = \left. n\frac{f_m}{2}(1+\frac{u^2}{c^2}) \right|_{(v,u)<<c} \approx$$

$$\approx \frac{mv^2}{2h} = f_B = f_B'' = \frac{\varepsilon_B}{h} = \frac{mZ^2e^4}{8n^2h^3\varepsilon_0^2} = \frac{n}{2}f_m \ , \tag{8.28}$$

-the wavelength of the orbital electron wave (see (4.7), (4.8), and (8.12)),

$$\lambda_s = \frac{h}{p} = \left. \frac{h}{c\sqrt{m^2 - m_0^2}} = \frac{2\pi r}{n} \right|_{v<<c} \approx \frac{2nh^2\varepsilon_0}{mZe^2} , \tag{8.29}$$

-and energy of the stationary electron wave (see T4.1 and (4.0.8)),

$$\varepsilon_s = hf_s = \varepsilon_k = (\Delta m)c^2 = muv = \left. \frac{mv^2}{1+\sqrt{1-\frac{v^2}{c^2}}} = nh\frac{f_m}{2}(1+\frac{u^2}{c^2}) \right|_{(u,v)<<c} \approx$$

$$\approx \frac{1}{2}mv^2 = \varepsilon_B = \frac{mZ^2e^4}{8n^2h^2\varepsilon_0^2} \ . \tag{8.30}$$

This analysis leads to the conclusion that the wave nature of matter, specifically in the case of a uniformly orbiting electron in natural, inertial motion, is determined solely by its motional energy. The electron's rest mass does not contribute to the accompanying wave energy. This conclusion contrasts with the prevailing interpretation in modern quantum theory, which considers both the total energy and the rest-mass of a particle as fundamental elements of its wave-energy equivalent.

Many misconceptions and errors surrounding the concept of particle-wave dualism, particularly in the atomic realm, arise from differences in the corresponding relative energy levels, which are also equal to the differences in their absolute energy levels. It is impossible to determine in advance which energy levels are absolute and which are relative since they are "mutually linearly translated or shifted" by a constant energy level.

As previously discussed, the relationship between group velocity and phase velocity (expressed in equations (8.23) and (8.24)) is crucial for identifying electron matter waves among various possibilities.

Furthermore, the results in equations (8.25) through (8.30) are in complete agreement with equation (8.9-2), mutually confirming, complementing, and supporting each other. Together, they fully describe the Particle-Wave Duality Code (PWDC). These same expressions also elucidate the essential relationship and intrinsic connection between mechanical rotation and the frequency of de Broglie matter waves, particularly when the electron mass is treated as the reduced mass, as shown in equation (8.9-1).

[♣ COMMENTS & FREE-THINKING CORNER: "Gravitostatic versus electrostatic analogy"

The results and conclusions discussed previously in sections (8.26) - (8.30) and (2.11.14) can be formulated more directly and efficiently by drawing an analogy between the solar system and an atomic structure. In this analogy, the Sun functions as the nucleus (like a proton), while the planets behave like electrons orbiting around it. This approach is valid when the solar system can be approximated as a "two-body problem," allowing us to exploit the mathematical similarities between the Coulomb force in the hydrogen atom and Newton's gravitational force. Using the following substitutions, we can systematically apply this analogy:

$$\left\{\frac{Ze^2}{4\pi\varepsilon_0 r^2} \Leftrightarrow \frac{GmM}{r^2},\ Ze \Leftrightarrow M,\ e \Leftrightarrow m,\ \frac{1}{4\pi\varepsilon_0} \Leftrightarrow G, h \Leftrightarrow H\right\} \Rightarrow \left\{\frac{Ze^2}{4\pi\varepsilon_0} \rightarrow GmM\right\}$$

Sun's mass (M) corresponds to the nucleus,
Planet's mass (m) corresponds to the electron,
Macrocosmic constant (H) analogically corresponds to Planck's constant (h), and
Gravitational constant (G) remains analogous to the electromagnetic context.

By using these analogical substitutions, we can reproduce all relevant results known from the analysis of the hydrogen atom and extend them to describe planetary motions within solar systems. For supporting arguments on the proportionality between mass and electric charge, refer to Chapter 2, sections (2.4-4.1) through (2.4-4.3).

A more detailed discussion on this analogy can be found in works by Arbab I. Arbab [63], Johan Hansson [67], and other related publications. The "gravito-static versus electrostatic analogy" should not be dismissed as merely a mathematical coincidence or an academic curiosity. It becomes particularly relevant when we consider the realistic possibility that the Sun and planets are electrically and magnetically polarized, behaving like interacting electric and magnetic dipoles or multipoles (since electromagnetic fields and forces are present around them). This hypothesis is further explored in Chapter 2 (see section 2.2: *Generalized Coulomb-Newton Force Laws*, equations (2.3) through (2.4-10)).

Here, the term "analogy" signifies more than just mathematical similarity. In the context of gravitation within planetary or solar systems, these results can be verified by astronomical measurements and theoretical analyses. Thus, in astronomy and gravitational phenomenology, we are dealing with verifiable scientific facts. This perspective suggests that gravitation and electromagnetic phenomena are interrelated and mutually coupled, at least within solar or planetary systems.

The intention is to establish that the source of gravitation is not merely mass, but rather a complex interplay of atomic forces, fields, and various mechanical and electromagnetic moments emanating from the internal structure of atoms. In this conceptualization, all atoms and masses in the universe are interconnected, synchronized, and communicate through matter waves, electromagnetic waves, and exchange of photons. Consequently, large-scale systems like solar systems may exhibit behavior analogous to that of atoms (see further discussion in Chapter 2: sections (2.11.14), T.2.3.3, T.2.3.3-a, and T.2.3.3-1).

This resemblance cannot be dismissed as a random or meaningless coincidence.

As we know, Niels Bohr's Planetary Atom Model was later refined by Schrödinger's equation and the particle-wave duality theory (referred to here as PWDC). Because of the applicability of the

"Gravito-static versus electrostatic analogy" (based on the analogy to Newtonian forces $\boxed{\frac{Ze^2}{4\pi\varepsilon_0} \to GmM}$), Schrödinger's equation and the associated wave functions can be analogically applied to similar planetary and astronomical contexts. However, this does not imply that the probabilistic methodology of quantum theory is necessarily relevant here, contrary to what some authors assert. Schrödinger's equation is applicable primarily because stable, closed orbits (whether of planets or electrons), which host periodic, standing waves, naturally create the conditions necessary for its formulation, without invoking probabilistic interpretations.

Chapter 2 (refer to equations from (2.4-4) to (2.4-4.3)) also indicates a direct proportionality between electric charge and the mass of an object. To illustrate this further, let us now apply the following analogical substitutions to the spatial quantization expressions for non-relativistic orbiting electrons (8.26 - 8.30), v<<c, to derive analogous standing matter wave expressions for orbiting planets (see (2.11.14) in Chapter 2), as follows,

$$\left\{\frac{Ze^2}{4\pi\varepsilon_0 r^2} \Leftrightarrow \frac{GmM}{r^2},\ Ze \Leftrightarrow M,\ e \Leftrightarrow m, \frac{1}{4\pi\varepsilon_0} \Leftrightarrow G, h \Leftrightarrow H\right\} \Rightarrow \left\{\frac{Ze^2}{4\pi\varepsilon_0} \to GmM\right\} \Rightarrow$$

phase velocity of an electron wave	***phase velocity of a planetary wave***
$u_s \approx \frac{1}{2} v = \frac{Ze^2}{4nh\varepsilon_0}$ ***(8.26)***	$u_n \approx \frac{1}{2} v = \frac{\pi GmM}{nH}$ ***(2.11.14)***

group velocity of the electron wave	***group velocity of the planetary wave***
$v_s \approx 2u_s = \frac{Ze^2}{2nh\varepsilon_0},$ ***(8.27)***	$v_n \approx 2u_n = \frac{2\pi GmM}{nH}$ ***(2.11.14)***

frequency of the orbital electron wave	***frequency of the orbital planetary wave***
$f_s \approx \frac{mZ^2e^4}{8\varepsilon_0{}^2 \cdot n^2h^3}$ ***(8.28)***	$f_n \approx \frac{2\pi^2G^2m^3M^2}{n^2H^3}$ ***(2.11.14)***

wavelength of the orbital electron wave	***wavelength of the orbital planetary wave***
$\lambda_s = \frac{h}{p} \approx \frac{2nh^2\varepsilon_0}{mZe^2}$ ***(8.29)***	$\lambda_n = \frac{H}{p} \approx \frac{nH^2}{GMm^2}$ ***(2.11.14)***

energy of the stationary electron wave	***energy of the stationary planetary wave***
$\varepsilon_s = hf_s \approx \frac{1}{2}mv^2 = \frac{mZ^2e^4}{8\varepsilon_0^2 \cdot n^2h^2}$ ***(8.30)***	$\varepsilon_n = \tilde{E}_n = Hf_n \approx \frac{1}{2}mv^2 = \frac{2\pi^2G^2m^3M^2}{n^2H^2}$ ***(2.11.14)***
radius of the electron orbit	***radius of the planetary orbit***
$r_n = \frac{n^2h^2\varepsilon_0}{\pi me^2Z}$ ***(8.4)***	$r_n = \frac{n^2H^2}{4\pi^2Gm^2M}$ ***(2.11.14)***

All results, parameters, and values on the left side of equations [8.26) - (8.30) and (8.4)] correspond to those of an orbiting electron, while the right side relates to analogous orbital parameters for planetary systems (for more details, see Chapter 2, section (2.11.14)). Since magneto-static forces between two magnets also obey Coulomb's force law—considering the spinning and rotating, electrically charged particles inside an atom—we can construct an analogous Niels Bohr-type atomic model based solely on the magnetic moments of these moving, charged elements.

*The analogies presented here emphasize that we are dealing with systems that are electromagnetically and/or electromechanically coupled and synchronized resonators. This resonance-like behavior is observed from the micro-scale of atoms to the macro-scale of galaxies. Such a unified perspective aligns closely with Nikola Tesla's concepts in his *Dynamic Theory of Gravity* (see [97]).*
Analogies between planetary systems and atoms share several common properties (see Chapters 2 and 10 for more details):

1. Size and Complexity: Planetary or solar systems are large, complex agglomerations or structures composed of atoms.

2. Structural Periodicities: Both systems are naturally characterized by many structural periodicities or periodic motions of their constituents, which are associated with matter-wave formations.

3. Self-Closed Structures: Both systems can be represented as hosting internally self-closed, circular, stationary, and standing matter-wave structures.

4. Coupling and Entanglement: Both systems feature constituents that are mutually coupled and synchronized through field coupling and entanglement relations.

5. Quantizing Relations: Both can be described by various analogous quantizing relations, such as standing waves and the spatial and temporal periodicities of matter states.

6. Electromagnetic Field Dominance: In both cases, the complexity of the involved electromagnetic fields is a dominant characteristic, suggesting that gravitation could be a consequence or derivative of the underlying electromagnetic nature.

7. Resonant System Synchronization: Due to the intrinsic tendency of resonant systems to synchronize spatially and temporally (over mutually overlapping resonant frequencies), we can intuitively, philosophically, and analogically visualize the internal structure of an atom as analogous to the structure of solar systems and relatively stable spiral galactic formations. ♣]

Bohr's second postulate (8.2) can now be fully expressed as a difference between absolute stationary energy levels:

$$\tilde{E}_f = hf_{12} = h(f_{s2} - f_{s1}) = \varepsilon_{s2} - \varepsilon_{s1} = \Delta\varepsilon_s = \Delta\varepsilon_k \, , \qquad (8.31)$$

where it clearly pertains to the (relativistic) motional energy of an electron and its orbital eigenfrequency (8.28). By applying Bohr's postulate (8.2), i.e., equation (8.31), to the frequency expression in (8.28) and using the approximation $v << c$, one arrives at the well-known result for the emission spectral lines of the hydrogen atom (8.9).

Shortly after Niels Bohr's formulation, researchers such as Sommerfeld, Louis de Broglie, and others significantly refined and enhanced the initial planetary atom model. Subsequently, Schrödinger, in a parallel effort, derived his famous wave equation—an endeavor that was both intuitive and axiomatic. Schrödinger's equation incorporated, expanded, and justified the quantization of particle-waves based on standing wave formations, providing a groundbreaking mathematical framework for analyzing atomic systems in a new light. However, this marked a turning point where further exploration and improvement of Bohr's original hydrogen atom model were prematurely abandoned, despite its untapped potential.

The accuracy and applicability of the spectral formula (8.9) are remarkably high, yet Bohr's model is often considered outdated and fully replaced by modern quantum mechanics practices. However, re-examining the foundational planetary atom model may reveal essential insights about particle-wave duality that have remained obscure. This re-evaluation opens the door to revisiting some long-standing assumptions formed during the development of wave-particle dualism, creating an opportunity to substantially refine and modernize Bohr's atom model and to confirm PWDC (more about PWDC in Chapter 10).

It's important to note that an electron orbiting the nucleus is not only revolving around its stationary path but also spinning around its axis with a spin frequency $f_s = \omega_s / 2\pi$ and an associated spin moment $\mathbf{L}_s = \mathbf{J}_s\omega_s$. This helical or spiral motion traces out a toroidal shape, as illustrated in (4.3-0) - (4.3-1.2), Fig. 4.1.1, Fig. 4.1.4, and T.4.3 in Chapter 4.1. Given this dynamic, we can express the total motional energy of the electron in its orbital state as follows:

$$\begin{aligned}
&\varepsilon_s = hf_s = \varepsilon_k = \tilde{E} = (\Delta m)c^2 = (\gamma - 1)mc^2 = muv = \frac{mv^2}{1+\sqrt{1-\frac{v^2}{c^2}}} = \\
&= \frac{pv}{1+\sqrt{1-v^2/c^2}} = \frac{\mathbf{L}_m\omega_m}{1+\sqrt{1-v^2/c^2}} = \frac{\mathbf{L}_s\omega_s}{1+\sqrt{1-v^2/c^2}} = \\
&= \frac{2\pi\mathbf{L}_m f_m}{1+\sqrt{1-v^2/c^2}} = \frac{2\pi\mathbf{L}_s f_s}{1+\sqrt{1-v^2/c^2}} = nh\frac{f_m}{2}\left(1+\frac{u^2}{c^2}\right)\Bigg|_{v<<c} \approx \\
&\approx \pi\mathbf{L}_m f_m = \pi\mathbf{L}_s f_s = \frac{1}{2}mv^2 = \varepsilon_B = nh\frac{f_m}{2} = \frac{mZ^2e^4}{8n^2h^2\varepsilon_0^{\,2}}, \qquad (8.30\text{-}1) \\
&pv = \mathbf{L}_m\omega_m = \mathbf{L}_s\omega_s, \ \mathbf{L}_s = \frac{h}{\pi} = \frac{f_m}{f_s}\mathbf{L}_m \approx \frac{n}{2}\mathbf{L}_m .
\end{aligned}$$

In equation (8.30-1), we begin to merge concepts introduced by M. Kanarev, as found in [44], and by Charles Lucas and his colleagues, as referenced in [16] through [22]. It's worth noting that the Lucas-Bergman atomic model has faced significant, yet arguably misguided, criticism from some mainstream leaders in contemporary quantum theory. While this model is often dismissed as lacking experimental verification, it produces better results and predictions than both the existing N. Bohr model and the official quantum theory.

A more accurate statement might be that all currently known atomic models, starting with Bohr's and including the elaborations in this book, represent different "Ptolemaic conceptualizations." These models describe systems with intrinsic structural periodicities and internally self-closed, mutually synchronized standing matter-wave structures. Despite their differences, they all yield sufficiently accurate and experimentally verifiable results due to the inherent periodicities and the mutual transformability or deterministic mapping within these systems.

The idea of isolated, orbiting, and spinning particles with rest masses (such as electrons, neutrons, and protons) existing inside atoms is increasingly difficult to accept. It seems more plausible that atoms, along with their internal components, are specific, self-stabilized, synchronized, and closed resonant formations of standing electromagnetic waves with certain internal dynamics. Electrons, protons, and neutrons are wave-particle dualistic formations, or equivalent states, that can be detected as particles only when they are excited, extracted, or expelled from atoms.

--------------- (This part, from here, should be additionally modified) --------------

Let us try to completely associate Bohr's atom model (in the context of the previous interpretation) with the general concept of the particle-wave dualism, which is presented in Fig.4.1, Chapter 4.1 and within relations from (4.9) to (4.22), Chapter 4.3. In the expression for the second Bohr's postulate, (8.31) figures a stationary orbital energy of the electron, which is, in fact, equal to the total (relativistic) motional energy of the electron (4.12). Of course, we may now consider that the electron has its orbital and stationary state (for which the main quantum number **n** is associated) with the total motional energy,

$$\varepsilon_{kn} = E_{kn} + \tilde{E}_n = \varepsilon_{sn} . \qquad (8.32)$$

If we now say that the electron moved from its stationary state $n = n_2 = 2$, to its second stationary state $n = n_1 = 1$, and there emitted a photon $\tilde{E}_f = hf_{12}$, the second Bohr's postulate (8.31) may be developed as,

$$\tilde{E}_f = hf_{12} = h(f_{s2} - f_{s1}) = \Delta\varepsilon_k = (E_{k2} + \tilde{E}_2) - (E_{k1} + \tilde{E}_1) =$$
$$= (E_{k2} - E_{k1}) + (\tilde{E}_2 - \tilde{E}_1) = \Delta E_k + \Delta\tilde{E} \qquad (8.33)$$

As the emitted photon, $\tilde{E}_f = hf_{12}$, is a form of wave energy that leaves the atom overall (and not only one of its stationary orbits), but we also have a case of action and reaction, or better to say, a case of shooting a missile from a gun called an atom. It is obvious that the atom will in such case experience something like a reactive recoil, i.e., then we must apply the general balance of energy and impulse of the particle-wave dualism, which are given by expressions (4.14) and (4.20), Chapter 4.3, but it is also clear that this general balance of energy will differ from the balance represented by (8.33). Of course, the difference between the total balance of the overall atom energy, and the case described by (8.33) is most likely, quantitatively, negligibly small, considering the relations of total energies among the electron, atom nucleus, and the emitted photon. Of course, this should not be forgotten (or neglected) due to the qualitative understanding of the situation in the full complexity of the particle-wave dualism.

Let us now determine the most general case of the balance of an excited atom energy that an outgoing photon will emit. Of course, an atom, i.e., some of its electrons, before emitting a photon, was excited (and according to the natural assumption, it will emit a photon in the next moment, and then return to its basic state). Such an atom may be characterized by its total motional energy that is equal to the sum of the motional energy of its excited electron, and the motional energy of the whole atom (let us say, almost the motional energy of its nucleus). So, state of an atom before de-excitation may be presented as:

$$\varepsilon_{ka2} = E_{k2} + \tilde{E}_2 + E_{ka2} + \tilde{E}_{a2}\,, \qquad (8.34)$$

where $E_{k2} + \tilde{E}_2$ is total stationary motional energy of the excited electron, as given in (8.33), and $E_{ka2} + \tilde{E}_{a2}$, is total motion energy of an atom as a whole (or let us say, significantly close to a motional energy of the atom nucleus), which was neglected until now (or we could start from an assumption that it was equal to zero). In a way analogous to the previous, the state of the whole situation after de-excitation (emitting a photon) may be expressed by the resulting motional energy:

$$\varepsilon_{ka1} = E_{k1} + \tilde{E}_1 + E_{ka1} + \tilde{E}_{a1} + hf_{12}\,, \qquad (8.35)$$

where $E_{k1} + \tilde{E}_1$ is a total stationary motional energy of an electron in a new stationary orbit, and $E_{ka1} + \tilde{E}_{a1}$ is a total motional energy of an atom (or let us say remarkably close to the motional energy of the atom nucleus) after de-excitation.

Let us now apply the law of conservation of total energy (4.14) to the cases given by (8.34) and (8.35), if the rest energy (or rest mass) of an atom (before and after de-excitation) did not change, wherefrom one will get:

$$\begin{aligned}\varepsilon_{ka2} - \varepsilon_{ka1} &= E_{k2} + \tilde{E}_2 + E_{ka2} + \tilde{E}_{a2} - (E_{k1} + \tilde{E}_1 + E_{ka1} + \tilde{E}_{a1} + hf_{12}) = \\ &= (E_{k2} - E_{k1}) + (\tilde{E}_2 - \tilde{E}_1) + (E_{ka2} - E_{ka1}) + (\tilde{E}_{a2} - \tilde{E}_{a1}) - hf_{12} = 0\,.\end{aligned} \qquad (8.36)$$

It is now possible from (8.36) to determine the energy of the emitted photon as,

$$\begin{aligned}\tilde{E}_f = hf_{12} &= (E_{k2} - E_{k1}) + (\tilde{E}_2 - \tilde{E}_1) + (E_{ka2} - E_{ka1}) + (\tilde{E}_{a2} - \tilde{E}_{a1}) \approx \\ &\approx (E_{k2} - E_{k1}) + (\tilde{E}_2 - \tilde{E}_1).\end{aligned} \qquad (8.37)$$

One may notice that the second Bohr's postulate (8.33) is identical to the situation described by (8.37), under a condition that all forms of the motional energy of an atom as a whole, $(E_{ka2} - E_{ka1}) + (\tilde{E}_{a2} - \tilde{E}_{a1})$, may be neglected or mutually annulled. This is correct (because the mass of an electron is negligible considering the mass of the atom nucleus, and rest masses of the electron and the nucleus did not change). If we now apply the law of conservation of the impulse (4.20), the atom (under the same assumptions as hither) would receive approximately the impulse that corresponds to the impulse of the emitted photon with an opposite sign.

The nature of the wavelength of the total stationary electron wave (that is expressed through the wave impulse of an electron) remains to be explained. During the previous analysis, de Broglie's definition of the wavelength was used for the wavelength of the electron wave (see (8.12), (8.25), (8.29)), compared with the fact that in this book we also exercise with the new definition of that same wavelength, through the wave impulse. **It is obvious that in the case of Bohr's atom model, the equality of the absolute values**

of the particle and wave impulse of an electron that is in motion in its stationary orbit, i.e., $|p|=|\tilde{p}|$ is achieved. We can explain this situation in the following way. If the atom is in the state of rest, or, let the velocity of its center of mass equals zero, then orbital and stationary revolution of an electron around the atom nucleus is very much balanced in the sense that some uniform, center-symmetrical, spatial distribution of the electron mass (across the whole area of the stationary orbit) exists. The total impulse of the electron, P_e, which is equal to the vector sum of its impulse as a particle p_e, and its wave impulse $\tilde{p}_e$, must be equal to zero (in the opposite case, if it is not equal to zero, the center of the atom mass would not be in the state of rest),

$$P_e = p_e + \tilde{p}_e = p + \tilde{p} = 0 \qquad (8.38)$$

From (8.38) we have:

$$\Delta p + \Delta \tilde{p} = 0 \,, \; p = -\tilde{p} \;\Rightarrow\; |p| = |\tilde{p}| \;\Rightarrow\; \lambda_s = \frac{h}{|\tilde{p}|} = \frac{h}{|p|} \qquad (8.39)$$

In reality, when we search for the de Broglie's wavelength (of some energy state) we should know that every stable, uniform and stationary state (or state of relative rest) regarding certain «Laboratory system» may be characterized either by its particle p*, or by its wave momentum,* $\tilde{p}$ *(* $\lambda = \frac{h}{\tilde{p}}$ and/or $\lambda = \frac{h}{p}$*. Substantial differences (as "vectors or scalar-like") between the wave and particle momentum originate in transient regimes when the certain state experiences a modulation of its motion*. According to (8.39), quantizing the orbital momentum of an electron, or applying the first Bohr's postulate, (8.1), in context with the wave attributes of an electron is more logical to present by quantizing its wave orbital momentum,

$$\tilde{m}vr = \tilde{p}r = n\frac{h}{2\pi} \,. \qquad (8.40)$$

Instead of (8.38), for the stable atom in the state of rest, we should consider the total momentum of the atom P_a (including the nucleus momentum $P_p = p_p + \tilde{p}_p$). The result or the conclusion above will approximately stay the same, since the nucleus of the stable atom has a much bigger mass than the electron, and it will be almost in the state of rest (comparing to a revolving electron: see also (8.9-1) and (8.9-2)),

$$\begin{aligned} &P_a = P_e + P_p = (p_e + \tilde{p}_e) + (p_p + \tilde{p}_p) = 0 \Rightarrow \\ &\Rightarrow P_e = p_e + \tilde{p}_e \cong 0 \text{ and } P_p = p_p + \tilde{p}_p \cong 0 \Rightarrow \\ &\Rightarrow \Delta p_e + \Delta \tilde{p}_e \cong 0 \text{ and } \Delta p_p + \Delta \tilde{p}_p \cong 0 \,. \end{aligned} \qquad (8.38\text{-}1)$$

As we know, an electron and an atom nucleus both revolve around their common center of mass. Consequently, the total orbital momentum of a stable and neutral atom $\mathbf{L_a} = L_p + L_e$ (in the state of rest, analogously to (8.38-1)) should also be equal to zero,

$$L_a = L_e + L_p = (\ell_e + \tilde{\ell}_e) + (\ell_p + \tilde{\ell}_p) = 0 \Rightarrow$$
$$\Rightarrow (\Delta\ell_e + \Delta\tilde{\ell}_e) + (\Delta\ell_p + \Delta\tilde{\ell}_p) = 0, \qquad (8.38\text{-}2)$$

implicitly accepting that between an atom nucleus and the revolving electron there is always certain energy-momentum coupling (with electromagnetic wave energy exchanges). Of course, here we should also consider angular spinning moments of involved participants (presently neglected).

With previous results and conclusions, N. Bohr's atom model fits into the general concept of the particle-wave dualism of this book, which was the purpose of former analysis. *In addition, here we are creating grounds showing that certain matter-waves, energy-momentum exchanges, and communications should naturally exist between electron and nucleus stationary states.*

8.1. New Aspects of Atom Configuration from the view of the Bohr's Model

By further exploration of the same problem, one may draw other interesting consequences that result from the richness of the content of N. Bohr's atom model. For example, frequency of the quantum of inter-orbital, electron exchange for stationary orbits with big quantum numbers is approximately equal to:

$$f_{12} = \frac{mZ^2e^4}{8h^3\varepsilon_0^{\ 2}}\left(\frac{1}{n_1^{\ 2}} - \frac{1}{n_2^{\ 2}}\right) \approx \frac{mZ^2e^4}{4n^3h^3\varepsilon_0^{\ 2}} = f_m \approx \frac{2f_s}{n} \ , \qquad (8.41)$$

implying that it is possible to use the following approximations:

$$n_1 = n_2 + 1 \ >> \ 1 , \quad n_1 \approx n_2 \approx n \approx \sqrt{n_1 n_2} \approx (n_1 + n_2)/2 , \quad v << c .$$

Looking from the other side, frequency of the stationary electron wave (in its orbit) is:

$$f_s = \left. \underline{nf_m \frac{1}{2}(1 + \frac{u^2}{c^2})} \right|_{v<<c} \approx \frac{mZ^2e^4}{8n^2h^3\varepsilon_0^{\ 2}} = \frac{n}{2} f_m$$
$$\frac{n}{2} \le \frac{f_s}{f_m} \le n \ , \ \frac{1}{2} \le \frac{f_s}{nf_m} \le 1 \ , \ 0 \le (u, v) \le c . \qquad (8.42)$$

One may notice that for <u>*border cases*</u> (8.41), when **n** is a great number, the frequency of the quantum of the electron emission is approximately equal to the corresponding orbital, mechanical (or rotational) frequency of an electron treated like a particle, f_m , (see (8.17)). Here, the quantum mechanical principle of correspondence is evoked, by which N. Bohr intended to show where the borders between a macroscopic and quantum treatment of the matter are in the world of physics. However, one should pay attention that wave frequency or electromagnetic quantum frequency does not have very much in common with the mechanical-revolution number of the particle, around some center, (8.42), and this again represents something else for a stationary electron wave (the only common term here is the term frequency, and both frequencies may be dimensioned using the same units). **In fact, N. Bohr's principle of correspondence is now corrected, and it differs from what Bohr claimed.**

Based on the expressions found for the group and phase velocity of an electron, (8.25), (8.26), and (8.27), as well as based on (4.0.27), (4.0.28), and (4.0.29), one can check that the following relations between group and phase velocity of an electron wave remain valid:

$$0 \le u_s \le v \le c, \ \ 0 \le u_s^2 \le u_s v \le v^2 \le c^2. \qquad (8.43)$$

The most interesting is that by solving differential equations that connect group and phase velocity of an electron wave (which may be found in the same form as in expressions (4.0.27), (4.0.28), and (4.0.29)) we may come to adequate spectral distributions of radiating energies that remind us of the Planck's law of heated blackbody emission.

Let us start with the expression for the group velocity of an electron wave (8.27) and let us connect it with (8.43), where we get:

$$\left. v = \frac{2u_s}{1+\frac{u_s^2}{c^2}} \right|_{(u,v)<<c} \approx 2u_s = \frac{Ze^2}{2nh\varepsilon_0} = \frac{Z}{n}c\alpha < \frac{Z_{max}e^2}{2n_{min}h\varepsilon_0} = Z_{max}c\alpha < c,\ n_{min} = 1,$$

$$\Rightarrow Z_{max} < \frac{2c\varepsilon_0 h}{e^2} = \frac{1}{\alpha} = \frac{2h}{\mu_0 ce^2} = 137.03604,\ \alpha = \frac{e^2}{2c\varepsilon_0 h},\ \varepsilon_0\mu_0 = \frac{1}{c^2} \qquad (8.44)$$

$$\Rightarrow \alpha Z_{max} < 1.$$

In (8.44), α is "thin (or fine) structure", the universal physical constant. Also, from (8.44) one may conclude that the maximal possible protons' number of certain (still not discovered) elements in our Periodic System of Elements is $Z_{max} \le 137$. Since 137 is not a much bigger number than hitherto known maximal protons number of the last known element from the Periodic Table, one may expect to discover around twenty new elements of the Periodic Table. Of course, concluding based on the previous (simplified) analysis is approximative by its character to be on the total proof-level, but it is logical, non-contradictory and interesting, at the same time putting some light on the nature of the thin structure constant α.

Since in Bohr's planetary atom model some circular (closed line) motion of electrons (or their stationary waves) is immanently present, it is important to determine mechanical and magnetic orbital moment of the electron (in the stationary orbit).

Mechanical orbital moment or torque of the electron is,

$$L = L_e = J_e\omega_m = \gamma mvr = \gamma m\omega_m r^2 = 2\gamma m\pi r^2 f_m,\ J_e = \gamma mr^2, \qquad (8.45)$$

and the magnetic moment of the electron is,

$$M = I_e S = I_e \pi r^2, \qquad (8.46)$$

where I_e is an elementary current of a rotating electron across its orbit, and S is the surface encompassed by that orbit.

If we now determine the elementary current of a rotating electric charge as the product of the electron charge **"e"** and corresponding frequency by which that charge really orbits f_e, there will be,

$$I_e = ef_e \Rightarrow M = \pi r^2 ef_e . \qquad (4.47)$$

The relation between the magnetic and mechanical moments of an electron is called the gyromagnetic ratio and it has great significance in describing the magnetic properties of materials:

$$\frac{M}{L} = \frac{\pi r^2 e}{2\pi r^2 \gamma m}\frac{f_e}{f_m} = (\frac{e}{2\gamma m})\frac{f_e}{f_m} . \qquad (8.48)$$

There remains an open question, how to treat the frequency f_e by which the charge orbits around the atom nucleus. In general cases, we have very few possibilities, such as:

a) That the electron charge and its mass are always concentrated in the same spatial spot, i.e., that there is some compact material object or wave-group that orbits around the atom nucleus by its mechanical frequency, so there will be $f_e = f_m$, or

b) That electron in the stationary orbit is a somewhat different form of spatially distributed mass and electric charge (across the whole orbit area) when the notion of the mechanical rotational frequency of the electron mass differs from the frequency by which the distributed electric charge of the electron orbits $f_e \neq f_m$. Consequently, we may consider that the orbiting frequency of the electron is equal to the frequency of its stationary electron wave $f_e = f_s$, (see (8.42)), and

c) That all previously mentioned frequencies are mutually dependent and different $f_e \neq f_m \neq f_s$.

In the first case, if the electron charge would orbit by its mechanical rotation frequency around the atom nucleus, $f_e = f_m$, the gyromagnetic ratio would be:

$$\frac{M}{L} = (\frac{e}{2\gamma m})\frac{f_e}{f_m} = \frac{e}{2\gamma m} = \frac{e}{2m}\sqrt{1-\frac{v^2}{c^2}} , \qquad (8.49)$$

$$0 < \frac{M}{L} \leq \frac{e}{2m} , \ 0 \leq v \leq c ,$$

which is the known (old Quantum theory) case found in the existing textbook literature.

In the second case, when the electron charge forms a stationary orbital electron wave and orbits by eigenfrequency of that wave around the atom nucleus, $f_e \approx f_s$, the gyromagnetic ratio will be:

$$\left\{\left(\frac{M}{L}=(\frac{e}{2\gamma m})\frac{f_e}{f_m}=\frac{n}{2}(\frac{e}{2\gamma m})(1+\frac{u_s^2}{c^2})\frac{f_e}{f_s}=\frac{n}{2}(\frac{e}{2m})(1-\frac{u_s^2}{c^2})\frac{f_e}{f_s}\right),(0\leq u_s\leq c)\right\}\Rightarrow$$

$$\Rightarrow\left\{\begin{array}{c}\boxed{0<\frac{M}{L}\leq\frac{n}{2}(\frac{e}{2m})\frac{f_e}{f_s}\cong\frac{n}{2}(\frac{e}{2m})},\\ \left(\begin{array}{l}f_s=nf_m\frac{1}{2}(1+\frac{u^2}{c^2}),f_m=2f_s/n(1+\frac{u^2}{c^2}),\frac{1}{2}nf_m\leq f_s\leq nf_m,\ 2\frac{f_s}{n}\leq f_m\leq\frac{f_s}{n},\\ L=n\hbar,\ \hbar=\frac{h}{2\pi},\mu_B=\frac{eh}{4\pi m}=\frac{e}{2m}\hbar\end{array}\right)\end{array}\right\}. \quad (8.50)$$

The treatment of the gyromagnetic ratio varies depending on how we approach the frequency of electron charge rotation around the atomic nucleus (as discussed in 8.49 and 8.50) and the relationship between the orbital eigenfrequency of the electron's de Broglie wave and the mechanical rotation frequency of its mass and charge. This area likely contains the weak points in Bohr's atomic model and early quantum theory, as the frequency of the electron's mechanical orbiting was incorrectly associated with the frequency of its orbital de Broglie wave.

The key issue is that the mechanical rotational frequency refers to the number of revolutions a material point makes around a center of rotation per unit of time, while the frequency of wave motion is characterized by relations (8.23) and (8.24). These are two distinct concepts and phenomena, yet they have sometimes been conflated under the same term—frequency, leading to subsequent corrections, additions, and the development of the correspondence principle in quantum physics. It can be challenging to distinguish between closely related concepts that are dimensionally equivalent.

Introducing the spin or rotation of the electron around its own axis complicates the situation with the gyromagnetic ratio. This spin generates a magnetic moment that adds to the total magnetic moment of the electron orbiting the nucleus. Analyzing the gyromagnetic ratio in this context suggests that the electron, when considered integrally as part of the atomic shell, is no longer merely a particle orbiting the nucleus. Instead, it is more likely a diffused and distributed particle-wave state—a mass-energy-momentum state, or matter-wave group that exhibits rotational characteristics, but not in the traditional sense of a strictly spatially localized particle, as in conventional quantum mechanics.

Furthermore, the distribution of electron mass and charge, involved in such orbital motion, likely follows different, yet interdependent, distribution laws. This perspective is supported by the fact that applying high pressures or low temperatures to masses or atoms leads to phase transformations in material properties and restructures the electron shells in terms of both body and energy. This suggests the possibility of different expressions for the electron's radius or the area it occupies. In (8.50) and earlier, we encounter velocity-dependent expressions that address the interval $0\leq(u,u_s,v)\leq c$, although we are still only discussing electrons. Briefly, we can give an intuitive and oversimplified understanding, or statement of electron states in relation to characteristic boundary velocities, such as: If or when $0\leq(u,u_s,v)<<c$, electron states are closer to

particle states, and when $0 <<< (u \cong u_s \cong v) \cong c$, such electron states should be kind of distributed electromagnetic field and matter-wave energy states.

Bohr's atomic model, however, lacks the depth to draw more precise conclusions. Nevertheless, electrons in different situations and energy-momentum states represent different formations or "packing states" of a specifically structured electromagnetic field. The structure of this electromagnetic field determines the creation of positive and negative electric charges, as well as familiar matter and antimatter entities.

The most significant implication of this discussion relates to the Particle Wave Duality Concept (PWDC), specifically the intrinsic coupling of linear motion and rotation (or spinning), analogous to the coupling between electric and magnetic fields.

[♣ The purpose of the detailed analyses presented in this chapter is to highlight that the issue isn't merely about the different conceptualizations of matter's structure from the perspectives of classical mechanics or quantum wave mechanics. The problem also lies in the fact that we often rely on insufficiently relevant facts, laws, concepts, and mathematical models, along with their logical connections and interdependencies. Some theoretical concepts that we still consider correct may have been incorrectly established from the outset, due to a lack of knowledge about certain links at the time of their creation. As a result, erroneous theoretical postulates may require the addition of new, equally flawed concepts to self-correct and align with experimental results, much like the updated Ptolemaic geocentric theory, unless we are prepared to discard the fundamental hypothesis altogether.

One of the most challenging aspects of writing books like this one is that they may face dogmatic criticism from proponents of existing, mainstream theoretical concepts and models. These models have become entrenched in the history of scientific thought, encyclopedias, and educational textbooks, and they appear to align with the experimental data we have. Administratively, we are often compelled, or forced by mainstream acceptance, to treat these models as stable and definitive references.

It is well-known that a model can be developed to connect a specific set of known facts adequately, but that same model may prove inadequate when later evaluated against a broader set of relevant facts discovered through new experimental practices. Intellectual rigidity, inertia of thought, and dogmatic adherence to long-held positions pose significant obstacles to the introduction of new ideas that challenge established models.

This is particularly true in physics, where some models or theories eventually reach a point of exhaustion, leading to their abandonment and replacement by new ones. When an old model no longer serves the purpose of scientific prediction, or when it fails to explain or predict all empirically known facts within certain domains as was the case with the Ptolemaic geocentric system, it is often set aside. However, this process has both advantages and disadvantages. Abrupt shifts in scientific thinking are risky and problematic, as the continuity of the cognitive process is essential. Scientific knowledge must evolve from simpler models toward more complex ones, which ideally build upon and refine their predecessors.

Citation concerning Historical Atom Models Evolution taken from Lucas-Bergman website,
http://www.commonsensescience.org/atom_models.html:

Bohr's Atom
In 1913, Niels Bohr proposed a theory of the hydrogen atom (the simplest of all atoms) consisting of one heavy proton in the center with one lighter electron in orbit around the proton. Bohr supposed,

- *That electrons move in circular orbits around the atomic nucleus.*
- *Only certain orbits are permitted.*
- *That in these permitted orbits, the electrons would not radiate (would not create radio waves).*
- *That light of certain colors (and wavelengths) would be created when the electron (of its own power) changed orbits.*

These postulates were entirely arbitrary and even violated the established laws of electricity and magnetism. In spite of this, physicists still use the Bohr model (when it is convenient).

Parson's Magneton Theory of the Atom

"By 1915, A. L. Parson knew that the Bohr model of the atom could not be real, so he developed and even experimented on a model of the atom where the electrons were not point-sized particles that orbit around the atomic nucleus. In Parson's atom, the electrons in the shells surrounding the nucleus were rings of charge (with the shape of a toroid or donut). Since the electrostatic charge at the surface of these rings is rotating, each electron is a tiny magnet. In 1918, Dr. H. Stanley Allen of the University of Edinburgh discussed the arguments in favor of the ring electron, showing how it removed many outstanding difficulties of other theories of the atom. In spite of its superiority, Parson's magneton model of the atom did not become popular (but a modern version, the Lucas model of the atom, has now been introduced).

De Broglie's Model of Matter: Particle-Wave Duality

"About 1924, Louis de Broglie proposed that all particles of matter (from single atoms to large objects) moving at some velocity would have the properties of a wave. Today, most physicists take this farther and say that all material objects are waves until they are measured or observed in some way. When this takes place, the wave is said to collapse and turn into an object. An example of this notion of reality is given by the famous Cornell physicist N. David Mermin who says, "We now know that the moon is demonstrably not there when nobody looks."

Schrodinger's Wave Model of the Atom

"In 1925, soon after de Broglie had put forward his ideas, Schrodinger used them [to write] a wave equation to describe this new mechanics of particles." Schrodinger's model of the atom is not a physical model (where an object has size, shape, and boundaries) but is a mathematical model (an equation where objects are point particles.) The equation is useful to predict some properties of objects (or atoms) but is not able to describe the object (or atom) itself.

Dirac's Model of the Atom

"The Dirac model is an equation that includes imaginary numbers. It is not an attempt to describe the objective reality of the physical electron but to predict the various levels of energy that the electron may have at various times. There appears to be a serious problem with the Dirac model, which has electrons orbiting the nucleus. Another major assumption of the Dirac theory is that the statistical version of quantum wave theory or quantum mechanics is valid. Unlike previous quantum theories of the atom that used real numbers (such as the Bohr model and Schrodinger model), the terms and imaginary numbers in Dirac's equation do not correspond to measurable quantities.

Standard Model of Elementary Particles

"The Standard Model of Elementary Particles is not a description of the atom. However, we must mention it now because, in modern theory, atoms are not only waves but also when measured, the atoms change into objects composed of elementary particles. In modern physics, the important components of atoms are electrons, protons, and neutrons. The Standard Model considers electrons to be true elementary particles, either waves or point particles with inherent properties of mass, magnetism, spin, and stability. However, in modern Quantum Theory, protons are supposed to be composed of quarks; and neutrons are thought to be composed of a different combination of quarks. Another important part of the Standard Model is that forces between these elementary particles are supposed to be carried by other particles that move back and force randomly between the material particles:

- *photons are supposed to be particles that carry forces between electrons,*
- *mesons are supposed to be particles that carry forces between protons and neutrons,*
- *gluons are supposed to be particles that carry forces between the quarks (which are supposed to be inside protons and neutrons).*

Lucas Model of the Atom

"In 1996, while still a high school student, Joseph Lucas introduced his model of the atom. In this model, electrons, protons, and neutrons are all based on Bergman's Spinning Charged Ring Model of Elementary Particles (a refinement of Parson's Magneton). The Lucas Model of the Atom is by far the most successful of all models of atom ever proposed. It is a physical model that shows where electrons are located throughout the volume of the atoms. This model predicts the "magic numbers" 2, 8, 18, and 32 of electrons in the filled shells and can predict why the Periodic Table of the Elements has exactly seven rows. The Lucas model also predicts the structure of the nucleus and correctly predicts hundreds of nuclide spins.

R.J. Haüy – G.P. Shpenkov – V. Christianto atom model (see [85] and [86])

As we know, all atoms are composed of electrons, protons, and neutrons. Proton and neutron have similar masses, almost equal to the mass of hydrogen atom, and to the sum of masses of an electron and a proton.

Mass of an electron	9.109389700 x 10^{-31} kg
Mass of proton	1.672623100 x 10^{-27} kg
Mass of an electron + proton	1.673534039 x 10^{-27} kg
Mass of hydrogen atom	1.673534000 x 10^{-27} kg
Mass of neutron	1.674928666 x 10^{-27} kg

It is hypothetically conceivable that a neutron could be presented as a specific combination of an electron and a proton. A hydrogen atom already consists of an electron and a proton, so within this imaginative framework, we might say that neutrons and hydrogen atoms are simply different combinations or "packings" of these two particles. Consequently, we could propose that since a hydrogen atom unites an electron and a proton, it could be viewed as a particular packing state of a neutron. Following this logic, all other atoms could be seen as various packing combinations of hydrogen-like atoms, or neutron-like structures.

This hypothetical perspective gains some support when we consider what happens during a photon-atom collision. For instance, we can observe the emission of photoelectrons (as explained by M. Maric-Einstein's Photoelectric Effect), a reduction in photon energy, the ejection of an electron (known as the Compton Effect), or the creation of an electron-positron pair if the photon has sufficient energy. These phenomena are commonly explained in Quantum Theory and Physics textbooks, but they can also be reimagined: inside atoms, electrons might represent different energy-momentum and spatial standing matter-wave formations, as opposed to the more corpuscular, concentrated nature of free electrons outside of atoms.

In this context, we could view electrons within atoms as standing waves of specific electromagnetic energy, like photon formations, distributed around the nucleus. When an external photon collides with an atom, the energy it transfers could expel this electromagnetic energy, effectively creating new electrons (Photoelectric Effect), new electrons and photons (Compton Effect), or electron-positron pairs. Essentially, this suggests that electrons and positrons are different structural formations or packings of photons, or specific forms of electromagnetic energy.

Empirical observations show that the behavior of electrons differ depending on their environment. Inside atoms, electrons exist as self-contained, standing electromagnetic waves. Once outside the atom, they take on a more corpuscular, spatially localized nature. Extending this imaginative hypothesis, we might say that specific wave packing of an electron and a proton could result in the formation of either a hydrogen atom or a neutron. Thus, all atoms could be viewed as specific packings and formations of hydrogen atoms, neutrons, or fundamentally, electromagnetic energy.

For a similar concept of composite atomic structures (though not explained in the same way), refer to sources [85] and [86].

Citation: A THEORETICAL PREDICTION OF MOLECULAR AND CRYSTAL STRUCTURES, ***Leonid G. Kreidik[1] and George P. Shpenkov[2]***

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Keywords: *Haüy's molecules, wave equation solutions, intra-atomic space, crystal structure*

Analyzing the structure of crystals at the end of 18th century, R.J. Haüy [1] has concluded that it is necessary to consider atoms as elementary molecules, an internal structure of which is closely related to the crystal shape of solids. Particles, constituents of these elementary molecules, must be coupled with strong bonds, which we call the multiplicative bonds. Then it is reasonable the ordinary molecules with relatively weak bonds to call the composite or additive molecules, e.g. if deuterium D is the multiplicative molecule then the hydrogen molecule H_2 is the additive one.

As masses of atoms are multiple to the neutron (hydrogen atom) mass then, following Haüy's ideas, it was reasonable to suppose that the atom, as the elementary Haüy's molecule, is the neutron multiplicative molecule. According to this model, using the wave equation, the problem on the distribution of matter (neutrons) in the elementary Haüy's molecules has been solved.
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…………………….
Based on elementary multiplicative Haüy's molecules, it is easy to predict the possible structures of both the crystals and additive molecules that is presented in the extended report.

1. *R.G. Haüy, Essai d'une theorie sur la structure des crystaux, Paris, (1784).*
2. *L.G. Kreidik, G.P. Shpenkov, Alternative Picture of the World, V.1-3, Bydgoszcz (1996).*
3. *S. Flugge, ed., Handbuch der Physik, Encyklopedia of Physics, V.30, Springer-Verlag (1957).*

Modern atomic physics correctly explains that within an atom, the interactions between positive and negative charges (such as between protons and electrons) are governed solely by electromagnetic forces, specifically Coulomb's electrostatic force between the positively charged nucleus and the surrounding electrons. Magnetic interactions, such as those between magnetic moments of atomic constituents, also fall under the umbrella of electromagnetic forces.

Given that the atomic nucleus, as conceptualized in contemporary physics, contains both protons and neutrons, and is highly stable in all stable atoms, one might logically search for a force stronger than the repulsive Coulomb force to account for the cohesion of protons and neutrons within such a small space. However, instead of addressing this problem with rigorous technical and scientific analysis, modern physics simply postulated the existence of a "strong nuclear force." This force, though essential, lacks reliable and verifiable mathematical modeling and was introduced as a dominant force that effectively negates the repulsive Coulomb force between protons. This oversimplified and intuitive assertion, while convenient, replaces a real problem with an unexplained concept that sounds plausible but remains hypothetical.

For unstable (radioactive) atoms, which decay over time, the introduction of a "weak nuclear force" followed a similarly simplistic approach, once again without robust mathematical modeling. This problem, too, was superficially addressed with an arbitrary label rather than a thorough solution. Over time, followers of these theories continued to use these hypothetical and incomplete concepts, developing modern atomic physics through new experimental results, mathematical practices, conservation laws, event probabilities, and uncertainty principles. These efforts have, in effect, masked the initial oversimplifications regarding nuclear forces.

In contemporary physics, we continue to teach that our universe is governed by four fundamental forces: Electromagnetic, Gravitational, Strong Nuclear, and Weak Nuclear forces. However, in practical calculations, we only have a firm grasp of electromagnetic forces. Gravitational forces are negligible at the atomic level, and the Strong and Weak Nuclear forces lack the well-developed field structures and mathematical formulations that characterize Maxwell's electromagnetic theory.

Moreover, some publications suggest that gravitational force might be an unusual manifestation of electromagnetic forces, though this remains speculative.

The author of this book believes that atomic constituents, electrons, protons, neutrons, etc., are essentially electromagnetic structures, specifically spatial standing matter-wave formations, particularly when coupled within atoms. This perspective suggests that the atomic nucleus is not composed of distinct particles glued together, but rather represents a complex electromagnetic field structure, akin to a well-combined matter-wave packet or group, which still manifests a net positive charge equal to the total charge of the protons involved.

Additionally, the author envisions that the electron cloud and the nucleus are, in some way, mirror-symmetrical or structurally similar, and are electromagnetically coupled and mutually synchronized, both internally and externally. This idea paves the way for conceptualizing a new theory of nuclear fields and forces within atoms.

For a more radical critique of contemporary concepts related to Strong and Weak Nuclear forces, and the existence of corpuscular neutrons within the atomic nucleus, refer to Charles W. Lucas, Jr.'s paper, "Is There Any Truth in Modern Physics?" (February 2015, Volume 18, Number 1, The Journal of Common-

Sense Science). Lucas presents evidence suggesting that a neutron is a specific formation or combination of an electron, and a proton based solely on electrodynamic interactions, without invoking Strong or Weak Nuclear forces. While some of Lucas's ideologically driven statements in the paper can be dismissed, his physics-related conclusions warrant consideration.

Citations from [91]: "A proper examination of atomic and nuclear data shows that there is no strong force holding the positively charged protons together in the nucleus and no weak force governing nuclear decay. There is only one force in the nucleus, which is the electrodynamic force.

... A second analysis of the NIST isotopic masses of various nuclei has discovered that there is no data in the entire table of isotopic masses to justify (1) the existence of neutrons inside nuclei, (2) the existence of the Strong Force inside nuclei to keep the protons bound together in the nucleus, and (3) the existence of the Weak Force controlling the decay of various nuclei. The only evidence for the electrodynamic force is found".

Citation from [93], F.A. Gareev, I.E. Zhidkova. "We come to the conclusion that all atomic models based either on the Newton equation and the Kepler laws or on the Maxwell equations or on the Schrödinger and Dirac equations achieved reasonable agreement with experimental data. We can only suspect that these equations are grounded on the same fundamental principle(s) which is (are) not known or these equations can be transformed into each other. … Bohr and Schrodinger assume that the laws of physics that are valid in the microsystem do not hold in the micro world of the atom. We think that the laws in macro and the micro world are the same". ♣]

8.2. Bohr Atom Model and the Stationary States of Nucleus

Until now, a somewhat static hydrogen atom model has been discussed, if electron orbits around the stable and immobile nucleus (what is almost correct, considering the ratio of the mass of the nucleus and the mass of the electron), but it is more precise to say that an electron and a proton orbit around the common center of mass.

As an illustration of the platform for merging de Broglie matter waves and N. Bohr's atom structure, let us analyze a hydrogen atom in the center-of-mass coordinate system, like the one shown in Fig.4.1 (from Chapter 4.1). Let us apply (4.1), (4.2), and (4.3) (from Chapter 4.1) to describe the movement of the electron and the proton around their common center of gravity, and to describe their associated de Broglie waves, assuming that the electron and atom nuclei are treated as rotating particles (creating slightly modified Bohr's model), having the following characteristics:
m_e-electron mass, m_p-nucleus or proton mass, $v_e = \omega_{me} r_e$ -electron velocity around the common center of mass, $v_p = \omega_{mp} r_p$ -proton velocity around the common center of mass, r_e-radius of revolving electron, r_p -radius of revolving proton, $\omega_{me} = \omega_{mp} = \omega_m = 2\pi f_m$-mechanical, revolving frequency of an electron and proton, $\lambda_e = h/\gamma_e m_e v_e = h/p_e$-de Broglie wavelength of electron wave, $\lambda_p = h/\gamma_p m_p v_p = h/p_p$ -de Broglie wavelength of proton wave, f_e -de Broglie frequency of an electron wave, f_p -de Broglie frequency of a proton wave, $u_e = \lambda_e f_e$ -phase velocity of the de Broglie electron wave, and $u_p = \lambda_p f_p$ -phase velocity of a de Broglie proton wave. In addition, the following basic relations also belong

to the above-described Bohr's hydrogen atom model: $\gamma_e m_e r_e^{\,2} = \gamma_p m_p r_p^{\,2}$, $\gamma_e m_e r_e^{\,2}\omega_m = \gamma_p m_p r_p^{\,2}\omega_m$, $\gamma_e m_e v_e r_e = \gamma_p m_p v_p r_p$, $n\lambda_e = 2\pi r_e$, $n\lambda_p = 2\pi r_p$ (see **[4]** regarding the same situation).

In a few steps, the following correct relations (between all the parameters described above, see (4.2) and (4.3) from Chapter 4.1), valid for Bohr's atom model, can easily be found (see also **[4]**):

$$\frac{m_p}{m_e} = \frac{\gamma_e}{\gamma_p}(\frac{r_e}{r_p})^2 = \frac{\gamma_e}{\gamma_p}(\frac{v_e}{v_p})^2 = \frac{\gamma_e}{\gamma_p}(\frac{\lambda_e}{\lambda_p})^2 = \frac{\gamma_e}{\gamma_p}(\frac{u_e}{u_p})^2 \cdot (\frac{f_p}{f_e})^2 = \frac{\gamma_e}{\gamma_p}(\frac{u_e}{u_p})^2 \cdot (\frac{\tilde{E}_p}{\tilde{E}_e})^2 =$$

$$= \frac{\gamma_e v_e \lambda_e}{\gamma_p v_p \lambda_p} = \frac{\gamma_e v_e r_e}{\gamma_p v_p r_p} = \frac{m_p c^2}{m_e c^2} = 1836.13,\ f_m = f_{me} = f_{mp} = f_{m(e,p)} = \omega_m / 2\pi,$$

or for $(v_e, v_p, u_e, u_p << c) \Rightarrow$

$$\frac{m_p}{m_e} = 1836.13 \cong \left[(\frac{r_e}{r_p})^2 = (\frac{v_e}{v_p})^2 = (\frac{\lambda_e}{\lambda_p})^2 = (\frac{u_e}{u_p})^2 \cdot (\frac{f_p}{f_e})^2 = \frac{v_e \lambda_e}{v_p \lambda_p} = \frac{v_e r_e}{v_p r_p} \right],$$

$$f_e = n\frac{f_m}{2}(1 + \frac{u_e^{\,2}}{c^2}) = n\frac{m_e Z^2 e^4}{8n^3 h^3 \varepsilon_0^{\,2}}(1 + \frac{u_e^{\,2}}{c^2}) \cong (f_p\text{, for } u_e << c\text{, or } u_e = c),$$

$$f_p = n\frac{f_m}{2}(1 + \frac{u_p^{\,2}}{c^2}) = n\frac{m_e Z^2 e^4}{8n^3 h^3 \varepsilon_0^{\,2}}(1 + \frac{u_p^{\,2}}{c^2}) \cong (f_e\text{, for } u_p << c\text{, or } u_p = c),$$

$$1 \le \frac{v_e}{u_e} = n \cdot \frac{f_m}{f_e} = \frac{2}{(1 + \frac{u^2{}_e}{c^2})} = 1 + \sqrt{1 - \frac{v^2{}_e}{c^2}} \le 2,$$

$$1 \le \frac{v_p}{u_p} = n \cdot \frac{f_m}{f_p} = \frac{2}{(1 + \frac{u^2{}_p}{c^2})} = 1 + \sqrt{1 - \frac{v^2{}_p}{c^2}} \le 2,\ n = 1, 2, 3, ... \tag{4.4}$$

$$\frac{f_p}{f_e} = \frac{1 + \frac{u^2{}_p}{c^2}}{1 + \frac{u^2{}_e}{c^2}} = \frac{1 + \sqrt{1 - \frac{v^2{}_e}{c^2}}}{1 + \sqrt{1 - \frac{v^2{}_p}{c^2}}} = \frac{1 + \frac{1}{\gamma_e}}{1 + \frac{1}{\gamma_p}} = \frac{v_e}{v_p}\frac{u_p}{u_e}.$$

It is important to underline that the revolving mechanical frequency of an electron and proton around their common center of mass, $\omega_{me} = \omega_{mp} = \omega_m = 2\pi f_m$, is something that should not be mixed or directly (quantitatively) associated to de Broglie matter-wave frequency of the (stationary or orbital) electron and/or proton wave/s, i.e., $\omega_m = 2\pi f_m \neq (\omega_e = 2\pi f_e \neq \omega_p = 2\pi f_p)$, $f_{e,p} \le n \cdot f_m = f_{e,p} \cdot (1 + \sqrt{1 - v^2{}_{e,p} / c^2}) \le 2f_{e,p}$. *From (4.4) we can also conclude that a wave energy of the stationary electron wave (* $hf_e = (\gamma_e - 1)m_e c^2 = \gamma_e m_e v_e u_e = p_e u_e = E_k$*) is fully equal to the electron's motional energy* *(meaning that the rest electron mass or its rest energy have no participation in this energy).* Obviously, a relation like (4.4) should be valid for planets rotating around their suns, except that a mass ratio is a different number (and planets in their solar systems should have their associated macrocosmic de Broglie waves). For instance, our planet Earth rotates around its sun and at the same time is spinning around its own planetary axis, performing similar motion as presented in Fig.4.1 (but numerical values of the de Broglie wavelength and frequency are meaningless for such big objects, if microworld Planck constant is relevant in such cases). For macrosystems like solar systems, there is another Planck-analog and much bigger constant $H >>> h$, with similar meaning as microworld Planck constant h (see more in Chapter 2.).

In any stable atom, there exists a relatively strong electromagnetic coupling and spatial (or spherical) mirror-imaging symmetry between electrons (or electron clouds) and protons in the atomic nucleus. This is governed by electric and magnetic Coulomb-type forces. Consequently, it is evident that the atomic nucleus also exhibits a complex structure of standing waves with inherent motional energy, including its rest energy.

Furthermore, it is well established that all atomic and subatomic entities, such as electrons, protons, and neutrons, possess both orbital and spin moments, along with associated magnetic moments, behaving like permanent magnets. These magnetic moments interact between the nucleus and the surrounding electron cloud, as magnets either attract or repel each other depending on their positions and the polarity of their magnetic fields, again following Coulomb-type forces.

It is therefore logical to conclude that the atomic nucleus and its surrounding electrons engage in deterministic electromagnetic communication, manifesting synchronized relationships characterized by mutual symmetry, imaging, and action-reaction effects, akin to a spherical mirror. These interactions involve relevant matter-wave groups or standing wave formations. Even if we argue that the Lucas-Kanarev atomic model is an improvement over Bohr's model, electromagnetic communication and structural symmetry (or mutual structural analogy) should exist in both.

Any phenomenon involving electron clouds, such as the absorption and emission of photons and electrons, should have a corresponding, balanced, and synchronous effect on the nucleus and its energy structure, due to the strong coupling of their electromagnetic fields. If we mechanically, electrically, or electromagnetically excite the electron cloud of an atom, its nucleus should experience a corresponding reaction, as all known conservation laws must be fully satisfied, and action-reaction forces and mutual inductions operate synchronously.

Whether we can sufficiently excite and destabilize nuclear states solely by stimulating the electron clouds remains an open question, though it might be possible by creating and inducing specific resonant states in the electrons. On a similar electromagnetic basis, all atoms and molecules within a macroscopic particle are mutually coupled (as described by van der Waals forces), meaning that if the macroscopic particle is externally agitated or excited, all associated nuclear particles will "feel" this excitation electromechanically and electromagnetically.

The differences between vibrational states of electrons and those of macromolecular structures lie in their characteristic spectrum, resonant, or modal frequencies. For instance, macromolecular vibrational states occupy a lower frequency band than electron states, while nuclear states cover a higher frequency band. Despite these differences, all states are continuously and electromagnetically coupled, communicating bidirectionally during excitations.

Analogously, this concept applies to planetary systems within a solar system. In fact, the bidirectional electromagnetic communication within and between stationary electron and nucleus states should extend both inward and outward, encompassing all atoms, masses, and cosmic formations. This creates closed circuit flows of electromagnetic and mechanical currents and streams. This is the concept we will follow as we remodel and extend Bohr's atomic model.

Let us now observe possible communication nucleus-electrons in the laboratory system, and in the center-of-mass system. Since we are interested only in aspects of photon emissions and absorptions, we know that in such cases there is not an involvement of the rest masses (similarly as in (8.36) and (8.37)). In this case, the law of conservation of the total energy is identical to the law of conservation of the motional energy, and we can apply it in the following way (as in (4.0.44) and (4.0.47)):

$$\varepsilon_{kj} + \varepsilon_{ke} = \varepsilon_{kc} + \varepsilon_{je} \Leftrightarrow E_{kj} + \tilde{E}_j + E_{ke} + \tilde{E}_e = E_{kc} + \tilde{E}_c + E_{je} + \tilde{E}_{je}, \quad (8.51)$$

This part, starting here, should be reformulated, and generalized.

In the following expressions, we will use the following symbols and values:

m_e, M_p -masses of an electron and a proton,

v_e, v_j, v_c -velocities of the electron, the nucleus, and the mass-center of the electrons-nucleus system,

$v_c = \frac{m_e v_e + M_p v_j}{m_e + M_p} \approx (\frac{m_e}{M_p}) v_e + v_j$ -the center of mass velocity.

Now, the corresponding energies (given by their non-relativistic representatives),

$\varepsilon_{kj} = E_{kj} + \tilde{E}_j$ - the total energy of the motion of the nucleus in the laboratory system,

$\varepsilon_{ke} = E_{ke} + \tilde{E}_e \approx \frac{1}{2} m_e v_e^2$ - the energy of the motion of the electron in the laboratory system,

$$\varepsilon_{kc} = E_{kc} + \tilde{E}_c \approx \frac{1}{2}(M_p + m_e)v_c^2 \approx \frac{1}{2}M_p v_c^2 \approx$$
$$\approx (\frac{m_e}{M_p})\frac{1}{2}m_e v_e^2 + \frac{1}{2}M_p v_j^2 + m_e \mathbf{v_e v_j} \approx (\frac{m_e}{M_p})\varepsilon_{ke} + \frac{1}{2}M_p v_j^2 + m_e \mathbf{v_e v_j}$$
- the energy of motion of the mass-center of the electron-nucleus system,

$\varepsilon_{je} = E_{je} + \tilde{E}_{je} = \varepsilon_r \approx \frac{m_e M_p}{2(m_e + M_p)}\left|v_e - v_j\right|^2 \approx \frac{1}{2}m_e v_e^2 \approx \varepsilon_{ke}$ -relative motional energy of an electron towards the nucleus, or electron-nucleus reaction energy (see (4.0.62)).

In non-relativistic case (when all figuring velocities of the nucleus and the electron are negligible regarding the light speed), the balance of energy (8.51) becomes,

$$\varepsilon_{kj} + \frac{1}{2}m_e v_e^2 \approx \frac{1}{2}(M_p + m_e)v_c^2 + \frac{m_e M_p}{2(m_e + M_p)}\left|\mathbf{v_e} - \mathbf{v_j}\right|^2 . \qquad (8.52)$$

In the case of the hydrogen atom, it is logical to assume that $v_j << v_e$, $m_e << M_p$, so (8.51) or (8.52) becomes:

$$\varepsilon_{kj} = E_{kj} + \tilde{E}_j \approx \frac{1}{2}M_p v_j^2 + (\frac{m_e}{M_p})\frac{1}{2}m_e v_e^2 + m_e \mathbf{v_e v_j} \approx$$
$$\approx \frac{1}{2}M_p v_j^2 + (\frac{m_e}{M_p})\varepsilon_{ke} + m_e \mathbf{v_e v_j}. \qquad (8.53)$$

In previous expressions, one should distinguish the total motional energy ε_k from the energy of particle motion E, in the same way, as differentiated in (4.12). If we start from the fact that photoexcitation of the atom has no interference with the rest mass of the nucleus, i.e., that it is relevant only for the electron stationary states in the shell, which is correct in essence, then one may transform (8.53) in the following way:

$$\varepsilon_{kj} = E_{kj} + \tilde{E}_j \approx \frac{1}{2}M_p v_j^2 + (\frac{m_e}{M_p})\varepsilon_{ke} + m_e \mathbf{v_e v_j} \approx$$
$$\approx E_{kj} + (\frac{m_e}{M_p})\varepsilon_{ke} + m_e \mathbf{v_e v_j}, \qquad (8.54)$$
$$\Rightarrow \tilde{E}_j = (\frac{m_e}{M_p})\varepsilon_{ke} + m_e \mathbf{v_e v_j} \approx (\frac{m_e}{M_p})\varepsilon_{ke} .$$

From (8.54) we can determine the total motional energy of an electron,

$$\varepsilon_{ke} \approx (\frac{M_p}{m_e})(\varepsilon_{kj} - E_{kj} - m_e \mathbf{v_e v_j}) . \qquad (8.55)$$

In the first approximation, starting from the fact that the atom nucleus observed as a particle, is effectively at rest. An imaginable velocity of the nucleus motion is almost equal to zero, and undoubtedly far smaller than the velocity of the electron (thereby, the particle energy of the motion of the nucleus, E_{kj}, will be equal to zero) the previous relation, (8.55), will become,

$$\varepsilon_{ke} \approx (\frac{M_p}{m_e})\varepsilon_{kj} \quad , \ (E_{kj} + m_e \mathbf{v_e v_j} \approx 0)$$

$$\varepsilon_{ke} \approx (\frac{M_p}{m_e})(\varepsilon_{kj} - E_{kj} - m_e \mathbf{v_e v_j}) . \tag{8.56}$$

We may now ask ourselves what happens if an atom is hit (excited) by a photon or emits a photon, i.e., we can apply the second Bohr postulate (8.31) on (8.56), so one gets,

$$\begin{aligned}
&\tilde{E}_f = hf_{e12} = h(f_{s1} - f_{s2}) = \varepsilon_{s2} - \varepsilon_{s1} = \Delta\varepsilon_s \ = \Delta\varepsilon_k = \\
&= \Delta\varepsilon_{ke} \approx (\frac{M_p}{m_e})\Delta\varepsilon_{kj} = 1836.13\Delta\varepsilon_{kj} \approx 1836.13 \cdot \frac{\gamma_p}{\gamma_e} \cdot \frac{(u_p\tilde{E}_e)^2}{(u_e\tilde{E}_p)^2}\Delta\varepsilon_{ke} \approx 1836.13 \cdot \frac{(u_p\tilde{E}_e)^2}{(u_e\tilde{E}_p)^2} \cdot \Delta\varepsilon_{ke} , \\
&(\frac{M_p}{m_e} \approx \frac{\Delta\varepsilon_{ke}}{\Delta\varepsilon_{kj}}) = 1836.13 = \frac{\gamma_e \cdot (u_e\tilde{E}_p)^2}{\gamma_p \cdot (u_p\tilde{E}_e)^2} \approx \frac{(u_e\tilde{E}_p)^2}{(u_p\tilde{E}_e)^2} , \\
&\Rightarrow m_e\Delta\varepsilon_{ke} \approx M_p\Delta\varepsilon_{kj} , \ \frac{\tilde{E}_e{}^2\Delta\varepsilon_{ke}}{\gamma_e u_e{}^2} = \frac{\tilde{E}_p{}^2\Delta\varepsilon_{kj}}{\gamma_p u_p{}^2} , \ 1836.13 \cdot \gamma_p \cdot (u_p\tilde{E}_e)^2 = \gamma_e (u_e\tilde{E}_p)^2 \ .
\end{aligned} \tag{8.57}$$

Starting from the fact that every change of motional energy is accompanied by the creation of the corresponding part of wave energy (neglecting the change of internal energy or rest mass of particles, which is at this time quite justified, we may formally transform (8.57) into

$$\begin{aligned}
&m_e\Delta\varepsilon_{ke} \approx M_p\Delta\varepsilon_{kj} \Leftrightarrow m_e hf_{e12} \approx M_p hf_{j12} \\
&\Leftrightarrow m_e f_{e12} \approx M_p f_{j12} .
\end{aligned} \tag{8.58}$$

Based on (8.57) and (8.58) a conclusion obtruded is that every inter-orbital change of stationary energy of an electron is accompanied by some similar response in a form of a coincident change of (some symmetrical) stationary energy levels of the nucleus (regardless of what it meant), especially because of strong Coulomb and magnetic moments forces acting between them. All those changes of the stationary levels are accompanied by emission or absorption of photons (in the electron shell, as well as around the atom nucleus). In fact, (8.58) represents the reaction or echo of the atom nucleus to the change that takes place in the atom electrons shell. This is logical and quite reasonable as well as from an angle of the law of conservation of impulses.

Of course, if there is some wave reaction of the atom nucleus to the change that hits its electron shell, this will be vectors (impulses) with the same direction, and with opposite senses.

Since the masses of the electron and the proton are known, it is possible to express (8.58) in a form of numerical relations,

$$f_{j12} \approx 5.446 \cdot 10^{-4} f_{e12} \ , \ f_{e12} \approx 1836.1 \cdot f_{j12} \ . \qquad (8.59)$$

From (8.58) and (8.59) we see that it is likely that there are radiating and absorptive spectra of the atom nucleus that are synchronous and coincident with absorptive and radiating spectra of the electron shell with known relations of proportionality between corresponding frequencies. Imaginatively, analogically, and creatively thinking, we could (one day) create LASERs based on stimulation and excitation of nuclear atom states, and since all atoms and masses should mutually communicate like coupled resonators, we could analogically extend LASER applications to gravitation.

There is another question; -how to detect previously predicted (still hypothetical) nucleus spectrum, and whether some interferences between the nucleus-spectrum and electron spectrum occur or not. **It is immediately noticeable that hypothetical nucleus spectrum (or spectral ECHO of the nucleus (8.59)) will be in the infrared and microwave part of the spectrum (partly overtaking the area of millimeter wavelengths *<u>and cosmic, microwave background or relict radiation</u>*). Another possible consequence is that Planck's law of blackbody radiation would be supplemented by a similar law of radiation, which is translated into a domain of lower frequencies for a factor $5.446 \cdot 10^{-4}$ (or $f_{j12} \approx \boxed{5.446 \cdot 10^{-4}} \cdot f_{e12}$), with a much smaller intensity and greater frequency density of corresponding wave components.** Perhaps, on this occasion, we could also pose a question about the existence of communication between stationary energy levels of an atom nucleus and its electrons' shell, manifesting as some quantized spectra of photons, which covers frequency interval of the background cosmic radiation.

We can now present (8.59) in a complete form that defines the combinational spectrum of the electromagnetic emission or absorption of the atom, using (8.9) or (8.41),

$$f_{e12} = \frac{m_e Z^2 e^4}{8h^3 \varepsilon_0{}^2} \left(\frac{1}{n_1{}^2} - \frac{1}{n_2{}^2} \right),$$

$$f_{j12} \approx \left(\frac{m_e}{M_p} \right) \frac{m_e Z^2 e^4}{8h^3 \varepsilon_0{}^2} \left(\frac{1}{n_1{}^2} - \frac{1}{n_2{}^2} \right). \qquad (8.60)$$

Knowing the relations between masses of an electron and a proton, as well as between frequency and the wavelength of the electromagnetic quantum (based on calculations from (8.60)), it is possible to obtain the following estimates for frequency and wavelengths of hydrogen spectrum,

$$\frac{f_{e12}}{f_{j12}} = \frac{\lambda_{j12}}{\lambda_{e12}} \approx (\frac{M_p}{m_e}) \approx 1836.1 \,,$$
$$f_{e12} \leq 3.2898417 \cdot 10^{15} Hz \,,\, \lambda_{e12} \geq 0.091126714 \mu m \,, \quad (8.61)$$
$$f_{j12} \leq 1.7917551 \cdot 10^{12} Hz \,,\, \lambda_{j12} \geq 167.31776 \mu m \,.$$

Since the hydrogen atom is one of the most prevalent elements in the cosmic universe, the previously given quantitative relations (8.61) should be almost universally valid. If we would like to include all possible atoms heavier than the hydrogen atom, we could broaden the meaning of the relation (8.58), considering that the nucleus mass of any atom is equal to the sum of masses of the protons and the neutrons that constitute the nucleus, so we shall have:

$$Z \cdot m_e f_{e12} \approx \left[ZM_p + (A - Z)M_n \right] f_{j12} \approx AM_p f_{j12} \approx AM_n f_{j12}. \quad (8.62)$$

A (=) Mass number (=) atomic mass number (=) nucleon number (=) total number of nucleons, meaning protons and neutrons together.
Z (=) Atomic number (=) A – N (=) Number of protons (=) Number of electrons
N (=) number of neutrons (=) A – Z.
$Z \leq N \leq 1.58Z < 2Z, 1 \leq N / Z \leq 1.58 < 2$
$A = Z + N \leq 2N \leq 1.58Z + N \leq 3.58Z < 2Z + N < 4Z$

If we assume that all possible atoms and their isotopes (known and still unknown) will be approximated within an atom number $A < \mathbf{4}Z$, $Z_{max.} \leq 137$, $A < \mathbf{4}Z < \mathbf{4 x 137 = 548}$ and (8.58), (8.62) and (8.44) will generate the following relations,

$$\frac{f_{e12}}{f_{j12}} = \frac{\lambda_{j12}}{\lambda_{e12}} \leq 4(\frac{M_p}{m_e}) \approx 4 \cdot 1836.1 = 7344.4 \,,$$
$$f_{e12} \leq Z \cdot 3.2898417 \cdot 10^{15} Hz \cong 4.507083129 \cdot 10^{17} Hz \,,\, \lambda_{e12} \geq \frac{1}{Z} \cdot 0.091126714 \mu m \cong 0.665158 nm, \quad (8.63)$$
$$f_{j12} \leq \frac{Z}{4} \cdot 1.7917551 \cdot 10^{12} Hz \cong 6.136761831 \cdot 10^{13} Hz, \lambda_{j12} \geq \frac{4}{Z} \cdot 167.31776 \mu m \cong 4.885190024 \mu m$$

One might ask: Is the cosmic background radiation truly a residual echo of the hypothesized primordial explosion—the Big Bang—or could it be related to a continuously present, intrinsic “blackbody-like radiation” originating from atomic nuclei or the very structure of matter itself (as roughly estimated in sections (8.60) – (8.63))? If we attempt to apply the general relations of “elementary certainty” from Chapter 4.0, equation (4.0.64), and the “Relations of Uncertainty” from Chapter 5, equation (5.1), to the spectral and temporal characteristics of the cosmic background radiation, as determined by current measurements and theoretical predictions, we find that these relations are not convincingly or mathematically satisfied.

In other words, the standard model of the Big Bang has considerable quantitative inconsistencies, despite being widely regarded as both conceptually sound and well-supported by empirical data. This leads us to question whether the Big Bang, as currently hypothesized, ever truly occurred in the manner described.

To summarize, we might speculatively consider whether we have sufficient grounds to doubt the traditional Big Bang narrative. Perhaps the cosmic background radiation and Planck's blackbody radiation law describe an ever-present, natural phenomenon that is fundamentally linked to the radiative and spectral properties of atoms and matter, rather than serving as the remnant of a single cataclysmic event.

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*Familiar analyzes, concepts, and conclusions (at least qualitatively) can be found in the works of Dr. George Shpenkov, addressing cosmic background radiation, and electromagnetic exchanges between atom nucleus and electrons' shell (see literature references under **[85],** organized and summarized by Victor Christianto. Similar and much wider problematic (also related to matter-wave equations and atom models) is discussed and elaborated by Victor Christianto in his article **[86]:** "Review of Schrödinger Equation & Classical Wave Equation". V. Christianto is an Independent Researcher; -URL: http://www.sciprint.org, Email: victorchristianto@gmail.com or admin@sciprint.org . Phone: (62) 341-403205 or (62) 878-59937095.).*

Citation from [86]: *"Kreidik & Shpenkov derive microwave background radiation of hydrogen atom based on Shpenkov's interpretation of classical wave equation. They conclude that the Microwave Background Radiation, observed in Cosmos, apparently is the zero-level (background) radiation of all atoms in the Universe. Following their dynamic model, the H-atom is a paired dynamic system with the central spherical micro-object of a complicated structure (proton) and the orbiting electron. The electron in H-atom under the wave motion exchanges energy with the proton constantly at the fundamental frequency* ω_e. *This exchange process between the electron and proton has dynamic equilibrium character. It is represented by a system of radial standing waves, which define "zero level exchange" in a dynamically stable state of the atom. At p = 0, they obtain* $\lambda = 0.106267\,\text{cm}$, *and then they can find an estimate of the absolute temperature of zero level of radiation:*

$$T = \frac{0.290\,\text{cm}\cdot\text{K}}{\lambda} = 2.7289\,\text{K} \cong \Delta\text{K} \qquad (22)$$

Where $\Delta = 2\pi \lg e = 2.7288$ *is the measure of the fundamental period (fundamental quantum measures). The temperature obtained coincides with the temperature of "relict" background measured by NASA's Cosmic Microwave Background Explorer (COBE) satellite to four significant digits (2.725 ±0.002K). The concept of zero level radiation of H-atoms question quantum mechanical probabilistic model, which excludes an electron's orbital motion along a trajectory as a matter of principle. "*

Bohr's hydrogen atom model is relatively simple and has been extensively tested and validated, particularly in the context of spectral characterizations. By integrating Bohr's planetary model with the concept of de Broglie matter waves (refer to Fig. 4.1 and equations (4.1), (4.2), and (4.3) in Chapter 4.1), we indirectly test and support the hypothesis presented in this book. This hypothesis posits that every linear motion is accompanied by a certain matter-field spinning, which naturally generates de Broglie matter waves, producing results consistent with equations under (4.4).

It is understood that electrons and positrons, like photons, are specific, self-standing, stabilized formations, analogous to structured, standing electromagnetic waves. These particles can be fully transformed into photons, as discussed in detail in Chapter 4.1. Moreover, based on the Compton and Photoelectric effects, we recognize that electrons, positrons, and photons exhibit both particle-like and wave-like behaviors, adhering to PWDC rules, as summarized in Chapters 4.1 and 10.

In this discussion, we explore various aspects of electromagnetic field phenomenology, focusing on the motion and formation of these fields. An atom, therefore, can be viewed as a structure composed of mutually coupled electromagnetic resonant states or resonators. One part of these resonant states is associated with the atomic nucleus, while the other, a mirror set of coupled energy states, is associated with the electron cloud. Additionally, there is a third component, the external space surrounding the atom, where gravitation arises from the exchange of radiant electromagnetic energy between the resonant states of atoms and other masses.

These resonant states can absorb or emit photons at very precise, matching frequencies, leading to discrete (quantized) internal and external energy exchanges. In these exchanges, photons, electrons, nuclear states, and their corresponding matter-waves are involved and can be transformed into one another. The essence of quantization in the microworld of atoms and subatomic structures is that every entity in this realm represents a set of specific, well-defined, and stable resonant states, which are spatially and temporally dependent. These resonant states can be likened to standing waves that generate oscillatory circuits.

These oscillatory configurations are internally and externally coupled and communicate through the emission, absorption, and exchange of specific electromechanical and electromagnetic matter waves, including photons and electrically charged entities. In our measurements and analyses, we observe these matter waves as spinning particles, such as electrons and photons. Stable resonant states can also be represented as self-contained standing wave formations. The quantum nature of this reality is a result of the coupling and interactions between excited resonant circuits and the matter waves they create, operating at specific resonant frequencies. This perspective on quantization differs slightly from the traditional interpretation in Quantum Theory. Nonetheless, energy exchanges between these self-standing resonators are finite and fixed, often quantized in specific amounts, whether by integers or other measures.

8.3. Structure of the Field of Subatomic and Gravitation related Forces

The objective here is to advance the concept of Niels Bohr’s atom model, or any similar atomic model, by incorporating elements of field theory that extends to macroscopic domain of gravitational forces. This involves introducing challenging mathematical and intuitive proposals that can later be refined and expanded. Atoms, the fundamental building blocks of macroscopic matter, possess non-zero rest masses and are composed of resonators or oscillating formations with self-stabilizing structures of standing matter waves.

Atoms continuously interact both internally and externally by exchanging photons. Among the forces acting between atoms and other masses, gravitation is present alongside electromagnetic forces. It is highly likely that gravitation has its deep ontological roots in electromagnetic fields and forces, which extend from internal atomic field forces.

All resonant and oscillating, mutually coupled structures with similar or overlapping spectral characteristics, and with intrinsic periodicity, are in constant communication. This communication occurs through acoustic, electromechanical, and electromagnetic coupling, leading to the creation of specific forces and fields both within and around these structures. This coupling activity synchronizes the atomic structures, fields, and forces, extending them toward larger agglomerated-masses and cosmic formations. These larger masses can be considered "macro-atomic structures," which are essentially unified, synchronized, and superimposed versions of individual atomic structures.

This synchronization is the fundamental connection between internal atomic field forces and gravitation, aligning with Rudjer Boskovic's concept of a universal natural force and Nikola Tesla’s Dynamic Gravity theory.

Starting from results as found in this chapter under (8.1) – (8.50), and in second Chapter, under (2.11.14), we can imaginatively and mathematically assemble and extrapolate what could be the structure of underlaying and universal, atomic-field force (acting internally inside atoms, and externally outside atoms). Let us assume that all occurrences connected with stationary energy states of electrons and an atom nucleus take place in space where a specific and complex structure of mentioned material force (for instance, described qualitatively as Rudjer Boskovic Universal Natural Force) exists. We will denote, in spherical coordinates, the function of such complex (atomic) force as $\mathbf{F(r,\theta,\phi,t)}$, where: $\mathbf{r}$ is the radius, $\mathbf{\theta,\phi}$ are angular coordinates, and $\mathbf{t}$ is a time coordinate.

We assume that the function of the force, F(X), of an atomic field is continuous and energy-limited (or finite):

$$\text{Lim } F(X)_{x \to A} = F(A), \quad \text{Lim } F(r)_{r \to R \leq \infty} = 0 \tag{8.64}$$

Let $R(r,\theta,\phi,t)$ be the radius vector of electrons' orbits of inter-atomic states in an atomic field (inside atoms). The element of an electron matter-wave orbit described by the radius vector $0 \leq \mathbf{r} \leq \mathbf{R(r,\theta,\varphi,t)} < \infty$ may be presented as,

$$\mathbf{dR^2 = dr^2 + r^2 \cdot d\theta + r^2 \cdot \sin^2\theta \cdot d\phi, \quad 0 \leq \theta \leq 2\pi, \quad -\frac{\pi}{2} \leq \phi \leq \frac{\pi}{2}.} \tag{8.65}$$

It is important to remember that any modern, still-applicable atomic model, starting with Bohr's model and extending back to concepts derived from Rutherford's scattering experiments, includes a central zone where the positively charged atomic nucleus resides. This nucleus, composed of protons and neutrons, can be understood as a structure of matter waves. Surrounding the nucleus is the "electron cloud zone," or an external envelope characterized by specifically structured, negatively charged electron field states.

Both the nucleus and the electron matter-wave states possess electric, magnetic, orbital, and spin moments, as well as electromagnetic dipole attributes. This indicates that atoms are dynamic structures, consisting of internally coupled resonant and standing-wave formations, which exhibit near-perpetual motion.

The electromagnetic nature of the electron and nucleus states ensures that they are mutually coupled and interacting. Consequently, any changes or behaviors within the electron cloud are expected to produce a corresponding mirror-image response or mapping within the atomic nucleus, reflecting similar structural and periodic properties. This implies that we should be able to describe the resonant states and structure of the atomic nucleus in a manner analogous to how we describe the structure of the electron cloud.

Let us start by describing the internal atom-field structure and properties. Now we can give a simplified definition of a stationary electron orbit $\mathbf{C_n}$. This is an orbit whose perimeter a_n is equal to the mean value of the wavelength $\bar{\lambda}_n$, of a stationary, or standing electron wave that covers orbit C_n, multiplied by the main quantum number n. This results from the first Bohr's postulate, as well as from the meaning of standing waves, and relates to de Broglie's wavelength; -see (8.10) - (8.11), and familiar conceptualization in Chapter 10.),

$$\begin{aligned} & a_n = \oint_{C_n} dR = \oint_{C_n} dr = n\bar{\lambda}_n = n\frac{h}{\tilde{p}_n}, \\ & \bar{\lambda}_n = \frac{1}{n}\oint_{C_n} dR = \frac{1}{n}\oint_{C_n} dr = \frac{a_n}{n} = \frac{1}{n}\sum_{i=1}^{n}\lambda_{ni} = \frac{h}{\tilde{p}_n}. \end{aligned} \tag{8.66}$$

If we imagine that states of an atom-nucleus should be a kind of spherical or spatial mirror image of electron states, because of Coulomb forces acting between them, a similar standing-waves situation within a nucleus (as described with (8.66)), should also exist there.

A much better replacement for (8.66), for any stable periodical motion on a closed and stationary orbit, was introduced by Wilson-Sommerfeld action integral (as the more general quantifying rule, see [9]), applied over one period of the motion. Here, the same Wilson-Sommerfeld action integrals will be formulated in a bit modified form (to be consistent with the concept of Particle-Wave Duality presented in this paper) as,

$$\begin{aligned} & \oint_{C_n} p_q dq = \left[\oint_{C_n} d(p_q q) - \oint_{C_n} q dp_q\right] = n_q h = \frac{\tilde{E}'_{nq}}{f'_q} = \frac{hf'_{nq}}{f'_q}, \tilde{E}'_{nq} = hf'_{nq} = hn_q f'_q = \left[\oint_{C_n} p_q dq\right]\cdot f'_q, \\ & \left[\oint_{C_n} d(p_q q) = 0, \oint_{C_n} q dp_q = -n_q h,\ f'_{nq} = n_q f'_q\right], \\ & \oint_{C_n} L_q dq = \left[\oint_{C_n} d(L_q q) - \oint_{C_n} q\, dL_q\right] = n_q h = \frac{\tilde{E}''_{nq}}{f''_q} = \frac{hf''_{nq}}{f''_q}, \tilde{E}''_{nq} = hf''_{nq} = hn_q f''_q = \left[\oint_{C_n} L_q dq\right]\cdot f''_q \\ & \left[\oint_{C_n} d(L_q q) = 0, \oint_{C_n} q\, dL_q = -n_q h, f_{nq} = n_q f''_q\right], \\ & q = (r,\ \theta,\ \varphi,...),\ r = r(x,y,z...),\ n_q = \text{integer } (=1,2,3,...). \end{aligned}$$

In the Wilson-Sommerfeld action integral we generalized different momentum states (concerning linear motion and rotation), by using electromechanical analogies (as established in the first chapter of this book), we can (at this time still analogically and hypothetically) extend the Wilson-Sommerfeld' action integral to electric and magnetic charges, as follows,

$$\left.\begin{array}{l}\oint_{C_n}\Phi_{electr.}d\Phi_{magn.}=\left[\oint_{C_n}d(\Phi_{electr.}\Phi_{magn.})-\oint_{C_n}\Phi_{magn.}d\Phi_{electr.}\right]=n_{electr.}h=\frac{\tilde{E}_{n-electr.}}{f_{electr.}}=\frac{hf_{n-electr.}}{f_{electr.}},\\ \oint_{C_n}\Phi_{magn.}d\Phi_{electr.}=\left[\oint_{C_n}d(\Phi_{electr.}\Phi_{magn.})-\oint_{C_n}\Phi_{electr.}d\Phi_{magn.}\right]=n_{magn.}h=\frac{\tilde{E}_{n-magn.}}{f_{magn.}}=\frac{hf_{n-magn.}}{f_{magn.}},\end{array}\right\rangle\Rightarrow$$

$$\oint_{C_n}d(\Phi_{electr.}\Phi_{magn.})=0,\oint_{C_n}\Phi_{magn.}d\Phi_{electr.}=-n_{electr.}h,\oint_{C_n}\Phi_{electr.}d\Phi_{magn.}=-n_{magn.}h$$

$$n_{electr.}=-n_{magn.},f_{n-electr.}=n_{electr.}f_{electr.},\ f_{n-magn.}=n_{magn.}f_{magn.},$$

$$\tilde{E}_{n-electr.}=hf_{n-electr.},\tilde{E}_{n-magn.}=hf_{n-magn.},(n_{electr.},n_{magn.})=\text{integer }(=\ 1,2,3,...)\ .$$

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This paper attempts to propose that Planck's quantization law $\tilde{E}=nhf$ has been incorrectly interpreted, even by Planck himself, and that a more accurate interpretation is possible as $\tilde{E}=hf(n)$. Specifically, it is emphasized here that the mechanical revolving frequency of a particle should not be conflated with the associated matter-wave frequency, particularly the frequency of a de Broglie matter wave (see equations (8.28) and (8.42)). This distinction is crucial for the correct treatment of phase and group velocity.

The same caution should apply to Planck's postulate regarding the energy of simple harmonic oscillators in the context of blackbody radiation. The argument here is that the concept of phase velocity in modern Quantum Mechanics, particularly concerning de Broglie matter waves, is not completely, adequately or consistently explained.

Fortunately, Planck and Einstein derived a mathematically accurate function for the energy density of a blackbody radiator through certain assumptions and curve-fitting, but this success led to the widespread acceptance of an essentially flawed interpretation of the quantization law. In this work, a simpler and more accurate understanding of the quantum nature of matter is presented, based on the idea that proper resonance and synchronization between internal and external field properties result in self-contained standing matter waves. These waves can be characterized, organized, or indexed by integers due to the mutually synchronized spatial-temporal periodicities of stable matter states.

The natural laws of quantization in physics should be rooted in the Kotelnikov-Shannon-Nyquist-Whitaker signal sampling and reconstruction methods (refer to literature sources [57], [109], [110], and [111] for more details).

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We may now define the energy of the stationary electron wave (i.e., reformulating the second Bohr's postulate) through the work of the atomic field force (4.1.64), as well as using the wave function of the stationary electron wave (see (4.0.1), to (4.0.5) from Chapter 4.0, and equations from (4.9) to (4.10-5) from Chapter 4.3), on the following way,

$$\tilde{\varepsilon}_{sn}=\oint_{C_n}\mathbf{F}d\mathbf{R}=\int_{C_n:[\Delta t]}\Psi_n{}^2(t)dt=\int_{C_n:[\Delta p]}v_n dp=\iint_{\sigma_n}(\nabla\times\mathbf{F})d\sigma=hf_{sn}\ \neq\ 0\ , \qquad (8.67)$$

$$\Rightarrow\nabla\times\mathbf{F}\neq0\ .$$

We shall determine the energy of an electromagnetic quantum of the inter-orbital exchange (by absorption or emission of photons), such as (see (4.0.1), to (4.0.5) from Chapter 4.0, and equations from (4.9) to (4.10-5) from Chapter 4.3),

$$\tilde{\varepsilon}_{m,n} = \oint_{[C_m - C_n]} \mathbf{F} d\mathbf{R} = \int_{[\Delta t]} \Psi_{m-n}{}^2(t)dt = \int_{[\Delta p]} v_{m-n} dp = h(f_n - f_m) = hf_{mn} \neq 0 , \quad \Rightarrow \nabla\mathbf{F} \neq 0 . \tag{8.68}$$

Again, it is good to say that stationary energy levels C_m and C_n could be inside of an atom among its electron and/or nucleus states.

Now, let us go back to the atomic force field $\mathbf{F}(\mathbf{r},\boldsymbol{\theta},\boldsymbol{\phi},\mathbf{t})$. From (8.67) and (8.68) we see that $\nabla \times \mathbf{F} \neq 0$ and $\nabla\mathbf{F} \neq 0$, so according to the general classification of fields in differential geometry, we must consider force field $\mathbf{F}(\mathbf{r},\boldsymbol{\theta},\boldsymbol{\phi},\mathbf{t})$ as a complex vector field, which may be always decomposed in two more elementary vector fields that are,

$$\mathbf{F}(r, \theta, \varphi, t) = \mathbf{F_1}(r, \theta, \varphi, t) + \mathbf{F_2}(r, \theta, \varphi, t) , \tag{8.69}$$

from which the first is **a potential field**:

$$\nabla \times F_1 = 0 , \nabla F_1 \neq 0 , \tag{8.70}$$

and the second is a **solenoidal field**:

$$\nabla \times F_2 \neq 0 , \nabla F_2 = 0 , \tag{8.71}$$

therefore, instead of (8.67) and (8.68), we may introduce new definition that replaces them,

$$\tilde{\varepsilon}_{sn} = \oint_{C_n} \mathbf{F}_2 d\mathbf{R} = \int_{C_n:[\Delta t]} \Psi_{2n}{}^2(t)dt = \int_{C_n:[\Delta p]} v_{2n} dp = \iint_{\sigma_n} (\nabla \times \mathbf{F}_2) d\sigma = hf_{sn} \neq 0 , \quad \Rightarrow \nabla \times \mathbf{F}_2 \neq 0 , \tag{8.72}$$

and

$$\tilde{\varepsilon}_{m,n} = \oint_{[C_m - C_n]} \mathbf{F}_1 d\mathbf{R} = \int_{[\Delta t]} S_{m-n}(t)dt = \int_{[\Delta t]} \Psi_{m-n}{}^2(t)dt = \int_{[\Delta p]} v_{m-n} dp = h(f_n - f_m) = hf_{mn} \neq 0 , \quad \Rightarrow \quad \nabla\mathbf{F}_1 \neq 0 . \tag{8.73}$$

Equations (8.72) and (8.73) suggest that atomic forces or fields, as described by equations (8.64) and (8.69), possess both axial (longitudinal) and radial (transversal) components, which penetrate both internally and externally. In this book, we propose that the external forces acting between atoms and other masses (or beyond the atom) are simply extensions or continuations of the internal atomic forces or fields. These extensions contribute to, and support, central forces such as gravitation and Coulomb-type electromagnetic forces. This concept aligns conceptually and intuitively with R. Boskovic's Universal Natural Force and N. Tesla's Dynamic Gravity.

Here, we should explore the possibility that mentioned potential and solenoidal vector fields (F_1 and F_2) are, when presented using Hilbert transform and an Analytic Signal model, mutually coupled and phase-shifted electric and magnetic fields. After applying proper mathematical processing and dimensional arrangements we could try to prove that wave functions of certain mutually-coupled electric and magnetic force or field function behave as $\Psi(t) \simeq F_1(t)$ ***and*** $\hat{\Psi}(t) \simeq F_2(t)$***, meaning creating certain electromagnetic-field Analytic signal (similar to Pointing vector) that is representing the total force function*** $\mathbf{F}(r, \theta, \varphi, t) = \mathbf{F_1}(r, \theta, \varphi, t) + \mathbf{F_2}(r, \theta, \varphi, t)$. *See more about Analytic Signals in Chapter 4.0.*

To treat an atom as a stable structure (both internally and externally), one should add the condition of balance of the potential energy of all attractive and all repulsive forces what secures atom stability, inwards and outwards, or internally and externally, such as,

$$\oiiint_{\left\{\begin{matrix} R\in[0,\infty], \\ \theta\in[0,2\pi], \\ \phi\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]\end{matrix}\right\}} \mathbf{FdR} = \mathbf{0} \, . \tag{8.74}$$

The balance of potential energy between all attractive and repulsive forces (as described in equation 8.74) within and around an atom can be further explained by hypothesizing a continuous exchange of electromagnetic wave packets between the stationary states of the nucleus and the electron shell of the atom. This exchange also occurs externally between atoms and other masses. The energy states involved should communicate synchronously and in both directions within the internal space of the atom. These interactions would be imperceptible in external space if the atom is entirely (electromagnetically) neutral, self-contained, stable, and unconnected to other atoms or external influences.

However, since atoms attract and agglomerate through inverse-square law forces ($1/r^2$) like gravity and Coulomb forces, it is evident that standing matter-wave field structures must also exist in the external space between atoms and separated masses. We can extend this concept by hypothesizing that the bidirectional, quantized electromagnetic exchanges between the atomic nucleus and its electron shell extend almost infinitely into outer space, connecting atoms with other atoms, masses, and cosmic bodies. In essence, all atoms and oscillating masses are synchronized and interconnected with the universe, continuously radiating and receiving electromagnetic waves, both externally and internally, while forming standing matter-wave structures.

This implies that all matters, whether metals, stones, monuments, pyramids, mountains, or living organisms like plants and animals—is interconnected, dynamic, and, at different levels, alive. Atoms and other masses serve as continuous sources and sinks of Tesla's radiative energy, qualitatively structured as Rudjer Boskovic's Universal Natural Force. Given that Newton's inverse-square law of gravitation mirrors the Coulomb force between electric charges, we can simplify this concept by visualizing gravitational attractions analogous to the attraction between two electric charges or permanent magnets. Here, each mass possesses a proportional number of electric dipoles or elementary permanent magnets, aligned to experience $1/r^2$, the inverse-square law of attraction (refer to Chapter 2, sections 2.1.1 and 2.2, for further details on Newtonian gravitation and its analogies).

The universe's response to this radiative energy and omnidirectional communication involves synchronous action and reaction forces. These external reaction forces resemble cohesion and adhesion forces and contribute to gravitational attraction and other electromagnetic interactions. Thus, all atoms, particles, larger objects, and the universe are constantly communicating through the radiation and exchange of electromagnetic energy. This exchange leads to the manifestation of gravitational force, as conceptualized in Nikola Tesla's Dynamic Gravity and Rudjer Boskovic's universal natural force theories. At the core of whole matter in the universe are electromagnetic fields, forces, and photons, each uniquely structured.

The Casimir effect may also be a measurable manifestation of this "radiant energy flow" or the fluctuations in the physical vacuum (see references [103] and [104]). The next chapter of this book, "9. Black Body Radiation & Photons," further explores these ideas.

*The foundational ideas for this interpretation of atomic force fields can be traced back to the works of Rudjer Boskovic on universal natural force, as outlined in his publication *Principles of Natural Philosophy* ([6]), and in various papers published in the *Herald of the Serbian Royal Academy of Science* between 1924 and 1940 (notably J. Goldberg 1924 and V. Žardecki 1940). Nikola Tesla's dynamic theory of gravitation ([97]) is also closely aligned with Boskovic's unified natural force and the concepts summarized in equation (8.74). Additional relevant ideas can be found in Reginald T. Cahill's *Dynamical 3-Space: Emergent Gravity* ([73]) and Jean de Climont's work ([117]).*

In summary, the field of electromagnetic forces $\mathbf{F}(r, \theta, \varphi, t)$ *at the subatomic and interatomic levels extends similarly outside atoms and between separated masses. These external fields are mutually synchronizing, coupling, and exchanging matter waves and photons between atoms and larger masses. This coupling, which is fundamentally electromagnetic in nature (as discussed in references [33] and [71]), results in the Newtonian force of gravitation. While the force field* $\mathbf{F}(r, \theta, \varphi, t)$ *described by equations (8.64) to (8.74) could be more complex and refined with advanced atomic models beyond the simplistic Bohr model, the core ideas about natural forces and the structure of our world remain consistent.*

Atoms inherently exhibit resonant and standing-wave structures, and the effects of mutual synchronization and coupling, including entanglement, are characteristic of such structures. Quantization in physics is intrinsically linked to these atomic and standing matter-wave formations. To support the notion of coupling and synchronization between atoms and masses, an intermediary medium, or a material carrier of matter waves (often referred to as the ether), is necessary.

Furthermore, our conceptualization of electromagnetic fields should evolve to incorporate fully symmetrical and mutually integrated electric and magnetic fields, as suggested in the third chapter of this book. Different induction laws and Lorentz forces (related to effects of inertial motions) also play a role in electro-mechanical couplings between atoms, masses, and mechanical and electromagnetic moments, which we can consider as effects of gravitation.

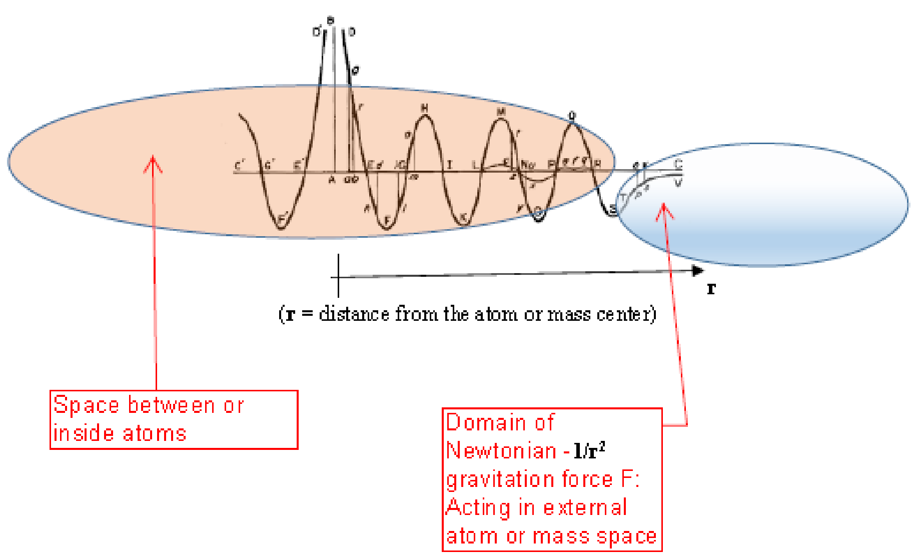
*Understanding gravitation involves extending or extrapolating these fields from within and around atoms to external macrocosmic masses. As discussed in the second chapter on gravitation, macrocosmic systems (such as planetary systems) exhibit analogous behavior and structure to atoms (see section 2.3.3, *Macro-Cosmological Matter-Waves and Gravitation*). This analogy effectively closes the conceptual loop, reinforcing the coherence of the underlying ideas.*

Imagine how much more vibrant and insightful physics could be if the ideas and hypotheses previously introduced had not been overlooked or disregarded in modern science.

In addition to stable and neutral atoms, we also recognize the existence of unstable, naturally radioactive atoms. This implies that the standing-wave structured electromagnetic atomic force or field $\mathbf{F}(\mathbf{r},\theta,\phi,\mathbf{t})$, as defined by equations (8.67) to (8.74), can sometimes be unstable. Consequently, we should be able to apply external, artificially created electromagnetic fields around naturally radioactive materials to manipulate (neutralize, dissipate, attenuate, compensate and eliminate) their inherent instability and radioactivity. By channeling, stabilizing, neutralizing, transmitting, or further destabilizing these atoms, we can influence their behavior and stimulate or eliminate radiation.

…………………………….

Citation from the internet; http://www.irb.hr/50g/eng/3/index.htm: "Rudjer Boskovic was the first to realize that the interatomic forces were something else than a sheer gravitation. In his main work, Theory of Natural Philosophy (Venice, 1763) he developed the concept of the atom as a point-like center of force. To explain cohesion at a microscopic level, he postulated the existence of forces between molecules, whose direction and intensity are distance dependent. According to Boskovic's ideas, forces (between solid particles and atoms) change the sign infinitely many times with decreasing distance (between solid particles and atoms); this requires oscillating potentials, or infinitely many relative minima, which is the essence of the nuclear picture and of quantum stationary states. His ideas exerted a strong influence on the leading physicists and chemists of the 19th century. Indeed, more than a century later such forces were introduced into modern physical chemistry and are known today as London-van der Waals forces. Rutherford took Boskovic's atom as a cornerstone of his, today canonic, picture of the atom".



Rudjer Boskovic's universal force function between solid particles and atoms

The traditional N. Bohr atom model primarily addresses electromagnetic interactions involving photons and electrons. It focuses on energy exchanges between different electron states and between electrons and the external atomic space. This model encompasses phenomena such as Compton and photoelectric effects, secondary emissions, lasers, diffraction, and ionization.

In contrast, the extended atomic model proposed here explores bidirectional electromagnetic or photonic energy exchanges between electron energy states and the corresponding nucleus states, as well as within the nucleus itself. This model suggests that cosmic and high-energy radiation, X-rays, gamma rays, electron-positron creation and annihilation, and spontaneous radioactive emissions are involved in such matter-wave energy exchanges.

The extended model also hypothesizes bidirectional electromagnetic matter-wave exchanges between all internal atomic energy levels, including those within the nucleus, and between atoms and external masses. This interaction supports or creates gravitational effects. Related to these electromagnetic exchanges are phenomena like infrared and microwave radiation, as well as other low-frequency electromagnetic phenomena. Further elaboration on these concepts is presented in Chapter 9 on Black Body Radiation.

We can summarize the following key analogies and intuitive elements supporting the idea that gravitation is a manifestation of the coupling of masses and atoms:

1. Electromagnetic Polarizability: Atoms can be polarized, and both electric and magnetic dipoles exist or can be created within them. Atoms, electrons, protons, and neutrons have magnetic, spinning, and orbital moments. Coulomb forces are similar for electric charges, dipoles, and magnets, and this analogy extends to planetary systems.

2. External Excitation and Communication: Atoms can be excited or ionized through electromagnetic radiation, indicating mutual communication between atoms and other masses.

3. Gravitational and Coulomb Forces: Newton's law of gravitation, which describes the attraction between distant masses, is mathematically analogous to Coulomb's law for electric charges.

4. Atomic Coupling and Synchronization: Compact masses are aggregates of mutually coupled and synchronized atoms. The "atomic fields and forces" involved are superimposed and integrated, reacting as a "united macro-atom" with averaged properties of individual atomic constituents.

*5. Elementary Particles and Matter Waves: Elementary particles like electrons, protons, and neutrons can be seen as simplified representations. These particles are better described as structured matter-waves or standing wave fields. The mapping between elementary particles and matter waves is explicable based on definitions of phase and group velocity, as discussed in Chapters 4.1, 10, and in this chapter. The analogy between Bohr's atomic model and planetary systems suggests that electromagnetic and electromechanical coupling extends from the microscopic to the macroscopic scale, aligning with Tesla's Dynamic Gravity Theory and Boskovic's Universal Natural Force conceptualization ([97]). This innovative concept posits that planetary masses are engaged in continuous electromagnetic and electromechanical energy exchanges, forming standing wave fields between the sun and its planets. Additional insights can be found in Poole's *Cosmic Wireless Power Transfer System and the Equation for Everything* ([144]) and through online resources about Tesla's Dynamic Gravity and Boskovic's Universal Natural Force. For more, see web links:* https://teslaresearch.jimdofree.com/dynamic-theory-of-gravity/ru%C4%91er-josip-bo%C5%A1kovi%C4%87-1711-1787/ *and* https://teslaresearch.jimdofree.com/dynamic-theory-of-gravity/

*6. Spinning Disks and Gyroscopes: Uniformly rotating spinning disks or gyroscopes maintain their spatial orientation and stability, even within a gravitational field. This behavior provides insights into the nature of the "extended atomic field" and its relation to gravitation and electromagnetic couplings. More details on this topic are discussed in Chapter 2, section 2.2.1, *What Gravitation Really Is*.*

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Citation from [144] "**Abstract**: By representing the Earth as a rotating spherical antenna several historic and scientific breakthroughs are achieved. Visualizing the Sun as a transmitter and the planets as receivers, the solar system can be represented as a long wave radio system operating at Tremendously Low Frequency (TLF). Results again confirm that the "near-field" is Tesla's "dynamic gravity", better known to engineers as dynamic braking or to physicists as centripetal acceleration, or simply (g). …

A new law of cosmic efficiency is also proposed that equates vibratory force and pressure with volume acceleration of the solar system. Lorentz force is broken down into centripetal and gravitational waves. …

Spherical antenna patterns for planets are presented and flux transfer frequency is calculated using distance to planets as wavelengths. The galactic grid operates at a Schumann Resonance of 7.83 Hz, …

The Sun and the planets are tuned to transmit and receive electrical power like resonating Tesla coils". …

....

Citation (from Common Sense Science, Charles W. Lucas, Jr. Statements related to irrelevant ideological items are simply omitted):

"**Experimental Evidence to Support Boscovich's Atomic Model**. ... It started with the discovery of solitons ... Solitons are long-lasting semi-permanent standing wave structures with a stable algebraic topology. [9] The soliton can exist in air or water as a toroidal ring. Solitons in water are usually formed in pairs known as a soliton and anti-soliton. Their structure is weak, and they decay after 10 or 20 minutes.

Bostick's Plasmons. Winston Bostick (1916-1991), the last graduate student of Nobel Prize winner Arthur Compton (1892-1962), experimentally discovered how to make "plasmons" or solitons from the electromagnetic field within an electromagnetic plasma. [10] These structures were strong compared to solitons in air and water. Solitons have very long lifetimes and cannot be destroyed by normal processes in nature. Bostick proposed that electrons were just simple solitons and positrons were contrary or anti-solitons. All other elementary particles were built of more complex geometrical structures, such as dyads, triads, quatrads, etc. All plasmons or solitons in the electromagnetic field are of the same shape, i.e., a toroidal ring. The plasmon was of very great strength. Bostick tried to create a bottle of plasmons to hold controlled thermo-nuclear fusion. All materials known to man up to that time slowly disintegrated when exposed to controlled thermonuclear fusion. Only the plasmon was strong enough, but Bostick failed to succeed in building a bottle of plasmons...

Hooper's Electromagnetic Field Experiments. The nature of the plasmon, electromagnetic soliton was more completely revealed by another modern-day scientist, William J. Hooper [11]. He discovered that charged elementary particles, such as the electron, were not only made of the electromagnetic field, but variations in the field around them due to their structure extending to great distances. This same feature is also observed about solitons in water.

Hooper [11] also discovered that there are three types of electric and magnetic fields. One of these types is due to velocity effects from Lenz's Law causing it to have property that cannot be shielded. Thus, portions of the electromagnetic field exist everywhere in the universe.

References:

9. May, J. P., **Stable Algebraic Topology**, p. 1 (1966). http://www.math.uchicago.edu/~may/PAPERS/history.pdf

10. Bostick, Winston H., "Mass, Charge, and Current: The Essence and Morphology," **Physics Essays, Vol. 4, No. 1**, pp. 45-59 (1991).

11. Hooper, W. J., **New Horizons in Electric, Magnetic, and Gravitational Field Theory** (Electrodynamic Gravity, Inc., 543 Broad Blvd., Cuyahoga Falls, OH 44221, 1974), preface. http://www.rexresearch.com/hooper/horizon.htm"

Of course, to complete, make more precise and revive the previously described atom model, it would be more proper to analyze the structure of an atomic field using the modified, generalized Schrödinger's equation as elaborated from (4.22) to (4.28), which is formulated in chapter 4.3 of this paper:

$$\left\{\begin{array}{l} \frac{\hbar^2}{\tilde{m}}(\frac{u}{v})\Delta\overline{\Psi} + L\overline{\Psi} = 0 \ ; \ \Delta\overline{\Psi} = (\frac{\tilde{E}}{L})\frac{1}{u^2}\frac{\partial^2 \overline{\Psi}}{\partial t^2} = jk(\frac{\tilde{E}}{L})\nabla\overline{\Psi} \ ; \ (\frac{L}{\hbar})^2\overline{\Psi} + \frac{\partial^2 \overline{\Psi}}{\partial t^2} = 0, \ \tilde{E} \Leftrightarrow E_k \\ \frac{L}{\hbar}\overline{\Psi} = j\frac{\partial\overline{\Psi}}{\partial t} = -\frac{\hbar}{L}\frac{\partial^2 \overline{\Psi}}{\partial t^2}; \ \frac{\partial\overline{\Psi}}{\partial t} + u\nabla\overline{\Psi} = 0; \overline{\Psi} = \overline{\Psi}(t,r), j^2 = -1 \\ (\frac{u}{v} = 1 + \frac{\lambda}{v}\frac{du}{d\lambda} = 1 + \frac{h}{\tilde{p}v}\frac{du}{d\lambda} = \frac{hf}{\tilde{p}v} = \frac{\hbar\omega}{\tilde{p}v} = \frac{\tilde{E}}{\tilde{p}v} \ , \ \tilde{E} = \tilde{m}uv = \tilde{p}u = hf = \hbar\omega = E_k, \ \lambda = \frac{h}{\tilde{p}}, u = \lambda f = \frac{\omega}{k}) \end{array}\right\} \Rightarrow$$

$$\begin{array}{l} \frac{\hbar^2}{\tilde{m}}(\frac{u}{v})\Delta = \frac{(\hbar u)^2}{\tilde{E}}\Delta = \frac{\tilde{E}}{k^2}\Delta = j(\frac{\tilde{E}^2}{kL})\nabla = \frac{\hbar^2}{L}\frac{\partial^2}{\partial t^2} \ , \Delta = jk(\frac{\tilde{E}}{L})\nabla, \ \tilde{E} - U_p \leq L < \infty, \\ S = \int_{t_1}^{t_2} L(q_i, \dot{q}_i ..., t)dt = \text{extremum} \ , \end{array} \qquad (4.25)$$

When using the wave equation (4.25) we should not forget that its wave function has a physical dimension related to power (and it does not represent only the probability of finding a certain energy state in some part of the atomic field space). Modified Schrödinger equation (4.25) is kind of Classical, second order, differential wave equation, where wave function $\overline{\Psi}$ is formulated as a Complex, Analytic Signal function, based on Hilbert transform, as elaborated in the chapters 4.0 and 4.3.

[♣ In solar systems, we observe orbiting and spinning planets, but within stable atoms, we only have spatial formations of electromagnetic fields and waves that adhere to periodic motions characteristic of standing waves. Despite its limitations, the traditional N. Bohr atom model, although inaccurate for describing atoms, offers useful mathematical analogies to solar systems. This model remains effective to a certain extent due to intrinsic periodicities and analogies, much like the Ptolemaic geocentric system was useful despite its inaccuracies. Other atomic models may offer improvements over Bohr's model, but they still produce similar results regarding spectral emissions and absorptions due to underlying periodicities.

There should be sources of vibrations and waves within atoms that support the internal standing-wave electromagnetic field structure, although we are still uncertain about their precise nature and location. A universal assumption could be that all atoms and macro masses in the universe are mutually coupled and synchronized with the universe functioning like a vast, low frequency rotating and oscillating system. Various perspectives on atom and planetary system modeling involve standing-wave formations, which are central to understanding these systems (see Chapter 2 for more details). However, many questions remain unanswered.

From a different perspective, modeling the atomic field (from equations (8.64) to (8.75)) suggests that similar approaches might apply to elementary particles, with due *consideration for their specific physical, mathematical, and quantum properties. For example, one might question whether the field of an elementary particle is "solenoidal," "potential," or "complex," and what type of communication exists between the particle's nucleus and shell, as well as with the external environment. This leads to an examination of group and phase velocities of matter-wave elements, amplitude and phase functions (in space-time and frequency domains), and the application of uncertainty principles (see Chapter 5).*

Modern quantum physics provides a complex and quantitatively predictable description of atoms and elementary particles, but this book proposes a different analysis. By axiomatically treating the wave function as a probability function, we align it with probability and statistical distributions, which describe domains of energy-momentum and field entities. This probabilistic treatment of wave functions, though not providing immediate temporal-spatial phase information, yields useful results by conforming to known conservation laws.

Empirically, we know that atoms absorb and emit electromagnetic energy in the form of single-frequency (or narrow-frequency band) photons. High-energy photons can create electron-positron pairs, and electron-positron collisions can result in annihilation and photon production. Compton and photoelectric effects demonstrate the mutual interaction of photons and electrons as particles and as dualistic matter-waves. These interactions, including energy dissipation related to infrared radiation, suggest that atoms are complex, self-stabilized forms of electromagnetic energy composed of standing waves and resonant structures. These structures communicate internally and externally through photon and electron exchanges. This scenario aligns with the concept of energy quantization, which is more about optimal energy packing and exchanges within multi-resonant standing-wave structures.

Regarding gravitation, analogies presented in the first chapter highlight that the linear momentum p and angular momentum L of mass-energy states (including associated electromagnetic moments and dipoles) are crucial. While a static mass m is not the sole source of gravitation, vibrating masses are significantly important. The analogy between Bohr's atomic model and solar systems underscores that wave-particle duality can be applied to both atomic and solar system models using wave functions and the Schrödinger equation and/or by defining appropriate atomic fields or forces (see equations (8.64) - (8.73)). This suggests that Newtonian and Einsteinian theories of gravitation, which should involve electromagnetic phenomena, may need updates. Furthermore, electric charges (such as electrons and protons) should be viewed as dynamic, motional energy states rather than static entities with fixed parameters. This perspective aligns with the idea of continuous electromagnetic energy exchange (or Tesla's radiant energy flow) between positive and negative charges or aligned electromagnetic dipoles, providing a foundation for explaining gravitation. Additionally, since both electromagnetic and gravitational waves exhibit transverse oscillations and follow a $1/r^2$ central force law, their origins might be interconnected, potentially related to electromagnetic interactions, heat radiation, and/or Tesla's radiant energy. ♣]

Download the last version here:
http://www.mastersonics.com/documents/revision_of_the_particle-wave_dualism.pdf

9. BLACKBODY RADIATION, GRAVITATION AND PHOTONS

(BRAINSTORMING DRAFT ... IN PROCESS)

Contemporary Physics description of the "Blackbody Radiation" phenomenology is something like:

> *Citation from: Black-body radiation - Wikipedia, "**Black-body radiation** is the thermal electromagnetic radiation within, or surrounding, a body in thermodynamic equilibrium with its environment, emitted by a black body (an idealized opaque, non-reflective body). It has a specific, continuous spectrum of wavelengths, inversely related to intensity, that depend only on the body's temperature, which is assumed, for the sake of calculations and theory, to be uniform and constant"...*[1][2][3][4]

In this book, we explore that Max Planck's law of blackbody radiation, while mathematically accurate, is essentially a well-executed "curve-fitting exercise" that aligns with the assumptions of contemporary quantum theory, despite incorporating some incomplete or flawed concepts. Planck's work produced a radiation curve that matches experimental measurements, but its underlying assumptions may benefit from revision. Also, classical Thermodynamics, including Entropy, should be updated, considering phenomenology of wave-particle duality and matter-waves properties, related to forms of energy and power transfer, relevant wave functions, impedances of involved electromechanical systems, active, reactive and complex heat sources and energy loads (see more about such mathematical modeling in Chapter 4.0). Anyway, in the background of Thermodynamics there is also electromagnetic radiation (especially in the infrared part of spectrum). Classical Thermodynamics mostly addresses mechanical aspects of electromagnetically neutral particle motions, and after a very long time of such engineering and theoretical practices, it is a time to make necessary updates and revisions. Particles in motions are gaining matter-wave properties (de Broglie hypothesis), and particles with magnetic spin moments are intrinsically linked to mechanical orbital and spin moments. Such phenomenology is naturally increasing complexity of Thermodynamics and meaning of Entropy. Thermodynamics defines temperature as directly proportional to the average kinetic energy of particles, gases, atoms, and molecules. *In the new, innovative approach to Thermodynamics, we should focus on wave energy, or total motional energy, rather than on purely mechanical kinetic energy, since the total wave energy of a system equals its total motional energy, including electromagnetic and other matter-waves contributions. Meaning of Entropy and second law of thermodynamics should also be linked to a total motional energy of certain evolutive "energy-moments-particles-waves" system, its geometry, and not only to, by Classical Thermodynamic considered, mechanical motions of involved particles. Another way to address heat energy is to treat it as the electromagnetically resistive and mechanically frictional power dissipation in all possible energy transfer situations and interactions between involved participants. Here we arrive again at the necessity of operating with definitions of active, reactive and apparent power, and complex electromagnetic and mechanical impedances (see more about such options in Chapter 4.0, under 4.0.11.0. Complex Wave Function, Energy, Power, and Impedance, and (4.0.82)).*

This discussion also focuses on the wave-particle duality of matter and its connection to gravitation, here in the context of Black-Body radiation. We start with the hypothesis that *gravitation arises from a spatially extended electromagnetic field phenomenon generated by the internal structure of atomic fields* (see more in chapter 8 and 10).

All over this paper are scattered small comments placed inside the squared brackets, such as: [♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making fast comments, and presenting challenging ideas, that could be some other time developed towards something much more meaningful and more properly integrated into Physics.

Gravitational masses typically consist of countless atoms that are synchronized and interact with one another, creating complex interferences and superpositions of their atomic and matter-wave fields. This interaction, including the creation of standing matter waves between masses, gives rise to an external macro-atomic field structure that manifests gravitational effects.

The motion of masses and atoms (including oscillations) is closely related to thermodynamics, particularly in relation to heat radiation. Consequently, if gravitation has an electromagnetic origin, heat radiation and Black-Body emissions could influence gravitational field. The synchronization and interference among oscillating atoms inside black body cavities externally lead to the formation of "macro-matter-wave structures", (being electromagnetic and other wave-particle oscillations). Such events result in macro-resonant standing waves between involved particles, matter-waves and masses. These phenomena further contribute to the effects of local gravitation field perturbation (and this phenomenology could also be in relation to Quantum Zero-Point Energy Fluctuations, Casimir attractive and repulsive forces [172], N. Tesla's Radiant energy, and Schumann resonances; -see lit. [144]).

One strategy for reinterpreting the Blackbody Radiation Law involves applying a form of "reverse engineering" in both mathematical and physical terms. This approach would seek to identify more natural concepts and better mathematical models that could replace Planck's original curve-fitting methodology combined with useful assumptions. The goal would be to provide a more coherent and elegant explanation of the peculiarities of blackbody radiation, ultimately arriving at the same or a similar result as Planck's law, but with conceptually updated and corrected foundations that address some of the initial weaknesses in quantum theory.

The flaw in the original development of Planck's Radiation Law lies in the misalignment between the time and frequency domains of the same radiation event (meaning we will either use a time domain or frequency domain for energy-related mathematical modeling, but not both at the same time). This mathematical and conceptual error was later compensated by additional assumptions during the curve-fitting process, which led to the successful formulation of Planck's Blackbody Radiation Law. Also, Quantum Theory uses non-dimensional wave function without considering its phase function. This book will explore these issues in greater detail, offering a clearer explanation.

Citation from: Black-body radiation - Wikipedia, "Calculating the black-body curve was a major challenge in theoretical physics during the late nineteenth century. The problem was solved in 1901 by Max Planck in the formalism now known as Planck's law of black-body radiation.[31] By making changes to Wien's radiation law (not to be confused with Wien's displacement law) consistent with thermodynamics and electromagnetism, he found a mathematical expression fitting the experimental data satisfactorily. Planck had to assume that the energy of the oscillators in the cavity was quantized, which is to say that it existed in integer multiples of some quantity. Einstein built on this idea and proposed the quantization of electromagnetic radiation itself in 1905 to explain the photoelectric effect. These theoretical advances eventually resulted in the superseding of classical electromagnetism by quantum electrodynamics. These quanta were called photons and the black-body cavity was thought of as containing a gas of photons. In addition, it led to the development of quantum probability distributions, called Fermi–Dirac statistics and Bose–Einstein statistics, each applicable to a different class of particles, fermions and bosons".

Let's first examine the broader natural context to which Blackbody Radiation belongs. This includes Thermodynamics (encompassing fluid dynamics and classical mechanics), the phenomena of Wave-Particle Duality, Electromagnetic theory, and other tangible effects from Quantum and Wave Mechanics, along with the associated mathematical processes. In essence, Blackbody radiation is closely related to the motion and interactions within fluids, particles, atoms, plasma states, molecular states, matter-waves, and the surrounding electromagnetic radiation inside a heated solid-state cavity.

To revise Planck's Radiation Law, a multidisciplinary and innovative approach to mathematical modeling and process simulation is required. This would involve integrating both old and new insights to produce a version of Planck's radiation law that maintains its present accuracy while achieving better conceptualization and greater mathematical and structural coherence. This approach would harmonize the innovative elements of a new theory with a more natural explanation of the same phenomenon.

An initial, promising result like a revolutionary mathematical approach has already been published (see [162]). It demonstrates that Thermodynamics, as an outgrowth of Classical Mechanics, also naturally emerges from Quantum Theory (QT) and Chaos Theory models when we progressively increase the mathematical level of Chaos. This finding also underscores that while contemporary Quantum Theory, as it presently stands, functions effectively well, but not because it is perfectly innovative and self-contained. Rather, it is structurally sound because it incorporates elements from pre-existing, successful, and powerful mathematical frameworks such as Signal Analysis, Statistics, Probability, Classical Mechanics, Conservation Laws, and the fundamental principles of Physics.

Here is the **citation or challenging, correct, but also provocative review related to thermodynamics**, formulated by Dr. Sorin Cezar Coşofreţ, https://www.pleistoros.com/en/newsletters:

.... "De facto, the first law of thermodynamics is an attempt to implement the energy conservation for thermodynamic systems, but early scientists fake it, and each generation of newcomers picked up the situation and perpetuated it without thinking...

As already presented, analytical mechanics and Noether theorem, represents the only parts of present physics which remain valid in the new proposed theory!

Everything else, even classical mechanic, need corrections up to complete refurbishing!
As consequence, the energy conservation law is not under question, but only the form this law was implemented in thermodynamics!

The present first law of thermodynamics states that a change in internal energy of a thermodynamic system equals the net heat transfer into the system minus the net work done by the system: $\Delta U = Q - W$.

The main objection to the present formulation of this law comes from the supposed equivalence between a form of energy and mechanical work! This is like a magic equivalence, but so far, no theoretical physicist ever advanced an argument why such equivalence should exist!

By comparison, If I take an electric circuit which perform some mechanical work, there is no equivalence between electrical energy and mechanical work, but only a conversion of some electric energy in mechanical work, with a certain yield!

A thermodynamic system should have a similar comportment as an electric circuit and, by using a thermodynamic machine, in certain conditions, it should be possible to **convert a gradient of pressures to mechanical work**, with a certain yield, of course!

Later on, it was observed that heat comes in three different ways, and these are: radiative, conductive and convective heat.
How can someone ever dream to have an equivalence between heat and mechanical work, when heat is not something unitary and it can come in different "flavours"!

A rational mind should have ruled out this equivalence between mechanical work and heat once this distinction between different types of heat was established!
Neither of these forms can be equivalent with mechanical work, but each of them, in certain conditions can generate some mechanical work!
As consequence, the first law of thermodynamics needs to be reformulated and, even worse for the entire field, the second and third laws have to be completely discarded!
…………………………..
Quantum theory was developed as a probabilistic and not a deterministic theory!
On the other hand, one cannot apply the Lagrangian and Hamiltonian approach to win at lottery!
In analytical mechanics, Lagrangian and Hamiltonian are continuous functions, and they cannot be used for making the characterization of a system with discrete variations for some specific parameters!
In the new theory, the entire atomic structure is going to be revised and built up as a deterministic theory!
The link to this newsletter: https://www.pleistoros.com/en/newsletters "

The remaining challenging and thought-provoking ideas that warrant further exploration include Nikola Tesla's concept of Radiant Energy, de Broglie's Wave-Particle Duality, Newton-Einstein Gravitation, and their potential connections to Thermodynamics and Electromagnetic fields. Experimental and theoretical evidence suggests that Gravity and Electromagnetism may be fundamentally or phenomenologically linked.

Given the well-established success of Max Planck's Blackbody Radiation Law, despite the complex path taken to develop it, we can now hypothetically and qualitatively revisit these ideas. This would involve reconsidering Planck and Einstein's work on Blackbody Radiation and electromagnetic wave quantization in the context of Nikola Tesla's Radiant Energy and the true origins of Gravitation. Additionally, this could be related to the Universal Natural Force-field concept elaborated by Rudjer Boskovic.

The most important references regarding Boskovic's Universal Natural Force [6], and Tesla's Dynamic Gravity [97] can be found online:
- [Rudjer Boskovic (1711-1787)] (https://teslaresearch.jimdofree.com/dynamic-theory-of-gravity/ru%C4%91er-josip-bo%C5%A1kovi%C4%87-1711-1787/)
- [Tesla's Dynamic Theory of Gravity] (https://teslaresearch.jimdofree.com/dynamic-theory-of-gravity/).

Recent studies [100] describe the familiar phenomenon of "radiative or radiant energy" as it relates to gravitational sources. The Casimir attractive and repulsive forces could be another manifestation of this "Radiant Energy activity"; -see [103], [104] and [172]. One of the most intriguing aspects of Radiant Energy and Gravitation, including Blackbody and other thermal radiation phenomena, relates to Tesla's hypothesis of a structurally resonant Universe. This theory posits that standing waves form between all cosmic masses and energy formations. These cosmic resonances create spatial-temporal fields and forces structures, leading to mutual resonant synchronization, entanglement effects, and other forms of local and non-local interactions between cosmic entities. These interactions could also be linked with Radiant and Heat energy formations, as discussed in Chapter 2, "Gravitation and Natural Forces based on Standing Waves, Resonating Universe."

In our universe, all objects are continuously exchanging "mass-energy-moments" states in the form of matter-waves, such as electromagnetic, cosmic rays, neutrinos, charged and neutral particles, mechanical vibrations, seismic waves, and thermal motions. These exchange interactions generate secondary matter-waves, Schumann resonances, and related oscillatory harmonics, which have measurable mechanical and electromagnetic manifestations, including heat radiation. Every atom, molecule, and mass produce and absorb a wide spectrum of these waves, vibrations, and oscillations, all of which could be causally linked to the Radiant Energy phenomena, most of which are of electromagnetic origin.

Atoms and their internal structures, along with the surrounding forces and fields, act as both "sources and sinks" of all natural forces, in line with the concept of Rudjer Boskovic's Natural Force. Atoms, molecules, agglomerated masses, and other forms of energy function as mechanical, electromechanical and electromagnetic resonators, exchanging energy through electromagnetic and mechanical wave-groups, such as photons, phonons, electrons, neutrinos ... This process of energy exchange occurs both internally and externally, including interactions with the atomic nucleus across various frequency ranges. This natural synchronization and coupling occur within and between spectrally overlapping atomic resonators and oscillators, as further detailed in Chapter 10.

The idea of photons as electromagnetic wave-packets was successfully applied by Mileva Marić and Albert Einstein in the context of the Photoelectric Effect, then in explaining Compton effect, and Max Planck also employed this formulation (E = hf) in creating his radiation law. Photons, assumed to be the way of energy quantization of electromagnetic energy or waves, are temporally, spatially, and spectrally narrow-band signals (meaning wave-packets enveloped by Gaussian or Bell curves). This concept is essential for understanding Wave-Particle Duality and the quantized photon energy E = hf (as elaborated in Chapters 4 and 10).

All-natural matter-waves, radiation, and oscillations are inherently coupled and superimposed, appearing simultaneously within and around the same radiating objects or masses. The practical distinction lies in how we detect specific signals within certain frequency ranges, depending on the sensors or measurement instruments available. Currently, our ability to measure the diversity of matter-waves across their full natural frequency ranges is limited by the technology at our disposal. When we analyze matter-wave radiating objects, we often overlook the fact that action-reaction forces are at play. The detection of mechanical or electromagnetic waves externally corresponds to synchronous electromagnetic, thermal, and mechanical oscillations or reactions internally within the object's atomic or molecular structure, and vice versa.

Atoms and molecules possess a multitude of electromagnetically charged, rotating, spinning, and oscillating resonant states across different frequency ranges. The Conservation Laws of Physics must be satisfied at both cosmic and atomic scales, resulting in universal synchronization. Depending on the frequency interval, it may be easier to detect mechanical signals in some cases, and electromagnetic signals in others, due to the limitations of current sensors and instruments. The various natural vibrations, emissions, and absorptions of matter-states and objects, whether characterized mechanically, acoustically, electromechanically, electromagnetically, or thermodynamically, are mutually coupled and spectrally overlapping, synchronizing within relevant temporal and spatial frequency ranges. This synchronization occurs within any interacting objects or masses.

When we monitor only a segment of a wide-band spectrum, such as thermal or blackbody (electromagnetic) radiation, it is essential to remember that this spectrum also includes contributions from other matter-wave and Radiant Energy events, whether mechanical, electromechanical, or electromagnetic. The Blackbody cavity is a natural environment for the creation, superposition, and interference of simple-harmonic matter waves (or wavefunctions) radiating from oscillating atoms. This leads to the formation of sinc (sin(x)/x) wavefunctions, and the superposition and interference of these sinc wavefunctions (or photons) generate the Blackbody Radiation curve as described by Planck. In this context, it is important to consider that the state of matter within the Blackbody cavity exhibits variable electromagnetic, electromechanical, mechanical, and thermodynamic properties over time and space. For instance, dielectric and magnetic permeability or susceptibility (ε, μ) within the Blackbody cavity are not constant and stable under these conditions. That means, relevant group and phase velocities of matter-waves are also mutually dependent and evolving (in relation to involved temperature and spectral content of surrounding EM radiation) and should comply with Rayleigh wave phase and group velocity formula: $v = u - \lambda(du / d\lambda) = d\tilde{E} / dp$, $u = \lambda f = \tilde{E} / p = 1 / \sqrt{\varepsilon\mu}$.

The author of this book suggests that Planck's Blackbody Radiation Law, particularly when applied to relatively low temperatures (below 500°C), closely resembles a typical Gaussian Normal Distribution. This similarity arises because the velocities of the dominant thermodynamic process participants at these temperatures are still relatively low. As temperatures rise, these velocities increase, causing non-linear and evolving relationships between group and phase velocities. This gradual deformation of the initial Gaussian distribution eventually leads to the modified shape observed in Planck's Blackbody Radiation Law.

In other words, the natural emissions and absorptions of radiative, electromagnetic, and acoustic fields by atoms and electrons, as discussed in Chapters 8 and 10, along with the excitation, charging and discharging of various resonators, produce matter-wave radiation (where Nikola Tesla's radiant energy also belongs). This process involves the creation and annihilation of active atomic, mechanical and electromagnetic mass-energy entities such as ionized atoms, photons, and other field charges. Similarly, certain excitations or vibrations of solid matter structures in the temporal domain produce corresponding "energy-mass-frequency effects" or emissions in the spatial domain, and vice versa. These aspects are not fully addressed by contemporary Blackbody theory.

In our oscillating and resonating Universe, all masses and atoms are interconnected, at least through the surrounding electromagnetic field. They tend to synchronize as matter-waves, while agglomerating, resonating, and exchanging "energy-moments." If we consider Planck's Blackbody electromagnetic radiation as "external radiation" (emitted from a blackbody cavity into the surrounding, almost empty space), we can analogically conceptualize the effects within heated cavities and masses as being influenced by "internal blackbody radiation" from the involved atoms. In both cases, Tesla's concept of radiant cosmic energy could play a role. Here, we generalize heat radiation, both external and internal, as originating from excited and oscillating electrons, atoms, molecules, and other particles and wave groups in various states of motion within and between surrounding macro masses, thereby facilitating mutual communication and synchronization among reaction participants.

Quantum entanglement effects among these matter entities also contribute to the universal tendency of resonant and standing matter-waves to synchronize. This synchronization should be understood within both temporal and spatial resonances.

All the mentioned motions and radiation, which act like matter-waves, can be conceptually (at least partially) identified as Tesla's Radiative, or Radiant Energy flow. Atoms, as predominantly electromagnetic multi-resonant structures, stabilize internally as standing matter-wave formations, while externally extending such micro-world field structures towards other macro-world atoms and masses. This interaction contributes to the phenomenon of Gravitation, as further explained in Chapter 8 under "8.3. Structure of the Field of Subatomic and Gravitation related Forces." When photons are emitted or absorbed, there is inevitably some activity and alignment of the involved electromagnetically charged particles or dipoles with electric, magnetic, and spinning properties. Situations where electromagnetic and/or thermal Blackbody Radiation is present, both internally and externally, are causally related to the force of Gravitation, as Tesla speculated. This connection is likely due to the commonality of involved masses, a resonating Universe, and the same Radiative Energy Flow, including the mathematical similarities between Coulomb-Newton central forces and Einstein's relation between mass and energy.

For example, considering our Sun, a "blackbody object" with a surface temperature of around 6000°C and its Corona zone temperature between one and two million degrees Celsius, we observe that this natural blackbody radiation is specifically "temperature-energy" focused or intensified. It is "spatially modulated" by the increasing velocity of structural standing macro matter-waves between the Sun and surrounding planets, extending into the high vacuum-state zone around the Sun. This concept is further explored in Chapter 2 under "Gravitation and Natural Forces based on Standing Waves, Resonating Universe."

We cannot dismiss Nikola Tesla's visionary ideas about Radiant Energy and Dynamic Gravity, despite our incomplete understanding of these concepts. Without Tesla's creativity, imagination, innovations, and specific engineering contributions, much of our contemporary technology and industry would not exist. The technically proven legacy of Tesla lends significant credibility to his less-understood visions and imaginative concepts about gravity, including the potential links between Radiative Energy and Gravitation. Tesla often spoke about his Dynamic Gravity theory based on standing waves between planets and Sun. Given Tesla's track record of materializing grand visions, it is likely that his insights into these phenomena still hold significant conceptual value. Tesla appeared to be uniquely and intuitively connected to the "knowledge data banks" of our Universe, far more so than others.

If we carefully examine Nikola Tesla's imaginative and hypothetical statements about Radiant Energy (see [97]), it appears that he envisioned this energy as extremely fine particles streaming or being a fluidic-mass flow with electromagnetic properties, behaving similarly to an ideal gas or ether. This "etheric energy," in the form of an ideal-fluid flow, circulates through or between all masses in the universe, manifesting gravitational force, like the attractive force experienced in Einstein's thought experiment about an upwardly accelerated elevator. This interpretation touches upon the conceptual foundations of Tesla's Dynamic Gravity theory, even though our understanding of Radiant Energy-flow and ether is still incomplete. However, we do know that in a vacuum, dielectric and magnetic permeability are measurable. Consequently, electromagnetic waves or photons travel through a vacuum, because ether behaves as fluid. The speed of light in vacuum depends solely on its static, fixed parameters, dielectric and magnetic permeability, $c = 1/\sqrt{\varepsilon_0 \mu_0}$. Hypothetically, just for the purpose of mathematical exercise, we could explore the idea that aether is kind of spatial, 3-dimensional (or multidimensional) mass-spring oscillators array or matrix, considering that involved masses are very small. This way, we could analogically connect speed of light $c = 1/\sqrt{\varepsilon_0 \mu_0}$ with the maximal velocity of mechanical mass-spring array oscillations (but this could be elaborated later).

Tesla's Radiant Energy likely includes electromagnetic radiation from heated masses, as well as various known forms of radiation such as photons, electrons, positrons, cosmic rays, neutrinos, different matter waves, plasma states, and other unspecified participants. Blackbody radiation and other forms of thermal radiation could thus be considered as familiar aspects of Tesla's cosmic Radiant Energy (see [128]). Consequently, heat-related radiation should naturally cause local perturbations in the surrounding gravitational field. Many hypothesize, as does author of this book, that gravitation has electromagnetic roots, given that Newton's law of gravitation is mathematically analogous to the electromagnetic Coulomb force between polarized charges, and both electromagnetic and gravitational waves are transverse and propagate in vacuum with the same speed C.

The "fluidic matter content around masses" could be influenced by the cosmic rotation and spinning of masses and other energy concentrations (see Chapters 1, 2, 10, and references

[114]-[128]). Thus, strongly heated masses might locally deviate from Newton's law of gravitation due to surrounding thermal and radiant field perturbations. Heating induces additional waves and particle streams, which influence the "Radiant Energy flow, and locally deform the gravitational field." Since heat is related to kinetic energy and wave behavior of involved particles, and creates blackbody electromagnetic radiation, gravitation is closely linked to relevant thermodynamic and electromagnetic field perturbations. These perturbations are interfering with standing matter-waves between gravitational masses and other energy formations in our resonating Universe (see Chapter 2 on "Natural Forces and Resonating Universe").

To analogically visualize Radiant Energy flow, consider a vessel with water on a heating plate. Water will evaporate at any low temperature due to the omnipresent Radiant Energy flow and other thermal motions and radiation. Natural liquid evaporation is partially influenced by this surrounding Radiant Energy. As water temperature rises, the temperature of evaporating particles increases until boiling occurs. The heated molecules gain enough kinetic energy to escape water, thereby influencing the Radiant Energy flow. Once boiling starts, the temperature of the remaining water stabilizes until all the water evaporates. This scenario provides an analogy for understanding and comparing relations between liquid evaporation, Radiant Energy flow, and Blackbody radiation. The "photomolecular effect," analogous to the photoelectric effect, leads to water evaporation beyond thermal limits (see [169]). This effect could be generalized as a "Radiant Energy molecular effect."

Even solid masses, like metals, undergo slight mass changes over time, as observed with the International Prototype of the Kilogram. These changes, some of which are inexplicable, might be related to environmental Radiant Energy flow and the Resonating Universe.

In essence, the heat-related phenomena of matter begin with the random mechanical motions and oscillations of atoms and molecules, leading to the creation of various matter-waves, fluids motion and electromagnetic radiation. As kinetic energy or temperature increases, the resulting electromagnetic or thermal radiation manifests as Blackbody radiation. These thermal and radiative effects, combined with the standing-wave effects of the resonating Universe, can locally influence gravitation by affecting the temperature, kinetic energy, and spatial distribution of involved particles and radiation. Analogous to liquid evaporation, intense heating of a mass generates additional Radiant and matter-waves energy flow, as vibrating molecules and atoms, being resonant objects, naturally emit matter waves when excited.

The surrounding electromagnetic field structure, which is hypothetically responsible for gravitational effects (as Tesla theorized), could be locally modified by heating and additional radiative energy streaming. It might be possible to measure deviations or perturbations in the local gravitational field as a function of temperature (as discussed in [101]). Forces of any kind are related to spatial gradients of relevant mass-energy density and/or temporal derivatives of involved mechanical and electromagnetic moments (see Chapter 10 for more about natural forces). Tesla primarily focuses on electromagnetic phenomena that produce and affect Radiant Energy flow. Since Blackbody radiation is a form of electromagnetic wave radiation, it follows that gravitation, heat, and electromagnetism are interrelated. One could hypothesize that stars, suns, and black holes are resonant nodes in a cosmic Radiant Energy flow, being part of a larger electromagnetic standing-wave field structure in the universe. Another source of Radiant Energy is the energy flow between electrically charged particles and electromagnetic dipoles (since electric charges should have dynamic properties).

In the first chapter of this book, we also analogically establish that the true sources of gravitation and other natural forces are mutually coupled natural field-charges, such as linear and angular moments, electric charges, and magnetic flux, or magnetic moments in various states of motion and oscillations. Thus, static, electromagnetically neutral masses are not the dominant sources of gravitation, as assumed by Newton and Einstein. Rather, dynamically active atoms, coupled with the extended atomic fields of other masses, are the true sources (see Chapters 1, 2 and 8). Blackbody radiation refers to the external electromagnetic radiation of any heated solid mass. Internally, such a heated Blackbody cavity contains many coupled, synchronized electromagnetic charges, plasma states, and other total energy participants. This continuous emission and absorption of photons inside a Blackbody cavity can be analogically likened to a fluid boiling. From this perspective, Blackbody radiation has a specific spectral signature, carrying part of information related to the local gravitational field.

It is time to update our understanding of heat energy, temperature, and associated Blackbody radiation. The current dominantly mechanistic view of thermodynamics might be incomplete (see [126] for more on this). Empirical evidence suggests that gravitational force between masses is not detectable at very small distances (micrometers or less). Gravitation is an external, macro-mass attraction effect, manifesting only between sufficiently large masses. Internal interactions and energy exchanges between atoms do not involve gravitational effects. From a global perspective, gravitation is causally linked to external radiative communications between masses and the oscillating universe, as Tesla speculated (see [128]). The mutual influence between heated masses and the surrounding gravitational force is already experimentally verified, indicating that heating can locally influence the force and spatial shape of the gravitational field (see the next Quote).

Quote *(The Negative Temperature Dependence of a Gravity is a Reality; Professor Alexander L. Dmitriev and Sophia A. Bulgakova, [101]): "Temperature dependence of force of gravitation - one of the fundamental problems of physics. The negative temperature dependence of the weight of bodies is confirmed by laboratory experiments and like Faraday phenomenon in electrodynamics is a consequence of natural "conservatism" of a physical system, its tendency to preserve a stable condition. Realization of experimental research of an influence of the temperature of bodies on their gravitational interaction is timely and, undoubtedly, will promote the progress of development of physics of gravitation and its applications".*

In this section, we aim to draw key conclusions from blackbody radiation and cosmic radiative energy exchanges, as factors influencing gravitation assuming that gravitation is the force primarily linked to electromagnetic phenomena and standing matter-waves of the resonating Universe (see Chapter 2 for more details). In these standing waves, only attractive forces exist in the nodal zones, while repulsive forces are confined to the anti-nodal zones of maximal velocity—a concept already tested with acoustic resonators and ultrasonic levitation cases (see [150-151] for further details).

Mathematically idealized, statistical sets and ensembles of mutually independent, non-interacting matter states cannot fully, or theoretically, capture the complexity of all natural matter states. Natural forces and field couplings create synchronizations between involved matter states, implying that motions (or wavefunctions) within these statistical sets and ensembles of similar or identical resonators (or objects, and matter states) always exhibit a resulting amplitude and phase function in their combined spatial-temporal domain when represented as Complex Analytic Signals or Phasors.

This observation challenges the foundations of contemporary Orthodox Quantum theory, suggesting that wavefunctions and wave equations should be understood as dimensional and measurable entities, not merely as statistical and probabilistic, non-dimensional constructs. Representing wavefunctions as Complex Analytic Phasors is the most general and universal approach to describing any wave phenomenon in physics. As we know, such a phasor consists of both a real and an imaginary part, two mutually phase-shifted wavefunctions that coexist, carry the same amounts of energy, and are mutually orthogonal or "energetically exclusive." There is no reason to consider one part more significant than the other, as both are equally important, much like the relationship between electric and magnetic field vectors in electromagnetic wave propagation, including photons.

From these dualistic and naturally dimensional wavefunctions (discussed further in Chapter 4.0), we can, through normalization, again create non-dimensional wavefunctions, allowing us to leverage the extensive mathematical tools of Quantum theory.

Now, let us continue with mathematical exploration (since the main idea here is to exercise Blackbody Radiation Law through Reversed Mathematical Engineering: We know the final, M. Planck's function of spectral content of the Blackbody emission, or electromagnetic radiation in its frequency (and wavelengths) domain, and we could also try to find its time-domain related wave-functions (or also to find spatial shapes of involved matter waves).

*The integral forms of Stefan-Boltzmann and Wien's radiation laws of a Blackbody heated to the temperature **T** presents a surface power density **R(T)** of emitted electromagnetic waves (or photons) from that Blackbody,*

$$
\begin{aligned}
&R(T) = R_T = \int_0^{\infty} dR(f,T) = d\tilde{P} / dS = \sigma \cdot T^4 (=) \left[\frac{W}{m^2}\right],\\
&\tilde{P} = \sigma \cdot T^4 \cdot S = R(T) \cdot S = \text{Power radiated in } [W] = \left[\frac{J}{s}\right],\\
&\lambda_{peak} = \frac{2.898 \cdot 10^{-3}}{T} (=) [m] (=) \text{Peak Wavelength at maximum intensity},\\
&\sigma = \text{Stefan's Constant} = 2\pi^5 k^4 / 15 c^2 h^3 = 5.67 \cdot 10^{-8} \left[\frac{W}{m^2 K^4}\right], \qquad (9.1)\\
&S = \text{Relevant (radiating) surface } [m^2],\\
&T = \text{Surface Temperature of the body } [K].
\end{aligned}
$$

Planck successfully created (or better to say mathematically fitted, based on experimental data and some intuitive assumptions) the electromagnetic, spectral radiance formula **R(f, T)** of a body at absolute temperature T as,

$$
\begin{aligned}
&dR(f,T) = \frac{2\pi f^2}{c^2} \cdot \frac{hf}{e^{hf/kT} - 1} df \, (=) \left[\frac{W}{m^2 \cdot sr}\right] \Rightarrow\\
&\frac{dR(f,T)}{df} = R_f(f,T) = \frac{2\pi f^3}{c^2} \cdot \frac{hf}{e^{hf/kT} - 1} (=) \left[\frac{Ws}{m^2 \cdot sr}\right], \qquad (9.2)\\
&\frac{dR}{d\lambda} = R_\lambda(\lambda,T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} (=) \left[\frac{W}{m^3 \cdot sr}\right]
\end{aligned}
$$

where **k** is the Boltzmann constant, **h** the Planck constant, and **c** the speed of light in the medium, or vacuum.

Citation (see http://en.wikipedia.org/wiki/Planck%27s_law): "The spectral radiance can also be measured per unit wavelength instead of per unit frequency. These distributions represent the spectral radiance of blackbodies—the power emitted from the emitting surface, per unit projected area of an emitting surface, per unit solid angle, per spectral unit (frequency, wavelength, wavenumber, or their angular equivalents). Since the radiance is isotropic (i.e. independent of direction), the power emitted at an angle to the normal is *proportional to the projected area, and therefore to the cosine of that angle as per Lambert's cosine law, and is non-polarized".*

Citation took from the internet (http://physics.info/planck/): "Let us try to derive the blackbody spectrum. Planck's law is the formula for the spectral radiance of an object at a given temperature as a function of frequency or wavelength. It has dimensions of power per solid angle per area per frequency or power per solid angle per area per wavelength.

$$\frac{dR(f,T)}{df} = R_f(f,T) = \frac{2\pi f^3}{c^2} \cdot \frac{hf}{e^{hf/kT} - 1} (=) \left[\frac{Ws}{m^2 \cdot sr}\right],$$

$$\frac{dR(\lambda,T)}{d\lambda} = R_\lambda(\lambda,T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} (=) \left[\frac{W}{m^3 \cdot sr}\right]$$

When these functions are multiplied by the total solid angle of a sphere (4π steradian) we get the spectral irradiance. This function describes the power per area per frequency or the power per area per wavelength.

$$\frac{dR(f,T)'}{df} = 4\pi R_f(f,T) = \frac{8\pi^2 f^3}{c^2} \cdot \frac{hf}{e^{hf/kT} - 1} (=) \left[\frac{W}{m^2 \cdot Hz}\right],$$

$$\frac{dR(\lambda,T)'}{d\lambda} = 4\pi R_\lambda(\lambda,T) = \frac{8\pi hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} (=) \left[\frac{W}{m^3}\right]$$

When these functions are integrated over wavelength, the result is either the irradiance or the power per area.

$$R(T) = \frac{2\pi^5 k^4}{15h^3c^2} T^4$$

The pile of constants in front of the temperature is known as Stefan's constant.

$$\sigma = \frac{2\pi^5 k^4}{15h^3c^2} = 5.67040 \times 10^{-8} \frac{W}{m^2 K^4}$$

Multiplying the irradiance by the area gives us the essence of Stefan-Boltzmann law.

$$\tilde{P} = \frac{2\pi^5 k^4}{15h^3c^2} \cdot S = 5.67040 \times 10^{-8} \cdot S = \sigma \cdot T^4 \cdot S = R(T) \cdot S = \text{Power radiated in } [W] = \left[\frac{J}{s}\right]$$

Then we apply the first derivative test to the wavelength form of Planck's law to determine the peak wavelength as a function of temperature.

$\lambda_{max} = \frac{hc}{k} \frac{1}{xT}$, *where x is the solution of the transcendental equation* $\frac{xe^x}{e^x - 1} - 5 = 0$, *x = 4.96511...*

Then we combine all the constants together and we get the Wien displacement law, $\lambda_{max} = \frac{b}{T}$, *b = 2.89777... mmK".*

The total power and *peak radiating force* of a heat radiation from the Blackbody Surface **S** can be found as,

$$\tilde{P}(t) = \Psi_{bb}^2(t) = d\tilde{E}/dt = \int_{[S]} R_T dS = R_T S = S\int_0^\infty dR(f,T) = S\sigma T^4 (=) [W]$$

$$\Rightarrow \tilde{P}(t) = d\tilde{E}/dt = \tilde{F} \cdot dr/dt = \tilde{F} \cdot v = \tilde{F} \cdot c = S\sigma T^4 \Rightarrow \quad (9.3)$$

$$\Rightarrow \text{Radiative Force } (=) \ \tilde{F} = S\sigma T^4 / c (=) [N],$$

and where $\Psi_{bb}(t) = a_{bb}(t)\cos\varphi_{bb}(t)$ is still unknown (assumed), time domain wave function of a Blackbody radiator (here and later recognizable by indexing "bb"). Radiated heat or electromagnetic energy from a Blackbody (based on Parseval's theorem), after a time-period Δt, would be,

$$\begin{aligned}
\tilde{E} &= \int_{-\infty}^{+\infty} \tilde{P}(t)dt = \int_{-\infty}^{+\infty} \Psi_{bb}^2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |U_{bb}(\omega)|^2 d\omega = \frac{1}{\pi}\int_{0}^{+\infty} |A_{bb}(\omega)|^2 d\omega = S\Delta t\int_{0}^{+\infty} dR(f,T) = \\
&= S\Delta t\int_{0}^{+\infty} \frac{dR(f,T)}{df} df = S\Delta t\int_{0}^{+\infty} \frac{2\pi f^2}{c^2}\cdot\frac{hf}{e^{hf/kT}-1} df = S\cdot\Delta t\cdot\sigma T^4 (=)[Ws = J], \\
d\tilde{E} &= \left[S\int_{0}^{+\infty} dR(f,T)\right]dt = \left[S\int_{0}^{+\infty} \frac{2\pi f^2}{c^2}\cdot\frac{hf}{e^{hf/kT}-1} df\right]dt = \Psi_{bb}^2(t)dt = S\cdot\sigma T^4 dt = \tilde{F}dr.
\end{aligned} \tag{9.4.1}$$

If the wave energy and wave function in question pertain to a single photon or a narrow-band wave packet, we can simplify the mathematical approach accordingly (as $\tilde{E} = hf \Rightarrow d\tilde{E} = h\cdot df$). Experimental evidence suggests that the time-domain wave function of blackbody electromagnetic radiation represents a stable, stationary, and continuous signal with a relatively long temporal-duration and limited frequency range, as predicted by the Uncertainty Principle. In this context, we aim to characterize blackbody wavefunctions as tangible signals with dimensions of power (analogous to the Poynting vector concept in electromagnetic energy), as introduced and modeled in Chapters 4.0, 4.3, and 10. This approach contrasts with QT treating them merely as stochastic and non-dimensional functions.

In sections 9.3 and 9.4.1, we introduced the concept of the matter-wave radiative force, which aligns with blackbody radiation. Given that action and reaction forces are always simultaneously present in every interaction, we should anticipate that the reaction and attractive force toward the radiating body corresponds to the gravitational force described by Newton, Coulomb, or as theorized by Nikola Tesla and Rudjer Boskovic. Thermal radiation, mass evaporation, and other forms of electromagnetic radiation collectively represent an energy flow or an equivalent mass flow. The internal balancing forces, which are repulsive and exhibit characteristics like standing waves, as described by Boskovic's Universal Natural Force Law, contribute to the formation of atoms and the aggregation of masses (discussed in detail in Chapter 8, particularly in section 8.3, "Structure of the Field of Subatomic and Gravitation-related Forces").

The existence of radiant energy flow can be likened to the experience in Einstein's accelerated elevator thought experiment, where a person inside the elevator feels a force equivalent to gravity (see references [6], [97], and [100]). Rudjer Boskovic, who supported Isaac Newton's formulation of the Law of Gravitation, further advanced the concept with his Universal Natural Force Function. Nikola Tesla, a strong advocate of Boskovic's ideas, often spoke of his own (unpublished) theory of Dynamic Gravity, which was likely inspired by R. Boskovic's work.

In section 9.4, it is theoretically possible to reverse-engineer or quantify the blackbody spectral amplitude function using principles discussed in Chapters 4.0 and 10, where relevant mathematical frameworks are provided.

$$A_{bb}^2(\omega) = \pi\ S\ \Delta t\left[\frac{dR(f,T)}{d\omega}\right] = \frac{1}{2}S\ \Delta t\left[\frac{dR(f,T)}{df}\right] = S\ \Delta t\frac{\pi f^2}{c^2}\cdot\frac{hf}{e^{hf/kT}-1} =$$

$$= S\cdot\Delta t\cdot\frac{\pi}{\lambda^2}\cdot\frac{\tilde{E}}{e^{\tilde{E}/kT}-1} = S\cdot\Delta t\cdot\frac{h}{8\pi^2c^2}\cdot\frac{\omega^3}{e^{h\omega/2\pi kT}-1} = \frac{1}{2}\frac{d\tilde{E}}{df} = \pi\frac{d\tilde{E}}{d\omega} =$$

$$= U_c^2(\omega) + U_s^2(\omega)\ (=)\ \left[Ws^2 = Js\right],$$

$$A_{bb}(\omega)e^{-j\Phi_{bb}(\omega)} = \iint_{[-\infty,+\infty]}\overline{\Psi}_{bb}(t)e^{j\omega t}dt = U(\omega) = U_c(\omega) - jU_s(\omega),$$

$$\overline{\Psi}_{bb}(t) = \Psi_{bb}(t) + j\hat{\Psi}_{bb}(t) = a_{bb}(t)e^{j\varphi_{bb}(t)} = \int_{(0,+\infty)}A_{bb}(\omega)e^{-j(\omega t+\Phi_{bb}(\omega))}d\omega,$$

$$\hat{\Psi}_{bb}(t) = H\left[\Psi_{bb}(t)\right] = a_{bb}(t)\sin\varphi_{bb}(t),\ \Psi_{bb}(t) = a_{bb}(t)\cos\varphi_{bb}(t), a_{bb}(t) = \sqrt{\Psi_{bb}^2 + \hat{\Psi}_{bb}^2},$$

\-\-\-\-\-\-\-\-\-\-\-\-\-\-

$$d\tilde{E} = \Psi^2(t)dt = \hat{\Psi}^2(t)dt = \left|\overline{\Psi}(t)\right|^2 dt = \left[\frac{a(t)}{\sqrt{2}}\right]^2 dt = \left|\frac{\overline{U}(\omega)}{\sqrt{2\pi}}\right|^2 d\omega = [\frac{A(\omega)}{\sqrt{\pi}}]^2 d\omega =$$

$$= P(t)dt = \frac{h}{2\pi}d\left[\frac{\Psi(t)\dot{\hat{\Psi}}(t) - \dot{\Psi}(t)\hat{\Psi}(t)}{a^2(t)}\right],\ h = \text{const.},$$

$$\tilde{E} = \int_{[T]}\Psi^2(t)dt = \int_{[T]}\left[a(t)\cos\varphi(t)\right]^2 dt = \int_{[T]}a^2(t)dt,$$

$$P(t) = \frac{d\tilde{E}}{dt} = \Psi^2(t)\ (\Leftrightarrow)\left[\frac{a(t)}{\sqrt{2}}\right]^2 (=)\left[W\right],\ t\in(-\infty,+\infty),$$

$$P(\omega) = \frac{d\tilde{E}}{d\omega} = \frac{\Psi^2(t)}{d\omega/dt} = \left[\frac{A(\omega)}{\sqrt{\pi}}\right]^2 (=)\left[Js = Ws^2\right],\ \omega\in(0,+\infty),$$

$$\Psi(t) = a(t)\cos\varphi(t) = -H\left[\hat{\Psi}(t)\right],\ \hat{\Psi}(t) = a(t)\sin\varphi(t) = H\left[\Psi(t)\right],$$

$$\overline{\Psi}(t) = a(t)e^{j\varphi(t)} = \Psi(t) + j\hat{\Psi}(t),$$

$$a(t) = \sqrt{\Psi^2(t) + \hat{\Psi}^2(t)},\ a^2(t) = \frac{\Psi(t)\dot{\hat{\Psi}}(t) - \dot{\Psi}(t)\hat{\Psi}(t)}{\overline{\omega}},\ \varphi(t) = \arctan\frac{\hat{\Psi}(t)}{\Psi(t)},$$

$$\dot{a}(t) = \frac{\partial a(t)}{\partial t} = \frac{\psi(t)\cdot\dot{\psi}(t) + \hat{\psi}(t)\dot{\hat{\psi}}(t)}{a^2(t)} = a(t)\cdot\text{Re}\left[\frac{\dot{\overline{\Psi}}(t)}{\overline{\Psi}(t)}\right], \tag{9.4.2}$$

$$f(t) = \frac{\omega(t)}{2\pi} = \frac{1}{2\pi}\frac{\partial\varphi(t)}{\partial t} = \frac{1}{2\pi}\dot{\varphi}(t) = \frac{1}{2\pi}\frac{\Psi(t)\dot{\hat{\Psi}}(t) - \dot{\Psi}(t)\hat{\Psi}(t)}{a^2(t)} = \frac{1}{2\pi}\text{Im}\left[\frac{\dot{\overline{\Psi}}(t)}{\overline{\Psi}(t)}\right].$$

In cases of single photons, with narrow frequency-band, it could be,

$$\left[\omega = \overline{\omega} = 2\pi\overline{f},\ c = \overline{\lambda f},\ \tilde{E} = h\overline{f},\ d\tilde{E} = hd\overline{f}\right]\Rightarrow$$
$$\Rightarrow A_{bb}^2(\overline{\omega}) = U_c^2(\overline{\omega}) + U_s^2(\overline{\omega}) = \frac{1}{2}\cdot\frac{d\tilde{E}}{df} = \pi\frac{d\tilde{E}}{d\omega} = \frac{h}{2}(=)\left[Js\right]. \tag{9.5.1}$$

Based on the amplitude spectral function $A_{bb}(\omega)$ we should be able to explore a family of possible time-domain wavefunctions $\Psi_{bb}(t) = a_{bb}(t)\cos\varphi_{bb}(t)$, or similar sinc and soliton wavefunctions, that comply with Planck's radiation law (as given in its frequency domain). In fact, we are searching for the most general analytic expressions of wavefunctions, or wave packets and photons (formulated in a time and frequency domain). In addition, from theoretical and experimental achievements of M. Planck,

A. Einstein, and Compton, we know that an elementary (narrow frequency band) wave packet in a form of a photon (in certain of its most elementary case) has the energy proportional to $\tilde{E} = hf$, or more correctly defined in its differential or infinitesimal form as $d\tilde{E} = h \cdot df$. We also know that a photon has certain mean-carrier-frequency and a narrow frequency bandwidth (or frequency duration). Based on experimental experiences, we also know that photons really have relatively short time and frequency durations, having Gaussian amplitude envelopes (being band-limited, temporally, spatially, and spectrally). We could also assume that (an external) blackbody radiation, or its wave function $\Psi_{bb}(t)$, is composed of many single photons, presented as simple-harmonic $\psi_i(t) = a_i(t)\cos\varphi_i(t)$ or similar sinc (=) sin(x)/x, elementary wavefunctions (including solitons). Anyway, the energy conservation here is defined by Parseval's theorem, and considering the quantum nature of photons, and matter-waves superposition, we can formulate the following simplified relations,

$$\Psi_{bb}^2(t) = \left[a_{bb}(t)\cos\varphi_{bb}(t)\right]^2 = \sum_{(i)} n_i \psi_i^2(t) \Rightarrow \tilde{E}_i = \int_{-\infty}^{+\infty} \psi_i^2(t)dt = p_i u_i\,,$$

$$\tilde{E} = \int_{-\infty}^{+\infty} \Psi_{bb}^2(t)dt = \frac{1}{\pi}\int_{0}^{+\infty} \left|A_{bb}(\omega)\right|^2 d\omega = \int_{-\infty}^{+\infty}\left[\sum_{(i)} n_i\psi_i^2(t)\right]dt = \sum_{(i)} n_i\tilde{E}_i\,, \qquad (9.5.2)$$

$$n_i \in \left[1,2,3,...\right].$$

It is far more realistic to assume that any emission, superposition, or interference effects involving matter-waves must adhere to the principles established by the "Kotelnikov-Shannon-Nyquist-Whittaker" theorem and the concepts of Analytic Signals. These principles govern signal analysis, synthesis, sampling, and reconstruction, using functions like the sinc and soliton waveforms.

Atoms, molecules, and other particles naturally generate clusters of superimposed sinc and Gaussian wavefunction components, either as discrete elements or as an integrated superposition of interacting atomic resonators. These narrow-band wave groups, defined across multiple domains, are governed by the Planck-Einstein photon quantization formula. Ultimately, the combined superposition of these energetic wave clusters, analogous to the dynamics of boiling fluid, gives rise to the total radiative output described by Planck's Radiation Law.

Here (in (9.3) – (9.5.2) and later), we are using not-normalized wavefunctions (having dimensions in SI units), without statistical and probabilistic assumptions, to stay connected with realistic, measurable, experimentally verifiable, and conceptually clear, tangible physics (as elaborated in Chapter 4.0).

If blackbody radiation wave functions are sufficiently close to Gaussian-envelope waves, we could say that in certain frequency interval $\Delta f = F$ equal to six times of involved frequency standard deviation ($F = \Delta f = 6\bar{\sigma}_f$), we will find 99% of mentioned radiative energy, as follows (see about PWDC in Chapters 10. and 4.0),

$$0.99 \cdot \tilde{E} = S\Delta t\int_{0}^{+\Delta f} dR(f,T) = S\Delta t\int_{0}^{6\bar{\sigma}_f} \frac{2\pi f^2}{c^2}\cdot\frac{hf}{e^{hf/kT}-1}df = 0.99\cdot S\cdot\Delta t\cdot\sigma T^4\,(=)\left[Ws = J\right],$$

and 100% of the same energy is equal to,

$$\tilde{E} = S\Delta t\int_0^{+\infty} dR(f,T) = S\Delta t\int_0^{+\infty} \frac{2\pi f^2}{c^2}\cdot\frac{hf}{e^{hf/kT}-1}df = S\cdot\Delta t\cdot\sigma T^4 (=)[Ws = J],$$

We should be able (with proper mathematical processing) to find (for 99%-energy content) the corresponding frequency interval $F = \Delta f = 6\overline{\sigma}_f$, and this will be almost the total frequency length (frequency duration or interval) of the radiative energy function. We could also select that the total time duration T^* of the same radiative function (for 99% of its energy) will be $\Delta t = T^* = 6\overline{\sigma}_t$. Now we can apply Uncertainty relations, and get the temporal duration of the radiative signal as,

$$\tilde{E}\cdot T^* = h\cdot F\cdot T^* \geq \frac{h}{2} \Leftrightarrow F\cdot T^* \geq \frac{1}{2} \Leftrightarrow T^* = 6\overline{\sigma}_t \geq \frac{1}{2F} = \frac{1}{12\overline{\sigma}_f},$$

and if we really measure the total radiated energy during the calculated time interval ($T^* \geq \frac{1}{2F} = \frac{1}{12\overline{\sigma}_f}$), we will get,

$$0.99\cdot\tilde{E} = 0.99\cdot S\cdot T^*\cdot\sigma T^4 \geq 0.99\cdot S\cdot\frac{1}{2F}\cdot\sigma T^4 \cong 0.5\frac{S}{F}\cdot\sigma T^4 (=)[Ws = J],\ (F = 6\overline{\sigma}_f \geq \frac{1}{2T^*}).$$

Here we use simbol T^* as the time interval, and T as the temperature.

Let us now imagine that a spherical "black body", with the external radius R, has certain (total) content of heat (and radiative) energy $\tilde{E}$, which can be considered as an internal matter-waves energy.

We could also approximate the total spatial length of the same body as being $L = 2R$. Resulting (total) linear momentum of the same "blackbody" radiator will be $\tilde{P}$. Proper time duration of the same "blackbody" object will be T^*. Now we can exploit relations between absolute domains durations or lengths, $T^*\cdot\tilde{E} = L\cdot\tilde{P}$ (see (10.2-2.2) from Chapter 10).

$$\begin{bmatrix} T^*\cdot\tilde{E} = L\cdot\tilde{P} \\ \tilde{E} \cong 0.5\frac{S}{F}\cdot\sigma T^4 = 0.5\cdot S\cdot T^*\cdot\sigma T^4 = 2\pi R^2\cdot T^*\cdot\sigma T^4 \\ L = 2R = C\cdot T^*[m],\ S = 4\pi R^2[m^2],\ V = \frac{4\pi}{3}R^3[m^3] \end{bmatrix} \Rightarrow$$

$$\Rightarrow T^*\cdot\tilde{E} = 2R\cdot\tilde{P} \Leftrightarrow T^*\cdot 2\pi R^2\cdot T^*\cdot\sigma T^4 = 2R\cdot\tilde{P}$$

$$\Rightarrow \pi R\cdot(T^*)^2\cdot\sigma T^4 = \tilde{P} \Leftrightarrow \frac{4\pi R^3}{C^2}\cdot\sigma T^4 = \tilde{P} \Rightarrow$$

$$\Rightarrow \tilde{P} = \frac{4\pi R^3}{3}\cdot\frac{3\sigma T^4}{C^2} = \frac{3\sigma}{C^2}\cdot V\cdot T^4[kg\cdot m]$$

We know that all "blackbody" objects are radiating matter-waves (or electromagnetic) energy, which has its own (resulting) linear momentum $\tilde{P}$. This should be the most important part of "radiant energy", as speculated by N. Tesla? On a similar way, instead of considering only a heat energy as a wave energy, we could conceptualize natural radioactivity, considering radioactive emission as an internal source of wave energy of unstable atoms.

....

Since we know universally valid relations between group and phase velocity, and we know de Broglie wavelength expression for wave packets and matter waves, including photons (see about PWDC in chapter 4.1), we should be able to explore velocity-dependent spectral distribution of quantized matter waves (that are inside a Blackbody cavity), for instance,

$$\left\{\begin{array}{l} \lambda = h/p,\ \tilde{E} = hf = pu = E_k\ ,\ u = \lambda f = \dfrac{v}{1+\sqrt{1-(\frac{v}{c})^2}} = \dfrac{\omega}{k} = \dfrac{\tilde{E}}{p}\ , \\ v = \dfrac{2u}{1-\frac{uv}{c^2}} = u - \lambda\dfrac{du}{d\lambda} = -\lambda^2\dfrac{df}{d\lambda} = \dfrac{d\omega}{dk} = \dfrac{d\tilde{E}}{dp}\ ,\ \omega = 2\pi f\ , \\ 0 \le 2u \le \sqrt{uv} \le v \le c \end{array}\right\} \Rightarrow \qquad (9.5.3)$$

$$\Rightarrow \boxed{d\tilde{E}_i = hdf_i = v_i dp_i = d(p_i u_i) = c^2\, d\tilde{m}_i = dE_{ki} = \psi_i^2(t)dt = \tilde{F}dr.}$$

The matter-waves and wave-particle duality foundational concepts discussed in Chapter 4.1 (see T.4.1) of this book have already laid the groundwork for this topic. Specifically, Planck's thermal radiation law should be reconsidered because of the interplay between group and phase velocities, along with the interactions, superpositions, diffractions, and interferences of the involved particles and matter-wave packets. For narrow-band, energy-quantized photons or wave packets, it is generally more appropriate to use differential energy balance relations $d\tilde{E}_i = h\cdot df_i = c^2\cdot d\tilde{m}_i = \psi_i^2(t)dt$. This approach offers a broader framework for understanding photon quantization.

In the context of a blackbody cavity, there are numerous "mass-energy-moments" and electromagnetic interactions at play, involving various matter waves. As a result, the dielectric and magnetic constants within and around the cavity $\varepsilon, \mu,\ c = 1/\sqrt{\varepsilon\cdot\mu}$ are unlikely to remain stable. Consequently, the speed of light c for the involved photons may also vary significantly. However, Planck's Radiation Law traditionally assumes that these photons travel at a constant, maximum value of c.

4.1. Interacting and coupled wave groups inside the Blackbody cavity

$\left\{\begin{array}{l} v = u - \lambda\dfrac{du}{d\lambda} = -\lambda^2\dfrac{df}{d\lambda}, \\ u = \dfrac{v}{1+\sqrt{1-v^2/c^2}} = \dfrac{u-\lambda\frac{du}{d\lambda}}{1+\sqrt{1-v^2/c^2}} = \dfrac{-\lambda^2\frac{df}{d\lambda}}{1+\sqrt{1-v^2/c^2}} \end{array}\right\} \Rightarrow$	$\left\{\begin{array}{l} \dfrac{du}{u} = -\left(\dfrac{d\lambda}{\lambda}\right)\sqrt{1-v^2/c^2} = \dfrac{df}{f}\cdot\dfrac{\sqrt{1-v^2/c^2}}{1+\sqrt{1-v^2/c^2}} \\ \dfrac{df}{f} = -\left(\dfrac{d\lambda}{\lambda}\right)(1+\sqrt{1-v^2/c^2}) \end{array}\right\}$
Non-relativistic group velocities: (Lower and moderate temperatures)	Relativistic group velocities: (Very high temperatures)

$v << c \Rightarrow v \cong 2u, \sqrt{1-v^2/c^2} \cong 1,$ $\left\{\frac{du}{u} \cong -\frac{d\lambda}{\lambda} \cong \frac{df}{2f}\right\} \Rightarrow$ $\left\{\ln\left\|\frac{u}{u_0}\right\| \cong -\ln\left\|\frac{\lambda}{\lambda_0}\right\| \cong \frac{1}{2}\ln\left\|\frac{f}{f_0}\right\|\right\} \Rightarrow$ $v \cong 2u = 2\lambda f$	$v \approx c \Rightarrow v \approx u \approx c, \sqrt{1-v^2/c^2} \cong 0$ $\frac{du}{u} \cong 0, \frac{df}{f} \cong -\frac{d\lambda}{\lambda}$

The matter-wave states inside a blackbody cavity are not limited to photons and electromagnetic standing waves. The cavity also contains electrons, ions, other charged and neutral particles (both micro and macro), matter waves, secondary emissions, scattering products, and plasma states. All structured and sufficiently stabilized matter, such as atoms, electrons, protons, and neutrons, exist in quantized states, forming well-packed, self-contained standing matter waves. This concept extends even to planetary and solar systems, as discussed in the second chapter of this book.

It is well known that the electrons and photons produced by heating are inherently synchronized, coupled, and mutually convertible. Electrons emit and absorb photons due to their quantized, resonant, and standing-wave structures. The internal and motional matter-wave states generated by thermal motions and associated electromagnetic activity naturally cover a broad range of velocities, temperatures, and frequencies.

Given this complex reality within the blackbody cavity, with its many different participants and their interactions, interferences, and superpositions, it is important to question whether the original assumptions underlying the development of Planck's radiation law (9.2) are fully conceptually clear, physically realistic, and entirely relevant. While Planck's final mathematical formula for blackbody radiation is highly accurate, the logic, assumptions, and mathematics leading to it should be viewed as practical and effective curve-fitting rather than a complete physical model. In essence, Planck's law represents a successful mathematical interpolation based on known results, but it does not provide a definitive answer to the nature of time, frequency, and spatial-temporal shapes and energy of the quantized wave functions generated by blackbody radiation (for more on this, see [161], *Hidden Variables: The Elementary Quantum of Light. A Significant Question in Quantum Foundations*).

Externally radiated photons outside the blackbody cavity have constant group and phase velocities $v \cong u \cong \lambda f \cong c$. However, this may not hold true inside the cavity. Therefore, the wave functions of quantized wave packets (including photons) are likely to differ in shape across space, time, and frequency domains, as well as in their physical nature, inside and outside the blackbody cavity.

In conclusion, the concept of blackbody radiation remains largely incomplete, oversimplified, and partially inaccurate. It continues to be an open and unresolved area of study.

The imaginative and challenging facts about blackbody radiation and possible relations to N. Tesla Dynamic Theory of Gravity

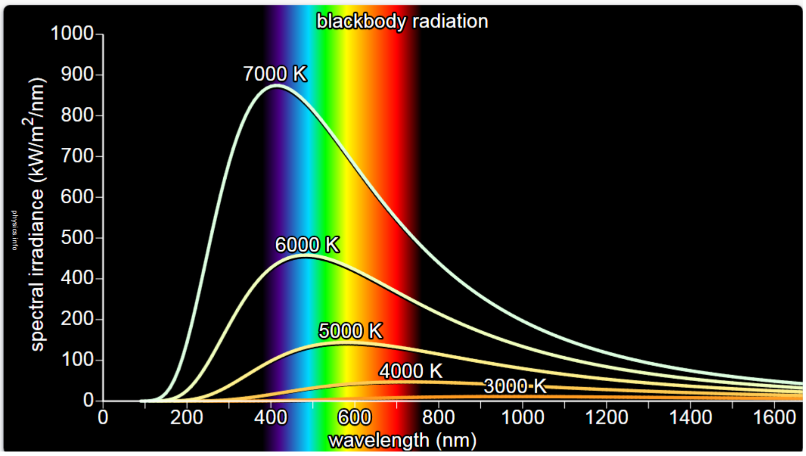
(taken from http://physics.info/planck/): The Physics Hypertextbook Opus in profectus

- We know the shape of the distribution,
- The peak shifts according to Wien's law,
- The Stefan-Boltzmann law describes the total power output.

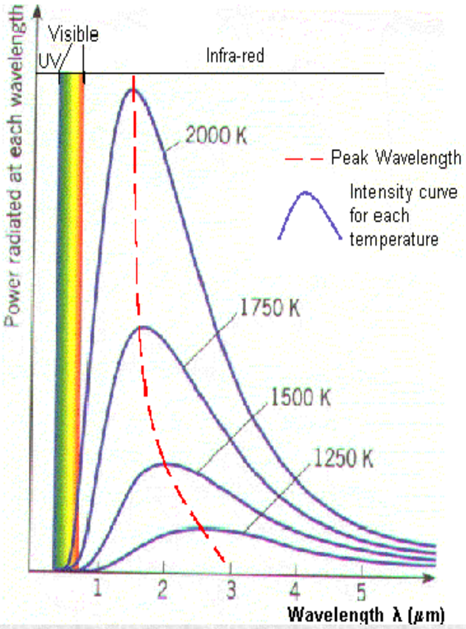
Wien's Displacement Law is in full mathematical agreement with Planck's Radiation Law (9.2) and accurately describes the peak wavelength $\lambda_{max.}$ of blackbody radiation, where the monochromatic emissive power reaches its maximum. However, while the Planck-Einstein photon quantization formula is widely used, it is only conditionally valid and has been incorporated into Blackbody Radiation modeling in a somewhat speculative manner (see illustrations below for reference). As a result, while the derived mathematical outcomes are practically correct and applicable, the underlying concepts and explanations for elementary energy quanta and their corresponding spatial-temporal signal shapes require significant re-evaluation.

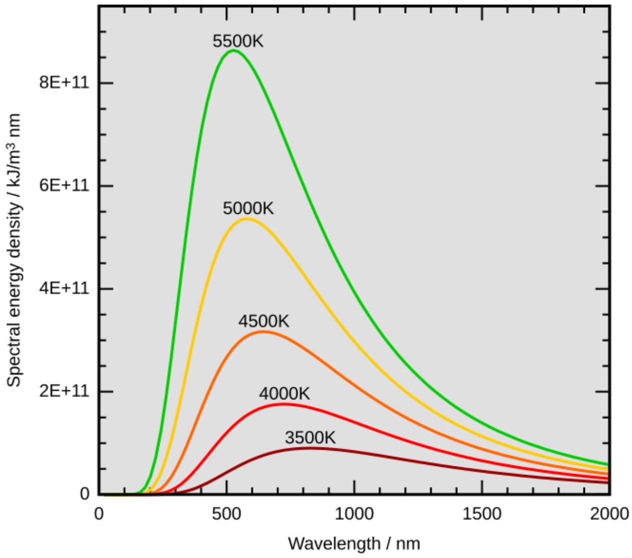
In essence, the current formulation of Blackbody Radiation Law appears to be incomplete in its conceptual foundation and may need a more robust theoretical reformation. Such an approach could potentially lead to transformative changes in the understanding and interpretation of Orthodox Quantum Theory.

$$\begin{aligned}
&\lambda_{max.} = \frac{2897.8 \cdot 10^{-6}}{T} = \frac{h}{\tilde{p}_{min}} = \lambda_{peak} (=) [m], \quad T = T_{min.} (=) [K] \\
&u = \lambda f \leq \lambda_{max.} \cdot f \leq c, \Rightarrow u = \frac{E_k}{p} = \frac{\tilde{E}}{\tilde{p}} \leq \frac{f}{T} \cdot 2897.8 \cdot 10^{-6} \leq c \, (=) \left[\frac{m}{s}\right], \; p = \tilde{p} = p_{min.}, \\
&\Rightarrow f \leq \frac{cT}{2897.8 \cdot 10^{-6}} = f_{max}, \; E_k = \tilde{E} = pu = hf \leq \frac{hcT}{2897.8 \cdot 10^{-6}} \leq c\tilde{p}_{max} = \tilde{E}_{max} (=) [J], \\
&\Rightarrow \tilde{p}_{max} \geq \frac{hT}{2897.8 \cdot 10^{-6}} = \frac{h}{\lambda_{max.}} = \tilde{p}_{min} \left(= \frac{hT}{2897.8 \cdot 10^{-6}} \right) \Rightarrow \\
&T = \frac{2897.8 \cdot 10^{-6}}{h} \tilde{p}_{min} = T_{min} \leq \frac{2897.8 \cdot 10^{-6}}{h} \tilde{p}_{max} = \frac{2897.8 \cdot 10^{-6}}{ch} \tilde{E}_{max} = \frac{2897.8 \cdot 10^{-6} \cdot f_{max}}{c} (=) [K]
\end{aligned} \tag{9.6}$$

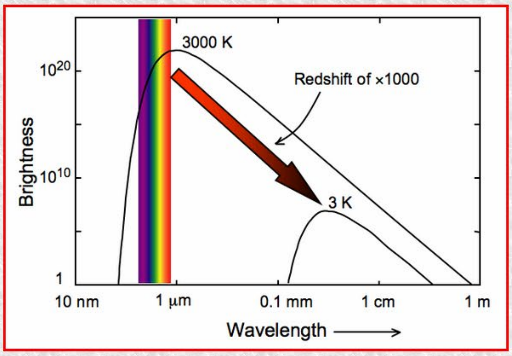


Taken from the Internet; -Blackbody Radiation
(http://physics.info/planck/): The Physics Hypertextbook Opus in profectus





Redshift Preserves a Thermal Spectrum



Redshift preserves a black body spectrum, simply lowering the temperature in proportion to the scale factor: $T_{now} = a_{emit} T_{emit}$. For example, the CMB was created at 3000K, at a scale factor of a = 10^{-3}, so today's CMB has spectrum T = 10^{-3} %#215; 3000 K = 3 K.

The graphs show:

- As the temperature increases, the peak wavelength emitted by the Blackbody decreases.
- As the temperature increases, the total energy emitted increases, because the total area under the curve increases.
- Blackbody radiation (https://en.wikipedia.org/wiki/Wien's_displacement_law) as a function of wavelength for various absolute temperatures. Each curve is seen to peak at a somewhat different wavelength; Wien's law describes the shift of that peak in terms of temperature.
- **Wien's displacement law** states that the Blackbody radiation curve for different temperatures peaks at a wavelength inversely proportional to the temperature. The shift of that peak is a direct consequence of the Planck radiation law, which describes the spectral brightness of Blackbody radiation as a function of wavelength at any given temperature. However, it had been discovered by Wilhelm Wien several years before Max Planck developed that more general equation, and describes the entire shift of the spectrum of Blackbody radiation toward shorter wavelengths as temperature increases.
- Formally, Wien's displacement law states that the spectral radiance of Blackbody radiation per unit wavelength peaks at the wavelength λ_{max} given by: $\lambda_{max.} = \frac{b}{T}$,

  where *T* is the absolute temperature in kelvin. *b* is a constant of proportionality called *Wien's displacement constant*, equal to 2.8977729(17)×10^{-3} m K[1].

Frequency-dependent formulation: For spectral flux considered per unit frequency df (in hertz), Wien's displacement law describes a peak emission at the optical frequency f_{max} given by:

$$f_{max} = \frac{\alpha}{h} kT \approx (5.879 \times 10^{10} \text{ Hz/K}) \cdot T$$

where *α* ≈ 2.821439... is a constant resulting from the numerical solution of the maximization equation, *k* is the Boltzmann constant, *h* is the Planck constant, and *T* is the temperature (in kelvin). This frequency does not correspond to the wavelength from the earlier formula, which considered the peak emission per unit wavelength.

Blackbody radiation does not depend on the type of object emitting it. An entire spectrum of blackbody radiation depends on only one parameter, the temperature, T.

A Blackbody is an ideal body which allows the whole of the incident radiation to pass into itself (without reflecting the energy) and absorbs within itself this whole incident radiation (without passing on the energy).

Wien's displacement law

- Shows how the spectrum of Blackbody radiation at any temperature is related to the spectrum at any other temperature. If we know the shape of the spectrum at one temperature, we can calculate the shape at any other temperature.
- A consequence of Wien's displacement law is that the wavelength at which the intensity of the radiation produced by a Blackbody is at a maximum, λ_{max}, it is a function only of the temperature.

Taken from the Internet (http://de.slideshare.net/FanyDiamanti/blackbody-radiation-11669156): BLACKBODY RADIATION. Authors: DISUSUN OLEH, Rahmawati Th. Diamanti, Ivone Pudihang, Recky Lasut, Aulinda Tambuwun, Deyvita Montolalu

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Conclusions and hypothetical proposals regarding N. Tesla Dynamic Force of Gravity:

1. The author speculates that Nikola Tesla's concept of radiative energy [97] is closely related to blackbody radiation, as well as to other forms of electromagnetic, cosmic, and electromechanical radiation. The Universe, or Cosmos, primarily exhibits electromagnetic and electromechanical oscillations, manifesting through various cosmic structures, particles, masses, atoms, and states of matter (solid, liquid, and plasma), with dualistic wave-particle and matter-wave behaviors.

2. Let us consider that all masses, atoms, and objects in the Universe are structurally resonating and communicating with each other, both internally and externally. This can be likened to well-coupled mechanical and electromagnetic resonators exchanging electromagnetic and mechanical radiation and vibrations. To simplify our understanding of gravitation, we can extend Niels Bohr's atomic model to include quantized electromagnetic energy exchanges, not just within electron orbits and clouds, but also between electrons and the atomic nucleus, as well as within the nucleus itself. Furthermore, these exchanges occur between any segment of an atom and its external environment, including other masses and atoms. This concept aligns with R. Boskovic's Universal Natural Force theory [6] (see Chapters 2 and 8 for more details).

Consequently, all atoms and masses in the Universe are synchronized, interconnected, and continuously exchanging electromagnetic and matter-wave energy, along with the associated mechanical and electromagnetic moments. It is speculated here that this radiative energy is closely related to blackbody radiation. Specifically, thermal electromagnetic radiation in the infrared domain may be generated by the stationary resonant states of the internal mechanical and electromagnetic resonances of heated masses and their atoms.

We understand that the spectral content and overall spectral complexity of any object can be manifested in at least two ways. The first is its electromagnetic spectrum, which is largely described by Planck's blackbody radiation law (via effective curve-fitting). This spectrum also extends to frequencies that are often difficult to detect, both exceptionally low and extremely high acoustic and electromagnetic oscillations. These are frequently overlooked because electromagnetic vibrations simultaneously produce mechanical or acoustic vibrations. While some mechanical, acoustic, and phonon-related vibrations exist within the domain covered by blackbody electromagnetic radiation, they are not always easily detectable or specifically measured.

The key point is that blackbody and electromagnetic radiation from a given body encompass the same frequency intervals as the associated acoustic, mechanical, and phonon vibrations, though they have different spectral distributions and amplitudes. As detailed in Chapters 6 and 10, the spatial and temporal dimensions, as well as the spectral characteristics of any energy-momentum state, are interrelated, correlated, and proportional. This means that all such states should cover the same frequency intervals, both temporally and spatially, although with different amplitude-frequency distributions and resonant modes. This concept can be analyzed using Finite Element Analysis to study the resonant states of a particular object. The electromagnetic and *mechanical aspects of the Universe are thus inherently united, even though a comprehensive unification theory for these fields is yet to be established.*

3. The domain where gravitational force, produced by radiative energy within stationary and standing planetary matter waves, intersects with blackbody radiation is significant for all matter waves with wavelengths greater than a specific threshold $\lambda_{max.} = \frac{b}{T} = \frac{2897.8 \cdot 10^{-6}}{T}$*. In this context, matter waves, including electromagnetic waves or blackbody radiation, which facilitate communication between atoms and other masses, exhibit lower frequencies compared to those of photons and electromagnetic exchanges within an atom. This phenomenon may be related to gravitation.*

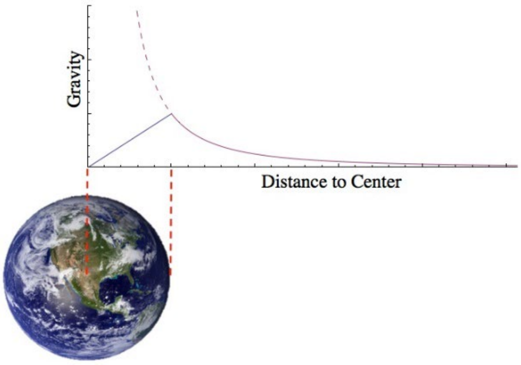
The accompanying images on the next pages illustrate the estimated domains of gravity-related electromagnetic, bidirectional radiative energy exchanges, as encircled in the diagrams. Additionally, see the extended Bohr atomic model in Chapter 8, under "8.3. Structure of the Field of Subatomic and Gravitation-related Forces," where similar concepts regarding inward and outward electromagnetic energy exchanges (inside and outside all atoms) are presented, reflecting the ideas of N. Tesla and R. Boskovic. Further arguments about the relationship between radiative energy and gravitation can be found in Chapter 2, "2.2.1. What Gravitation Really Is."

....

Black body radiation and Gravitation force comparisons

Let us start with the following Citation from Internet:
https://www.askamathematician.com/2018/06/q-why-does-gravity-pull-things-toward-the-center-of-mass-whats-so-special-about-the-center-of-mass/
Q: Why does gravity pull things toward the center of mass? What's so special about the center of mass? | Ask a Mathematician / Ask a Physicist

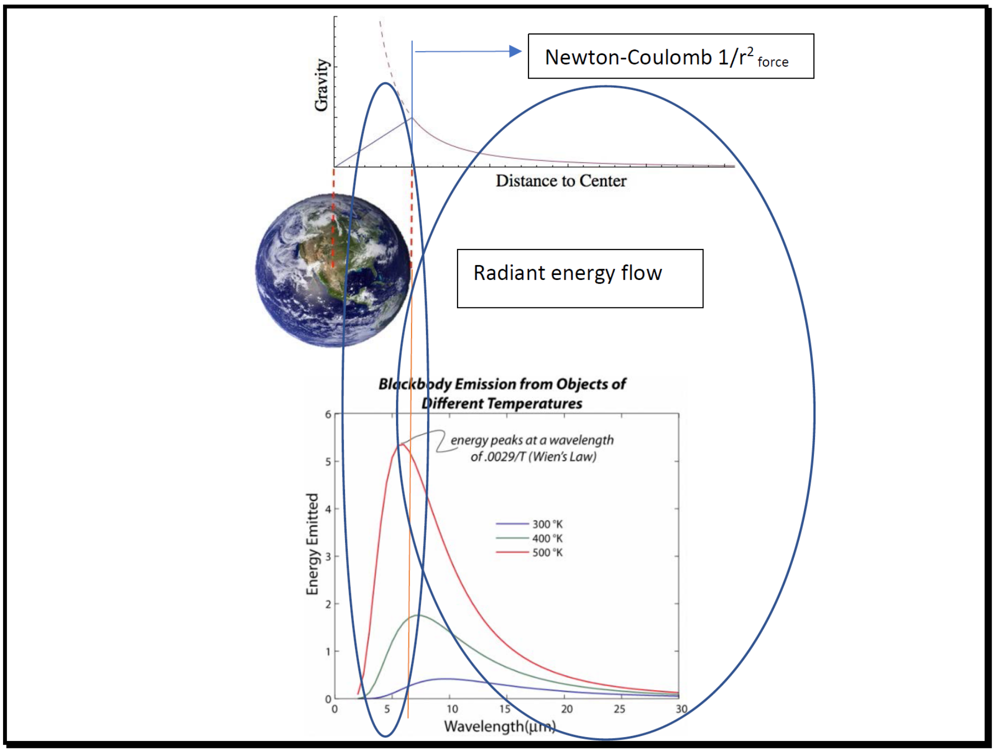
.... "If you're inside of a sphere of mass, only the layers below you count toward the gravity you feel. So, the closer you are to the center, the less mass is below you, and the less gravity there is. If you were in an elevator that passed through the Earth, you'd know you were near the center because there'd be no gravity. With the same amount of mass in every direction, every atom in your body would be pulled in every direction equally, and so not at all. A sort of gravitational tug of war stalemate.
Newton's Universal Law of Gravitation applies to everything (hence the "universal"). But instead of applying to the centers of mass of pairs of big objects, it applies to every possible pair of pieces of matter. In the all-to-common-in-space case of spheres, we can pretend that entire planets and stars are points of mass and apply Newton's laws to those points, but only as long as we don't need perfection (which is usually). As soon as things start knocking into each other, or the subtle effects of their not-sphere-ness become important, you're back to carefully tallying up the contribution from every chunk of mass".



"As you approach a sphere of mass gravity increases by the inverse square law but drops linearly inside the sphere. If the density varies (which it usually does), that straight line will bow up a bit."

It is well-established that both electromagnetic and gravitational waves are transverse oscillations and that both fields adhere to the "1/r² central force" law. This common characteristic suggests that their origins might be related—potentially electromagnetic in nature and possibly connected to heat radiation or Tesla's concept of Radiant Energy.

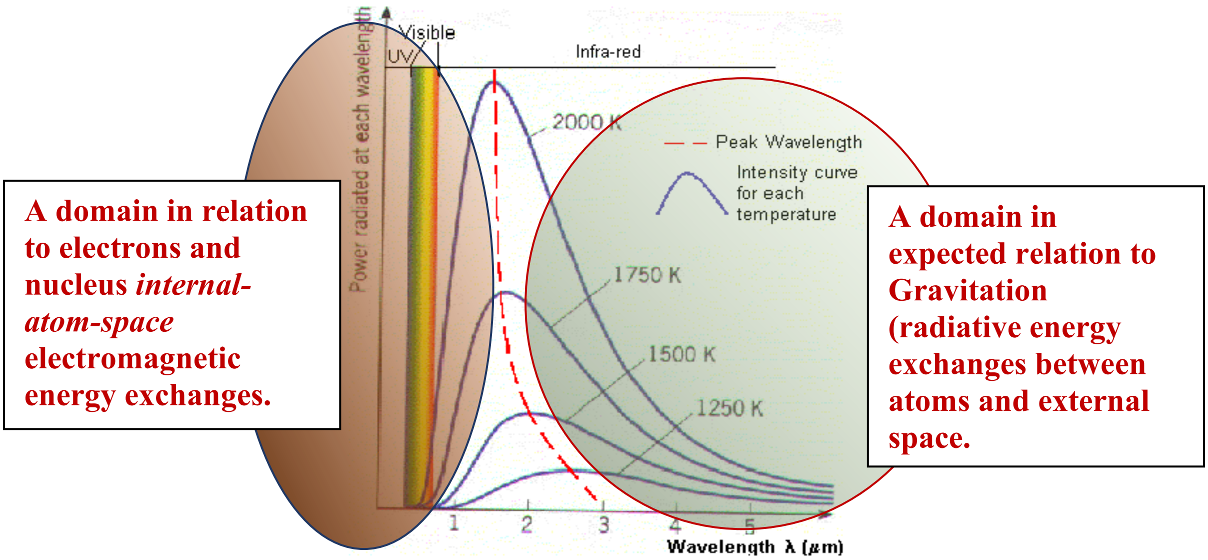
Let us now engage in a creative and imaginative exploration, with a flexible intellectual approach, to draw an indicative analogy between the gravitational force curve, the curve of blackbody electromagnetic emission, and the cosmic background radiation from the Big Bang. The comparison is illustrated in the accompanying images below. The goal here is to identify potential imprints of gravitational influences within the curves of blackbody radiation.

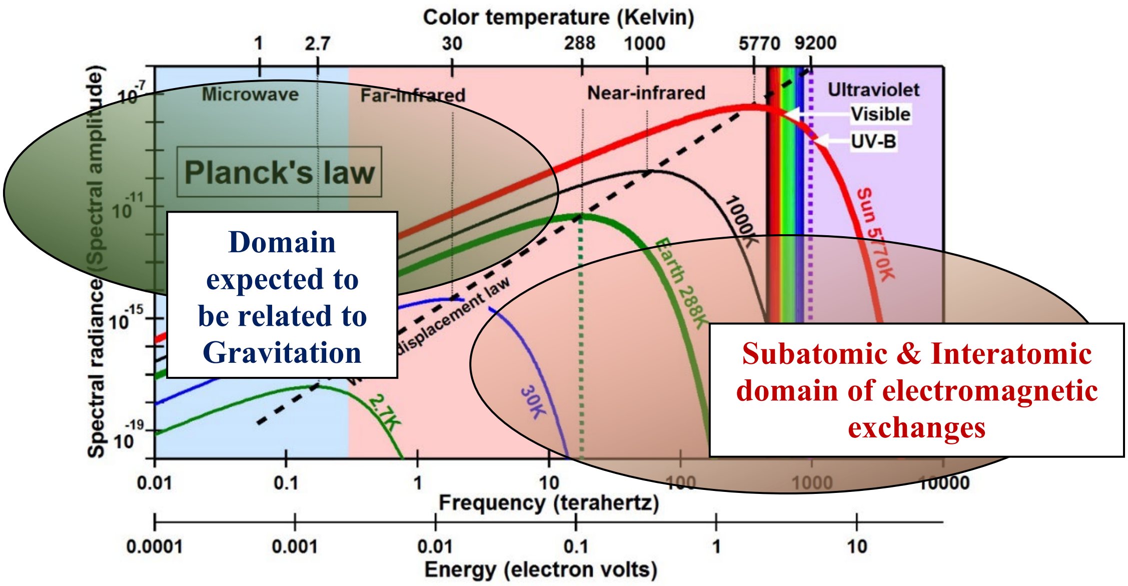


Of course, in case of gravitation, horizontal axis is the distance between *attracting masses centers*, and Blackbody emission curve is related only to wavelengths of radiated emission, and comparison and correlation between relevant curves presents a challenging task, but intuitively we can feel some encouragement for establishing certain conceptual comparison. ***Basically, here we are hypothetically suggesting that Blackbody radiation is involved in Tesla's Radiant energy flow***.

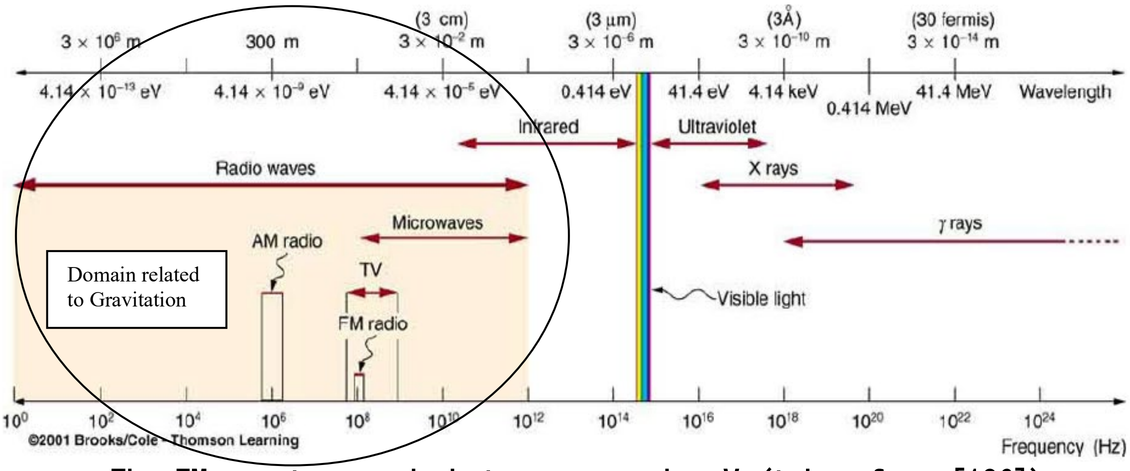
By continuing with similar brainstorming, we can intuitively (and hypothetically) create some conceptual and analogical assignments, or assumptions, indicating where we could expect detection of gravitation-related electromagnetic emissions (see the following illustrations with relevant and indicative comments).

Read more at http://phys.org/news/2013-07-blackbody-stronger-gravity.html#jCp http://phys.org/news/2013-07-blackbody-stronger-gravity.html, **[100]** M. Sonnleitner1,2, M. Ritsch-Marte2, and H. Ritsch1:
"Perfectly non-reflective objects, called blackbodies, produce blackbody radiation when at a uniform temperature. Although the properties of blackbody radiation depend on the blackbody's temperature, this radiation has always been thought to have a net repulsive effect. Now in a new study, scientists have theoretically shown that blackbody radiation induces a second force on nearby atoms and molecules that are usually attractive and, quite surprisingly, even stronger than the repulsive radiation pressure. Consequently, the atoms and molecules are pulled toward the blackbody surface by a net attractive force that can be even stronger than gravity. The new attractive force—which the scientists call the "blackbody force"—suggests that a variety of astrophysical scenarios should be revisited".

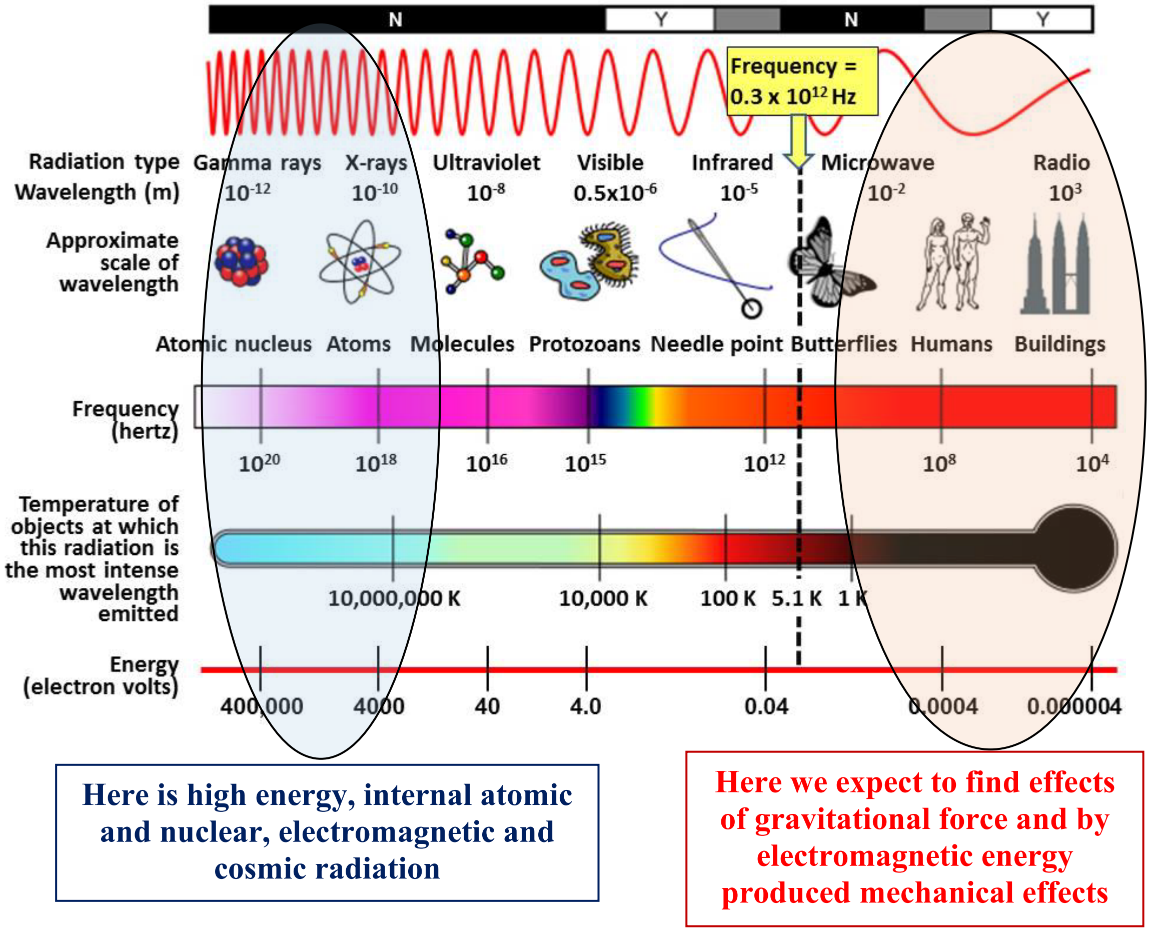




Blackbody Radiation and Gravitation
(taken from https://ozonedepletiontheory.info/energy-not-additive.html)



The EM spectrum and photon energy in eV (taken from [126])



Properties of the Electromagnetic Spectrum in relation to Gravitation
(From http://ozonedepletiontheory.info/ImagePages/EM-spectrum-properties.html
https://ozonedepletiontheory.info/energy-not-additive.html)

Citation from https://ozonedepletiontheory.info/energy-not-additive.html: "Planck's law describes the spectral radiance (the energy contained in electromagnetic radiation at each frequency) radiated by a Blackbody of matter as a function of the temperature of the surface of the body (in degrees Kelvin). Note that when a temperature is increased, the spectral radiance is increased for each frequency and the peak spectral radiance moves to a higher frequency".

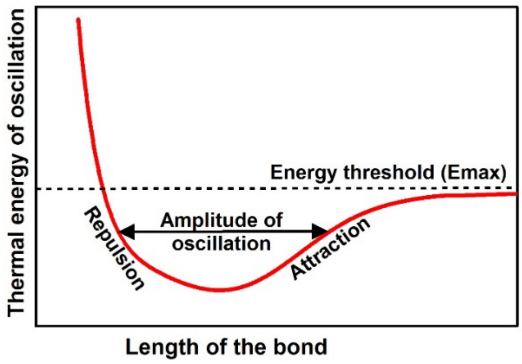
Thermal energy

The chemical bonds that hold atoms together to form matter are not rigid. They are observed to oscillate about an energy minimum between electrodynamic repulsive forces pushing the atoms apart and electrodynamic attractive forces pulling the atoms together. As the thermal energy of oscillation increases, the amplitude of oscillation increases until at some energy threshold (E_{max}) the bond comes apart. Energy (frequency) may increase in discrete steps as higher and higher normal modes (frequencies) of oscillation are activated. One such anharmonic atomic oscillator exists for every normal mode of oscillation of every degree of freedom of every bond in the matter. Each oscillator has a characteristic resonant frequency and an amplitude of oscillation, the latter of which increases with increasing temperature (thermal energy). Because the oscillator is spatially asymmetric, the average length of the bond *increases with increasing thermal energy so that the volume of the matter expands with increasing temperature, something that is well observed.*

An atomic oscillator of this type is asymmetric (anharmonic) because the force of repulsion increases very rapidly as charges are pressed together whereas the force of attraction decreases much more slowly with distance as opposite charges separate.

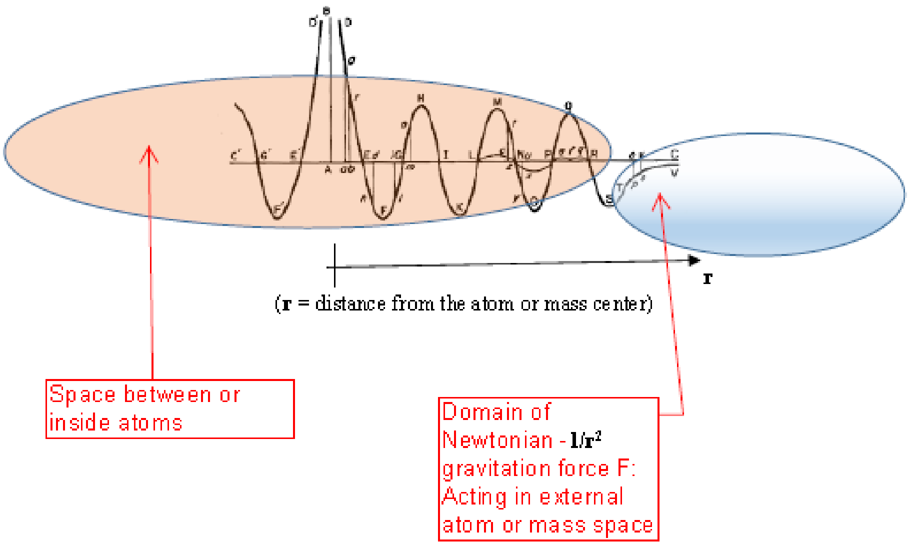
An atomic oscillator of this type is frictionless and thus can oscillate for an exceptionally long time. The only way to add energy to such an oscillator, or to subtract energy from it, is through resonance. When the amplitude of oscillation at a specific frequency of one bond is larger than the amplitude of oscillation at the same frequency of an adjacent bond, the bond with the larger amplitude will "give up" energy to the bond with the lower amplitude until the amplitudes for both bonds are equal. The rate of energy transfer increases with increasing difference in amplitudes.

We can find that atomic and molecular chemical bonds between matter particles are also surprisingly familiar and like R. Boskovic Natural Force (see the pictures below).



The chemical bonds that hold atoms together to form matter are observed to oscillate about an energy minimum between electrodynamic repulsive forces pushing the atoms apart and electrodynamic attractive forces pulling the atoms together. As the thermal energy of oscillation increases, the amplitude of oscillation increases until at some energy threshold (E_{max}) the bond comes apart (taken from the Internet: https://ozonedepletiontheory.info/energy-not-additive.html).

R. Boskovic Universal Natural Force Law (see below) also describes a familiar kind of standing waves structured forces.



Rudjer Boskovic's Universal Natural Force Function

If we consider gravitation as a form of electromagnetic force, it is natural that both forces would adhere to the same Coulomb-Newton $1/r^2$ law, as they could be manifestations of the same underlying force. When we focus specifically on what we recognize as gravitation, it appears to be the attractive component of R. Boskovic's universal natural force curve (depicted on the right side of the accompanying image). Typically, we expect natural forces to have both positive and negative, or attractive and repulsive, components. However, in the case of Newtonian gravitation, the repulsive part remains unidentified.

In contrast, Rudjer Boskovic's universal natural force theory suggests that repulsive, balancing forces exist within the oscillatory structure of masses and atoms (shown on the left side of the image). These forces are organized as periodic, resonant, standing matter-waves or "standing masses and energy agglomerations." The existence of such attractive and repulsive forces can also be experimentally demonstrated using ultrasonic resonators, where nodal zones exhibit only attractive forces, and anti-nodal zones display only repulsive forces. For more on gravitation and R. Boskovic's theories, refer to the second chapter of this book.

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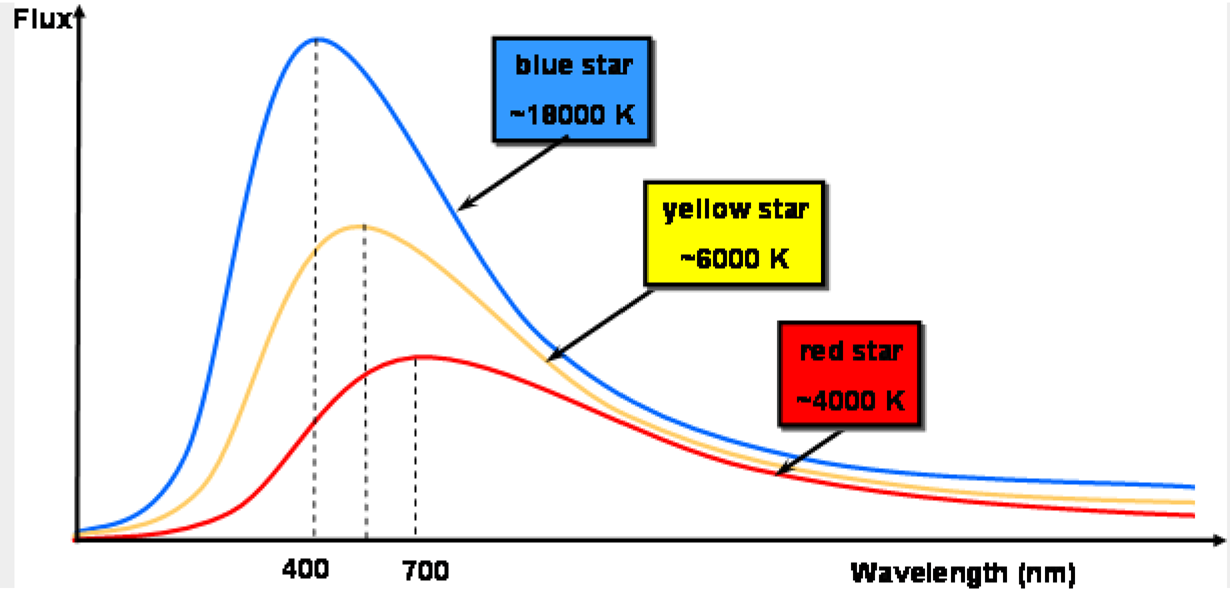
The Uncertainty Relations (or inequalities between original and spectral domains) in mathematics are universally valid and apply to both the micro and macro realms of physics. They are not necessarily limited to the probabilities and statistical distributions of microphysical events, as contemporary Quantum Theory suggests. By applying these Uncertainty Relations appropriately, we can gain new insights into the durations and spatial extents of Blackbody radiation concerning its energy content, time, frequency, and spatial dimensions.

The implications of properly understanding and applying these Uncertainty Relations could lead to innovative perspectives in Cosmology, Background or Relict Radiation, Big Bang theories, natural forces, matter structure, and Gravitation, among other areas. Further exploration of these topics can be found in Chapters 6 and 10.

Citation from, [100]:
http://phys.org/news/2013-07-blackbody-stronger-gravity.html#jCp
Scientific Article: M. Sonnleitner, et al. "Attractive Optical Forces from Blackbody Radiation." *PRL* 111, 023601 (2013). DOI: 10.1103/PhysRevLett.111.023601
The blackbody objects are perfect not-reflectors that produce constant radiation when they are at a uniform temperature. Thus, the properties of a blackbody depend on its temperature, thinking that this radiation would have a repulsive effect. Now a new, scientists have demonstrated theoretically that blackbody radiation induces a second force in atoms and molecules that are near its surface, which is attractive and stronger than the repulsive radiation pressure. Consequently, the atoms and molecules are pulled to the surface of the blackbody by a force which may be greater than gravity. The new attractive force - what scientists describe as a "blackbody *force*" - suggests that a variety of astrophysical scenarios need to be revisited.



Scientists identify a few interesting results in their formulation. First, this force decays with the third power of the distance to the blackbody ($F \propto 1/r^3$). Second, it is stronger for small objects. Third, the force is stronger for warmer objects. Above a few thousand degrees Kelvin, the force of attraction changes to repulsion.
In their study, scientists demonstrated that the strength of the blackbody in a grain of dust, at a temperature of 100 K, is much stronger than the gravitational force in this grain. However, for a massive star at a temperature of 6000 K blackbody force is much weaker than the gravitational force.

9.1. Wave Function and dimensions of Photon as an Energy Quantum

Since Planck's radiation law suggests that thermal radiation consists of discrete wave packets or electromagnetic energy quanta, known as photons, it is valuable to determine the most probable waveforms that describe such photons across temporal, spatial, and frequency domains. This approach allows us to assess the mathematical feasibility and physical significance of these descriptions, given that the Planck-Einstein photon energy formula, $\tilde{\mathbf{E}} = \mathbf{hf}$, is not universally applicable to arbitrarily assumed quantization scenarios. Therefore, it is crucial to understand why it performs well in certain cases and fails in others.

In Quantum Theory, it is generally accepted that a single photon's energy is given by the product of Planck's constant h and the photon's frequency f. However, defining, isolating, and describing a single photon as a specific waveform remains a challenge. This section aims to explore the mathematical underpinnings of Planck's radiation law by analyzing the nature of photons with energies represented as $\tilde{\mathbf{E}} = \mathbf{hf} = \frac{\mathbf{h}}{2\pi}\omega$.

Historically, the energy form of Planck's radiation law was derived through empirical curve fitting to match the experimentally observed blackbody radiation spectrum. The concept of a photon as an electromagnetic wave packet was later confirmed through the pioneering works of Planck, Compton, Einstein, de Broglie, and Bohr, establishing the photon as a carrier of a single quantum of electromagnetic energy ($\tilde{\mathbf{E}} = \mathbf{hf}$).

Given this understanding, a photon should also be representable by a time-domain wavefunction such as $\psi(\mathbf{t}) = \mathbf{a}(\mathbf{t})\cos\varphi(\mathbf{t})$, modeled as an Analytic signal with a narrow frequency band and a Gaussian or bell-curve-shaped amplitude envelope (as elaborated in Chapters 4.0 and 10). This analysis seeks to define a family of photon wavefunctions or wave-packet representations that naturally embody a single quantum of energy, thereby supporting Planck's radiation law and clarifying the circumstances under which the concept of quantized photons is valid and applicable.

For further details on photon conceptualization, refer to Chapter 4.1 under "4.1.1.1. Photons and Particle-Wave Dualism" and "T.4.0. Photon – Particle Analogies".

The Analytic Signal presentation (of wave packets) gives the chance to extract immediate (or instant) signal amplitude $\mathbf{a}(\mathbf{t})$, phase $\varphi(\mathbf{t})$, and frequency $\omega(\mathbf{t}) = \frac{\partial\varphi(\mathbf{t})}{\partial \mathbf{t}} = 2\pi\mathbf{f}(\mathbf{t})$ (see also (4.0.47) - (4.0.54) from the Chapter 4.0), and all of that is analogically applicable in a spatial domain. Here we also consider that Planck's energy quantum effectively relates to the mean matter-wave, or electromagnetic wave frequency, $\overline{\omega} = \langle\omega(\mathbf{t})\rangle = 2\pi\langle\mathbf{f}(\mathbf{t})\rangle = 2\pi\overline{\mathbf{f}}$, of certain narrow-band photon wavefunction $\Psi(\mathbf{t})$. By considering immediate (or instant) frequency definition $\omega(\mathbf{t}) = \frac{\partial\varphi(\mathbf{t})}{\partial \mathbf{t}}, \varphi(\mathbf{t}) = \mathbf{arctg}\frac{\hat{\Psi}(\mathbf{t})}{\Psi(\mathbf{t})}$, we could also analyze the meaning of the analogous time-duration function in a frequency domain, $\tau(\omega) = \frac{\partial\Phi(\omega)}{\partial \omega}, \Phi(\omega) = \mathbf{arctan}\frac{\mathbf{U}_s(\omega)}{\mathbf{U}_c(\omega)}$, and try to find how they are mutually related. For instance, $\omega(\mathbf{t}) = \frac{\partial\varphi(\mathbf{t})}{\partial \mathbf{t}}$ $(\leq ? \geq)$ $\tau(\omega) = \frac{\partial\Phi(\omega)}{\partial \omega}$ (what should be related to Uncertainty relations, well known in Quantum Theory and Spectrum Analysis, for instance, $\omega(\mathbf{t}) \cdot \tau(\omega) \geq \pi$, or $\Delta\omega(\mathbf{t}) \cdot \Delta\tau(\omega) \geq \pi$, or $\sigma_{\bar{\omega}} \cdot \sigma_{\mathbf{T}} \geq \pi \ldots$).

In other words, we assume that photons or elementary energy quanta, as wave packets, possess finite durations in time, spatial length, and frequency. This approach suggests that a wave energy quantum, or wave packet, should be represented using frequency band-limited wave functions. Consequently, we can view these time-domain elementary wavefunctions, each carrying a single quantum of energy $\tilde{\mathbf{E}} = \mathbf{h}\bar{\mathbf{f}}$, as “basis functions” for Spectral Signal Analysis and Synthesis.

Currently, the exact analytic forms of photon wavefunctions in the temporal, spatial, or frequency domains are still unknown. What we do know is that these wavefunctions must contain a specific energy content equal to $\tilde{\mathbf{E}} = \mathbf{h}\bar{\mathbf{f}}$ and should be optimally concentrated in their time, space, and frequency domains. This implies that they likely belong to the family of narrow-band Gaussian pulses or Gaussian window functions, which include Gaussian and Analytic Signal wavelets. Such waveforms are ideal because they are optimally concentrated and finite across all three domains, as discussed in Chapters 4.0 and 10.

To accurately represent photon energy, we define it as the energy of a photon’s analytic signal wavefunction in both time and frequency domains, determining the mean photon frequency and applying Planck’s formula for photon energy. For further details, see the derivation and discussion in equations (10.2-3).

Photon energy we can define as an energy of a photon’s analytic signal wavefunction both in time and frequency domains (meaning, we determine a mean photon frequency, and later, Planck photon energy), as follows (see the same in (10.2-3)).

$$\tilde{E} = \int_{[T]} \Psi^2(t)dt = \int_{[T]} [a(t)\cos\varphi(t)]^2 dt = \frac{1}{2}\int_{[T]} a^2(t)dt = \frac{1}{\pi}\int_0^{\infty} A^2(\omega)d\omega = \frac{h}{2\pi}\overline{\omega} = h\overline{f} = h\frac{u}{\lambda} = pu = \tilde{m}c^2,$$

$$\omega(t) = \frac{\partial\varphi(t)}{\partial t} = 2\pi f(t) = \dot{\varphi}(t) = \frac{\Psi(t)\dot{\hat{\Psi}}(t) - \dot{\Psi}(t)\hat{\Psi}(t)}{a^2(t)} = \mathrm{Im}\left[\frac{\dot{\overline{\Psi}}(t)}{\overline{\Psi}(t)}\right] = 2\pi f(t).$$

$$\Rightarrow \overline{\omega} = \left\langle \frac{\partial\varphi(t)}{\partial t} \right\rangle = 2\pi\langle f(t)\rangle = 2\pi\overline{f} = \frac{1}{T}\int_{[T]} \omega(t)dt \;\; (\cong) \;\; \frac{\frac{1}{T}\int_{[T]} \omega(t)\cdot a^2(t)\cdot dt}{\int_{[T]} a^2(t)\cdot dt} \;\; (\cong)$$

$$(\cong)\,\tilde{E}\frac{2\pi}{h} = \frac{2}{h}\int_0^{\infty}[A(\omega)]^2 d\omega = \frac{2\pi}{h}\int_{[t]} a^2(t)dt,\;\; \frac{\tilde{E}}{\overline{\omega}} = \frac{d\tilde{E}}{d\omega} = \frac{h}{2\pi},$$

$$\overline{f} = \frac{\frac{1}{\pi}\int_0^{\infty} f\cdot[A(\omega)]^2 d\omega}{\frac{1}{\pi}\int_0^{\infty}[A(\omega)]^2 d\omega} = \frac{\frac{1}{\pi}\int_0^{\infty} f\cdot[A(\omega)]^2 d\omega}{\tilde{E}} = \frac{\frac{1}{2}\int_{-\infty}^{+\infty} f(t)\cdot a^2(t)\cdot dt}{\frac{1}{2}\int_{-\infty}^{+\infty} a^2(t)\cdot dt} = \frac{\frac{1}{2}\int_{-\infty}^{+\infty} f(t)\cdot a^2(t)\cdot dt}{\tilde{E}} = \frac{\overline{\omega}}{2\pi},$$

$$\Rightarrow \left[\begin{array}{l} \dfrac{\tilde{E}}{\overline{\omega}} = \dfrac{\left[\int_{[T]} a^2(t)\cdot dt\right]^2}{\frac{1}{T}\int_{[T]} \omega(t)\cdot a^2(t)\cdot dt} = \dfrac{\int_{[T]} a^2(t)\cdot dt}{\frac{1}{T}\int_{[T]} \omega(t)dt} = \dfrac{\left[\int_0^{\infty}[A(\omega)]^2 d\omega\right]^2}{\pi\int_0^{\infty}\omega\cdot[A(\omega)]^2 d\omega} = \\ = \dfrac{h}{2}\dfrac{\int_{[t]} a^2(t)dt}{\int_0^{\infty}[A(\omega)]^2 d\omega} = \dfrac{a^2(t)\int_{-\infty}^{+\infty} a^2(t)dt}{2\left[\Psi(t)\dot{\hat{\Psi}}(t) - \dot{\Psi}(t)\hat{\Psi}(t)\right]} = \dfrac{h}{2\pi} = \mathrm{Const.} \end{array}\right]$$

(9.8), (9.9), (9.10)

The principal objective here is to find the family of elementary wave functions that could present quantized matter wave packets. In fact, Quantum theory is already successfully using the concept of quantized energy wave packets. This mathematically works well in explaining the number of experiments in Physics (some of them well known, are Compton and Photoelectric effect and Blackbody Radiation law), although we still do not know their analytic signal forms in a temporal and/or spatial domain (and the fact is that in many cases we do not need to know them). In fact, mathematics already has much more generally valid quantization or Signal Analysis and Synthesis processing based on "Kotelnikov-Shannon-Nyquist-Whitaker-Gabor" theory, useful to describe photons or wave-packets. Eventually, we should be able to determine a family of wavefunctions, as $\boldsymbol{\Psi(t) = a(t)\cos\varphi(t)}$, or sinc wavefunctions $\boldsymbol{\Psi(t) = \frac{\sin\varphi(x,t)}{\varphi(x,t)}}$, or $\boldsymbol{\Psi(x,t) = a(x,t)\frac{\sin(\underline{\Delta\omega}\,t - \underline{\Delta k}\,x)}{(\underline{\Delta\omega}\,t - \underline{\Delta k}\,x)}\cos(\omega t - kx)}$ that can describe photons. All such basic wave functions should have at least one common characteristic, which is that each of them has the energy content equal to a Planck's narrow-band photon energy, $\boldsymbol{\tilde{E} = h\overline{f} = \int_{[t]} \Psi^2(t)dt = \int_{[t]} a^2(t)dt}$ (also knowing that Planck-Einstein energy formula is correct. This is also related to Parseval's identity; - see familiar elaborations in [161], Hidden Variables: The Elementary Quantum of Light. A Significant Question in Quantum Foundations).

The energy relation $\tilde{E} = Hf$ is also applicable to planetary and macro systems, like its micro-world counterpart $\tilde{E} = hf$. However, in this context, the constant H is not equivalent to Planck's constant h (for more details, refer to Chapter 2.3.3: "Macro-Cosmological Matter-Waves and Gravitation"). Interestingly, similar energy-frequency relationships (using a corresponding constant H) are also observed in vortex flow meters, where they are linked to fluid flow vortices. This is explored further in Chapter 4.1, see equations (4.3-0), (4.3-0)-a, and (4.3-0)-b.

A key question arises: how and why can a single characteristic (central) frequency, when multiplied by Planck's constant h or another constant H, accurately represent the motional wave-energy of a narrow-band wave group? What does this frequency signify, and what kind of wave group represents? The answer lies in considering that this central frequency is simply the *mean frequency* $\bar{f}$ of the associated narrow-band elementary matter-wave group, calculated in relation to its energy. In this context, the wave group (or wave packet, such as de Broglie matter wave) is constructed from an infinite number of elementary waves that span a relatively small frequency interval: $0 \leq f_{min.} < \bar{f} < f_{max.} < \infty$.

The energy and mean frequency of such a narrow-band wave group can be rigorously derived by connecting Parseval's theorem with Planck's energy formula, as demonstrated in equations (9.8), (9.9), and (9.10). We can further validate Planck's energy formula $\tilde{E} = h\bar{f}$ using a straightforward approach based on one of the most general formulas for all definite integrals. Applying this to the wave energy expressions in (9.8), (9.9), and (9.10) provides additional support for the fundamental relationship:

$$\left\{\begin{array}{l} \int_a^b f(x)\cdot g(x)dx = f(c)\int_a^b g(x)dx \,, a < c < b, g(x) \geq 0, \\ f(x) \text{ and } g(x) \text{ are continuous in } [a \leq x \leq b], \\ \text{Let us consider: } f(x) = f,\ g(x) = [A(\omega)]^2 > 0, x = \omega \in (0,\infty) \end{array}\right\} \Rightarrow$$

$$\begin{array}{l} \tilde{E} = \frac{1}{\pi}\int_{f_{min.}}^{f_{max.}} [A(\omega)]^2 d\omega = h\bar{f} = h\frac{\frac{1}{\pi}\int_0^\infty f\cdot[A(\omega)]^2 d\omega}{\tilde{E}} \cong \bar{A}\cdot\Delta f, \Rightarrow \\ \tilde{E}^2 = \frac{h}{\pi}\int_0^\infty f\cdot[A(\omega)]^2 d\omega = \frac{h}{\pi}\cdot\bar{f}\cdot\int_0^\infty [A(\omega)]^2 d\omega = h\bar{f}\cdot\tilde{E} \Rightarrow \\ \Rightarrow \boxed{\tilde{E} = h\bar{f}}, \bar{A}\cdot\Delta f \cong h\bar{f} \Leftrightarrow \frac{\Delta f}{\bar{f}} \cong \frac{h}{\bar{A}}, dE = h\cdot df, h \cong \bar{A}\cdot\frac{\Delta f}{\bar{f}}, \\ f_{min.} < \bar{f} < f_{max.}, \bar{f} = 0.5(f_{min.} + f_{max.}), \Delta f = (f_{max.} - f_{min.}). \end{array} \quad (9.11)$$

....

Another supporting ground for Planck's photon energy formula is related to the nature (and mathematics) of matter-waves creation and propagation in real matter media, explicable based on the following facts:

A) *Any kind of temporal or pulsing, random, non-periodical, and periodical excitations of real matter states (based on Fourier Analysis) produce simple harmonic oscillations or vibrations (meaning sinusoidal and cosines elementary waves). The mentioned vibrations are mutually interfering and superimposing, creating, or presenting spectral complexity of relevant matter-wave state.*

B) *Discrete superposition of (limited number of) simple harmonic wavefunctions (matter waves, or oscillations) within certain* *frequency-band limited* *interval* $\Delta f = (f_{max.} - f_{min.})$ *will produce specific narrow-band wave-group that has the form of a sinc function [sinc f(x) = sin f(x)/f(x)]. We will also get similar sinc function using continual superposition of countless number of simple-harmonic wave components, covering the same, limited frequency interval* Δf *(see the same in (10.2-3)).* ***Every sinc function in a time domain has a rectangular, frequency band limited picture in its frequency (or spectral) domain.*** *That means, superposition of sinc wavefunctions (in a time domain), considering energy of such functions, is the summation (or integration) of Planck energy amounts* $\tilde{E} = \left[\sum_{[\Delta f_i]} h \cdot \bar{f}_i \text{ , or } \int_{[\Delta f]} h \cdot df\right]$, *or* $\tilde{E} = \sum_{[f_{min.}, f_{max.}]} A_i \cdot \Delta f_i$. *Rectangular frequency-response shapes are very much indicative as something that has limited, finite or quantum nature (since a surface of any rectangular form presents certain discrete and limited signal energy).*

C) *One of differences between Fourier and Kotelnikov-Shannon-Nyquist signal analysis and synthesis is in the fact that Fourier theory is using only simple harmonic wave functions, and Kotelnikov-Shannon-Nyquist theory is using superposition of convenient sinc functions, meaning that Kotelnikov-Shannon signal analysis and synthesis is faster, and more natural.* *See more in Chapter 4.0 under (4.0.30) - (4.0.44).*

D) *The nature of matter-waves that are relatively stable wave-particle dualistic objects is that such wave-packets are band and energy limited (or finite) in all their original, and spectral domains, being bell-curve or Gaussian envelope shaped signals (see more in Chapter 10.).*

To illustrate differently what a finite and limited duration (of an elementary and narrow-band) wave function or wave packet means in time and frequency domains, let us imagine that we can approximate an equivalent, averaged wavefunction, which will replace real wave-packet function, $\Psi(\mathbf{x}, \mathbf{t})$. Let us realize this by placing new wave function into a rectangular-shape amplitude borders (in time, space, and frequency "rectangular frames" or molds). We shall also request that this elementary, finite, narrow-band wave function (or wave packet $\Psi(\mathbf{x}, \mathbf{t})$) has energy equal to one energy quantum $\tilde{\mathbf{E}} = \mathbf{hf}$. For instance, the real signal amplitude in a time domain, $\mathbf{a(t)} / \sqrt{\mathbf{2}}$, will be replaced by its effective and constant amplitude, $\bar{\mathbf{a}} / \sqrt{\mathbf{2}}$. The real signal duration, $\mathbf{T}$, in a time domain will be replaced by effective signal duration, $\bar{\mathbf{T}}$. The real signal amplitude in a frequency domain, $A(\omega)/\sqrt{\pi}$, will be replaced by its effective and constant amplitude, $\bar{\mathbf{A}} / \sqrt{\pi}$. The real signal duration, $\mathbf{F}$, in a frequency domain will be replaced by effective signal duration, $\overline{\mathbf{F}}$. Also, the signal mean (central, or carrier) frequency, $\mathbf{f}$, will be replaced by its effective central frequency $\bar{\mathbf{f}}$, (4.15), placed in the middle (effective mass or center of gravity) point of the interval $\overline{\mathbf{F}}$. Instead of rectangular signal frames or molds (as a method of an effective signal averaging), we could also say that both time and frequency domain expressions of the same wave function would be certain equivalent Gaussian pulses (Gaussian window functions) because the Gaussian function is optimally concentrated in the joint time-frequency domain.

Now, based on wave energy expressions found in (5.14), and on "Kotelnikov-Shannon-Whittaker-Nyquist Sampling and Signals Recovery Concepts" all relevant parameters of an effective (and averaged), rectangular band-limited wave-packet signal (being one-quantum of wave energy $\tilde{\mathbf{E}} = \mathbf{h}\bar{\mathbf{f}}$), can be presented as certain sinc wavefunction:

$$\Psi(x,t) = a(x,t)\frac{\sin(\underline{\Delta\omega}\, t - \underline{\Delta k}\, x)}{(\underline{\Delta\omega}\, t - \underline{\Delta k}\, x)}\cos(\omega t - kx)\ (\Leftrightarrow)\ \text{wave packet},$$

$$\omega = 2\pi f\ (=)\ \text{carrier frequency}\ (>>>\underline{\Delta\omega})$$

$$\underline{\Delta\omega} = 2\pi\Delta f\ (=)\ \text{wave-packet frequency bandwidth}\ (<<<f)\ .$$

If this is an elementary wave packet and presents energy of narrow-band photons, we will have,

$$\tilde{E} = \int_{[\Delta t]} \Psi^2(t)dt = \frac{1}{\pi}\int_0^{+\infty} |A(\omega)|^2 d\omega = \int_{[\Delta\omega]} |A(\omega)|^2 d\omega = h\bar{f}\ .$$

Certain conceptualization of photons is already elaborated in Chapter 4.1, under "4.1.1.1. Photons and Particle-Wave Dualism" and summarized in "T.4.0. Photon – Particle Analogies".

We also know (see Chapter 5. "Quantizing and Kotelnikov-Shannon, Whittaker-Nyquist Sampling Theorem") that time and frequency duration of such elementary and optimal sampling amount should be,

$$\Delta t \cdot \Delta f = T \cdot F = \frac{1}{2} \Rightarrow$$

$$\tilde{E} = \int_{[\Delta t]} \Psi^2(t)dt = \int_{[\Delta\omega]} |A(\omega)|^2 d\omega = = \frac{\bar{a}^2}{2}\bar{T} = \frac{\bar{A}^2}{\pi}\bar{F} = \frac{\bar{A}\bar{a}}{2\sqrt{\pi}} = h = \overline{pu} = A \cdot \Delta f\,,$$

$$\bar{T} = \frac{\bar{A}}{\bar{a}\sqrt{\pi}} = \frac{1}{2\bar{F}}\,,\ \bar{F} = \frac{\bar{a}\sqrt{\pi}}{2\bar{A}} = \frac{1}{2\bar{T}}\,,\ \overline{TF} = \frac{1}{2}, \frac{\Delta f}{\bar{f}} = \frac{F}{\bar{f}} = \frac{h}{A}$$

$$\bar{f} = \frac{\bar{\omega}}{2\pi} = \frac{1}{2\pi^2\tilde{E}} \int_0^{+\infty} \omega |A(\omega)|^2 d\omega = \sqrt{\frac{1}{2\pi^2 h} \int_0^{+\infty} \omega |A(\omega)|^2 d\omega} \,,$$
$$\bar{\lambda} = \frac{h}{\bar{p}},\ \bar{u} = \bar{\lambda}\bar{f} = \left\langle \frac{\omega}{k} \right\rangle = \left\langle \frac{\tilde{E}}{p} \right\rangle,\ \ \bar{v} = \left\langle \frac{\Delta\omega}{\Delta k} \right\rangle = \left\langle \frac{d\tilde{E}}{dp} \right\rangle, \qquad (5.14\text{-}1)$$
$$h = \frac{\bar{A}\bar{a}}{2\bar{f}\sqrt{\pi}} = \frac{\bar{A}^2}{\pi} \cdot \frac{\bar{F}}{\bar{f}} = (\frac{\bar{a}}{2})^2 \frac{1}{\bar{f} \cdot \bar{F}} = 6.62606876 \text{ x } 10^{-34} \text{Js} \,,$$
$$\bar{F} = (\frac{\bar{a}}{2})^2 \frac{1}{h\bar{f}} = \frac{\bar{a}\sqrt{\pi}}{2\bar{A}} = \frac{1}{2\bar{T}},\ \frac{\bar{F}}{\bar{f}} = (\frac{\bar{a}}{2})^2 \frac{1}{h\bar{f}^2} = \frac{\bar{a}\sqrt{\pi}}{2\bar{A}\bar{f}} = \frac{1}{2\bar{T}\bar{f}}.$$

As shown in equation (5.14-1) in Chapter 5, if a quantized narrow-band matter-wave packet has a higher mean frequency $\bar{f}$, its frequency width or total duration $\bar{F}$ is shorter. Consequently, for matter-wave packets with low or extremely low mean frequencies (as in phenomena related to gravitation, planetary, and galactic systems), the total duration $\bar{F}$ of these wave packets becomes exceptionally long.

Equation (5.14-1) provides conditions for defining the boundaries of elementary matter-wave domains, or “signal atomization,” by using optimally concentrated waveforms such as Gaussian and analytic signals in all mutually conjugate domains. These conditions are analogous and mutually consistent with the expressions found in (5.2.1), (5.3), and (5.4.1). This critical analysis addresses the limitations of the traditional Planck-Einstein single-photon energy quantization, typically expressed as $\tilde{\mathbf{E}} = \mathbf{hf}$, suggesting that the concept of quantization is more nuanced than initially proposed by M. Planck and A. Einstein, which laid the foundations of Quantum Theory (for a similar perspective, see [161]: *Hidden Variables: The Elementary Quantum of Light. A Significant Question in Quantum Foundations*).

The wave functions of elementary quanta of energy (e.g., photons) should ideally belong to a family of finite-energy, narrow-band Gaussian pulses or Gabor wavelets. Such waveforms ensure optimal time-frequency resolution, allowing these wave packets to be well-defined and localized in both their time and frequency domains, qualities that should correspond to the characteristics of a photon.

Moreover, since the blackbody radiation spectrum $A^2(\omega) = S\,\Delta t \frac{\pi f^2}{c^2} \cdot \frac{hf}{e^{hf/kT} - 1}$ can be considered a specific superposition of elementary narrow-band energy quanta (i.e., photons), we should be able to apply the “Kotelnikov-Nyquist-Shannon-Whitaker” sampling and reconstruction principles. This would enable us to derive analytic forms of space-time-dependent wave functions for these elementary quanta. Each individual photon or corresponding wave packet is essentially a narrow-band signal, making such an approach appropriate.

It is also reasonable to expect that these elementary wave-packet functions should satisfy both the Classical and Schrödinger wave equations. In some cases, the energy content of these wave-packet functions can be represented as $\tilde{E}_i = hf_i$, as this conceptualization has been successfully applied in the context of phenomena like Compton and Photoelectric effects. Consequently, the wave function for such a wave packet should be expressible as $\psi(t,x)=\psi(\omega t-kx)$ or using a similar form like the sinc function $\frac{\sin\varphi(x,t)}{\varphi(x,t)}$, or soliton solutions. These analytic-signal waveforms satisfy the Classical and Schrödinger equations and provide group and phase velocities derived from such wave equations:

$$u=\lambda f=\frac{\omega}{k}=\frac{\tilde{E}}{p}=\frac{v}{1+\sqrt{1-(\frac{v}{c})^2}}\ ,\ v=\frac{d\omega}{dk}=\frac{d\tilde{E}}{dp}=u-\lambda\frac{du}{d\lambda}=-\lambda^2\frac{df}{d\lambda}=\frac{2u}{1-\frac{uv}{c^2}},$$
$$\lambda=h/p,\ \tilde{E}=hf=pu\,,\omega=2\pi f$$

If we additionally explore relations (5.14-1), we can get a group velocity of an elementary matter wave packet or photon as,

$$h\cdot\Delta t\cdot\Delta f=h\cdot T\cdot F=\Delta x\cdot\Delta p=L\cdot P=\frac{h}{2},P=\frac{hf}{c}\Rightarrow L\cong\frac{c}{2f}=\frac{\lambda}{2}$$
$$\Rightarrow\frac{\Delta x}{\Delta t}=\frac{h\Delta f}{\Delta p}=\frac{\Delta\tilde{E}}{\Delta p}=\frac{h}{2\Delta p\cdot\Delta t}=v=c,\Rightarrow\frac{h}{2c}=\Delta p\cdot\Delta t. \qquad (5.14\text{-}2)$$

In equation (5.14-2), Δx represents the optimal elementary spatial sampling length, Δp is the elementary sampling momentum, and L and P denote the total spatial signal length and total signal momentum, respectively. From equations (5.14-1) and (5.14-2), we can see that a single energy quantum (such as a matter-wave packet or photon) should possess predictable and calculable characteristics. These characteristics are derived directly from fundamental mathematical principles in Signal and Spectral Analysis, Parseval's theorem, and Uncertainty (or Certainty) relations.

The key question here is whether Physics truly deals with a universal law governing naturally quantized wave packets (as initially conceptualized by Planck and Einstein) or with discrete energy exchanges between resonant, standing wave building blocks that represent matter constituents. Such standing waves, in various contexts, absorb or emit wave packets as discrete amounts of energy, giving the impression that wave packets are inherently and universally quantized. However, this may simply be a result of stable matter structures forming as complex standing wave configurations, naturally countable in terms of wavelengths, time periods, and other discrete quantities, which resemble quantization. Consequently, the exchange of energy between these structures may only appear quantized to an external observer, without implying a universal quantization for all energy exchanges, impacts, or radiation phenomena.

The position taken in this book is that current Quantum Theory concepts regarding quantization are oversimplified and overly generalized, sometimes extending to scenarios where such quantization does not apply universally. A more robust theoretical foundation for understanding quantization would be through the "Kotelnikov-Nyquist-Shannon-Whittaker" sampling and signal reconstruction concepts.

The potential implications of these ideas are profound. For example, by controlling the flow of matter-wave energy, it may be possible to influence the structural integrity and stability of matter. This could be achieved by stimulating or modulating specific resonant frequencies or characteristic natural frequencies, thereby affecting the mechanical and structural properties of a given object. This approach forms the basis of the MMM (Matter Modulation and Manipulation) technology developed by the author (see [140], European Patent Application related to MMM technology).

Additionally, Parseval's identity allows us to connect the temporal, spatial, and relevant frequency domains of signal energy, thereby providing another mathematical form of the Energy Conservation Law. A particularly intriguing aspect of Parseval's identity involves the known transformation of photons or wave packets (which have no rest mass) into particles with non-zero rest mass (such as electrons and positrons). This suggests that by exploring the wave function and energy spectral characteristics, we approach the fundamental ontological nature of matter structure and energy quantization.

Another universal principle involves the "mathematical Uncertainty Relations," which establish a fundamental link between the total durations of mutually conjugate original and spectral domains. These relations define "Certainty" conditions under which standing matter waves begin to form stable particles, $\Delta t \cdot \Delta f = T \cdot F = \frac{1}{2}$. This principle is explored in detail in Chapters 5 and 10. Current interpretations of Uncertainty principles in Physics, which rely heavily on Probability and Statistics, need to be updated. Instead, universally valid mathematical Uncertainty Relations, independent of statistical assumptions, should serve as the primary foundation for concepts such as Heisenberg's Uncertainty Principle.

Finally, another key idea presented in this book is a reevaluation of the interpretation of the famous Cosmic Microwave Background (CMB) radiation, which is traditionally seen as a relic of the Big Bang. It is argued here that the CMB may instead be the result of ongoing, natural, real-time thermal and electromagnetic radiation emitted by all matter and atoms throughout the universe (see Chapter 8 for more details).

Download the last version here:
http://www.mastersonics.com/documents/revision_of_the_particle-wave_dualism.pdf

10. PARTICLES AND SELF-CLOSED STANDING MATTER WAVES

This chapter serves as the foundational theoretical and conceptual framework for the entire book, providing ontological clarification for all subsequent discussions. It builds upon the universally valid assumptions, principles, and concepts introduced in Chapters 2, 4.0, 4.1, 4.2, and 4.3, particularly those related to Matter Waves and Wave-Particle Duality. Briefly summarizing, contemporary Quantum Theory (QT) could be significantly, mathematically and conceptually upgraded and simplified with here-elaborated mathematical modelling options, reducing its strong dependance from probabilistic foundations.

By addressing the key concepts, foundational principles, and interrelations within wave-particle duality, this chapter connects the ideas presented in all other chapters in a coherent mathematical and logical framework. The discussion focuses on the energy-momentum aspects of various motions and states of matter. We assume (or conceptualize) that matter, or mass, consists of self-stabilized elementary matter waves arranged in standing wave formations. These formations create elementary particles, atoms, and other stable matter structures, including larger masses. In other words, this chapter explores the nature of how standing matter waves pack or format, and how this relates to the structure of matter, matter waves, and wave-particle duality.

Key principles discussed include universally applicable conservation laws, continuous symmetries, and the principle of least or stationary action, along with Hamiltonian and Lagrangian mechanics. To explore this framework, we proceed with the following options:

1. We can consider that our Universe is already unified and "temporally-spatially-spectrally" synchronized across various mutually compatible, complementary, and often analogous entities, structures, and states of matter. This is true even though we have yet to develop a sufficiently robust General Unified Field Theory in physics. Mathematics, which naturally arises from the exploration of physics and nature, serves as the universal language, framework, skeleton, and logic of our Universe. It represents, describes and process tangible (experimentally manageable) phenomena without relying on postulated or artificially created mathematical constructs. Mathematics provides basic, universally applicable structures and models where everything we explore in physics can be placed. Deviations from these natural scientific and mathematical frameworks often lead to paradoxical, unnatural, and confusing insights into the reality of our Universe.

Artificially constructed mathematical theories, those dealing mainly with virtual, imaginative, abstract, symbolic, and stochastic entities, as seen in Game Theory, can be misleading and confusing when applied to physics. Unfortunately, such approaches are frequent in contemporary physics. This process involves mapping, imaging, and transforming realistic, tangible objects and measurable phenomena into an abstract domain filled with virtual entities. Operating within such artificially assembled mathematical theory, or using game tools, we eventually attempt to revert to the real world of tangible physics through inverse mathematical transformations, practical prescriptions, assumptions and convenient imaging. The results, because these mappings and transformations are often non-deterministic or not one-to-one, the results could be arbitrary, unclear, and difficult to interpret. In some cases, we are unable to fully and accurately revert to the realm of realistic, tangible physics.

When this happens, we lose the ability to apply natural and universally valid mathematical logic, tangible conceptualization, healthy intuition, visualization and creativity. As a result, our conclusions are often limited and plagued by absurd questions, dilemmas, and unrealistic statements, issues that are evident within the current framework of Quantum Theory (QT). Despite its abstract and complicated nature, QT works surprisingly well within its self-defined boundaries, as evidenced by the results and praise from its founders, followers, and most contemporary physicists. QT appears to be a sophisticated, mathematically rich, and efficient theory, but it is also unnecessarily abstract and complicated. It employs generally valid mathematical tools, descriptions, postulations, and abstract definitions that are trivially applicable to almost everything in physics but offers little that is uniquely specific or new. It mimics conservation laws and universal principles of physics while being almost tautological in nature. To produce meaningful results, QT must always strongly reference well-established deterministic theories, laws, and foundations of Mathematics and Classical Physics, as well as universally valid conservation laws of nature, modified in mathematically manageable ways, where Signal Analysis, Statistics, Probability, and Mechanics dominate such framework.

Contemporary QT unnecessarily complicates our understanding of physics and the Universe, creating the false impression that it is a self-standing, original, and stable theory with strong, unique, and universally valid ontological probabilistic foundations. QT addresses events stochastically and on average terms, when dealing with large numbers of participants. However, many of QT's achievements were first discovered experimentally, intuitively, and through other deterministic and tangible methods rooted in Classical Physics, and only later retroactively assigned to or explained by Orthodox QT.

This situation is reminiscent of the well-known tale of the emperor who walked naked, while everyone around him, following the tailors' instructions, claimed he was well-dressed fearing that only the ignorant would fail to see his clothes. It took an innocent, naive child to point out that the emperor was, in fact, naked, though everyone else dismissed this observation. Natural beauty lies in elegance and simplicity, but in this case, the "tailors" of QT maybe had something else in mind.

*Citation from https://en.wikipedia.org/wiki/The_Emperor%27s_New_Clothes: **"The Emperor's New Clothes"** (Danish: Kejserens nye klæder) is a short tale written by Danish author Hans Christian Andersen, about two weavers who promise an emperor a new suit of clothes that they say is invisible to those who are unfit for their positions, stupid, or incompetent – while in reality, they make no clothes at all, making everyone believe the clothes are invisible to them. When the emperor parades before his subjects in his new "clothes", no one dares to say that they do not see any suit of clothes on him for fear that they will be seen as stupid. Finally, a child cries out, "But he isn't wearing anything at all!" The tale has been translated into over 100 languages.[1]*

In both society canonized and contemporary mainstream physics, adopting an ambiguous, compromising, and apologetic approach to "ignorant or overly progressive statements" has often been a safer route than experiencing outright dismissal, punishment, or persecution. History provides ample evidence of this. For instance, when someone in the past argued, based on solid evidence, that the Sun was at the center of our planetary system, they risked severe punishment because the prevailing ideology insisted that Earth was the center of the Universe. This was especially true during the period of the Catholic Church's Inquisition and the dominance of Ptolemy's geocentric model, which persisted for nearly a thousand years before modern science gradually corrected it.

In physics, we must be cautious when associating a domain with abstract, mathematically rich, and superficially grandiose concepts. Such approaches, while appearing complex and sophisticated, often contribute little that is new or significant. Quantum Theory (QT), for instance, has creatively and imaginatively merged Statistics and Probability with already well-established concepts and laws like Signal and Spectral analysis, Conservation laws, and the Hamilton-Lagrange mechanics framework. This approach, largely shaped by voting and consensus and (among young founders of Orthodox QT), benefits from what is already known, correct, and verifiable in natural sciences and mathematics, while offering few groundbreaking insights.

QT has constructed an abstract, unnatural, and complicated framework for mapping the real world to its virtual and abstract creations. This framework, glorified by its founders and followers as eternally valuable and unique, often claims that even absurdities and miracles are statistically possible to happen, albeit with infinitesimally small probabilities. While the mathematical complexity of modern QT is undeniably high, this sophistication often serves to mask the trivialities it includes. Despite this, many significant advances in understanding the micro-world of physics have come from empirical discoveries, guided by deterministic intuition, logic, and analogical thinking rooted in Classical Physics, or seldom from results of arbitrary experimentation.

Some aspects of contemporary Orthodox Quantum Theory (QT) resemble the creations of game theory, combined with various mathematical techniques from Classical Mechanics, Signal Analysis, Curve Fitting, and other established modeling methods. Additionally, QT incorporates "consensus-based instructions, definitions, and patchwork attachments" that are adopted through voting.

To address critical questions from curious or skeptical audiences, QT often resorts to oversimplified explanations. One common approach is the use of narratives, like the well-known "Alice and Bob" scenarios in photon counting, which illustrate probability-related concepts through simple scenarios. However, these narratives often fail to consider all relevant participants in the experimental situation. Instead of using advanced mathematical models that account for all elements of the experiment, QT relies on probabilities and simplified assumptions. This approach overlooks the potential for much better modeling of wavefunctions and wave equations, which could more naturally explain experimental results without significant relying on probabilistic interpretations.

Contemporary QT often lacks its own robust, innovative, and deeply founded natural science principles. Despite this, it works remarkably well in conceptual frames of probability, compelling us to accept it as it is. However, this acceptance should also prompt us to recognize the need for better mathematical conceptualization and modeling within QT. This book concludes that to advance our understanding of QT, we must focus on developing more sophisticated mathematical frameworks.

2. Theories about nature that are both grand and extraordinarily successful should exhibit elegance, simplicity, and a seamless integration with the broader, tangible, natural, and deterministic body of science. For example, thermodynamics, while heavily reliant on statistics and probability theory, remains fundamentally deterministic.

   The founders of QT, along with many of their modern successors, approached critical questions from skeptics with a mix of prudence, honesty, and a touch of satire. They often responded to such inquiries with statements that can be freely interpreted as:

   > "*Nobody really understands why QT works well*", but anyway it works very well, and based on number of correct results we see that its foundations and predictions should also and always be totally correct, and in agreements with all measurements and experiments.
   > Niels Bohr, said, "Anyone who is not shocked by QT has not understood a single word." Richard Feynman said, "It is safe to say that nobody understands quantum mechanics." John Wheeler, Feynman's mentor, also a key player in the way quantum physics developed, said, "If you are not completely confused by quantum mechanics, you do not understand it." Roger Penrose, one of the leading modern-day thinkers on the meaning of quantum physics, wrote, "Quantum mechanics makes absolutely no sense."

   Mainstream physics currently insists that we accept Orthodox QT as it is, primarily because it works exceptionally well. The success of QT lies in its effective integration of Classical and modern Mechanics, Mathematics, Statistics, Probability theory, Signal and Spectral analysis, Conservation laws and Universal Variational Principles. These elements, when appropriately and mathematically structured, contribute to QT's robustness, even if they are artificially assembled. However, what QT lacks is a natural, immediate, and joint "spatial-temporal-spectral" non-probabilistic wavefunction modeling, such as using Complex Analytic Signals or Phasors (as discussed in Chapter 4.0). For instance, this approach could conceptually, mathematically, and naturally explain quantum entanglements.

   Throughout its development, Orthodox QT has also incorporated several exotic concepts, including the implicit importance of "divine inspiration" in essential discoveries (as seen in the case of Schrödinger's equation). Additionally, QT has produced unique and powerful mathematical tools, concepts, models, and functions, often hailed as "brilliant minds' revelations" (such as the contributions of P. Dirac and Max Born). However, the leaders and adherents of Orthodox QT typically discourage any critical reassessment of its foundations, arguing that these issues have already been addressed and resolved. Consequently, attempts to revisit or challenge these foundations are often ignored, dismissed, or outright rejected.

   The sheer volume of publications in physics, where numerous concepts are repeatedly mentioned, superficially explained, and interdisciplinary mixed, creates an overwhelming sense of familiarity. This leads to the perception that everything, whether old or new, seems similar or already known. Innovators who seek to introduce new models or fundamentally challenge Orthodox QT are often unwelcome and dismissed as ignorant or merely repackaging existing knowledge. Only those who fully embrace Orthodox QT are allowed to contribute, usually with minor, insignificant extensions of existing teachings.

   A good, natural physics-related theory should not be artificially constructed; rather, it should be discovered, understood, and described in the simplest, most natural, and elegant mathematical terms. This approach contrasts sharply with contemporary QT, where mathematics should be grounded in physics and repeatable experimental results, serving to model, predict, and process real natural events.

Another pearl of short and very much indicative, correct formulation regarding problematic aspects of contemporary Orthodox QT can be found in the following citation:

"**Laszlo Petruska,** Philosopher of Natural Sciences1y
Do physicists really understand quantum physics or do they just follow the math and pretend to understand it?
(6) Do physicists really understand quantum physics or do they just follow the math and pretend to understand it? - Quora

How could anyone understand quantum physics or even relativity when the basic definitions are violated.

For example: wave is energy propagating in a medium without the significant and permanent displacement of the medium in the direction of propagation.

Yet we conceive light emitted as photons, traveling electromagnetic waves in vacuum (no medium) where the EM wave itself is traveling just to be absorbed as photons again. Light is its own frame of reference (very unscientific). Light regulates its own finite speed (oxymoron).

Quantum mechanics fails to conceive the field-like (wave-like) behavior of its particles. Quantum field theory is not clear about the particles (localized) behavior of its fields.

Due to these and other misconceptions quantum physics still hasn't been able to fully explain simple things like the double-slit experiment, entanglement, how can the light speed up after leaving the prism.

I believe the fundamental properties of matter, energy and nature of physical processes apply any level from the quantum through every day to cosmic. The formalism of quantum physics is fundamentally wrong."

For instance, there are also challenging, inspiring, provocative, indicative, and sufficiently well-supported comments and fundamental critics regarding weak sides of contemporary (orthodox) Quantum and Nuclear physics publicized by Dr. Sorin Cezar Cosofret (see more under [126]), but such, a little bit aggressive, non-diplomatic, simplified and essentially correct statements, are being a priory and immediately dismissed, without being analyzed. Here, (as an illustration) are listed some shocking or surprising (but essentially correct) statements and comments regarding our modern Physics, formulated by Dr. Sorin Cosofret (citation from the author's email):

"-Any form of energy must have a carrier.
-Vacuum cannot be a carrier nor a reservoir for energy.
-Dark matter and dark energy are either incomplete or nonsensical concepts.
-There is no similitude between the comportment of a solid body and the comportment of a gas.
-Only a „supplementary" central force can explain the discrepancies between the observed and calculated motion of a particle around a center of force.
-Any material body which performs a periodic motion around a center of force must be acted compulsory by the force of a vortex or something similar.
-Gravitational force cannot generate or maintain a rotational motion alone.
-The so-called electric force cannot generate or maintain a rotational motion alone.
-No signal before dark age epoch can be observed by a later observer (assuming that a Big Bang existed, of course)
-It is demonstrated that no credible dark matter distribution can explain the rotational motion in the outskirts of galaxies.
-Dark matter is found to be in complete contradiction with experimental reality.
-The recombination process of a mixture of elements cannot be a single step and single stage temperature process.
-The variation of physical parameters for a gas mixture generates only a mass transfer if the specific conditions for deposition or condensation are not reached".

3. Wave-Particle Duality and Energy States

Wave-Particle Duality (here PWDC, as introduced in Chapter 4.1) is a fundamental concept in physics that explores the relationships, transformations, interferences, and equivalences between various forms of energy. These forms can be categorized as follows:

1° Kinetic Energy of Particles: The energy associated with the motion of particles.
2° Energy of Wave Motions: This includes the energy of waves, which are also a form of any motion and/or kinetic energy.
3° Combined Energy States: Situations (and moment-energy states) where both kinetic energy of particles and wave energy coexist and interact.
4° Stable Energy States: This encompasses states of rest, agglomerated energy states, and static or stable energy of particles.

PWDC addresses how these different energy forms (a, b, c, and d) can be related, transformed, mapped, and considered equivalent. Further details on PWDC are discussed later in this chapter.

****The Physical Medium and Material Form****

All these motions and energy states require a physical medium or spatial matrix, a material form or carrier where they manifest. Whether it's an etheric, ideal, or real fluid, or natural fields, this medium must exist, even if its nature still remains unknown in some cases.

Various states of matter, such as solid, crystalline, fractal, liquid, and plasma states are combinations of the energy-moments states and their corresponding material forms.

****Wave Phenomena and optimal Mathematical Framework****

All phenomena known as waves, vibrations, and oscillations adhere to the same physical framework and can be described using common mathematical models and wave equations, such as Fourier Analysis, the Complex Analytic Signal model, and the Classical second-order partial differential Wave equation (discussed in Chapters 4.0 and 4.3).
Stable matter states, including particles, atoms, molecules, and other stable masses (with non-zero rest masses), are essentially structured as standing matter-waves. These structures are composed of self-contained, spatial standing wave formations, and various physical oscillators and resonators. On a microscopic level, these standing-wave formations are superpositions and interferences of more elementary matter-waves or wavefunctions, likely electromagnetic waves that are coupled with, or create, mechanical waves.

****Electromagnetic and Mechanical Couplings****

Electromechanical and electromagnetic couplings and the mutual synchronization of matter-waves penetrate from the micro-world of atoms to the macro-universe. Stochastic, probability-based, and "non-dimensional waves" (as proposed in modern QT are not included here, except as useful mathematical abstractions, because such QT models often neglect or fail to address real-time wave-phase information. However, due to universal synchronization effects, even statistical systems of particles and other matter states (including ideal gaseous states) will gradually synchronize, creating a joint macro-system amplitude and phase function that conveys resulting information about velocities, frequencies, and wavelengths.

****Misconceptions in Quantum Theory****

The frequent and sometimes arbitrary use of terms like quantum theory, quantum physics, quantum phenomenology, quantum nature, and quantum effects imply that all microphysics should inherently possess discrete, quantized properties. In this book, however, "quantizing" primarily refers to finite, resonating matter structures where stable standing matter-waves are involved. This concept is analogous to analyzing the spectrum of natural and/or modal resonant frequencies (and associated harmonics) of a solid body, which can be visualized using Finite Element Analysis methods.

****Matter Structures as Oscillatory Systems****

All matter structures, such as atoms, molecules, and other matter states, can be conceptually, mathematically, and effectively represented as ensembles of real, physical, oscillatory, and resonant systems. These systems include electromagnetic, electromechanical, and mechanical oscillators, akin to mass-spring or inductance-capacitance oscillators (with damping or energy dissipation elements). This situation is comparable to Fourier spectral analysis, where any spatial-temporal wavefunction (or matter state in motion) can be decomposed into simple-harmonic sinusoidal, and/or “sinc (=) sin x/x” components.

****Electromagnetic and Mechanical Oscillations****

Mechanical or acoustic oscillations, vibrations, and audio signals in different material media (not vacuum) can also be created using various signal-modulation techniques on laser beams and plasma states. This indicates that electromagnetic and mechanical oscillations and waves of matter are fundamentally coupled. For more details, refer to sections (10.2-2.4) and references [133] to [139].

****Elementary Physical Oscillators and Resonators****

Elementary physical oscillators or resonators (mostly atom states) naturally interact by exchanging quantized amounts of energy, moments, and different field-charges through various forms of matter-waves. This can be mathematically represented through Fourier Analysis, involving a series of integer-indexed elementary harmonic waves. In cases of stable resonant structures, standing waves naturally consist of integer numbers of wave-groups, wavelengths, or half-wavelengths.

****Standing Waves and Resonant Structures****

The mutual communication and resonance-couplings between specific resonating structures are optimized in spectral areas where natural resonant frequencies of the involved participants overlap. This interaction can create discrete or quantized states, or discrete amounts of energy, charges, wavelengths, and mechanical and electromagnetic moments. Each finite, well-defined geometry or resonator shape has a limited, integer-countable number of resonant frequencies and wavelengths in both temporal and spatial domains.

****Quantizing in Physics and Signal Analysis****

Focusing on Fourier Analysis, we find that the most fundamental matter waves, or the harmonic function bases used to construct all matter waves, are simple harmonic (sinusoidal) functions. Mathematics has evolved to replace purely sinusoidal wave components with sinc function bases (such as solitons), thereby optimizing Fourier Signal Analysis, and it looks that Nature is also or intrinsically using such methods.

This refined theory of Signal Analysis and Synthesis explains quantization in terms of signal sampling, discretization, and reconstruction across both temporal and spatial domains. This forms a solid mathematical foundation for quantum phenomenology and the Uncertainty Relations in Physics, drawing upon concepts from Kotelnikov, Shannon, Nyquist, Whittaker, Fourier, and Analytic Signal Analysis (see references [57, 58, and 59]).

In physics, what we often refer to as quantization involves discretized (integers-related) frequencies and wavelengths of spatial and temporal harmonic components of specific wave functions or signals. Additionally, or consequently, the energy and momentum components of certain phenomena can also be quantified in both space and time, and interactions or energy and moments exchanges between such quantized systems are in a similar way discretized (being not typically continual and linear). All these quantization options are interrelated and effectively connected to the formation of standing waves, resonant structures, periodic patterns, stable, uniform, stationary and inertial motions, crystalline and fractal-related systems.

****Application to the Macro-Universe****

We will explore how standing waves and energy-moment quantizing apply to all material structures, from atoms and masses to crystals, fractals, solar systems, and galaxies (see more in Chapter 2). From this perspective, the entire tangible and stable Nature around us can be quantized, decomposed, and again synthesized. However, there are also "open-space, fields and forces-related situations" where standing waves are not involved.

****Deterministic vs. Probabilistic Models****

The quantized or quantum nature of our Universe represents deterministic manifestations of multi-resonant assemblies of different resonant states and standing wave formations, both in temporal and spatial domains. Probabilistic and statistically conceptualized models of these structures, while opposite to any deterministic background, are effective when natural and mathematical conditions allow for such stochastic modelling.

Conclusion

In essence, all stable matter states in our Universe are structural combinations or superpositions of more elementary, vibrating, and resonant states with dualistic wave-particle properties. Some of these are considered elementary particles in physics, and all can be mathematically synthesized as summations of "sinc" elementary wavefunctions, including solitons. This understanding brings us closer to the concepts of String Theory, where strings are the simplest elementary vibrating entities used to address and compose other elementary particles, atoms, and mass structures.

Citation from: strings theory - Search (bing.com)
"String theory is a theoretical idea that ***everything in the universe is made of tiny, vibrating strings instead of point-like particles***[12345]*. These strings are smaller than atoms, electrons or quarks, and they vibrate at different frequencies in many dimensions of space*[2345]*. String theory tries to explain phenomena that are not currently explainable by the standard model of quantum physics, and to unite general relativity and quantum mechanics*[34]*."*

Since String Theory represents the most promising platform for the unification of universal natural fields and forces, it would be highly beneficial to integrate and theoretically unify contemporary concepts of matter structures based on "sinc" and soliton wave functions, resonant and standing matter waves with the advancements in String Theory. The mathematical foundations for this unification are well-supported by the "Fourier-Kotelnikov-Shannon-Nyquist-Whittaker" theory and the Complex and Hypercomplex Analytic Signal analysis theory.

The dualistic wave-particle nature of matter, a foundational concept in our understanding of the universe, began to emerge, albeit unintentionally, when Jean-Baptiste Joseph Fourier developed the principles of Fourier analysis and harmonic analysis. Fourier's work demonstrated that:

A) Arbitrary **time-domain** functions can be decomposed into elementary sinusoidal wave components, leading to the creation of relevant ***temporal-frequency*** spectral functions. This concept was later refined and expanded by the "Shannon-Kotelnikov-Nyquist-Whittaker" theory, which introduced signal sampling and reconstruction methods (see more in [57, 58, 59]), and by Dennis Gabor's Analytic Signal model (see references [7], [57]). These developments provided a robust mathematical framework for understanding and processing wave-particle duality and matter waves. Concurrently, scientists like Louis de Broglie, Albert Einstein, Max Planck, Erwin Schrödinger, Werner Heisenberg, and others advanced our understanding that matter oscillates, creating stabilized structures of matter-waves that interact in various forms, emitting and absorbing wave energy (as "sinc (=) sin x/x" elementary functions).

B) Similarly, **spatial-domain** functions, regardless of their shape or geometric relation, can be decomposed into elementary sinusoidal or sinc functions, generating relevant ***spatial-frequency*** spectral distributions. This process mirrors time-domain analysis, where time is replaced by spatial length. Here again, the “Fourier-Shannon-Kotelnikov-Nyquist-Whittaker” signal sampling and reconstruction theory and Dennis Gabor’s Analytic Signal model can be applied by substituting time with a relevant spatial dimension. The work of de Broglie, Einstein, Planck, Schrödinger, Heisenberg, and others has deepened our understanding of the intrinsic connection between space and time, leading to a more generalized theoretical framework for matter waves and wave-particle duality. Structures with certain geometric periodicities, such as crystals, fractals, pyramids, or mountains ranges, serve as sources, emitters, and receivers of matter waves, with temporal periodicity producing spatial periodicity and vice versa.

C) The **combined spatial-temporal-frequency domain** provides the most realistic and effective method for mathematically processing wavefunctions. Matter waves and motion states, characterized by energy and momentum, can be equally well generated by perturbations, excitations, and variations in geometric shape, size, and oscillations occurring in either the temporal or spatial domain, since these domains are anyway inherently connected and synchronized.

An important implication of this is that an elementary narrowband wave packet, often modeled as a Gaussian or Bell-curve, envelope-shaped quant of electromagnetic energy, could be more comprehensively defined than by the Planck-Einstein relation alone $\tilde{\mathbf{E}} = \mathbf{hf}$. Thus, the Planck-Einstein energy-quant should be considered more universally applicable as a differential or infinitesimal energy quantity $\mathbf{d\tilde{E}} = \mathbf{h} \cdot \mathbf{df}$. For further details, see Chapter 9, later sections in this chapter (relations under 10.2-2.4), and literature references [133-139].

4. **Statistics and Probability Theory** are universally applicable mathematical tools, particularly effective in analyzing large sets of identical or similar elements. These tools are indispensable for understanding numerical and functional structures, distributions, and trends across various systems of events, making them valuable in both natural and social sciences. However, it is important not to overemphasize the ontological significance or universal modeling power of statistics and probability, particularly in the field of microphysics. Despite their impressive numerical fitting and modeling capabilities, these tools should not serve as the significant foundational concepts of modern Quantum Theory (QT) or microworld physics.

   Instead, Probability and Statistics should be applied as a final step, used to present or model results and trends after thorough analysis, rather than as the conceptual basis of QT. The conservation laws, well-established in physics, should not be replaced by statistical and probabilistic laws. Unfortunately, this has occurred in the current framework of QT. The future of QT and microworld physics should dominantly focus on natural and proper applications of conservation laws, Lagrangian-Hamiltonian Mechanics, the Parseval theorem, and the principles of least or stationary action. Additionally, modeling matter waves using complex analytic signals or phasors, including complex Classical and Schrödinger wave equations based on Analytic Signal wavefunctions, should be prioritized. Also Electromagnetism is on some fundamental way underlaying and creating states of atoms, masses, mechanics and gravitation. These approaches, discussed further in this chapter under "10.00 DEEPER MEANING OF PWDC," are still not fully integrated into contemporary QT.

When the foundational steps of Orthodox QT were established, the omitting or exclusion of temporal-spatial phase information from the analysis of motional states led to premature and exclusive reliance on probabilistic and statistical interpretations of wavefunctions. This approach, which aims to describe matter waves, limits the mathematical modeling, conceptualization, and imaginative potential of physics. To make the theory operational, conservation laws were substituted with the conservation of total probability, and corrective mathematical steps were introduced (or postulated) to align such QT with the probability and statistics framework.

While Probability and Statistics are indeed universal tools for describing situations involving large numbers of identical or similar elements, they should not serve as the primary, ontological foundations of natural sciences that deal with motions and waves. By preserving temporal-spatial phase information and applying complex analytic signal phasors for wavefunction modeling, as introduced in Chapter 4.0, we retain the ability to later use statistical tools when appropriate. Furthermore, this approach allows us to better understand and describe the universal tendency of matter toward resonant or vibrational synchronization and the "extended meaning of entanglement" between different objects, matter states, and systems (including idealized statistical systems). These relationships, which arise from interactions or processes that produce similar or overlapping spectral signatures, are perfectly manageable mathematically when wavefunctions are represented as complex analytic signals or phasors.

Considering the substantial and sophisticated development of contemporary QT over the past century, it would be unwise to dismiss it outright. Instead, to enhance its natural applicability to both micro and macro worlds of physics, the most effective strategy is to upgrade, reconstruct, and update QT in line with the principles outlined in this book, particularly those related to PWDC, analytic signal wavefunctions, and wave equations.

*Citation from https://en.wikipedia.org/wiki/Bose%E2%80%93Einstein_condensate: A **Bose–Einstein condensate (BEC)** is a state of matter (sometimes called the fifth state of matter) which is typically formed when a gas of bosons at low densities is cooled to temperatures very close to absolute zero (-273.15 °C). Under such conditions, a large fraction of bosons occupy the lowest quantum state, at which point microscopic quantum phenomena, particularly wavefunction interference, become apparent macroscopically. A BEC is formed by cooling a gas of extremely low density, about one-hundred-thousandth (1/100,000) the density of normal air, to ultra-low temperatures. This state was first predicted, generally, in 1924–1925 by Albert Einstein[1] following a paper written by Satyendra Nath Bose, although Bose came up with the pioneering paper on the new statistics.[2]*

5. The total quantity of linear, rotational, and other motions of different forms of matter and waves in the Universe are always, on a macro scale, mutually neutralized, balanced, and compensated in cases of vectors. This balance results in a net effect that can be considered zero (or neutral) when represented as different moments, forces, vectors, or opposing charges. Additionally, every action is always equal to the corresponding reaction, being in accordance with various induction laws known in mechanics, electromagnetism, and other fields of physics.

   Solutions to all well-known second-order partial differential wave equations consistently appear in pairs of signals or wave groups that propagate inwards and outwards, or in opposing spatial and temporal directions. This is due to the inherent coupling and transformability of time and space domains, with the principle that "action equals reaction" is universally applicable.

   Natural wave phenomena, vibrations, and oscillations in different media can be described using similar wave equations, including the Schrödinger equation, all of which originate from the same Classical wave equation (as discussed in Chapter 4.3). The Classical wave equation, when utilizing complex analytic signals as wavefunctions, consistently yields solutions that consist of at least two mutually coupled and phase-shifted wave groups propagating in opposite spatial and temporal directions (mutually transformable using integral Hilbert transform).

   The phenomenon of quantum entanglement may be closely related to action-reaction dynamics, various induction laws, and spatial-temporally symmetric and coupled vibratory states. CPT Symmetry is in the background of such phenomenology. This entanglement relationship becomes evident within innovative models of wavefunctions presented as complex and hypercomplex analytic signal phasors. The synchronization of matter states and associated entanglements is directly proportional to the extent of overlapping spectral characteristics of mutually communicating or coupled resonant states, always in accordance with conservation laws and other universal principles of physics.

   It is important to recognize that solutions of relevant classical wave equations consistently address at least two spatially and temporally opposed wave groups. This consideration also raises the question of how we conceptualize and understand the arrow of time. For instance, the spatial positions of certain events can be located between positive and negative infinity in any combination of (x, y, z) coordinates. Since spatial and temporal coordinates are interrelated, as conceptualized in the theory of relativity, the hypercomplex analytic wavefunction of a phasor could, or perhaps should, have at least three time-coordinates (t_x, t_y, t_z). This leads to the intriguing possibility that the arrow or direction of time could be spatially distributed or oriented in multiple directions (see more in Chapter 5, "5.0.1. Meaning of Time and Certainty Relations").

6. Our Universe may be part of a Multiverse composed of multiple phase-shifted worlds, where the phase shifting relates to spatial and temporal dimensions. These dimensions behave like mutually orthogonal wave functions that could represent matter-waves and energy-momentum carrying entities. The tangible,

experimentally verifiable world of physics is the primary and tangible reality we currently understand, while everything else, which belongs to a domain of probabilistic, non-detectable, phase-shifted, orthogonal, or higher dimensions of matter, could be imaginatively considered as an ether or as a residual spatial and temporal background structure or matrix of our Universe.

For example, an absolute vacuum state, or an extremely empty space in our 3+1-dimensional world, could be filled with such a fluidic ether, the content of which still remains unknown. A reasonable assumption is that this ether might possess properties like an ideal gaseous fluid. Additionally, this ether is likely to have measurable electric and magnetic permeability or susceptibility, confirming the constant speed of electromagnetic waves in a vacuum. This suggests that the Universe is fundamentally electromagnetic in nature, as dielectric and magnetic susceptibility and permeability can be considered intrinsic properties of this "spatial-temporal electromagnetic body" of the Universe.

The Casimir effect (or attractive and/or repulsive Casimir force) is related to the same phenomena involving vacuum states, matter state couplings, and communications, which this book describes as the universal temporal-spatial tendency of matter toward mutual synchronization between spectrally overlapping resonant atomic states. This synchronization creates a resulting phase function applicable to isolated particles, as well as micro and macro systems, sets, and ensembles (including ideal gases).

The hypothesized ether or fluidic, etheric background may be composed of dielectric and magnetic entities that participate in resonant state coupling and energy transfer, as Nikola Tesla speculated and experimented with his "magnifying transmitters" [97]. Similar ideas and concepts are increasingly discussed online; see, for example, “Jean de Climont, Editions d'Assailly” [117]. Understanding and exploiting this hidden reality of fluidic and etheric matter, which remains a perplexing problem, will likely continue to be an ongoing project. Determining other detectable or measurable parameters of an ether, if they exist, remains a challenging question. Perhaps such an ether is not completely analogous to real fluids but still represents the spatial-temporal body of our Universe with measurable electromagnetic properties.

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The following citations present a kind of universally applicable, advanced, state of the art position of the contemporary Physics (taken apart from not so much conceptually clear and artificially assembled statistical, Orthodox QT):

*“**Principles of Physics,** From Quantum Field Theory to Classical Mechanics*
https://doi.org/10.1142/9056 | February 2014. Pages: 444
Author: Jun Ni (Tsinghua University, China). ISBN: 978-981-4579-39-1

We start with the following five basic principles to construct all other physical laws and equations. These five basic principles are: (1) Constituent principle: the basic constituents of matter are various kinds of identical particles. This can also be called locality principle; (2) Causality principle: the future state depends only on the present state; (3) Covariance principle: the physics should be invariant under an arbitrary coordinate transformation; (4)

Invariance or Symmetry principle: the spacetime is homogeneous; (5) Equi-probability principle: all the states in an isolated system are expected to be occupied with equal probability. These five basic principles can be considered as physical common sense. It is very natural to have these basic principles. More important is that these five basic principles are consistent with one another. From these five principles, we derive a vast set of equations which explain or promise to explain all the phenomena of the physical world.

Citation "*Karen Markov, Ph.D. Physics, Yale University (1991). Answered Oct 29 2019* ·

"What are some fundamental principles of physics every person should learn?

This is a remarkably difficult and profound question. It asks for the 10 most important pillars of physics. My personal view on it would be

1. ***Principle of least action:*** *this principle in variation theory defines how equation of motion look like and how they are related to symmetries of a so-called action function(al). Principle of least action - Wikipedia Newton's equation F=ma is a direct consequence of it.*
2. ***Conservation of energy and momentum:*** *a consequence of (1) if one assumes time and spatial homogeneity. Laws of physics and outcome of experiments don't depend when and where the experiment happened. This relationship of symmetries and conservation law is the essence of Noether's theorem. I would call this the second especially important principle. Noether's theorem - Wikipedia*
3. ***Constant speed of light****: defines how 4 dimensional space-time works and is the basis of special relativity. Speed of light - Wikipedia*
4. ***Strong equivalence of gravitational and acceleration (mass):*** *one of the foundations of general relativity. Stating that gravitation and acceleration is the same (on small scales). Equivalence principle - Wikipedia*
5. *Growth of entropy with time or* ***Second Law of Thermodynamics****: the foundation of thermodynamics and statistical physics. It links probability, reversibility and macroscopic variables. It also states that time flows in one direction(!). Equivalence principle - Wikipedia*
6. ***Heisenberg's uncertainty principle****: stating that you cannot measure certain quantities accurately at the same time. It describes the quantum nature of the universe on small scales. Uncertainty principle - Wikipedia*
7. ***CPT symmetry:*** *all physical laws and systems are invariant under reversal of time, parity and charge. This symmetry is well observed and limits the possible theories of the universe. It states that antimatter and matter behave in the same way. CPT symmetry - Wikipedia*

These 7 principles are the pillars of modern physics. They shape how the world works on a very basic level. If the world were a computer, this would be the lowest level of kernel code".

Critique of Quantum Theory: A Reassessment of Contemporary Views

Too often, discussions of Quantum Physics or Orthodox Quantum Theory (QT) are filled with glorifying language and elaborate descriptions. These narratives, often espoused by ardent QT advocates, assert that QT works well mathematically, produces accurate predictions, and that the underlying reality and conceptual framework are of little concern. The author of this book addresses these modern narratives and idealized statements with the following commentary.

Quantum Theory is frequently described as the ultimate explanation for how everything in the physical world works and the best description we have of matter and natural forces. However, this statement is overly simplistic and ambitious. Quantum Theory is more about specifically tailored mathematics than physics. If signal analysis, statistics, and probability theory—which are well-established mathematical disciplines—work well, then so does QT. QT is an artificial hybrid, assembled from independently developed mathematical theories, including elements of game theory, with ad hoc assumptions and postulations accepted by consensus among certain physicists (or QT founders). Additionally, QT relies on all conservation laws of physics, which are universally valid across natural sciences.

Alternative and potentially superior mathematical models and concepts could replace contemporary QT, but mainstream QT authorities often suppress or ignore these attempts. Many arbitrary statements are formulated by QT proponents, who offer optimistic but vague and overly broad descriptions. These statements lack specific and unique meanings and are intended to be universally applicable, resulting in verbal glorification without substantive explanations. Numerous independent, alternative, and complementary theories and models already exist or could be further developed to replace QT.

Present-day QT was first developed and postulated in the 1920s by Niels Bohr, Werner Heisenberg, Erwin Schrödinger, and others. There is a common opinion that QT should become more comprehensible and universal, combining with other physics elements, mainly Einstein's special theory of relativity, which describes the behavior of fast-moving objects and leads to quantum field theories. To achieve such unification, QT must be integrated with electromagnetic theory (EM), mechanics, thermodynamics (TD), and fluid dynamics. Concurrently, EM should be enhanced to generate an updated relativity theory (RT). Eventually, the new QT could be assembled, and thermodynamics should evolve to account for the total motional energy in processes, incorporating both kinetic and matter-wave energy.

Current quantum field theories address three of the four fundamental forces responsible for all matter interactions: electromagnetism, the strong nuclear force, and the weak nuclear force. However, much about the strong and weak nuclear forces is likely incorrect, offering only descriptive labels without real explanations. Electromagnetism (EM) is tangible and well-supported by mathematical models and experimental evidence. Other nuclear forces, including gravitation, may be derivatives or consequences of an updated EM. Understanding force effects around nodal and anti-nodal zones of structured standing waves of matter is another complementary approach.

Modern QT claims the Standard Model, based on quantum field theories, is the most accurately tested depiction of matter's workings. However, the Standard Model contains many arbitrary established constants and presents a disorganized picture of matter. If any of the four fundamental forces are fundamentally incorrect, the Standard Model will fail. Better mathematical models and conceptual frameworks are still missing. As it stands, QT is full of paradoxes, illogical, ambiguous, and confusing statements, such as Schrödinger's cat and entanglement effects. Emerging technologies like ultra-secure quantum cryptography and ultra-powerful quantum computing highlight the need for innovative theoretical foundations to replace contemporary QT.

Einstein's general theory of relativity remains the leading gravity theory, though it is non-quantum and particle-free, akin to Newton's gravitation. Efforts to integrate gravity with QT to form a unified "theory of everything" have been unsuccessful. EM needs to be updated to a level that can generate a new RT, followed by establishing a new QT, and eventually a general unified field theory. Cosmological measurements suggesting that over 70-95% of the universe consists of dark matter and dark energy remain hypothetical assumptions not explained by the Standard Model. Incomplete concepts and improper mathematical modelling in cosmology lead to arbitrary, contradictory statements and strange insights. A better understanding of gravitation, possibly related to electromagnetic forces and fields, is essential for progress in this area.

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[♣ From the following citation, and by author of this book created comments, we will see what challenging aspects of contemporary QT are, and why we need to upgrade its foundations.

Citation from: Mark John Fernee. How does quantum field theory (QFT) contribute to our understanding of gravity? Is there a unified theory that explains both QFT and quantum mechanics?

A) "Quantum mechanics was developed to model empirical outcomes, the most well-known being the double slit experiment. To do this you need to let go of the notion of what objects are. Rather you work backwards from the empirical observation of interference. In essence, interference is understood using wave theory, so we need a wavefunction."

Comment: Correct, but incomplete and too much simplified framework, to be considered as a good starting and conceptual platform. Here we need to consider relevant a kind of intelligently upgraded two-body or multibody system, and from such setup to draw other conclusions. The same problematic should also be

analyzed as spatial-temporal evolving situation, where we need to consider mutually interacting (or united) temporal and spatial spectral characteristics of all involved participants.

B) *"The next step is to note that in the low energy limit, the interference pattern is built up through the accumulation of point-like detections. This is distinctly particle like."*

Comment: Again incomplete, simplified and partially wrong consideration. Here we need to better understand why particle-like detections are eventually creating wave-like interference patterns. Something is missing in this concept. Relevant wavefunction is incomplete (without phase function) and not considering comments under A). If emission of photons (or electrons) has randomized spatial-temporal phase shifts, this creates randomized particle-like detections, which are eventually (after longer time interval) completing or accumulating as wave-like interference pattern.

C) *"Combining the wave-like propagation with particle-like detection, we have the wavefunction describing a probability amplitude from which the probability of detection can be calculated using the Born rule. This is something that you could easily guess given the empirical data."*

Comment: Combining two incomplete and oversimplified setups (as A) and B)) and introducing hypothetical assumption about always valid and applicable probability and statistics related modelling, we only complicate the same situation. Parseval theorem or identity is always valid and applicable in similar forms in signal analysis and probability theory, naturally being in the background of such speculations. It effectively mimics energy conservation law, by "framing, smoothing and binding" proposed incomplete and hypothetical modelling concepts.

D) *"The next step is the collapse hypothesis, wherein the wavefunction collapses to a point on detection. The combination of these factors describes the double slit experiment but does not explain it. There is no known mechanism for the collapse hypothesis. This has given rise to the various interpretations of quantum mechanics."*

Comment: Collapse hypothesis is a useful, speculative nonsense that is artificially and imaginatively completing the erroneous conceptual picture. It works sporadically until we find something much better.

E) *"However, what is apparent from the classical theory is that the field can be decomposed into a superposition of modes, each of which can be modelled as a simple harmonic oscillator. The quantum simple harmonic oscillator is a textbook problem that introduces Planck's quanta as the energy packets representing the successive excitation of the mode. These quanta are called photons. However, given the level of abstraction, the properties of the photon are far removed from the properties of a classical particle."*

Comment: This is only one good statement among possible, relevant and complementary explanations of quantizing. It should be appropriately extended and generalized considering achievements of "Fourier-Shannon-Kotelnikov-Nyquist-Whittaker-D. Gabor" signal analysis. ♣]

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WAVE EQUATIONS

In this section, we will focus on characterizing progressive and standing plane waves, which are fundamental components of most matter-waves. Plane waves provide a simple, clear and straightforward way to describe and define the essential properties of matter-waves and wavefunctions. The wavefunction considered here is based on the Complex Analytic Signal modeling and formulations as discussed in Chapter 4.0, specifically about equations (4.0.1), (4.0.4), and (4.0.82).

Mathematically, self-contained standing matter waves will be associated with relatively stable matter structures such as atoms, various particles, and planetary systems. On the other hand, progressive waves will represent the wavelike energy-momentum communication between these stable matter structures. This approach forms the foundation for later addressing other types of waves using the same mathematical structure and methodology established here with plane waves and the referenced formulations (4.0.1) – (4.0.82).

Plane waves are characterized by a wavefront (or displacement) function that depends only on the spatial coordinates (here, x) in the direction of wave propagation, and on time (t). This simplification allows us to clearly understand and represent the behavior of matter-waves in various contexts, such as $\Psi(x,t)$, $p(x,t), v(x,t)$ … For instance, we can present a plane wave, which is an oscillatory displacement wave function as,

$$\Psi(x,t) = A\sin\left[\frac{2\pi}{\lambda}(x - u\cdot t)\right] = A\sin\left[2\pi(\frac{x}{\lambda} - \frac{u}{\lambda}\cdot t)\right] =$$
$$= A\sin\left[2\pi(\frac{x}{\lambda} - f\cdot t)\right] = A\sin\left(k\cdot x - \omega\cdot t\right).$$

A more general form of a wave function $\Psi(x,t)$ with an initial phase constant Φ that shifts the same wave is, $\Psi(x,t) = A\sin\left(k\cdot x - \omega\cdot t + \Phi\right)$. The wave number k and the angular frequency $\omega = 2\pi f = 2\pi\frac{u}{\lambda}$ are analogically defined as being directly dependent on relevant spatial and temporal periodicities (or periods) such as wavelength λ and time period $T = \frac{1}{f}$, meaning $k = 2\pi f_x = \boxed{2\pi\cdot\frac{1}{\lambda}}, \omega = 2\pi f_t = 2\pi f = \boxed{2\pi\cdot\frac{1}{T}}$.

Plane waves could also be considered as being good approximations of spherical waves, at large distances from a point wave source. Plane waves $\Psi(x,t)$ are known to be solutions of a second-order, homogenous, Classical, partial differential wave equation (equally known in acoustics, electromagnetism, mechanical oscillatory motions like a string and membranes oscillations, in fluids and plasma electromagnetic waving and oscillations, etc.). The generally known form of Classical wave equation (valid and applicable to any kind of wave and oscillatory motions in Physics) is,

$$\Delta\Psi(x,t) - \frac{1}{u^2}\frac{\partial^2\Psi(x,t)}{\partial t^2} = 0.$$

Schrödinger equation

The Schrödinger wave equation can be naturally and smoothly derived from the classical second-order differential wave equation when the wavefunction is modelled as an Complex Analytic Signal function (as discussed in Chapter 4.3). However, Schrödinger's original approach involved a more complex development process, relying on what could be described as "intuitive and intricate mathematical patchwork." Despite this, his equation remains one of the most significant equations in the study of matter waves in physics.

Recent publications have demonstrated the mutual transformability or equivalence between the Classical wave equation and the Schrödinger equation, including their relation to electromagnetic wave equations. This suggests that the Classical wave equation is a fundamental and universally applicable equation in both the micro and macro realms of physics, being particularly interesting when the wavefunction is modelled as a non-probabilistic Complex Analytic Signal.

Supporting this perspective, several studies, such as [105] Himanshu Chauhan, Swati Rawal, and R.K. Sinha's "Wave-Particle Duality Revitalized: Consequences, Applications, and Relativistic Quantum Mechanics," and [85] George Shpenkov and [86] Victor Christianto's "Review of Schrödinger Equation & Classical Wave Equation," reinforce the idea that the classical wave equation is the most foundational tool for wave analysis.

Although the Schrödinger equation has been remarkably successful in predicting outcomes in numerous cases, it does have limitations. For instance, it fails to accurately predict certain spectral lines of the hydrogen atom. As cited by Joseph Lucas and Charles W. Lucas, Jr. in their work, "A Physical Model for Atoms and Nuclei - Part 3" [145], there are instances where the Schrödinger equation falls short in its predictive power. Read citation below (from **[145],** Joseph Lucas and Charles W. Lucas, Jr. A Physical Model for Atoms and Nuclei - Part 3):

Citation (J. Lucas and C. Lucas): The Toroidal (atom) model is extended in this article to describe the emission spectra of hydrogen and other atoms. ... The resulting model accurately predicts the same emission spectral lines as the Quantum Model. including the fine structure and hyperfine structure. Moreover it goes beyond the Dirac Quantum Model of the atom to predict 64 new lines or transitions in the extreme ultraviolet emission spectra of hydrogen that have been confirmed by the extreme Ultraviolet Physics Laboratory at Berkeley from its NASA rocket experiment data [5]...

Schrödinger wave equation (with probability wave function interpretation) works very well mostly for relatively small and sufficiently isolated micro-world items. In cases of big compounds of atoms and other forms of matter it works in principle, but with increased complexity and difficulties regarding mathematical processing, and in some cases, it would not produce useful results (read the following citation from **[146]**).

*Citation: **Quantum theory cannot consistently describe the use of itself. Daniela Frauchiger1 & Renato Renner1***
"Quantum theory provides an extremely accurate description of fundamental processes in physics. It thus seems likely that the theory is applicable beyond the mostly microscopic domain in which it has been tested experimentally. Here, we propose a Gedankenexperiment to investigate the question whether quantum theory can, in principle, have universal validity.

The idea is that, if the answer was yes, it must be possible to employ quantum theory to model complex systems that include agents who are themselves using quantum theory.
Analyzing the experiment under this presumption, we find that one agent, upon observing a particular measurement outcome, must conclude that another agent has predicted the opposite outcome with certainty. The agents' conclusions, although all derived within quantum theory, are thus inconsistent. This indicates that quantum theory cannot be extrapolated to complex systems, at least not in a straightforward manner".

The probabilistic (and non-dimensional) interpretation of a wave function, along with its associated mathematical processing, serves more as a smoothing and averaging tool, rather than a direct, real-time representation of Reality. This approach, while useful in some contexts, can introduce limitations and paradoxes when it comes to fully understanding Reality, because Reality cannot be completely and naturally captured by an artificial mathematical model based on probability wave functions.

In this book, the wave function is modeled as an Analytic Signal, which relates directly to the real-time power of matter waves and is grounded in the Parseval theorem. This approach offers a more natural, richer, and more productive framework for modeling matter waves compared to the probabilistic method. For a detailed exploration of this non-probabilistic wavefunction model, see Chapters 4.0 and 4.3 of this book.

Wavefunction motional properties

The **wave function phase velocity $\mathbf{u}$** is the velocity of a point on the wave $\Psi(x,t)$ that has certain constant phase (for example, its crest), and is given by relations,

$$u = \lambda \cdot f = \frac{\lambda}{T} = \frac{\omega}{k} = \frac{\tilde{E}}{p},$$

and on a similar way we define and develop **group wave-function velocity $\mathbf{v}$** as the velocity of the envelope of a wave $\Psi(x,t)$ (that has certain constant amplitude), and it is given by relations as (see more in Chapters 4.0 and 4.1),

$$v = \frac{d\omega}{dk} = \frac{dE}{dp} = \frac{d\tilde{E}}{dp} = \frac{dE_k}{dp} = u - \lambda\frac{du}{d\lambda} = u + k\frac{du}{dk} = -\lambda^2\frac{df}{d\lambda}.$$

It is important to highlight that solutions of classical wave equations consistently involve at least two wave components or wave groups that are intrinsically coupled, often referred to as entangled. These wave packets propagate in opposite directions, either towards or away from a vibration or signal source, due to the simultaneous manifestation of action and reaction effects, as well as the principles of induction and inertia. This dual (inwards and outwards) propagation reflects the universally valid conservation laws and symmetries in physics (see Chapter 4.3 for more details). Later, we will represent these wave groups or functions, which move in opposing directions, as $\psi^{+} = \sum_{[n]} \psi_i^{+}$ and $\psi^{-} = \sum_{[n]} \psi_i^{-}$.

From a point of view of Spectral, Fourier and Analytic Signals analysis, and wave functions modeling, plane waves (when being solutions of Classical wave equation) are (for instance) presentable as,

$$\Psi(x,t)=\Psi(kx-\omega t)+\Psi(kx+\omega t)=$$
$$=\sum_{[n]}\psi_i(kx-\omega t)+\sum_{[n]}\psi_i(kx+\omega t)=\sum_{[n]}\psi_i^{+}+\sum_{[n]}\psi_i^{-},$$
$$\text{or, } p(x,t)=p(kx-\omega t)+p(kx+\omega t),$$
$$\text{or, } v(x,t)= v(kx-\omega t)+v(kx+\omega t), \ \ldots$$

General solutions of Classical wave equations (in any form, and when $\Psi(x,t)$ is formulated as a Complex, Analytic signal wave function $\overline{\Psi}(x,t)$) are also presentable as a linear summation or integral superposition of (at least two, or "many in-pairs") elementary plane-wave elements in a complex form, such as,

$$\overline{\Psi}(x,t)=a(x,t)\cdot e^{I(kx-\omega t)}+b(x,t)\cdot e^{I(kx+\omega t)}=\overline{\psi}^{(+)}(x,t)+\overline{\psi}^{(-)}(x,t),\ \overline{\psi}^{(+,-)}(x,t)=\left|\overline{\psi}^{(+,-)}\right|\cdot e^{I\left[\varphi_{(+,-)}\right]}$$
$$\Rightarrow \overline{\Psi}(x,t)=\left|\overline{\psi}^{(+)}\right|\cdot e^{I\left[\varphi_{(+)}\right]}+\left|\overline{\psi}^{(+)}\right|\cdot e^{I\left[\varphi_{(-)}\right]}+\left|\overline{\psi}^{(-)}\right|\cdot e^{I\left[\varphi_{(+)}\right]}+\left|\overline{\psi}^{(-)}\right|\cdot e^{I\left[\varphi_{(-)}\right]},\ I^2=-1.$$

In this book, we consistently model complex wave functions as Complex Analytic Signal functions $\overline{\Psi}(x,t)$ *(see Chapter 4.0, specifically sections "4.0.11. Generalized Wave Functions and Unified Field Theory" and "4.0.12. Evolution of the RMS Concept" for more details). This approach provides a natural and general method for describing and mathematically defining all wave parameters and properties in both space-time and corresponding frequency domains. This modeling applies to all types of waves, including de Broglie matter waves (see more on this under section 4.0.82 in Chapter 4.0).*

The square of such a wave function, which is dimensional and tangible (meaning not-probabilistic), represents the power of the signal or matter-wave, or its temporal energy flow. This function has a rich mathematical structure that allows for complex mathematical processing. It can be transformed or normalized to align with the probabilistic wave functions as found in orthodox Quantum theory, providing a similar mathematical framework with several advantages and new insights (see Chapters 4.0 and 4.3, especially section "4.3.3. Probability and Conservation Laws," for more on power-related wave functions).

Arbitrary-shaped and energy-finite wave functions can always be represented through Fourier analysis, as summations or integral superpositions of elementary, planar, simple harmonic, and sinusoidal waves. In physics, particular attention should be paid to narrow frequency-band signals, wave packets, or wave groups—such as Gaussian-Gabor amplitude-shaped signals. These signals, including photons and familiar matter waves among elementary particles, are prevalent because Nature tends to synthesize signals in an energy-efficient, discretized, and rapid manner, often using narrow-band wave groups (as known in Kotelnikov-Nyquist-Shannon-Whittaker signal analysis).

Gaussian or bell-curve-shaped wave packets are well-localized, energy-finite, and band-limited in both their temporal and spatial (or original and spectral) domains. This implies that the relevant uncertainty relations for such signals approach certainty relations, akin to "standing wave atomized" relations between corresponding domains, where an equality sign holds between domain durations (see more in Chapter 5).

A "Gaussian-Gabor pulse", limited in both time and frequency, has a finite spectral content in all domains. For instance, a signal with a certain frequency carrier, such as a chirp or burst, has total (or absolute, non-statistical) time and frequency-limited *widths, denoted as T and F. Given that we are dealing with a Gaussian-Gabor pulse, its frequency-domain amplitude will also have a Gaussian bell-curve envelope. This pulse is energy- and domain-duration-limited, allowing us to determine or measure the relevant temporal and frequency signal durations, T and F. The generally valid relation between the total temporal and frequency durations of such Gaussian-Gabor pulses is* $T \cdot F \approx 1$*. In other cases, where the analyzed signal is energy-finite but not band-limited, the relation would be* $T \cdot F > 1$.

If a Gaussian signal propagates through a non-dispersive, sufficiently homogenous medium and it is detected by sensors at different locations, each sensor will detect signals with Gaussian envelopes like the incident source signal*. For each specific sensor, the measured temporal and frequency widths or durations of the received signals will again satisfy the basic uncertainty domain relation*

$$T_i \cdot F_i \approx 1 \, , \, i = 1,2,3,...,n \;\Rightarrow\; F_i \approx \frac{1}{T_i} .$$

Consequently, measured temporal durations T_i could be different, but in every new situation we will always have, $T_i \cdot F_i \approx 1$, $i = 1,2,3,...,n$. If we now summarize the same situation for one specific sensor, we can define the *propagating-media structure quality factor* as,

$$Q_s = \frac{F_{initial}}{F_{measured}}, \begin{pmatrix} T_{initial} \cdot F_{initial} = T_{measured} \cdot F_{measured} \approx 1, \\ \frac{T_{initial}}{T_{measured}} \cdot \frac{F_{initial}}{F_{measured}} = 1 \end{pmatrix} \Rightarrow 0 < \left(Q_s = \frac{F_{initial}}{F_{measured}} = \frac{T_{measured}}{T_{initial}} \right) \leq 1 .$$

If Q_s is close to one (1), monitored structure is homogenous, isotropic, and stable, and if Q_s would rich certain small-value-threshold, this could be considered as having dissipative and/or anisotropic media. For instance, we could conveniently present mentioned narrow-band wave-groups or wave-packets also as Analytic Signals, being the superposition of elementary (simple-harmonic and sinusoidal) plane waves, such as,

$$\overline{\Psi}(x,t) = \begin{Bmatrix} \sum_{(n)} \left[a(k_n) \cdot e^{I(k_n x - \omega_n t)} + b(k_n) \cdot e^{I(k_n x + \omega_n t)} \right] = \sum_{(n)} \overline{\psi}_n \\ \text{or} \\ \int_{[\Delta k]} \left[a(k) \cdot e^{I(kx - \omega t)} + b(k) \cdot e^{I(kx + \omega t)} \right] dk = \int_{[\Delta k]} \overline{\psi} dk \end{Bmatrix} = \overline{\Psi}^{+}(kx - \omega t) + \overline{\Psi}^{-}(kx + \omega t) = \left| \overline{\Psi} \right| \cdot e^{I\Phi}$$

$$R_e\left[\overline{\Psi}(x,t)\right] = \Psi(x,t) = A(x) \frac{\sin(\underline{\Delta\omega}\, t - \underline{\Delta k}\, x)}{(\underline{\Delta\omega}\, t - \underline{\Delta k}\, x)} \cos(\omega t - kx) + B(x) \frac{\sin(\underline{\Delta\omega}\, t + \underline{\Delta k}\, x)}{(\underline{\Delta\omega}\, t + \underline{\Delta k}\, x)} \cos(\omega t + kx)$$

$$\omega = 2\pi f \,(=) \text{carrier frequency } (>>> \underline{\Delta\omega})$$

$$\underline{\Delta\omega} = 2\pi\Delta f \,(=) \text{wave-packet frequency bandwidth } (<<< f) .$$

Wave-particle duality can be conceptualized in a similar manner by envisioning any stable particle with a non-zero rest mass as a tangible, spatial assembly of many elementary oscillators or resonant circuits, mechanical, acoustic, electromagnetic, and electromechanical, as illustrated in Fig. 1.1 of the first chapter.

An Analytic Signal Wave Function $\overline{\Psi}(x,t)$, based on the D. Gabor model using the Hilbert transform (refer to [7], [57]), and the corresponding Complex Classical Wave equation, provide a natural and convenient framework for representing de Broglie matter waves. This is particularly effective when combined with the properties and facts of the Particle Wave Duality Concept (PWDC). For further details on PWDC, see Chapters 4.1 and 4.3, as well as the continuation of this chapter.

$$\Delta\overline{\Psi}(x,t)-\frac{1}{u^2}\frac{\partial\overline{\Psi}^2(x,t)}{\partial t^2}=0 \Leftrightarrow \left[\Delta\Psi-\frac{1}{u^2}\frac{\partial^2\Psi}{\partial t^2}=0\,,\ \Delta\hat{\Psi}-\frac{1}{u^2}\frac{\partial^2\hat{\Psi}}{\partial t^2}=0\right]$$

$$\left[\begin{array}{l}\overline{\Psi}(x,t)=\left|\overline{\Psi}\right|\cdot e^{I\Phi}=\overline{\Psi}^{+}(x,t)+\overline{\Psi}^{-}(x,t)=\Psi(x,t)+I\cdot\hat{\Psi}(x,t)=\\ =\left[\Psi^{+}(x,t)+I\cdot\hat{\Psi}^{+}(x,t)\right]+\left[\Psi^{-}(x,t)+I\cdot\hat{\Psi}^{-}(x,t)\right]=\\ =\left[\Psi^{+}(x,t)+\Psi^{-}(x,t)\right]+I\cdot\left[\hat{\Psi}^{+}(x,t)+\hat{\Psi}^{-}(x,t)\right],\ \hat{\Psi}(x,t)=H\left[\Psi(x,t)\right],\ I^2=-1\,.\end{array}\right]$$

The complex wave function based on the Analytic Signal naturally and effortlessly produces Schrödinger and other well-known wave equations from Quantum Theory when integrated with the Particle Wave Duality Concept (PWDC). This occurs without the need for assumptions or artificial mental and mathematical hybridization, where missing elements must be supplemented by intuition or inspiration. For more on PWDC, refer to Chapters 4.1, 4.3, 8, and the current chapter. The interpretation and mathematics of wave functions in this book are introduced in Chapter 4.0, under "4.0.11. Generalized Wave Functions and Unified Field Theory." While distinct from the probabilistic wave functions of Quantum Theory, these wave functions yield equivalent or even richer results than traditional quantum mechanical wave functions.

It's important to emphasize that solutions of Classical and Complex second-order partial differential Wave Equations, including all forms of Schrödinger equations, inherently involve at least two Complex Analytic-Signal wave functions, or wave packets, propagating in opposite directions (inward and outward) $\overline{\Psi}^{+}(x,t)$, $\overline{\Psi}^{-}(x,t)$. These wave functions respect CPT Symmetry, being resonantly or entanglement-coupled, reflecting the principle that action equals reaction, combined with various induction laws. Unfortunately, due to specific wave-emitting conditions, we often consider only one of these wave components (typically the one propagating outward or in a positive direction).

Spatially and omnidirectionally, wave groups exist around and within a wave source, propagating in pairs in opposite temporal and spatial directions, positive, negative, inward, and outward. Solutions of Ordinary and Complex, Classical Wave, and Schrödinger equations (including Dirac's equation) are combinations of waves propagating in opposite directions. Some of these waves propagate along negative time axis, challenging our conceptual understanding of wave motions. Nikolai Kozyrev's work [158] provided deep insights into the nature of time, space, matter waves, and cosmological events, experimentally documenting that matter-wave signals can travel through past, future, and present time, even exceeding the speed of light (all of that again related to CPT Symmetry). Modern Cosmology and Astronomy, particularly concerning Doppler effects or redshift and blueshift measurements, relate to these phenomena. These frequency shifts, caused by linear motions, are further modulated by the angular motions of cosmic objects, matter states, and fields (see more in Chapter 2, and references [37], [40], [41], and [43]).

By conceptualizing the unity of complex, synchronized matter motions across all temporal and spatial directions, we lay the foundation for understanding imaginative phenomena such as innovations, visions, predictions, and telepathy, if humans and other living species act as both receivers and emitters of electromagnetic and other matter waves. If we can travel in any direction within spatial coordinates, and given the close relationship between time and space, it raises the question: why haven't we mastered omnidirectional time travel? Nikola Tesla, with his boundless imagination and successful exploration of spatial-temporal concepts, serves as a prime example of such intellectual and creative freedom.

Additionally, this perspective prompts us to consider antimatter states as wave groups traveling in a negative, past-time direction, phase-shifted similarly to how Feynman depicted them in his famous interaction diagrams. We might speculate that matter and antimatter states (or their corresponding wave groups) are mutually orthogonal functions, akin to the real and imaginary components of a Complex Analytic Signal function. This could explain why we inhabit a universe dominated by ordinary matter, without interference or overlapping with an antimatter counterpart.

Citation taken from: https://simple.wikipedia.org/wiki/Feynman_diagram.
A **Feynman diagram** is a diagram that shows what happens when elementary particles collide.[1] Feynman diagrams are used in quantum mechanics. A Feynman diagram has *lines* in different shapes—straight, dotted, and squiggly—which meet up at points called *vertices*. The vertices are where the lines begin and end. The points in Feynman diagrams where the lines meet represent two or more particles that happen to be at the same point in space at the same time. The *lines* in a Feynman diagram represent the probability amplitude for a particle to go from one place to another.

In Feynman diagrams, the particles are allowed to go both forward and backward in time. When a particle is going backward in time, it is called an antiparticle. The meeting points for the lines can also be interpreted forward or backwards in time, so that if a particle disappears into a meeting point, that means that the particle was either created or destroyed, depending on the direction in time that the particle came in from.

Feynman diagrams are named after Richard Feynman, who won the Nobel Prize in Physics. His diagrams are very simple in the case of quantum electrodynamics (QED), where there are only two kinds of particles: electrons (little particles inside atoms) and photons (particles of light). In QED, the only thing that can happen is that an electron (or its antiparticle) can emit (or absorb) a photon, so there is only one building block for any collision. The probability of amplitude for the emission is very simple, -it has no real part, and the imaginary part is the *charge* of the electron.

[♣ Inside the Classical, second order, partial differential wave equation, relevant wave velocity is phase velocity $\mathbf{u}$,

$$\Delta\Psi(x,t)-\frac{1}{u^2}\frac{\partial^2\Psi(x,t)}{\partial t^2}=0\,,\ u=\lambda\cdot f=\frac{\omega}{k}=\frac{\tilde{E}}{p}.$$

If the same wave equation relates only to electromagnetic waves or photons (in an open space and vacuum), then, relevant wave equation is,

$$\Delta\Psi(x,t)-\frac{1}{c^2}\frac{\partial^2\Psi(x,t)}{\partial t^2}=0\,,\ c=\frac{1}{\sqrt{\varepsilon_0\mu_0}}=\lambda\cdot f=\frac{\omega}{k}=\frac{\tilde{E}}{p}=const.$$

Let us now imagine that analyzed waves (either electromagnetic, or some of other nature) are passing through specific (spatial-temporal) media where dielectric and magnetic constants $\varepsilon_r=\varepsilon(\mathbf{r}),\ \mu_r=\mu(\mathbf{r})$ are for some physical reason *spatial and/or material parameters dependent*, meaning that involved phase and group velocity would also be spatial and/or material parameters dependent (all of them also being causally dependent of participants' masses, energies and moments distributions). This will influence Classical wave equation to consider other relevant factors, as,

$$\Delta\Psi(r,t) - \frac{1}{u(r)^2}\frac{\partial^2\Psi(r,t)}{\partial t^2} = 0,\ r = r(x,y,z),\ u(r) = \frac{1}{\sqrt{\varepsilon_r\mu_r}} = \lambda\cdot f = \frac{\omega}{k} = \frac{\tilde{E}}{p} = \frac{\partial r}{\partial t},$$

$$\varepsilon_r = \varepsilon(r),\ \mu_r = \mu(r),\ v = \frac{d\omega}{dk} = \frac{dE}{dp} = \frac{d\tilde{E}}{dp} = \frac{dE_k}{dp} = u - \lambda\frac{du}{d\lambda} = u + k\frac{du}{dk} = -\lambda^2\frac{df}{d\lambda},$$

$$\left(\tilde{E} \cong E_k\right) = E - E_o = \int_{[\Delta t]} \Psi^2(t)dt = \int_{-\infty}^{+\infty} \hat{\Psi}^2(t)dt = \int_{[\Delta t,\Delta p]} v dp = \int_{[m]} c^2 d\tilde{m} =$$

$$= \int_{-\infty}^{+\infty} \Psi^2(r)dt = \int_{-\infty}^{+\infty} \hat{\Psi}^2(r)dt = \dots,\ \tilde{m} = \gamma m, E_o = mc^2 = const.,$$

$$F = \frac{dE}{dr}(=)\nabla_{(x,y,z)}E = \frac{dp}{dt},\ \tau = \frac{dE}{d\theta}(=)\nabla_{(\theta\dots)}E = \frac{dL}{dt},\ \overline{P}_4 = P(\vec{p}, \frac{E}{c}),\ \lambda = \frac{h}{p}.$$

Such conceptualization (of Classical Wave Equation, group, and phase velocity, and involved dielectric and magnetic constants) in its later evolution would create grounds for better explanation of all natural forces (including gravitation), since forces are related to gradients of involved energy (and masses) distributions, $F = \frac{dE}{dr} = \nabla_{(x,y,z)}E = \frac{dp}{dt}$. See more about natural forces in the same Chapter of this book, under "10.02 MEANING OF NATURAL FORCES".

Also, in cases of different interactions (between particles and mater waves, or photons, in any combination) like impacts, waves refractions, interferences, superpositions and passing by or through massive objects, we could expect that dielectric and magnetic constants of the media where mentioned interactions are happening, $\varepsilon_r = \varepsilon(r,\dot{r}),\ \mu_r = \mu(r,\dot{r}), r = r(x,y,z)$, will significantly change during interaction, since relevant phase and group velocity would also change.

Additionally, the understanding of blackbody radiation, the relationship between temperature and electromagnetic or light emissions, and the Cosmic Microwave Background (CMB) radiation can be better explained compared to contemporary physics. Specifically, the CMB may be (still hypothetically) interpreted not as evidence of the Big Bang, but rather as an intrinsic property of matter and atoms.

The current thermodynamic definition of temperature, which is primarily based on the average kinetic energy of involved particles, will be expanded to include other forms of motional and matter-wave energy. As a result, the Planck Blackbody Radiation Law will undergo significant revisions, potentially leading to a complete remodeling of its explanation.

Through this line of thinking, as explored in this chapter, we arrive at the conclusion that electromagnetic phenomena in various forms underpin all matter, fields, and forces in the universe. Orthodox Quantum Theory (QT) fails to capture this rich interplay of matter and energy in motion because it relies on "fuzzy, incomplete, and averaged mathematical models" that are conceptually limited. Specifically, QT lacks in-depth analysis of crucial factors such as electromagnetic properties of reaction participants, wave-particle velocities, acting forces, signal phases, and energy.

In developing mathematics and wave functions, we humans naturally began with real numbers and eventually progressed to complex numbers and functions. However, in nature, physics, and the universe, complex (and hypercomplex) numbers, functions, and spaces are the starting point for mastering matter waves and other motions. The real numbers domain is merely a lower-dimensional projection or subset of this broader complex-numbers domain. To illustrate, consider the difference between analyzing a situation in a laboratory coordinate system (analogous to the real numbers domain) and analyzing it in the center-of-mass system (analogous to the complex numbers domain).

Humans tend to begin their intellectual processes in the real numbers domain and gradually expand to the complex numbers' domain through inductive, analogical, and intuitive thinking. However, in nature, the universe, and physics, the complex numbers domain is fundamental. Thus, the true meaning and nature of wave functions and wave equations should be defined initially in the complex and hypercomplex domain. Only afterward can this complex-numbers domain be reduced to the real numbers and functions domain.

A similar analogy can be drawn with Minkowski-Einstein Complex 4-vectors. On one hand, conservation laws of energy and other relevant vectorial quantities (such as moments, velocities, and forces) are traditionally formulated separately in the domain of real numbers, ordinary vectors, and functions. On the other hand, Minkowski-Einstein Complex 4-vectors unify and generalize these quantities within the domain of complex numbers and functions. ♣]

Matter waves and matter states synchronization

Synchronization between various motional objects, matter states, wave groups, masses, fluids, or atoms occurs naturally in many ways. In this context, we refer to phase-frequency resonant synchronization between motional states, which involves mutual temporal-spatial coupling through relevant matter-wave communications (since all motional "energy-momentum states" generate matter waves). Even biological entities and living matter, including their internal matter waves, synchronize in various ways, internally and externally, locally and non-locally, thereby becoming an integral part of the surrounding universe. These matter-waves may exhibit both electromagnetic and mechanical wave-particle characteristics. The distinction between organic, living matter and inorganic, non-living matter lies in their overall spatial and temporal activity and the degree of mutual synchronization.

Other forms of matter waves and wave-particle duality behaviors, not yet mentioned or understood, may also play a role in synchronization, such as the concept of Morphic Resonance (see more at [Morphic Resonance],
(https://www.sheldrake.org/research/morphic-resonance) , [153]).

In any of these cases of matter-state synchronization, it is important to reach a critical mass of identical or similar resonators, matter states, objects, or living species (like atoms, crystals, or biological cells) to stimulate and accelerate system-wide synchronization. This occurs because these matter states, which are of the same kind, have identical or similar spectral characteristics, functioning as resonant structures that naturally couple and communicate through superposition and interference effects between their respective matter-waves, thereby creating joint and resulting amplitude and phase functions. This is also valid for mathematically idealized thermodynamic and statistical systems, like ideal gases, plasma-states etc.

Examples of synchronization also include the mixing, alloying, and homogenization of all kinds of fluids. Small pieces of metal, for instance, have specific electrical, mechanical, and chemical properties. If we add another piece of the same metal to a quantity of liquid metal, through fusion or melting, the new larger piece (after mixing and solidification) will retain the same electric and mechanical properties as the original. In this process, the atoms of the new mass will be well-fitted, packed, and electromagnetically and mechanically coupled or synchronized. This internal or interatomic coupling is also an act of electromagnetic attraction and synchronization between the involved atoms. When atoms are of the same kind, synchronization and mutual coupling between the metal pieces (or their atoms) occur quickly and easily during melting and homogenization, because they share identical (atomic and molecular) spectral content.

This natural synchronization is responsible for the formation of macro masses, planetary systems, and galaxies. We often overlook that such phenomena are a form of communication and electromagnetic coupling between resonators with identical, similar, or overlapping spectral content. Synchronization within complex macro systems means that a certain resulting amplitude and wave-phase function will emerge for these systems. Since gravitation always acts between masses, it implies a certain level of synchronization between any mutually separated masses. We know that all solar, planetary, and galactic systems are also in some form of mutual, periodic, or resonant synchronization, as elaborated in Chapter 2 of this book.

If "internal" or interatomic synchronization is essentially electromagnetic, this suggests that any "external" non-mechanical or non-contact synchronization between separated masses could also involve some form of electromagnetic coupling. The intensity of coupling between different, separated masses (aside from Newtonian attraction, which depends on the distance between them) also relates to the spectral or resonant content of the masses involved. Since atoms are ensembles of resonators or resonant states, the coupling or synchronization level is strongest where the spectral characteristics of both masses overlap in spatial and temporal frequency.

In conclusion, different aspects of synchronization between atoms and masses are constantly active in our universe, constituting a natural fact or law. Wave-particle duality concerning relevant matter waves extends from atoms and subatomic entities to macro-systems, including all forms of living and biological matter, which are created when the involved micro-participants enter mutually synchronized resonant states, thereby creating joint and resulting phase functions of their matter-waves.

We can also conceptualize the boundary between the micro and macro cosmos as lying at the atomic level. Everything smaller than or within atoms largely belongs to the micro world of physics, while larger agglomerations and compounds of atoms represent the macro world of physics, including planetary systems and galaxies. This simplified and analogical understanding serves as a good starting or indicative platform, though it can be further elaborated.

It is reasonable to assume that all atoms and masses in our universe are somehow connected, coupled, and inclined to synchronize. Entanglement effects can be understood as a form of immediate communication between coupled energy-momentum states, where entanglement (in its broader and still hypothetical meaning) may also function as an active, real-time information-carrying channel. Gravitation and electromagnetic fields are currently the best candidates for explaining non-mechanical couplings between masses, and it is plausible that gravitation is a specific manifestation of electromagnetic field interactions between masses.

All of this resonates with Nikola Tesla's concept of Resonant Electromagnetic coupling, using a magnifying transmitter, and the associated Radiant (or electromagnetic) energy flow between involved masses, formulated as "Dynamic Gravity Theory," including ideas from Rudjer Boskovic's "Universal Natural Force" theory. Tesla's wireless energy transfer, using a magnifying transmitter, is based on combined temporal and spatial resonance effects, where the electromagnetic resonance of Tesla's transformer is coupled with a receiver tuned to the same resonant frequency, using the planet as a common conductor or ground electrode. *Here associated effects of* ***<u>synchronization, resonant coupling, and entanglements</u>*** *(between different matter-wave entities and resonators with mutually overlapping spectral characteristics, created by the same initial interaction) are explicable based on Analytic Signal modeling (see more in Chapter 4.0). For instance, real part of the wave function* $\overline{\Psi}(\mathbf{x},\mathbf{t})$ *is composed of two wave components propagating in mutually opposed (temporal and spatial) directions* $\Psi^{+}(\mathbf{x},\mathbf{t})+\Psi^{-}(\mathbf{x},\mathbf{t})$ *and its imaginary part also has two of such wave components* $\hat{\Psi}^{+}(\mathbf{x},\mathbf{t})+\hat{\Psi}^{-}(\mathbf{x},\mathbf{t})$, *meaning that what we detect, analyze, measure on one side of waves propagation, it will be directly and immediately transferred or present on the opposite side of propagation.*

The resulting phase function of such complex ensembles of atoms and/or particles will inherently contain information about all relevant velocities, frequencies, and wavelengths of the corresponding matter waves. By applying Parseval's theorem to this scenario (to determine the total energy of all wave groups involved), we find that these wave groups, with their positive and negative indices $\mathbf{x}\in(-\infty,+\infty),\ \mathbf{t}\in(-\infty,+\infty)$ *corresponding to mutually opposed wave propagation, synchronously contribute to the total energy in real, present time. This effectively creates a superposition of all past and future matter-wave states for a given event in the present.*

In other words, any present-time state or event is a result of the superposition of its past and future states, with spatial and temporal domains extending infinitely in both positive and negative, or inwards and outwards, directions. This concept provides a strong foundation for understanding temporal and spatial synchronization, resonant coupling, "action-equal-to-reaction" phenomena, induction effects, and an extended meaning of entanglement. These principles, or phenomena are part of universal CPT Symmetry, being valid and applicable across both the micro and macro worlds of physics, including biological systems, and are based on the real-time satisfaction of conservation laws, as well as mechanical and electromagnetic action-reaction and induction laws.

Coherence between two wave functions or signals is a measure of their resonant synchronization (see more about coherence factors in Chapter 4.0, around definitions (4.0.83), (4.0.87), and (4.0.109)). Developing an understanding of these principles and related technologies could lead to advancements such as resonant wireless power transfer, like what Nikola Tesla experimented with.

Further support for the explanation of entanglement and synchronization effects comes from the creation of an Analytic Signal function through the application of the Hilbert transform, which generates two mutually orthogonal functions that are phase-shifted by $\pi/2$*. By creatively combining Minkowski space-time 4-vectors with Complex Analytic Signal modeling (see more at the end of this chapter under "Minkowski Space Foundations, 4-Vectors, and Hypercomplex Analytic Signal"), we find that the Hilbert transform effectively penetrates higher dimensions within a given multidimensional space, addressing additional dimensions relevant to the analyzed matter-wave situation. Since Minkowski energy-momentum 4-vectors have been* *incredibly successful in describing and mastering all kinds of impact and scattering interactions, we can reasonably conclude that the implications of these spatial-temporal dimensional relationships, synchronizations, and other mathematical extensions are fundamentally correct.*

We can begin conceptualizing de Broglie matter waves from the premises established by Particle-Wave Duality Concepts (PWDC). These premises are based on analyses of the Compton effect, the Photoelectric effect, Bragg diffraction, the continuous spectrum of X-rays, and other various phenomena such as secondary emissions, blackbody radiation, and diffractions realized with atomic and molecular rays. All these effects and facts contribute to the formulation of PWDC, an abbreviation coined in this book to represent the matter-wave and wave-particle duality concept, as elaborated in Chapters 4.1, 4.3, 2.3.3, 8, and elsewhere. For additional historical and logical background on the foundations, deviations, and necessary rectifications of wave-particle duality concepts, see references [92] and [105].

Consequently, we can represent every moving particle (in terms of its kinetic energy attributes, disregarding its rest mass if it exists; see the picture below) as a superposition of elementary wave functions $\psi(x,t)$ or $\overline{\psi}(x,t)$, *characterized by its energy-momentum properties and PWDC principles (see more in Chapters 4.1 and 4.3, and later in this chapter under (10.1)). This superposition of simple harmonic and plane-wave elements creates a dualistic, narrow-band wave-group particle equivalent—a type of Gaussian wave packet that represents the moving particle.*

$$\left\{\begin{array}{c} \psi(x,t) = a(x,t)\cdot\cos 2\pi(\frac{x}{\lambda}-\frac{t}{T}) + b(x,t)\cdot\cos 2\pi(\frac{x}{\lambda}+\frac{t}{T}) = a(x,t)\cdot\cos(kx-\omega t) + b(x,t)\cdot\cos(kx+\omega t), \\ \text{or} \\ \overline{\psi}(x,t) = a(x,t)\cdot e^{I(kx-\omega t)} + b(x,t)\cdot e^{I(kx+\omega t)} = \overline{\psi}^{+}(x,t) + \overline{\psi}^{-}(x,t),\ I^2 = -1,\ \psi(x,t) = R_e\left[\overline{\psi}(x,t)\right] \end{array}\right\} \Rightarrow$$

$$\Rightarrow \overline{\Psi}(x,t) = \left\{\begin{array}{l} \sum\limits_{(n)} \overline{\psi}_n,\ \text{or}\ \int\limits_{[\Delta k]} \overline{\psi} dk, \\ \text{or summation of wave-groups} \\ \left|\overline{\psi}(x,t)\right| \frac{\sin(\underline{\Delta\omega}\, t - \underline{\Delta k}\, x)}{(\underline{\Delta\omega}\, t - \underline{\Delta k}\, x)} e^{I(\omega t \pm kx)} \end{array}\right\} = \overline{\Psi}^{+}(kx-\omega t) + \overline{\Psi}^{-}(kx+\omega t) = \left|\overline{\Psi}\right|\cdot e^{I\Phi} .$$

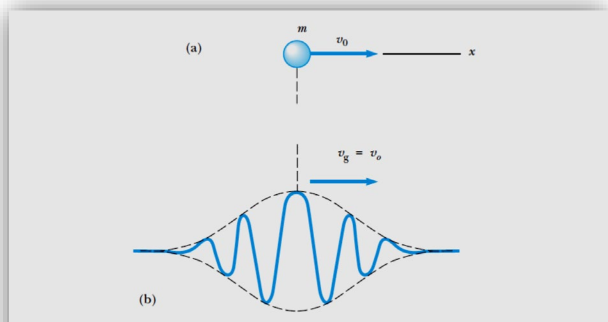

Figure 5.17 Representing a particle with matter waves: (a) particle of mass m and speed v_0; (b) superposition of many matter waves with a spread of wavelengths centered on $\lambda_0 = h/mv_0$ correctly represents a particle.

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When we conceptualize de Broglie, matter waves, or wave packets that are associated to moving particles, we should always have in mind to follow analogical mathematical conceptualization of photons compared to particles, almost in the same way as already elaborated in Chapter 4.1. under "4.1.1.1. Photons and Particle-Wave Dualism ".

We could also present the same, simple or Hypercomplex Analytic Signal, and Complex Wavefunctions, or wave packets, as finite or infinite multiplications of mutually coupled and mutually modulating, harmonic wavefunctions (see more in Chapter 4.0), as,

$$\overline{\Psi}(t) = \Psi_n(t)\prod_{(i=0)}^{n-1}\cos\varphi_i(t) + I\Psi_n(t)\prod_{(i=0)}^{n-1}\sin\varphi_i(t) = \Psi_n(t)\prod_{(i=0)}^{n-1}e^{I\cdot\varphi_i(t)},\ I^2 = -1 .$$

This approach to mathematical representation and processing of wave functions could, in some cases, offer advantages over the traditional summation of waves based on "Fourier-Shannon-Nyquist-Kotelnikov" harmonic elements. However, it is not yet sufficiently developed or integrated with modern signal processing techniques. The use of hypercomplex (or quaternion-modeled) Analytic Signal Waveforms $\overline{\Psi}(t)$, with at least three mutually orthogonal imaginary units (or axes), presents a promising mathematical modeling platform. This framework can represent elementary particles and micro-world matter states (such as photons, electrons, neutrinos, and quarks, including their antimatter counterparts) and can be extended to higher spatial-temporal dimensions and supersymmetry concepts. It allows for more natural modeling of new elementary particle partners, consisting of higher energy levels or short-lived particles or matter states (such as muons and leptons). For more on the mathematical foundations and energy-related structural hierarchy of matter waves, see Chapter 6, specifically "6.1. Hypercomplex, In-depth Analysis of the Wave Function."

Contemporary supersymmetry theory is by some intellectual inertia grounded in binary, bipolar, or dualistic concepts of symmetry, such as using + (plus) and - (minus) signs for field charges, moments, and spin numbers, or waves moving in opposite directions through space and time. These models often rely on mirror symmetry between an object and its image, rotational symmetry in both directions, and similar dualistic principles for conceptualizing matter and antimatter particles, along with other matter states (being part of CPT symmetry).

However, supersymmetry based on quaternions or hypercomplex analytic wavefunctions, which incorporate three imaginary units or three mutually orthogonal spatial-temporal domains, immediately suggests more imaginative and challenging ideas. This framework introduces "ternary," "tripolar," or "three-way" spatially-temporally structured matter-wave concepts of symmetry (here we miss better or commonly accepted terminology). The far-reaching implications of this approach could fundamentally alter and significantly evolve our understanding of the structure of matter, natural forces, elementary particles, the multidimensionality of our universe, and the nature of supersymmetry.

Additionally, our understanding of the direction or "arrow" of time, within the context of hypercomplex analytic signal or phasor modeling, would also undergo significant evolution.

10.00 DEEPER MEANING OF PWDC

The true pioneer of Wave-Particle Duality, though not officially recognized as such, is Jean-Baptiste Joseph Fourier. Fourier's creation of Fourier Analysis demonstrated that all temporal and spatial functions, whether periodic or aperiodic, can be decomposed into sums of simple harmonic, sinusoidal functions. These functions represent motional states and geometric formations, and they can also be recomposed from these components, provided that relevant mathematical conditions and procedures are respected.

Later, *Hungarian American* scientist Denis Gabor significantly enhanced and optimized Fourier Analysis by introducing the concept of the Analytic Signal. This innovation became a universally applicable mathematical model (or wavefunction) for all kinds of matter waves, oscillatory motions, and other wave phenomena known in physics. Using this universal wavefunction, combined with the principles of Particle-Wave Duality Concept (PWDC), one can develop both the Classical Wave Equation and the Schrödinger Equation without resorting to ad hoc assumptions and postulations.

Additionally, Luis de Broglie discovered the dualistic wave-particle relationship between the properties of moving particles and waves, analogous to the behavior of photons. PWDC effectively defines the functional mapping and mathematical equivalence, or transformations, between moving particles and their corresponding matter-wave groups (as detailed in section 10.1). PWDC also fundamentally describes the relationship and unity between linear and angular (or rotational) motions, based on the translational and rotational symmetry of matter within our universe.

Of course, the historical and chronological development of these theoretical and experimental contributions, from Fourier to Luis de Broglie and beyond, was far from straightforward. Many other key figures, such as Schrödinger, Heisenberg, Niels Bohr, Sommerfeld, Max Planck, Max Born, Albert Einstein, Mileva Marić, and Rudjer Boskovic, among others, also played significant roles in the advancement of this field.

Citation from https://en.wikipedia.org/wiki/Wave%E2%80%93particle_duality : "In the late 17th century Sir Isaac Newton had advocated that light was particles, but Christiaan Huygens took an opposing wave approach.[3] Thomas Young's interference experiments in 1801, and François Arago's detection of the Poisson spot in 1819, validated Huygen's wave models".

Now we know that specific **PWDC** properties and relations (as introduced in Chapter 4.1 and summarized in "T.4.0. Photon – Particle Analogies") are connecting a moving particle, or its matter-wave wavefunction, and its (limited frequency-band, and energy-finite) matter-wave equivalent (or wave-packet), with de Broglie wavelength λ, narrow-band matter-wave or particle kinetic energy $E_k = \tilde{E}$, with an equivalent wave-packet group velocity V, its phase velocity $u = \lambda f$, and mean carrier frequency $f = \frac{\omega}{2\pi} = \frac{1}{T}$, being briefly summarized as follows.

$$\begin{Bmatrix} \text{PWDC} \\ \text{particle} \\ \text{wave} \\ \text{duality} \\ \text{code} \end{Bmatrix} (\Leftrightarrow) \left\{ \begin{array}{l} \overline{\Psi}(x,t) = \left|\overline{\Psi}\right| \cdot e^{I\varphi}, \lambda = \frac{h}{p},\ \tilde{E} = E_k = hf = \hbar\omega = \tilde{m}c^2 = m^* vu = pu = h\frac{\omega}{2\pi} = \frac{h}{T} = (\gamma - 1)mc^2,\ k = \frac{2\pi}{\lambda} = \frac{2\pi}{h}p, \\ d\tilde{E}_i = c^2 d(\gamma\tilde{m}_i) = hdf_i = v_i d\tilde{p}_i = d(\tilde{p}_i u_i) = -dE_{ki} = -c^2 d(\gamma m_i) = -v_i dp_i = -d(p_i u_i), \\ \omega = \frac{d\varphi}{dt} = \frac{2\pi}{T} = \frac{2\pi}{h}\tilde{E} = 2\pi f, u = \lambda f = \frac{\tilde{E}}{p} = \frac{\omega}{k},\ \mathbf{L}\omega = pv,\ v = v_g = u - \lambda\frac{du}{d\lambda} = \frac{d\tilde{E}}{dp} = \frac{d\omega}{dk},\ m^* = \gamma m, p = \gamma mv = m^* v, \\ m^* = m + \tilde{m} = \gamma m = m/\sqrt{1 - v^2/c^2}, \tilde{m} = m^* - m = \gamma m\left(1 - \sqrt{1 - v^2/c^2}\right) = \tilde{E}/c^2, p = m^* v = (m + \tilde{m})v = mv + \tilde{m}v, \\ \\ \Delta\overline{\Psi}(x,t) - \frac{1}{u^2}\frac{\partial\overline{\Psi}^2(x,t)}{\partial t^2} = 0 \Leftrightarrow \left[\Delta\Psi - \frac{1}{u^2}\frac{\partial^2\Psi}{\partial t^2} = 0,\ \Delta\hat{\Psi} - \frac{1}{u^2}\frac{\partial^2\hat{\Psi}}{\partial t^2} = 0\right] \Rightarrow \\ \\ \left\{ \begin{array}{l} R_e\left[\overline{\Psi}(x,t)\right] = \Psi(x,t) = A(x)\frac{\sin(\underline{\Delta\omega}\ t \pm \underline{\Delta k}\ x)}{(\underline{\Delta\omega}\ t \pm \underline{\Delta k}\ x)}\cos(\omega t - kx) + B(x)\frac{\sin(\underline{\Delta\omega}\ t \mp \underline{\Delta k}\ x)}{(\underline{\Delta\omega}\ t \mp \underline{\Delta k}\ x)}\cos(\omega t + kx) = \\ = A(x)\frac{\sin(x \pm v\cdot t)}{(x \pm v\cdot t)}\cdot\cos\left[\frac{2\pi}{\lambda}(x - u\cdot t)\right] + B(x)\frac{\sin(x \mp v\cdot t)}{(x \mp v\cdot t)}\cdot\cos\left[\frac{2\pi}{\lambda}(x + u\cdot t)\right] = \\ = a(x,t)\cdot\cos\left[\frac{2\pi}{h}(p\cdot x - \tilde{E}\cdot t)\right] + b(x,t)\cdot\cos\left[\frac{2\pi}{h}(p\cdot x + \tilde{E}\cdot t)\right] = a(x,t)\cdot\cos\left[\frac{2\pi}{\lambda}(x - u\cdot t)\right] + b(x,t)\cdot\cos\left[\frac{2\pi}{\lambda}(x + u\cdot t)\right] \end{array} \right. \end{array} \right\}, \tag{10.1}$$

See also relevant background, later in the same chapter under (10.2-3), and in Chapter 4.0, under (4.0.34) and (4.0.35), including the situation regarding an equivalence and coupling between a spinning object and its "*linear-moment thrust-force,* $\mathbf{L}\omega = pv$", around equations (2.11.3-1) and (2.11-4), in Chapter 2):

Probabilistic Wave function, $\Psi(x,t)$*, as presently used and interpreted in Orthodox Quantum theory, has no immediate amplitude and phase information that is dependent on spatial and temporal variables. To get certain relevant (but still not immediate) spatial or combined spatial-temporal distribution, we need to rely on probabilistic and statistical expectations, based on a countless number of repetitive events, until we start recognizing some averaged wave-like shapes, or wave properties, produced as superposition, diffraction, and interference effects between such matter wave* $\Psi(x,t)$ *and its surroundings.* $\Psi(x,t)$ *is a wave packet that is replacing moving particles, where we associate PWDC properties described by (10.1).*

Contrary, if we treat a wave function $\overline{\Psi}(x,t)$ *as an Analytic Signal, which is a complex function or* ***Phasor*** *(as established in this book, meaning being non probabilistic,* $\Psi^2(x,t)\ (=)\ \text{Power}$*), we know from the very beginning that such wave function explicitly has very rich and immediate, spatial and temporal amplitude and phase information in all of its domains, as described by* ***PWDC*** *properties. The Mentioned* ***Phasor*** *will be causally related to or connected with* ***PWDC*** *facts as summarized under (10.1). In addition, mentioned phase information (of an Analytic signal wavefunction) is essentially and completely supporting formulation of de Broglie hypothesis, and gives a platform for an easy development of Schrödinger equation without any exotic and artificial postulation (see more in Chapter 4.3). From such instant phase function, we can extract de Broglie wavelength and other matter-wave parameters (see more, later in the same chapter under (10.1.1), and in "10.1 Hypercomplex Analytic Signal functions and interpretation of energy-momentum 4-vectors in relation to matter-waves and particle-wave duality"). Sooner or later, we need to accept that an ideally empty space or total vacuum still has certain fluidic matter content, or an ether, which is the carrier of electromagnetic and other matter waves, facilitating matter waves propagation and interactions with other matter states, and contributing to dualistic matter-wave properties.*
PWDC, as elaborated in this book, has a lot of common-sense grounds (and it could be creatively extended) with similar ideas as "Many worlds Interpretation" of Quantum theory, and with de Broglie-

Bohm and Both-particle-and-wave united view (which is already quoted in Chapter 4.1, as taken from https://en.wikipedia.org/wiki/Wave%E2%80%93particle_duality#cite_note-6).
PWDC also has strong ontological and analogical relations with the String theory concepts.

Within the PWDC framework, as summarized in section 10.1, it is crucial to emphasize that all relevant parameters and attributes of wave packets, wave groups, matter waves, moving particles, and their corresponding wave functions, whether in the micro or macro realms of physics, can be effectively modeled and presented as follows:

1. Complex Analytic Signal Functions or Phasors: These functions are directly related to relevant signal-power functions. Examples of such modeling are provided in Chapter 4.0, specifically around section 4.0.82. When merged with the PWDC concepts from Chapter 10, section 10.1, this approach almost directly yields Schrödinger's equation and other familiar wave equations in micro-physics. These equations all originate from the classical, second-order partial differential wave equation, eliminating the need for guesswork, patchwork postulates, or reliance on statistics and probability. Analytic Signal modeling of wavefunctions offers the most natural mathematical environment for conceptualizing de Broglie matter waves and wave-particle duality. Further details can be found in Chapters 4.0, 4.1, and 4.3.

2. Einstein-Minkowski 4-Vectors and Complex Energy-Momentum Relations: These can also be understood and conveniently represented as Complex Analytic Signals or Phasors, analogous to the Phasors used in electrical circuit theory. The methodology for establishing Complex Phasor modeling is discussed in Chapters 4.0, 6, and later in this chapter under section 10.1, "Hypercomplex Analytic Signal Functions and Interpretation of Energy-Momentum 4-Vectors in Relation to Matter Waves and Wave-Particle Duality." From these Complex Analytic Signal Phasors and Wave Functions, de Broglie Matter-Wave properties, as outlined in section 10.1, can be directly supported and derived. In fact, Einstein-Minkowski 4-vectors and associated Phasors can serve as a robust complement or even a replacement for the de Broglie matter-wave hypothesis. Relevant equations are provided from sections 10.1.1 to 10.1.5. These Phasor functions, derived from Minkowski-Einstein 4-vectors, allow us to construct Complex Analytic Signal wave functions and develop a family of second-order partial differential equations equivalent to the Classical and Schrödinger equation. More information on this can be found in Chapters 4.0 and 4.3.

In Chapter 2, which addresses gravitation and the macro world of physics, it is shown that the de Broglie matter-wave concept and the PWDC elaborated here are analogically applicable to solar systems. This is explored in section 2.3.3, "Macro-Cosmological Matter-Waves and Gravitation."

3. From the Phase Function of matter-wave Phasors, we can also find temporal and spatial frequencies and signal-duration relations between corresponding spatial and temporal domains (or we can find relevant spatial-temporal periodicity parameters). In addition, it will become clear that spatial-temporal integrity and mutual, direct, and stable domains proportionality $\boxed{\Delta x = C \cdot \Delta t}$ should naturally exist concerning stable and mutually synchronized and stabilized energy-momentum entities (as in A. Einstein Relativity theory), as follows,

$$\begin{bmatrix}\Psi(t)\to A(\omega)\\ \Psi(x)\to A(k)\end{bmatrix}\Rightarrow\begin{bmatrix}\Psi(x,t)\to A(k,\omega),\\ \varphi(=)\text{phase}(=)\omega t\pm kx\end{bmatrix}\Rightarrow\overline{\Psi}(x,t)=\left|\overline{\Psi}\right|\cdot e^{I\varphi}\Rightarrow$$

$$\left.\begin{matrix}\omega=\omega_t=\left|\frac{\partial\varphi}{\partial t}\right|=\left|\frac{\partial\varphi_1}{\partial t}\right|=\left|\frac{\partial\varphi_2}{\partial t}\right|=2\pi f_t=\frac{2\pi}{T}=2\pi f,\\ k=\omega_x=\left|\frac{\partial\varphi}{\partial x}\right|=\left|\frac{\partial\varphi_1}{\partial x}\right|=\left|\frac{\partial\varphi_2}{\partial x}\right|=2\pi f_x=\frac{2\pi}{\lambda}=2\pi\frac{p}{h}\end{matrix}\right\}\Rightarrow\begin{Bmatrix}u=\frac{\omega}{k}=\frac{f_t}{f_x}=\frac{\lambda}{T}=\lambda f_t=\frac{\tilde{E}}{p}=\frac{\partial x}{\partial t}\\ v=v_g=\frac{d\omega}{dk}=\frac{d\tilde{E}}{dp}=\frac{dx}{dt},\ \lambda=\frac{h}{p}\end{Bmatrix}$$

$$\Rightarrow d\varphi=\frac{\partial\varphi}{\partial t}dt+\frac{\partial\varphi}{\partial x}dx=\omega_t dt+\omega_x dx\Rightarrow\boxed{d\varphi=\sum_i\omega_i ds_i\ ,s_i\in[t,x,...]}\Rightarrow$$

$$\Rightarrow\begin{Bmatrix}dk\cdot dx=d\omega\cdot dt\\ \frac{2\pi}{h}dp\cdot dx=2\pi df\cdot dt\\ \frac{\omega}{k}=\frac{\partial x}{\partial t},\frac{d\omega}{dk}=\frac{dx}{dt}\end{Bmatrix}\Rightarrow\begin{Bmatrix}dp\cdot dx=hdf\cdot dt=dE\cdot dt\Leftrightarrow\Delta p\cdot\Delta x=\Delta E\cdot\Delta t,\\ dE=hdf=d\tilde{E}=dE_k\ ,\\ \omega\cdot dt=k\cdot dx=2\pi f\cdot dt=\frac{2\pi}{\lambda}\cdot dx\Rightarrow\lambda f\cdot dt=dx,\\ \text{if }\lambda f=u=C=\text{Const.}\Leftrightarrow dx=C\cdot dt\Rightarrow\boxed{\Delta x=C\cdot\Delta t}\end{Bmatrix}.\qquad(10.1.1)$$

Or, if we have some additional angular motion added to the same moving object (see (4.8-4) from Chapter 4.2), we could analogically find group and phase angular velocity of such motion as,

$$\omega_{ph.}=\frac{\tilde{E}}{\tilde{L}}=2\pi f_{ph}(\equiv)\text{ angular phase velocity},\quad\omega_g=\frac{dE}{dL}=\frac{d\tilde{E}}{d\tilde{L}}=2\pi f_g(\equiv)\text{ angular group velocity}.$$

Anyway, from (10.1.1), we see that wavefunction phase $\varphi(x,t)$ carries all relevant information about group and phase velocity, temporal and spatial frequencies, and wavelengths. If we operate with non-dimensional or probabilistic wavefunctions, without phase function, we will not have direct (or any) insight in just mentioned items.

Since spatial and temporal dimensions or durations of any energy-moments state are always mutually linked and related, it is logically and analogically natural that “Planck-Einstein” energy-formula $\tilde{E}=hf,\ dE=hdf=d\tilde{E}=dE_k$, initially defined for a time domain, with Planck constant $h_t=h$, has its spatial domain quantized energy defined on a similar way, meaning that another Planck-like constant h_x is also involved here, as follows,

$$\tilde{E}=\begin{Bmatrix}hf=h_t f_t,\ dE=hdf=h_t df_t=d\tilde{E}=dE_k\\ hf=h_x f_x,\ dE=hdf=h_x df_x=d\tilde{E}=dE_k\end{Bmatrix},\begin{Bmatrix}(h_t,h_x)=\text{constants}\\ h=h_t=\mathbf{6.62607004}\times\mathbf{10^{-34}}\ m^2\,kg/s\\ c=\mathbf{299\,792\,458}\ m/s\end{Bmatrix}\Rightarrow$$

$$\Rightarrow d\tilde{E}=h_t df_t=h_x df_x=h_x d\left(\frac{1}{\lambda}\right)=-h_x\frac{d\lambda}{\lambda^2}\Leftrightarrow\frac{h_t}{h_x}df_x=-\frac{d\lambda}{\lambda^2},\ \alpha=\frac{h_t}{h_x}=\frac{h}{h_x}=\frac{1}{c}=\text{Const.}\Rightarrow$$

$$\Rightarrow h_x=ch_t=ch,\ \alpha(f_t-f_{t0})=\alpha(f-f_0)=(\frac{1}{\lambda}-\frac{1}{\lambda_0})=(f_x-f_{x0})\Rightarrow$$

$$\Rightarrow h_x=h_t\frac{df_t}{df_x}=h\frac{df}{df_x}=-h\frac{df}{d\lambda}\lambda^2=hv\le h\cdot c=\mathbf{1.985445824}\times\mathbf{10^{-25}}\left[m^3\,kg/s^2\right]\Rightarrow\qquad(10.1.2)$$

$$\Rightarrow\frac{df_t}{df_x}=\frac{h_x}{h_t}=\frac{1}{\alpha}=\text{Const}\Rightarrow\alpha\cdot df_t=df_x\Rightarrow\alpha(f_t-f_{t0})=(f_x-f_{x0}),\ (f_{t0},f_{x0})=\text{constants}.$$

In conclusion, spatial and temporal narrow-band wave-packet, or energy quant of the same “energy-moments event”, (or a photon) are mutually equal and analogically respecting the same “Planck-Einstein” wave-energy law in its differential form, $(hf = h_x f_x) \Rightarrow dE = hdf = h_x df_x = d\tilde{E} = dE_k$. In addition, modelling and explanation of Photoelectric and Compton effects also benefit from just described conceptualization, confirming **PWDC** ... See similar elaborations and problems including critical conclusions regarding photons and wave energy quantizing in [161], Hidden Variables: The Elementary Quantum of Light. A Significant Question in Quantum Foundations.

4. **To explain the meaning of de Broglie matter-waves**, and to additionally support Planck-Einstein wave-energy quant ($\tilde{\mathbf{E}} = \mathbf{hf}$), let us begin by considering uniform and inertial, linear motion of a particle with mass m, velocity v = Const., linear momentum $\mathbf{p} = \gamma \mathbf{mv}$, and kinetic energy E_k. Since this is in the same time a wave-particle dualistic-energy system, with a wave energy $\tilde{\mathbf{E}}$, these two forms of energy, kinetic energy E_k and wave energy $\tilde{\mathbf{E}}$, are mutually equal.

Here we also assume applicability of the natural law of linear momentum conservation, meaning that $\mathbf{p} = \tilde{\mathbf{p}} = \gamma \mathbf{mv} = \mathbf{const.}$.

For the purposes of this discussion, we treat the particle and its wave energy formation as a mutually equal or dualistic wave-particle state where moving particle can be presented as wave-group, which can be conceptualized as an "energy-cloud" in the form of a spinning disk, ring, or spinning toroidal form (as a helicoidal or solenoidal envelope-shape, around the path of the particle in linear motion). Since $\mathbf{p} = \tilde{\mathbf{p}} = \gamma \mathbf{mv} = \mathbf{const.}$, such associated matter-wave formation should also have an angular momentum $\mathbf{L} = \tilde{\mathbf{L}} = \mathbf{J}\boldsymbol{\omega} = \mathbf{const}$, and angular or spinning velocity $\boldsymbol{\omega} = \mathbf{2\pi f} = \text{constant}$, since natural law of angular momentum conservation should also be satisfied, meaning that assumed matter-wave energy $\tilde{\mathbf{E}}$ also presents uniform and inertial, angular motion.

This interpretation brings us closer to the Planck-Einstein formulation of elementary wave-packets, or energy quanta ($\tilde{\mathbf{E}} = \mathbf{hf}$), as follows,

$$\left.\begin{array}{l} E_k = (\gamma - 1)mc^2 = \dfrac{pv}{1+\sqrt{1-(v/c)^2}} = \left.\dfrac{\gamma m v^2}{1+\sqrt{1-(v/c)^2}}\right|_{v<<c} \cong \dfrac{1}{2}pv = \dfrac{1}{2}mv^2, \\ \tilde{E} = E_k = \begin{bmatrix}\text{spinning} \\ \text{wave energy}\end{bmatrix} = (\gamma - 1)J\omega_c^2 = \dfrac{L\omega}{1+\sqrt{1-(v/c)^2}} = \left.\dfrac{\gamma J \omega^2}{1+\sqrt{1-(v/c)^2}}\right|_{v<<c} \cong \\ \cong \dfrac{1}{2}L\omega = \dfrac{1}{2}J\omega^2 = \pi L f, \quad \tilde{L} = \left.\dfrac{\gamma J\omega}{1+\sqrt{1-(v/c)^2}}\right|_{v<<c} \cong J\omega = const \end{array}\right\} \Rightarrow \qquad (10.1.3)$$

$$\Rightarrow \left\{\begin{array}{l} pv = L\omega,\ mv^2 = J\omega^2, L = pv/\omega,\ \omega = 2\pi f,\ dE = dE_k = d\tilde{E},\ p = \tilde{p}, \\ u = \lambda f = \omega / k = \tilde{E}/p = \dfrac{v}{1+\sqrt{1-(v/c)^2}},\ \boxed{\lambda = \dfrac{h}{p} = \dfrac{h}{L}\sqrt{\dfrac{J}{m}}},\ \omega_c = c\sqrt{\dfrac{m}{J}}, \\ v = d\omega/dk = dE/dp = u - \lambda d(u/d\lambda) = -\lambda^2 (df/d\lambda),\ \gamma = 1/\sqrt{1-(v/c)^2} \end{array}\right\}$$

The most interesting thing here is to support Planck-Einstein wave-energy quantum,

$$\tilde{E}_i = h \cdot f_i = \pi L_i \cdot f_i \Leftrightarrow h = \pi L_i \,,\; L = \sum_{[i]} L_i$$

$$\tilde{E}_{total} = \tilde{E} = \sum_{[i]} \tilde{E}_i = h \sum_{[i]} n_i \cdot f_i = H \cdot \bar{f} \Rightarrow H = h \frac{\sum_{[i]} n_i \cdot f_i}{\bar{f}} = N \cdot h = N \cdot \pi L \,. \qquad (10.1.4)$$

$$N = \frac{\sum_{[i]} n_i \cdot f_i}{\bar{f}} \,,\; \bar{f} = \text{mean, central frequency},\; n = 1, 2, 3, ...$$

At this point of indicative conceptual improvisations, we still face the challenge of explaining why the microworld constant h (Planck's constant), and macroworld H might exhibit some velocity dependence, or alternatively, how we might reconcile this with a framework where these constants remain strictly constant.

From our analysis, we also observe that a mass at rest, i.e., a mass in a state of standstill, can be viewed as a composition of elementary spinning wave packets. These packets can be conceptualized as numerous tiny "wave-energy gyroscopes" or configurations of spinning disks and/or toroids, each contributing to the overall energy of the mass. ***In other words, the whole matter in our Universe are dualistic wave-particle states, and solid and liquid masses are kind of solidified and stabilized condensates of matter-wave states.***

This insight also provides essential, though still incomplete, theoretical support for the concept discussed in section 2.3, titled "How to Account for Rotation in Relation to Gravitation?" from the second chapter, which deals with gravitation (see equations (2.5), (2.10), (2.10.1), (2.11), (2.11-1) through (2.11.3), and (2.4-4.9) - (2.4-4.11)). Familiar issues are explored in chapter 4.1, under sections "4.1.1.1. Photons and Particle-Wave Dualism" and "T.4.0. Photon – Particle Analogies." These topics are also revisited in chapter 9, under "9.1. Wave Function and Dimensions of Photon as an Energy Quantum." All these ideas need to be unified and better harmonized, but the core concept (about spinning matter-wave energy equivalent to linear particle kinetic energy…) is already becoming clear. ***The essential insight here is that uniform, linear inertial motion of any mass is united with the uniform inertial spinning motion around the same mass, manifesting as a "matter-wave, spinning energy-cloud".*** *After certain development of such ideas and concepts, most probably we will advance in mastering gravitation control, or flying vehicle technology based on artificial gravity. In addition to such flying-machine ideas, we could also exercise hypothetical transformation of the equation* $pv = L\omega$ *from (10.1.3) into similar vectors equation* $\vec{p} \cdot \vec{v} = \vec{L} \cdot \vec{\omega}$*, intended to be useful in producing and controlling complex (x, y, z) spatial motions. Complexity and energy related content of gravitational force we can find in Chapter 2, under "**2.2. Ideas and proposals regarding Generalized Coulomb-Newton Force Laws.**"*

Now we can create the following resume about Inertial states and matter-wave energy:

States of Rest

a) **Corpuscular point of view without mechanical spinning**: A particle with a mass m is standstill or in the state of rest in its Laboratory System of coordinates. Particle linear velocity V = 0. The same particle has certain inertia proportional to its mass. We can classify this Inertia as Linear Inertia. This particle has the total, internal and state of rest energy $\mathbf{E_{t0} = mc^2}$.

b) **Corpuscular point of view with mechanical spinning**: If the same particle is mechanically spinning with constant spinning velocity ω_s = Const.(being in the state of "linear rest", V = 0), this particle will have kinetic spinning energy E_s, and its new "linear state of rest" energy will increase as, $(\mathbf{E_{t0} = mc^2}) \rightarrow (\mathbf{m} + \frac{\mathbf{E_s}}{\mathbf{c^2}})\mathbf{c^2}$.

c) **Undulatory point of view without mechanical spinning**: Associated matter-wave particle attributes or internal spinning states are fully inside the particle, being externally not visible, since V = 0. Consequently, the same particle has the same angular or spinning inertia, as a manifestation of number of internal, atomic spinning states, since it has the same rest mass m. This, internal like "solidified", particle wave-energy again has the same total energy $\mathbf{\tilde{E}_{t0} = E_{t0} = mc^2}$.

d) **Undulatory point of view with mechanical spinning**: If the same particle is mechanically spinning with constant spinning velocity ω_s = Const.(being in the state of "linear rest", V = 0), this particle will have kinetic spinning energy E_s, and its new state of rest, "internal matter-wave energy" will increase as, $(\mathbf{\tilde{E}_{t0} = E_{t0} = mc^2}) \rightarrow (\mathbf{m} + \frac{\mathbf{E_s}}{\mathbf{c^2}})\mathbf{c^2}$.

States of uniform linear motions

e) **Corpuscular point of view without mechanical spinning**: Now the same particle with a mass m is in its Laboratory Coordinate System, in a stable, uniform, linear motion, and it has linear velocity, V = const. The same particle now has certain Linear Inertia, which is the same as under case 1 (meaning proportional to mass m). This particle has internal rest energy $\mathbf{E_{t0} = mc^2}$, motional total energy $\mathbf{E_{tv} = \gamma mc^2 = E_{t0} + E_{kv}}$, and kinetic energy $\mathbf{E_{kv} = (\gamma - 1)mc^2}$.

f) **Corpuscular point of view with mechanical spinning**: If the same particle is mechanically spinning with constant spinning velocity ω_s = Const.(being in the state of stable, uniform, linear motion, V = const.), this particle will have kinetic spinning energy E_s, and its new "linear kinetic energy" will increase as,

$$[E_{kv} = (\gamma - 1)mc^2] \rightarrow (\gamma - 1)(m + \frac{E_s}{c^2})c^2 .$$

g) **Undulatory point of view without mechanical spinning**: The same particle with a mass m in linear motion with V = const, in its Laboratory Coordinate System has at the same time associated external, rotational or spinning matter-wave energy $\tilde{E}$. This spinning or angular wave energy is equal to the particle kinetic energy,

$$\tilde{E} = E_{kv} = (\gamma - 1)mc^2 = \frac{pv}{1+\sqrt{1-(v/c)^2}} = \left.\frac{\gamma mv^2}{1+\sqrt{1-(v/c)^2}}\right|_{v<<c} \cong \frac{1}{2}pv = \frac{1}{2}mv^2,$$

$$\tilde{E} = \tilde{E}_{k\omega} = (\gamma - 1)J\omega_c^2 = \frac{L\omega}{1+\sqrt{1-(v/c)^2}} = \left.\frac{\gamma J\omega^2}{1+\sqrt{1-(v/c)^2}}\right|_{v<<c} \cong \frac{1}{2}L\omega = \frac{1}{2}J\omega^2 = \pi Lf,$$

$$\omega_c = c\sqrt{\frac{m}{J}}.$$

h) **Undulatory point of view with mechanical spinning**: If the same particle is mechanically spinning with constant spinning velocity ω_s = Const. (while being in the state of stable, uniform linear motion, V = const), this particle will have its "linear kinetic and wave energy" $\tilde{E} = E_{kv} = E_{k\omega}$ and in addition it will have kinetic, mechanical spinning energy E_s, meaning that new "external spinning matter-wave energy" will increase as, $(\tilde{E} = E_{kv} = E_{k\omega}) \rightarrow \tilde{E} + E_s$.

5. We also need to consider that "**PWDC-relevant" wavefunctions** always have, at least, two (mutually coupled) **analytic signal wave components**, (such as $\overline{\Psi}^{+}(x,t) + \overline{\Psi}^{-}(x,t)$), propagating in mutually opposite, or inwards and outwards space-time directions (as solutions of Classical and Schrödinger wave equations), such as,

$$\overline{\Psi}(x,t) = \Psi + I \cdot \hat{\Psi} = |\overline{\Psi}| \cdot e^{I\varphi} = |\overline{\Psi}_1| \cdot e^{I\varphi_1} + |\overline{\Psi}_2| \cdot e^{I\varphi_2} = \overline{\Psi}^{+}(x,t) + \overline{\Psi}^{-}(x,t),$$

$$\hat{\Psi} = H[\Psi], \varphi_{1,2} = kx \mp \omega t, I^2 = -1 ,$$

and this (about mutually opposed waves, wavefunctions, or wave-packets) is kind of manifestation of all action-reaction and induction laws known in physics, also in relation to entanglement between them.

Here is the place to notice similarities of Analytic Phasors modeling with de Broglie and D. Bohm pilot wave model. Any Phasor or Complex Analytic Signal wavefunction can be created when we take an original signal function $\Psi(x,t)$ *and add its imaginary part,* $\hat{\Psi}(x,t) = H[\Psi(x,t)]$. *Complex Phasors could also be made compatible with Minkowski 4-vectors from relativity theory. This way modeling, we are generating the matter-wave phase function* $\varphi(x,t) = \mathrm{arctg}(\hat{\Psi}/\Psi) = \omega t \mp kx + \varphi_0$, *which generates all PWDC properties (as summarized in (10.1)), and serves in developing non-probabilistic, power-related wavefunction and Schrödinger equation (without artificial postulations and assumptions; -see more in Chapters 4.0 and 4.3). This shows that Schrödinger equation is also the direct consequence, or product of Classical second order, partial differential wave equation. This* *way conceptualizing, de Broglie and D. Bohm pilot wave model could be significantly upgraded (since it is not necessary to assume or postulate existence of pilot-waves and universal applicability of artificially postulated Schrödinger equation), meaning that Quantum Theory (QT) should not be essentially probabilistic and mathematically artificial or specifically packed and postulated. D. Bohm already proved that his deterministic QT can completely replace the orthodox, probabilistic QT.*

Citation from: https://en.wikipedia.org/wiki/Wave%E2%80%93particle_duality#cite_note-6. “The pilot wave model, originally developed by Louis de Broglie and further developed by David Bohm into the hidden variable theory proposes that there is no duality, but rather a system exhibits both particle properties and wave properties simultaneously, and particles are guided, in a deterministic fashion, by the pilot wave (or its "quantum potential") which will direct them to areas of constructive interference in preference to areas of destructive interference. This idea is held by a significant minority within the physics community.[39]

De Broglie himself had proposed a pilot wave construct to explain the observed wave-particle duality. In this view, each particle has a well-defined position and momentum, but is guided by a wave function derived from Schrödinger's equation. The pilot wave theory was initially rejected because it generated non-local effects when applied to systems involving more than one particle. Non-locality, however, soon became established as an integral feature of quantum theory and David Bohm extended de Broglie's model to explicitly include it”.

In the resulting representation, also called the de Broglie–Bohm theory or Bohmian mechanics,[18] the wave-particle duality vanishes, and explains the wave behavior as a scattering with wave appearance, because the particle's motion is subject to a guiding equation or quantum potential.

....

A pragmatic and realistic approach to Physics, especially within the PWDC framework, requires that relevant wave functions be finite, energy-content limited, and well-defined (or localized) in their temporal, spatial, and spectral domains. These functions should resemble band-limited Gabor-Gaussian Wave Packets. The size, duration, integrity, and stability of these signals across all conjugate domains should adhere to Parseval’s Theorem and relevant mathematical Uncertainty Relations, as discussed in Chapter 5 of this book. For example, the spatial and temporal dimensions of a solid particle in motion, such as a metal ball with a well-defined shape will differ when analyzed from the perspective of Classical Mechanics versus an equivalent Matter-Wave Packet. According to Einstein’s theory of relativity, the real spatial-temporal size, dimensions, and shape of an object are motion-parameters-dependent.

6. Closed Electromagnetic and Mechanical Systems:
Every kinematic process involving energy-moments, such as currents, voltages, forces, and velocities, should be part of a closed electromagnetic, mechanical, or fluidic circuit (see Chapter 1 for more details). It's essential to identify the input or source-energy elements that supply a "black box system" with electro-mechanical excitation and understand where its output or load elements are located. However, in many theoretical and practical analyses in Physics, we often conceptualize systems as isolated, open-frame, or freely floating circuits, which contradicts the conservation laws requiring balanced outputs for any given input. Traditional Physics often treats angular and linear moments of the same motion as independently conserved, overlooking the fact that these are interrelated in cases of inertial and stationary motions, special cases of orbital and periodic matter-wave phenomena. Mechanical, electromechanical, electromagnetic, and fluidic systems, especially during temporal and spatial fluctuations or transformations, produce and receive matter waves in various forms (e.g., acoustic waves, mechanical vibrations, electromagnetic waves, etc.). While we often focus on dominant oscillatory phenomena, it's crucial to consider associated waves. Notably, solutions to Classical wave equations (and various forms of Schrödinger equations) always come in pairs of oppositely propagating wave groups.

7. Wave-Particle Duality as a Bridge:
The Wave-Particle Duality phenomenon serves as a "bridge" between linear motions with spatial translational symmetry and associated rotational motions with spatial rotational

symmetry, or a “bridge” between motional particles and associated matter waves. Both cases require considering temporal-spatial unity or integrity. Linear motions of masses can be analogously linked to the linear motions of electrically charged particles in an electric field. Similarly, rotational motions of masses can be associated with the rotational (spinning and helical) motions of electrically charged particles in a magnetic field. The spatial and temporal dimensions or intervals of involved masses, electric charges, and associated matter waves are respecting CPT Symmetry, being interdependently connected, as addressed in Relativity theory. Contemporary theories on matter-states, gravitation, planetary systems, galaxies, black holes, neutron stars, and other natural forces often focus on a mechanical, electrically neutral, Newtonian perspective, neglecting the electromagnetic properties of the universe. However, this book argues that the mechanical, gravitational, electromagnetic, and wave-particle duality aspects of our universe are inseparable. There are strong indications that gravitation might be a form of structured electromagnetic phenomenon.

8. Energy and Wave-Packet Quantization:
Energy and wave-packet quantization in Physics should be grounded in universally valid mathematical concepts of signal sampling and reconstruction, as established by the "Kotelnikov-Shannon-Whittaker-Nyquist" theorem. Further refinements, such as non-dimensional mathematical normalizations or probabilistic and statistical modeling, can be explored as needed. The PWDC foundation should be established based on these principles, not on arbitrary axiomatic postulates designed for limited purposes. The micro-world matter-wave concept is also applicable to macro matter waves of astronomical objects, as elaborated in Chapter 2, particularly in section 2.3.3, "Macro-Cosmological Matter-Waves and Gravitation."

Interpretations of what a matter-wave function should conceptually, dimensionally, and qualitatively be continuing to spark debate and uncertainty in modern physics. The concept of non-dimensional probability and possibility waves in Quantum Theory (QT) is seen by some as an artificial mathematical construct, an effective yet unconventional tool. However, the mathematical and physical similarities between the wavefunctions used in QT and Classical wave equations suggest that matter waves cannot exist without a carrier medium, fluid, or spatial matrix to propagate through. Matter waves, including those described in this book and QT wavefunctions, are inherently tied to the kinetic or motional energy of a dynamic undulatory process or interaction. The existence of a carrier medium, whether known or hypothetical, is necessary. This medium, potentially analogous to the concept of ether, could be viewed as a manifestation of a higher-dimensional reality with electromagnetic properties, like the idealized vacuum which also exhibits measurable electromagnetic characteristics.

Regardless of the existence of an ether, universally applicable mathematical models have been developed to address wavefunctions and wave motions. The Complex Analytic Signal model, exclusively practiced in this book, remains the most natural and effective mathematical framework for modeling all types of waves known in Physics. In contemporary physics literature, complex wave functions are often regarded as auxiliary mathematical tools that simplify and expedite the solution of Classical and other wave equations, which is indeed correct.

If we will use only, by an Analytic signal modeled wave function, or wave group, such as,

$$\begin{aligned}
&\bar{\Psi}(x,t) = \left|\bar{\Psi}\right| \cdot e^{I\Phi} = \bar{\Psi}^{+}(x,t) + \bar{\Psi}^{-}(x,t) = \Psi(x,t) + I \cdot \hat{\Psi}(x,t) = \\
&= \left[\Psi^{+}(x,t) + I \cdot \hat{\Psi}^{+}(x,t)\right] + \left[\Psi^{-}(x,t) + I \cdot \hat{\Psi}^{-}(x,t)\right] = \\
&= \left[\Psi^{+}(x,t) + \Psi^{-}(x,t)\right] + I \cdot \left[\hat{\Psi}^{+}(x,t) + \hat{\Psi}^{-}(x,t)\right],\ \hat{\Psi}(x,t) = H\left[\Psi(x,t)\right],\ I^{2} = -1\ ,
\end{aligned}$$

we will come to the conclusion that both, real part ($\mathbf{Re}\left[\overline{\Psi}(x,t)\right] = \Psi^{+}(x,t) + \Psi^{-}(x,t)$ $= \mathbf{\Psi(x,t)}$), and corresponding imaginary part ($\mathbf{Im}\left[\overline{\Psi}(x,t)\right] = \hat{\Psi}^{+}(x,t) + \hat{\Psi}^{-}(x,t) = \hat{\Psi}(x,t)$), of the same complex wave function, or Complex Analytic signal ($\overline{\Psi}(x,t)$), are realistic, natural matter-wave components, also existing coincidently and synchronously within the same time and space (see much more about Analytic signal properties in Chapter 4.0). The reasons for this statement are:

a) Total signal energy of $\mathbf{\Psi(x,t)}$ is equal to the total signal energy of $\hat{\Psi}(x,t)$, or to the half of the total signal energy of $\overline{\Psi}(x,t)$ (see (4.0.4)). Here are the grounds for understanding self-organization, auto-synchronization, entanglement effects, and double-slit interference and diffraction in physics.

b) The common signal-phase, or wavefunction-phase, and immediate frequency, group, and phase velocity, can be found only by knowing (and using) both $\mathbf{\Psi(x,t)}$ and $\hat{\Psi}(x,t) = \mathbf{H}\left[\Psi(x,t)\right]$, meaning that both $\mathbf{\Psi(x,t)}$ and $\mathbf{\Psi(x,t)}$ are equally realistic existing as synchronous and coincident matter waves.

c) Energy and power of an Analytic signal wavefunction $\overline{\Psi}(x,t)$, is dynamically balanced, coupled and synchronously fluctuating between $\Psi(x,t)$ and $\hat{\Psi}(x,t)$, and also between $\Psi(x, t = const.) = \Psi(x)$ and $\Psi(x = const., t) = \Psi(t)$, including on a similar way $\hat{\Psi}(x)$ and $\hat{\Psi}(t)$, based on the relativistic spatial-temporal unity and transformability. Or formulating the same analogically, this is as electromagnetic energy in resonating L-C circuits (meaning energy fluctuating between electric inductance and capacitance), or in mechanically resonating circuits, as oscillatory, <u>*potential, and kinetic*</u> energy exchanges, as well as in mass-spring and pendula systems, or as coupling of electric and magnetic field components within an electromagnetic wave or photon, while a total involved energy is conserved (meaning constant). Simplifying this, we can say that both spatial, temporal, or united spatial-temporal systems with intrinsic periodicities have certain static and dynamic, active, and potential, mutually fluctuating energy content. Specific spatial matter-forms can also radiate and receive matter waves.

d) Consequently, both wave packets $\Psi(x,t)$ and $\hat{\Psi}(x,t)$ should always and coincidently exist (on some self-organized, auto-synchronous, mutually-coupled way) in all oscillatory, resonant, and matter-wave phenomena, since Conservation laws, universal laws of inertia, and <u>***associated action and reaction laws or forces***</u>, including induction related forces, currents, and voltages, are always working, both in mechanics and electrodynamics. If we see, measure, and consider as relevant only one of the mentioned wave components ($\Psi(x,t)$ or $\hat{\Psi}(x,t)$), this is only saying that our modeling and understanding of the same matter-wave and particle-wave duality situation is still incomplete, or only partially implemented. Here (within (10.1), (10.2), a), b), c) and d)) are the most important grounds of Wave-Particle Duality (in this book summarized as **PWDC** facts). If this was properly considered before analyzing double-slit experiments, exchanges of tons of words between Bob and Alice will be avoided.

e) N. Tesla and R. Boskovic's ideas about gravitation, universal natural force, radiant energy, and here-formulated PWDC implicate that all atoms and masses in our Universe should mutually communicate externally and internally (outwards and inwards) being parts of closed circuits of involved energy, mass, moments, currents, and voltages (as explained in the first chapter about analogies).

Citation from: https://www.quora.com/What-did-Nikola-Tesla-mean-by-his-quote-if-you-wish-to-understand-the-universe-think-of-energy-frequency-and-vibration Nikola Tesla quote, "**If you wish to understand the universe, think of energy, frequency and vibration**," highlights his belief that everything in the universe is made up of energy that is constantly vibrating at different frequencies. According to Tesla, by understanding and harnessing these vibrations, we can better understand and manipulate the world around us. This concept is still relevant in modern physics, particularly in the study of quantum mechanics."

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****Mechanical and Acoustic Energy Transmission****

Mechanical and acoustic energy, including oscillations, vibrations, audio signals, and music, can be created and transferred through various signal-modulating techniques in both temporal and spatial domains. These techniques can be applied to laser beams and dynamic plasma states (or signals) (see relations in sections 10.2-2.4 and references [133] to [139]). Newton's action-reaction forces, electromagnetic induction laws, and quantum entanglement effects align with solutions to classical wave equations. These solutions demonstrate that mutually opposed, self-synchronized waves, propagating in pairs and in different spatial-temporal directions, are consistently created, as detailed in Chapter 4.2.

****Spontaneous and Natural Synchronization****

Spontaneous matter-states and structural system vibrations synchronization is observed in mechanical, electromechanical, electromagnetic, and biological systems, which supports the concept of closed circuits. For further details, refer to the first chapter of this book and Chapters 4.1, 4.3, 8, and 9 (e.g., Fig. 4.1.6 illustrates closed-circuit energy flow). Additionally, biological entities, including humans, exhibit intrinsic self-synchronization effects driven by structural memory in liquids or electromagnetic excitation from light, mechanical vibrations, or music. Coherence between two wave functions or signals, which measure resonance and mutual synchronization, is discussed in Chapter 4.0, particularly around definitions (4.0.83), (4.0.87), and (4.0.109).

****Human Synchronization and Technological Influences****

In humans, synchronization and entanglement manifest as complex mental interactions and couplings related to group ideologies, sports competitions, mass trends in music, art, creativity, politics, and other motivations. Today, the Internet and Artificial Intelligence are powerful tools that support creativity and facilitate connections and synchronizations among humans, their environment, used equipment, and technological processes. Local and non-local synchronizations and entanglements involve overlapping temporal-spatial spectral characteristics of connected entities, resulting in joint amplitude and phase functions.

9. Understanding Wave-Particle Duality

Wave-particle duality is a fundamental concept in Physics. For example, photons exhibit particle-like behavior in phenomena such as the photoelectric effect, Compton scattering, and Bremsstrahlung. To determine if a photon (which has zero rest mass) behaves as a particle, compare its wavelength with the wavelength, width, or physical size of a real interacting particle (which has non-zero rest mass). A photon with a short wavelength and high frequency *will interact with particles as a particle. Conversely, if a photon's wavelength is much larger than the interacting particle's physical size, it will behave as a wave, with minimal mechanical interactions.*

Photons are always narrow-band wave packets (Gaussian-Gabor-Kotelnikov-Shannon-Nyquist). Whether they act as particles or waves depends on their wavelength in relation to the physical size of the interacting particles. This concept applies to all matter-waves or wave groups. The wave-particle duality of these entities is closely related to the wavelength and physical size of interacting participants. Depending on the spatial size relation, a photon or wave group may exhibit more particle-like or wave-like behavior, but the Planck-Einstein energy relation $\tilde{E} = hf$ remains applicable to all narrow-band wave packets and photons (see Chapter 4.1 for further discussion on diffraction patterns).

Limitations of Probability Waves

Imaginary or virtual probability waves, which are not measurable or dimensional, belong to abstract or game-theory models. While they are mathematically useful within the boundaries of Quantum Theory, they often create unrealistic or metaphysical situations and outcomes.

*[♣ **Freethinking comments**: Here is a very convenient place to try to address or explain how light or photons' speed is always constant and equal to $C \cong 300'000\, km/s$. Let us imagine a certain omnidirectional light source (or lamp) that is (mechanically) moving in one direction with certain constant, linear speed equal to V. Such light source is naturally radiating mutually entangled (or coupled) photons in a direction of its mechanical motion and in the opposite direction, $\bar{\Psi}^{+}(x,t)$, $\bar{\Psi}^{-}(x,t)$, like we already stated for the solutions of Classical wave equations. If we insist that only Galilean velocity transformations should be naturally applicable here (as everywhere else), we will (for instance) have that photonic wave group $\bar{\Psi}^{+}(x,t)$ is propagating with the speed $(C+V)[km/s]$, and its opposite, and coupled photonic wave group, $\bar{\Psi}^{-}(x,t)$, will propagate with $(C-V)[km/s]$. In fact, when using an Analytic signal model, we will have four wave-groups, such as, $\bar{\Psi}(x,t) = [\Psi^{+}(x,t) + \Psi^{-}(x,t)] + I \cdot [\hat{\Psi}^{+}(x,t) + \hat{\Psi}^{-}(x,t)]$. Let us be interested only for real wave elements $[\Psi^{+}(x,t) + \Psi^{-}(x,t)] = R_e[\bar{\Psi}(x,t)]$, and let us now calculate the average wave group's speed of $R_e[\bar{\Psi}(x,t)]$ as $\langle V \rangle = 0.5 \cdot \lfloor (C-V) + (C+V) \rfloor = C$. We see that regardless of (mechanical motion) speed V, the resulting wave-groups' speed (of electromagnetic waves) is always C. Let us now determine relevant center of mass velocity of $R_e[\bar{\Psi}(x,t)]$ (if we consider that light source or lamp will be in the center of mass position, and an independent observer (in the state of rest in a laboratory system) is monitoring and/or calculating mentioned center of mass velocity. Again, we will get that the relevant center of mass velocity V_c will be equal to C, regardless of V, since,*

$$V_c = \frac{(hf/c^2)\cdot(C-V) + (hf/c^2)\cdot(C+V)}{2\cdot(hf/c^2)} = C.$$

If we take an imaginary part of the same complex wave function, $[\hat{\Psi}^{+}(x,t) + \hat{\Psi}^{-}(x,t)]$ we will draw the same conclusions. As we know, Albert Einstein intuitively and logically, or by elimination (since nothing else worked well) concluded the same. Different consequences and dubious complexity of such axiomatic postulations and thinking are resulting in what we presently have as Relativity theory (meaning, mathematically, a certain satisfactory and well-working solution is created, using Lorentz transformations, but this is still conceptually not completely explained theory). The biggest problem here (in a contemporary Physics literature) is that in most of the wave motions analyzes is systematically forgotten or omitted that Classical wave equation should always have (at least) two wave groups, mutually coupled, synchronized, and propagating in mutually opposite spatial-temporal directions), such as $\bar{\Psi}(x,t) = |\bar{\Psi}| \cdot e^{I\Phi} = \bar{\Psi}^{+}(x,t) + \bar{\Psi}^{-}(x,t)$, what could count a lot for conceptual understanding of *matter waves. Most probably, that Michelson-Morley experiment should also be analyzed using elaborated ideas here (and maybe the old ether concept could be revitalized or better explained). ♣]*

The larger framework for addressing particles interactions, motional states, rest masses, and all involved energies and moments is supported with well-known relativistic theory 4-vector energy-momentum relations, and perfectly connected and compatible (or mutually provable as a cause and consequence) with all **PWDC** relations of Matter Waves (10.1), such as,

$$\left[\begin{array}{l}\left(\begin{array}{l}\left\{P_4^2=(p,\frac{E}{c})^2=\text{invariant}\right\}\Rightarrow p^2-\frac{E^2}{c^2}=-\frac{E_0^2}{c^2},\ p=\gamma mv,\ \frac{E}{c}=\gamma mc,\\ E=E_{tot.}=\gamma mc^2=E_0+E_k=E_0+\tilde{E},\ E_0=mc^2,\ E_k=E-E_0=(\gamma-1)mc^2,\ E=\gamma E_0,\\ v_g=v=\frac{dE}{dp}=\frac{dE_k}{dp}=\frac{d\tilde{E}}{dp}=\frac{d\omega}{dk}=u-\lambda\frac{du}{d\lambda}=-\lambda^2\frac{df}{d\lambda}=\frac{df}{d(\frac{1}{\lambda})}=\frac{2u}{1+\frac{uv}{c^2}},\frac{df}{v}=d(\frac{1}{\lambda}),\\ u=\lambda f=\frac{\omega}{k}=\frac{E_k}{p}=\frac{\tilde{E}}{p}=\frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}}=\frac{\tilde{E}}{\tilde{p}},\ \omega=2\pi f,\ \gamma=\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\end{array}\right)\Rightarrow\\ \qquad\Rightarrow\left(\begin{array}{l}\lambda=\frac{h}{p}=\frac{h}{\gamma mv}=\frac{h}{\tilde{m}v}=\frac{h}{\tilde{p}},\ k=\frac{2\pi}{\lambda}=\frac{2\pi}{h}p=\frac{2\pi}{h}\tilde{p},\\ dE=dE_k=d\tilde{E}=vdp=hdf=c^2d(\gamma m)=c^2d\tilde{m}\end{array}\right).\end{array}\right]\Leftrightarrow$$

$$\Leftrightarrow\left[\begin{array}{l}\vec{p}^2c^2+E_0^2=E^2,\ (p\to p\pm\Delta p)\Leftrightarrow(E\to E\pm\Delta E)\Rightarrow\\ \left\{\begin{array}{l}(p+\Delta p)^2c^2+E_0^2=(E+\Delta E)^2\\ (p-\Delta p)^2c^2+E_0^2=(E-\Delta E)^2\\ \overline{v}=\frac{\Delta x}{\Delta t}=\frac{\Delta\omega}{\Delta k}\end{array}\right\}\Leftrightarrow\left\{c^2\cdot p\,\Delta p=E\,\Delta E\Leftrightarrow\frac{\Delta E}{\Delta p}=c^2\frac{p}{E}=\overline{v}\right\}\Rightarrow\\ \qquad\Rightarrow\overline{v}=\frac{\Delta x}{\Delta t}=\frac{\Delta E}{\Delta p}=c^2\frac{p}{E}=h\frac{\Delta f}{\Delta p}=\frac{\Delta\tilde{E}}{\Delta p}=\frac{\Delta\omega}{\Delta k}\le c.\end{array}\right]\Rightarrow$$

$$\left[\begin{array}{l}\left\{\begin{array}{c}u=\frac{\omega}{k},\ \omega=ku\\ (k\to k\pm\Delta k/2)\Leftrightarrow(u\to u\pm\Delta u/2)\end{array}\right\}\Rightarrow\Delta\omega=(k+\frac{1}{2}\Delta k)(u+\frac{1}{2}\Delta u)-(k-\frac{1}{2}\Delta k)(u-\frac{1}{2}\Delta u)=\\ \Rightarrow=k\Delta u+u\Delta k\Leftrightarrow\overline{v}=\text{Lim}\left\{\frac{\Delta\omega}{\Delta k}=u+k\frac{\Delta u}{\Delta k}=\left(\frac{\Delta x}{\Delta t}=\frac{\Delta E}{\Delta p}=c^2\frac{p}{E}=h\frac{\Delta f}{\Delta p}=\frac{\Delta\tilde{E}}{\Delta p}\right)\right\}_{\Delta\to 0}\Rightarrow\\ \qquad\Rightarrow\left\{v=u+k\frac{du}{dk}=\frac{d\omega}{dk}=\frac{d\tilde{E}}{dp}=\frac{dx}{dt}=\text{instant group velocity}\right\}.\end{array}\right] \quad (10.2)$$

Relations within (10.2) are demonstrating that matter waves and particle-waves duality concepts are mathematically mutually well integrated and fully compatible with relativistic or Minkowski particle “energy-momentum” picture. This is mutually reinforcing and confirming the conceptual and theoretical power of both entities, meaning being applicable to matter waves (or wave groups) and to relativistic properties of moving particles (see much more about the same items in chapter 4.1). The significance of relations (10.2) is even bigger because wave-group velocity and matter waves formulas (10.1) are now independently developed, not considering any of relativistic theory concepts and mathematical practices (see about group and phase velocity in chapter 4.0). ***Very familiar concepts, ideas, and conclusions (concerning PWDC) can be found in [105], Himanshu Chauhan, Swati Rawal, and R K Sinha. WAVE-PARTICLE DUALITY REVITALIZED CONSEQUENCES, APPLICATIONS, AND RELATIVISTIC QUANTUM MECHANICS.***

Let us (for instance) make a digression towards Signals and Spectrum analysis (based on Integral Fourier and Analytic Signal related transformations) and consider that one of the relevant time-domain signals (or function) for addressing particle-wave duality is the linear moment, being a time-dependent function, $p = mv = p(t)$. Direct Fourier transform of such signal will be, $F[p(t)] = \overline{P}(f)$, where $P(f)$ presents "*temporal*" spectrum of $p(t)$. The inverse Fourier transformation will again reproduce the original time-domain signal $F^{-1}[\overline{P}(f)] = p(t)$ (see basic integral signals transformations and associated definitions, starting from (4.0.1) to (4.0.5-1), T.4.0.1 and (4.0.13), as presented in Chapter 4.0).

Before we make the next step, let us introduce definitions (or just symbols) of mutually analogical "*temporal and spatial*" frequencies ω_t, ω_x as, $\omega_t = \frac{2\pi}{T} = \frac{2\pi}{h}\tilde{E} = 2\pi f_t = \omega$, $\omega_x = \frac{2\pi}{\lambda} = \frac{2\pi}{h}p = 2\pi f_x = k$, having the same meanings as in (10.1) and (10.2). Now we can define Fourier pairs of time and frequency domain functions as,

$$F[p(t)] = \overline{P}(\frac{\omega_t}{2\pi}) = \overline{P}(f_t) = \int_{-\infty}^{\infty} p(t)e^{-j2\pi f_t t}dt,\ \omega = \frac{2\pi}{T} = \frac{2\pi}{h}\tilde{E} = 2\pi f_t = \omega_t,$$

$$F^{-1}[\overline{P}(f_t)] = p(t) = F^{-1}\left[\overline{P}(\frac{\omega_t}{2\pi})\right] = \int_{-\infty}^{\infty}\overline{P}(\frac{\omega_t}{2\pi})e^{j2\pi f_t t}df_t = \int_{-\infty}^{\infty}\overline{P}(f_t)e^{j2\pi f_t t}df_t .$$

We can also claim that linear moment is dependent on its spatial coordinate x, being the path (or axis) in a direction of its propagation, $p = mv = p(x)$ (by simply replacing t by x). Direct Fourier transform of such spatial-domain signal will be, $F[p(x)] = \overline{P}(f_x)$, where $\overline{P}(f_x)$ presents "*spatial spectrum*" of $p(x)$. The inverse Fourier transform will again reproduce the original spatial-domain signal $F^{-1}[\overline{P}(f_x)] = p(x)$.

$$F[p(x)] = \overline{P}(\frac{\omega_x}{2\pi}) = \overline{P}(f_x) = \int_{-\infty}^{\infty} p(x)e^{-j2\pi f_x x}dx,\ \omega_x = 2\pi f_x = k = \frac{2\pi}{\lambda} = \frac{2\pi}{h}p,$$

$$F^{-1}[\overline{P}(f_x)] = p(x) = F^{-1}\left[\overline{P}(\frac{\omega_x}{2\pi})\right] = \int_{-\infty}^{\infty}\overline{P}(\frac{\omega_x}{2\pi})e^{j2\pi f_x x}df_x = \int_{-\infty}^{\infty}\overline{P}(f_x)e^{j2\pi f_x x}df_x .$$

Linear moment should be dependent both on its temporal and spatial dimensions t and x, as $\mathbf{p = p(x,t)}$. Consequently, a linear moment will also be more generally presentable as a two-dimensional Fourier integral transform pair,

$$\overline{P}(f_x, f_t) = F[p(x,t)] = A_p(f_x, f_t)e^{j\Phi(f_x, f_t)} = A_p(f_x, f_t)e^{j\frac{2\pi}{h}(p\cdot x - \tilde{E}\cdot t)} = A_p(f_x, f_t)e^{j(k\cdot x - \omega\cdot t)}$$

$$p(x,t) = F^{-1}[\overline{P}(f_x, f_t)] ,$$

and since temporal and spatial integrity, signal shape and its basic properties (at least for stable, non-dispersive energy-momentum matter states) should be in a certain way united, stable, and preserved (based on always valid Conservation Laws of Physics), it is expectable that temporal and spatial dimensions and periods of certain signal should also be mutually dependent, proportional, and causally related. This can be demonstrated or realized by using mathematical Uncertainty Relations and relevant group and phase velocity relations (of the particle or wave group in question), as we can see in (10.1) and (10.2), and Chapter 4.0, starting from (4.0.60) until (4.0.72), and in Chapter 5.0, from (5.1) until (5.4.1). Of course, we can (on a similar way) expose or formulate the spatial, spectral, and periodicity related importance among angular motions and angular and spinning moments, especially in cases of spatial-temporal standing waves (as practiced or interpreted in the Fourier and Statistical Optics, and in this book in the first Chapter by "T.1.8 Generic Symmetries and Analogies of the Laws of Physics", and in the fifth chapter by "T.5.4. "Wavelength analogies in different frameworks").

The true pioneers behind the wave-particle duality concept, or its essential mathematical foundations, are Jean Baptiste Joseph Fourier and, later, Dennis Gabor (see Chapter 4.0 for more details). Fourier's work, along with the more refined Analytic Signal transformations introduced by Denis Gabor, describe the mutually conjugate domains of original and spectral signals. These are not just abstract mathematical constructs; they reflect fundamental aspects of Nature, the Universe, and Physics, and are consistently demonstrated in the analysis and synthesis of waves.

Analytic Signal spectral functions are particularly valuable because they focus exclusively on positive frequencies, making them more practical and realistic than Fourier spectral functions. Mathematics, when not reduced to an abstract game of imposed rules, is often considered the best natural and exact language of Physics. Therefore, mathematical conclusions, implications, predictions, and extrapolations should be regarded as either already known or awaiting experimental confirmation in Physics. This holds true for Fourier transforms and the later development of Analytic Signal spectral functions.

For example, consider a signal representing linear momentum p(x, t) in stable, uniform inertial motion (meaning it remains constant and preserved). The spectral functions associated with this signal reveal an intrinsic periodicity in both temporal and spatial domains. This is a manifestation of the particle-wave duality of matter states, a concept independently discovered and confirmed in Physics.

This raises the question: why attempt to explain wave-particle duality by constructing or postulating a probability-based framework when it can be explained more naturally, simply, and powerfully through real (not artificial, abstract or imaginary) Mathematics? Spectral distributions and transformations are typically understood in the context of time-frequency-periodicity or spectral domains. However, similar periodicity exists in spatial, configurational, and geometrical terms, related to the shapes of atoms, microparticles, and larger masses. Moreover, spatial and temporal dimensional properties, velocities, and periodicities are interrelated and deterministically transformable, respecting CPT Symmetry, and ensuring the unity and stability of our Universe.

The "Double Slits Interference and Diffraction" experiment is a cornerstone of Particle-Wave Duality theory (abbreviated as PWDC in this book). From a spatial perspective, double or multiple slit situations can be understood as cases of spatial or topological periodicity, like the structures found in crystals, fractals, and other naturally occurring patterns. These structures, without any probabilistic assumptions, manifest spatial interference and superposition effects, exhibiting wave-like signal patterns and spatial spectral signatures.

As a result, other artificial, imaginative, or game-theory-based concepts, such as ontologically stochastic and probability-based matter-waves or certain aspects of Orthodox Quantum Theory (QT), are conditionally valid within their predefined frameworks. Despite its success *and integration with many grand, undisputed mathematical and physics-related theories, modern QT remains limited by these artificial constructions.*

Probability theory and statistics are indeed powerful, universally applicable tools for analyzing processes involving large numbers of identical participants or matter states. However, they should not be mistaken for having essential, exclusive, or unique ontological significance, particularly in the realm of Quantum Theory. Particle-wave duality manifestations, such as impacts, scatterings, diffractions, and interferences, should be primarily understood within a spatial and temporal periodicity framework, based on relevant spectral distributions.

For accurate mathematical analysis, these interactions should be examined in both the Laboratory system (which describes what we observe) and the Center of Mass system (which reveals what Physics or the Universe respects). Conservation laws of Physics must always be satisfied in a real space-time environment. It is also important to consider that static frames, diffraction plates, slits, prisms, and other experimental apparatus are active participants in the interaction, especially from the perspective of spatial and temporal periodicity or spectral content.

Signals or wave functions with temporal periodicity generate multiple spectral, frequency-dependent harmonic signals. Similarly, spatial 2D and 3D objects with geometric or spatial periodicity, such as crystals, fractals, metals, stone-blocks, pyramids and other solid structures, when properly agitated, produce matter-wave effects with linear and/or angular momentum properties, all without relying on probability-based modeling.

Furthermore, we can speculate that certain geometrically complex structures, when properly structured and excited, could serve as matter-wave energy generators or geophysical machines. This might explain the hidden, hypothetical functionality of macro-objects, such as pyramids and hollowed megalithic structures found on our planet. Temporal and spatial periodicities are coupled and synchronized, a relationship that can be explained through Uncertainty Relations, group and phase velocity relations, or PWDC relations, as elaborated later in this book.

It is also crucial to determine whether the particle-wave duality manifestations are electromagnetically neutral (like neutral masses) or involve additional electromagnetic charges and fields. The presence of such charges and fields, especially when in motion, significantly alters the analysis of matter-wave interactions. Additionally, the material properties of surrounding frames, targets, and diffraction plates, whether they are metallic, dielectric, magnetic, or possess electrostrictive or magnetostrictive properties, must be considered.

Experiments involving electron diffraction, scattering, superposition, and interference are often cited as evidence that particles behave like waves. However, it is likely that electrons are already specific electromagnetic-energy (or photons) packed states before any interaction occurs. In this context, it is no surprise that these photonic states manifest wave-like properties under certain conditions.

The foundations of Orthodox Quantum Theory and its analysis of matter-waves and particle-wave duality are oversimplified, neglecting the signal-phase function and real and immediate space-time-frequency interactions mentioned here. This has led to incomplete models and concepts, propped up by imposed postulates and metaphysical assumptions. Consequently, modern QT raises unnatural and perplexing questions, logical uncertainties, and dilemmas, while promoting probability and statistics-based modeling as the primary mathematical approach, despite its limited ontological significance.

In the contemporary QT, we consider $[\Psi(x,t)]^2$ as a probability that certain matter-wave or energy state can be found in certain segment of space. Higher probability here means that chances to detect certain states (or particles) in certain positions are higher. The sum of all probabilities related to the same event are equal to one (1), and this is anyway equivalent to satisfying the total energy conservation law. We could also and much more naturally consider $[\Psi(x,t)]^2$ as a power (or spatial power density) distribution, and where such power density has its peaks, there we should expect to detect relevant matter states. Summing of all such space-time-integrated power functions (related to the same event) correspond to relevant total energy, and this way we could make analogy or parallelism between probabilistic and power related wavefunctions (of course, after applying convenient mathematical normalization).

In reference [106], Brigham, E. Oran presents several Fourier transform time-frequency domain pairs that are closely associated with realistic scenarios in the understanding of basic particle-wave duality (see Fig. 10.1). By replacing the time domain on the left side with a similar curve in the space domain and the frequency domain on the right with its analogous spatial-frequency domain, we can encompass all single, double, and multiple diffraction, interference, and superposition scenarios. These scenarios involve variables or functions (such as linear momentum) that are constant, periodic, sinusoidal, or impulse-like, and whose spectral domains exhibit wave-like shapes, or vice versa (as illustrated in Fig. 10.1).

These conclusions, along with the integral transformation curve shapes in both directions, remain valid if we simply swap the original and spectral domain images (or exchange the temporal t and spatial x variables) and label them accordingly. The mutual shapes of original and spectral signals in both directions, based on Fourier integral transformation and shown in Fig. 10.1 (encircled in red), potentially represent matter-wave or wave-group states (or packets) that are similar or equivalent to stable, particle-like states with finite energy and limited temporal, spatial, and spectral durations across all domains. In such cases, Uncertainty Relations can be reframed as Certainty Relations (using equality instead of inequality; see Chapter 5 for more details). The energy states referenced here could represent photons, elementary particles, atoms, and so on.

If we create similar original-to-spectral representations based solely on Analytic Signal modeling, negative frequencies will be excluded. However, we will arrive at the same qualitative conclusions and signal shapes as depicted in Fig. 10.1. From this figure, we can draw the following important conclusions:

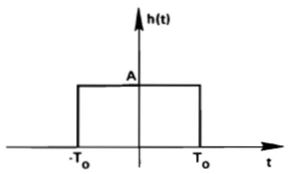
a) **Rectangular wave shapes** in the temporal or spatial domain correspond to sinc-function-defined spectral functions.

b) **Bell-curve or Gaussian distribution curves**, when presented as temporal and spatial wavefunctions, maintain similar bell-curve amplitudes or envelopes in both their original and spectral domains. These curves are well-defined, limited, and energy-finite across all domains, suggesting how quantized wave-packets and particles might be created or modeled.

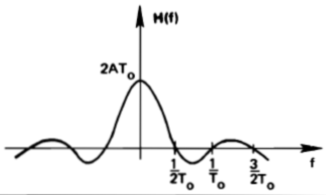
c) The "Kotelnikov-Shannon-Nyquist-Whitaker**"** signal sampling, analysis, synthesis, and reconstruction methods utilize sinc-shaped wavefunctions $\frac{\sin\varphi(x,t)}{\varphi(x,t)}$, indicating that sinc and similar soliton wavefunctions, or narrow-band wave packets, form the optimal basis for modeling dualistic elements in Physics and Nature. These functions effectively replace and optimize the Fourier basis of simple harmonic or sinusoidal functions (see summation formulas with sinc wavefunction basis in Chapter 4.0, under sections (4.0.30) - (4.0.36)). For instance, in digital signal processing, Nyquist-Shannon-Kotelnikov sampling collects short-duration rectangular pulses or wavefunction samples (in the time domain), with their corresponding frequency-domain spectral functions being sinc functions (and vice versa). The same holds true if Gaussian or bell-curve enveloped narrow-band signals are used instead of rectangular samples. Fig. 10.1 visually confirms these statements, showing that narrow-band wave-packets are similar (or equivalent) to sinc function waves. Additionally, narrow-band wave-packets can be constructed by the integral or discrete superposition of simple harmonic sinusoidal elementary waves (see Chapter 4.0, sections (4.0.30) - (4.0.36)). This suggests the conditions under which certain wave-groups/packets/functions can be narrow-band, limited in duration, energy-finite, and well-defined across all domains, aiding in the modeling of particles and quantized waveforms, including photons. In such scenarios, the Planck-Einstein formula for the energy of an (electromagnetic) wave-packet (or photon) is particularly useful in explaining phenomena like the Photoelectric and Compton effects, as well as wave-particle impacts and scatterings in all combinations. However, this formula should not be treated as a universal wave quantization rule, as is often done in Quantum Theory (QT). Instead, it simply quantifies the energy of any narrow-band, Gaussian, or Bell-curve shaped electromagnetic wave-packet, characterized by a specific carrier and mean frequency, and a certain limited frequency bandwidth. The dimensions (or size) of energy quanta, such as photons, can be estimated as discussed in Chapter 9, under "9.1. Wave Function and dimensions of Photon as an Energy Quantum." For more on different aspects of quantizing in Physics, see Chapters 5 and 8 of this book.

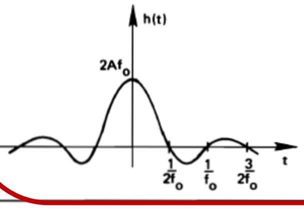
d) In physics, the most interesting **dualistic objects and narrow-band wave-packets** are photons and electrons, which are known to be related or connected in various ways, including situations of mutual transformability. These elementary particles or wave-packets also possess spin attributes, though we cannot directly observe or visualize any mechanical spinning. What we identify as spin attributes are intrinsically connected with associated magnetic moments. Instead of searching for mechanical spinning moments in wavefunctions, we should focus on how natural wavefunctions are created in the real world of Physics (see section (4.0.82) in Chapter 4.0). For photons, electrons, and other charged particles, the natural wavefunction is a combination of electric and magnetic field vectors, akin to the Poynting vector concept. This leads to the conclusion that the spin property of these wave-packets is a type of specific magnetic field vector. The gyromagnetic ratio in such cases is constant, indicating that spin or angular moments can effectively be replaced by specific magnetic field vectors. Thus, the most accurate modeling of natural, Physics-related matter waves should be based on Complex (and Hypercomplex) Analytic Signal modeling (of relevant Phasors or Complex wavefunctions), as introduced in Chapter 4.0. Transforming this deterministic analysis into QT operations involving statistical and probabilistic elements is merely a matter of normalizing wavefunctions (to make them dimensionless).

Time Domain		Frequency Domain

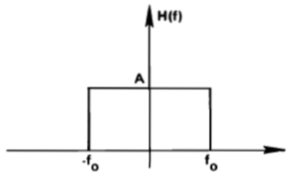


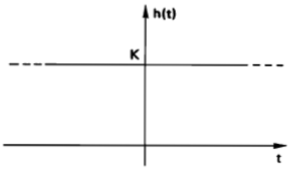
$$h(t) = A \quad |t| < T_0$$
$$= \frac{A}{2} \quad |t| = T_0 \qquad \Longleftrightarrow \qquad H(f) = 2AT_0 \frac{\sin(2\pi T_0 f)}{2\pi T_0 f}$$
$$= 0 \quad |t| > T_0$$



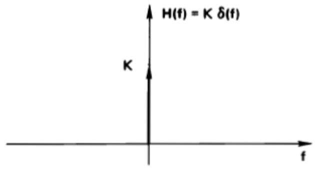


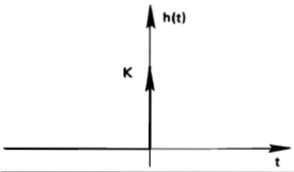
$$h(t) = 2Af_0 \frac{\sin(2\pi f_0 t)}{2\pi f_0 t} \qquad \Longleftrightarrow \qquad H(f) = A \quad |f| < f_0$$
$$= \frac{A}{2} \quad |f| = f_0$$
$$= 0 \quad |f| > f_0$$



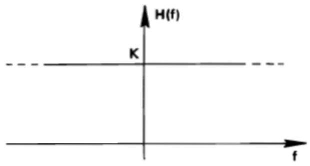


$$h(t) = K \qquad \Longleftrightarrow \qquad H(f) = K\delta(f)$$

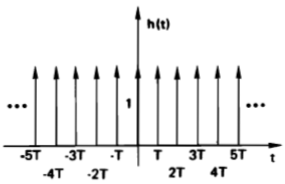




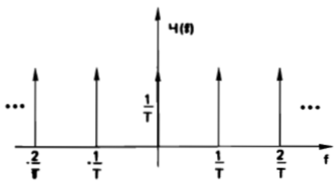
$$h(t) = K\delta(t) \qquad \Longleftrightarrow \qquad H(f) = K$$

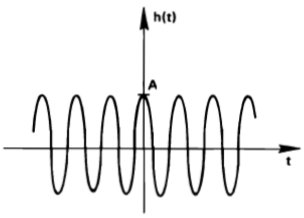


Time Domain		Frequency Domain

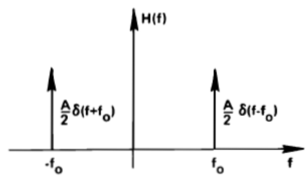


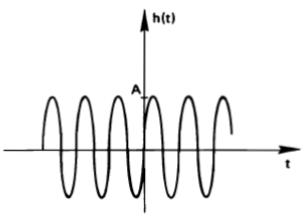
$$h(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT) \qquad \Longleftrightarrow \qquad H(f) = \frac{1}{T} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T}\right)$$



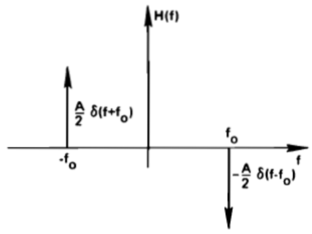


$$h(t) = A\cos(2\pi f_0 t) \qquad \Longleftrightarrow \qquad H(f) = \frac{A}{2}\delta(f - f_0) + \frac{A}{2}\delta(f + f_0)$$





$$h(t) = A\sin(2\pi f_0 t) \qquad \Longleftrightarrow \qquad H(f) = -j\frac{A}{2}\delta(f - f_0) + j\frac{A}{2}\delta(f + f_0)$$



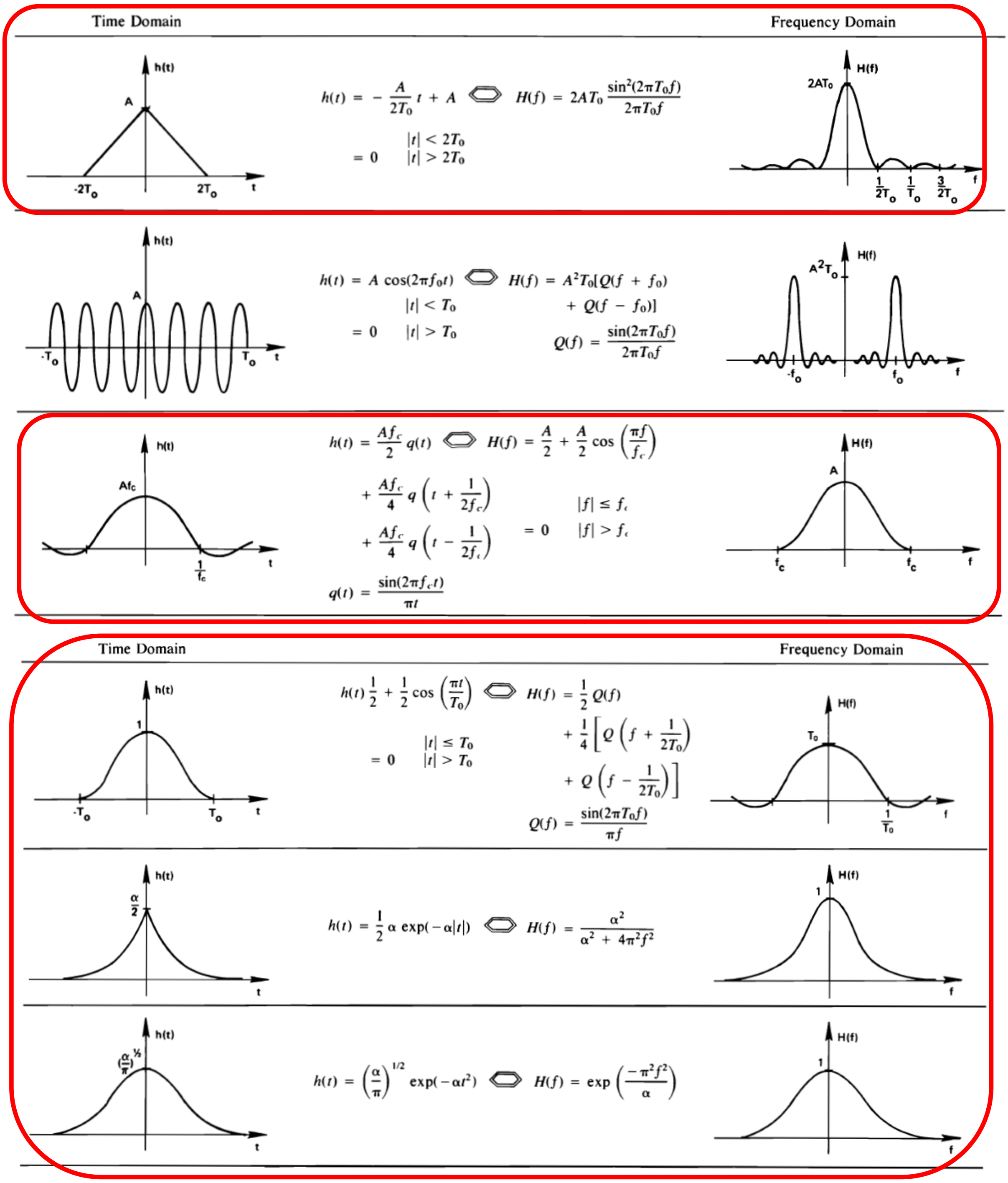


Fig.10.1, Fourier transform with time-frequency domain pairs
(Taken from [106], Brigham, E. Oran**)**

If a matter-wave or wave-packet is presented as an Analytic Signal (introduced by D. Gabor, [57]; see more in Chapter 4.0), we can determine its instantaneous amplitude, natural frequency, and phase functions across all relevant domains. From the phase function, we can also derive the corresponding wavelength, revealing that these wave-related parameters can be treated as properties of de Broglie matter waves, as illustrated in (10.1) and (10.2). Essentially, the Analytic Signal and de Broglie matter waves belong to the same Particle-Wave Duality Concept (PWDC) framework and should be unified within this most natural model.

The Analytic Signal wave function is closely linked to the kinetic energy or wave energy and power of the motion being analyzed. This wave function always consists of two mutually phase-shifted components (Real and Imaginary parts), both of which are equally real and coexist within the same wave motion. For example, in an electromagnetic field or its matter-wave formations, the electric and magnetic field vectors are orthogonal, coupled, and complementary, forming the Poynting vector, which is directly related to the energy of the radiated or received electromagnetic signal. More examples of wave functions in the form of Analytic Signals can be found at the end of Chapter 4.0, under "4.0.11. Generalized Wave Functions and Unified Field Theory."

Notably, the concepts of the Analytic Signal, de Broglie matter waves, and PWDC scenarios apply universally across both the micro and macro scales of physics, indicating no fundamental difference in the underlying mathematical modeling. What must be distinguished, however, are the localization, dimensions, shapes, and sizes of matter waves and their associated equivalent particles or bodies. This differentiation allows us to apply the same Uncertainty Relations across both small particles and large masses, recognizing that mass is not solely defined or localized as a static, stable and solid object with a fixed geometry.

The entire family of Schrödinger and related wave equations in Quantum Theory (QT) can be developed seamlessly by formulating the relevant wave functions as complex Analytic Signal functions and phasors (see more in Chapter 4.3).

Additionally, there is a more elementary and universally applicable "wave-mechanical" method to derive the same expressions for the group and phase velocity of a wave packet. This method is explored in Chapter 4.0 (see "4.0.5. Wave Packets and Mathematical Strategies in Formulating Wave Velocities," with equations from (4.0.6) to (4.0.33)). The existence of multiple independent methods that converge on the same results regarding wave velocities underscores the importance and universality of these findings, further reinforcing the foundational concepts of particle-wave duality and matter waves. This approach also facilitates theoretical cross-platform connections and unification within Physics.

It is crucial to remember that Physics and Nature are already harmoniously and smoothly united in many ways; -it is our specific theories, tools, and concepts that remain disjointed. The properties of Particle-Wave Duality, along with the interconnectedness and mutual transformability of spatial-temporal reality, significantly contribute to this unity.

Within the concept that wave, and kinetic energy are mutually equal ($\tilde{E} = E_k = hf$ *) we can find that matter-wave phase speed,* u *is always lower than relevant wave-group or particle speed v (see more in chapter 4.1), because,*

$$\left\{\begin{array}{l}\left[\begin{array}{l}E_k = \tilde{E} = hf = \dfrac{pv}{1+1/\gamma}, \gamma = 1/\sqrt{1-\dfrac{v^2}{c^2}}, m = \gamma m_0, \\ f = \left(\dfrac{E_k}{h}\right)_{v<<c} = \dfrac{\frac{mv^2}{2}}{h}, \lambda = \dfrac{h}{mv} \Rightarrow u = \lambda f = \dfrac{v}{2}, \text{ or} \\ f = \left(\dfrac{E_k}{h}\right)_{0<v<c} = \dfrac{(\gamma-1)m_o c^2}{h} = \dfrac{(m-m_0)c^2}{h}, \lambda = \dfrac{h}{\gamma mv} \Rightarrow u = \lambda f = \dfrac{v}{1+\sqrt{1-\dfrac{v^2}{c^2}}} \\ (0 \le v \cong 2u << c) \Rightarrow E_k = \tilde{E} = hf \cong \dfrac{1}{2}mv^2 = \dfrac{1}{2}pv \end{array}\right] \\ \left(u = \dfrac{E_k}{p} = \dfrac{v}{1+\sqrt{1-\dfrac{v^2}{c^2}}} = \dfrac{\frac{dE_k}{dp}}{1+\sqrt{1-\dfrac{v^2}{c^2}}}, v = \dfrac{dE_k}{dp}\right) \Rightarrow \\ \dfrac{dE_k}{E_k} = \dfrac{dp}{p}\left(1+\sqrt{1-\dfrac{v^2}{c^2}}\right) = \left\{\begin{array}{l}\left(\dfrac{dE_k}{E_k} \cong 2\dfrac{dp}{p}\right)_{v<<c} \Rightarrow E_k \cong \dfrac{mv^2}{2} \\ \left(\dfrac{dE_k}{E_k} \cong \dfrac{dp}{p}\right)_{v\approx c} \Rightarrow E_k \approx cp \end{array}\right\}, \\ \left[\begin{array}{l}\left(\begin{array}{c} v \cong u \cong c, \tilde{m} = \gamma m_0 = m \\ E_0 = m_0 c^2, p_0 = m_0 c) = \text{constants}\end{array}\right) \Rightarrow \left(\dfrac{\tilde{E}}{p} \cong \dfrac{d\tilde{E}}{dp}\right) \Leftrightarrow \dfrac{dp}{p} \cong \dfrac{d\tilde{E}}{\tilde{E}} \Rightarrow \\ \tilde{E} = hf = E_k \cong \dfrac{E_0}{p_0}p \cong cp, \; v = \dfrac{d\tilde{E}}{dp} \cong \dfrac{E_0}{p_0} \cong \dfrac{E_a}{p_a} = \dfrac{E_b}{p_b} ... \cong u = \lambda f = \dfrac{\tilde{E}}{p} \cong c \end{array}\right]\end{array}\right\} \Rightarrow$$

$$\Rightarrow 0 \le u < 2u \le \sqrt{uv} \le v \le c. \qquad \textit{(10.2-1)}$$

This looks like a certain part of the matter wave is retarded or traveling behind moving particle, what is somewhat strange because intuitively we are closer to the concept that moving particle, and its matter wave should mutually overlap within the same space-time-frequency intervals.

*From the same exercise, we can also conclude that an optimal localization (and maximal concentration) of all relevant particles and wave attributes and characteristics is achievable only when group and phase velocity are mutually similar and close to the maximal speed **c.** This is valid for photons and fast moving microparticles), and this implicitly indicates which self-stabilized and self-closed matter waves are creating stable particles.*

We could hypothetically try (just for comparison) to see what will happen if a particle's total energy and its matter-wave energy are mutually equal, $E_t = \tilde{E} = \gamma mc^2 = \tilde{m}c^2$ *, as for example,*

$$f = \frac{E_t}{h} = \frac{\gamma mc^2}{h}, \lambda = \frac{h}{\gamma mv} \Rightarrow \left(u = \lambda f = \frac{c^2}{v}, 0 \le v \le c\right) \Rightarrow u > c.$$

This time, the situation gets very strange because phase speed will be higher than particle speed, and even higher than light speed c, and this is one reason more to eliminate such option as unrealistic, meaning that total particle energy is not equal to the matter-wave-packet energy that represents the same particle. The other basic and strong reasons to consider that only kinetic particle energy creates a relevant wave group are consequences of analyzes of Compton and familiar impact and scattering effects, and satisfaction of basic conservation laws (See chapter 4.2 of this book, "4.2.2. Example 3: Elastic collision photon-particle").

Obviously, we have much more arguments (as elaborated in the chapters 4.1 and 4.2) to consider that kinetic and matter-wave energy of moving particles is mutually equal, $(\tilde{E} = E_k = hf) \Rightarrow (0 \le u < 2u \le \sqrt{uv} \le v \le c)$. ***The way to understand what kind or part of matter-***

wave group is retarded (or delayed), and following the particle or wave group, is to take into consideration that all moving particles should be in certain (mutual) field-force and matter waves relations and couplings with their environment, creating kind of two or multi-body situations. Even if we do not see (or measure) any presence of a second body, certain well-hidden matter-wave and fields relation should exist between a single moving particle and its environment. This way, it is imaginable that some part of a matter wave in question is propagating behind moving particle, where phase speed u ***is lower than particle or group speed,*** $0 \leq u < 2u \leq \sqrt{uv} \leq v \leq c$.

Another supporting situation to explain the existence of a retarded part of a matter wave group is that linear and spinning or angular motions (of the same particle) are mutually complementary and united, having natural tendency to create closed-line, circular paths (like in cases of atoms), this way potentially hosting standing waves formations (like in cases of stable planetary systems; -see "2.3.3. Macro Cosmological Matter Waves and Gravitation" in chapter 2.). Within mentioned self-closed lines and standing waves formations, the meaning of retarded waves is becoming an important conceptual part of involved structural matter-wave groups' periodicity.

We also know that general solutions of differential, Classical Wave Equations (including similar complex, analytic signal, and generalized Schrödinger equations) are always presented with two wave functions propagating in mutually opposed spatial-temporal directions, what we are, too often and simply neglecting as not enough operational and not well conceptually supported in Physics (see much more in chapter 4.3). Such two-component wave group can explain velocity and spatially retarded matter wave's nature. In addition, we could imagine (or exercise) that mentioned wave groups also have mutually opposed (or mutually canceling) angular and/or spin moments, this way coming closer to the real nature of matter waves and wave-particle duality).

What should be much more general and realistic, united modeling of particle-wave duality is the framework of Analytic Signal functions (as established by D. Gabor; - see much more in chapters 4.0 and 4.1). By using the Analytic Signal model creatively and with certain mathematical and dimensional arrangements, we should be able to present every matter-wave function of certain motion, its power, field, and force as a couple of mutually phase-shifted and Hilbert transform related wave functions $\Psi(t)$ ***and*** $\hat{\Psi}(t)$. ***Mentioned couple creates a complex analytic signal function*** $\overline{\Psi}(t) = \Psi(t) + j\hat{\Psi}(t) = (1+jH)\cdot\Psi(t)$, ***where both*** $\Psi(t)$ ***and*** $\hat{\Psi}(t)$ ***present real, natural, and detectable items, like combination and coupling of electric and magnetic field vectors that is creating an electromagnetic field. On a similar way, every linear motion of particles and waves presented with a wave function*** $\Psi(t)$, ***should be automatically followed by synchronously created analytic signal couple*** $\hat{\Psi}(t)$, ***again creating a complex analytic signal*** $\overline{\Psi}(t) = \Psi(t) + j\hat{\Psi}(t)$, ***which has all matter-wave properties that are products of Analytic Signal modeling (such as de Broglie wavelength, frequency, phase etc.). Both*** $\Psi(t)$ ***and*** $\hat{\Psi}(t)$ ***should present detectable physics items. For instance, a very good example of*** $\Psi(t)$ ***and*** $\hat{\Psi}(t)$ ***as coupled matter waves is where*** $\Psi(t)$ ***presents certain linear motion, and*** $\hat{\Psi}(t)$ ***presents a spiral, spinning or helix, de Broglie matter wave of a field existing in a space around the path of the original linear motion of*** $\Psi(t)$. ***De Broglie matter waves have the frequency and phase described by the corresponding Analytic Signal model, as defined in chapter 4.0, by equations (4.0.2). From the Analytic Signal frequency and phase, we can determine de Broglie matter-wave wavelength*** $\lambda = h/p = u/f$ ***and belonging group and phase velocity*** $v = u - \lambda du/d\lambda = -\lambda^2 df/d\lambda$. ***Relevant Analytic Signal wave functions that naturally describe de Broglie matter waves are power and motional energy-related functions, including corresponding field and force functions. By creating normalized, non-dimensional wave functions, we will be able to address Quantum Theory approach to***

the same problematic. This will be the background for explaining matter-waves and particle-wave duality in this book.

Now we can summarize matter waves and particle-wave duality concepts such as:

1. ***Propagating matter waves can create momentum-energy entities (such as atoms, nuclear and elementary particles and other energy states) with stable (non-zero) rest masses in cases if and when such matter waves will transform to self-closed and stabilized standing-waves structures*** (having an integral number of half-wavelengths, $n\frac{\lambda}{2}$, $n = 1, 2, 3...$, or $n\bar{\lambda} = 2\pi\bar{r}$, $n \cdot \theta = 2\pi...$, as summarized in T.4.2. "Wavelength analogies in different frameworks" in chapter 4.1). Freely propagating photons (as wave packets) could become electrons or other particles (with non-zero rest masses) if under certain conditions such wave formation is forced to become self-closed, stabilized, and standing waves structured formation. This way, motional wave energy (without rest mass attributes) could be transformed into a moving particle with a rest mass. This is usually related to elementary and subatomic particles formations).

   Certain relatively stable dynamic state of matter, or a system of particles, like an inertial or uniform motion that has number of intrinsic and structural periodicities, (including orbital, circular, spinning, and repetitive, periodical motions, and standing waves formations), can be relatively well described, or approximated, with number of mutually isomorphic, analogous, and equivalent mathematical models (being like Geocentric, Ptolemaic concepts). Anyway, only one of them will be the most correct and realistic model. Quantizing in Physics is mostly related to energy-finite, spatially-temporally limited entities (wave packets or groups), and to different counting and quantifications of atomized (or discretized) matter states, such as number of half-wavelengths, number of revolutions, number of periods, spinning frequency... (where we are using integers), and to simple arithmetic relations between integers-characterized states of matter in mentioned systems with intrinsic and standing-waves related periodicities. As examples for quantized structures, we could take Bohr's and Lucas-Bergman atom models (see Chapter 8.), and Planetary systems (see chapter 2; -2.3.3.).

We propose an extended understanding of ***inertial motions*** *as those defined by mutually coupled and interchangeable linear and angular momenta. These motions are characterized as natural, self-sustaining, stable, smooth, periodic in spacetime, and self-contained (see Chapters 2 and 4.1 for details). Linear and angular inertial, mechanical and electromagnetic motions consistently and synchronously produce stable, self-contained, spinning, rotating, and standing matter-wave formations along their respective helicoidal paths.*

This leads to the view that stable rest masses can be understood as ensembles of "internally frozen or captured" inertial spinning states, where angular and linear momenta are internally and externally combined, conserved, or complemented by *intrinsically generated matter-waves. For masses in motion, their kinetic energy is equivalent to the energy of their associated matter-waves. In this framework, matter-waves, in relation to gravitation, may be interpreted as a type of inertial and electromagnetic waves.*

Standing-matter-waves, which always manifest as an integer (or countable) number of wavelengths and time periods, form the foundation of quantization in physics. The integral role of natural inertial motions in maintaining stable orbital motions is explored in detail in [36] and throughout this book (e.g., Chapter 2, Section 2.3.3, "Macro-Cosmological Matter-Waves and Gravitation"). This principle is applicable, under negligible friction effects, to systems ranging from the micro-world of atoms to the macro-world of planetary systems.

In summary, this perspective connects stable, rotating, standing matter-wave formations (representing rotational inertial motions) to the natural, linear inertial motions of the involved particles (see more about inertia and inertial states or motions in chapters 1, 2, and 4.2).

2. Any wave function can be presented and analyzed in its time and frequency domain, and its minimal space-time-frequency durations in mentioned domains are respecting "Uncertainty Relations" (see Chapter 5. UNCERTAINTY). Regarding the size of elementary particles and other (standing waves) stabilized particles, mathematical Uncertainty (between corresponding time, space, and frequency domain-intervals) is effectively transformed into a Certainty, where an inequality sign, "$\geq$, or $\leq$" is transformed into an equality sign, "=". ***As elaborated in Chapter 5, we can conclude that metrics and energy formatting of Nature, regarding its elementary parts (such as atoms and elementary particles), have come to certain, conditionally non-divisible (and minimal) units of building blocks, this way realizing optimal matter waves packing (or formatting) with minimal and finite domain intervals.*** Such elementary matter building blocks (or narrow-banded matter-wave packets, effectively being like Gabor-Gaussian Wave Packets) should satisfy the following "resonant gearing and fitting, CERTAINTY relations, or optimal packing conditions" (for more explanations see (5.3) in the chapter 5.),

$$\begin{aligned}&\left(\Delta q_{mag.}\cdot\Delta q_{el.}\right)_{min.}=\left(\Delta\alpha\cdot\Delta L\right)_{min.}=h\cdot\left(\Delta t\cdot\Delta f\right)_{min.}=\left(\Delta x\cdot\Delta p\right)_{min.}=\\&=\left(\Delta t\cdot\Delta\tilde{E}\right)_{min.}=c^{2}\left(\Delta t\cdot\Delta m\right)_{min.}=\left(\Delta s_{1}\cdot\Delta s_{2}\right)_{min.}=h/2,\ (\Delta x)_{min.}=\frac{\lambda}{2}.\end{aligned} \qquad (5.3)$$

In sections (5.3) and beyond, we focus solely on the total and absolute durations of a signal or wave function across all its original and spectral domains, without considering relevant probabilities or statistical standard deviations.

From the Uncertainty Relations discussed in Chapter 5 and specifically in section (5.3), we can also infer a direct proportionality between space and time. For example, the relationships relevant to a given energy-moment state effectively ensure their structure, integrity, stability, and relatively constant shape. This is because the original and spectral functions in the time and frequency domains are interchangeable, much *like how the original and spectral domains of spatial (or geometrically related) signals are connected through group and phase velocity relations. For further supporting arguments, refer to sections (10.1), (10.2), and (4.2) from Chapter 4.1.*

$$\begin{pmatrix} \Delta x \cdot \Delta p = \Delta t \cdot \Delta E = h \cdot \Delta t \cdot \Delta f \geq h/2 \Leftrightarrow \Delta x \cdot \Delta\tilde{p} = \Delta t \cdot \Delta\tilde{E} = h \cdot \Delta t \cdot \Delta f \geq h/2, \\ \Delta q_{mag.} \cdot \Delta q_{el.} = \Delta\alpha \cdot \Delta L = h \cdot \Delta t \cdot \Delta f = \Delta x \cdot \Delta p = \Delta t \cdot \Delta\tilde{E} = c^2 \Delta t \cdot \Delta m \end{pmatrix} \Rightarrow$$

$$v = \frac{\Delta x}{\Delta t} = \frac{\Delta\tilde{E}}{\Delta p} = \frac{\Delta\omega}{\Delta k} = h\frac{\Delta f}{\Delta p} \text{ (= group velocity)} \Leftrightarrow v\Delta t = \Delta x = \frac{\Delta\omega}{\Delta k}\Delta t = h\frac{\Delta f}{\Delta p}\Delta t.$$

Probability and statistics in this context should be considered secondary or supplementary tools, primarily useful for enhancing final modeling and presentation of results when appropriate conditions are met. In sections (5.3), (10.1), and (10.2), the relationships between different functions, variables, and parameters are deterministic, clear, analytic, and explicit, without any inherent probabilistic nature. However, these situations and relationships can also be interpreted statistically or probabilistically by applying certain mathematical adjustments, which may involve artificial hybridization.

It is important to note that not all matter states and matter-wave packets are stable, maintaining consistent spatial and temporal durations or shapes in relevant variables. Some states may exist as short-lived transients or dispersive waves with varying energy-moment characteristics. Ultimately, any stochastic conceptualization or modeling of particle-wave duality should be more statistically deterministic (like in thermodynamics, where the law of large numbers applies) rather than relying on the probabilistic and non-deterministic concepts often associated with orthodox quantum theory.

A common mistake in analyzing and modeling uncertainty relations is the assumption that these relations are mostly applicable in a statistical sense, limited to the micro-world of physics. This assumption lacks mathematical foundation, as mathematics is universally applicable and serves as the common framework, language, and logic for the entire universe, both micro and macro. Concepts of quantization and quantum nature in physics can also be universally represented and applied when they are ontologically based on signal analysis and synthesis as developed by Fourier, Gabor, Kotelnikov, Shannon, Nyquist, and Whittaker.

To additionally expose the space-time symmetry, related to mutual replacements of original and spectral domains, let us analyze analogically formulated consequences starting from Einstein-Minkowski 4-vectors. The most useful, and in a countless number of cases verified, is an energy-momentum 4-vector,

$$\overline{\mathbf{P}}_4 = (\mathbf{p}, \frac{\mathbf{E}}{\mathbf{c}}) \Rightarrow \mathbf{p}^2 - \frac{\mathbf{E}^2}{\mathbf{c}^2} = -\frac{\mathbf{E}_0^2}{\mathbf{c}^2} = \textbf{ invariant (= constant)}.$$

We will also exploit the same uncertainty relations (as formulated in Chapter 5) applicable on certain wave group, or wave packet, or simply on the same $\overline{\mathbf{P}}_4 = (\mathbf{p}, \frac{\mathbf{E}}{\mathbf{c}})$, narrow-banded energy-momentum matter-wave group,

$$\mathbf{\Delta x \cdot \Delta p = \Delta t \cdot \Delta E} \left(\geq \frac{\mathbf{h}}{\mathbf{2}}\right).$$

To expose and underline analogies and symmetry (between original and spectral domains), we can formulate or expand the following relations,

$$\left\{\begin{array}{l}\omega_x = k = 2\pi f_x = \boxed{2\pi\cdot\frac{1}{\lambda}} = 2\pi\cdot\frac{\tilde{p}}{h} = \frac{2\pi}{h}\cdot\tilde{p} = \frac{\tilde{p}}{\hbar}\\ \omega_t = \omega = 2\pi f_t = \boxed{2\pi\cdot\frac{1}{T}} = 2\pi\cdot\frac{\tilde{E}}{h} = \frac{2\pi}{h}\cdot\tilde{E} = \frac{\tilde{E}}{\hbar}\\ \Delta x\cdot\Delta\tilde{p} = \Delta t\cdot\Delta\tilde{E} \geq \frac{h}{2},\ v = \frac{\Delta x}{\Delta t} = \frac{\Delta\tilde{E}}{\Delta\tilde{p}}\\ F(=\text{force}) = \frac{\Delta\tilde{p}}{\Delta t} = \frac{\Delta\tilde{E}}{\Delta x},\ \Delta E = \Delta\tilde{E},\ \Delta p = \Delta\tilde{p}\end{array}\right\} \Leftrightarrow \left\{\begin{array}{l}\Delta\tilde{p} = \hbar\cdot\Delta k,\ \Delta\tilde{E} = \hbar\cdot\Delta\omega\\ \Delta x\cdot\Delta k = \Delta t\cdot\Delta\omega \geq \pi = \frac{h}{2\hbar},\\ \Delta k = 2\pi\Delta f_x,\ \Delta\omega = 2\pi\Delta f_t,\\ \Delta x\cdot\Delta f_x = \Delta t\cdot\Delta f_t \geq \frac{1}{2},\\ v = \frac{\Delta x}{\Delta t} = \frac{\Delta f_t}{\Delta f_x} = \frac{\Delta\omega}{\Delta k} = \frac{\Delta\tilde{E}}{\Delta\tilde{p}} = \frac{d\omega}{dk} = \frac{dE}{dp}\\ \frac{\Delta\tilde{p}}{\Delta t} = \frac{\Delta\tilde{E}}{\Delta x} = \frac{d\tilde{p}}{dt} = \frac{d\tilde{E}}{dx}\ (=)\text{Force}\end{array}\right\}.$$

As we can see, force fields around "moment-energy-states" are also generally related to all Uncertainty and Certainty relations applicable on here relevant states (both in micro and macro world physics).

Let us now (based on analogical assumptions) exercise space-time symmetry, connectivity, proportionality, and possibility of mutual original and spectral domain replacements (here underlined with $\boxed{\Delta x} \leftrightarrow \boxed{\Delta t}$ and $\boxed{\Delta p = \hbar\Delta k} \leftrightarrow \boxed{\Delta\tilde{E} = \hbar\Delta\omega}$). We already know that energy and momentum are mutually related ($E \leftrightarrow p$) or connected within invariance of $\overline{P}_4 = (p, \frac{E}{c})$. From A. Einstein-Minkowski Relativity mathematics we also know that space-time interval should be invariant regarding different reference or coordinates (and inertial) systems presentations, $(\Delta s)^2 = c^2(\Delta t)^2 - (\Delta x)^2 = \text{const.}$. Now, we will create a couple of new, analogical (look-like, somewhat hypothetical) 4-vectors that are only formally like $\overline{P}_4 = (p, \frac{E}{c})$, and draw interesting consequences about space and time intervals' relations, as follows,

$$\left\{\begin{array}{l}\overline{P}_4 = (p, \frac{E}{c}) \Rightarrow p^2 - \frac{E^2}{c^2} = -\frac{E_0^2}{c^2} = \text{invariant}\\ \boxed{\Delta x}\cdot\Delta p = \boxed{\Delta t}\cdot\Delta E,\ E_0 = mc^2\\ (\Delta s)^2 = c^2(\Delta t)^2 - (\Delta x)^2 = \text{const.}\end{array}\right\} \Rightarrow \left\{\begin{array}{l}\overline{X}_4 = (x, ct) \Rightarrow (\Delta x)^2 - c^2(\Delta t)^2 = (\Delta x_0)^2 - c^2(\Delta t_0)^2 = -(\Delta s)^2\\ \left[\begin{array}{l}\overline{T}_4 = (t, \frac{x}{c}) \Rightarrow (\Delta t)^2 - \frac{(\Delta x)^2}{c^2} = (\Delta t_0)^2 - \frac{(\Delta x_0)^2}{c^2} = \text{invariant}\\ \overline{E}_4 = (E, cp) \Rightarrow E^2 - c^2p^2 = E_0^2 = \text{invariant}\end{array}\right]\\ \frac{(\Delta s)^2}{(\Delta t)^2} = c^2 - \frac{(\Delta x)^2}{(\Delta t)^2} = c^2(1 - \frac{v^2}{c^2}), \Delta s = c\Delta t\sqrt{1 - \frac{v^2}{c^2}},\ v = \frac{\Delta x}{\Delta t}\end{array}\right\} \Rightarrow$$

$$c^2 = \frac{(\Delta x_0)^2}{(\Delta t_0)^2} + \frac{(\Delta s)^2}{(\Delta t_0)^2} = \frac{(\Delta x)^2}{(\Delta t)^2} + \frac{(\Delta s)^2}{(\Delta t)^2} = v^2 + \frac{(\Delta s)^2}{(\Delta t)^2} \Rightarrow (1 - \frac{v^2}{c^2}) = \frac{(\Delta t_0)^2 - \frac{(\Delta x_0)^2}{c^2}}{(\Delta t)^2} = \frac{1}{c^2}\frac{(\Delta s)^2}{(\Delta t)^2} \cong \frac{(\Delta t_0)^2}{(\Delta t)^2} \Rightarrow$$

$$\Delta t = \frac{\Delta x}{v} = \frac{\sqrt{(\Delta t_0)^2 - \frac{(\Delta x_0)^2}{c^2}}}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{\Delta t_0\sqrt{1 - \frac{1}{c^2}\frac{(\Delta x_0)^2}{(\Delta t_0)^2}}}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{\frac{\Delta s}{c}}{\sqrt{1 - \frac{v^2}{c^2}}} \cong \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}},\ \Delta s = c\Delta t_0\sqrt{1 - \frac{1}{c^2}\frac{(\Delta x_0)^2}{(\Delta t_0)^2}} \cong c\Delta t_0$$

Indexing with "0" indicates that the corresponding variable belongs to the standstill rest state (or relevant center of mass, inertial system). On some way, here we implicitly consider that moving energy-momentum state in question is like a certain narrow-banded matter-wave packet with finite and short durations, both in its time and frequency domain. For instance, if such motional matter-state has its total spatial length (or size) in the direction of its propagation equal to Δx, and if we observe the same state between its temporal points T_1, T_2 and corresponding spatial points X_1, X_2, where $\Delta X = X_2 - X_1 \gg \Delta x,\ \Delta T = T_2 - T_1 \gg \Delta t$, we will, approximately (or in average) consider that,

$$\left(\frac{\Delta x}{\Delta t} \cong \frac{\Delta X}{\Delta T}\right) \cong \left(v = \frac{dx}{dt} = u - \lambda \frac{du}{d\lambda}\right), \text{ or } \sqrt{\left(\frac{\Delta x}{\Delta t}\right)^2 + \left(\frac{\Delta y}{\Delta t}\right)^2 + \left(\frac{\Delta z}{\Delta t}\right)^2} \cong$$

$$\cong \sqrt{\left(\frac{\Delta X}{\Delta T}\right)^2 + \left(\frac{\Delta X}{\Delta T}\right)^2 + \left(\frac{\Delta X}{\Delta T}\right)^2} \cong \left(v = \frac{dr}{dt} = u(1 + \sqrt{1 - v^2 / c^2})\right).$$

Practically, here we are in some way dealing mostly with linear and non-dispersive, ***stable structure and shape*** matter-states, where we can experience regular (or linear and simple) interference and superposition effects, and symmetry between original and spectral signal domains, known in Analytic Signal and Fourier integral transformations.

In fact, here we still need to think more creatively and imaginatively, and develop or give proper formulations of spatial, temporal, and spectral items, and formulate an overall (or better) explanation, but we can already see that the idea about spatial and temporal proportionality and connectivity and CPT Symmetry is very much incorporated in Relativity theory. Here we also present another (analogical and symmetry-related) insight that is testing the skeleton of Relativity theory. For instance, we could pose a question if (supposed to be) an invariant space-time interval from Relativity theory is "*only approximately invariant"* or always constant, since $\Delta s \cong c\Delta t_0$. A larger and richer mathematical framework to address matter structure will be to create similar Minkowski 4-vectors or n-vectors and corresponding Phasor's mathematics with Hypercomplex Analytic Signals.

Spatial-temporal and joint time-space-frequency reality,

duality, proportionality, and mutual spatial-temporal transformability can also be imaginatively addressed if we consider that, at least local, infinitesimal, or differential space-time connections and relations are like relations between an Analytic signal, real and imaginary wavefunctions (which are by Hilbert transform mutually conjugate and phase shifted) couple, including that PWDC conditions are also satisfied (see (10.2) and later in this chapter).

We still do not know what the time essentially is, but we can easily operate with time intervals or time intervals durations in relation to corresponding spatial intervals. In the four-dimensional space-time world (x, y, z, t) we know that all spatial and temporal axes are mutually orthogonal. The orthogonality of time axis we realize mathematically based on significant success and applicability of Minkowski 4-vectors from Relativity theory, where time axis is an imaginary numbers axis. If instead of ordinary complex function with one real and one imaginary member, we apply hypercomplex dimensional framework (with three imaginary units or axes), we will create the platform that time could also have three temporal and mutually orthogonal components or axes, and every event position is such space-time reality will be specified as (x, y, z, t_x, t_y, t_z). This will obviously require that we evolve with our conceptualization and understanding of the time arrow, or time direction (based on more-complex relativistic relations between spatial and temporal coordinates and intervals).

Here, still like an intuitive and over-simplified brainstorming (before establishing better mathematical foundations), time domain is considered as a kind of Hilbert transform of a space-domain (or vice versa), what means that temporal and spatial domains are (on certain way), mathematically (or functionally) mutually orthogonal and phase shifted for $\frac{\pi}{2}$ (similar as practiced in Minkowski space 4-vectors), as for instance, intuitively simplified and improvised on the following way,

$$\begin{bmatrix} x = ut,\ \Delta x = u\Delta t, dx = udt \\ u = \lambda f\ (=) \text{phase velocity} \\ \partial x^2 = \partial (ut)^2 = u^2 \partial t^2 \\ \partial t^2 = \partial (\frac{x}{u})^2 = \frac{1}{u^2}\partial x^2 \end{bmatrix}, \begin{bmatrix} \Delta \overline{x} = \Delta x + I \cdot \Delta \hat{x} = \Delta x + I \cdot H[\Delta x] = \Delta x + I \cdot u \cdot H[\Delta t] \\ \Delta \hat{x} = H[\Delta x] = H[\Delta (ut)] = u \cdot H[\Delta t],\ I^2 = -1 \\ \overline{x} = \|\overline{x}\| \cdot e^{I\varphi(t)} = \sqrt{(\Delta x)^2 + (\Delta \hat{x})^2} \cdot e^{I\varphi(t)} = I \cdot \|\overline{x}\| \\ \varphi(t) = \text{arctg}\frac{u \cdot H[t]}{ut} = \text{arctg}\frac{H[t]}{t} = \frac{\pi}{2} \end{bmatrix}.$$

If we apply proposed spatial-temporal relations, or t and x replacements, such as, $\Delta x = u\Delta t,\ dx = udt$, $\partial x^2 = \partial (ut)^2 = u^2\partial t^2$, $\partial t^2 = \partial (\frac{x}{u})^2 = \frac{1}{u^2}\partial x^2$, effectively meaning that spatial and temporal durations and/or dimensions are mutually replaceable, as $x \rightleftarrows t$, in the Complex Classical Wave equation (or in the Complex Schrödinger equation), we can prove that in a few simple mathematical steps we will again get the same Classical wave equation, as follows,

$$\left\{ \begin{matrix} \boxed{\Delta \overline{\Psi}(x,t) - \frac{1}{u^2}\frac{\partial \overline{\Psi}^2(x,t)}{\partial t^2} = 0} \Leftrightarrow \left[\Delta \Psi - \frac{1}{u^2}\frac{\partial^2 \Psi}{\partial t^2} = 0,\ \Delta \hat{\Psi} - \frac{1}{u^2}\frac{\partial^2 \hat{\Psi}}{\partial t^2} = 0 \right] \\ \& \\ \boxed{x = ut,\ \partial x^2 = \partial (ut)^2 = u^2\partial t^2,\ \partial t^2 = \partial (\frac{x}{u})^2 = \frac{1}{u^2}\partial x^2, x \rightleftarrows t} \end{matrix} \right\} \Rightarrow \boxed{\Delta \overline{\Psi}(x,t) - \frac{1}{u^2}\frac{\partial \overline{\Psi}^2(x,t)}{\partial t^2} = 0}.$$

(10.2.1a)

The deeper meaning of presented brainstorming exercise is that the same matter waves could be created, synchronously and coincidently, both in a time and in a spatial domain (since Classical Wave Equation did not change its form when we mutually replaced temporal and spatial variables t and x). Consequently, we could say that time and space are very much symmetrical, connected and mutually transformable, and we can also draw similar conclusions for relations between relevant temporal and spatial spectral domains. We know that the same Classical wave equation as known in mechanics is also describing electromagnetic waves. *Such kind of thinking could lead us to explain and understand better "double-slit interference effects" and the nature of wave-particle duality (see more in Chapter 4.1).*

Let us go back to basics of dimensionality where all relevant dimensions or axes should be mutually orthogonal. Analytic Signal model (or Hilbert integral transformation) is also and naturally producing mutually orthogonal functions. For instance, let us start from spatial-temporal wave function $\Psi(x,t)$. This function or signal, based on our detection or measurements methodology we can present as two wave functions (once only in a spatial, and second time only in a temporal domain), as follows,

$$\Psi(x,t) \rightarrow \begin{bmatrix} \Psi(x),\ t = \text{const} \\ \Psi(t),\ x = \text{const} \end{bmatrix}.$$

Here we assume (or accept) that spatial and temporal dimensions should be mutually orthogonal. Consequently, if we know certain time-domain, mathematically formulated function $\Psi(t)$, its corresponding or associated (phase shifted and orthogonal), spatial-domain function is Hilbert transform of the original time-domain function, producing $\Psi^*(x)$ and vice versa.

One of indicative ways how this could be mathematically formalized or still intuitively and hypothetically presented using Hilbert transform and the Analytic Signal model, is as follows,

$$\left\{\begin{bmatrix}\Psi(t)\\ \text{temporal domain}\\ \dots\dots\dots\\ \Psi(x)\\ \text{spatial domain}\end{bmatrix}\to H\begin{bmatrix}\Psi(t)\\ \Psi(x)\end{bmatrix}=\begin{bmatrix}\Psi^*(t)\\ \Psi^*(x)\end{bmatrix}(=)\begin{bmatrix}\Psi^*(x)\\ \overline{\Psi}(x)=\Psi(x)+I\cdot\Psi^*(x)\\ \text{spatial domain}\\ \dots\dots\dots\\ \Psi^*(t)\\ \overline{\Psi}(t)=\Psi(t)+I\cdot\Psi^*(t)\\ \text{temporal domain}\end{bmatrix}\Rightarrow\begin{bmatrix}\overline{\Psi}(x,t)=\Psi^*(t)+I\cdot\Psi^*(x)\\ \text{spatial}-\text{temporal}\\ \dots\dots\dots\\ \overline{\Psi}(x,t)=\Psi^*(x)+I\cdot\Psi^*(t)\\ \text{spatial}-\text{temporal}\\ \dots\dots\dots\\ \overline{\Psi}(x,t)=\Psi(x,t)+I\cdot\Psi^*(x,t)\end{bmatrix}\right\}$$

or

$$\left\{\begin{matrix}\Psi(x,t)\to H\left[\Psi(x,t)\right](=)\Psi(x)+\Psi(t)\to H\left[\Psi(x,t)\right]=\Psi(t,x)\Rightarrow\\ \Rightarrow\overline{\Psi}(x,t)=\Psi(t)+I\cdot\Psi(x)=\Psi_x(t)+\Psi_t(t)+I\cdot\left[\Psi_t(x)+\Psi_x(x)\right]\end{matrix}\right\}$$

or

$$\left\{\begin{matrix}\begin{bmatrix}\Psi(x,y,z)\\ \text{spatial domain}\end{bmatrix}\to H\left[\Psi(x,y,z)\right]=\begin{bmatrix}\Psi(t_x,t_y,t_z)\\ \text{temporal domain}\end{bmatrix}\Rightarrow\\ \Rightarrow\begin{bmatrix}\overline{\Psi}(x,y,z,t_x,t_y,t_z)=\Psi(t_x,t_y,t_z)+I\cdot\Psi(x,y,z)\\ \text{spatial-temporal}\end{bmatrix}\end{matrix}\right\},\ I^2=-1\ .$$

(10.2.1b)*

Temporal and spatial orthogonality between mutually related functions $\Psi(t)$ and $\Psi(x)$, and $\Psi^*(t)$ and $\Psi^*(x)$ is also associating on mutual orthogonality between an electric and magnetic field functions, as in cases of electromagnetic waves, what could be an additional (still hypothetical, or indicative) insight into understanding what time and space are. In fact, everything in our temporal-spatial world is in some way well-packed, integrated and mutually dependent regarding temporal and spatial spectral pictures of the same event, and there is certain causal, functional, or integral-transformations explicable mapping between them (here realized using Hilbert transform).

To be more indicative, let us analogically address the energy of the same spatial-temporal wave function from different points of view (of course using Parseval's identity, or wave energy definition; -see about Parseval identity in Chapter 4.0, equations (4.0.4)).

$$\Psi(x,t) \rightarrow \begin{bmatrix} \Psi(x),\ t = const \\ \Psi(t),\ x = const \end{bmatrix} \rightarrow \tilde{E} = E_k = \begin{bmatrix} \int\limits_{[\Delta t]} \Psi^2(t)dt = \frac{1}{\pi} \int\limits_{[\Delta\omega_t]} \left|A(\omega_t)\right|^2 d\omega_t \\ \int\limits_{[\Delta x]} \Psi^2(x)dx = \frac{1}{\pi} \int\limits_{[\Delta\omega_x]} \left|A(\omega_x)\right|^2 d\omega_x \\ \int\limits_{[\Delta x, \Delta t]} \Psi^2(x,t)dx \cdot dt = \frac{1}{\pi} \int\limits_{[\Delta\omega_x, \Delta\omega_t]} \left|A(\omega_x)\right|^2 d\omega_x \cdot d\omega_t \end{bmatrix}$$

$$d\tilde{E} = \begin{bmatrix} \Psi^2(t)\,dt = \frac{1}{2}\left|\overline{\Psi}(t)\right|^2 dt = \frac{1}{\pi}\left|A(\omega_t)\right|^2 d\omega_t = \frac{1}{2}a^2(t)dt \\ \Psi^2(x)\,dx = \frac{1}{2}\left|\overline{\Psi}(x)\right|^2 dx = \frac{1}{\pi}\left|A(\omega_x)\right|^2 d\omega_x = \frac{1}{2}a^2(x)dx \\ \Psi^2(x,t)\cdot dx \cdot dt = \frac{1}{2}\left|\overline{\Psi}(x,t)\right|^2 dx \cdot dt = \frac{1}{\pi}\left|A(\omega_x,\omega_t)\right|^2 d\omega_x \cdot d\omega_t = \frac{1}{2}a^2(x.t)\cdot dx \cdot dt \end{bmatrix} = \begin{bmatrix} h_t \cdot df_t \\ \boxed{\begin{matrix} p = h \cdot df_x, \\ f_t = f = \omega / 2\pi, \\ f_x = 1/\lambda = p/h = k/2\pi \end{matrix}} \end{bmatrix} = \begin{bmatrix} v_t dp_t \\ v_x dp_x \\ \omega_t dL_t \\ \omega_x dL_x \end{bmatrix} = c^2 d\tilde{m}$$

$$\begin{bmatrix} \frac{\partial \tilde{E}}{\partial t} = \Psi^2(t) \\ \frac{\partial \tilde{E}}{\partial x} = \Psi^2(x) \\ \frac{\partial^2 \tilde{E}}{\partial x \cdot \partial t} = \Psi^2(x,t) \end{bmatrix} = Power(=)\left[W\right],\quad \omega_t = \omega,\ \omega_x = k,$$

(10.2.1b)**

What is creatively and analogically presented with (10.2.1b)* and (10.2.1b)** in a very much simplified way, could become the inspiration for number of challenging ideas and projects regarding wave-particle duality and spatial-temporal nature of our Universe understanding. Here we also find that energy can be stored in spatial configurations (what in Mechanics presents potential energy), and as a motional kinetic energy of particles and matter-waves. Static, spatial configuration dependent energy is well connected and balanced with other motional energy types.

In addition, we can be very certain that electromagnetic and associated mechanical or spatial vibrations and waves are also synchronously created, coupled and/or detectable (in parallel with mechanical motions or oscillations of masses), giving us a platform to develop concepts and technologies related to extended meaning of CPT Symmetry and entanglement effects, gravitational waves, and maybe new methods of communications and masses transport. Mechanical, ultrasonic, or acoustical energy, moments, oscillations and vibrations, or audio signals and music, can also be created and transported by applying different signal-modulating techniques (both in spatial and temporal domain) on laser beams and dynamic plasma states (or signals); - See relations under (10.2-2.4), literature references from [133] until [139], and familiar relations (3.7-1) and (3.7-2) from the third chapter of this book.

We could also (and still hypothetically) imagine that by producing convenient conditions of specifically created electric and magnetic fields around certain heavy or macro-mass, combined with an external, vibrational excitation, effects of gravitational attraction, could be weakened or cancelled (thanks to complexity of electromagnetic and mechanical interactions based on (3.1). (3.2), (3.3) and (3.4), as elaborated in Chapter 3.). The orientation of internal electric and magnetic dipoles and moments of threated mass should be changed on a way to become perpendicular to a direction of natural gravitational force from mentioned mass towards the biggest mass in its neighborhood (to weaken natural gravitation). The mentioned external vibrational mass excitation could be realized mechanically, acoustically, ultrasonically and using, by frequency-range well-selected infrared and/or microwaves radiators. Mechanical and/or acoustic excitation should cover the lower frequency range of natural, mechanical resonant frequencies of the mass under vibrational treatment. External electromagnetic infrared excitation should have wavelengths that are of the same order as an average-size of internal mass constituents, which are atoms, molecules, and other agglomerated mass-grains. In addition, we should conveniently apply low frequency amplitude and *frequency modulation on the carrier, infrared, or ultrasonic electromagnetic wave/s, to cover the range of dominant natural (or mechanical) frequencies of the macro-mass in question, this way mechanically resonating the same mass. Such effects of vibrational agitation belong to "MMM technology", - see [140], European Patent Application (related to MMM technology); -EP 1 238 715 A1. Multifrequency ultrasonic structural actuator).*

Let us now address ***signal modulations*** from the platform of Uncertainty relations (see (4.2) from Chapter 4.1, (5.3) from Chapter 5., and (10.1) and (10.2) in this chapter). Gabor-Gaussian Wave Packets or "energy-space-time-frequency finite signals" satisfy the following "resonant, spatial-temporal gearing and fitting", or CERTAINTY relations" between total and absolute durations of the same signal or wave function in all its mutually original and spectral domains,

$$\begin{aligned}&\left(\Delta q_{mag.}\cdot\Delta q_{el.}\right)_{min.}=\left(\Delta\alpha\cdot\Delta L\right)_{min.}=h\cdot\left(\Delta t\cdot\Delta f\right)_{min.}=\left(\Delta x\cdot\Delta p\right)_{min.}=\\&=\left(\Delta t\cdot\Delta\tilde{E}\right)_{min.}=c^2\left(\Delta t\cdot\Delta m\right)_{min.}=\left(\Delta s_1\cdot\Delta s_2\right)_{min.}=h/2,\ (\Delta x)_{min.}=\frac{\lambda}{2}.\end{aligned} \quad (5.3)$$

To simplify new idea presentation, let us reduce (5.3) to, (5.3-1),

$$\Delta x\cdot\Delta p=\Delta t\cdot\Delta\tilde{E}=h\cdot\Delta t\cdot\Delta f=c^2\Delta t\cdot\Delta m=\Delta\alpha\cdot\Delta L=\Delta q_{mag.}\cdot\Delta q_{el.}=\text{const}. \quad (5.3\text{-}1)$$

If we multiply this extended relation (on both sides) with certain oscillatory or harmonic, modulating function f(r,t), or f(x,y,z,t), we will again get a new and valid extended relation, such as,

$$\begin{aligned}&\left(\Delta x\cdot\Delta p\right)\cdot f(r,t)=\left(\Delta t\cdot\Delta\tilde{E}\right)\cdot f(r,t)=h\cdot\left(\Delta t\cdot\Delta f\right)\cdot f(r,t)=c^2\left(\Delta t\cdot\Delta m\right)\cdot f(r,t)=\\&=\left(\Delta\alpha\cdot\Delta L\right)\cdot f(r,t)=\left(\Delta q_{mag.}\cdot\Delta q_{el.}\right)\cdot f(r,t)=\text{const}\cdot f(r,t)\end{aligned} \quad (5.3\text{-}2)$$

In fact, to explain the idea of modulation, here is enough to apply $f(r,t)$ only on $\Delta x \text{ or } \Delta p$, and respectively the same way on other mutually conjugate parts in (5.3-2), since all of them are mutually and naturally coupled, and when we modulate one of them, its conjugate couple would also be modulated (to maintain extended Uncertainty or Certainty relations valid). The extended relation (5.3-2) by itself is always valid, obvious, and almost trivial, but the consequences of such triviality are that if we (for instance) modulate only Δx with $\sin(\omega t\pm kr)$, the same modulation will directly and synchronously affect $\Delta\tilde{E},\Delta m,\Delta L$,... as follows,

$$\begin{aligned}&\Delta p\cdot\left[\Delta x\cdot\sin(\omega t\pm kr)\right]=\Delta t\cdot\left[\Delta\tilde{E}\cdot\sin(\omega t\pm kr)\right]=c^2\Delta t\cdot\left[\Delta m\cdot\sin(\omega t\pm kr)\right]=\\&=\Delta\alpha\cdot\left[\Delta L\cdot\sin(\omega t\pm kr)\right]=\text{const}\cdot\left[\sin(\omega t\pm kr)\right],\end{aligned}$$

or similar modulation could also be described with some oscillatory δ intervals, such as.

$$\Delta p \cdot [\Delta x \pm \delta x] = \Delta t \cdot [\Delta\tilde{E} \pm \delta\tilde{E}] = c^2 \Delta t \cdot [\Delta m \pm \delta m] = = \Delta\alpha \cdot [\Delta L \pm \delta L] \qquad (5.3\text{-}3)$$

Of course, within (5.3-3) we have number of modulating and synchronizing combinations (or consequences) between or among all involved delta intervals (Δ, δ), what initiates number of ideas for new scientific and engineering explorations. One of almost obvious and illuminating insights here is that when any of (Δ, δ) intervals is being agitated or modulated, all other (Δ, δ) intervals will be immediately influenced (what is related to matter states coupling, synchronization effects, gravitation related field-force effects, different communications techniques, and in certain cases, maybe faster than speed of light signal propagation). The mentioned external modulation (excitation or manipulation) can be of vibratory, acoustic, electromagnetic, mechanical, or of any other radiant energy nature. This also means that matter waves, moments and energy can be created both with temporal and/or spatial modulations or excitations of different matter states.

Already known phenomena (in relation to similar domain-durations modulation or variations) are electrostrictive and magnetostrictive effects, being very much exploited in engineering practices. Another domain of fantastic (and still very much imaginative and hypothetic) engineering applications could be to exploit certain electroconductive crystals on a way to make them be parts of "*Radiant, cosmic energy streaming*", this way influencing modulating effects (5.3-3), and extracting electric energy from such devices. *Crystal radio receiver* is one of such "*multiple-domains-coupling"* applications (in addition to its diode, AC signal rectifying function).

Our Micro and Macro-Universe are anyway coupled, synchronized, and vibrating, giving an additional power and weight to relations in (5.3-3). If we can detect and conveniently transform such temporal-spatial spectral complexity into acoustic and visual signals, the result would be a certain Cosmic Symphony of our Universe. Here familiar are also unusual and futuristic ideas and theories about physical properties of time, torsional fields, possibility that some signals or measurable effects are traveling faster than light, coming from NIKOLAI KOZYREV and Gennady Shipov (see more in references under [158] and [159]).

Citation from [158]: "Kozyrev's simultaneous time "is a different concept than Einstein's concept of relative time. Kozyrev's experiments have already proven that torsion waves travel at vastly greater speeds than light, perhaps instantaneously, while Einstein claimed the speed of light was the ultimate speed in the universe."

....

From generally valid Uncertainty relations between all important signal domain (or moving particle and its equivalent wave-group) total and absolute, non-statistical durations (see (5.2) in Chapter 5.), we can conclude ***what the nature of time-duration is***, as follows,

$$\begin{aligned}&\left[\Delta q_{mag.}\cdot\Delta q_{el.}=\Delta\alpha\cdot\Delta L=h\cdot\Delta t\cdot\Delta f=\Delta x\cdot\Delta p=\Delta t\cdot\Delta\tilde{E}=c^2\Delta t\cdot\Delta m\geq h/2,\ \Delta E=c^2\Delta m,\ i_{el.}=\frac{\Delta q_{el.}}{\Delta t}\right]\Rightarrow\\&\Rightarrow\left[\Delta E=\frac{\Delta q_{mag.}\cdot\Delta q_{el.}}{\Delta t}=\frac{\Delta\alpha\cdot\Delta L}{\Delta t}=\frac{\Delta x\cdot\Delta p}{\Delta t}=\bar{v}\cdot\Delta p=h\Delta f=c^2\Delta m=P\Delta t=\Delta\tilde{E}\geq\frac{h}{2\Delta t}\right]\cdot\frac{\Delta t}{\Delta E}\Rightarrow\\&\Rightarrow\Delta t=\frac{\Delta q_{mag.}\cdot\Delta q_{el.}}{\Delta E}=\frac{\Delta\alpha\cdot\Delta L}{\Delta E}=\frac{\Delta x\cdot\Delta p}{\Delta E}\geq\frac{h}{2\Delta E}=\frac{h}{2c^2\Delta m}=(\Delta t)_{min.}\,.\end{aligned}\tag{5.2}$$

The concept of spatial-temporal unity, transformability, and coupling of mutually associated, phase-shifted matter waves is profound. However, contemporary physics and natural philosophy need to better address the methods of developing and applying these ideas. In general, when any intervals or delta differences (Δ) in (5.2) or (5.3-3) are disturbed, oscillated, or set in resonance, they generate different yet synchronized matter waves.

Our current understanding of time and space remains overly simplified, often viewed as binary and linear. We perceive time as an evolving state with a past, present, and future, akin to positive and negative values on an infinite scale. The present, or "zero point," exists along this continuum, implying that any point could be seen as the present. Similarly, in spatial terms, we intuitively grasp positive and negative directions in a one-dimensional context, while understanding space as a three-dimensional entity defined by orthogonal (mutually perpendicular) axes. This allows for an infinite number of spatial directions, far beyond a simple linear framework. Since space and time are interconnected, this multidimensional nature must also apply to time. Just as we have three spatial dimensions, it is plausible that time possesses multiple dimensions.

Our current mathematical framework extends our three-dimensional spatial world into four dimensions by introducing time as an orthogonal dimension to space. In Relativity theory and Minkowski space-time, this new temporal dimension is represented by an imaginary axis, orthogonal to the three spatial dimensions. Consequently, the traditional notion of time as only past, present, and future becomes insufficient, as time can now occupy multiple spatial-temporal directions. By extending this concept using hypercomplex numbers (with at least three imaginary units), our understanding of time and its multidimensional complexity becomes much richer, opening infinite, omnidirectional spatial-temporal possibilities, just as we have in the spatial domain.

In classical wave equations, matter waves are often generated in pairs, propagating in opposite spatial-temporal directions. This creates a deeper mathematical and conceptual unity between space and time. While we always experience the present, certain spatial, electromagnetic, or matter wave components exist and interact around us, continuously evolving. These waves cancel out past and future states, leaving only the dynamic present.

Spatial and temporal states are interconnected, allowing us to define positive, zero, and negative directions in both domains. Any spatial or temporal wave function with a finite duration, like a particle, can be decomposed into elementary sinusoidal waves that extend infinitely in both directions. Through Fourier analysis, these waves can combine to form real, finite signals in all domains (temporal, spatial, and spectral). This understanding aligns with the equations in Chapter 4.0, *(4.0.1) - (4.0.5),* which show that a real object or wave packet only materializes after the superposition of its elementary components, covering both spatial and temporal infinities, $x\in(-\infty,+\infty),\ t\in(-\infty,+\infty)$.

If we consider a multidimensional wave function or phasor with a Gaussian amplitude, the concept of positive, zero, and negative spatial-temporal events becomes much more intricate, involving four-dimensional or hypercomplex phasors. In such a framework, the arrow of time becomes a new, multidirectional concept that requires further exploration.

Another perspective on time and space emerges from how we construct spatial and temporal wave functions, wave groups, or signals, representing motional particles. In all cases, Fourier or analytic signal superposition plays a role, combining simple harmonic waves traveling with phase velocity $u=\frac{\partial x}{\partial t}=\frac{\omega}{k}=\frac{\tilde{E}}{\tilde{p}}$ in both positive and negative spatial-temporal directions. However, once these waves combine, the resulting wave group exhibits group velocity $v=\frac{dx}{dt}=\frac{d\omega}{dk}=\frac{d\tilde{E}}{d\tilde{p}}$, and only spatial dimensions retain positive and negative directions. Time, in this case, progresses along the positive temporal axis, from a defined zero-point towards infinity. This situation can be mathematically described when observing motion with constant phase and group velocity (refer to Chapter 4.0 for more details).

$$u=\frac{\partial x}{\partial t}=\frac{\omega}{k}=\frac{\tilde{E}}{\tilde{p}}\Rightarrow\begin{pmatrix}\partial x=u\partial t\\ u\cong const.\end{pmatrix}\Rightarrow x=x_0+ut,\ x\in(-\infty,+\infty),t\in(-\infty,+\infty),\ \omega\in\begin{bmatrix}(-\infty,+\infty)\\ or\\ [0,+\infty)\end{bmatrix}$$

$$v=\frac{dx}{dt}=\frac{d\omega}{dk}=\frac{d\tilde{E}}{d\tilde{p}}\Rightarrow\begin{pmatrix}dx=vdt\\ v\cong const.\end{pmatrix}\Rightarrow x=x_0+vt,\ x\in(-\infty,+\infty),t\in[0,+\infty),\ \omega\in[0,+\infty) \qquad (10.2.1c)$$

$$0\le u\le\frac{v}{2}\le\sqrt{uv}\le v\le c,\ v=u-\lambda\frac{du}{d\lambda}=-\lambda^2\frac{df}{d\lambda},\ x_0=Const.,\ \tilde{E}=hf=E_k,\ v<<<c\Rightarrow v\cong 2u$$

$$\Rightarrow 0\le dx\le udt\le\frac{v}{2}dt\le\sqrt{uv}\cdot dt\le vdt\le cdt.$$

Infinities in Physics

The meaning of infinities, or what is finite or infinite in Physics depends on our theoretical, mathematical, and observational point of view. This effectively depends on which system of reference we are analyzing certain situation. For instance, if we can describe certain process or state of matter with certain function f(x, n) = f[x(n)], n = 1, 2, 3, … we may be able to find $\lim_{n\to\infty} f(x(n))=L$. This could also be related to multidimensional set of functions describing certain more complex situation as, $\lim_{n\to\infty} f_i(x,y,z,t,n)=L_i, i=1,2,3,...$ If we (as an observer) are "submerged inside such multidimensional set of variables", for us some of coordinates or dimensions (x,y,z,t,n) could appear as being unlimited, but in reality, outside of the same dimensional frame, and for another observer, mentioned infinities could be finite.

For instance, meaning of a time for us (within our spatial dimensional matter state f(x,y,z)) appears as something eternal, being unlimited, and/or endlessly spreading towards infinity, but if somebody from a higher dimensional world could observe us, our total time related interval or domain could be limited. We also know that temporal and spatial dimensions in our Universe are mutually related. Consequently, we still do not know if our time-domain is unlimited.

Functional transformations and mapping could also describe such situations where certain coordinate is unlimited in one system of reference or coordinates, and limited in the other, like in Möbius transformations and Riemann Sphere concepts (described with linear coordinate transformations between z and z', where z' = (az + b) / (cz + d)). That means that our understanding of what is limited, or it spreads towards infinity, cannot be concluded, or analyzed based only on different subjective feelings and different philosophical elaborations. We need to use sophisticated mathematics for proper infinity understanding.

If we are not sure what the time is, we can still mathematically operate and make conclusions based on time intervals (or time differences) and on relevant velocities, as something what we can precisely measure.

If we are not sure how to understand a time as one additional and specific dimension of our 4-dimensional Universe (x, y, z, t), we could follow and extend the logic pattern knowing that (x, y, z) are mutually orthogonal, and consequently, the time dimension should also be orthogonal to mentioned spatial coordinates or dimensions, and we know that Hilbert transform is creating such mutually orthogonal functions.

For making valid analysis, modeling, predictions, descriptions, and conclusions in Physics, we need to rely on Mathematical Logic, and on "Natural Mathematics". Natural Mathematics here means mathematics directly or causally developed or deduced based on measurements, experiments and concepts that are already confirmed during longer time, and from different conceptual platforms (meaning this is mathematics universally applicable in all domains of Physics). Orthodox Quantum theory mathematics is largely artificially and unnaturally constructed, and hybridized with grandiose, natural, and self-standing theories like Signal and Spectral Analysis, Statistics, Probability, and Mechanics, to serve certain purpose, mostly in its own, self-defined domain. This is still a questionable, and challenging situation, or unique example in our modern Natural Sciences.

Now we have more real facts to think about how to understand the meaning of time, and this problematic is becoming much more tangible, interesting, and challenging. *At least, we can conclude that only philosophical understanding and explanations of the nature and meaning of time (based only on our intuition, perception, verbal, and intellectual constructions) is not enough and not scientifically acceptable, being inferior compared to relevant mathematical analyses.*

For instance, we know that some humans have sufficiently indicative and correct, predictive, and visionary insights, both related to past and future situations. Here, we could speculate that such "specifically assembled", on some way mentally exceptional members of humankind could behave as very unusual, selective, and sensitive receivers of matter (or electromagnetic) waves that belong to a past or future time-space events, this way manifesting extraordinary and exotic, telepathic, and visionary insights and predictions which are in some cases shown to be sufficiently or surprisingly correct. As we know, Nikola Tesla was one of such people.

Now, we could also mathematically and hypothetically speculate about time-traveling options. For instance, R. Feynman, with his interactions' diagrams successfully conceptualized antimatter states (on a practical, simple, and useful way) as wave groups traveling in a negative, past-time direction (see more here: https://www.britannica.com/science/Feynman-diagram).

Explaining the nature of time is a particularly challenging and ongoing project in modern Physics. Let us analyze circular, closed-line motion in a two-dimensional Descartes space (x, y). In such spatial domain (on a flat surface) we start a motion from a certain point on a closed circular line, and following the same line, after certain time, we will again arrive to the same starting point. This is obvious and easy to understand or conceptualize for a self-closed spatial domain. Since we know that temporal and spatial domains are mutually connected and directly proportional, we can analogically create certain "time-traveling circular line", and start moving along this line in one direction, passing through *present, past, and future time zones*, and eventually we will arrive to the same present-time-related starting point. Let us mathematically exercise with such imaginative and exciting options. The equation of a closed circular line in a (x, y) spatial plane is producing the following relations,

$$\left(x^2+y^2=R^2=const.\right)\Rightarrow\begin{pmatrix}xdx+ydy=0\\ x\Delta x+y\Delta y=0\end{pmatrix}\Rightarrow\begin{pmatrix}x\dfrac{dx}{dt}+y\dfrac{dy}{dt}=0\\ x\dfrac{\Delta x}{\Delta t}+y\dfrac{\Delta y}{\Delta t}=0\\ x\cdot v_x+y\cdot v_y=0\\ v^2=v_x^2+v_y^2\end{pmatrix}$$

If we now create a fully analogical, closed "*circular timeline*", since spatial and temporal domains are directly proportional, we will get,

$$\begin{pmatrix}t_x^2+t_y^2=T^2=const.\\ x=ct_x, y=ct_y, c=const\\ \Delta x=c\Delta t_x, \Delta y=c\Delta t_y\end{pmatrix}\Rightarrow\begin{pmatrix}t_x dt_x+t_y dt_y=0\\ t_x\Delta t_x+t_y\Delta t_y=0\end{pmatrix}\Rightarrow\begin{pmatrix}t_x dx+t_y dy=0\\ t_x\Delta x+t_y\Delta y=0\\ t_x\cdot v_x+t_y\cdot v_y=0\\ v^2=v_x^2+v_y^2\end{pmatrix}\Rightarrow\begin{bmatrix}t_x^2+t_y^2=\left(\dfrac{R}{c}\right)^2\\ R=cT\end{bmatrix}$$

We see here that time, like a space (x, y), should have associated time components (t_x, t_y). Since we also know that our temporal and spatial domains are mutually orthogonal (existing synchronously), we could place a time domain on an imaginary axis of the united space-time domain, as very successfully practiced in the Minkowski-Einstein 4-vectors and Relativity theory mathematics.

$$x=Ict_x, y=Ict_y, R=IcT, \Delta x=Ic\Delta t_x, \Delta y=Ic\Delta t_y, I^2=-1.$$

Analogical time-space relations and conclusions can be now extended and summarized for the four-dimensional, united spatial and temporal domain, (x, y, z), (t_x, t_y, t_z), as follows,

$$\left(x^2+y^2+z^2=R^2=const.\right)\Rightarrow\begin{pmatrix}xdx+ydy+zdz=0\\ x\Delta x+y\Delta y+z\Delta z=0\end{pmatrix}\Rightarrow\begin{pmatrix}x\cdot v_x+y\cdot v_y+z\cdot v_z=0\\ v^2=v_x^2+v_y^2+v_z^2\end{pmatrix}$$

$$\begin{pmatrix}t_x^2+t_y^2+t_z^2=T^2=const.\\ x=ct_x, y=ct_y, z=ct_z, c=const\\ \Delta x=c\Delta t_x, \Delta y=c\Delta t_y, \Delta z=c\Delta t_z\end{pmatrix}\Rightarrow\begin{pmatrix}t_x dt_x+t_y dt_y+t_z dt_z=0\\ t_x\Delta t_x+t_y\Delta t_y+t_z\Delta t_z=0\end{pmatrix}\Rightarrow$$

$$\Rightarrow\begin{pmatrix}t_x\cdot v_x+t_y\cdot v_y+t_z\cdot v_z=0\\ v^2=v_x^2+v_y^2+v_z^2\end{pmatrix}\Rightarrow\begin{bmatrix}t_x^2+t_y^2+t_z^2=\left(\frac{R}{c}\right)^2\\ R=cT\end{bmatrix}$$

$$x=ict_x, y=jct_y, z=kct_z, R=IcT, \Delta x=ic\Delta t_x, \Delta y=jc\Delta t_y, \Delta z=kc\Delta t_z, I^2=i^2=j^2=k^2=ijk=-1, ij=k, jk=i, ki=j\ ...$$

Of course, here, instead of using an ordinary and simple complex analytic signal function with one imaginary unit, we exploit a Hypercomplex analytic signal function (or quaternions) model with three mutually orthogonal complex units i, j and k, to present a Hypercomplex Minkowski space with three temporal components. This way, we open a window towards new understanding of Time, Symmetry, and higher dimensional spaces in Physics.

As we can see, thanks to Hypercomplex numbers or functions modelling, here we gradually introduce and explain the concept that, if our World has three spatial dimensions (x, y, z), and if spatial and temporal dimensions are mutually related, then the same World should also have three temporal dimensions (t_x, t_y, t_z), three phase velocities (u_x, u_y, u_z), three group velocities (v_x, v_y, v_z), and three maximal speed limits (c_x, c_y, c_z) analogue to maximal velocity **c** of electromagnetic waves.
Of course, such concepts (about temporal, spatial and by velocity characterizations of motions) could be elaborated much better, and in different ways established. Consequently, our four-dimensional "Space-Time" World could evolve to six-dimensionality, also being on some way anisotropic, because some of velocity components of maximal speed of matter waves and "energy-moments" states, (c_x, c_y, c_z), within the same World could be both higher and/or lower compared to maximal speed **c** (from the Relativity Theory), as follows,

$$x=ic_x t_x, y=jc_y t_y, z=kc_z t_z, R=IcT,\ I^2=i^2=j^2=k^2=ijk=-1, ij=k, jk=i, ki=j\ ...$$

$$t_x^2+t_y^2+t_z^2=\left(\frac{R}{c}\right)^2, R=cT,\ v^2=v_x^2+v_y^2+v_z^2,\ u^2=u_x^2+u_y^2+u_z^2,\ c^2=c_x^2+c_y^2+c_z^2,$$

$$t_x\cdot v_x+t_y\cdot v_y+t_z\cdot v_z=0,\ t_x^2+t_y^2+t_z^2=\left(\frac{R}{c}\right)^2,\ v=u-\lambda\frac{du}{d\lambda}...$$

Nikola Tesla said that maximal speed of electromagnetic waves (in some cases) can be much higher than what, in his time and still, consider as c. Since we know that in homogenous and isotropic media c depends on relevant dielectric and magnetic constants as, $c=1/\sqrt{\varepsilon\mu}$, or in a vacuum state $c=1/\sqrt{\varepsilon_0\mu_0}=c_0$, that means in some specific, unusual, and anisotropic media, dielectric and magnetic constants could also be characterized as $(\varepsilon_x,\mu_x),(\varepsilon_y,\mu_y),(\varepsilon_z,\mu_z)$, what will generate (c_x, c_y, c_z) with specific maximal values different from c.

*Here we arrived close to an exciting and challenging possibility to analyze what **space-time travelling** means in any positive, negative, or differently specified direction (what will remain as an open and ongoing project)*. Since we know that matter waves and any other spatial-temporal signal shape can be decomposed on elementary sinusoidal components, such sinusoidal components spread or penetrate towards infinity, both in any positive and negative direction. That means what we consider past ***or future time*** is intrinsically and coincidently (or synchronously) included in Fourier and Analytic signals decomposition (where both-side infinite durations, sinusoidal functions are involved). If certain devices, or somebody of us, or other living species, can selectively extract and detect mentioned elementary, simple harmonic matter wave components, this could (combined with convenient signal processing and an artificial intelligence software) create ***visionary penetrations in the past and/or future temporal realities***. Nikola Tesla said, and one of his technical assistants confirmed later, that he experimented with familiar, bidirectional interplanetary communicating devices; -see more in [160]. Nikolai Kozyrev with his theory of time, [158], also speculated with similar ideas. Better to stop here by evoking related imaginative options until such kind of thinking, analyses and technologies would naturally evolve and mature. See more of similar challenging and hypothetical speculations and analyzes at the end of this chapter under: "10.1 Hypercomplex Analytic Signal functions and interpretation of energy-momentum 4-vectors in relation to matter-waves and particle-wave duality".

Another, almost like science fiction prediction, is that periodicity-characterized spatial structures like crystals and fractals, can analogically exist, or be created in a time domain (since time and space domains, and their relevant spectral domains are mutually strongly related and causally linked; -see more in **[129]**). We could metaphorically say that music signals are oscillations in a time domain, and crystals are different forms of music produced in a spatial domain. Consequently, certain crystals, most probably, can be used as detectors of coupled mechanical, gravitational and/or electromagnetic, or Radiant energy waves (not only based on an old fashion amplitude demodulation method using a diode detector).

*Citation (from [129], **Frank Wilczek,** "Crystals in Time" in Scientific American 321, 5, 28-36 (November 2019), doi: 10.1038/scientificamerican1119-28, View This Issue*

- ***"Crystals are orderly states of matter in which** the arrangements of atoms take on repeating patterns. In the language of physics, they are said to have "spontaneously broken spatial symmetry."*
- ***Time crystals, a newer concept, are states of matter** whose patterns repeat at set intervals of time rather than space. They are systems in which time symmetry is spontaneously broken.*
- ***The notion of time crystals was first proposed in 2012**, and in 2017 scientists discovered the first new materials that fully fit this category. These and others that followed offer promise for the creation of clocks more accurate than ever before. …….*

The next step in our understanding of crystals is occurring now, thanks to a principle that arose from Albert Einstein's relativity theory: space and time are intimately connected and ultimately on the same footing. Thus, it is natural to wonder whether any objects display properties in time that are analogous to the properties of ordinary crystals in space. In exploring that question, we discovered "time crystals." This new concept, along with the growing class of novel materials that fit within it, has led to exciting insights about physics, as well as the potential for novel applications, including clocks more accurate than any that exist now". (End of citation)

Now we could better understand somewhat hidden and unusual theoretical grounds of Quantum Physics, Richard Feynman diagrams, anti-matter states and imaginative P. Dirac fluctuations of elementary and virtual particles in any vacuum state.

Citation from: https://en.wikipedia.org/wiki/Dirac_sea "The **Dirac sea** is a theoretical model of the vacuum as an infinite sea of particles with negative energy. It was first postulated by the British physicist Paul Dirac in 1930[1] to explain the anomalous negative-energy quantum states predicted by the Dirac equation for relativistic electrons (electrons traveling near the speed of light).[2] The positron, the antimatter counterpart of the electron, was originally conceived of as a hole in the Dirac sea, before its experimental discovery in 1932.[nb 1]

In whole theory, the solutions with negative time evolution factors[*clarification needed*] are reinterpreted as representing the positron, discovered by Carl Anderson. The interpretation of this result requires a Dirac sea, showing that the Dirac equation is not merely a combination of special relativity and quantum mechanics, but it also implies that the number of particles cannot be conserved.[3] Dirac sea theory has been displaced by quantum field theory, though they are mathematically compatible".

P. Dirac creatively and imaginatively hypothesized that micro-world, elementary particles and different matter-wave entities originating from an infinite sea of particles could be found on many different places at the same time (of course, with certain probability), as spontaneously *and randomly appearing and disappearing events, and R. Feynman, also imaginatively and very successfully, presented such situations including interactions among elementary particles and different matter-waves with his diagrams.*

Citation from: https://en.wikipedia.org/wiki/Feynman_diagram "In theoretical physics, **Feynman diagrams** are pictorial representations of the mathematical expressions describing the behavior of subatomic particles. The scheme is named after its inventor, American physicist Richard Feynman, and was first introduced in 1948. The interaction of sub-atomic particles can be complex and difficult to understand intuitively. Feynman diagrams give a simple visualization of what would otherwise be an arcane and abstract formula. As David Kaiser writes, "since the middle of the 20th century, theoretical physicists have increasingly turned to this tool to help them undertake critical calculations", and so "Feynman diagrams have revolutionized nearly every aspect of theoretical physics".[1] While the diagrams are applied primarily to quantum field theory, they can also be used in other fields, such as solid-state theory".

Something like that (very much virtual, arbitrary, exotic, and imaginative, but anyway productive), what Dirac and Feynman created, is much better explicable mostly thanks to fields-coupled, resonant and entanglement, self-synchronization effects between mutually opposed (inward and outward propagating) waves, when modeled using Complex Analytic Signal Phasors. Coupled waves or wave groups (created in pairs) are solutions of all relevant Complex and Ordinary, second order, Classical Wave equations (including Schrödinger equation). In addition, we know that even an empty space or an idealized total vacuum state are still presenting certain waves-carrier-medium with measurable electromagnetic properties (analogically behaving like an ideal fluid, being familiar to old ideas about ether). If the "old ether" is correctly confirmed as being non-existent, as conceptualized in the original version of Relativity theory (what even A. Einstein, much later renounced), it is still necessary to have in mind that certain more imaginative and maybe fluidic matter, or material background state with electromagnetic properties should exist (in a state of an ideal vacuum), being the carrier of electromagnetic and other matter-waves. It is very instructive and good to know that Maxwell-Faraday Electromagnetic theory is analogically, essentially, and intuitively developed by having in mind real-world fluid-dynamics (see more in Chapter 3. of this book and in publications from Tsutomu Kambe [123]). Waves without a carrier fluid or certain carrier matter state are simply impossible.

10.01 CERTAINTY AND UNCERTANTY RELATIONS IN PHYSICS

Uncertainty Relations and Their Applications

Uncertainty relations, often referred to as the quantum uncertainty principle, are fundamental concepts rooted in standard Signals and Spectral Analysis, and by extension, universally valid mathematics. In essence, these relations describe the mathematical connections between the temporal, spatial, and spectral durations of the same signal (or wave packet) across different, mutually conjugated domains (original and spectral). While quantum theory has adapted these mathematical uncertainty relations, the core principles remain grounded in well-established mathematics (see Chapter 5 for further discussion).

Uncertainty relations manifest in various forms within mathematics and physics, all of which are interconnected and essentially equivalent. They relate to concepts such as optimal signal sampling and reconstruction (as in the Shannon-Kotelnikov-Nyquist-Whittaker theory) or are "on average" connected with statistical definitions of domain durations, as in contemporary quantum theory. Furthermore, these relations can be linked to finite differences and error analysis, helping us define and understand minimal sizes or atomized and discretized domains-durations in physics, concepts we refer to as Certainty Relations (see Chapter 5 for more details).

Uncertainty Relations in the Macro World

The challenges in understanding Uncertainty Relations at the macro scale (as well as at other scales) arise from our interpretation of size, length, spatial boundaries, geometry, and temporal durations of real masses and matter waves. The spatial size of a “mass-energy-momentum” matter-wave that represents a stable or solid mass is often significantly larger than the perceptual and directly measurable spatial or geometric boundaries of the same mass. This discrepancy arises because the group and phase velocities of any matter-wave are different. Consequently, the wave-like spread of a moving particle is much broader and fuzzier than the solid, finite-boundary particle itself, naturally synchronized with surrounding waves, particles, and masses.

This wave-like nature of matter explains why Uncertainty Relations apply equally to both the micro and macro realms of physics. We apply these relations to relevant matter-waves, not directly to solid particles, where spatial or geometric boundaries are more sharply defined within a smaller spatial zone. For wave-like formations or wave packets, it is natural to comply with universally valid mathematical uncertainty relations, regardless of the sizes or dimensions involved.

Common Misinterpretations and the Macro-Cosmological Perspective

One common mistake is the mechanical and geometrical interpretation of Uncertainty Relations from the macro world of solid particles (with precisely defined spatial dimensions) to the realm of matter-wave formations. Many naïve interpretations of the uncertainty behaviors of our world stem from this initial misinterpretation, as the spatial boundaries of solid bodies are usually smaller than the matter-wave and electromagnetic boundaries of the same body.

It's crucial to understand that for micro and subatomic entities, we always use Planck’s constant, h. However, for analogous situations in the macro world, such as planetary and galactic systems, which also form self-closed, standing matter-wave formations, we must consider much larger Planck-like constants, H (see Chapter 2, Section 2.3.3, "Macro-Cosmical Matter-Waves and Gravitation"). This perspective underscores that the continuous-symmetries and conservation laws of physics apply universally to both micro and macro particles, real masses, and matter waves. Our conceptual and mathematical approaches must be adjusted accordingly.

Moreover, all natural or electromechanical systems and motions belong to closed and multidimensional networks with defined sources and loads, much like electric circuits (see Chapter 1 for more details). The probabilistic framework of contemporary quantum theory is merely a convenient method of mathematical modeling that accounts for the conservation laws of the universe and the complexity of wave-particle duality by averaging values and statistical properties. We should avoid ontological mystifications in relation to probability and statistics as currently practiced in quantum theory. This understanding of Uncertainty Relations aligns more closely with the theoretical concepts of Nikola Tesla and Rudjer Boskovic regarding the structure and forces of our universe (see references [97] and [117]).

Connecting Uncertainty Relations with Signal Analysis Theory

The framework of realistic quantized energy exchanges and particle formation in physics is closely related to signal analysis and synthesis, as described by the

Kotelnikov-Shannon-Nyquist-Whittaker concepts of signal discretization and information recovery. Additionally, Parseval's identity, which connects the time and frequency domains of a wave function or signal, is equivalent to energy conservation in physics and is always valid.

We also know that spectral (or frequency) analysis of signals is causally related to various physical and mechanical effects, such as motion energy, power, and momentum, which manifest as superposition, interference, refraction, reflection, and scattering of wave functions or wave groups. These brilliant mathematical concepts are universally valid and are currently employed in various communication and energy transfer technologies, including the operation of electric circuits in electrotechnology, where complex voltage and current functions, or phasors, are used.

To integrate these concepts into particle-wave duality and quantum theory, we could consider a signal or wave packet with a total time-domain duration T and total frequency spectrum duration F. We could approximate that 99% or 99.99% of the signal energy is contained within these intervals and apply the Kotelnikov-Shannon-Nyquist-Whittaker theory to discretize or quantize the signal in both its time and frequency domains under appropriate conditions, as follows,

$$\begin{aligned} &\left\{TF \geq \frac{1}{2},\ (\Delta t)_{max.} = \Delta t \leq \frac{1}{2F},\ (\Delta f)_{max.} = \Delta f \leq \frac{1}{2T},\ F = F_t\right\} \Rightarrow \\ &\left\{0 < \Delta t \cdot \Delta f \leq \frac{1}{2} \leq TF \leq \frac{1}{4\Delta t \cdot \Delta f}\right\} \Rightarrow \left(\Delta t \cdot \Delta f\right)_{min.} = TF_t = \frac{1}{2}. \end{aligned} \tag{10.2-2}$$

Here, Δt and Δf are time and frequency sampling intervals necessary to make total signal recovery after operating with Kotelnikov-Shannon-Nyquist-Whittaker discretization in both domains (see more in [57, 58, and 59]). Consequently, we could say that proper (elementary) quantum of certain arbitrary and energy finite signal (effectively being like Gabor-Gaussian Wave Packet) are signal elements or signal elementary wave functions with Δt and Δf, as time and frequency sampling durations. Based on Kotelnikov-Shannon-Nyquist-Whittaker theory we can also formulate the family of analytic, elementary wave functions of such quantized signals, considering such signals as wave groups with group and phase velocity mutual relations as $v = u - \lambda \frac{du}{d\lambda} = \frac{d\tilde{E}}{dp},\ u = \lambda f = \frac{\tilde{E}}{p} ...$ (see more in Chapter 5.). When we rich absolute limits such as, $\left(\Delta t \cdot \Delta f\right)_{min.} = TF_t = \frac{1}{2}$, we are addressing elementary particles, or relatively stable elementary matter domains, masses, and other stable or standing matter wave groups (or building blocks of our Universe); -this way practically explaining the real nature or meaning of Uncertainty Relations and Quantization in Physics.

Since spatial and temporal matter states and motions (and their relevant spectral domains) are always mutually coupled and dependent, we can analogically imagine that $\mathbf{L}$ and $\mathbf{F_x}$ are the total spatial duration and total spatial frequency interval of the same matter state, and Δx, Δt and Δp are its minimal sampling and signal reconstructing lengths and momentum, and we can extend (10.2-2) to,

$$(\Delta t\cdot\Delta f)_{min.}=TF_t=\frac{1}{2}\Rightarrow$$

$$\begin{bmatrix} h(\Delta t\cdot\Delta f)_{min.}=\frac{h}{2\pi}(\Delta t\cdot\Delta\omega)_{min.}=(\Delta t\cdot\Delta E)_{min.}=\boxed{(\Delta x\cdot\Delta p)_{min.}}= \\ =\frac{h}{2\pi}(\Delta x\cdot\Delta k)_{min.}=hTF_t=\boxed{hLF_x}=\frac{h}{2} \end{bmatrix}_{\Delta\to\, d\to 0}\Rightarrow \tag{10.2-2.1}$$

$$\Rightarrow\left[h(dt\cdot df)=\frac{h}{2\pi}(dt\cdot d\omega)=(dt\cdot dE)=\boxed{(dx\cdot dp)}=\frac{h}{2\pi}(dx\cdot dk)\right]/\,dt=$$

$$=\left[h\cdot df=\frac{h}{2\pi}\cdot d\omega=dE=d\tilde{E}=v\cdot dp=dE_k=\frac{h}{2\pi}v\cdot dk=c^2dm\right]$$

To be more general, we could safely consider that stabilized matter states (like stable particles and atoms) should qualitatively and conceptually present matter waveforms or wave-packets with certain (total) motional or wave energy content equal to $\tilde{E}=E_k$, with certain resulting momentum equal to $\tilde{P}=P>>>\Delta p$, with the total spatial length $L>>>\Delta x$, with the total temporal duration $T>>>\Delta t$, and be presentable with the following domains, and "Nyquist-Shannon-Kotelnikov" sampling Δ-intervals relations,

$$h(\Delta t\cdot\Delta f)_{min.}=(\Delta t\cdot\Delta E)_{min.}=\boxed{(\Delta x\cdot\Delta p)_{min.}}=hTF_t=\boxed{hLF_x}=\boxed{T\cdot\tilde{E}=L\cdot\tilde{P}}=\text{constant}\Rightarrow$$

$$\begin{cases}(\Delta t\cdot\Delta f)_{min.}=TF_t=LF_x=\overline{\text{constant}},\, f=f_t=\omega/2\pi,\rightarrow\boxed{L=\frac{F_t}{F_x}\cdot T\cong V^*\cdot T,\ V^*=\text{const.}}\\ (\Delta t\cdot\Delta E)_{min.}=(\Delta x\cdot\Delta p)_{min.}=\underline{\text{constant}},\end{cases}$$

$$\begin{cases}(\Delta E)_{min.}=\left(\frac{\Delta x}{\Delta t}\cdot\Delta p\right)_{min.}=\overline{v}\cdot\Delta p\Leftrightarrow dE=v\cdot dp,\overline{v}=\frac{\Delta x}{\Delta t}=\frac{\Delta E}{\Delta p}=\frac{\Delta\omega}{\Delta k}\Leftrightarrow v=\frac{dx}{dt}=\frac{dE}{dp}=\frac{d\omega}{dk}\\ (\Delta E)_{min.}=\left(\frac{\Delta x}{\Delta t}\cdot\Delta p\right)_{min.}=(\Delta x\cdot\Delta p)_{min.}\cdot\frac{1}{\Delta t}(=)h\cdot f\Rightarrow dE=hdf=vdp,\\ \left(\frac{\Delta E}{\Delta t}\right)_{min.}=\left(\frac{\Delta x}{\Delta t}\cdot\frac{\Delta p}{\Delta t}\right)_{min.}=\overline{v}\cdot\frac{\Delta p}{\Delta t}\Leftrightarrow\frac{dE}{dt}=v\cdot\frac{dp}{dt}=v\cdot\mathbf{F}(=)\text{Power, where }\mathbf{F}(=)\text{force,}\\ \mathbf{F}=\frac{\Delta E}{\Delta x}=\frac{\Delta p}{\Delta t}=\nabla E=\dot{p}=m^*\dot{v}\ \begin{pmatrix}\text{analog to torque}\\ \Leftrightarrow\end{pmatrix}\ \tau=\frac{\Delta E}{\Delta\theta}=\frac{dL}{dt}=\dot{L}=J\dot{\omega}\end{cases} \tag{10.2-2.2}$$

Respecting mobility-type electromechanical analogies (see more in Chapter 1.), velocity v corresponds to certain voltage $u=\frac{\Delta\Phi}{\Delta t}$, and force $\mathbf{F}=\frac{\Delta p}{\Delta t}$ (or torque $\tau=\frac{\Delta E}{\Delta\theta}=\frac{dL}{dt}$) corresponds to certain electric current $i=\frac{\Delta Q}{\Delta t}$, and we could qualitatively, dimensionally (and when reasonable, or as a brainstorming and indicative exercise), extend (10.2.2) to,

$$\text{Power }(=)\ \frac{\Delta E}{\Delta t}=\overline{v}\cdot\frac{\Delta p}{\Delta t}\Leftrightarrow\frac{dE}{dt}=v\cdot\frac{dp}{dt}=v\cdot\mathbf{F}=\omega\cdot\tau(=)u\cdot i=\frac{\Delta\Phi}{\Delta t}\cdot\frac{\Delta Q}{\Delta t}\Rightarrow$$
$$\Rightarrow\Delta E=v\cdot\Delta p=\omega\cdot\Delta J=\frac{\Delta\Phi\cdot\Delta Q}{\Delta t}, \tag{10.2-2.3}$$

where Φ and Q are relevant magnetic flux and involved electric charge (when applicable).

It is important to note that only Gaussian (or similar bell curve) amplitude-shaped matter-wave packets, or energy-momentum states, are well-localized in both their original and spectral domains. Since temporal, spatial, and relevant spectral domains of all motional states and wave functions are interconnected and proportional, we can reasonably discuss temporal and spatial periodicity, as well as temporal and spatial resonant states.

One particularly significant resonant state is the combined temporal-spatial resonant and standing wave condition, where mutually synchronous temporal and spatial resonances occur simultaneously. Such standing-wave states can create stable particles with non-zero rest masses when they form self-closed structures. If the standing matter-wave structure takes the form of open-ended waves (i.e., not self-closed), these formations become emitters of matter waves.

In standing waves, nodal and antinodal zones locally manifest effects of attractive and repulsive forces. This occurs because gradients of vibrational energy and mass density are created toward the nodal and/or antinodal zones, potentially offering an alternative explanation for the natural forces we observe in physics.

If any of the delta intervals, or total duration values from references (5.2), (10.2-2) - (10.2-2.4), begin to exhibit oscillatory modulation, perturbation, or vibration, all other delta intervals (or signal domain durations) will immediately and synchronously generate matter waves, vibrate, and resonate. Consequently, new vibrating spots may form based on the object's total spectral complexity, both temporal and spatial. For instance,

$$\left\{\begin{array}{l}(\Delta x \text{ or } \Delta t)\Rightarrow\left[(\Delta x\pm\delta x)\text{ or }(\Delta t\pm\delta t)\right]\\ h(\Delta t\cdot\Delta f)_{min.}=(\Delta t\cdot\Delta E)_{min.}=\boxed{(\Delta x\cdot\Delta p)_{min.}}=hTF_t=\boxed{hLF_x}=\boxed{T\cdot\tilde{E}=L\cdot\tilde{P}}=const\end{array}\right\}\Rightarrow$$

$$\Rightarrow\left\{\begin{array}{l}h\left[(\Delta t\pm\delta t)\cdot(\Delta f\mp\delta f)\right]_{min.}=\left[(\Delta t\pm\delta t)\cdot(\Delta E\mp\delta E)\right]_{min.}=\\ =\boxed{\left[(\Delta x\pm\delta x)\cdot(\Delta p\mp\delta p)\right]_{min.}}=h(T\pm\delta T)\cdot(F_t\mp\delta F)=\boxed{h(L\pm\delta L)\cdot(F_x\mp\delta F_x)}=\\ =\boxed{(T\pm\delta T)\cdot(\tilde{E}\mp\delta E)=(L\pm\delta L)\cdot(\tilde{P}\mp\delta\tilde{P})}=const\end{array}\right\}\Rightarrow \quad (10.2\text{-}2.4)$$

$$\Rightarrow h(\delta t\cdot\delta f)_{min.}=(\delta t\cdot\delta E)_{min.}=\boxed{(\delta x\cdot\delta p)_{min.}}=hTF_t=\boxed{hLF_x}=\boxed{T\cdot\tilde{E}=L\cdot\tilde{P}}=const$$

We can also extend in a similar way the same Uncertainty or Certainty perturbative and oscillatory relations, considering total (non-statistical, or absolute) associated electromagnetic charges and angular moments and displacements, as already presented in Chapter 5., under (5.2) and later,

$$\Delta q_{mag.}\cdot\Delta q_{el.}=\Delta\alpha\cdot\Delta L=h\cdot\Delta t\cdot\Delta f=\Delta x\cdot\Delta p=\Delta t\cdot\Delta\tilde{E}=c^2\Delta t\cdot\Delta m\geq h/2. \quad (5.2)$$

In engineering practice, we have such effects of significant mutual and bidirectional energy transformations in relation to piezoelectric and magnetostrictive effects. Oscillations, *perturbations,* or absolute durations in any of domains from (10.2-2.4) and (5.2) should also produce EMP (electromagnetic pulse) effects, like in cases of nuclear explosions.

Citation taken from: https://www.businessinsider.fr/us/nukes-electromagnetic-pulse-electronics-2017-5 “Nuclear blasts trigger an effect called electromagnetic pulse, or EMP. EMP can disrupt or even destroy electronics from miles away. Blasts miles above a country like the US might severely damage its electric and telecommunications infrastructure.
A nuclear detonation creates plenty of terrifying effects, including a blinding (and burning) flash of light, a building-toppling blast wave, an incendiary fireball, and radioactive fallout that can drift for hundreds of miles.
But there's a lesser-known consequence of a nuclear explosion that can drastically expand its damage zone: an electromagnetic pulse, or EMP. EMPs are rapid, invisible bursts of electromagnetic energy. They occur in nature, most frequently during lightning strikes, and can disrupt or destroy nearby electronics. However, nuclear EMPs — if a detonation is large enough and high enough — can cover an entire continent and cripple tiny circuits inside modern electronics on a massive scale, according to US government reports. The power grid, phone and internet lines, and other infrastructure that uses metal may also be prone to the effects, which resemble those of a devastating geomagnetic storm “.

[♣ By analyzing based on references (10.2-2.4) and (5.2), we can explore how small spatial, mechanical, electromechanical, and electromagnetic vibrations within certain signal-domain durations can amplify oscillations across other interconnected and conjugate domains. In some cases of matter states, this can lead to significant displacements, forces, and mechanical or electromagnetic moments, ultimately generating different matter waves, like phenomena observed in electrostriction and magnetostriction.

This process can be compared to mechanical systems like a lever or hydraulic press, which function as force or amplitude amplifiers. For instance, specific spatial configurations of macro elements composed of large masses with a polycrystalline structure, when suitably vibrated or excited—whether mechanically, electromechanically, by light beams, cosmic rays, thermal and seismic vibrations, electromagnetic discharges, or other planetary oscillations—can produce secondary matter-wave radiation with measurable mechanical and electromagnetic effects.

*Russian professor Viktor Stepanovich Grebenikov described several phenomena akin to gravitation, antigravitation, and temporal-spatial matter-wave interactions in geometrically periodic structures, such as natural beehive cells, dry honeycomb structures, or similarly assembled objects. Other examples include coincidental, continuous mechanical, ultrasonic, and electromagnetic waves with geometry-specific frequencies, observed around large mountains and pyramids. For more information, see [141], *Electromagnetic Properties of Pyramids from Positions of Photonics*, and [142], Hrvoje Zujić’s work on the electromagnetic mechanism of ultrasound on the Bosnian Pyramid of the Sun (Visoćica Hill).*

In other words, natural emissions of radiative electromagnetic, mechanical, and acoustic fields from atoms and other masses, as discussed in Chapter 8, and the excitation of specific geometric, crystalline, fractal, or spatial matter forms with structural spatial periodicity (acting as resonators), can produce or amplify dynamic effects of matter-wave radiation. This process can result in the creation of active mechanical and electromagnetic oscillations, wave moments, ionized atoms, and oscillating field charges, and vice versa.

Similarly, excitation or vibration of solid matter structures, including planets and galaxies, in the temporal domain can produce corresponding energy-mass-frequency effects or emissions of different matter waves, and vice versa. Schumann resonances, along with related mechanical, seismic, acoustic, and electromagnetic oscillations, provide good examples of such phenomena. ♣]

*Citation from Wikipedia, https://en.wikipedia.org/wiki/Schumann_resonances. “The **Schumann resonances (SR)** are a set of spectrum peaks in the extremely low frequency (ELF) portion of the Earth's electromagnetic field spectrum. Schumann resonances are global electromagnetic resonances, generated and excited by lightning discharges in the cavity formed by the Earth's surface and the ionosphere.[1] “*

$\begin{pmatrix} \Delta t \cdot \Delta f = \Delta x \cdot \Delta p \geq 1/2 \\ \Rightarrow \frac{\Delta x}{\Delta t} = \frac{\Delta f}{\Delta p} \geq \frac{1}{2\Delta p \cdot \Delta t} \end{pmatrix} \Rightarrow \begin{pmatrix} \Delta t \to 0, \Delta f \to \infty \\ \Delta x \to 0, \Delta p \to \infty \\ \Delta f \to 0, \Delta t \to \infty \\ \Delta p \to 0, \Delta x \to \infty \end{pmatrix} \Rightarrow$	*Relations between durations of mutually conjugated domains, as follows from (5.2), (10.2-2)-(10.2-4), (5.3)-(5.3-3) are part of the explanation why big spatial objects (as high Rocky Mountains and pyramids) can manifest associated electromagnetic and acoustic, oscillatory and resonance effects.*

*Here are the reasons and explanations why we detect such matter-waves effects around and inside of different spatial forms of micro and macro masses, and other structural resonators, such as spatial crystalline forms, pyramids, high and big stones, and mountains ... Enormously big crystalline-matter forms or objects like rocky mountains, pyramids, obelisks and sarcophaguses are creating, both internally and externally, specific standing matter-waves field-structures (of electromagnetic and mechanical or acoustic nature), and when living species are placed inside (or around) such objects, we could expect unusual healing, well-being and rejuvenation effects (or effects of natural synchronization between small objects with internal disorders and big objects with stabilized structural order). One good example of such mutually coupled spatial-temporal, electromagnetic and acoustic domains-durations, perturbations, and matter waves emissions is also manifesting as the old-fashioned **Crystal Radio Receiver** or detector.*

See more about Crystal Radio Receivers here https://en.wikipedia.org/wiki/Crystal_radio. Citation from Wikipedia.

………………….. Citation: "A **crystal radio receiver**, also called a **crystal set**, is a simple radio receiver, popular in the early days of radio. It uses only the power of the received radio signal to produce sound, needing no external power. It is named for its most important component, a crystal detector, originally made from a piece of crystalline mineral such as galena.[1] This component is now called a diode.

…………..

Crystal radios receive amplitude modulated (AM) signals, and can be designed to receive almost any radio frequency band, but most receive the AM broadcast band.[20] A few receive shortwave bands, but strong signals are required. The first crystal sets received wireless telegraphy signals broadcast by spark-gap transmitters at frequencies as low as 20 kHz.[21][22]

As a lesser known feat, a different design of crystal radios can also been constructed specifically for receiving frequency modulated (FM) VHF broadcast signals.[23]

……………………………

Crystodyne:

In early 1920s Russia, Oleg Losev was experimenting with applying voltage biases to various kinds of crystals for manufacture of radio detectors. The result was astonishing: with a zincite (zinc oxide) crystal he gained amplification.[33][34][35] This was negative resistance phenomenon, decades before the development of the tunnel diode. After the first experiments, Losev built regenerative and superheterodyne receivers, and even transmitters. A crystodyne could be produced in primitive conditions; it can be made in a rural forge, unlike vacuum tubes and modern semiconductor devices. However, this discovery was not supported by authorities and soon forgotten; no device was produced in mass quantity beyond a few examples for research.

………………….

The earliest crystal receivers did not have a tuned circuit at all, and just consisted of a crystal detector connected between the antenna and ground, with an earphone across it.[1][68] Since this circuit lacked any frequency-selective elements besides the broad resonance of the antenna, it had little ability to reject unwanted stations, so all stations within a wide band of frequencies were heard in the earphone[50] (in practice the most powerful usually drowns out the others). It was used in the earliest days of radio, when only one or two stations were within a crystal set's limited range". End of citation.

……………………..

[♣ Free thinking, brainstorming zone.

To explore the concept of "thinking outside the box" (where the "box" represents our Universe), we might imagine our Universe as a vast electromechanical system or machine, automatically controlled by a powerful computer running advanced Artificial Intelligence software. This hypothetical software operates according to a complex algorithm, which we can partially understand through natural laws of conservation, analogies (as discussed in the first chapter of this book), recognizable patterns, and sophisticated mathematical methods.

This Universe-as-computer analogy suggests that the Universe has a clock or carrier frequency that synchronizes all its modules, functional blocks, processors, and operations. The algorithm governing it could be fixed and finite, characterized by repeating patterns and predefined procedures. With sufficient intellectual and mathematical effort, it might be possible to derive or calculate universal constants known in physics, including the masses of planets, galaxies, atoms, and elementary particles. While this idea remains within the realm of metaphysics or science fiction, it warrants attention and further investigation.

Several empirically or theoretically known patterns describe how natural matter structures, masses composed of atoms and molecules are created, organized, grow, communicate, and propagate. These patterns include:

1. Growth as crystalline structures,
2. Expansion as fractals,
3. Following Fibonacci numbers and the "Golden Ratio" for structural expansion (see more here:
[link](http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fibonacci/fibnat.html#section2)),
4. Evolving towards certain spatial symmetry and periodicity,
5. Being internally structured as self-closed, spatial, standing matter-wave resonant formations that synchronize externally,
6. Being composite structures based on the patterns (1-5),
7. Adhering to electromechanical analogies and symmetries,
8. Being interconnected by matter-created fields and matter-wave communication, while conforming to the classical second-order differential wave equation.

All these structural manifestations (1-8) originate at the atomic level. Atoms, and their subatomic components, function as ensembles of self-closed standing matter-wave resonators, which are mutually synchronized. These atoms are systematically organized in the Periodic Table of Elements, constructed from a limited set of elementary particles (e.g., electrons, protons, neutrons) and characterized by intrinsic periodicities and analogies.

This leads to the conclusion that microstructures composed of atoms are directly and synchronously connected in real-time with macrostructures also composed of atoms. This connection is crucial for maintaining the stability of structural matter properties. In simplified terms, mechanical objects represent boundary or asymptotic cases of "matter-wave energy packing," likely involving electromagnetic energy. Consequently, matter in our Universe is a composition of mutually synchronized structures, with transformability between energy, mass, and matter waves. These matter-waves convey information, signals, or wavefunctions. The well-known relation between mass and energy is given by Einstein's equation, $E = mc^2$*, and in this book, matter waves are described as dualistic wave-particle formations, characterized by the "PWDC set of properties" (discussed further in section "10.00 Deeper Meaning of PWDC").*

This understanding might also explain how and why water and other liquids can exhibit specific structuring and memory effects.

We can now explore how synchronization and communication between masses and atoms occur, extending our analysis to wave-particle duality, the creation of matter waves, entanglement effects, and the coupling of resonant (temporal-spatial) spectral domains. Known natural forces and fields are causally linked to the same underlying nature. It is evident that everything we address in physics such as fields, forces, and other "mass-

energy-moment-charges" formations originates from atoms, spreading synchronously and spatially-temporally, both internally and externally.

The general rules and mathematical models applicable here include all conservation laws, as well as spectral analysis and synthesis theory, based on the work of J. Fourier, D. Gabor, and others (refer to [8], [57], [58], [59], [79], [109], [110], and [111] for relevant achievements). Quantization in physics relates to the discretization of "matter-wave packets" or "wave groups of energy and momentum exchanges" between stabilized, oscillating matter domains and structures, such as atoms. These stabilized matter domains (or masses) are always created from self-closed standing matter waves, which act as spatial resonant structures. These structures communicate by exchanging quantized (discretized) energy packets, such as photons. Probability and statistics serve as valuable mathematical tools when conditions allow for such processing.

....

In human cultural heritage we also find the **hypothetical and philosophical concept about our Universe being (metaphorically formulated) as a kind of natural supercomputer** (or an automate with its axiomatic rules, following an algorithm with fixed structure) elaborated as Samkhya. *Samkhya-yoga or Sankhya is an ancient axiomatic, natural, Vedic philosophy also addressing AXIOMATIC PRINCIPLES IN RELATION TO PHYSICS: It unifies the two contentious concepts of materiality and spirituality by demonstrating through precise mathematics that all world manifestation is a hologram or the very embodiment of spirituality. Samkhya has extraordinary scientific content (most probably created about 30'000 to 40'000 years ago). In our cultural and historical heritage, it is described within 70 verses, addressing axiomatically (and using the most simple, basic mathematical operations) our knowledge about the Universe, Mathematics and Physics.* Read the following exiting citations from [148]:

"The confidence that this theory is precise and correct, comes only from the fact that its numerical parameters match those of physics with a better-than-acceptable order of accuracy*. These comparative numbers are shown later in a tabulated form.*

....

The holographic mode of manifestation is proved mathematically by showing that all phenomenon is bound simultaneously by a spectrum of seven states and released sequentially by one mode. The enigma in science, why sound, light, particulate, molecular, atomic, nuclear, and sub-particle level have a periodicity spectrum, is resolved axiomatically. The above parameters are some of the 'easy to understand' aspects that differ from physics. ***The mathematical aspects of this theory are all encompassing, profound and complete in all respects. The unified solutions are derived internally and matched accurately to provide numerical answers to every known and unknown stable parameter in physics and cosmology. It has its own system of internal proof by matching six alternate derivations to 25 decimal places.***

Sankhya enables the tabulation of the entire cosmic manifestation parameters like any mathematical log table or almanac, with tie certainty there will be no phenomena found falling outside it. This aspect is not possible in science today. How do we know Sankhya is right? Differentiating the Sankhya-derived-mass of the universe by its smallest displacement leaves a precise single unit-angular displacement value of the very first interaction. Such accuracy is possible only in the realm of the divine! Sankhya also gives equally accurate numerical solutions to both scientific and holistic problems in phenomenon. The latter process by itself is an extraordinary confirmation of Sankhyan supremacy, for science has deliberately closed its eyes to *it and believes holistic perception does not exist! An outstanding feature of Sankhya is that no measured or empirical inputs are required, and the axiomatic theory starts by manipulating the interactions between two objects in various ways. To explain briefly, Sankhya is based on counting only oscillatory interactions as a ratio of a standard & axiomatic cycle of 10 counts which are dimensionless, scale-invariant, coherent, synchronous, reflection invariant and symmetric. Though space has substantial qualities identified as the Purusha State, it cancels out, as all measurements are relative comparisons through its smallest unit, the moolaprakriti. So, the Purusha's basic qualities are not mathematically relevant in defining phenomenon. Any Sankhya equation is always the algebraic sum of three gunas as tama (strong force), raja (weak force and gravity) and satwa (electromagnetic force) or a ratio of tama/(raja into satwa.). Hence, all equations compare only three real dimensions. There are three cyclic states to define time and are governed by three principles, simultaneity, self-similarity, and relativity and these have scalar (full force), tensor (stress dependent force) and vector (time dependent force) characteristics respectively. All of space is always in a dynamic oscillatory state, at an axiomatic rate of 296575967 oscillations per cycle of 10 oscillations or 299792458 oscillations at a meter wavelength/second, which equals the velocity of light in vacuum. The extraordinary fallout from deriving the holographic oscillatory state is that it corrects velocity of light in physics relativistically by the solar orbital velocity in the galaxy by the factor 1.010845. Michelson & Morley detected this corrected value, but no one realized that it was relevant and though the experiments failed, their results displayed a doppler blue shift in frequency. Hence, the frequency of light in the solar system cannot be constant. ....”*

Modern days concepts, ideas and creations, on certain way slightly reminding to Sankhya philosophy, but much more elaborated, sophisticated, and scientifically useful can be found in [167] Fundamental Theory of Physics, and in [168] Metamathematics: Foundations & Physicalization, by Stephen Wolfram, indicating that our Universe could be considered as a kind of natural super computer operating based on well-defined algorithms and software ... ♣]

…………………………..

Let us now summarize, and later visually represent, the concepts of Wave-Particle Duality, Stability, and Unity in relation to Wavefunctions and Quantization, and explore when, where, and how these concepts are valid and applicable.

The true originator of the Wave-Particle Duality theory is Jean-Baptiste Joseph Fourier, rather than Louis de Broglie, Schrödinger, Heisenberg, or Niels Bohr. While these later physicists contributed significantly to the theory, they primarily adapted Fourier's mathematical concepts for use in physics. Another key contributor to the development of Wave-Particle Duality was the Hungarian American scientist Denis Gabor, who established a broadly applicable mathematical model for waves, signals, and oscillations in the form of Complex Analytic Signals, models not inherently tied to statistics and probability theory.

So far, we have concluded that the Planck-Einstein photon energy formula, $\tilde{\mathbf{E}} = \mathbf{hf}$, is effectively applicable (as an energy form) only to narrow-band, energy-finite wave packets (or groups). In this formula, the frequency f represents the mean or central carrier frequency of the photon-wave-packet. When this elementary wave packet is considered an equivalent particle, as seen in analyses of Compton scattering, the photoelectric effect, and various particle-wave interactions, this approach has proven successful.

Thus, we conclude that the same wave packet (or photon) must also be energy-finite and narrow-banded in its spatial-temporal and frequency-related domains. The validity of $\tilde{E} = hf$ is mathematically supported only under these conditions. This is elaborated upon in Chapter 4.0, particularly around equations (4.0.47) to (4.0.54), and in Chapter 5, particularly section 5.14-1, where these principles are applied analogically to all original and spectral narrow-band, Gaussian, and Bell-curve wavefunction domains. Further discussion can be found in Chapter 9, section 9.1, "Wave Function of Photon or Energy Quantum."

It is crucial to understand that the universal equivalency between a wave packet and its particle equivalent state requires that their envelope functions in all temporal, spatial, and frequency domains must also be Gaussian-shaped or resemble a Bell curve (though not necessarily identical). Under these conditions, the wave packet and its particle equivalent dualistic state are well-defined or localized in all relevant domains. In such cases, the mathematical Uncertainty relations between signal domains or total signal durations can be treated as Certainty, or mutual equivalency, relations.

At this point, we are not addressing wavefunction modeling related to statistics and probability. In this book, wavefunctions are formulated as Complex Analytic Signal functions or Phasors, as established by Denis Gabor (see Chapter 4.0 for more details). These functions are later fully developed within the Kotelnikov-Shannon-Whitaker-Nyquist signal analysis and synthesis theories and concepts, rather than relying solely on Fourier Spectral Analysis and stochastic modeling. However, these Analytic Signal Wavefunctions can be easily transformed into normalized, non-dimensional functions, like those used in quantum theory for probabilistic wavefunction interpretation.

The concepts of quantization discussed here, within the context of Wave-Particle Duality (or PWDC), are more closely related to Parseval's theorem and Kotelnikov-Shannon-Whitaker-Nyquist signal analysis than to the more artificial and often misinterpreted concept of quantization ($\tilde{E} = hf$) as practiced in contemporary quantum theory.

In our spatial-temporal world or universe we still operate (in different ways) with 3+1 or 4-dimensional presentations of matter states and motions. That means, certain matter-wave function is presentable as $\Psi(x,y,z,t) = \Psi(r,t)$, $r = r(x,y,z)$, or much better in its Complex, Analytic Signal (or Phasor) form as: $\overline{\Psi}(x,y,z,t) = \overline{\Psi}(r,t) = \overline{\Psi}$, $r = r(x,y,z)$. In cases when we need to address (measure and localize in absolute lengths or duration terms) energy transfer or energy flow (or current of energy) of a finite-energy and narrow-banded matter wave, we will operate with relevant (still not at all probabilistic and/or statistical meaning) power wavefunction $\left|\overline{\Psi}\right|^2$. Graphically, this can be roughly illustrated as on the 4D to 3D orthogonal projections below (to give some visual and qualitative impression about four-dimensionality):

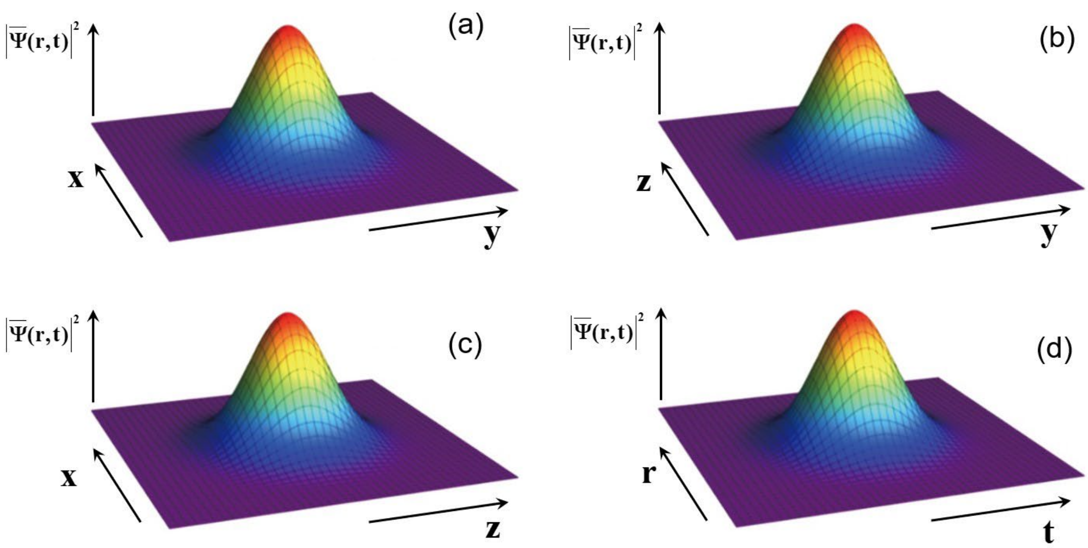


Fig. 10.2, Three-dimensional projections of a narrow band wavefunction $\left|\overline{\Psi}(r,t)\right|^2$, $r = r(x,y,z)$ in a space-time continuum

Let us now create certain graphical associations (on Figs. 10.3 and 10.4) about Gaussian or Bell-curve shaped wave-packet (or photon) and its equivalent (or dualistic) particle, that corresponds to **PWDC**, as elaborated in this book,

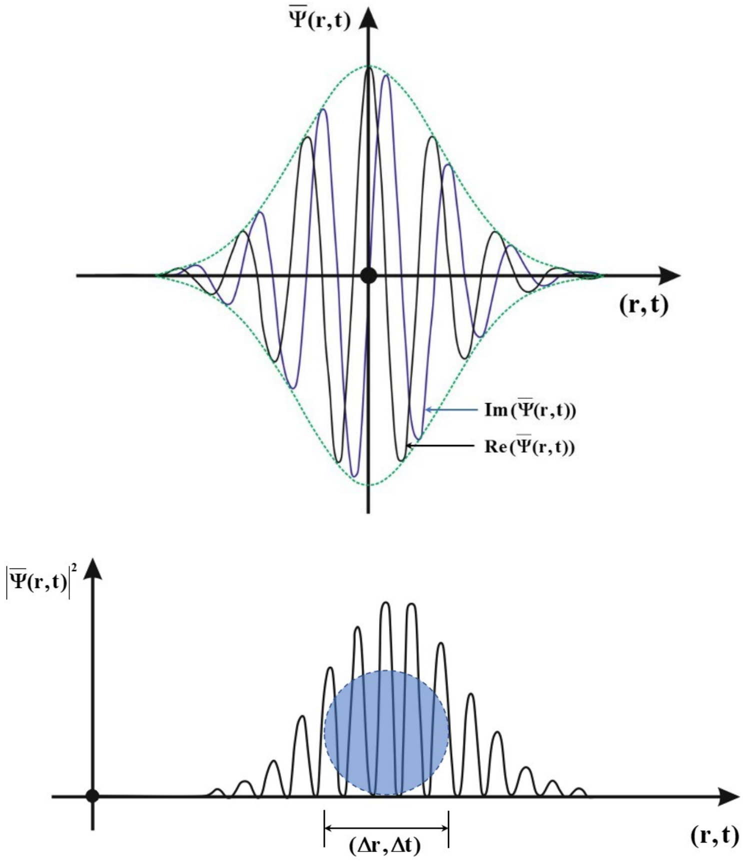


Fig. 10.3, Two-dimensional orthogonal projection of a narrow band wavefunction and its particle-equivalent in motion, in a space-time continuum (delta-intervals for space and time are of course different).

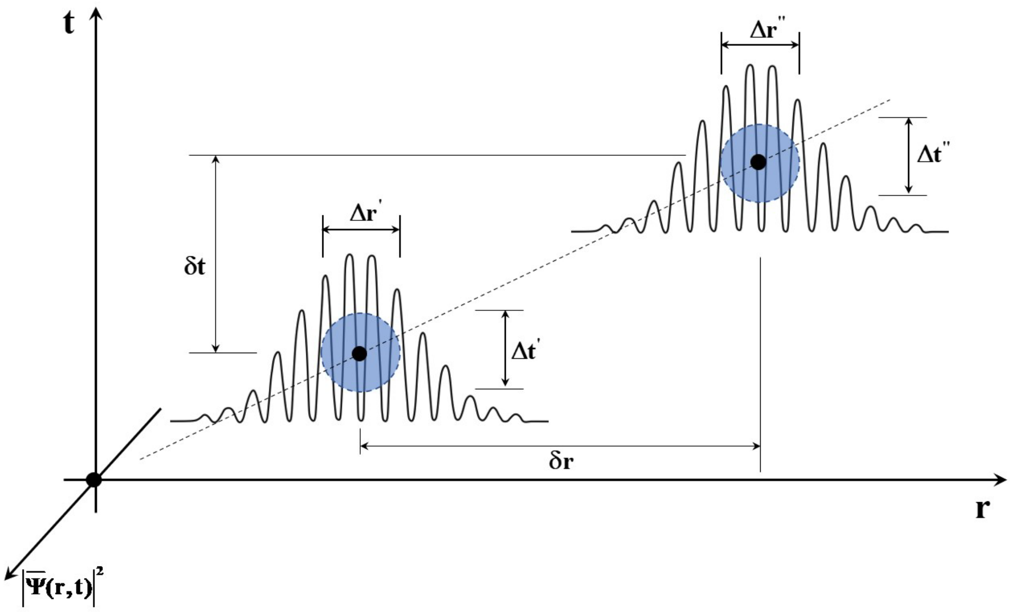


Fig. 10.4, Motion of a wavefunction or wave-packet $\left|\overline{\Psi}(r,t)\right|^2$ and its particle-equivalent in a space-time (r, t).

On the picture Fig. 10.4 are presented two successive motional positions of certain narrow-band wave-packet (like photon) or its equivalent particle. We assume that this wave packet is sufficiently stable (like soliton), that it has Gaussian or Bell-curve envelopes in all its domains (during its motion), has finite, absolute lengths, or durations $\Delta r' \cong \Delta r'' \cong \Delta r$, $\Delta t' \cong \Delta t'' \cong \Delta t$, and travels between two successive states passing small absolute intervals $\delta r, \delta t$. In such cases are valid Certainty intervals relations such as $\Delta t \cdot \Delta\tilde{E} \cong \Delta r \cdot \Delta p \ (=) \text{const.}$ (see more about CERTAINTY relations (5.2), (5.3), … (10.2-2.1) - (10.2-2.4), in Chapters 5. and 10, and in Chapter 4.0, especially around equations (4.0.76)). Now, based on the picture from Fig. 10.4, we can extract the following _motional-object group velocity_, this way underlining an additional criterion about matter-wave motional integrity and continuity, such as,

$$\left\{\begin{array}{l} r = k \cdot t,\ k = \delta r / \delta t = v = \Delta r / \Delta t,\ \Delta r = v \cdot \Delta t, \delta r = v \cdot \delta t \\ \Delta t \cdot \Delta\tilde{E} \cong \Delta r \cdot \Delta p \ (=) \text{const.} \Rightarrow \Delta r \cong \Delta t \cdot (\Delta\tilde{E} / \Delta p) \cong v \cdot \Delta t \end{array}\right\} \Rightarrow$$
$$\Rightarrow v \cong \Delta r / \Delta t \cong \delta r / \delta t \cong \Delta\tilde{E} / \Delta p,\ v = dr / dt = d\tilde{E} / dp (= u - \lambda \cdot (du / d\lambda) = -\lambda^2 df / d\lambda).$$

Here we have the situation happening in a 4-dimensional (x, y, z, t) time-space, but we can reduce it also on the 2-dimensional (r, t) space, where r = r (x, y, z). Such 2-dimensional motions could be controlled using convenient engineering methods based on automatic, 3-gyroscopes spatial position control, this way dominantly paying (associated mathematical) attention only to a motion in a 2-dimensional (r, t) space.

How or why, we can treat wave-packets presented on Figs. 10.2, 10.3 and 10.4 as equivalent particles, it is already indicatively explained in Chapter 4.0 by superposition of a continuum of simple-harmonic waves within certain narrow-band frequency interval, around equations

(4.0.30) – (4.0.44), what creates sinc wavefunction-groups (sinc(x) = sin(x)/x). The same type of sinc functions basis is used in Kotelnikov-Shannon waves analysis and synthesis (already many times mentioned in this chapter and in Chapter 4.0). Based on such conceptually indicative grounds we can understand Planck-Einstein photons or wave-packets quantizing.

Since here we address only matter-wave function $\left|\overline{\Psi}(\mathbf{r},\mathbf{t})\right|^2$ with Gaussian or Bell-curve (or similar and narrow-band) envelopes, we can determine its effective (also approximate) statistical "*size/duration/dimensions/energy or lengths*" of a corresponding and equivalent (dualistic) particle, based on the following graphical presentation, as we see on Fig. 10.5,

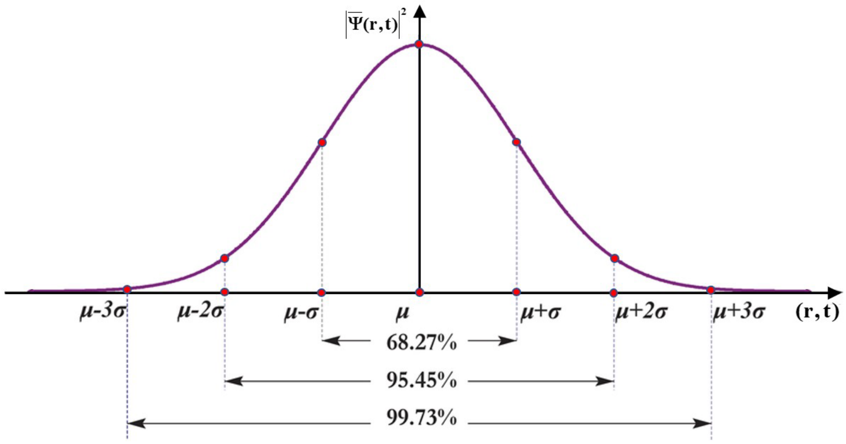


Fig. 10.5, Statistical interval lengths of wavefunction related to percentual energy content.

Practically, from Fig. 10.5 we can say that six-sigma (or 6σ), standard deviation, or $\pm 3\sigma$ statistical interval around the mean amplitude μ of Gaussian amplitude $\left|\overline{\Psi}(\mathbf{r},\mathbf{t})\right|^2$, captures 99.7% of the total wave-packet, photon, or better to say of its equivalent wave-particle, dualistic and motional energy. This is the most realistic and statistically tangible estimation if we like to know the size, durations, or domain lengths of certain Wave-Particle, or Dualistic Matter-Wave state (not implicating or ontologically imposing that everything about wave functions should be based only on Probability and Statistical modeling). Of course, this is also the situation when we will need to mathematically normalize and conveniently adjust mentioned Bell curve $\left|\overline{\Psi}(\mathbf{r},\mathbf{t})\right|^2$, or our complex phasor wavefunction, to be presentable as an Orthodox QT "*statistical or probability function*", and to be able to estimate its *energy content distribution*.

In addition, we can try to creatively address a wavefunction collapse, frequently used in contemporary Orthodox (or probabilistic) QT. Such cases, related to experiments' observations and measurements in QT, here based on **PWDC** concepts from this book, should correspond to spatial-temporal evolution of certain arbitrary shaped wavefunction to its new states being narrow-banded, Gaussian or Bell-curve enveloped signals or wave-groups.

Citation from Wikipedia, the free encyclopedia:
Wave function collapse
https://en.wikipedia.org/wiki/Wave_function_collapse

*In quantum mechanics, **wave function collapse**, also called **reduction of the state vector,**[1] occurs when a wave function—initially in a superposition of several eigenstates—reduces to a single eigenstate due to interaction with the external world. This interaction is called an observation, and is the essence of a measurement in quantum mechanics, which connects the wave function with classical observables such as position and momentum. Collapse is one of the two processes by* *which quantum systems evolve in time; the other is the continuous evolution governed by the Schrödinger equation.*[2]

Calculations of quantum decoherence show that when a quantum system interacts with the environment, the superpositions apparently reduce to mixtures of classical alternatives. Significantly, the combined wave function of the system and environment continue to obey the Schrödinger equation throughout this apparent collapse.[3] *More importantly, this is not enough to explain actual wave function collapse, as decoherence does not reduce it to a single eigenstate.*[4][5]

Historically, Werner Heisenberg was the first to use the idea of wave function reduction to explain quantum measurement.[6] [citation needed]

In the next table (T.10.00), we will expose analogical comparisons between temporal and spatial signals' presentations, with spectral content parameters of relevant wavefunctions (See also T.4.0, T.4.0.1 and T.4.0.2 from Chapters 4.0 and 4.1., contributing to Wave-Particle Duality understanding.)

T. 10.00.

<table>
<tr><th colspan="2">Temporal aspect of motion</th><th colspan="2">Spatial aspect of motion</th></tr>
<tr><th colspan="4">CERTAINTY RELATIONS & WAVE-PARTICLE DUALITY</th></tr>
<tr><td colspan="2">$\Delta E \cdot \Delta t \cong$
$dE \cdot dt =$</td><td colspan="2">$\cong \Delta p \cdot \Delta r, \quad (r = r(x,y,z))$
$= dp \cdot dr$</td></tr>
<tr><td colspan="4">$\psi_t = \psi(r,t)_{r=const.} = \psi(t), \; \psi_s = \psi(r,t)_{t=const.} = \psi(r)$
$\psi(t) \leftrightarrow \Psi(\omega), \psi(r) \leftrightarrow \Psi(k), \psi(r,t) \leftrightarrow \Psi(k,\omega)$
Mutually conjugate or original and spectral functions
$$\tilde{E} = E_k = \begin{bmatrix} \int_{[\Delta t]} \Psi^2(t)dt = \frac{1}{\pi}\int_{[\Delta\omega_t]} \left|A(\omega_t)\right|^2 d\omega_t \\ \int_{[\Delta r]} \Psi^2(x)dr = \frac{1}{\pi}\int_{[\Delta\omega_s]} \left|A(\omega_s)\right|^2 d\omega_s \\ \int_{[\Delta r,\Delta t]} \Psi^2(x,t)dx \cdot dt = \frac{1}{\pi}\int_{[\Delta\omega_s,\Delta\omega_t]} \left|A(\omega_s)\right|^2 d\omega_s \cdot d\omega_t \end{bmatrix}, \; p = \tilde{p} = \frac{1}{v}\tilde{E}$$</td></tr>
<tr><td>$\tilde{E} = h\overline{f} = \frac{h}{T}$
$d\tilde{E} = hdf =$
$= vdp = c^2 d\tilde{m}$
Planck-Einstein wave energy relation in a temporal domain (not real, integers related quantizing).</td><td>Temporal spectrum or temporal periodicity (=) temporal frequency (=)
$\omega_t = \frac{2\pi}{T} = \frac{2\pi}{h}\tilde{E} = 2\pi f_t = \omega = 2\pi f$
(=) number of temporal cycles or periods per unit of time, or per s.
(Here, index “t” means temporal)
$f_t = \frac{\tilde{E}}{h} = \frac{\omega}{2\pi} = \frac{1}{T} = f$</td><td>$p = h\overline{f}_s = \frac{h}{\lambda}$
$d\tilde{E} = \dot{p}dr = Fdr =$
$= vdp = c^2 d\tilde{m} = hdf$
Planck-Einstein wave energy relation in a spatial domain (is not a real, integers and indexing related quantizing).
$\frac{f_t}{f_s} = \frac{\tilde{E}}{p} = \frac{\lambda}{T}$</td><td>Spatial spectrum or spatial periodicity (=) spatial frequency (=)
$\omega_s = \frac{2\pi}{\lambda} = \frac{2\pi}{h}p = 2\pi f_s = k$
(=) number of spatial cycles or periods per unit of length or per m.
(Here, index “s” means spatial. It could also be with index r)
$f_s = \frac{p}{h} = \frac{k}{2\pi} = \frac{1}{\lambda} = \frac{\omega_s}{2\pi}$</td></tr>
<tr><td>See more in Chapter 9. under 9.1. “Wave Function of Photon or Energy Quantum</td><td>Any band limited and/or with Gaussian or bell curve shaped envelopes in a temporal domain can be synthesized, decomposed, or reconstructed (based on Fourier Signal Analysis, or better based on Denis Gabor Analytic Signal analysis, including Kotelnikov-Shannon-Nyquist theories) on number of simple harmonic or sinc wavefunctions in a time domain, being presentable as $\psi_t = \psi(\omega,t)$.
Typical natural manifestations or examples here are related to sound, music, and mechanical vibrations (including voltages, currents, velocities, and forces having described properties in a temporal domain.</td><td>$F = grad\, E = dp/dt$
(=) Force
$d\tilde{E} = h\frac{df_s}{dt}dr = h \cdot v \cdot df_s$
$f_s = k/2\pi = 1/\lambda$
$v = dr/dt = group\ vel. =$
$= u - \lambda(du/d\lambda) =$
$= u + p\frac{du}{dp} = -\lambda^2(df/d\lambda) =$
$= d\omega/dk = d\tilde{E}/dp$
$u = \lambda f = \omega/k = \tilde{E}/p =$
$= hf/p = f/f_s = \omega_t/\omega_s$
$= v/\left(1+\sqrt{1-\frac{v^2}{c^2}}\right)$</td><td>Any band limited and/or with Gaussian or bell curve shaped envelopes in a spatial domain can be synthesized, decomposed, or reconstructed (based on Fourier Signal Analysis, or better based on Denis Gabor Analytic Signal analysis, including Kotelnikov-Shannon-Nyquist theories) on number of simple harmonic or sinc wavefunctions in a spatial domain, being presentable as $\psi_s = \psi(k,r)$. Typical natural manifestations or examples here are crystalline and fractals structures. In other words, we could conclude based on analogies that excited crystalline structures are emitting matter waves with mechanical moments properties.</td></tr>
<tr><td colspan="4">Of course, in real spatial-temporal situations it is important to understand here presented relations in their integral, mutually united, and synchronized form (with mutually and deterministically coupled</td></tr>
</table>

temporal and spatial spectral characteristics), such as, $v = (\Delta r / \Delta t \cong \Delta E / \Delta p), \Psi = \Psi(\omega, t, k, r)$,

$$d\varphi = \frac{\partial \varphi}{\partial t} dt + \frac{\partial \varphi}{\partial r} dr = \omega_t dt + \omega_r dr \Rightarrow \boxed{d\varphi = \sum_i \omega_i ds_i , s_i \in [t, r, ...]}.$$

This summary explains wave-particle duality and Quantum Theory based on **wavefunction modeling:**

1. Wavefunctions and Natural Signals:
Every natural (dimensional, non-statistical) wavefunction or signal has both an amplitude and a phase function, which exist across various domains, whether original or spectral, temporal, or spatial. The most comprehensive and natural mathematical modeling of these wavefunctions is achieved using Complex Analytic Signals, as established by Denis Gabor (see Chapter 4.0).

2. Quantum Theory and Wavefunctions:
In Quantum Theory (QT), wavefunctions are non-dimensional, normalized constructs, specifically assembled for statistical and probabilistic analysis. These functions, widely accepted as a foundation of Orthodox QT, were developed by the “Copenhagen team”, a group of creative and intelligent scientists, mathematicians, and physicists. Although Orthodox QT wavefunctions are probabilistic, they can always be derived from natural, dimensional wavefunctions based on Complex Analytic Signals. This process involves simplifying the natural wave functions, effectively reducing their properties by removing phase functions. Orthodox QT was established before the formulation of Complex Analytic Signal modeling, requiring several assumptions and modifications to approximate the advantages of Analytic Signals and to develop the Schrödinger equation.

3. Wavefunction Analysis and Synthesis:
Any wavefunction can be analyzed, synthesized, decomposed, and reconstructed using Fourier and Analytic Signal modeling with simple harmonic sinusoidal functions. The efficiency of this mathematical process was greatly enhanced through the work of Kotelnikov, Shannon, and Nyquist, who employed sinc functions instead of simple harmonics. Sinc functions, which are naturally band-limited and exhibit a bell-curve envelope, are analogous to wave groups or wave packets, concepts that can be associated with photons through proper modeling. For instance, if we synthesize, compose, or create a wavefunction in its temporal domain using sinc functions, $\Psi(t) = \frac{\sin \omega t}{\omega t} = \text{sinc } (\omega t)$, since temporal and spatial domains are also mutually connected, the same wave function would be analogically presentable in its spatial domain as $\Psi(x) = \frac{\sin kx}{kx} = \text{sinc } (kx)$, or both of them could be analogically presented as, $\Psi(x,t) = \frac{\sin(\omega t \pm kx)}{\omega t \pm kx} = \text{sinc } (x,t)$. Every sinc function in its original domain (for instance temporal-spatial domain) will have (perfectly limited) rectangular or square shaped form in relevant spectral or frequency domains (meaning it will be defined with certain frequency interval, signal amplitude, and certain mean, average or central frequency; see such cases on Fig.10.1.). Based on such thinking or conceptualization, we can understand why and how Planck-Einstein or photon energy quant, or wave packet is presentable as $\tilde{E} = hf$. Starting from $\Delta E \cdot \Delta t \cong \Delta p \cdot \Delta x$, $dE \cdot dt = dp \cdot dx$ we can analogically and indicatively conclude that every motion or perturbation in a temporal domain will produce discrete or integral superposition of time-dependent simple-harmonic or sinc elementary wavefunctions. It is analogously valid that any motion or perturbation in a spatial domain will result in the discrete or integral superposition of spatial coordinates dependent of simple-harmonic or sinc elementary wavefunctions. The same applies to their corresponding spectral domains, as confirmed by the Parseval theorem.

<u>**WAVEFUNCTIONS OVERVIEW**</u>

1. Wavefunctions and the Complex Analytic Signal Model:
When wavefunctions are modeled using Complex Analytic Signals, they represent the most elegant solutions to classical second-order partial differential wave equations, including the Schrödinger equation. Any form of the Schrödinger equation can be logically derived step-by-step from the Classical

Wave Equation, without the need for artificial assumptions or mathematical patchwork, when the wavefunction is expressed as a Complex Analytic Signal (more in Chapter 4.0).

2. Gaussian Wavefunctions and Certainty Relations:
Gaussian and Bell-curve-shaped envelopes of wavefunctions are uniquely well-defined, narrow-band, and energy-finite across all relevant domains, including spatial, temporal, and spectral (frequency) domains. This characteristic satisfies "CERTAINTY RELATIONS" as discussed in Chapter 5. Conversely, the famous "UNCERTAINTY RELATIONS" (not limited to Heisenberg's) apply universally to all other non-Gaussian wavefunctions, whether in micro or macro physics. These non-Gaussian wave functions experience shape deformation and energy dispersion along their wave-motion path. Gaussian wavefunctions are particularly significant in technology because they possess well-localized, maximal spatial-temporal and surface energy density, and narrow frequency bands across all spectral domains, making them analogous to particles. Thus, quantization in physics is primarily related to Gaussian wavefunctions, where "UNCERTAINTY RELATIONS" evolve towards "CERTAINTY RELATIONS." When addressing wave-particle duality, such as in the Compton and Photoelectric effects, this duality is only valid when photons or electromagnetic wave packets are finite, narrow-band, Gaussian or Bell-curve shaped wavefunctions. The same logic applies to other narrow-band wave-packets, elementary particles, and stable matter states (see Chapter 8 for more on Quantization in Physics).

3. Energy Flow and the Amplitude Function:
The flow of motional energy in relation to a wavefunction is naturally and mathematically tied to its Amplitude function. The phase function, on the other hand, does not carry the energy content of a wavefunction; it instead relates to the spatial-temporal and spectral properties, signal velocities, and interactions between involved entities.

4. Parseval's Theorem and Wavefunctions:
Parseval's Theorem in Signal Spectral Analysis applies universally to any kind of wavefunction and mirrors the conservation of motional “energy-moments” in both temporal and spatial domains.

5. Total Probability Rule and Energy Conservation:
In the context of large statistical sets of events, the Total Probability Rule closely resembles Parseval’s theorem and effectively describes and replaces the law of energy conservation.

6. Law of Large Numbers and Synchronization:
The Law of Large Numbers and the resonant synchronization of universal matter states are integral components of the same reality. These principles apply to natural wavefunction modeling or any other wavefunction within a certain matter-state system (including ideal gases and other statistical systems).

7. Conservation Laws and Variational Principles:
Fundamental conservation laws and variational principles in physics universally apply to all wave functions, provided that mathematical modeling is properly defined and applied (see Chapter 4.1).

8. Thermodynamics and Its Evolution:
Thermodynamics, a masterpiece of Classical Mechanics, naturally evolves from Quantum Theory (QT) and Chaos theory modeling as we gradually increase the level of mathematical chaos. This evolution has been demonstrated through computer simulations (Reference: Vienna University of Technology. How Chaos Theory Mediates Between Quantum Theory and Thermodynamics, December 2022, retrieved from https://phys.org/news/2022-12-chaos-theory-quantum-thermodynamics.html).

This comprehensive overview (points 1 to 8) provides a clearer understanding of the ontological, philosophical, and conceptual similarities, ambiguities, and differences between natural and Quantum Theory (QT) wavefunctions. The described conceptualization, modeling, and applicability of natural, dimensional wavefunctions (compared to QT’s non-dimensional, probabilistic wavefunctions) offer significant mathematical advantages when using Analytic Signal modeling (Chapter 4.0). However, the gatekeepers of modern physics and QT remain reluctant to engage in such challenging discussions or initiate revisions to QT. The key message of this book is that while QT is valuable, it could be significantly improved.

There are many pseudo sciences or populistic and spiritual teachings and practices describing vibrational complexity, radiative and healing power of geometric and crystalline forms. This is like

promoting very much seducing, exotic, and magic, soul-motivating recommendations on how to profit from such holistic and universal cosmic, geometrical, biological, and psychological vibrations or waves. Often in the same package of universal natural vibrations teaching we find people selling different objects, pictures, pyramids, and crystals with some of magical powers ... Anyway, such popularized and arbitrary practices also show some desirable effects or healing results, partially confirmable as placebo situations, and partially acceptable because there are also realistic benefits of such accumulated and exotic insights (accumulated during thousands of years, within many different cultures). See, for instance, presentations of Dr. Robert Gilbert ("CONNECT TO THE DIVINE FREQUENCY" | Hidden Ancient Knowledge of VIBRATION (ugetube.com), www.vesica.org).
In fact, (from the point of view as presented in this book) the common grounds of such pseudo-science practices and teachings is essentially related to temporal and spatial, spectral complexity of matter forms in our permanently oscillating and resonating Universe, to PWDC, to universally applicable Uncertainty and Certainty relations, and to unity, synchronizations, and couplings between electromagnetic and mechanical phenomenology.

10.02 MEANING OF NATURAL FORCES

Now, (based on (10.2-2.2) and (10.2-2.3)), it is possible to elaborate the wider and mutually analogical meaning of linear force $F=\frac{\Delta E}{\Delta x}=\frac{\Delta p}{\Delta t}=\frac{dp}{dt}$, and angular force or torque $\tau=\frac{\Delta E}{\Delta \theta}=\frac{dL}{dt}$, and to create united concept of natural forces.

1. Any kind of force (or current of certain field charges, or moment entities) is analogically presentable as the first temporal derivation of involved linear $F=\frac{dp}{dt}$, angular $\tau=\frac{dL}{dt}$, or electromagnetic moment, and/or corresponding field charges (see more about ***force-current*** analogies in the first chapter).

2. Any kind of force and torque is also (analogically and dimensionally) presentable as the spatial gradient of relevant *motional energy distribution*, such as, $F=\frac{dE}{dx}(=)\nabla_{(x,y,z)}E$, $\tau=\frac{dE}{d\theta}(=)\nabla_{(\theta...)}E$. Energy here is effectively kinetic energy, since $dE_{tot.}=dE=dE_k$.

3. Of course, based on 1. and 2. we can easily develop less general, traditional, Newtonian force definition as a mass multiplied by corresponding acceleration, $F=m\cdot\dot{v}=m\cdot a=dp/dt$, $\tau=J\cdot\dot{\omega}=J\cdot\alpha=dL/dt$.

4. In cases of mechanically-spinning objects like gyroscopes (with zero initial linear moment), we need to consider (still hypothetically) that such spinning (if accelerated, having a torque $\vec{\tau}=\frac{d\vec{L}}{dt}$) could also generate certain linear (or axial) force-thrust $\vec{F}=\frac{d\vec{p}}{dt}$, and vice versa (what is also analog to an electromagnetic photon situation, as summarized in Chapter 4.1, and this also supports the conceptual meaning of wave-particle duality, and presents a part of explanation of Gravitation, as elaborated in Chapters 2. and 8.), as follows,

$$\boxed{\vec{L}\vec{\omega} = \vec{p}\vec{v}} \Rightarrow d\vec{L}\cdot\vec{\omega} + \vec{L}\cdot d\vec{\omega} = \vec{p}\cdot d\vec{v} + \vec{v}\cdot d\vec{p} \Leftrightarrow \frac{d\vec{L}}{dt}\cdot\vec{\omega} + \vec{L}\cdot\frac{d\vec{\omega}}{dt} = \vec{p}\cdot\frac{d\vec{v}}{dt} + \vec{v}\cdot\frac{d\vec{p}}{dt} \Leftrightarrow$$

$$\vec{\tau}\cdot\vec{\omega} + \vec{L}\cdot\vec{\alpha} = \vec{p}\cdot\vec{a} + \vec{v}\cdot\vec{F} \Rightarrow \vec{\omega}\left(\vec{\tau} + \frac{L}{\omega}\cdot\vec{\alpha}\right) = \vec{v}\left(\frac{p}{v}\cdot\vec{a} + \vec{F}\right) \Leftrightarrow \vec{\omega}\left(\vec{\tau} + J^*\cdot\vec{\alpha}\right) = \vec{v}\left(m^*\cdot\vec{a} + \vec{F}\right)$$

$$\Rightarrow \omega\tau = vF = v\frac{dp}{dt} = \omega\frac{dL}{dt},\ L\cdot\alpha = p\cdot a,\ \vec{\tau} = \frac{d\vec{L}}{dt},\ \vec{F} = \frac{d\vec{p}}{dt}, \vec{\alpha} = \frac{d\vec{\omega}}{dt},\ \vec{a} = \frac{d\vec{v}}{dt},$$

where $\vec{\alpha} = \frac{d\vec{\omega}}{dt}, \vec{a} = \frac{d\vec{v}}{dt}$ are angular and linear accelerations of the same spinning body. Here we should not forget that *magnetic field effects* will follow spinning of masses, explaining some of the unusual and extraordinary behaviors of gyroscopic motions of interatomic and planetary formations.

5. We also understand that force manifestations are associated with standing-wave field structures, as seen in phenomena like ultrasonic or acoustic levitation. In these cases, attractive forces are observed near nodal zones, where oscillation amplitude is minimal or zero. Conversely, repulsive forces are observed near zones of maximal amplitude. This principle applies to both mechanical and electromagnetic oscillations and waves.

   High-power mechanical, ultrasonic, or acoustic energy, along with moments, forces, oscillations, vibrations, audio signals, and music, can be generated and transmitted by applying various signal-modulating techniques to laser beams, dynamic plasma states, or other carriers. These carriers can then transfer lower-frequency mechanical vibrations or signals. Relevant details can be found in sections (10.2-2.4) and literature references [133] through [139], as well as in the European Patent Application "MMM Technology, EP 1 238 715 A1" [140].

   Additionally, mechanical or ultrasonic vibrations can be induced in solid metal wires and beams, like how electric and magnetic fields or photons propagate. These vibrations, which are analogous to photon propagation, can travel as acoustic waves along metal wires of any shape, whether in coils, solenoids, zig-zag forms, etc., that oscillate perpendicularly to the direction of propagation. These acoustic waves can efficiently transfer significant amounts of energy over long distances with minimal attenuation, regardless of the wire's length.

Standing waves generate varying accelerations around nodal and anti-nodal zones, which naturally create energy and mass gradients, or "temporal moments' derivations," toward these nodal zones. As a result, conditions 1 and 2 of the basic force definition are met through spatial-temporal coupling and symmetry.

The effects observed in acoustic and ultrasonic levitation in fluids serve as a demonstration of these forces. Additionally, in our macro-universe, similar effects occur between atoms and masses. These are caused by specific standing-wave structures that extend internal atomic fields and forces to external atoms and masses, resulting in gravitational effects. For further details, see Chapter 8, particularly section 8.3, "Structure of the Field of Subatomic and Gravitation-Related Forces."

Citation from https://en.wikipedia.org/wiki/Second_law_of_thermodynamics
*"The **second law of thermodynamics** is a physical law based on universal experience concerning heat and energy interconversions. One simple statement of the law is that heat always moves from hotter objects to colder objects (or "downhill"), unless energy is supplied to reverse the direction of heat flow".*

Another way to explain or support the second law of thermodynamics, which states that heat moves from hotter to colder objects, is by considering the spatial gradients of mass or energy density involved. This approach provides a more technical, tangible, and clear explanation than the empirical statement that heat spontaneously transfers from a hotter object to a colder one.

Typically, colder objects are naturally and spatially denser than hotter objects (when such a comparison is possible). In the contact zone between the two, a spatial gradient of energy density (equivalent to the mass density gradient) exists, which acts as a force driving heat from the hotter to the colder object. However, it is important to note that the concept of temperature should be updated. The total motional energy of a thermodynamic process includes both the kinetic energy of the particles and the energy of the associated matter waves.

This perspective allows for a more natural or mechanical explanation of the second law of thermodynamics, where the unidirectional transfer of heat energy is driven by the forces created until the energy or mass density and temperature distributions between the two objects equalize. This framework explains natural forces, torques, and vortices—including gravitation—as linear and angular spatial gradients of energy, mass, and temperature.

Additionally, the effects of vibrations and standing waves contribute to these spatial gradients and the associated forces observed in nodal and anti-nodal zones. The laws of conservation of linear and angular momentum, along with energy conservation, further support this unified concept of thermodynamics, natural forces, gravitation, spinning, vortices, and other "energy-moment" interactions, including wave-particle duality. This concept aligns with ideas proposed by Nikola Tesla and Rudjer Boskovic regarding natural forces and gravitation, suggesting that thermodynamics arises from wave-particle duality, or PWDC.

6. Natural Forces and Their Implications

Natural forces can be understood as the attraction and repulsion between like or opposite charges and moments, both mechanical and electromagnetic. This perspective includes Newtonian and Coulomb $1/r^2$ forces. For more on electromechanical analogies, refer to Chapter 1 of this book.

In the context of gravitational $1/r^2$ fields and forces, linear momentum (p) and angular momentum (L) are particularly important. Interestingly, there is no clear indication that static mass (m) should be the primary source of gravitation. Instead, evidence suggests that an intrinsically vibrating mass may play a crucial role in creating gravitational effects. This insight implies that the theories of gravitation proposed by Newton and Einstein may one day require significant updates.

For example, cohesion, adhesion, and Van der Waals forces are fundamentally based on electromagnetic field effects. However, to a macro-world observer, these forces may appear as purely attractive mechanical forces. Additionally, there is reason to believe that electric charges (such as electrons and protons) are dynamic, motional energy states, like mechanical and electromagnetic moments, rather than static entities. Currently, contemporary physics incorrectly considers electric charges as fixed and stable (measured in Coulombs). This dynamic nature of charges suggests a continuous electromagnetic energy exchange, like Nikola Tesla's concept of radiant energy flow within an etheric fluid, which could help explain gravitation.

7. The Fundamental Role of Energy in the Universe

Energy is the fundamental substance of everything in the universe. "Energy-moment relations" describe and channel motions within the cosmos. The conservation laws of physics establish the ultimate boundaries and rules of order, governing the exchange of mass, energy, and momentum. Variational principles in physics, which arise from these conservation laws, describe the tendencies, behaviors, and states of matter.

Everything in the universe adheres to the mathematical principles of analysis and synthesis, as outlined by the Kotelnikov-Shannon-Nyquist-Fourier signal processing theory. It is likely that different formats and packings of electromagnetic energy give rise to various forms of energy, matter-waves, and masses. Consequently, our universe is a structurally resonant system, where different energy states are packed and oscillate.

The formation of standing waves in the universe creates stable resonant structures of masses and other energy states. In the nodal zones of these resonant oscillations, masses, planets, and other cosmic formations accumulate, manifesting as gravitational attraction. This concept can also be applied to other natural forces. A practical consequence of this understanding is that, by using specific emitters and projectors of modulated electromagnetic energy (such as laser beams or matter-waves), we may be able to influence the behavior, motion, and structural stability of mass formations. This applies equally to resonator techniques based on mechanical elements and motions. All resonators within the universe are interconnected, tending to synchronize within spectral domains where their resonant frequencies overlap.

Natural forces may also be deeply and fundamentally connected to the Uncertainty Principle, which is valid in both the micro and macro cosmos (see sections 10.2-2, 10.2-2.2, and 10.2-2.3). While we often rely on the historical Newtonian definition of force (as the product of mass and acceleration), it is more accurate and general to consider spatial and temporal energy and moment gradients or derivatives. The ideas discussed in sections 1 through 7 provide a better foundation for explaining gravitation and other natural forces (including nuclear forces) than oversimplified assumptions or unverifiable axioms.

Nuclear forces (both weak and strong) are often explained in ways that seem unnatural or superficial when compared to the richer concept of natural forces outlined here. So far, we have addressed the simplest forms of linear, angular, central, oscillatory, and standing wave-related forces, with a focus on their analogical, dimensional, and hypothetical aspects. These forces often involve potential fields and locally concentrated energy states, which also produce force effects, further enriching the complexity of natural forces. A more general definition of forces, along with mathematical processing, is based on Lagrangian and Hamiltonian mechanics.

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Citation from: https://www.sciencedaily.com/releases/2014/12/141219085153.htm, literature reference [107].

"Patrick Coles, Jedrzej Kaniewski, and Stephanie Wehner made the breakthrough while at the Centre for Quantum Technologies at the National University of Singapore. They found that 'wave-particle duality' is simply the quantum 'uncertainty principle' in disguise, reducing two mysteries to one.

Wave-particle duality is the idea that a quantum object can behave like a wave, but that the wave behavior disappears if you try to locate the object. It is most simply seen in a double slit experiment, where single particles, electrons, say, are fired one by one at a screen containing two narrow slits. The particles pile up behind the slits not in two heaps as classical objects would, but in a stripy pattern like you'd expect for waves interfering. At least this is what happens until you sneak a look at which slit a particle goes through -- do that and the interference pattern vanishes.

The quantum uncertainty principle is the idea that it is impossible to know certain pairs of things about a quantum particle at once. For example, the more precisely you know the position of an atom, the less precisely you can know the speed with which it is moving. It is a limit on the fundamental knowability of nature, not a statement on measurement skill. The new work shows that how much you can learn about the wave versus the particle behavior of a system is constrained in the same way.

Wave-particle duality and uncertainty have been fundamental concepts in quantum physics since the early 1900s.

In earlier papers, Wehner and collaborators found connections between the uncertainty principle and other physics, namely quantum 'non-locality' and the second law of thermodynamics. The tantalizing next goal for the researchers is to think about how these pieces fit together and what bigger picture that paints of how nature is constructed".

From (10.2-2.1), (10.2-2.2) and (10.2-2.3), we can, on some way philosophically and conceptually, conclude that stable, self-standing and sustainable or long-lasting recognizable (motional) matter states (or relevant wave packets) have certain predictable, micro, and macro domains unity and flexibility, which could be summarized as:

a) If the wave packet in question will experience certain temporal modulation or perturbation during the time-duration ($\Delta t, T$), this will produce corresponding energy perturbation ($\Delta E, \tilde{E} = E_k$) and vice-versa, keeping their product stable or constant, like $\Delta t \cdot \Delta E = T \cdot \tilde{E} = const.$. Similar is valid for the product of relevant total domains durations, such as $\left(T \cdot F_t,\ L \cdot F_x,\ L \cdot \tilde{P}\right) \cong const. ...$

b) If certain spatial modulation or perturbation Δx will happen, this will produce predictable momentum perturbation Δp and vice-versa, keeping their product stable or constant, $\Delta x \cdot \Delta p = L \cdot P = Const.$. Similarity is valid for the product of relevant total intervals durations L and F_x. Temporal and spatial total lengths (or dimensions) of certain stable and motional matter state are anyway mutually dependent and strongly or causally linked (like $L \cong V^* \cdot T$, $V^* = const.$), what is also established on a different way in A. Einstein Relativity theory. Let us explore the relation between total signal durations or lengths of mutually conjugated spatial and temporal domains of certain signal or wave packet, $T \cdot \tilde{E} = L \cdot \tilde{P}$. For having stable moving objects, signals, and wave packets, we know that the condition of stability is to have direct proportionality between relevant temporal and spatial durations (such as $L = const \cdot T$). When we make signals' sampling and signals reconstruction, based on Kotelnikov-Nyquist-Shannon theory, we know that selected (and maximal) sampling intervals Δt and Δx should be,

$$\begin{pmatrix} \Delta t = \frac{1}{2f_{t-max}} = \frac{1}{2F_t}, \Delta x = \frac{1}{2f_{x-max}} = \frac{1}{2F_x} \\ T = N \cdot \Delta t, L = N \cdot \Delta x, N (=) \text{ integer} \end{pmatrix} \Rightarrow \begin{pmatrix} T \cdot \tilde{E} = L \cdot \tilde{P} \Leftrightarrow \Delta t \cdot \tilde{E} = \Delta x \cdot \tilde{P} \Rightarrow \\ V^* = \frac{L}{T} = \frac{\Delta x}{\Delta t} = \frac{\tilde{E}}{\tilde{P}} = \frac{F_t}{F_x} = u = \text{const.} \end{pmatrix}.$$

This is supporting constant phase speed $V^* = u = \text{const.}$ of all elements of the stable wave packet in question. If $u = \text{const.} = C (=) \text{ speed of light}$, this is on some way implicating involvement of an electromagnetic nature in the background. In fact, here we could imaginatively establish that “***characteristic, proper-time duration***” T of a moving particle (that has its wave packet replacement with spatial length L, and the same temporal duration T), can be uniquely defined as, $T = L / C,\ L = C \cdot T,\ C = \text{const.}$. The same idea is later expanding towards 4-dimensional spatial-temporal basis, such as, (x, y, z, iCt), $i^2 = -1$.

c) In cases of combined and accelerated motions, if temporal ($\Delta t, T$) and spatial durations ($\Delta x, L$) would be affected synchronously, then all other ΔE, Δp, T, F_t, L, and F_x would synchronously experience directly proportional perturbations (and vice-versa).

d) We can apply similar reasoning to masses, electric charges, magnetic fluxes, and electric voltages and currents, where applicable. This approach can also be extended to orbital, rotational, and spinning motions, along with their associated angular moments (see supporting literature sources in [36], Anthony D. Osborne, Department of Mathematics, Keele University, Keele, Staffordshire ST5 5BG, UNITED KINGDOM & N. Vivian Pope, 'Llys Alaw', 10 West End, Penclawdd, Swansea, West Glamorgan SA4 3YX, UNITED KINGDOM).

It appears that all forms of angular, orbital, spinning, and circular motions in the universe might be interconnected through specific, immediate, synchronous cosmic and holistic links. These connections could act almost instantaneously, regardless of the distances between such entities. If this hypothesis is correct and verifiable, it suggests that under certain favorable conditions, we could harness these 'energy-angular-momentum connections' for practical applications.

This could enable direct and immediate connections with countless spinning, rotating, and orbiting systems—ranging from atoms to solar systems and galaxies. Such connections could potentially be leveraged in the development of new devices for communication or energy harvesting.

In essence, all mechanical, electromagnetic, and other perturbations of matter are synchronously interconnected on a cosmic scale. The universe resonates and responds to every temporal, spatial, electromagnetic, mechanical, and other perturbation. The matter around us is unified, manifesting through various electromagnetic, electromechanical, mechanical, spatial, and temporal aspects. This fundamental unity gives rise to the many analogies discussed in the first chapter of this book.

Fourier analysis teaches us that any wave group can be decomposed into a series of elementary, simple-harmonic sinusoidal waves or functions. However, the signal analysis and synthesis processes occurring naturally in our universe employ different and optimized signal bases, such as 'sinc' functions. These functions are effectively *used to atomize and quantize signals, especially when different particles, wave groups, and states are created or when they interact and communicate.*

In this context, our goal is to identify or formulate the optimal and natural signal shape of an elementary wave packet or wave energy quantum. This would involve using Analytic Signal modeling as the basis for elementary waves in tasks such as signal analysis, quantization, decomposition, reconstruction, qualification, quantification, and various presentations across spatial, temporal, and frequency domains.

When these elementary wave packets naturally combine, self-stabilize, and form certain standing waves or self-closed structures, they can give rise to particles (or atoms) with stable rest masses. Visualizing these scenarios is more akin to finite element analysis of resonant states in mechanical structures under stationary vibrations, rather than the oversimplified quantization concepts currently employed in contemporary quantum theory.

By considering wave energy forms (5.14) and the principles of 'Kotelnikov-Shannon, Whittaker-Nyquist Sampling and Signal Recovery' (see references in [57, 58, and 59], and Chapter 4.0 under equations (4.0.34), (4.0.5-2), (4.0.35)), we can formulate all relevant parameters of an effective, elementary band-limited wave packet or signal, such as the wave energy of a narrow-band photon as follows,

$$\begin{aligned}
&\Psi(x,t) = a\sum_{(i)}\cos(\omega_i t - k_i x) = a\cdot R_e\left[\int_{\left(k_0-\frac{\Delta k}{2}\right)}^{\left(k_0+\frac{\Delta k}{2}\right)} e^{j(\omega t - kx)}dk\right] = \\
&= a(x)\frac{\sin(\underline{\Delta\omega}\ t - \underline{\Delta k}\ x)}{(\underline{\Delta\omega}\ t - \underline{\Delta k}\ x)}\cos(\omega t - kx)\ (\Leftrightarrow)\ \text{band limited wave packet}, \\
&\omega = 2\pi f\ (=)\ \text{carrier frequency}\ (>>>\underline{\Delta\omega}), \\
&\underline{\Delta\omega} = 2\pi\underline{\Delta f}\ (=)\ \text{wave-packet frequency bandwidth}\ (<<<f), \\
&v = d\omega/dk = \underline{\Delta\omega}/\underline{\Delta k},\ u = \omega/k.
\end{aligned} \tag{10.2-3}$$

If $\Psi(x,t)$ *is an elementary narrow-frequency-band, like Gaussian Wave Packet (which has a mean frequency* $\bar{f}$*), we should also consider that its energy is (see more in Chapter 9., under "9.1. Wave Function of Photon or Energy Quantum"),*

$$\boxed{\begin{matrix}\tilde{E} = E_k = h\bar{f} \\ (h = \text{const.})\end{matrix}} = \int_{[\Delta t]}\Psi^2(t)dt = \frac{1}{2}\int_{[\Delta t]}a^2(t)dt = \frac{1}{\pi}\int_0^{+\infty}|A(\omega)|^2 d\omega = \int_{[\Delta t, \Delta p]} vdp = \int_{[m]} c^2 d\tilde{m}\ ,\ \tilde{m} = \gamma m.$$

Kotelnikov-Shannon signal decomposition and/or synthesis also has similar Gaussian elementary wave components or basis wave-functions. This looks like having a superposition of elementary sinc matter-wave packets, like (10.2-3)), such as (see more in chapter 4.0):

$$\Psi(t)=a(t)\cos\varphi(t)=\sum_{n=-\infty}^{+\infty}\Psi(n\cdot\delta t)\frac{\sin\Omega(t-n\cdot\delta t)}{\Omega(t-n\cdot\delta t)}= \\ =\sum_{n=-\infty}^{+\infty}a(n\cdot\delta t)\frac{\sin\Omega(t-n\cdot\delta t)}{\Omega(t-n\cdot\delta t)}\cos\varphi(n\cdot\delta t),\ \Psi(n\cdot\delta t)=a(n\cdot\delta t)\cos\varphi(n\cdot\delta t), \tag{4.0.34}$$

while Fourier analysis is dealing with pure alternating or sinusoidal elementary waveforms (such as simple harmonic currents and voltages, and other analogical values known in mechanics).

Where Ω *is the highest frequency in the spectrum of* $\Psi(t)$, *and we could consider that* Ω *is the total frequency duration of the signal* $\Psi(t)$.

Since sampling frequency-domain of signal-amplitude, Ω_L, *is always in a lower frequency range than the frequency range of its phase function* $\Omega=\Omega_H$, *and since the total signal energy is captured only by the signal amplitude-function, we should also be able to present the same signal as summation of relevant sinc-functions* $\frac{\sin\varphi(x,t)}{\varphi(x,t)}$:

$$\Psi(t)=a(t)\cos\varphi(t)=\sum_{n=-\infty}^{+\infty}a(n\cdot\delta t)\frac{\sin\Omega(t-n\cdot\delta t)}{\Omega(t-n\cdot\delta t)}\cos\varphi(n\cdot\delta t)= \\ =\left[\sum_{n=-\infty}^{+\infty}a(n\cdot\delta t_L)\frac{\sin\Omega_L(t-n\cdot\delta t_L)}{\Omega_L(t-n\cdot\delta t_L)}\right]\cdot\left[\sum_{n=-\infty}^{+\infty}\cos\varphi(n\cdot\delta t)\frac{\sin\Omega_H(t-n\cdot\delta t)}{\Omega_H(t-n\cdot\delta t)}\right], \\ \overline{\Psi}(t)=\sum_{n=-\infty}^{+\infty}a(n\cdot\delta t)\frac{\sin\Omega(t-n\cdot\delta t)}{\Omega(t-n\cdot\delta t)}e^{j\varphi(n\cdot\delta t)}=a(t)e^{j\varphi(t)}, \tag{4.0.35}$$

We also know (see (5.14-1), *Chapter 5. "Quantizing and Kotelnikov-Shannon, Whittaker-Nyquist Sampling Theorem") that effective time duration* $\overline{T}=\Delta t$, *frequency duration* $\overline{F}=\Delta f$, *of an elementary and optimal sampling energy quant,* $\Delta\tilde{E}=\tilde{E}=hf$, *could be approximated as certain rectangular pulse shape (being extendable to all of its domains),*

$$\left\{\begin{array}{l}\Delta t\cdot\Delta f=T\cdot F=\overline{TF}=\frac{1}{2},\ \Delta t=\overline{T},\ \Delta f=\overline{F},\ \ \overline{\Psi}(t)=a(t)\cdot e^{j\varphi(t)},\ \Delta\omega=2\pi\Delta f \\ d\tilde{E}=\Psi^2(t)\,dt=\frac{1}{2}\left|\overline{\Psi}(t)\right|^2dt=\frac{1}{\pi}\left|A(\omega)\right|^2d\omega=\frac{1}{2}a^2(t)dt=h\,df=vdp=c^2d\tilde{m} \\ \Delta\tilde{E}=\frac{1}{2}\overline{a}^2(t)\Delta t=\frac{1}{\pi}\left|\overline{A}(\omega)\right|^2\Delta\omega=h\overline{f}=v\Delta p=c^2\Delta m=\tilde{E}=\text{"one quant"}\end{array}\right\}\Rightarrow$$

$$\Rightarrow\tilde{E}=E_k=\int_{[\Delta t]}\Psi^2(t)dt=\frac{1}{\pi}\int_{[\Delta\omega]}\left|A(\omega)\right|^2d\omega=\frac{\overline{a}^2}{2}\overline{T}=2\overline{A}^2\overline{F}=\frac{\overline{A}\overline{a}}{2\sqrt{\pi}}=h\overline{f}=\overline{pu},$$

$$\Rightarrow\overline{T}=\left(\frac{2\overline{A}}{\overline{a}}\right)^2\overline{F}=\frac{1}{2\overline{F}},\overline{F}=\left(\frac{\overline{a}}{2\overline{A}}\right)^2\overline{T}=\frac{1}{2\overline{T}},\ \boxed{\frac{\Delta f}{\overline{f}}=\frac{\overline{F}}{\overline{f}}=\frac{h}{2\overline{A}^2}},$$

$$\begin{aligned} & h = 2\overline{A}^2 \cdot \frac{\overline{F}}{\overline{f}} = 2\overline{A}^2 \cdot \frac{\Delta f}{\overline{f}} = 6.62606876 \text{ x } 10^{-34} \text{Js}, \ \Delta f = \frac{h\overline{f}}{2\overline{A}^2} \\ & \overline{f} = \frac{\overline{\omega}}{2\pi} = \frac{1}{2\pi^2 \tilde{E}} \int_0^{+\infty} \omega |A(\omega)|^2 d\omega = \sqrt{\frac{1}{2\pi^2 h} \int_0^{+\infty} \omega |A(\omega)|^2 d\omega} = \frac{2\overline{A}^2 \Delta f}{h} \end{aligned} \tag{5.14-1}$$

$$\overline{\lambda} = \frac{h}{\overline{p}}, \ \overline{u} = \overline{\lambda}\,\overline{f} = \left\langle \frac{\omega}{k} \right\rangle = \left\langle \frac{\tilde{E}}{p} \right\rangle, \ \overline{v} = \left\langle \frac{\Delta\omega}{\Delta k} \right\rangle = \left\langle \frac{d\tilde{E}}{dp} \right\rangle.$$

Understanding the Quantum Nature of the Universe

At its core, the quantum nature of our Universe can be explained by considering that all stable energy-momentum states, wave packets, and masses in motion are quasi-resonant, self-closed standing-matter wave structures. These spatial-temporal states communicate and interact by exchanging wave packets composed of "Gaussian-Kotelnikov-Shannon" sinc functions $\overline{\psi}(x,t) = |\overline{\psi}(x,t)| \frac{\sin(\underline{\Delta\omega}\, t - \underline{\Delta k}\, x)}{(\underline{\Delta\omega}\, t - \underline{\Delta k}\, x)} e^{I(\omega t \pm kx)}$*, which carry energy and momentum.*

Aspects of Matter Quantization

Quantizing matter involves two main aspects related to the spatial and temporal periodicities of a given matter state:

1. Discrete Energy-Momentum Exchanges:
- Communications between all matter states, both micro and macrocosmic, are not continuous. Instead, they occur through the exchange of discrete packets of energy. In some cases, mass can also be exchanged.

2. Integer Associations with Wave Quanta:
- By associating integers with these wave energy quanta, we can relate them to the counting of wavelengths in standing waves or simply to the number of involved wave packets.

Comparison with Contemporary Quantum Theory

The actual mathematical theory of matter quantization aligns more closely with "Kotelnikov-Nyquist-Shannon" sampling, analysis, and signal synthesis (or reconstruction) than with the current quantum theory, which often relies on verbal descriptions. Contemporary quantum theory tends to oversimplify and lacks precision in its explanation of quantization in nature.

Stable Structures as Standing Matter-Wave Formations

Many stable structures in the Universe, such as solar systems, atoms, and elementary particles, are self-closed and spatially self-stabilized standing matter-wave formations. These structures exhibit multiple structural, temporal, and spatial periodicities that can be described using integers. These self-stabilized and self-closed structures communicate by exchanging energy, mass, and momentum quanta through "Gaussian-Kotelnikov-Shannon-Nyquist" and soliton wave packets.

Implications and Further Research

The current presentation of quantization in quantum theory remains oversimplified and imprecise, merely acknowledging that quantization exists ubiquitously in nature. For a more comprehensive understanding, refer to sources [57, 58, and 59], as well as critical analyses such as [161], which discusses erroneous concepts related to photons and wave energy quantization in "Hidden Variables: The Elementary Quantum of Light. A Significant Question in Quantum Foundations."

By applying Euler-Lagrange-Hamilton equations and concepts based on the Variational Principle of Least or Stationary Action, we can arrange realistic and optimal scenarios within this framework. The favored "Gaussian-Kotelnikov-Shannon-Nyquist" elementary wave packets likely play a significant role in explaining phenomena such as Nikola Tesla's wireless, pulsing energy transfer.

Consequences of particle-wave duality conceptualization (as summarized under 1., 2., and 3.) are that certain particle (such as an electron, positron, proton…) could be created from a specific matter-wave group (initially without having energy-momentum elements with rest masses), on a spatially finite, closed line or self-closed multi-dimensional domain, where matter-wave in question will create stable standing-waves field structure. The mean (or median) perimeter C_n of mentioned self-closed spatial domain (taken as an oversimplified case) will have an integral number of matter-waves wavelengths, $C_n = n\lambda$ and integral number of associated, elementary time periods $n\tau = \frac{n}{f}$, where $n = 1, 2, 3, ...$ In reality, such spatially closed, energy-finite and spatially limited standing-waves domains will not be created only around ideal circles or perfect toroid. We should also consider that values of relevant matter wavelengths and belonging elementary time periods (as conceptualized here) should be conveniently treated only as average values,

$$\lambda = \frac{h}{p} \Rightarrow d\lambda = -h\frac{dp}{p^2} \Rightarrow \bar{\lambda} = \frac{C_n}{n} = \frac{1}{n}\oint_{C_n} d\lambda = -\frac{h}{n}\oint_{C_n}\frac{dp}{p^2} = \frac{h}{\bar{p}},$$
$$\tilde{E} = hf = \frac{h}{\tau}(=)\tilde{m}c^2 \Rightarrow \tau = \frac{h}{\tilde{E}} \Rightarrow \bar{\tau} = \frac{1}{\bar{f}} = \frac{1}{n}\oint_{C_n} d\tau = -\frac{h}{n}\oint_{C_n}\frac{d\tilde{E}}{\tilde{E}^2} = \frac{h}{\bar{\tilde{E}}}. \tag{10.3}$$

****Quantization and the Formation of Particles in Microphysics****

In simple cases where self-closed standing waves form a circular structure with a radius r, the perimeter of this circle becomes $C_n = n\bar{\lambda} = 2\pi\bar{r} = n\frac{h}{\bar{p}} \Rightarrow \bar{L} = \bar{p}\cdot\bar{r} = n\frac{h}{2\pi}$.
Quantization and particle formation in the realm of microphysics are directly related to the packing of "integer numbers of half $\frac{\lambda}{2}$ (or whole) matter-wavelengths." This occurs when stable, self-closed standing waves and resonant structures are formed.

Atoms, elementary particles, photons, and other stabilized matter-wave formations naturally communicate by exchanging similar quantized and minimal energy-momentum wave packets. The physical size of elementary particles is directly proportional to the integer number of half wavelengths $\frac{\lambda}{2}$ of the relevant matter wave or wave group.

The Role of Planck's Constant in Modeling

It is noteworthy that whenever Planck's constant h (or a macrocosmic equivalent H) can be successfully applied to model a process or describe an interaction, it indicates that we are dealing with spatially closed structures. These structures may be circular, elliptical, toroidal, spherical, or other forms where matter waves create standing waves.

****Analogies with Modal and Finite Element Analysis****

This concept becomes much clearer and more intuitive when we draw analogies with Modal and Finite Element Analysis, which are used to identify natural resonant frequencies or resonant states of a given body. These analyses typically reveal structures of standing waves corresponding to different resonant situations.

In the case of atoms and their minimal or elementary constituents, we also encounter standing waves and self-stabilized resonant structures. This aspect is a significant part of the Quantum World of Physics, supported by both theoretical and experimental practices.

****Matter-Wave Communication and Sampling Theory****

Additionally, matter-wave communication between these standing waves and resonant structures adheres to the "Kotelnikov-Shannon, Whittaker-Nyquist" sampling and signal reconstruction rules. This further emphasizes the interconnectedness of these phenomena within the quantum framework.

****Understanding Matter Waves and the Creation of Elementary Particles****

One of the most credible explanations for the formation of stable elementary particles (such as electrons, protons, positrons, etc.) involves a specific type of electromagnetic wave. These waves are composed of intricately structured and mutually coupled electric and magnetic field components, given their well-known interconvertibility and the significant interactions between them.

In essence, elementary particles with measurable rest masses (like electrons, positrons, and protons) can be formed from these specifically structured electromagnetic matter-wave groups, which initially possess no rest mass. The rationale behind this statement is elaborated in Chapter 4.1 of this book, as well as in references [17] to [22], [44], and [84].

****Supporting Evidence from Experiments****

Numerous experiments have confirmed the particle-wave duality and the existence of matter waves both theoretically and mathematically. Examples include the Compton effect, the photoelectric effect, secondary emissions, blackbody radiation, and diffraction phenomena with atomic and molecular rays. In many of these cases, the participating particles exhibit electric charges, magnetic moments, and intrinsic spin characteristics with constant gyromagnetic ratios.

However, in most contemporary physics analyses of these dualistic matter manifestations, the electromagnetic properties, such as charges, magnetic moments, and spinning are often overlooked. Instead, the focus is typically on the kinematic and mechanical parameters, such as velocities, masses, and energies involved in linear motion. Yet, spinning and magnetic moments are crucial for understanding matter waves and should not be neglected.

****Matter Quantization and Standing Waves****

The creation of stabilized, self-sustaining n-dimensional particles or bodies is inherently related to the packing of n-dimensional standing waves, including both angular and electromagnetic components. This multidimensional and discretized packing forms the foundation of matter quantization. The Planck constant (h) is fundamentally involved in all cases of quantization related to the microphysics of atoms, photons, and subatomic entities.

Moreover, a similar form of quantization, governed by a macrocosmic Planck-analog constant (H), exists in macrophysics, where stable and inertial astronomical and planetary systems also behave as self-closed standing matter-wave formations (as discussed in Chapter 2, “2.3.3. Macro-Cosmological Matter-Waves and Gravitation”).

In physics, quantization is intrinsically related to the inertial and stabilized (mechanical and electromagnetic) structures of matter, where standing waves contribute to the granulation and packing of matter. The relevant mathematics for such quantization aligns with the "Kotelnikov-Shannon, Whittaker-Nyquist Sampling Theory," further detailed in [57, 58, and 59], and in Chapter 8 of this book.

****Quantum World and the Role of Standing Waves****

In modern microphysics, the terms "Quantum World" and "Quantum Physics" are often used to describe the realm of atoms and subatomic particles. However, the entire universe, both micro and macro, is a quantum world made up of interconnected resonating structures of standing matter waves and mass agglomerations. The "Kotelnikov-Shannon, Whittaker-Nyquist Sampling, and Signal Recovery Theory" governs the rules and logic of quantization and energy-momentum communications within this universe.

At least two of the four fundamental forces of our Universe, as currently understood, are likely the result of attracting and repelling effects within different nodes of these standing waves and the structuring of electromagnetic fields. As a result, our understanding of the fundamental forces is expected to evolve significantly or even undergo a complete transformation.

****Transformation of Energy and Mass in Particles****

Once a stable particle is formed, the motional matter-wave energy of these closed standing waves is converted into the particle's rest mass or captured by it. Any excess non-consumed energy-momentum will either contribute to the particle's macro motion or be radiated in the form of matter waves. This process illustrates the concept of energy formatting or packing, as discussed in "4.3.8. Mass, Particle-Wave Duality, and Real Sources of Gravitation" in Chapter 4.3 of this book.

Extending the Concept of Standing Waves

We can now combine the concept of closed-domain standing waves (Section 10.3) with other significant relations and wavefunctions, beginning with Section 10.1, related to PWDC. This approach will highlight the advantages of working with wavefunctions and extend our elementary understanding of standing waves, from simple circularly closed lines in one plane to more complex three-dimensional or multi-dimensional spatial coordinates and time-dependent standing wave structures. See Chapter 4.3, starting from the equations under Section 4.9-0, for further elaboration.

$$
\begin{aligned}
&\overline{\lambda} = \frac{h}{\overline{p}} = \frac{C_n}{n} = \frac{1}{n}\oint_{C_n} d\lambda = -\frac{h}{n}\oint_{C_n}\frac{dp}{p^2} = -\frac{h}{n}\oint_{C_n}\frac{d\tilde{E}}{vp^2} = -\frac{h}{n}\oint_{C_n}\Psi^2\frac{dt}{vp^2} = -\frac{h}{n}\oint_{C_n}\Psi^2\frac{vdt}{(pv)^2},\\
&\overline{\tau} = \frac{1}{\overline{f}} = \frac{h}{\overline{\tilde{E}}} = \frac{1}{n}\oint_{C_n} d\tau = -\frac{h}{n}\oint_{C_n}\frac{d\tilde{E}}{\tilde{E}^2} = -\frac{h}{n}\oint_{C_n}\frac{vdp}{(pu)^2} = -\frac{h}{n}\oint_{C_n}\Psi^2\frac{dt}{(pu)^2},\ n = 1,2,3,...,\\
&(v <<< c, p = mv, m \cong const.) \Rightarrow \overline{\lambda} = \frac{h}{m\overline{v}}, \overline{\tau} = \frac{h}{(\frac{m\overline{v}^2}{2})} = \frac{1}{\overline{f}},\ \overline{u} = \overline{\lambda f} = \frac{\overline{v}}{2},\\
&(v \approx c, p \approx \frac{hf}{c}, m \approx \frac{hf}{c^2}) \Rightarrow \overline{\lambda} = \frac{c}{\overline{f}}, \overline{\tau} = \frac{1}{\overline{f}},\ \overline{u} = \overline{\lambda f} \approx c,\\
&\overline{u} = \overline{\lambda f} = \frac{\oint_{C_n}\Psi^2\frac{vdt}{(pv)^2}}{\oint_{C_n}\Psi^2\frac{dt}{(pu)^2}} = \frac{\overline{\tilde{E}}}{\overline{p}},\ \Psi = \Psi(x,y,z,t) = \Psi(r,t).
\end{aligned}
\tag{10.4}
$$

The meaning of the wavefunction $\Psi = \Psi(x,y,z,t) = \Psi(r,t)$, $\Psi^2(t) = \frac{d\tilde{E}}{dt}$, from (10.1) – (10.4) and elsewhere in this book, is essentially related to relevant signal or matter-wave Power, and not at all significantly related to QT, or statistics and probability theory practices (see much more about Power in the chapter 4.0, under equations (4.0.82) and later). Power and Analytic signal related wave function (see below) is much richer and more informative regarding understanding and describing particle-wave duality and matter wave states, than QT probabilistic and statistics-based concepts.

$$\left\{\begin{array}{l} \{d\tilde{E} = hdf = dE_k = c^2 d(\gamma m) = vdp = d(pu)\} / dt \\ \Leftrightarrow \left\{\Psi^2(t) = \frac{d\tilde{E}}{dt} = h\frac{df}{dt} = c^2\frac{d(\gamma m)}{dt} = \frac{d(pu)}{dt} = v\frac{dp}{dt}\ (=)[W]\right\}, \\ v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} = u + p\frac{du}{dp} = \frac{d\omega}{dk} = \frac{d\tilde{E}}{dp} = h\frac{df}{dp},\ u = \lambda f = \frac{\omega}{k} = \frac{\tilde{E}}{p} = \frac{hf}{p} \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left\{\begin{array}{l} POWER = \frac{d\tilde{E}}{dt} = \frac{1}{2}\left|\overline{\Psi}(t)\right|^2 = \frac{1}{\pi}\left|A(\omega)\right|^2\frac{d\omega}{dt} = \frac{1}{2}a^2(t) = \frac{1}{2}\left|\overline{s}_1\cdot\overline{s}_2\right|^2 = \frac{1}{2}(a_1\cdot a_2)^2 = \\ = h\frac{df}{dt} = v\frac{dp}{dt} = c^2\frac{d\tilde{m}}{dt} = s_1\cdot s_2\ (=)[W], \\ \overline{\Psi}(t) = \left|\overline{\Psi}(t)\right|\cdot e^{I\varphi} = a(t)\cdot e^{I\varphi} = \overline{s}_1\cdot\overline{s}_2 = a_1\cdot a_2\cdot e^{I(\varphi_1+\varphi_2)},\ \overline{s}_{1,2} = a_{1,2}\cdot e^{\varphi_{1,2}}, \\ \left|\overline{\Psi}(t)\right| = a_1\cdot a_2 = a(t),\ \varphi = \varphi_1 + \varphi_2, \\ \omega = 2\pi f = \frac{d\varphi}{dt} = \frac{d\varphi_1}{dt} + \frac{d\varphi_2}{dt} = \omega_1 + \omega_2 = 2\omega_c,\ \omega_1 = \omega_2 = \omega_c. \end{array}\right\} \Rightarrow$$

Mathematical Transformations and Their Role in Quantum Theory

It is important to note that certain carefully chosen isomorphic mathematical transformations, mappings, normalizations, and simplifications, when applied to the wave function, can be structured to align with universally valid conservation laws. These transformations help to realize contemporary probabilistic quantum theory (QT) concepts, which are both operational and predictive in analyzing various interactions, matter structures, and particle-wave duality.

To leverage the extensive mathematical resources available in signal analysis, we will consider the matter-wave function to be, at a minimum, an Analytic Signal function $\Psi(r,t)$. This approach is related to Hilbert Transform, as introduced and detailed in Chapters 4.0 and 4.3 of this book.

Now we can extend self-closed standing waves quantized packing concept (on same closed line C_n) to be applicable to arbitrary shaped closed domains, as follows,

$$\begin{array}{l} \overline{\lambda} = \frac{h}{\overline{p}} = \frac{C_n}{n} = \frac{1}{n}\oint_{C_n} d\lambda \Rightarrow \overline{p}C_n = \overline{p}\oint_{C_n} d\lambda = nh \Rightarrow \\ \Rightarrow \oint_{C_n} pdr = -h\overline{p}\oint_{C_n}\frac{dp}{p^2} = -h\overline{p}\oint_{C_n}\frac{d\tilde{E}}{vp^2} = -h\overline{p}\oint_{C_n}\Psi^2\frac{dt}{vp^2} = -h\overline{p}\oint_{C_n}\Psi^2\frac{vdt}{(pv)^2} = nh, \\ \overline{\tau} = \frac{1}{\overline{f}} = \frac{h}{\overline{\tilde{E}}} = \frac{1}{n}\oint_{C_n} d\tau \Rightarrow \overline{\tilde{E}}\oint_{C_n} d\tau = nh \Rightarrow \oint_{C_n}\tilde{E}dt = nh \Rightarrow \oint_{C_n} pdr = \oint_{C_n}\tilde{E}dt = nh, \\ \Psi^2(t) = \frac{d\tilde{E}}{dt} \Rightarrow \overline{\tilde{E}} = \oint_{C_n}\Psi^2(t)dt = h\overline{f} = -\frac{(\overline{\tilde{E}})^2}{n}\oint_{C_n}\frac{vdp}{(pu)^2} = -\frac{(\overline{\tilde{E}})^2}{n}\oint_{C_n}\Psi^2\frac{dt}{(pu)^2} = \\ = \frac{\overline{f}}{n}\oint_{C_n} pdr = \frac{\overline{f}}{n}\oint_{C_n}\tilde{E}dt = -\frac{(\overline{\tilde{E}})^2}{n}\oint_{C_n}\frac{d\tilde{E}}{\tilde{E}^2} = \tilde{m}c^2 \Rightarrow h\overline{f} = \tilde{m}c^2 \Leftrightarrow \frac{c}{\overline{f}} = \frac{h}{\tilde{m}c} = \overline{\lambda}_c. \end{array} \quad (10.5)$$

What we can find in (10.4) and (10.5), in addition to quantizing conditions, is that Compton wavelength $\overline{\lambda}_c$ of certain elementary particle corresponds to the averaged wavelength of its internally packed standing wave, under the condition that such internal standing wave should propagate (on its self-closed orbit) with the speed of photons,

$$\overline{\lambda}_c = \frac{c}{\overline{f}} = \frac{\overline{\lambda f}}{\overline{f}} = \frac{h}{\tilde{m}c} = \overline{\lambda} \,. \tag{10.6}$$

Modeling Matter Waves with Toroidal Structures

In this framework, self-closed domains of standing waves are modeled as structures around thin, large-diameter toroid. This assumption is based on the helical-spinning nature of matter waves and provides several advantages, including simplified mathematical processing and clearer connections to fundamental physics principles. Similar concepts are explored in the works of C. Lucas, David L. Bergman, Paul J. Wesley, Dennis P. Allen, Dr. Günther Landvogt, and M. Kanarev, as discussed in Chapter 4.1 and references [17], [18], [22], [36], [44], and [84]. The matter-wave function $\Psi = \Psi(x, y, z, t) = \Psi(r, t)$, as described in equation (10.5), models stabilized elementary particles distributed around a thin toroidal envelope. Here, C_n represents the median or central line of the toroid. To accurately model this structure, spatial (multidimensional) integration should be applied to equation (10.5). This method acknowledges that the geometry of self-closed standing waves is both complex and multidimensional, leading to additional quantum numbers that support angular and spatial quantization. This approach parallels the use of Wilson-Sommerfeld action integrals, discussed in Chapter 5, particularly around equations (5.4.1) to (5.4.3).

Originally developed within the framework of Niels Bohr's atomic model, Wilson-Sommerfeld action integrals can be analogously applied to matter waves that form elementary particles, as illustrated in equation (10.5). Electrons and protons, with neutrons representing a coupled state between an electron and a proton, are fundamental building blocks of all atoms, molecules, and matter in the universe. Other particles and quasi-particles observed in the micro-world are typically short-lived and unstable, resulting from collisions, field interactions, and the superposition of these elementary particles and matter waves. These represent transient states and sub-structural formations.

Application to Solar and Planetary Systems

The principles of Wilson-Sommerfeld action integrals are also applicable to stable solar and planetary systems, as detailed in Chapter 2 of this book.

Conservation Laws and Quantum Theory

All standing-wave entities must satisfy energy-momentum conservation and continuous symmetry laws, as discussed in Chapter 1. This framework enables the introduction, structuring, and understanding of various real and virtual particles and interaction products. The challenge lies not in the sheer number of elementary particles and quasiparticles, but in addressing potentially confusing or inadequately structured concepts and mathematical models related to particle-wave duality.

Mathematical Modeling and Signal Processing

From a mathematical and modeling perspective, elementary particles, photons, and relevant matter waves should exhibit finite spatial, temporal, and frequency localizations. These should be represented as Gaussian and Analytic Signal pulses and processed using techniques such as the windowed Fourier transform or Gabor transform, as discussed in reference [79].

Wilson-Sommerfeld Action Integrals and Quantization

Wilson-Sommerfeld action integrals, as detailed in reference [9] and Chapter 5 ("Uncertainty"), particularly around equations (5.4.1) to (5.4.3), provide a general rule for quantifying self-closed standing waves that function as energy-carrying structures. These integrals are applied over one complete period of periodic motion on a self-closed stationary orbit and were instrumental in supporting Niels Bohr's Planetary Atom Model.

By extending the Wilson-Sommerfeld action integrals to include all "CHARGE" elements identified in sections T.5.3 and (5.2.1), we can derive quantizing expressions that relate mutually conjugate variables. These expressions align with the "Periodicity of de Broglie Wave Intervals" outlined in T.5.3 and offer significant metrics for various elementary energy states.

Metrics of Elementary Particles:

$$\left\{\begin{array}{l} 2\left|\Delta q_{mag.}\cdot\Delta q_{el.}\right|_{min.} = 2\left|\Delta\alpha\cdot\Delta L\right|_{min.} = 2h\cdot\left|\Delta t\cdot\Delta f\right|_{min} = 2\left|\Delta x\cdot\Delta p\right|_{min.} = 2\left|\Delta t\cdot\Delta\tilde{E}\right|_{min.} = \\ = 2c^2\left|\Delta t\cdot\Delta m\right|_{min.} = 2\left|\Delta s_1\cdot\Delta s_2\right|_{min.} = h, \\ \left\{\lambda = \frac{h}{p} = \tilde{\lambda},\ \tilde{E} = hf = h\frac{1}{\tilde{T}} \Leftrightarrow \tilde{T} = \frac{h}{\tilde{E}}\right\} \Rightarrow \left\{\tilde{\theta} = \frac{h}{L},\ \tilde{\Phi}_{el.} = \frac{h}{\Phi_{mag.}},\ \tilde{\Phi}_{mag.} = \frac{h}{\Phi_{el.}}, \tilde{s}_1 = \frac{h}{s_2}\right\}, \\ [X] = [q_{mag.}, q_{el.}, x, \alpha],\ [Q] = [q_{el.}, q_{mag.}, p, L],\ \Phi_{el.} = q_{el.},\ \Phi_{mag.} = q_{mag.}. \end{array}\right\} \Leftrightarrow$$

$$\Leftrightarrow \left\{\begin{array}{l} \tilde{\Phi}_{mag.}\cdot\Phi_{el.} = \tilde{\Phi}_{el.}\cdot\Phi_{mag.} = \tilde{\lambda}\cdot p = \tilde{\theta}\cdot L = \tilde{T}\cdot\tilde{E} = ... = \tilde{s}_1\cdot s_2 = h, \\ (\tilde{\Phi}_{mag.}\cdot\Phi_{el.})_1 + (\tilde{\lambda}\cdot p)_2 + (\tilde{\theta}\cdot L)_3 + (\tilde{T}\cdot\tilde{E})_4 + ... + (\tilde{s}_1\cdot s_2)_m = mh,\ m = 1,2,3... \end{array}\right\} \Rightarrow$$

$$\Rightarrow [\tilde{X}][Q] = [h] \Rightarrow$$

Wilson-Sommerfeld action integrals

$$\left\{\begin{array}{l} \oint_{C_n} p_\lambda d\lambda = n_\lambda h,\ \oint_{C_n} L_\theta d\theta = n_\theta h, \\ (n_\lambda, n_\theta) = \text{integers } (=1,2,3,...) \end{array}\right\} \wedge \left\{\begin{array}{l} \oint_{C_n} \Phi_{el.} d\Phi_{magn.} = n_{el.}h,\ \oint_{C_n} \Phi_{mag.} d\Phi_{el.} = n_{mag.}h, \\ \oint_{C_n} \tilde{E}_n dt = nh,\ (n_{el.}, n_{mag.}, n) = \text{integers} \end{array}\right\} \Rightarrow$$

$$\Rightarrow \begin{cases} \oint_{C_n} [X]d[Q]^T = \oint_{C_n} [Q]d[X]^T = \oint_{C_n} \tilde{E}dt = n\cdot[X]\cdot[Q]^T = [n]\cdot[Q]\cdot[X]^T = [n]h,\ n = 1,2,3,... \\ [X]^T = [X(t)]^T = \begin{bmatrix} q_{mag.} \\ q_{el.} \\ x \\ \alpha \end{bmatrix}, [Q]^T = [Q(t)]^T = \begin{bmatrix} q_{el.} \\ q_{mag.} \\ p \\ L \end{bmatrix}, [n] = \begin{bmatrix} n_{el.} \\ n_{mag.} \\ n_{\lambda} \\ n_{\theta} \end{bmatrix} \end{cases} \tag{5.4.1}$$

Addressing Macro Particles and Their Characterization

Up to this point, we have primarily discussed the creation of elementary particles from pure standing matter-waves. These initial discussions focus on the internal properties of matter waves, such as their linear momentum and speed, which are not directly observable or measurable. These parameters pertain to the internal, orbital, or self-closed spatial domains where the matter waves form standing waves. We have not yet addressed complex particles, such as atoms or their agglomerations, nor have we considered particles with already existing and stable rest masses.

Once an elementary particle is formed as a stable entity with a standing-wave structure, it becomes externally measurable as a particle with a rest mass. At this point, it can move as a compact object within a surrounding external or laboratory space.

The implications of Wilson-Sommerfeld action integrals, along with the development of the Schrödinger equation as outlined in Chapter 4.3, support the idea that stable elementary particles and matter-wave groups (e.g., electrons, protons, and neutrons) likely possess a circular, self-closed, or toroidal spatial structure. This concept aligns with the work of Bergman and Lucas in “Physical Models of Matter” and “Spinning Charged Ring Model of Elementary Particles” (see references [16] to [22]).

When such a particle is created, it exhibits dualistic properties: it possesses both “internal” standing-wave characteristics and “external” matter-wave properties. The particle is characterized by measurable parameters such as mass, linear and angular momentum, and kinetic energy. In some cases, additional properties like electric and magnetic charges, potentials, and currents may also be relevant.

The particle's external parameters, including speed, momentum, kinetic energy, and matter-wave wavelength (as detailed in equation (10.7)), relate to its motion in a laboratory system. These should not be confused with the internal, orbital, and standing-wave parameters discussed in equations (10.1) to (10.6).

$$\begin{aligned} & m = \tilde{m}, p = \gamma mv, E_k = \tilde{E} = (\gamma - 1)mc^2 = E - E_0 = hf, \lambda = \frac{h}{p}, \gamma = (1 - \frac{v^2}{c^2})^{-\frac{1}{2}} \\ & P_4(p, \frac{E}{c}) = \text{invariant to ref. system} \Rightarrow p^2 - \frac{E^2}{c^2} = -\frac{E_0^2}{c^2}, E_0 = mc^2, E = \gamma mc^2. \end{aligned} \tag{10.7}$$

Until we have only certain matter narrow-band wave (still not transformed or structured in a particle that has rest mass), we can apply the total energy equivalency between wave and effective mass-energy in some of the forms as,

$$\left(E_{total} = \tilde{E} = E_k = \tilde{m}c^2 = hf\right) \Rightarrow dE_{total} = d\tilde{E} = dE_k = c^2 d\tilde{m} = hdf. \tag{10.8}$$

Once when the same wave is structured as a stabilized, self-closed (circular), standing waves formation, we can consider that rest mass is created (meaning that former $\tilde{m}$ will start its new, kinematic existence as $m_0 = \tilde{m} = m$, having also certain (external) speed and kinetic energy, and if we monitor such particle as a whole, externally (in a Laboratory system), we will consider that $dm_0 = dm = 0$. The kinetic energy of such particle is now presenting associated, de Broglie, or matter wave energy,

$$\begin{aligned}&\left(E_{total} = mc^2 + \tilde{E} = mc^2 + E_k,\ \tilde{E} = E_k\right) \Rightarrow \\ &dE_k = d\tilde{E} = dE_{total} = vdp = hdf,\ p = \gamma mv.\end{aligned} \tag{10.9}$$

Matter wave frequency in (10.9) is no more comparable or equal to the frequency from (10.8), meaning that now (10.9) starts to be a relevant framework. In most of our analyses of particles motions and interactions with other particles (in Physics or Mechanics), we are starting from (10.7), meaning from already created particles (and not asking what kind of standing waves structures are packed inside).

For macro-mass formations like stabilized planetary and galactic systems with circular, elliptic or other self-closed orbits, and similar to toroidal or helical motions, Planck constant h is likely to be replaced by another macro-world $H >> h$ constant, respecting validity of similar matter-waves and particle-wave duality concepts and mathematics known for micro-world situations (see Chapter 2., " 2.3.3. Macro-Cosmological Matter-Waves and Gravitation").

The most significant (experimentally confirmed) examples supporting concepts elaborated here are the creation of an electron + positron couple from sufficiently high-energy photons, and total annihilation of an electron + positron creating only photons. Other supporting examples are related to familiar phenomenology of Compton and Photoelectric effects, secondary emissions, and atoms excitations by absorbing or emitting photons, presenting foundations and step stones of Wave-Particle Duality of matter.

For instance, in chapter 4.2 of this book ("4.2.2. Example 3: Elastic collision photon-particle (Compton Effect)"), we can find experimentally verifiable arguments ***why and how matter-wave energy is equal to a relevant particle kinetic energy***.

The process which is opposite to (or inverse of) Compton Effect is the continuous spectrum of x-rays (of photons) emission, caused by impacts of electrons (accelerated in the electrical field between two electrodes) with the anode as their target. The emission of x-ray photons starts when the electrons are abruptly stopped on the anode surface. If the final, impact electron speed is non-relativistic, $\mathbf{v} << \mathbf{c}$, the maximal frequency of the emitted x-rays is found from the relation: $hf_{max.} = \frac{1}{2}mv^2$, and in cases of relativistic electron velocities, we have $hf_{max.} = (\gamma - 1)\, mc^2$ (and both of them are experimentally confirmed to be correct, being also kinetic or motional energy, fully in compliance with other PWDC relations, as (10.1), (10.2) and 10.2-1). If we now consider electrons (before the impact happen) as matter waves, where the electron matter-wave energy corresponds only to a kinetic or motional energy, without rest-mass energy content, we will be able to find de Broglie, matter wave frequency of such electrons (just before their impact with the anode). With such impact execution, the electrons are fully stopped, and the energy content of their matter waves is fully transformed and radiated in the form of x-ray photons (or into another form of waves, or motional energy), whose frequency corresponds to the matter wave frequency of electrons in the moment of the impact. This equality of the frequencies of radiated x-ray photons and electron matter-waves (in the moment of impact) explains the essential nature of electron matter waves (eliminating the possibility that the rest mass belongs to matter-wave energy content, except in particle-antiparticle annihilations). Similar experimental observations (about the kinetic energy of incident electrons and energy of generated x-rays) are also known for elastic scatterings or bremsstrahlung x-rays. An excellent theoretical background that explains equivalency between kinetic and matter waves (motional) energy is presented in [105], Himanshu Chauhan, Swati Rawal, and R K Sinha. WAVE-PARTICLE DUALITY REVITALIZED, CONSEQUENCES, APPLICATIONS, AND RELATIVISTIC QUANTUM MECHANICS. *Contemporary Quantum Theory is dominantly supporting wave equations, matter waves, and wave-particle duality concepts based on Classical Mechanics, non-relativistic motions, and assumptions, when v <<< C, and when involved rest mass is included in a matter wave content, what is producing bizarre and useless (or incorrect) result that phase velocity could be higher than speed of light C. See an explanation under (4.10-5) in chapter 4.3.*

Before we can fully explore the conceptual and mathematical possibilities related to wave functions and wave equations, it is essential to establish a stable and accurate foundation for particle-wave duality, as summarized in sections 10.1 to 10.9. Using the Analytic Signal model, we can then extend our understanding in ways that align more closely with contemporary quantum theory (QT). Unfortunately, generations of scientists have approached microphysics from a perspective clouded by statistical and probabilistic assumptions. These assumptions have led to the creation of an artificially complex and ad-hoc QT, without a clear foundation rooted in elementary physics. While the founders of QT crafted a mathematically coherent theory, the result is far from an easily understandable, realistic, and intuitively manageable physics theory. Such a theory should not rely on numerous ad-hoc rules, freestanding parameters, and assumptions.

We now recognize that de Broglie's matter-wave wavelength and other wave-particle duality concepts are not just abstract ideas but are consequences of unity, coupling, synchronization, and complementary relationships between linear and angular (or spinning) motions in two- or multi-body systems (as discussed in Chapter 4.1). During de Broglie's time, it was perhaps necessary to postulate wave-particle duality based on intuition and experimental fits, but today we can explain these phenomena more naturally and deterministically using the Analytic Signal framework, which is grounded in clear concepts of basic physics and mathematics. Unfortunately, many QT textbooks still present these ideas dogmatically, as they were during de Broglie's time.

We now understand that the Schrödinger equation, along with other well-known wave equations in contemporary quantum theory such as the Klein-Gordon and Dirac equations, can be seen as natural, logical extensions of d'Alembert's and the Classical Wave Equation. In this framework, the wave function is treated as a Complex Analytic Signal function (see more in Chapter 4.3). However, in most quantum theory textbooks, the Schrödinger equation is still presented as a fundamental, postulated step, often portrayed as a unique, almost divine inspiration and invention of E. Schrödinger. While this perspective may have been justified in Schrödinger's time, we now know that even electromagnetic wave equations, as classical wave equations, can be easily transformed into the Schrödinger equation (see Victor Christianto's "Review of Schrödinger Equation & Classical Wave Equation"). Many forms of classical wave equations, similar or identical to those used in quantum theory, are also found in acoustics, fluid dynamics, and mechanical oscillations (see Hubert & Lumbroso), suggesting that all matter waves and oscillations in different physical phenomena share the same mathematical and physical foundations.

It is also important to remember that the same equations describing group and phase velocity for all matter waves, as shown in sections 10.2 and 10.4, are directly related to all Classical Wave Equations. The Complex Analytic Signal-based Schrödinger wave equation is fully equivalent to the Classical Wave Equation, and it can be seen as integrated into or derived from it. All concepts, results, and relationships regarding the family of Schrödinger wave equations are summarized in this chapter (for instance, around sections 10.1 to 10.5 and in Chapter 4.3 summarized by equation 4.25).

$$\begin{gathered}
\Delta\overline{\Psi}-\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=\left(\frac{L}{\hbar u}\right)^2\overline{\Psi}+\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=j\frac{L}{\hbar u^2}\frac{\partial\overline{\Psi}}{\partial t}+\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=0,\\
-\frac{\hbar^2}{\tilde{m}}\left(\frac{u}{v}\right)\Delta\overline{\Psi}+L\overline{\Psi}=0\;;\;\Delta\overline{\Psi}=\frac{1}{u^2}\frac{\partial^2\overline{\Psi}}{\partial t^2},\;\left(\frac{L}{\hbar}\right)^2\overline{\Psi}+\frac{\partial^2\overline{\Psi}}{\partial t^2}=0,\;\frac{L}{\hbar}\overline{\Psi}=j\frac{\partial\overline{\Psi}}{\partial t},\\
L\in\left[\ldots\boxed{\tilde{E}-U_p}\text{ or }\boxed{\tilde{E}}\text{ or }\boxed{\tilde{E}+E_0-U_p}\text{ or }\boxed{\tilde{E}+E_0}\text{ or }\boxed{\tilde{E}+E_0+U_p}\text{ or }\boxed{\tilde{E}+U_p}\ldots\right]\\
S=\int_{t_1}^{t_2}L(q_i,\dot{q}_i\ldots,t)dt=\text{extremum}\,.
\end{gathered}\tag{4.25}$$

Starting from equation (4.25), we can derive all other wave equations known in Quantum Theory (QT) by translating the basic Lagrangian function for a specific, relatively fixed or stable energy level. This translation can be done in both directions, either inwards or outwards, depending on the energy configuration we wish to analyze. For a more detailed discussion on these topics, refer to Chapter 4.3.

As we can conclude, it is not necessary to rely on probabilistic or statistical assumptions, ontological conjectures, or artificial mathematical postulates to develop the well-known equations of QT. These equations can be derived smoothly and causally, remaining largely within the framework and principles of Classical Physics and Mathematics.

However, to align with Orthodox Quantum Theory, we can reinterpret the derived wave equations and wave functions through appropriate mathematical normalization (to remove their dimensionality). This normalization process allows us to assign a probabilistic meaning to these equations, as the Parseval theorem mimics the law of energy conservation. From the perspective of Classical Physics and the deterministic approach favored here, this reinterpretation opens the door to conceptualizing a

Unified Fields and Forces Theory by integrating QT with String Theory and Quantum Field Theory.

[♣ COMMENTS AND FREE-THINKING CORNER (still working on this project):

Project Title: Real Matter-Wave and Mass Formation Explained by the Effects of Ultrasonic and Mechanical Excitation of Weakly Coupled Electron States in Electro-Conductive Materials, in Relation to Wave-Particle Duality.

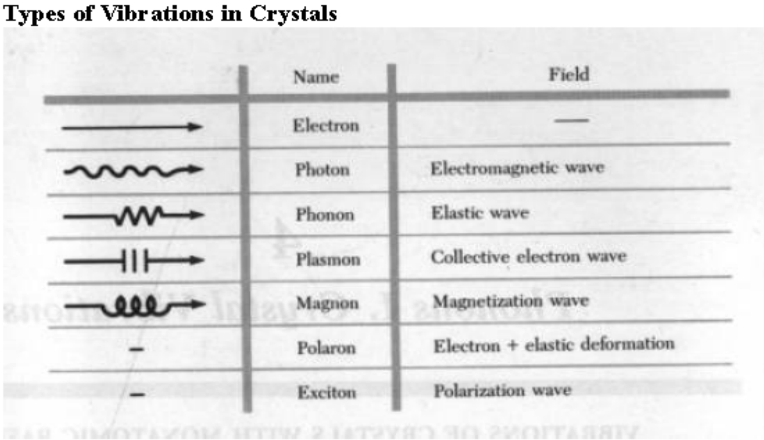
This project explores the hypothesis that it is possible to influence, excite, and enhance the mobility of weakly coupled and relatively free electron states through ultrasonic methods, both acoustically and mechanically. The proposal suggests that by vibrating certain electrically conductive materials, such as electrodes, galvanic or electrochemical cells, and similar metal components, where electric charges, voltages, and currents are generated, consumed, charged, or discharged, we can achieve these effects.

It is currently understood that ordinary mechanical, acoustic, or ultrasonic excitation of a solid object does not typically produce these results. However, the mechanical or electromechanical ultrasonic excitation of specific natural, internal, parametric, and other resonant states within electro-conductive materials, at specific resonant frequencies, could indeed increase the mobility of free electrons and generate additional electric current, while also influencing associated voltages and charges.

We will begin by referencing the following citation (see below), which outlines an approximate model of metals, crystals, electrically conductive materials, and similar solid structures ([119]).

Chapter 4 : Phonons and Crystal vibration

Types of Vibrations in Crystals



	Name	Field
	Electron	—
	Photon	Electromagnetic wave
	Phonon	Elastic wave
	Plasmon	Collective electron wave
	Magnon	Magnetization wave
–	Polaron	Electron + elastic deformation
–	Exciton	Polarization wave

Normal modes of crystal vibrations

Many solid materials, including all metals, are composed of atoms arranged in a lattice arrangement called crystals. There are a variety of crystal structures like cubic, hexagonal, cubic with an atom in the center of the cube, called body centered cubic, cubic with an atom in the center of each face of the cube, called face centered cubic, and others. The particular structure depends on the relative sizes of the atoms that are nestled together to form the crystal. The reason that materials take crystal form is that these neat geometrical structures represent the lowest energy configuration of the collection of atoms making up the material. To dislodge an atom from the crystal structure requires the addition of energy.

Theoretically, at a temperature of absolute zero, the atoms of a crystal lie at their lowest energy position without moving at all. As thermal energy is added to the crystal it is manifest by vibration of the atoms about this equilibrium location. Within the limits of fairly small vibrations the electric forces bonding the atoms together stretch or compress a bit to a higher energy configuration. Each atom acts as though it were connected to its neighbors by little springs. The added energy is stored in the crystal as the kinetic energy of the atoms in motion and the potential energy of the compressed or stretched springs.

Assumptions and Hypotheses Foundations

1. Matter-Wave Groups as Wave-Packets:

Electrons, photons, phonons, plasmons, magnons, solitons, and similar entities are all matter-wave groups or wave-packets of specific forms of matter and fields. These wave-packets possess motional energy, akin to the energy-momentum properties of moving particles. Up to a certain extent, these wave-packets can be analogously treated as particles, characterized by group and phase velocity, wavelength, frequency, angular momentum, and linear momentum. This concept aligns with the wave-particle duality described in contemporary physics.

2. Unified Mathematical Treatment:

The properties of these matter-waves and their analogy with corresponding particles can be managed using the same mathematical framework, particularly through Analytic Signal modeling. This approach supports the idea of wave-particle duality without necessitating separate mathematical or physical theories for each entity (e.g., electrons, photons, phonons, etc.). While probabilistic and statistical methods are useful for modeling large sets of identical entities and events, they are not essential or exclusive for the fundamental understanding of these wave-packets. In complex real-world structures and motions, many of these matter-wave packets can coexist and interact, often being electromagnetically and electromechanically coupled.

3. Electromechanical Coupling in Matter:
Mechanical and acoustic vibrations in solid, liquid, and gaseous media involve the oscillatory motion of masses, molecules, and atoms, which inherently possess electromagnetic properties due to their charged particles and magnetic moments. Consequently, any mechanical, acoustic, or thermodynamic vibration or motion is electromechanically coupled with the internal electromagnetic nature of atoms, thereby involving or generating electrons, photons, phonons, plasmons, magnons, solitons, and other matter-waves. This coupling also extends to interactions with external electromagnetic radiation, as seen in phenomena like the photoelectric effect, Compton scattering, and photovoltaic electricity generation. When exposed to electromagnetic radiation, particularly in the infrared-to-microwave frequency range, these interactions result in heating and thermodynamic effects, causing random oscillations in atoms and molecules, which are themselves electromechanical perturbations of existing resonant states within the material.

4. Mass-Spring Matrix of Solid Structures:
Metals, solid bodies, and crystals can be modeled as mass-spring systems, where the masses represent atoms or molecules, and the springs represent attractive forces such as Van der Waals forces, cohesion, adhesion, and electrostatic and magnetostatic interactions. This mass-spring matrix exhibits natural, parametric, and resonant frequencies that are electromechanically coupled. The spatial periodicity of this structure is akin to standing electromagnetic waves, and it extends to larger scales in the universe, as conceptualized by thinkers like Nikola Tesla and Rudjer Boskovic.

5. Simplified Modeling of Metals:
In simplified models, atoms within solid structures are stable particles (or nodes) connected by springs, enveloped by clouds of relatively free electrons or weakly connected electron states. This model underlies the phenomenon of electrical conductivity and other matter-wave interactions within metals. Many matter-wave packets coexist and interact within the same solid body, and solutions to classical wave equations often appear in pairs, corresponding to inward and outward traveling waves. These wave-packet pairs, such as electrons and photons, are created as coupled entities that communicate with each other, suggesting a broader concept of entanglement that contemporary physics has yet to fully embrace.

6. External Excitation of Mass-Spring Structures:
By applying external mechanical or ultrasonic excitation to the mass-spring matrix of an electro-conductive material, such as through a piezoelectric transducer or laser pulsing, we can excite its resonant states. The most effective excitation will occur at frequencies that match the natural and parametric resonant states of the material, typically in the MHz range. This resonance will cause the atoms or masses to oscillate with large amplitudes, velocities, and forces, which in turn increases the mobility of surrounding free electron clouds and affects electrical properties such as current and voltage. This process also induces heating and structural changes in the material.

7. Experimental Evidence and Applications:

Experimental evidence already shows that MHz-domain ultrasonic excitation at the appropriate resonant frequencies significantly enhances the current and voltage produced by electrodes in batteries and optimizes charging and discharging properties. Similar effects are observed when ultrasonic vibrations are applied to electrolytes in electrochemical and galvanic cells, as well as when laser or photonic excitation is used on liquids and solids, resulting in acoustic, thermal, and matter-wave effects.

8. Mass-Spring Structures as Standing Waves:

The mass-spring structure of matter can be viewed as a standing wave formation, where atoms occupy stable nodal positions and springs represent balanced electromagnetic attractions. The distance between neighboring atoms can be related to the half-wavelength of a mechanical or acoustic resonance λ_{em}. External ultrasonic excitation at the appropriate resonant frequency f_{us} will significantly influence this structure, generating heat and other effects. Similarly, external electromagnetic radiation, when its wavelength matches the mechanical resonance wavelength, will produce analogous heating effects due to strong electromechanical coupling.

This framework allows us to estimate the relationship between the relevant ultrasonic frequency and the frequency f_{em} of external electromagnetic radiation needed to produce the same heating effects within the mass-spring electromechanical resonators.

$$\left\{\begin{array}{l} C_i = \lambda_i f_i \neq C_j \,,\;\; \lambda_i = C_i / f_i = C_j / f_j = \lambda_j (=) \text{ intuitive and initial assumption} \\ C_{em} = 3\cdot 10^8 \left[\frac{m}{s}\right] (=) \text{ speed of electromagnetic waves in vacuum} \\ C_i = C^*_{em} \in \left(10^8 \,,\, 3\cdot 10^8\right)\left[\frac{m}{s}\right] (=) \text{ range of estimated speeds of electromagnetic waves in} \\ \qquad\qquad\qquad\qquad \text{different materials} \\ C_j = C_{us} \in \left(10^3 \,,\, 5\cdot 10^3\right)\left[\frac{m}{s}\right] (=) \text{ estimated speed of ultrasonic waves in solids and liquids} \\ f_{em} (\approx) 10^{12} \text{ Hz } (=) \text{ some frequency between infrared and microwave radioation} \\ \lambda = \frac{C_{us}}{f_{us}} \cong \frac{C^*_{em}}{f_{em}} \approx \lambda_{em} \approx \lambda_{us} \end{array}\right\} \Rightarrow$$

$$\Rightarrow f_{us} \cong \frac{C_{us}}{C^*_{em}} f_{em} \in f_{em}\cdot(0.333 \,,\, 5)\cdot 10^{-5} \text{ Hz } (\approx)\; 10^{12}\cdot 10^{-5}\cdot(0.333 \,,\, 5)\text{Hz } (\approx)$$

$$(\approx)\;(0.333 \,,\, 5)\cdot 10^{7}\,\text{Hz} \approx (3.33, \, 50)\,\text{MHz} \,.$$

Multifrequency Resonance Effects and Their Applications

It is well-established that mechanical, ultrasonic, electromechanical, electromagnetic, molecular, and atomic resonance effects in certain objects occur over different frequency ranges. For instance, when an object is subjected to high-frequency electromagnetic radiation that aligns with its molecular and atomic natural resonant frequencies—especially when this radiation is periodically frequency-modulated within the same range—and simultaneously exposed to low-frequency modulation (such as amplitude or pulse-width modulation) targeting the lower mechanical or ultrasonic resonant frequencies, we can expect unusual effects. These effects may remain stable under low to moderate oscillation amplitudes, but once a certain amplitude threshold is exceeded, the multilevel resonance could become nonlinear and destructive. This may lead to the decomposition or softening of the material, akin to what is described in the John Hutchison effect (assuming this effect is independently verified, it would serve as an excellent example; see [143]).

The ideas presented here could spark the development of new applications and technologies. By using acoustics and electromagnetic waves—either separately or in combination at different resonant frequencies—we can achieve effects such as mass heating, stress relief, and the transformation and modification of the electrical and mechanical properties of solid objects.

Applications and Expected Outcomes

1. Enhanced Electrical Properties in Batteries and Electrochemical Systems:
We should anticipate improvements in electrical properties when applying acoustic or ultrasonic enhancements in contexts like battery charging and discharging, optimizing electrochemical and galvanic reactions, electrolysis, and even maximizing the Compton and photoelectric effects. Similar enhancements can be expected in laser technologies, semiconductors, and photovoltaic cells. For example, electro-conductive metals and other materials subjected to resonant excitation in the MHz range might exhibit variations in electrical resistance, like NTC (Negative Temperature Coefficient) and PTC (Positive Temperature Coefficient) resistive elements. In other words, the internal mechanical resonant states of a material are inherently coupled with similar electromagnetic resonant states.

2. Electrostrictive and Magnetostrictive Effects:
Electrostrictive (piezoelectric) and magnetostrictive properties of materials are closely related to the electromechanical couplings and spatial mass-spring structures discussed here. In the case of resonant excitation of photovoltaic (solar) cells, we could hypothetically expect an increase in electron flow generated by incident photons. This would occur because the electromechanical and parametric resonance effects might periodically reduce the gaps between nonconductive and conductive zones, thereby increasing the electron current. As a result, the efficiency of solar cells could be significantly enhanced.

3. Optimization of Energy Efficiency:
The positive effects of ultrasonic resonant optimization and the resulting improvements in energy efficiency, or electrochemical activity, of batteries, accumulators, and galvanic electric sources, are already well-documented and experimentally confirmed.

Thermal and Ultrasonic Activity Interactions

There is a notable similarity between thermal motions and ultrasonic matter agitation. Thermal activity can be described as chaotic, random, and omnidirectional oscillatory motion, while ultrasonic agitation tends to be more organized, harmonic, and periodic. Nonetheless, the effects of thermal and ultrasonic activities on materials can be similar and may even interfere with one another. By lowering the temperature of a material (including liquids and gases) and/or applying ultrasonic resonance, especially when specific resonant states of the material are activated, we could potentially induce effects like superconductivity, superfluidity, and Bose-Einstein condensation. These phenomena are indeed experimentally observed, indicating that such interactions are feasible.

Let us now demonstrate how we could compare or unify wave functions concept and Classical wave-equation with electric circuit analysis. For instance, in chapter 1. (elaborating about analogies in physics), for the series connection R-L-C circuit, Kirchoff's Voltage Law states (see equation (1.1): "**The sum of all the *across element voltage differences* in a loop is equal to zero.**" Based on the situation in Fig.1.1 a), we shall have:

$$\sum \mathbf{u_i} = 0,\ \mathbf{u} = \mathbf{u_L} + \mathbf{u_R} + \mathbf{u_C} = \mathbf{L}\frac{\mathbf{di}}{\mathbf{dt}} + \mathbf{Ri} + \frac{1}{\mathbf{C}}\int \mathbf{idt} = \mathbf{L}\frac{\mathbf{d^2q}}{\mathbf{dt^2}} + \mathbf{R}\frac{\mathbf{dq}}{\mathbf{dt}} + \frac{\mathbf{q}}{\mathbf{C}}, \qquad (1.1)$$
$$(\mathbf{i} = \mathbf{i_L} = \mathbf{i_R} = \mathbf{i_C} = \frac{\mathbf{dq}}{\mathbf{dt}}),$$

To create deterministic and dimensional (power-related) wave function (as conceptualized in this book; -see equations (4.0.82) in chapter 4.0), let us multiply both sides of the equation (1.1) with the relevant current i. This way we will get,

$$[u]\cdot i = \psi^2 = \left[L\frac{di}{dt} + Ri + \frac{1}{C}\int idt\right]\cdot i = \left[L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C}\right]\cdot i \Rightarrow$$

$$\psi^2 = Li\frac{di}{dt} + Ri^2 + \frac{i}{C}\int idt = Li\frac{d^2q}{dt^2} + Ri\frac{dq}{dt} + i\frac{q}{C} = L\frac{dq}{dt}\frac{d^2q}{dt^2} + R\left(\frac{dq}{dt}\right)^2 + \frac{q}{C}\frac{dq}{dt}$$

$$\Rightarrow 2\psi\frac{\partial\psi}{\partial t} = \frac{\partial}{\partial t}\left[L\frac{dq}{dt}\frac{d^2q}{dt^2} + R\left(\frac{dq}{dt}\right)^2 + \frac{q}{C}\frac{dq}{dt}\right] \Rightarrow$$

$$\frac{\partial\psi}{\partial t} = \frac{1}{2\psi}\frac{\partial}{\partial t}\left[L\frac{dq}{dt}\frac{d^2q}{dt^2} + R\left(\frac{dq}{dt}\right)^2 + \frac{q}{C}\frac{dq}{dt}\right] = \frac{1}{2\psi}\frac{\partial\psi^2}{\partial t}$$

$$\frac{\partial^2\psi}{\partial t^2} = \frac{1}{2}\frac{\partial}{\partial t}\left(\frac{1}{\psi}\right)\frac{\partial\psi^2}{\partial t} + \frac{\partial\psi}{\partial t} = -\frac{1}{2}\left(\frac{1}{\psi^2}\frac{\partial\psi}{\partial t}\right)\frac{\partial\psi^2}{\partial t} + \frac{\partial\psi}{\partial t} = \frac{\partial\psi}{\partial t} - \frac{1}{\psi}\left(\frac{\partial\psi}{\partial t}\right)^2$$

We already know that Classical wave equation (see more in chapter 4.3) has the following form, $\frac{1}{c^2}\frac{\partial^2\Psi}{\partial t^2} = \frac{\partial^2\Psi}{\partial r^2} (=)\nabla^2\Psi,\ c = \omega/k\,(=)$ phase velocity ,
where c should not be mixed with the constant velocity of electromagnetic waves or photons. The usual symbol for a phase velocity in this book is u, but since here we are using u for voltage, we selected c for a phase velocity. Of course, again we will consider mutual relations between phase and group velocity as overwhelmingly present in this book, $c = \omega/k,\ v = d\omega/dk,\ v = c - \lambda dc / d\lambda$.

In fact, we know that temporal and spatial wave-function domains (and relevant spectral functions) of the same event are mutually coupled or related (as already elaborated in this chapter). This way, judging mostly by intention to establish an analogy, we will create an extension of Kirchoff's Voltage Law to a certain (here relevant, but still not specifically described) spatial coordinate system. We also know that the same example of series connection of capacitive, inductive, and resistive elements (R-L-C), has its natural resonant frequency where relevant current and voltages are oscillating simple-harmonically (meaning that in a time domain we already have oscillations or matter waves). Corresponding, analogically produced Classical wave equation (for the same example) will be,

$$\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} = \frac{1}{c^2}\left[\frac{\partial\psi}{\partial t} - \frac{1}{\psi}\left(\frac{\partial\psi}{\partial t}\right)^2\right] = \frac{\partial^2\psi}{\partial r^2} (=) \nabla^2\psi = \Delta\psi = -k^2\psi,\ c = \omega/k,$$

where ψ describes kind of flow of relevant electric power, charge, or electric field, since here,

$$\psi^2 = Li\frac{di}{dt} + Ri^2 + \frac{i}{C}\int idt = L\frac{dq}{dt}\frac{d^2q}{dt^2} + R\left(\frac{dq}{dt}\right)^2 + \frac{q}{C}\frac{dq}{dt}.$$

Of course, additional elaborations will be necessary to develop, extend, and generalize here introduced ideas about wave functions and relations between spatial and temporal domains (since we will need to organize the same electric circuit as an electric waveguide line), but intuitive and analogical grounds for such projects are already paved.

Everything relevant in our physics is about networks and circuits of electric and mechanical active signals and waves, such as currents, voltages, forces, velocities, power, energy etc. This is the largest platform of relevance for analyzing and using wave functions and wave equations in physics. This way we can extend our conceptualizations in physics from simple and easy, perceptually tangible events and objects in motion, towards a higher level of mathematically constructed systems and models.

Uncertainty Relations and Quantum Theory: A Critical Perspective

It is now understood that various forms of uncertainty or inequality relations are closely related to ordinary mathematics, specifically in the context of signal sampling, signal reconstruction, and spectrum analysis. These relations are universally valid and applicable to any signal or wave function, extending from the micro to the macro universe. Heisenberg's Uncertainty Principle, often heralded as a groundbreaking insight, is a straightforward mathematical consequence of well-known inequality relations between the durations of original and transformed domains of a wave function or signal. This reveals natural unity and effective mathematical modeling between physics and mathematics. However, many quantum theory textbooks incorrectly present Heisenberg's discovery as a unique and divine insight, applicable only to the microphysical world. For a more detailed discussion, refer to Chapter 5.

Probability Theory and Quantum Theory

The success of contemporary quantum theory (QT) is largely attributed to the universal applicability of Parseval's theorem, along with the conceptual compatibility between normalized and averaged forms of conservation laws in physics (such as energy and momentum conservation). This is underpinned by the trivial fact that the sum of all probability of an event equals one, representing the total probability of the event. The surrounding mathematical framework, which underpins the probabilistic foundation of QT, has been complex and demanding, yet necessary for making the theory work effectively (see more in "4.3.3. Probability and Conservation Laws" from Chapter 4.3).

Signal processing, spectrum analysis, statistics, and probability theory have developed independently as robust and well-established fields, making them ideal tools for integration into contemporary QT. When conditions naturally favor the application of these fields to events involving large numbers of similar or identical entities, they provide a framework for accurate modeling and conclusions in physics and other natural sciences. Parseval's theorem, for instance, is universally applicable in such situations, effectively mimicking the energy conservation law through the normalization of the wave function, akin to a total probability law.

Another important consideration is that stable molecular, atomic, and subatomic structures are typically spatially structured standing matter waves with quantized, relatively stable periodicity and internal symmetry. When such natural periodicity and causality exist, it is possible to construct mathematically operational but conceptually flawed theories, akin to the geocentric Ptolemaic system, that fit the observed phenomena by inventing and associating rules that complement the natural periodicity. A prime example of this is Bohr's model of the hydrogen atom, which, despite being conceptually flawed, yields many useful results (as do other atomic models; see Chapter 8).

The Limitations of Contemporary Quantum Theory

Many of the abstract and seemingly magical consequences of contemporary QT arise from an invented, hybridized, and partly artificial mathematical framework. Although this framework works well in practice, it should not be glorified as a divine or ultimate achievement of modern physics. The founders of QT successfully manipulated their subjects by cleverly combining several grandiose, independent, and universally valid mathematical theories. However, presenting this combination as a unique and brilliant product of QT misrepresents the true nature of these mathematical tools. While these tools are undoubtedly powerful, their successful application does not guarantee the creation of a perfect and eternally valid theory.

Despite the frequent claims that contemporary QT is continuously and fully confirmed, these confirmations are, in fact, equivalent to saying that statistics and probability theory, when appropriately merged with signal and spectral analysis, function as expected. This is a trivial statement, universally valid across all scientific disciplines wherever these mathematical tools are correctly applied.

Rethinking Quantum Theory

In summary, it was not necessary to hastily and exclusively conclude that everything in the microphysical world is fundamentally probabilistic and statistical in nature. This approach led to the creation of an artificial, hybridized mathematical theory, which then required the reintroduction of well-known physical laws, such as the principle of least action and Hamilton-Lagrange formalism, to make it work effectively. However, by applying signal analysis and particle-wave conceptualization, as elaborated in this book, we find that ordinary mathematics, which directly represents tangible physical phenomena, provides much clearer and more explicit insights into wave-particle duality and matter waves than probability-based intermediary transformations and concepts.

Probability and statistics are indeed powerful mathematical tools, but they should be applied primarily in contexts where they naturally belong, such as in thermodynamics, where they describe and model large statistical sets. When creating artificial mathematical concepts based on statistics, we can always normalize the involved wave functions, creating non-dimensional functions that allow for the application of probability and statistical methods. This approach, when combined with Parseval's theorem, can produce a robust theory that mirrors important classical physics laws.

Orthodox Quantum Theory vs. the Concepts in This Book

The differences and similarities between orthodox QT and the concepts promoted in this book can be summarized as follows:

1. Foundation of Quantum Theory: Orthodox QT, as developed by the Copenhagen interpretation, relies heavily on statistics and probability, merging these concepts with Fourier signal analysis. In contrast, this book promotes a mathematical model based on power-related wave functions and energy-moments functions, grounded in analytic signals and the Kotelnikov-Whittaker-Nyquist-Shannon sampling theorem (see Chapters 4.0, 4.1, and 4.3). This approach provides a clearer and more causal framework for analyzing wave-particle duality.

2. Quantization: Orthodox QT intuitively postulates and generalizes quantization to fit the laws of Fourier signal analysis and statistics. This book, however, links quantization to standing-wave segments and energy-moments formations, respecting the sampling and signal reconstruction concepts from the Kotelnikov-Whittaker-Nyquist-Shannon theorem (see Chapters 4.0 and 8).

3. Energy and Matter-Waves: Orthodox QT often implies that the total energy of a particle represents its total matter-wave content, leading to the erroneous conclusion that phase velocity can exceed the speed of light. This book clarifies that only kinetic energy corresponds to matter-wave energy, ensuring that both group and phase velocities remain below the speed of light. The failure to incorporate complex analytic signal functions into the analysis of motional masses and particles leaves classical and quantum physics "theoretically blind" to the complexity of matter structures (see the end of Chapter 4.0).

4. Wave-Particle Duality: Orthodox QT merges wave-particle duality with statistics, probability, and Heisenberg's Uncertainty Principle. In contrast, this book emphasizes the Particle-Wave-Duality-Concept (PWDC), which is developed without relying on statistical and probability-based interpretations (see Chapter 5). By focusing on mathematical inequalities between original and spectral signal durations, this book offers a more natural and deterministic understanding of wave-particle duality.

5. Schrödinger Wave Equation: Orthodox QT treats the Schrödinger wave equation as a divine insight of E. Schrödinger. This book demonstrates that Schrödinger's wave equation, like other wave equations in quantum physics, can be derived from the universally applicable classical wave equation when the wave function is treated as a complex, analytic signal (see Chapter 4.3).

In many publications related to QT, the similarities and differences between traditional and innovative concepts are often creatively mixed, leading to confusion. Only through careful and selective analysis can we determine when and why the concepts promoted in this book offer a better understanding of wave-particle duality.

Self-organizing synchronizations of motions and matter states in Physics (or in our Universe), including living species, are noticeably clear and significant, empirical (experimentally verifiable) manifestations of matter waves and particle-wave duality.

Citation from Wikipedia: https://en.wikipedia.org/wiki/Self-organization .
Self-organization, also called (in the social sciences) spontaneous order, is a process where some form of overall order arises from local interactions between parts of an initially disordered system. The process is spontaneous, not needing control by an external agent. It is often triggered by random fluctuations, amplified by positive feedback. The resulting organization is wholly decentralized, distributed over all the components of the system. As such, the organization is typically robust and able to survive or self-repair substantial perturbation. Chaos theory discusses self-organization in terms of islands of predictability in a sea of chaotic unpredictability.

Self-organization occurs in many physical, chemical, biological, robotic, and cognitive systems. Examples of self-organization include crystallization, thermal convection of fluids, chemical oscillation, animal swarming, and artificial and biological neural networks.

Self-organization is realized[2] in the physics of non-equilibrium processes, and in chemical reactions, where it is often described as self-assembly. The concept has proven useful in biology,[3] from molecular to the ecosystem level.[4] Cited examples of self-organizing behavior also appear in the literature of many other disciplines, both in the natural sciences and in the social sciences such as economics or anthropology. Self-organization has also been observed in mathematical systems such as cellular automata.[5] Self-organization is not to be confused with the related concept of emergence.[6]
Self-organization relies on three basic ingredients:[7]
strong dynamical non-linearity, often though not necessarily involving positive and negative feedback
a balance of exploitation and exploration
multiple interactions

The cybernetician William Ross Ashby formulated the original principle of self-organization in 1947.[8][9] It states that any deterministic dynamic system automatically evolves towards a state of equilibrium that can be described in terms of an attractor in a basin of surrounding states. Once there, the further evolution of the system is constrained to remain in the attractor. This constraint implies a form of mutual dependency or coordination between its constituent components or subsystems. In Ashby's terms, each subsystem has adapted to the environment formed by all other subsystems.[8]
The cybernetician Heinz von Foerster formulated the principle of "order from noise" in 1960.[10] It notes that self-organization is facilitated by random perturbations ("noise") that let the system explore a variety of states in its state space. This increases the chance that the system will arrive into the basin of a "strong" or "deep" attractor, from which it then quickly enters the attractor itself. The thermodynamicist Ilya Prigogine formulated a similar principle as "order through fluctuations"[11] or "order out of chaos".[12] It is applied in the method of simulated annealing for problem-solving and machine learning.[13] The thermodynamicist Adrian Bejan formulated the constructal law as the law of physics of design emergence and evaluation in nature bio and non-bio.[14][15][16]

Citation from: *https://bettstetter.com/research/sync/ , from Christian Bettstetter, Professor, Networked and Embedded Systems, Klagenfurt;* ***Self-Organizing Synchronization:***

“Time synchronization algorithms that operate in a decentralized manner in large wireless networks are being developed and evaluated. It is demonstrated that the stochastic nature of interactions between devices is a key ingredient for convergence to synchrony. This convergence has been mathematically proven for arbitrarily connected network topologies and variably changing interaction delays. Implementation and experiments on programmable radios show synchronization precisions below one microsecond when applying automatic phase rate correction. The developed solutions were patented and expected to be employed in a resource-limited embedded sensor and robot networks in smart factories and other emerging applications.

There is a broad spectrum of work on pulse-coupled oscillators to model synchronization phenomena in biology, physics, and other basic sciences. A prominent example is swarms of fireflies that synchronize their blinking behavior. The beauty of these synchronization phenomena lies in the fact that system-wide synchrony emerges among the participating entities in a completely nonhierarchical manner without the need for central entities. Such synchronization is scalable with the number of entities and robust against complete failure of entities or appearance of new entities. It is considered as a prime example of self-organization in nature, like the flocking of birds or shoaling of fish—where simple rules in each entity and localized interactions between neighboring entities lead to pattern formation of the entire system”.

There is an excellent, complementary, and brief or bottom-line resume, how founders of modern Quantum Theory decided to take Probability and Statistics as the fundamental framework for introducing and explaining quantizing in Physics, and what they, really and mistakenly, created (see [89]).

***Citations (on the next pages) taken from [89]**:*
*"The Universal Force Volume 1, from **Mr. Charles W. Lucas Jr.***
(See below)

2.2 Quantum Mechanics

I think it is safe to say that no one understands Quantum Mechanics.
Richard Feynman [44]

Can the world possibly be as absurd as it appears to us in our scientific theories? Werner Heisenberg [45]

When quantum effects were first noticed in the emission spectra of the atom and black body radiation, physicists had a choice. The primary options were (1) the physical internal structures of the electron and the atom were the source of quantum effects or (2) all objects in nature are point particle quantum harmonic oscillators which can only be described by a mathematical quantum wave function which provides information about the probability amplitude of position, momentum, energy and other physical properties of the oscillator. The decision made by the leaders of the atomic physics community was to go with option (2).

In 1927 quantum mechanics was standardized on the Copenhagen interpretation formulated by Niels Bohr and Werner Heisenberg while collaborating in Copenhagen. Bohr and Heisenberg extended the probabilistic interpretation of the quantum mechanical wave function originally proposed by Max Born. In this interpretation questions like "Where was the particle before I measured its position?" are meaningless. The measurement process randomly picks out exactly one of the many possible states allowed by the quantum state's wave function in a manner consistent with the defined probabilities that are assigned to each possible state. According to this interpretation, the interaction of an observer or apparatus that is external to the quantum system is the cause of wave function collapse to a specific state. Thus, according to Heisenburg, "reality is in the observations, not in the structure (of the electron or the atom)". [46]

The Copenhagen interpretation of quantum mechanics departs from classical physics primarily at the atomic and subatomic scales where the dynamics of the systems can be described in terms of Planck's constant h. It provides a mathematical description of the dual particle and wave-like behavior and interactions of matter and energy. The name "quantum mechanics" is derived from the observation that some physical quantities can only change by discrete amounts, or quanta in Latin. The angular momentum of an electron bound to an atom is quantized. The energy of an atomic electron is quantized resulting in discrete atomic emission line spectra.

This version of quantum mechanics is significantly different from classical science in that there is no basis for the law of cause and effect. All of nature is based 100% on random statistical probabilities. Since no internal physical structure of quantum oscillators is considered in this approach, all elementary particles are treated as point oscillators with no internal structure. Within the Copenhagen version of quantum mechanics the wave-particle duality of energy and matter and the uncertainty principle attempt to provide a unified view of the behavior of photons, electrons and other atomic-scale objects.

The history of quantum mechanics [47] began with a number of different scientific experiments. In 1838 Michael Faraday discovered the existence of cathode rays later identified as beams of electrons by J. J. Thompson in 1897. Next in the winter of 1859-1860 Gustav Kirchoff made statements about black body radiation. In 1877 Ludwig Boltzmann suggested that the energy states of a physical system could be discrete instead of continuous. Then in 1887

48

Heinrich Hertz discovered the photoelectric effect. Finally in 1900 Max Planck made the quantum hypothesis that any radiating atomic system could be divided into a number of discrete energy elements such that the energy ε of each of these energy elements is proportional to the frequency ν with which each of them individually radiate energy, i.e. $\varepsilon = h\nu$ where h is called Planck's constant.

Then in 1905 Albert Einstein explained the photoelectric effect previously reported by Heinrich Heinz in 1887 by postulating that light itself is made of individual quantum particles in order to be consistent with Max Planck's quantum hypothesis. In 1926 these quantum particles were called photons by Gilbert N. Lewis. The phrase "quantum mechanics" was first used in Max Born's 1926 paper "Zur Quantenmechanik der Stoßvorgänge". [48]

In the field of physics quantum mechanics has made a significant impact on the science of the elementary particles, the atom, the nucleus and molecules. Some of the fundamental idealizations in the assumptions or postulates or axioms of the Copenhagen version of quantum Mechanics are

1. **The state of any system is determined by an idealized universal wave function $\Psi(x,t)$**
2. **$\Psi^* \Psi$ gives rise to an idealized statistical interpretation of the universe**
3. **There is no Law of Cause and Effect in the universe**
4. **The Heisenberg Uncertainty Principle $\sigma_x \sigma_p \geq h/4\pi$**
5. **All particles are point oscillators**
6. **There is no deformation or elasticity of particles**
7. **All processes obey the linear superposition principle**

These are all idealizations. Problems with these idealizations are reviewed below.

1. **Experiments have not been able to detect any sort of physical aether to support a universal wave. Thus the universal wave function has been declared non-physical and just mathematical, i.e. probabilistic in nature.**
2. **A non-physical mathematical universal wave function cannot give rise to anything but a statistical interpretation of the universe. The statistical interpretation of the universe is in disagreement with the well-established law of cause and effect.**
3. **The primary purpose of science is to explain effects in terms of causes.**
4. **The quantum uncertainty arises in quantum mechanics due to the idealized matter wave nature of all quantum objects. The uncertainty relationship between any pair of non-commuting self-adjoint operators such as position and momentum are subject to uncertainty limits. These are entirely mathematical in origin, since the universal wave function is not a physical wave. It allows violations of conservation of energy and momentum for a period of time.**
5. **Accelerator particle scattering experiments have shown that massive particles such as the proton and neutron have both a finite-size and an internal charge**

structure consisting of at least three primary structures. See Hofstadter's electron scattering data in Figure 2-1 above. [9]

6. **All experimentally observed massive physical particles have finite-size and are elastic and can deform. Only idealistic unphysical point particles are not deformable.**
7. **Many physical phenomena, such as that giving rise to the laser (Light Amplification by Stimulated Emission of Radiation), are based on non-linear electrodynamic processes. However idealistic point particles cannot participate in non-linear processes.**

Besides having problems with its fundamental axioms, Quantum Mechanics now has problems with the fundamental experiments upon which the theory was initially developed such as the photoelectric experiment, the Blackbody Radiation experiment and the atomic emission spectra. For the Photoelectric experiments see section 2.1.2 above. The Blackbody Radiation experiment and atomic emission spectra are reviewed in the next two sections.

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Reflections on Matter-Waves, Blackbody Radiation, and the Big Bang Concept

Based on universal electromechanical and physical analogies (see Chapter 1), all wave motions and oscillations in our universe can be classified as forms of matter waves. These waves, whether they involve electric, magnetic, electromagnetic fields, or mechanical displacements, velocities, and forces, are described by similar mathematical models. A fundamental aspect of matter waves is the requirement for a carrier fluid, spatial matrix, or medium through which they propagate, even if we are sometimes unaware of the precise nature of this medium.

The classical wave equation (a second-order partial differential equation) applies to all matter waves, including quantum mechanical ones, when the wave function is formulated as a complex analytic signal (see Chapter 4.3 for more details). Schrödinger's equation, as well as other wave equations in quantum theory, can be derived directly and causally from this analytic signal without resorting to patchwork methods. These wave equations typically yield two sets of solutions, corresponding to waves propagating in opposite spatial-temporal directions, both inwards and outwards. The phenomenon of quantum entanglement is closely linked to these solutions, all the while adhering to the conservation and induction laws of physics.

Thermal and blackbody radiation, including the Planck formula and the Stefan-Boltzmann law, originate from the thermal motion of atoms and other particles, with excited electrons and photon exchanges playing a key role in the resulting spectral distribution (see Chapters 8 and 9 for more details). Since atoms have electron clouds strongly coupled with the positive charges in their nuclei, the blackbody radiation curve associated with electrons should have a corresponding electromagnetic analog within the atomic core. Among the positive charges within the nucleus, a similar, albeit frequency-shifted, radiation or spectral distribution should exist, given the strong coupling between electrons and protons. Consequently, the cosmic background radiation, often attributed to the Big Bang, is more likely related to these electromagnetic interactions between electrons and protons, rather than being evidence of a singular, cataclysmic event.

It's important to note that the blackbody radiation associated with the atomic core is significantly less intense and frequency-shifted compared to that of the electron clouds, due to the mass difference (protons being almost 2000 times heavier than electrons) and the smaller size and oscillation amplitudes of the nucleus. While the radiation curves for electrons and protons are similar in shape (both adhering to the Planck formula), the concept of the Big Bang as the absolute beginning of the universe remains overly simplistic and incomplete.

All energy-related events, motions, and transformations in the universe must adhere to conservation laws, involving sources and sinks akin to the principles governing electrical circuits (see Chapter 1). The Big Bang, as currently hypothesized, represents an open-ended scenario lacking identifiable sources or sinks, making it conceptually problematic to take seriously.

Furthermore, the phenomena of redshifts, the Doppler effect, and cosmic expansion apply universally to all matter waves and oscillatory motions. Wave-Particle Duality, in fact, serves as the most broadly applicable unifying concept within our universe (see Chapters 4.0, 4.1, 4.2, and 4.3 for a detailed discussion).

Finally, while it may be premature to assert with certainty, it seems likely that the four fundamental forces of nature will eventually be understood as manifestations of electromagnetic field effects and resonant standing wave structures, reflecting the vibrational, spinning, and rotating nature of the universe on both microscopic and macroscopic scales.

[♣ COMMENTS & FREE-THINKING CORNER**: **The remaining question to answer is ***what de Broglie, matter waves really are.***

Contemporary quantum theory often ventures into complex abstractions, exploring multiple virtual realities, “probabilities of possibilities,” and speculative entities. However, what we observe, measure, and know in our physical world, where the duality of matter waves and particles is evident, are primarily manifestations of electromagnetic energy. These include photons, light beams, heat waves, and the interactions between electrons, protons, photons, and other particles with electromagnetic charges and spin attributes. We can also detect the effects of matter waves in mechanical motions, oscillations, waves, and similar phenomena in fluids.

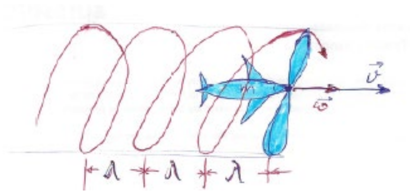
At the core of what we identify as particles and matter waves lies an essential electromagnetic phenomenology as combination and organization of electromagnetically charged entities. Different configurations or “packaging” of these electromagnetic entities give rise to everything else in the universe, including electrons, protons, atoms, and larger masses. Therefore, gravity may be a secondary, macroscopic effect of an underlying electromagnetic nature that we do not yet fully understand or accurately describe. Some aspects of the electromagnetic field that remain unknown could contribute to what we currently describe as gravity (see Chapter 2 for more details).

For instance, all masses and atoms in the universe are composed of elementary constituents like electrons, protons, and neutrons, which possess spin and associate angular and magnetic moments, effectively forming elementary magnets. While these elementary magnets are usually randomly distributed and self-neutralized, any residual, uncompensated electromagnetic properties within these masses could be a source of gravity. This idea aligns with the descriptions provided by Newton's and Coulomb's laws. In Chapter 2, striking analogies are drawn between planetary motions and Niels Bohr's atomic model, suggesting that electromagnetic phenomena are integral to gravity, or vice versa (see Section 2.3.3: “Macro-Cosmological Matter Waves and Gravitation”).

Chapter 3 of this book further explores electromagnetic fields and waves, conceptualizing them as a perpetually coupled, self-exciting, and fully symmetrical combination of electric and magnetic field vectors. This understanding of Maxwell’s equations could lead to the development of a new, clearer, and more generalized Special Theory of Relativity, potentially extending even further. A notable and symmetrical reformulation of Maxwell’s equations, as discussed in the works of Prof. Jovan Djuric, explains the origins of gravity based on the creation of electromagnetic dipoles. This creation results from the separation between the center of mass and the center of self-gravitation.

To conceptualize matter waves, one could argue that all waves and oscillations known in physics, including acoustic and electromagnetic waves, are forms of matter waves. These waves are not merely probability functions or distributions of possible events. The mathematics used to describe all matter waves is consistent across different types of waves. Particles themselves can be viewed as specific, stabilized formations of matter waves. The author of this book posits that a stable particle's creation from a complex combination of matter waves requires the involvement of some electromagnetic components. These electromagnetic matter waves act as a "binding and gluing medium," connecting all other (non-electromagnetic) energy-momentum entities in the process of forming stable particles.

One of the oversimplified analogical ways to visualize de Broglie matter waves creation is to imagine that particle in a straight and uniform linear motion that has mass $\mathbf{m}$ and kinetic energy E_k presents a propeller-driven airplane (we could also take an example of a propeller-driven submarine; -see the picture below). Inside of an airplane is its engine that is responsible for supplying kinetic energy to its motion. The more kinetic energy our airplane engine will produce will be directly proportional to the rotational (spinning) speed of its propeller. ***We can formulate the relation between rotational propeller speed*** ω ***and airplane kinetic energy*** E_k ***as,*** $\omega = k \cdot E_k , k = \text{constant.}$ ***(of course, naturally valid on average, for one full propeller revolution).***



Matter waves analogical visualization will be to imagine certain helix-line or helix-surface guided (or like a spiral mixer) waving motion behind a propeller that, in case of constant kinetic energy per one period (or constant spinning speed of the propeller, $\omega = k \cdot E_k , k = \text{constant.}$) will have elements of periodicity such as wavelength and frequency. Now we will exploit other necessary relations connecting de Broglie matter waves' properties with propeller-created waves (see chapter 4.1), such as,

$$\left\{\begin{array}{l} \omega = 2\pi f = k \cdot E_k , k = \text{constant.} \\ f = \frac{\omega}{2\pi} = \frac{k}{2\pi} E_k \ , \ \lambda = \frac{h}{p}, E_k = \tilde{E} = hf \\ u = \lambda f = \frac{h}{p}\frac{k \cdot E_k}{2\pi} = \frac{E_k}{p} \Rightarrow hk = 2\pi, \ k = \frac{2\pi}{h} \end{array}\right\} \Rightarrow \omega = k \cdot E_k = \frac{2\pi}{h} E_k = \frac{2\pi}{h}\tilde{E} = \frac{2\pi}{h} hf = 2\pi f . \quad (10.10)$$

Analogies in Matter Waves and Electromagnetic Phenomena

In the framework of PWDC (as described in relations 10.10), there is a clear analogical and mathematical complementarity between the rotation of a propeller and the kinetic energy associated with linear motion. To illustrate this, one might imagine replacing the airplane model with an analogically spinning bullet or gyroscope, where the spin serves as a stabilizing factor that contributes to the stability of the primary linear motion. It's important to note that in such scenarios, we typically work with average parameter values because the accumulation of energy is a process that requires time to quantify, typically measured over at least one full revolution of the propeller. In more complex situations involving large numbers of similar entities, it becomes natural to apply statistical and probabilistic methods, much like in modern quantum theory (QT).

An intriguing project could involve analyzing the rotational speed of a real airplane propeller in relation to the airplane's kinetic energy, considering various realistic conditions. This analysis would provide valuable insights into the interplay between rotational and linear motion.

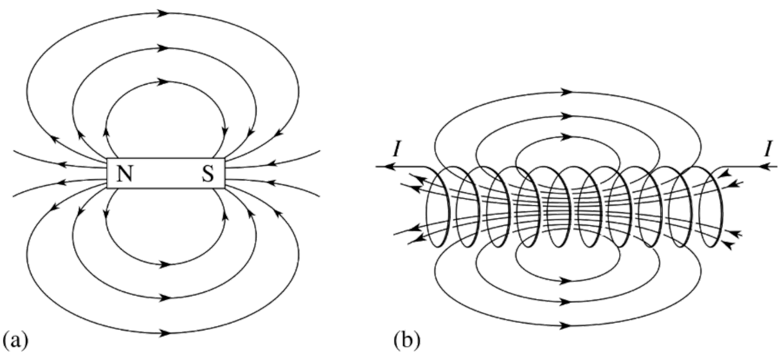
A crucial question then arises: what would be the analogous "propeller" for real moving particles, such as those associated with de Broglie matter waves, which are not propeller-driven airplanes or spinning bullets? Additionally, what is the nature of the surrounding "fluid" in which these particles move, especially considering that matter-waves for micro and elementary particles, like electrons, are observed even in extreme vacuum conditions?

There may be multiple answers to these questions, but the author suggests that spinning within electromagnetically charged and polarized dipoles (or multipoles), coupled with corresponding electromagnetic and mechanical moments, plays a key role. These elements are already intrinsic to the structure of moving particles and are electromagnetically coupled with their surrounding environment, including vacuum conditions. On a broader scale, our universe is characterized by slow movement and rotation at both microscopic and macroscopic levels. The centers of mass, centers of self-gravitation, and global centers of angular momentum do not perfectly overlap. This misalignment creates electromechanical tension, developing potential and stresses, which in turn produce electric and magnetic dipoles (and possibly multipoles). These phenomena manifest as what we perceive as gravitational effects (see Chapter 2 and the works of Prof. Dr. Jovan Djuric).

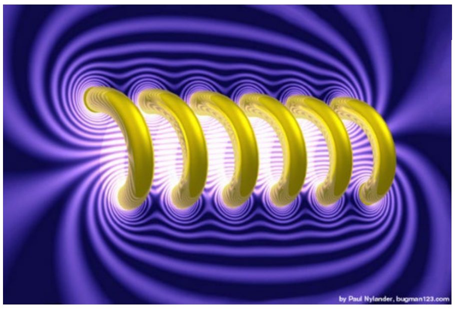
In essence, the deep, fundamental nature of de Broglie matter waves is closely tied to electromagnetic phenomena.

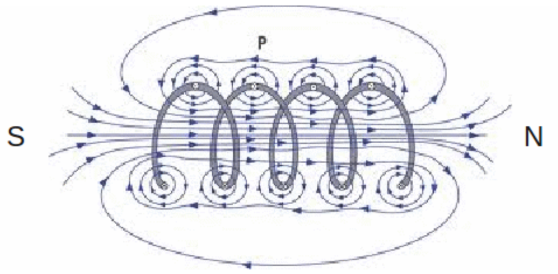
……………………..

Another example that is visually and analogically implicating existence of a ***unity of linear and spiraling motions*** is related to the geometry of surrounding magnetic fields in a vicinity of conductive wires and solenoids when (helical) electric current (or helical spiral motions of charged particles) is circulating such wires. See self-explaining pictures below.



(a) (b)





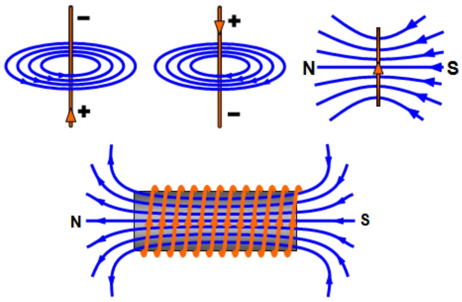
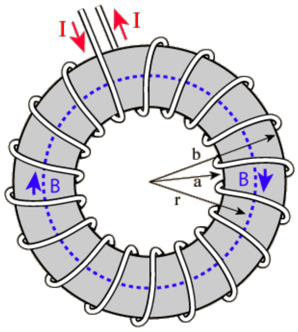
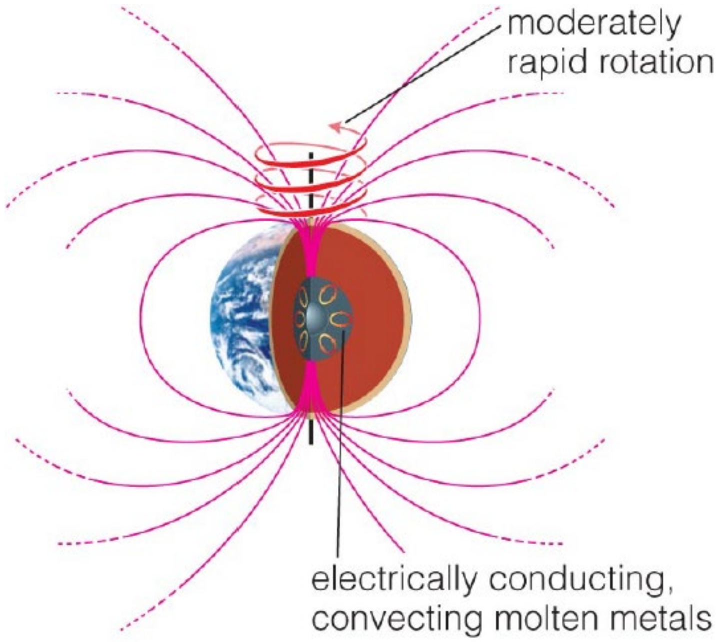


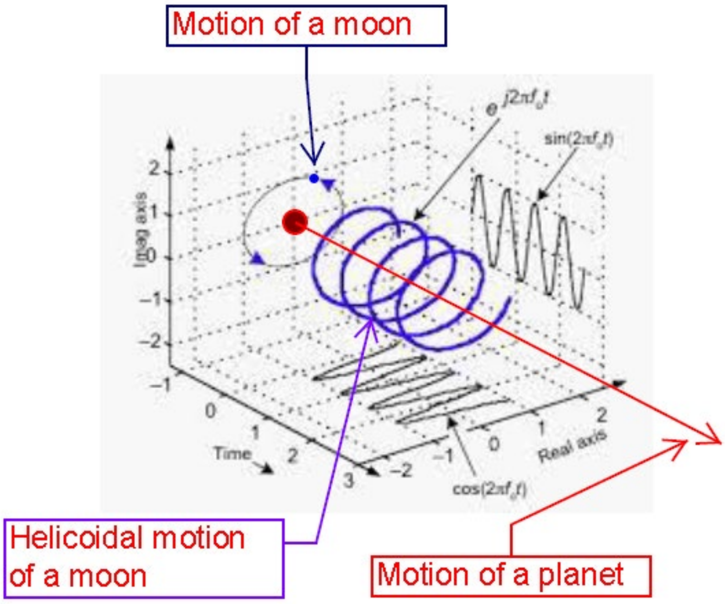
Figure 2





Citation and pictures are taken from the Internet (source or author were not mentioned): A planet can have a magnetic field if charged particles are moving helically inside. There are 3 requirements: 1° Molten interior (such as liquid core), 2° Convection (up/down), and 3° Moderately rapid rotation (around).

If we take into consideration an orbiting planet of a certain solar system that has its moon (or satellite orbiting the same planet), and if we place an observer in the center of such solar system, the moon's orbiting will look like a helical motion (see the picture below). Of course, electric current and mass motion are not directly comparable, but in both cases, we have certain energy flow because electric charges and masses in motion effectively carry certain energy and power.



An excellent and familiar modeling of the helical electron (very much compatible to here conceptualized helical matter waves, associated to linear motions) can be found in [108]; -Oliver Consa. "g-factor and Helical Solenoidal Electron Model"

Helix and Vortex Phenomena in Fluids

Helical and vortex spinning, rotations, waves, and oscillations in fluids are natural phenomena that vividly demonstrate the presence of matter waves, particularly when there is relative motion between a solid body and the surrounding fluid, whether in open space or within a tube. Fluids tend to spontaneously follow the helical or spinning field structures associated with matter waves. This behavior can be observed in various contexts, such as the operation of a vortex flowmeter, which measures the frequency of vortices in a flowing liquid. The frequency of these vortices is directly and linearly proportional to the fluid's velocity, an effect that can be analogously viewed as a manifestation of de Broglie matter waves (see Chapter 4.1 for further details).

The following images provide intuitive support for the idea that vortex phenomena in fluids, along with the associated spinning and helical motions, are causally linked to the manifestations of matter waves. Some of these images are sourced from Martin Roth's dissertation, "Automatic Extraction of Vortex Core Lines and Other Line-Type Features for Scientific Visualization" (ETH Zurich, Institute of Scientific Computing, Computer Graphics Group, Diss. ETH No. 13673, Hartung-Gorre Verlag Konstanz, 2000), while others are taken from various online sources. These visuals serve as nearly self-explanatory examples, reinforcing the connection between vortex behavior in fluids and matter wave phenomena.

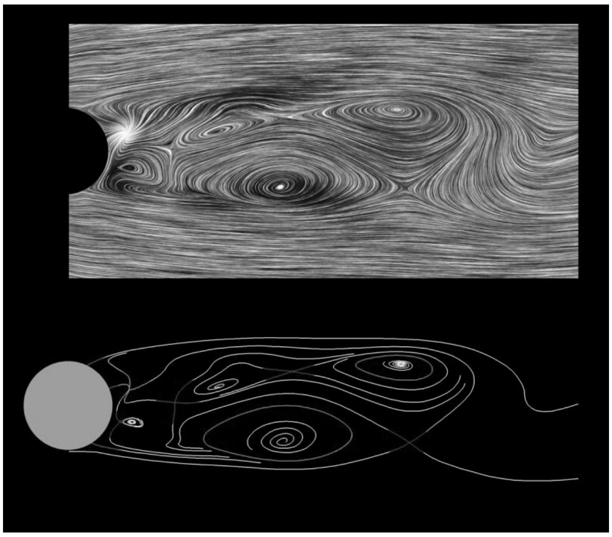


Fig. 2.13: Vector field topology (below) and LIC (above) of the flow past a cylinder (von Karman vortex street).

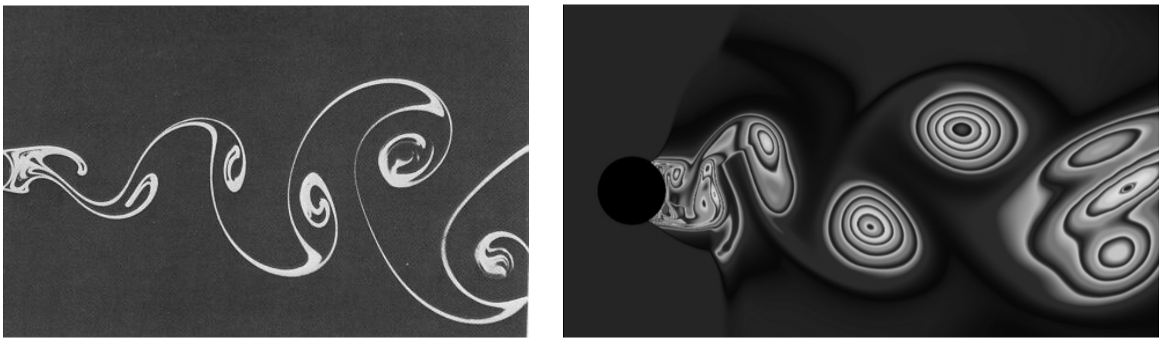


Fig. B.1: Flow behind a cylinder: von Karman vortex street (flow direction to the right).
Left: Photo of dye injected near the cylinder in an experiment
Right: Visualization of computer simulation: contours of vorticity magnitude.

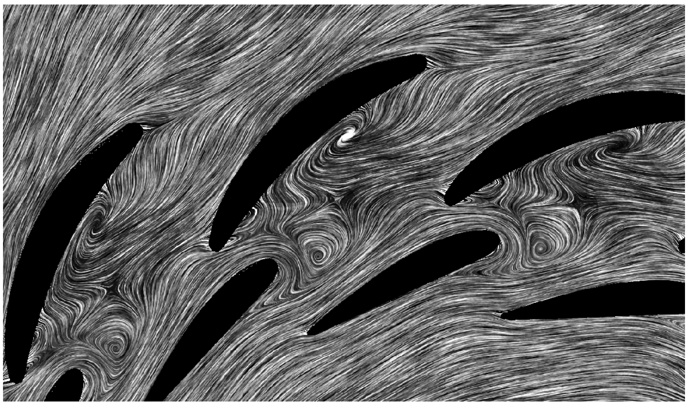
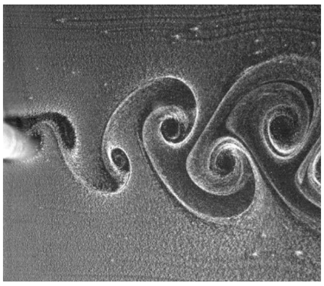
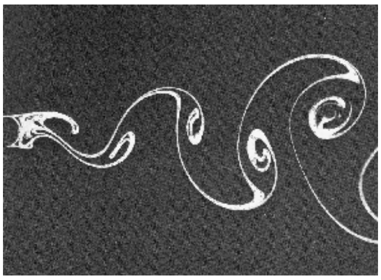
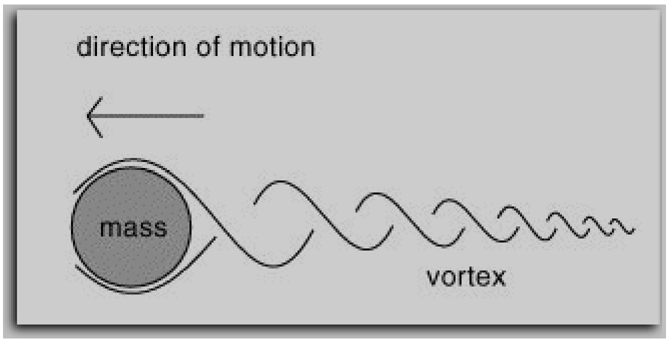


Fig. 2.9: Line Integral Convolution (LIC) image of 2D flow in the stator of a pump-turbine in pump mode (reversed); main flow direction is from lower left to upper right.







Gyroscopic Motion and Matter Waves

The conceptualization of both micro and macro matter waves is significantly supported by the study of gyroscopic and spinning-disk motions. A spinning disk, for example, naturally stabilizes and maintains its orientation. When friction is minimized or eliminated, this spinning motion can be viewed as a form of inertial motion, distinct from the linear inertial motions described by Newtonian mechanics. Furthermore, inertially spinning disks (ideally without friction) can be internally part of a mass content, or externally as spinning matter-waves associated with inertial linear masses motions, much like the wave-particle dualistic behavior observed in microphysics and in planetary systems.

Similarly, fast-moving bullets, satellites, and other objects in linear motion achieve greater stability and path control when they are spinning, much like gyroscopes. This observation suggests that linear motion and spinning are naturally complementary and mutually stabilizing. Every inertial and linear motion can, therefore, be associated with a corresponding spinning inertial motion, like the vortex motion created when water drains from an open sink. This vortex motion illustrates that fluids are sensitive to the dynamics of surrounding fields, matter-waves, and gravitational and inertial phenomena.

In certain cases, the spinning associated with linear motion may be less obvious or overlooked. However, this hidden spinning stabilizes the linear motion and creates helical or spiraling matter-waves, which contribute to the particle-wave duality effects discussed in this book. Most experimental evidence of particle-wave duality involves electromagnetically *charged particles, such as electrons and protons, which possess intrinsic spin and magnetic moment attributes like gyroscopes or spinning disks.*

Given that matter is composed of molecules, atoms, electrons, protons, neutrons, and other subatomic particles, many of which have intrinsic orbital and spin moments, we can infer that these internal spins act as the "propellers and gyroscopes" essential for supporting particle-wave duality and matter wave phenomena. When at rest, these internal and intrinsic spin vectors are randomly oriented in all directions. Externally, we do not see or measure resulting spinning effects (because of mutual vectors cancellations). When certain mass is changing its state of linear inertial motion, and accelerating, its ensemble of internally spinning elements $\vec{L}_s$ *is somehow getting progressively aligned (as vectors), being in a direct proportionality with the mass velocity or its linear momentum* $\vec{p} = \frac{\omega_s}{v}\vec{L}_s , \vec{L}_s = \frac{v}{\omega_s}\vec{p}$ *and also valid vice versa (see more in the chapter 4.1; -equations (4.3-0)-(4.3-0)-h, and later in this chapter (10.1.3)-(10.1.5)). Practically, the kinetic energy of a moving mass* E_k *is being fully balanced (or followed) with an internal and intrinsic, effective spinning (or vortex) energy* $\tilde{E} = E_s = hf_s = hf , \omega = \omega_s = 2\pi f_s = 2\pi f$ *being also valid in an opposite order, which here presents matter waves energy (see (10.11)). This way we can imagine or creatively visualize the formation of helically spinning matter-waves, as the resulting "thrust force effect of many aligned, elementary spinning elements", each of them having magnetic moments (earlier imaginatively described as gyroscopes, spinning discs, spinning toroid and propellers), where basic matter-waves and particle-wave duality relations (10.1) and (10.2) are naturally applicable.*

$$E_k = \tilde{E} = E_s = hf_s = \begin{Bmatrix} \frac{1}{2}mv^2 \\ \frac{1}{2}J\omega_s^2 \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} \frac{mv^2}{1+\sqrt{1-\frac{v^2}{c^2}}} \\ \frac{J\omega_s^2}{1+\sqrt{1-\frac{v^2}{c^2}}} \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} \frac{pv}{1+\sqrt{1-\frac{v^2}{c^2}}} \\ \frac{L_s\omega_s}{1+\sqrt{1-\frac{v^2}{c^2}}} \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} \frac{\vec{p}\vec{v}}{1+\sqrt{1-\frac{v^2}{c^2}}} \\ \frac{\vec{L}_s\vec{\omega}_s}{1+\sqrt{1-\frac{v^2}{c^2}}} \end{Bmatrix} \Rightarrow \quad (10.11)$$

$$\Rightarrow \boxed{pv = L_s\omega_s \Rightarrow \vec{p} = \frac{\omega_s}{v}\vec{L}_s , \vec{L}_s = \frac{v}{\omega_s}\vec{p}, \vec{F} = \frac{d\vec{p}}{dt}, \vec{\tau}_s = \frac{d\vec{L}_s}{dt} = \vec{p}\cdot\frac{d}{dt}\left(\frac{v}{\omega_s}\right) + \frac{v}{\omega_s}\cdot\vec{F}}$$

In summary, for stable, uniform and complex inertial motions, it is likely that all linear and angular (or orbital) moments are interrelated and mutually transformable, because of associated effects of wave-particle duality. Therefore, the conservation laws for different moments cannot be applied independently. Instead, we must consider inertial motions where angular (orbital and spinning) moments are predominantly conserved, linear moments are also conserved, or where a combination of both is present, always considering associated magnetic fields. This integrated approach likely describes the formation of particles, atoms, and solar systems, which result from the specific combination of linear and angular motions with intrinsic electromagnetic entities.

Intrinsic spinning moments are inherently linked with corresponding magnetic moments. Due to the holistic rotation present in our universe, matter tends to exhibit dipoles and multi-poles electrical and magnetic polarizations. This phenomenon contributes to the electromagnetic complexity and helix field structures discussed.

These concepts extend naturally to gravitation and planetary systems, as detailed in Chapter 2 of this book, specifically section 2.3.3. Historical perspectives on gravitation, differing from

Newtonian and Einsteinian theories, can be found in the works of Nikola Tesla and Rudjer Boskovic (see references [6], [97], and [117]).

When examining matter and wave interactions, it is important to consider that real-world frameworks, laboratories, and significant interaction zones are often best described using mathematically manageable coordinate systems. These systems include local centers of mass, centers of inertia, centers of self-gravitation, centers of rotation, and centers of electromagnetic charges. While laboratory coordinate systems provide initial insights and intuitive understanding, they are not always sufficient for a comprehensive analysis of matter, particles, and waves.

Wave-particle duality and probabilistic quantum theory originated from complex experimental observations, such as the wave-like interference and superposition effects seen in two-slit experiments with particles like electrons. These experiments suggest that there are additional, often invisible or effective, reaction participants beyond what is observable in a local laboratory setting. These hidden elements can be mathematically managed using concepts such as the center of mass and reduced mass systems. ♣]

10.1 Hypercomplex Analytic Signal Functions and Energy-Momentum 4-Vectors in Relation to Matter Waves and Particle-Wave Duality

In analyzing particle motion and interactions, energy and momentum conservation laws are well-established and experimentally verified. We will rely on these energy-momentum and velocity-dependent relativistic concepts and formulas until we reach a more general and profound understanding.

This book does not treat mass as the sole or primary source of gravitation, as suggested by the analogical conclusions in Chapter 1. Instead, we propose that the true sources of gravitation are the energy-momentum properties of moving masses, including their linear, angular, electromagnetic, and spinning moments, along with associated vibrations. Consequently, mass should be re-conceptualized and quantified in a manner that reflects these complex interactions. The following theoretical foundations provide a basis for addressing this issue:

1. Relative Motion and Field Interactions:
All masses in the universe are in relative motion and are interconnected both externally and internally. These connections are mediated by various natural fields, forces, and matter waves.

2. Nature of Rest Mass:
Rest mass can be viewed as a relatively stabilized and internally neutralized aggregation of spinning and electromagnetic states within atoms. For complex macro masses, this includes a unified set of atoms, molecules, and particles in various energy-momentum states. These internal constituents often possess spinning attributes and magnetic and electric moments. When these attributes are balanced or neutralized, the mass appears electromagnetically neutral. However, no truly neutral mass exists in the universe since all masses are in relative motion, with internal temperatures and resulting spinning moments. For elementary particles, mass is estimated by their total energy, often without additional heat or vibrational components. In some matter-wave states, where stable rest mass may not be present, only motional energy mass equivalents are observable, but these still interact with particles having non-zero rest masses.

3. Interconnected Masses:

Masses are holistically interconnected in the universe. These connections, including the synchronization of resonant states, adhere to conservation laws and can be conceptualized through mutual radiant energy exchanges or mass fluctuations. Known forms of these fluctuations include electromagnetic waves, cosmic rays, radioactive emissions, and particle annihilations. Such exchanges and fluctuations may affect the rest mass quantity, although in most cases, these variations are minor.

4. Nikola Tesla's Contributions:

Nikola Tesla's work on radiant energy and the concept of ether offers valuable insights. He proposed that even in a near-absolute vacuum, there exists a fine, two-component fluid composed of electro-conductive and electrically dipole-polarizable masses, mixed with an insulating fluid. Tesla's experiments with electric discharges in evacuated vacuum tubes supported the idea of a residual electro-conductive matter or ether, although its exact nature remains unclear. For further details, refer to sources [97] and [117].

5. Einstein-Minkowski 4-Vectors:

The current state of the art in addressing the complexity of mass and energy-momentum interactions is effectively captured by the Einstein-Minkowski 4-vectors framework from Relativity Theory. This book will extend and refine this 4-vector approach to incorporate matter-wave properties, offering a more comprehensive description and quantification of mass and energy-momentum states.

$$
\begin{aligned}
&\overline{P}_4 = (\vec{p}, \frac{E}{c}) = (\vec{p}, \frac{E_o + E_k}{c}), \\
&E_o = E_{oo} + E_{ov} + E_{os} = m_{oo}c^2 + m_{ov}c^2 + m_{os}c^2 = m_o c^2 = mc^2, \\
&E_k = (\gamma - 1)mc^2 = \frac{\vec{p}\vec{v}}{1 + \sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{\vec{L}\vec{\omega}}{1 + \sqrt{1 - \left(\frac{v}{c}\right)^2}}, \\
&E = E_o + E_k = \gamma mc^2 \Leftrightarrow m_o = m_{oo} + m_{ov} + m_{os} = m, \vec{p} = \gamma m\vec{v}, \vec{L} = \gamma J\vec{\omega}, \vec{p}\vec{v} = \vec{L}\vec{\omega},
\end{aligned}
$$

where the meaning of mass and energy components is:

$m_{oo} = \frac{E_{oo}}{c^2}$ (=) fully compensated part of the standstill rest mass (meaning neutral mass that has total zero charges and zero-vectors properties). In cases of complex macro masses (with many atoms and/or molecules), this is the dominant total mass part.

m_{ov} (=) internal vibrational and heat-energy exited mass equivalent,

$E_{ov} = m_{ov}c^2 = \sum_{(i)} \frac{1}{2} m_{ov-i} v_i^2 = \sum_{(i)} \frac{1}{2} \vec{p}_{ov-i}\vec{v}_i$ (=) total, internal and active vibrational and heat energy,

m_{os} (=) total, not-compensated, active, and externally measurable ***spinning energy mass equivalent*** (or part of externally detectable spinning, where spinning moment vector is non-zero).

$E_{os} = m_{os}c^2 = \sum_{(i)} \frac{1}{2} J_{os-i}\omega_{s-i}^2 = \sum_{(i)} \frac{1}{2} \vec{L}_{os-i}\vec{\omega}_{s-i} = \frac{1}{2} J_{os}\omega_s^2 = \frac{1}{2} \vec{L}_{os}\vec{\omega}_s$, (=) total, active and externally detectable self-spinning energy.

The resulting linear mass (or particle) moment is,

$$
\vec{p} = \gamma m\vec{v} = \gamma(m_{oo} + m_{ov} + m_{os})\vec{v} = \gamma(\frac{E_{oo}}{c^2} + \frac{E_{ov}}{c^2} + \frac{1}{2c^2} J_{os}\omega_s^2)\vec{v} .
$$

Here we implicitly accept (or assume) that all masses or particles' motions are curvilinear, and locally like rotational and orbital motions (of course, apart from self-spinning), since,

$$E_k = (\gamma - 1)mc^2 = \frac{\vec{p}\vec{v}}{1 + \sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{\vec{L}\vec{\omega}}{1 + \sqrt{1 - \left(\frac{v}{c}\right)^2}} \Rightarrow$$

$$\vec{p}\,\vec{v} = \vec{L}\vec{\omega} = \gamma(m_{oo} + m_{ov} + m_{os})v^2 = \gamma(\frac{E_{oo}}{c^2} + \frac{E_{ov}}{c^2} + \frac{1}{2c^2}J_{os}\omega_s^2)v^2 = \gamma m v^2 .$$

Integrating Matter-Waves and Energy-Momentum 4-Vectors

To this point, we have established or assumed that all masses in the universe are engaged in curvilinear (e.g., orbital) motions, and that spinning contributes to the total rest mass. Linear motions can be viewed as cases of angular motions with arbitrarily large radii. The conservation laws for linear and angular momentum, along with translational and rotational spatial-temporal symmetries, appear to be valid. Although these laws might seem contradictory at times, they are unified by de Broglie matter waves and the principle of total energy conservation, which may also involve electromagnetic phenomena.

Given the well-established principles of Classical Mechanics and Physics, we have temporarily overlooked external energy-mass exchanges, mass flow, and radiant energy-mass transfer between masses. Instead, we have focused on relatively stable particles with balanced mass-moments. It is now time to enhance Classical Mechanics by incorporating concepts from Matter-Waves and Wave Mechanics.

Let us delve deeper into the formulas and relations governing motional mass-energy-moments, starting with Minkowski-Einstein 4-vectors. Albert Einstein's theory of Relativity, despite its complex and sometimes challenging concepts, provides practical and useful foundational elements. For instance, concepts such as "inertial frame time," "proper time," and Einstein-Riemann-Minkowski 4-vectors are invaluable for analyzing mechanical interactions in physics. While Relativity Theory has faced criticism, its 4-vector framework remains unparalleled and indispensable. However, a more comprehensive understanding of these concepts is still forthcoming.

The central idea here is to demonstrate that rest mass is not the sole or primary source of gravitation. Instead, mass, when associated with linear and orbital moments and electromagnetic charges (including vibrational states), plays a causal role in gravitation. Furthermore, motional mass generates matter waves or can be represented as a matter-wave packet. Gravitation may simply be a consequence of the attractive properties within the standing matter waves' field structure, particularly around the nodal zones of mass agglomerations.

Additionally, the cosmos or universe should be conceptualized as a multidimensional electromechanical network, with inputs and outputs, or sources and loads, where all conservation laws are naturally satisfied. This perspective aligns with Nikola Tesla's ideas on dynamic gravity [97, 98, 99, 117] and Rudjer Boskovic's description of universal natural forces [6].

To illustrate these concepts, we can analogically create a table of matter attributes (T.10.1), which can be imaginatively referred to as "4-scalars" or complex numbers based on known 4-vectors. This table would mathematically mimic the formalism of Minkowski-Einstein 4-vectors, providing a structured framework for understanding the complex interactions of mass, energy, and gravitational phenomena.

T.10.1, (T.2.2-3)

	Ref. Frame Values	Symbolic 4-Vectors	Invariant Expressions
Momentum	$p = \gamma M_0 v$ $pc^2 = Ev$	$\overline{P}_4 = (p, \frac{E}{c}) = M_0 \cdot \overline{V}_4$ $= M_0 \cdot (\gamma v, \gamma c)$	$\overline{P}_4^2 = p^2 - \frac{E^2}{c^2} =$ $= -\frac{E_0^2}{c^2} = inv.$
Velocity	$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \frac{\Delta\mathbf{r}}{\Delta t}$	$\overline{V}_4 = (\gamma v, \gamma c)$	$\overline{V}^2 = \gamma^2 v^2 - \gamma^2 c^2 =$ $= -c^2 = inv.$
Distance (space interval)	$\Delta\overline{S}_4 = \overline{V}_4 \cdot \Delta t$	$\Delta\overline{S}_4 = (\gamma\Delta\mathbf{r}, \gamma c\Delta t)$	$(\gamma\Delta\mathbf{r})^2 - (\gamma c\Delta t)^2 = (\Delta\mathbf{r}_0)^2 - (c\Delta t_0)^2$
Time	$\Delta t_0 = \gamma\Delta t$ $\Delta t_0 -$ measured time	$\Delta\overline{T}_4 = \frac{1}{v}\Delta\overline{S}_4 = (\gamma\Delta t, \gamma\frac{c}{v}\Delta t)$	$\Delta t_0 = \gamma\Delta t$ $\Delta t -$ proper time
Energy		$E = E_t = \gamma E_0 = \gamma M_0 c^2 = \frac{pc^2}{v}$	

We could now start introducing and exercising grounds of new “Hyper-Complex Relativity”, or ***“Hyper-Complex Analytic-Signal-Phasors” of an extended Relativity theory***, based on different mathematical options within 4-vectors from T.10.1 (see more about Complex Analytic Signals in chapter 4.0).

Let us first find what kind of complex and analytic signals (or **phasors**) functions are already present behind 4-vectors of the contemporary Relativity theory (regarding energies, masses, and linear and angular moments). We will effectively come close to the familiar situation that is already known in electronics and electro-technique regarding complex functions or **phasors** representation of currents, voltages, and electric impedances (see different elaborations regarding electromechanical analogies and symmetries in the first chapter). All such results (or complex phasors, developed from 4-vectors of Relativity theory) are briefly presented below:

$$E_k = E_t - E_0 = (\gamma - 1)E_0 = \frac{\gamma - 1}{\gamma} E_t = \int_{-\infty}^{+\infty} [\Psi(t)]^2 \, dt \ldots = \tilde{E}$$

$$POWER = \frac{d\tilde{E}}{dt} = \frac{dE_k}{dt} = [\Psi(t)]^2 = \frac{1}{2}|\overline{\Psi}(t)|^2 = \frac{1}{\pi}|A(\omega)|^2 \frac{d\omega}{dt} = \frac{1}{2}a^2(t) = h\frac{df}{dt} = v\frac{dp}{dt} = c^2\frac{d\tilde{m}}{dt} (=) [W],$$

$$\overline{\Psi}(x,t) = \begin{cases} \sum_{(n)} \overline{\Psi}_n, \text{ or } \int_{[\Delta k]} \overline{\psi} dk, \\ \text{or summation of wave-groups} \\ |\overline{\psi}(x,t)| \frac{\sin(\underline{\Delta\omega}\, t - \underline{\Delta k}\, x)}{(\underline{\Delta\omega}\, t - \underline{\Delta k}\, x)} e^{I(\omega t \pm kx)} \end{cases} = \overline{\Psi}^+(kx - \omega t) + \overline{\Psi}^-(kx + \omega t) = |\overline{\Psi}| \cdot e^{I\Phi}$$

$$\left\{ \begin{array}{l} \overline{P}_4 = P(p, \frac{E_t}{c}) = P(p, I\frac{E_t}{c}) \Rightarrow p^2 - \frac{E_t^2}{c^2} = -\frac{E_0^2}{c^2} \Leftrightarrow E_t^2 = E_0^2 + c^2p^2 = E_0^2 + E_p^2 = \\ (\gamma M_0 c^2)^2 = (E_0 + E_k)^2 = E_0^2 + 2E_0E_k + E_k^2, \; E_p = cp = \gamma M_0 vc \Rightarrow \\ \Rightarrow p^2 = (\gamma M_0 c)^2 - \frac{E_0^2}{c^2} = (\gamma M_0 c)^2 - (M_0 c)^2 = \gamma^2 M_0^2 v^2 \\ (\gamma M_0)^2 = (\frac{E_0}{c^2})^2 + (\frac{p}{c})^2, E_0 = mc^2 + E_s = M_0c^2, \gamma = 1/\sqrt{1 - (\frac{v}{c})^2} \end{array} \right\} \Rightarrow$$

$$\overline{P} = p \cdot \cos\theta \pm Ip \cdot \sin\theta = \gamma M_0 c - I\frac{E_0}{c} = \gamma M_0 v e^{-I\theta} = pe^{-I\theta}$$
$$\left|\overline{P}\right| = p = \gamma M_0 v,\ \theta = \operatorname{arctg}\sqrt{1 - v^2/c^2} = \theta(x,t)$$

$$\left.\begin{matrix} \omega = \omega_t = \left|\frac{\partial\theta}{\partial t}\right| = 2\pi f_t = \frac{2\pi}{T} = 2\pi f, \\ k = \omega_x = \left|\frac{\partial\theta}{\partial x}\right| = 2\pi f_x = \frac{2\pi}{\lambda} = 2\pi\frac{p}{h} \end{matrix}\right\} \Rightarrow \left\{\begin{matrix} u = \frac{\omega}{k} = \frac{f_t}{f_x} = \frac{\lambda}{T} = \frac{\partial x}{\partial t} = \lambda f \\ v = \frac{d\omega}{dk} = \frac{dx}{dt} = u - \lambda\frac{du}{d\lambda} \end{matrix}\right\} \Rightarrow \left\{\begin{matrix} dk \cdot dx = d\omega \cdot dt \\ \frac{2\pi}{h} dp \cdot dx = 2\pi df \cdot dt \end{matrix}\right\} \Rightarrow$$
$$\Rightarrow dp \cdot dx = hdf \cdot dt = dE \cdot dt \Leftrightarrow \Delta p \cdot \Delta x = \Delta E \cdot \Delta t, \qquad (10.1.1)$$
$$(I^2 = -1, I = (i,j,k), i^2 = j^2 = k^2 = -1, ij = k, jk = i, ki = j...).$$

Analyzing similar cases, when a motional particle will get an additional, mechanical spinning moment, ($\vec{L}_{i-ms} \neq \vec{0}$) in certain interaction, we will be able to reproduce new results, like (10.1.1),

$$\left\{\begin{matrix} \left(\begin{matrix} (\vec{L} \to \vec{L} + \vec{L}_{ms} = \vec{L}^*) \Rightarrow (M \to M + m_{ms} = M^*) \Rightarrow \\ \vec{p} = \gamma M\vec{v} \to \gamma(M + m_{ms})\vec{v} = \vec{p} + \vec{p}_{ms} = \vec{p}^* = \gamma M^*\vec{v} \\ E = \gamma Mc^2 \to E + E_{ms} = E_0 + E_k + E_{ms} = E_0 + E_s + E_{ms} = E^* = \gamma M^* c^2 \\ E_0 = Mc^2 \to E_0 + E_{ms} = (M + m_{ms})c^2 = E_0^* = M^* c^2 \\ E_k \to E_k^* = (\gamma - 1)M^* c^2, E_{ms} = \text{mechanical spinning energy} \end{matrix}\right) \Rightarrow \\ \overline{P}_4 = \left(\vec{p}, \frac{E}{c}\right) \to \left(\vec{p}^*, \frac{E^*}{c}\right) \Leftrightarrow \left(\vec{p}^*, I\frac{E^*}{c}\right) \Rightarrow (\vec{p}^*)^2 - (\frac{E^*}{c})^2 = -(\frac{E_0^*}{c^2})^2 \Leftrightarrow (E_0^*)^2 + (\vec{p}^*)^2 c^2 = (E^*)^2 = (E_0^* + E_k^*)^2 \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left[\begin{matrix} \overline{E}^* = E^* \cdot e^{\pm I\theta^*} = (E_0^* + E_k^*) \cdot e^{\pm I\theta^*} = \sqrt{(E_0^*)^2 + (\vec{p}^*)^2 c^2} \cdot e^{\pm I \cdot \operatorname{arctg}\frac{p^* c}{E_0^*}} = \gamma M^* c^2 \cdot e^{\pm I\theta^*} = \gamma \overline{M}^* c^2, \\ E_0^* = M^* c^2,\ \overline{M}^* = M^* \cdot e^{\pm I\theta},\ I^2 = -1, \\ \gamma M^* c^2 = \sqrt{(E_0^*)^2 + (\vec{p}^*)^2 c^2} = E_0^* + E_k^* = E^*, \\ \theta^* = \operatorname{arctg}\frac{p^* c}{E_0^*} = \operatorname{arctg}(\gamma\frac{v}{c}) = \operatorname{arctg}\frac{\frac{v}{c}}{\sqrt{1 - (\frac{v}{c})^2}} = \theta,\ 0 \le (\theta = \theta^*) \le \frac{\pi}{2} \end{matrix}\right] \Rightarrow \qquad (10.1.2)$$
$$\Rightarrow \overline{P}^* = \gamma\overline{M}^*\vec{v} = \gamma M^*\vec{v} \cdot e^{\pm I\theta^*} = \vec{p}^* \cdot e^{\pm I\theta^*} = \vec{p}^* \cdot e^{\pm I\theta}$$

Complex, Hypercomplex and Analytic Signal function interpretation of energy-momentum vectors and scalars (like in (10.1), (10.1.1) and (10.1.3)) is revealing energy-momentum phase function "θ", which is the new quality parameter, still not exploited (or known) within the Minkowski 4-vectors concept, and in the contemporary theoretical physics. Linear momentum $\overline{P} = \gamma\overline{M}v = p \cdot e^{\pm I\theta}$ is now presentable as a Complex or Hypercomplex and Analytic Signal function, like **Phasors and complex impedances** in electric sciences).

Here is the place to address de Broglie or matter waves. Matter-wave and particle-wave duality concept is in a certain specific way connected to a moving particle having a linear moment $\vec{p}$. Moving particles are also presentable as a wave-packet or wave group, which has an angular moment $\vec{L}_s$, spinning frequency $f_s = \omega_s / 2\pi = \tilde{E}/h$, (or for macro objects $\tilde{E}/H$), and helical wave nature. We could simply postulate (or exercise) that the mentioned phase angle θ should have all essential properties of de Broglie matter waves, as for instance, $\theta = \operatorname{arctg}\sqrt{1 - v^2/c^2} = \theta(x,t) = \omega_s t \pm kx + \theta_0,\ \theta_0 = \text{const.}$, presentable as,

$$\begin{bmatrix} \vec{p} = \frac{\omega_s}{v}\vec{L}_s = \gamma M\vec{v}, \vec{L}_s = \frac{v}{\omega_s}\vec{p} = \frac{v}{\omega_s}\gamma M\vec{v} \\ \overline{P} = \gamma\overline{M}\vec{v} = \gamma M\vec{v}\cdot e^{\pm I\theta} = \vec{p}\cdot e^{\pm I\theta} \\ \lambda = \frac{H}{p}, u = \lambda f_s = \frac{\tilde{E}}{p}, v = \frac{d\tilde{E}}{dp}, k = \frac{2\pi}{\lambda} = \frac{2\pi}{H}p \\ \omega_s = 2\pi f_s, \Psi^2(t) = \frac{d\tilde{E}}{dt} = \frac{dE_k}{dt} \end{bmatrix} \Rightarrow \begin{bmatrix} \vec{L}_s = \frac{v}{\omega_s}\vec{p} = J_s\vec{\omega}_s, \\ \overline{L}_s = \frac{v}{\omega_s}\overline{P} = \frac{v}{\omega_s}\vec{p}\cdot e^{\pm I\theta} = \vec{L}_s\cdot e^{\pm I\theta} = J_s\vec{\omega}_s\cdot e^{\pm I\theta} = \overline{J}_s\vec{\omega}_s, \\ \tilde{E} = \int_{-\infty}^{+\infty}\Psi^2(t)dt = \int_{-\infty}^{+\infty}\hat{\Psi}^2(t)dt = \int_{-\infty}^{+\infty}\left|\frac{\overline{\Psi}(t)}{\sqrt{2}}\right|^2 dt = \int_{-\infty}^{+\infty}\left[\frac{a(t)}{\sqrt{2}}\right]^2 dt = \int_{-\infty}^{+\infty}\left|\frac{\overline{U}(\omega)}{\sqrt{2\pi}}\right|^2 = E_k \end{bmatrix}. \quad (10.1.3)$$

(See more in Chapters 4.1 and 4.3).

If we consider results and formulations from (10.1.1) - (10.1.3) as relevant, (of course, after appropriate theoretical "brushing"), complex linear momentum p, or its Phasor function $\overline{P}$, in relation to matter-waves' and particle-wave duality, should effectively present kind of replacement or complement for de Broglie matter waves concept. Phasor $\overline{P} = \gamma\overline{M}v = p\cdot e^{\pm I\theta}$, which is in the form of Complex Analytic Signal, is a mixed particle-wave, or "*particle and surrounding spinning-filed*" entity (since it is combining elements of a particle linear motion, and certain kind of associated spinning, like helix or solenoidal field-tail), like turbulent and waving motion behind a particle moving on a fluid surface. See (4.3-0), in Chapter 4.1, and (4.41-1) - (4.41-4), in Chapter 4.3).

It looks like we will come closer to understanding natural or proper time flow (time scale, or time dimension) in relation to mass motion and associated helix matter-wave, if we could find a connection between energy-momentum phase "θ" and local time "t". Apparently, certain (local and dominant) matter-wave, spinning frequency ω_s , associated to linear momentum of a dominant particle, could serve as the time clock, or proper time-reference signal, for registering (measuring or representing) real-time flow (of a motional particle in question). This way, we can represent instantaneous time-dependent, matter waves functions (of linear and angular moments) as cosines and sinusoidal functions with a phase equal to, $\theta = \omega_s t \pm kx + \theta_0$.

$$\left\{\begin{matrix} \overline{P}(t) = \gamma M\vec{v}\cdot e^{\pm I(\omega_s t\pm kx+\theta_0)} = \vec{p}\cdot e^{\pm I(\omega_s t\pm kx+\theta_0)} = \frac{\omega_s}{v}\vec{L}_s\cdot e^{\pm I(\omega_s t\pm kx+\theta_0)} = \frac{\omega_s}{v}\overline{L}_s(t) \\ \overline{L}_s(t) = \vec{L}_s\cdot e^{\pm I(\omega_s t\pm kx+\theta_0)} = \frac{v}{\omega_s}\vec{p}\cdot e^{\pm I(\omega_s t\pm kx+\theta_0)} = \frac{v}{\omega_s}\overline{P}(t), \quad \theta_0 = \text{const., or} \\ \overline{P}(t,x) = \vec{p}\cdot e^{+I\cdot\theta(t,x)} + \vec{p}\cdot e^{-I\cdot\theta(t,x)}, \overline{L}_s(t,x) = \vec{L}_s\cdot e^{+I\cdot\theta(t,x)} + \vec{L}_s\cdot e^{-I\cdot\theta(t,x)} \end{matrix}\right\} \Rightarrow \gamma M = \frac{p}{v} = \frac{\omega_s}{v^2}L_s, \quad (10.1.4)$$

or, by generalizing cases of time-space propagating matter-waves in a direction r = r(x, y, z), we will analogically have the following linear and angular moments, or complex Phasors, including relevant wave function,

$$\overline{P}(r,t) = \gamma M\vec{v}\cdot e^{\pm I(\omega_s t\pm kr+\theta)} = \vec{p}\cdot e^{\pm I(\omega_s t\pm kr+\theta)}, \ \overline{L}_s(r,t) = \vec{L}_s\cdot e^{\pm I(\omega_s t\pm kr+\theta)} = \frac{v}{\omega_s}\vec{p}\cdot e^{\pm I(\omega_s t\pm kr+\theta)} \quad . \quad (10.1.5)$$

$$\text{POWER } (=) d\overline{E}/dt = v\cdot\left(d\overline{P}/dt\right) = \overline{\Psi}^2(r,t) = \text{Wave function.}$$

Regarding the mentioned time-reference signal, as described in (10.1.3) - (10.1.5) and earlier in (4.3-0)-r, we could try to understand it in the following way (by giving an example in relation to our planetary system). For us, dominant and natural time flow is related to the orbital motion of our planet around our local Sun, or about the local and dominant center of mass. Planet Earth is at the same time self-rotating and has a certain spin. This is creating associated, helix matter-wave, characterized by its spiraling frequency $\vec{\omega}_s$. Since our masses and sizes (related to humans and other living species on our planet Earth) are negligible compared to the local planetary mass (and local center of mass), it is logical that our real-time flow is dominated by the "signal carrier" time flow belonging to the orbiting mass of our planet. We also know that our SI unit of time is really extracted from the parameters of the orbital motion of our planet Earth. For the micro world, something similar is valid in a different scale of time-space relations.

From (10.1.4) and (10.1.5) we can also see that mass **M** has an explicit relation to its resulting or effective and intrinsic, angular or spin moment $\gamma M = \frac{p}{v} = \frac{\omega_s}{v^2} L_s$. Now the meaning of Mobility type analogies is becoming much clearer (as elaborated in the first chapter of this book; -see T 1.8), from where it is evident that real charges or sources of gravity (based on electromechanical analogies) should be relevant linear and angular momenta. For instance, Newton force of gravitation between two masses (see the second chapter of this book) should be more correctly written as,

$$F_g = G\frac{M_1 \cdot M_2}{R^2} \Leftrightarrow G\frac{\gamma_1 M_1 \cdot \gamma_2 M_2}{R^2} = \left[\frac{G}{v_1 v_2}\right]\frac{p_1 \cdot p_2}{R^2} = \left[\frac{G\omega_{s1}\omega_{s2}}{v_1^2 v_2^2}\right]\frac{L_{s1} \cdot L_{s2}}{R^2}. \quad (10.1.6)$$

Linear and angular moments and corresponding velocities, $p_1, p_2, v_1, v_2, L_{s1}, L_{s2}, \omega_{s1}, \omega_{s2}$, we can also consider as mutually collinear vectors in relation to stable inertial motions of involved participants.

It is also clear that every mass (even in a relative state of rest) should have certain resulting intrinsic spin, and certain minimal rest mass,

$$\begin{aligned}
&\left[\gamma M = \frac{p}{v} = \frac{\omega_s}{v^2} L_s\right]_{v \to 0} \Rightarrow M = M_0 = \left(\frac{\omega_{s0}}{v_0^2}\right) L_s = \left(\frac{\omega_{s0}}{v_0^2}\right) L_{s0} \Rightarrow \\
&\Rightarrow F_g = G\frac{\gamma_1\left(\frac{\omega_{s0}}{v_0^2}\right) L_{s01} \cdot \gamma_2\left(\frac{\omega_{s0}}{v_0^2}\right) L_{s02}}{R^2} = \left(\frac{G\omega_{s0}^2}{v_0^4}\right)\frac{\gamma_1 L_{s01} \cdot \gamma_2 L_{s02}}{R^2} \\
&\Rightarrow \left(F_g\right)_{v_{1,2} << c} \cong \left(\frac{G\omega_{s0}^2}{v_0^4}\right)\frac{L_{s01} \cdot L_{s02}}{R^2} = G\frac{M_1 \cdot M_2}{R^2}.
\end{aligned} \quad (10.1.7)$$

Consequently, our universe should have certain intrinsic or hidden and holistic, linear, and angular velocity parameters (v_0 and ω_{s0}, probably not always and everywhere the same), because gravitational attraction exists also between two masses in relative states of rest. In fact, here we need to admit that masses are not primary and only sources of gravitation, but masses in motion (having linear and angular moments, with associated and properly oriented electric and magnetic dipoles and moments) are most relevant for gravitational attraction (see the second chapter of this book, "2. GRAVITATION", where such problematic is analyzed more profoundly). Similar intuitive conclusions we could draw if we expand our creative freedom and exercise what will be the consequences if we present Newton law of "*complex gravitational force*" in terms of complex masses, such as,

$$\overline{F}_g = G\frac{\overline{M}_1 \cdot \overline{M}_2}{R^2} = G\frac{\gamma_1 M_1 \cdot \gamma_2 M_2}{R^2} \cdot e^{\pm I(\theta_1 + \theta_2)} = G\frac{\gamma_1 M_1 \cdot \gamma_2 M_2}{R^2} \cdot e^{\pm I[(\omega_{s1} + \omega_{s2})t + \theta_0]}. \quad (10.1.8)$$

Relating Angular Moments, Magnetic Fields, and Gravitation

Angular momentum and intrinsic spin parameters are intrinsically linked to magnetic field moments. This connection suggests that gravitational forces could be related to these intrinsic magnetic properties. Since moving elements with magnetic flux generate electric currents, it is feasible to establish a systematic relationship between gravitation and electromagnetism.

The insights gained from analyzing mass, momentum, and energy, as described in equations (10.1.1) - (10.1.5), reveal a complex nature in these quantities. We have used complex mathematical functions with both real and imaginary parts, like those in Complex and Analytic Signal functions. These insights are built upon the foundation of 4-vectors and Einstein-Minkowski energy-momentum invariance, as established in general Relativity Theory (refer to

T.10.1 or T.2.2-3). The effectiveness and accuracy of these mathematical frameworks have been validated through experimental results across macro and microphysics.

The complex nature of mass, moments, and energy (as discussed in (10.1) and (10.1.1) - (10.1.5)) is a natural and essential aspect of physics. The challenge remains to interpret this complexity and understand the meaning of imaginary and apparent components of these quantities. Contemporary discussions on cosmic phenomena such as dark matter, dark energy, and zero-point vacuum states might share commonalities with the concepts of Analytic Signal, Complex, Real, and Imaginary values presented in (10.1.1) - (10.1.5). The background of complex and hypercomplex functions could support the existence of dark matter and energy by representing these phenomena as imaginary or apparent mass components, given that the mass in our universe is in constant motion. This motion involves both linear and angular components and manifests as particle-wave duality, a concept extensively discussed in this book.

Currently, the most compelling theories on gravitation are derived from Rudjer Boskovic's universal Natural Force theory [6] and Nikola Tesla's Dynamic Theory of Gravity [97]. These theories suggest that gravitation may be a specific manifestation of the universal electromagnetic force associated with resonant oscillations in the universe, creating attractive forces in nodal zones where gravitational masses are located. According to this book's perspective, our universe might only have two fundamental forces: electromagnetic forces and forces related to standing waves structured as matter waves. If all matter waves have an electromagnetic nature or origin, then there would effectively be only one acting family of forces—electromagnetic forces. For more detailed discussions, see [117].

*In (10.1.1) - (10.1.5), and earlier in (4.3-0)-p,q,r, we can recognize the kind of combination between complex and vector functions. Circular spinning frequency (or a matter wave frequency) $\vec{\omega}_s$ is a vector collinear with the particle linear moment $\vec{p}$, and hyper-complex imaginary unit **I** also has the structure like vectors, since it is composed of three, mutually orthogonal, more elementary imaginary units **i, j, k** (see more in the chapter 6. around equations (6.8) – (6.13)). Ordinary vectors can naturally be related (or fixed) to certain observer system of reference, but hyper-complex imaginary units, here naturally coupled with particle linear momentum and associated spinning, could also get other dynamic and structural meanings, and have specific and rich relations with observer's system of reference.*

In parallel to contemporary 4-vectors of Relativity theory, we can try to formulate analogical and equivalent Hypercomplex Analytic functions or Hypercomplex 4-vectors (see chapter 4.0 where Hypercomplex Analytic Signal functions are introduced). On the following brainstorming example (see below) in relation to linear momentum 4-vector (from Relativity Theory), it is possible to demonstrate (or propose) how we could analogically create Hypercomplex 4-vectors, especially when certain micro-world entity is composed of three simpler energy-momentum 4-vectors (like quarks, antiquarks, and familiar triadic structures and combinations).

***[♣ COMMENTS & FREE-THINKING CORNER**:*

***Particularly challenging opportunity or project** (combined with the facts under (10.1), (10.1.1) - (10.1.5)) will be to test, develop, or exercise the idea that Minkowski-Einstein 4-vectors as remarkably successful concept (or mathematical model applied in Physics of different interactions, impacts and scatterings), could be compatible and complementary to corresponding, **Complex Analytic Signal Phasors and relevant Wavefunction, as well as with de Broglie matter-waves framework** (as already introduced in Chapter 4.0). In other words, we will try to create Complex (and Hypercomplex) Analytic Signal Phasors from Minkowski 4-vectors. This will establish strong foundations of Wave-Particle Duality and Matter-Waves concept (after applying convenient mathematical makeup, what is still hypothetical), as follows,*

$$\left\{\left(\begin{array}{l}\overline{P}_4 = P(p, \frac{E}{c}), (p, H[p]) = (p, \hat{p}) \\ \overline{P}_4 = \left(\vec{p}, \frac{E}{c}\right) \Leftrightarrow \left(\vec{p}, I\frac{E}{c}\right), \ \hat{p} = H[p] \\ p = -H\left[\frac{E}{c}\right] = -H\left[\frac{E_k}{c}\right] = -H[\hat{p}] \\ \frac{E}{c} = H[p] = \frac{E_0}{c} + \frac{E_k}{c}, \ H[const.] \cong 0 \\ H(=)\text{Hilbert transform}\end{array}\right) \Rightarrow p^2 - \frac{E^2}{c^2} = -\frac{E_0^2}{c^2}\right\} \Rightarrow$$

$$\Rightarrow \begin{cases} \overline{P} = p + I\hat{p} = \sqrt{p^2 + \hat{p}^2} \cdot e^{I\theta} = \gamma\overline{M}v = p \cdot e^{\pm I\theta} = p \cdot \cos\theta \pm Ip \cdot \sin\theta, \ |\overline{P}| = p = \gamma M v, \\ \overline{M} = M \cdot e^{\pm I\theta} = |\overline{M}| \cdot e^{\pm I\theta} = M_r \pm IM_i, M_r = M \cdot \cos\theta, \ M_i = M \cdot \sin\theta, \\ \overline{E} = \overline{E}_{tot} = E + I\hat{E} = \sqrt{E^2 + \hat{E}^2} \cdot e^{I\theta} = E_0 \pm I \cdot cp = E_{tot} \cdot e^{\pm I\theta} = (E_0 + E_k) \cdot e^{\pm I\theta} = \\ = \sqrt{E_0^2 + c^2 p^2} \cdot e^{\pm I\theta} = \gamma M c^2 \cdot e^{\pm I\theta} = \gamma \overline{M} c^2, |\overline{E}_{tot}| = E_{tot} = \gamma M c^2, \ I^2 = -1, \\ \theta = \operatorname{arctg}\left(\frac{\hat{p}}{p}\right)(\Leftrightarrow)\left[\omega t \mp kx = 2\pi(f \cdot t \mp \frac{x}{\lambda})\right], \\ \left.\begin{array}{l}\omega = \omega_t = \left|\frac{\partial\theta}{\partial t}\right| = 2\pi f_t = \frac{2\pi}{T} = 2\pi f, \\ k = \omega_x = \left|\frac{\partial\theta}{\partial x}\right| = 2\pi f_x = \frac{2\pi}{\lambda} = 2\pi\frac{p}{h}\end{array}\right\} \Rightarrow \left\{\begin{array}{l} u = \frac{\omega}{k} = \frac{f_t}{f_x} = \frac{\lambda}{T} = \frac{\partial x}{\partial t} = \frac{v}{1 + \sqrt{1 - v^2/c^2}} \\ v = \frac{d\omega}{dk} = \frac{dx}{dt} = u - \lambda\frac{du}{d\lambda}, \ \lambda = \frac{h}{p} = \frac{2\pi}{k}\end{array}\right\}, \\ E = E_0 + E_k = E_0 + \frac{pv}{1 + \sqrt{1 - v^2/c^2}} = E = E_{tot}, \ E_k = \tilde{E} = \frac{pv}{1 + \sqrt{1 - v^2/c^2}} = hf \\ \hat{E} = H[E], p = -H[\hat{p}], \ I^2 = -1, \ (\text{here}, H(=) \text{ Hilbert transform}) \ \Rightarrow \\ \boxed{dE = vdp \Rightarrow \text{Power} = dE/dt \Rightarrow \left(d\overline{E}/dt\right) = v\left(d\overline{P}/dt\right) = \overline{\Psi}^2 \ (=) \text{wavefunction squared}}. \end{cases}$$

Extending Energy-Momentum 4-Vectors to Hypercomplex 4-Vectors

A promising avenue for advancing the theory of energy-momentum 4-vectors from Relativity is to analogically extend these concepts to Hypercomplex numbers and "Analytic Signal Phasors" or wavefunctions. This extension could enhance the already impressive applicability of 4-vectors, providing a robust mathematical foundation for understanding Matter-Waves and Wave-Particle Duality.

Given the established analogies between linear and rotational motions and electromagnetic phenomena (as discussed in the first chapter of this book) and leveraging the successful mathematical treatment of complex phasor functions in electromagnetism (such as voltages, currents, power, and impedances), we can analogically apply the same phasor methodology to mechanical systems and matter waves. Just as we analyze phase shifts and transformations in electrical currents and voltages due to various loads, and define different types of power (active, reactive, apparent, complex, reflected), we can similarly apply these concepts to linear and angular motions in mechanics.

Furthermore, the wave-function concept, as extended in this book to encompass temporal-spatial signals, velocities, forces, and moments (see Chapter 4.0 and (4.0.82)), aligns naturally with phasor notation. In this context, dark or invisible mass and energy might be represented by the imaginary components of moments, masses, and energies when expressed as complex functions and Analytic Phasors.

By adopting this approach, we could significantly advance our understanding of mechanical interactions and Wave-Particle Duality theory. However, current methods often rely on incomplete and conceptually limited probabilistic and statistical strategies. While Probability and Statistics are crucial for mass-data processing and modeling across various scientific and practical fields, they should not overshadow the potential of more rigorous theoretical extensions.

Einstein's General Relativity theory describes how space and time are intertwined and curved around massive objects, creating the illusion of gravitational force. Similarly, this concept of curved space can be analogically applied to electromagnetically charged entities. By employing suitable mathematical models, we can achieve accurate representations of gravitation, wave-particle duality, electromagnetic forces, fields, and interactions with charged particles.

To explore this further, consider the hypothetical exercise of extending Einstein-Minkowski 4-vectors to Hypercomplex 4-vectors:

$$\left\{\begin{array}{l}\overline{P}_4=\left(\vec{p},\frac{E}{c}\right)\Leftrightarrow\left(\vec{p},I\frac{E}{c}\right)=\left(\vec{p}_i,i\frac{E_i}{c_i}\right)+\left(\vec{p}_j,j\frac{E_j}{c_j}\right)+\left(\vec{p}_k,k\frac{E_k}{c_k}\right)\Leftrightarrow\\ \Leftrightarrow\left(\vec{p},I\frac{E}{c}\right)=\left(\vec{p}_i+\vec{p}_j+\vec{p}_k,\frac{iE_i+jE_j+kE_k}{c}\right)=\left(\vec{p}_i+\vec{p}_j+\vec{p}_k,I\frac{E}{c}\right)\\ \overline{P}_4=\left(\vec{p},\frac{E}{c}\right)=\text{invariant}\Rightarrow\vec{p}^2-(\frac{E}{c})^2=-(\frac{E_0}{c^2})^2\Leftrightarrow E_0^2+p^2c^2=E^2=(E_0+E_k)^2,\\ \vec{p}=\vec{p}_i+\vec{p}_j+\vec{p}_k=\vec{p}_t, E=E_{0t}+E_{kt}, E_0=E_{0t}, E=E_t,\\ E_i=E_{0i}+E_{ki}, E_j=E_{0j}+E_{kj}, E_k=E_{0k}+E_{kk},\\ I^2=i^2=j^2=k^2=-1, ij=k,\ jk=i, ki=j, ji=-k, kj=-i, ik=-j\end{array}\right\}\Rightarrow$$

$$\Rightarrow\left\{\begin{array}{l}I\frac{E}{c}=i\frac{E_i}{c_i}+j\frac{E_j}{c_j}+k\frac{E_k}{c_k},\\ (\vec{p}_i+\vec{p}_j+\vec{p}_k)^2-(\frac{E}{c})^2=-(\frac{E_0}{c^2})^2,\\ \left(\frac{E_{0-tot}+E_{k-tot}}{c}\right)^2=\left(\frac{E_{0i}+E_{ki}}{c_i}\right)^2+\left(\frac{E_{0j}+E_{kj}}{c_j}\right)^2+\left(\frac{E_{0k}+E_{kk}}{c_k}\right)^2\end{array}\right\}\Rightarrow$$

$$\Rightarrow\left\{\begin{array}{l}\left(\frac{E}{c}\right)^2=\left(\frac{E_i}{c_i}\right)^2+\left(\frac{E_j}{c_j}\right)^2+\left(\frac{E_k}{c_k}\right)^2\\ \left(\frac{E_0}{c}\right)^2=\left(\frac{E_{0i}}{c_i}\right)^2+\left(\frac{E_{0j}}{c_j}\right)^2+\left(\frac{E_{0k}}{c_k}\right)^2\\ \left(\frac{E_{k-tot}}{c}\right)^2=\left(\frac{E_{ki}}{c_i}\right)^2+\left(\frac{E_{kj}}{c_j}\right)^2+\left(\frac{E_{kk}}{c_k}\right)^2\\ \frac{E_{0-tot}E_{k-tot}}{c^2}=\frac{E_{0i}E_{ki}}{c_i^2}+\frac{E_{0j}E_{kj}}{c_j^2}+\frac{E_{0k}E_{kk}}{c_k^2}\end{array}\right\}\Rightarrow\left\{\begin{array}{l}p^2=\frac{E^2-E_0^2}{c^2}=\frac{E_i^2-E_{0i}^2}{c_i^2}+\frac{E_j^2-E_{0j}^2}{c_j^2}+\frac{E_k^2-E_{0k}^2}{c_k^2}=\vec{p}_i^2+\vec{p}_j^2+\vec{p}_k^2\\ \vec{p}_i^2=\frac{E_i^2-E_{0i}^2}{c_i^2},\vec{p}_j^2=\frac{E_j^2-E_{0j}^2}{c_j^2},\vec{p}_k^2=\frac{E_k^2-E_{0k}^2}{c_k^2}\\ \text{or, if}\quad c=c_i=c_j=c_k\rightarrow\\ p^2=E^2-E_0^2=E_i^2-E_{0i}^2+E_j^2-E_{0j}^2+E_k^2-E_{0k}^2=\vec{p}_i^2+\vec{p}_j^2+\vec{p}_k^2\\ \vec{p}_i^2=\frac{E_i^2-E_{0i}^2}{c^2},\vec{p}_j^2=\frac{E_j^2-E_{0j}^2}{c^2},\vec{p}_k^2=\frac{E_k^2-E_{0k}^2}{c^2}\\ E^2=E_i^2+E_j^2+E_k^2, E_0^2=E_{0i}^2+E_{0j}^2+E_{0k}^2\end{array}\right\}$$

(4.3-0)-s, (10.1.6)

In (10.1.6), constants c_i, c_j, c_k *that have dimensions of speed (like the universal speed constant* $c\cong 3\cdot 10^8 m/s$*), could be equal to* $c=c_i=c_j=c_k$ *(but here involved mathematics is also giving chances for other options). Energies indexed with "0" and "k" in (10.1.6) are effectively paving the way to explain creations of big numbers of products in impact reactions (zeroes are indicating particles with rest masses, and k-indexing stands for kinetic energy states. Here we have a small indexing problem, since k is the imaginary unit in the Hypercomplex number definition, and such confusing indexing should be (one day) corrected, but here it was easier and faster to neglect such small problems since main intention has been to present the idea about Hypercomplex 4-Vectors). Such extended energy-momentum framework can be later merged with universal Complex and Hypercomplex Analytic Signal representation of wavefunctions (leading to all famous wave equations of QT), and with novel* *foundations of multidimensional Universe (see Chapter 4.3 and chapter 6, equations (6.10) - (6.13)). Presence of three imaginary units in (10.3.6) is intuitively igniting ideas about mutually coupled energy triplets such as three quarks, three anti-quarks etc., what could create another, more general and more precise concept of Super-Symmetry in the world of microphysics (and significantly or essentially enrich and simplify the Standard Model).*

Minkowski space foundations, 4-vectors, and Hypercomplex Analytic Signal

*Citation from: "https://en.wikipedia.org/wiki/Minkowski_space: In mathematical physics, **Minkowski space** (or **Minkowski spacetime**) is a combination of three-dimensional Euclidean space and time into a four-dimensional manifold where the spacetime interval between any two events is independent of the inertial frame of reference in which they are recorded. Although initially developed by mathematician Hermann Minkowski for Maxwell's equations of electromagnetism, the mathematical structure of Minkowski spacetime was shown to be an immediate consequence of the postulates of special relativity.[1]*

Minkowski space is closely associated with Einstein's theory of special relativity and is the most common mathematical structure on which special relativity is formulated. While the individual components in Euclidean space and time may differ due to length contraction and time dilation, in Minkowski spacetime, all frames of reference will agree on the total distance in spacetime between events.[nb 1] Because it treats time differently than it treats the 3 spatial dimensions, Minkowski space differs from four-dimensional Euclidean space.

In 3-dimensional Euclidean space (e.g., simply space in Galilean relativity), the isometry group (the maps preserving the regular Euclidean distance) is the Euclidean group. It is generated by rotations, reflections and translations. When time is amended as a fourth dimension, the further transformations of translations in time and Galilean boosts are added, and the group of all these transformations is called the Galilean group. All Galilean transformations preserve the 3-dimensional Euclidean distance. This distance is purely spatial. Time differences are separately preserved as well. This changes in the spacetime of special relativity, where space and time are interwoven.

Spacetime is equipped with an indefinite non-degenerate bilinear form, variously called the Minkowski metric,[2] the Minkowski norm squared or Minkowski inner product depending on the context.[nb 2] The Minkowski inner product is defined so as to yield the spacetime interval between two events when given their coordinate difference vector as argument.[3] Equipped with this inner product, the mathematical model of spacetime is called Minkowski space. The analogue of the Galilean group for Minkowski space, preserving the spacetime interval (as opposed to the spatial Euclidean distance) is the Poincaré group.

In summary, Galilean spacetime and Minkowski spacetime are, when viewed as manifolds, actually the same. They differ in what further structures are defined on them. The former has the Euclidean distance function and time (separately) together with inertial frames whose coordinates are related by Galilean transformations, while the latter has the Minkowski metric together with inertial frames whose coordinates are related by Poincaré transformations".

....

We know that Minkowski space and 4-vectors concept is extremely productive and useful mathematical processing in Physics and Relativity theory (proven as working without problems). Let us reestablish and explain it (again) on a quite simple, elementary, and step by step way, as follows.

1° We consider that everything what is happening in the world of Physics or Nature belongs to the united spatial-temporal domain. We start with the concept (or assumption) that spatial and temporal domains are coexistent, and on some direct and natural way mutually linked, related, replaceable, and transformable (not identical, but mutually directly proportional, $\mathbf{r} \rightleftarrows \mathbf{t}$*).*

2° The second assumption is that spatial and temporal domain are mutually orthogonal (meaning mutually phase shifted for $\pi/2$ or for $90°$*). This is giving us a chance to consider that the Time Domain is the Hilbert transform of the Space Domain, or vice versa (since we do not have direct destructive interferences and overlapping between them). Of course, here we only conceptually and intuitively describe how spatial and temporal domains are mutually orthogonal and phase shifted, and how we could manage this mathematically using Hilbert transform. More rigorous and much better* *mathematical elaboration about the mentioned concept is still missing here, being an evolving project. For instance, if we describe certain motion with its (spatial) radius vector* $r = r(x, y, z),\ x = x(t), y = y(t),\ z = z(t)$, *and with its temporal position* $t = t(t_x, t_y, t_z)$, *then we can exercise to assemble four-dimensional, or Minkowski-space 4-vector situation, such as,*

$$R\left[r(x, y, z), t(t_x, t_y, t_z)\right] = \mathbf{R}(\mathbf{r}, \mathbf{t}),\ \ \mathbf{dx} = \mathbf{c}_x \cdot \mathbf{dt}_x,\ \mathbf{dy} = \mathbf{c}_y \cdot \mathbf{dt}_y,\ \mathbf{dz} = \mathbf{c}_z \cdot \mathbf{dt}_z,$$
$$\mathbf{r} = \mathbf{u} \cdot \mathbf{t},\ \mathbf{u} = \lambda \mathbf{f} \le \mathbf{c} = \mathbf{const.},$$
$$H[r] = H[r(x, y, z)] = \mathbf{u} \cdot t(t_x, t_y, t_z) = \mathbf{u} \cdot \mathbf{t}\ (\cong \mathbf{c} \cdot \mathbf{t}\ \text{, for photons}),$$
$$\bar{R}_4 = R\left[r(x, y, z), t(t_x, t_y, t_z)\right] = \mathbf{R}(\mathbf{r}, \mathbf{Ict}),\ \mathbf{I}^2 = -\mathbf{1}.$$

Spatial $\mathbf{r}$, *and temporal* $\mathbf{Ict}$ *positions (dimensions, coordinates, domain lengths) are mutually orthogonal, like in some right-angle triangle, and we can simply apply Pythagoras' Theorem in order to get relevant hypotenuse length, which is A. Einstein, relativistic space-time interval,* ΔS *(see more here: https://www.sciencedirect.com/topics/physics-and-astronomy/minkowski-space).*

3° If we assume that in different referential systems, spatial and temporal lengths $\Delta\mathbf{r},\ \Delta\mathbf{t}$ *are different, but mutually linked, mutually orthogonal and proportional on the same way, making that in a higher dimensional (spatial-temporal) framework this always presents the same hypotenuse length or duration* $\Delta S = \text{const.}$, *we will have the following invariance relation (as in Special Relativity theory),*

$$\begin{bmatrix} \Delta\bar{\mathbf{S}}_4 = \bar{V}_4 \cdot \Delta t = (\gamma v \cdot \Delta t, \gamma c \cdot \Delta t) = (\gamma\Delta r, \gamma c \cdot \Delta t) \\ \Leftrightarrow\ \Delta\bar{S}(\gamma\Delta r, I\gamma c\Delta t) \Rightarrow \Delta\bar{S} = \left|\Delta\bar{S}_4\right| \cdot \mathbf{e}^{\mathbf{I}\cdot\mathbf{arctg}\frac{c}{v}} \\ \bar{V}_4 = (\gamma v, \gamma c) = (\gamma\frac{\Delta r}{\Delta t}, \gamma c)\ \Leftrightarrow\ (\gamma\frac{\Delta r}{\Delta t}, \mathbf{I}\gamma c), \\ \mathbf{I}^2 = -\mathbf{1},\ \mathbf{c}^2\mathbf{t}^2 = \mathbf{c}_x^2\mathbf{t}_x^2 + \mathbf{c}_y^2\mathbf{t}_y^2 + \mathbf{c}_z^2\mathbf{t}_z^2, \\ (\Delta\mathbf{r})^2 = (\Delta\mathbf{x})^2 + (\Delta\mathbf{y})^2 + (\Delta\mathbf{z})^2. \end{bmatrix} \Rightarrow \begin{bmatrix} \left|\Delta\bar{S}_4\right|^2 = (\gamma\Delta r)^2 - (\gamma c \cdot \Delta t)^2 = -(c\Delta t_0)^2 \Rightarrow \\ \Rightarrow (\gamma c \cdot \Delta t)^2 - (c\Delta t_0)^2 = (\gamma\Delta r)^2 \Rightarrow \\ \Rightarrow \boxed{\Delta t_0 = \gamma\Delta t} \\ \Delta t_0 = \text{measured time},\ \ \Delta t = \text{proper time} \\ \Psi(r,t) = \gamma\Delta r, \hat{\Psi}(r,t) = \mathbf{H}[\Psi(r,t)] = c\gamma\Delta t, \\ H[\gamma\Delta r] = c\gamma\Delta t,\ H[c\gamma\Delta t] = c \cdot H[\gamma\Delta t] = -\gamma\Delta r, \\ \gamma\Delta r = -c \cdot H[\gamma\Delta t], \gamma\Delta t = \frac{1}{c}H[\gamma\Delta r] \end{bmatrix}$$

4° The reference system invariant formulation of the complex 4-vector, which describes certain spatial-temporal length is on some intuitive way comprehensible as,

$$\Delta S_4 = \Delta S(\boxed{\gamma\Delta r}, I\boxed{\gamma c\Delta t}) = \Delta S\left(\boxed{\begin{matrix}\text{spatial interval that}\\ \text{motional particle passed}\end{matrix}}, \boxed{\begin{matrix}\text{spatial or time interval, which a}\\ \text{photon would cover}\end{matrix}}\right).$$

By analogy, we can now formulate similar 4-vector for a particle that has linear momentum p,

$$P_4 = P(\boxed{p}, I\boxed{\frac{E_{tot}}{c}}) = P\left(\boxed{\begin{matrix}\text{moment}\\ \text{of a particle}\end{matrix}}, \boxed{\begin{matrix}\text{moment of the same particle}\\ \text{which is effectively transformed}\\ \text{in a "photonic wave-group"}\end{matrix}}\right) = P(p, I\frac{E_{tot}}{c}).$$

Analogically (also a bit hypothetically), applying Pythagoras' Theorem, we can get the following, reference systems invariance relations for energy-momentum 4-vectors,

$$P_4 = P(p,\ I\frac{E_{tot}}{c}) \Rightarrow p^2 - \left(\frac{E_{tot}}{c}\right)^2 = p_i^2 - \left(\frac{E_{tot-i}}{c}\right)^2 = ... = p_0^2 - \left(\frac{E_{tot-0}}{c}\right)^2 = -\left(\frac{E_{tot-0}}{c}\right)^2 = const.,$$

$$p_0 = mv = p(v=0) = 0,\ i = 1,2,3,...\ \ v = \frac{\Delta E_{tot}}{\Delta p} = \frac{\Delta E_k}{\Delta p}, \Delta E = \Delta E_{tot} = \Delta E_k = v\Delta p$$

$$\Delta\overline{P}_4 = \overline{P}(\Delta p,\ \frac{\Delta E_{tot}}{c}) \Leftrightarrow \overline{P}(\Delta p,\ I\frac{\Delta E_{tot}}{c}) = m\overline{V}_4(\gamma\Delta v, \gamma c) = m\frac{\Delta\overline{S}_4}{\Delta t} \Rightarrow$$

$$\Delta\overline{P}_4 = \left|\Delta\overline{P}_4\right| \cdot e^{-I\mathrm{arctg}\frac{\Delta E_{tot}}{c\Delta p}} = \left|\Delta\overline{P}_4\right| \cdot e^{-I\cdot\mathrm{arctg}\frac{v}{c}} = \sqrt{(\Delta p)^2 - \left(\frac{\Delta E_{tot}}{c}\right)^2} \cdot e^{-I\cdot\mathrm{arctg}\frac{v}{c}} \Rightarrow \begin{bmatrix} \Delta\overline{E}_4 = v\Delta\overline{P}_4 = \overline{E}(v\Delta p,\ \frac{v}{c}\Delta E_{tot}) \\ \overline{E}_4 = v\overline{P}_4 = \overline{E}(vp,\ \frac{v}{c}E_{tot}) \end{bmatrix}.$$

By creating complex Phasors from Minkowski 4-vectors, such as $\Delta\overline{P}_4 = \left|\Delta\overline{P}_4\right| \cdot e^{-I\theta}$, we are directly addressing Wave-Particle Duality and Matter Waves, since phase function θ has the information about relevant wave velocities, frequencies, and wavelengths.

We can analogically formulate a force 4-vector as,

$$\overline{F}_4 = \frac{\Delta\overline{P}_4}{\Delta t} = \overline{P}(\frac{\Delta p}{\Delta t}, \frac{1}{c}\frac{\Delta E_{tot}}{\Delta t}) = m\overline{V}_4(\gamma\frac{\Delta v}{\Delta t}, \frac{\gamma c}{\Delta t}) = m\frac{\Delta\overline{S}_4}{(\Delta t)^2}.$$

Extending Minkowski's 4-Vectors to Hypercomplex Space-Time

Minkowski's concept of 4-vectors has proven immensely successful and influential in Physics, forming a crucial part of the Special Relativity theory. Both Mileva Maric and Albert Einstein, along with other proponents of Relativity, adopted Minkowski's mathematical framework, which was essential for the development and understanding of Special Relativity. Without Minkowski's 4-vectors, Special Relativity would lack the clarity and rigor that have made it a cornerstone of modern physics. Minkowski himself did not assert claims on his intellectual contributions, and it is perhaps time to more fully acknowledge his monumental role in shaping Relativity theory. Similar recognition is due to Henri Poincaré for his contributions to theory as well. Additionally, Lorentz transformations and the foundational aspects of Relativity theory can be traced back to an evolved version of Maxwell-Faraday electromagnetic theory, as discussed in Chapter 3 of this book.

Minkowski's 4-dimensional space-time, which incorporates a single imaginary unit (i.e., $I^2=-1$), can be expanded to a higher-dimensional Hypercomplex space-time with at least three imaginary units (see Chapter 6 for a detailed discussion). This extension opens new avenues for exploring multidimensional structures and the potential complexity of the time domain. Hypercomplex and Analytic Phasor models, along with Minkowski's space-time concept and Hilbert transforms, provide a robust theoretical framework for understanding these multidimensional insights.

In a multidimensional universe, it is logical to expect an equal number of spatial and temporal dimensions, given that temporal and spatial domains are mutually proportional, phase-shifted, orthogonal, and transformable. If some spatial dimensions exhibit unusual non-linearity or anisotropy, we might anticipate similar properties in the corresponding temporal dimensions. This becomes particularly intriguing when considering interactions between temporal and spatial domains, such as effects of anti-gravity, where periodicity parameters in both domains are comparable.

Furthermore, each imaginary unit in a hypercomplex space-time can be thought of as comprising multiple new imaginary units. This conceptualization allows for the exploration of increasingly higher-dimensional spatial-temporal worlds. While this progression can become complex and seemingly infinite, it remains mathematically feasible and provides a clearly structured framework for further theoretical exploration.

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Signals representation with Hypercomplex functions and relevant Hyper Complex Analytic Signals or Phasors has rich and challenging structure, and can be developed to represent many of complex and *elementary matter states, as we can imaginatively and creatively judge from the following brainstorming exercises,*

$$\overline{Z} = \overline{Z}_{si} + \overline{Z}_{sj} + \overline{Z}_{sk} = \overline{Z}_{pi} \cdot \overline{Z}_{pj} \cdot \overline{Z}_{pk} = \left|\overline{Z}\right| \cdot e^{I\cdot\Theta},$$
$$i^2 = j^2 = k^2 = I^2 = -1,\ ij = k, jk = i, ki = j, ik = -j, kj = -i, ji = -k,$$

$$\begin{bmatrix} \overline{Z}_{si} = A_{si} \cdot e^{i\cdot\theta_{si}} = a_{si} + i \cdot b_{si} = \sqrt{a_{si}^2 + b_{si}^2} \cdot e^{i\cdot arctg(\frac{b_{si}}{a_{si}})}, \\ \overline{Z}_{sj} = A_{sj} \cdot e^{j\cdot\theta_{sj}} = a_{sj} + j \cdot b_{sj} = \sqrt{a_{sj}^2 + b_{sj}^2} \cdot e^{j\cdot arctg(\frac{b_{sj}}{a_{sj}})}, \\ \overline{Z}_{sk} = A_{sk} \cdot e^{k\cdot\theta_{sk}} = a_{sk} + k \cdot b_{sk} = \sqrt{a_{sk}^2 + b_{sk}^2} \cdot e^{k\cdot arctg(\frac{b_{sk}}{a_{sk}})} \end{bmatrix},$$

$$\begin{bmatrix} \overline{Z}_{pi} = A_{pi} \cdot e^{i\cdot\theta_{pi}} = a_{pi} + i \cdot b_{pi} = \sqrt{a_{pi}^2 + b_{pi}^2} \cdot e^{i\cdot arctg(\frac{b_{pi}}{a_{pi}})}, \\ \overline{Z}_{pj} = A_{pj} \cdot e^{j\cdot\theta_{pj}} = a_{pj} + j \cdot b_{pj} = \sqrt{a_{pj}^2 + b_{pj}^2} \cdot e^{j\cdot arctg(\frac{b_{pj}}{a_{pj}})}, \\ \overline{Z}_{pk} = A_{pk} \cdot e^{k\cdot\theta_{pk}} = a_{pk} + k \cdot b_{pk} = \sqrt{a_{pk}^2 + b_{pk}^2} \cdot e^{k\cdot arctg(\frac{b_{pk}}{a_{pk}})} \end{bmatrix} \Rightarrow$$

$$\Rightarrow \begin{bmatrix} \overline{Z} = \overline{Z}_{si} + \overline{Z}_{sj} + \overline{Z}_{sk} = (a_{si} + i \cdot b_{si}) + (a_{sj} + j \cdot b_{sj}) + (a_{sk} + k \cdot b_{sk}) = (a_{pi} + i \cdot b_{pi}) \cdot (a_{pj} + j \cdot b_{pj}) \cdot (a_{pk} + k \cdot b_{pk}) = \\ = A_{pi} \cdot A_{pj} \cdot A_{pk} \cdot e^{i\cdot\theta_{pi} + j\cdot\theta_{pj} + k\cdot\theta_{pk}} = \left|\overline{Z}\right| \cdot e^{I\cdot\Theta} = \left|\overline{Z}\right| \cdot \cos\Theta + I \cdot \left|\overline{Z}\right| \cdot \sin\Theta = \\ = (a_{si} + a_{sj} + a_{sk}) + i \cdot b_{si} + j \cdot b_{sj} + k \cdot b_{sk} = A_s + I \cdot B_s, A_s = \left|\overline{Z}\right| \cdot \cos\Theta,\ B_s = \left|\overline{Z}\right| \cdot \sin\Theta \end{bmatrix} \Rightarrow$$

$$\Rightarrow \begin{bmatrix} I \cdot \Theta = i \cdot \theta_{pi} + j \cdot \theta_{pj} + k \cdot \theta_{pk}, I = i \cdot \frac{\theta_{pi}}{\Theta} + j \cdot \frac{\theta_{pj}}{\Theta} + k \cdot \frac{\theta_{pk}}{\Theta}, \\ \left|\overline{Z}\right| = A_{pi} \cdot A_{pj} \cdot A_{pk} = \frac{a_{si} + a_{sj} + a_{sk}}{\cos\Theta} = \sqrt{A_s^2 + B_s^2}, \\ A_s = a_{si} + a_{sj} + a_{sk} = \left|\overline{Z}\right| \cdot \cos\Theta, I \cdot B_s = i \cdot b_{si} + j \cdot b_{sj} + k \cdot b_{sk} = I \cdot \left|\overline{Z}\right| \cdot \sin\Theta, \\ I \cdot \left|\overline{Z}\right| \cdot \sin\Theta = i \cdot b_{si} + j \cdot b_{sj} + k \cdot b_{sk}, B_s^2 = (i \cdot b_{si} + j \cdot b_{sj} + k \cdot b_{sk})^2 = (b_{si}^2 + b_{sj}^2 + b_{sk}^2), \\ i \cdot j \cdot b_{si} \cdot b_{sj} + i \cdot k \cdot b_{si} \cdot b_{sk} + j \cdot k \cdot b_{sj} \cdot b_{sk} = k \cdot b_{si} \cdot b_{sj} - j \cdot b_{si} \cdot b_{sk} + i \cdot b_{sj} \cdot b_{sk} = 0, \\ I = i \cdot \frac{b_{si}}{\left|\overline{Z}\right| \cdot \sin\Theta} + j \cdot \frac{b_{sj}}{\left|\overline{Z}\right| \cdot \sin\Theta} + k \cdot \frac{b_{sk}}{\left|\overline{Z}\right| \cdot \sin\Theta} = i \cdot \frac{\theta_{pi}}{\Theta} + j \cdot \frac{\theta_{pj}}{\Theta} + k \cdot \frac{\theta_{pk}}{\Theta} = i \cdot \frac{b_{si}}{B_s} + j \cdot \frac{b_{sj}}{B_s} + k \cdot \frac{b_{sk}}{B_s}, \\ \frac{\theta_{pi}}{\Theta} = \frac{b_{si}}{\left|\overline{Z}\right| \cdot \sin\Theta}, \frac{\theta_{pj}}{\Theta} = \frac{b_{sj}}{\left|\overline{Z}\right| \cdot \sin\Theta}, \frac{\theta_{pk}}{\Theta} = \frac{b_{sk}}{\left|\overline{Z}\right| \cdot \sin\Theta}, \\ \frac{\sin\Theta}{\Theta} = \frac{b_{si}}{\left|\overline{Z}\right| \cdot \theta_{pi}} = \frac{b_{sj}}{\left|\overline{Z}\right| \cdot \theta_{pj}} = \frac{b_{sk}}{\left|\overline{Z}\right| \cdot \theta_{pk}}, \frac{\theta_{pi}}{\Theta} = \frac{b_{si}}{B_s}, \frac{\theta_{pj}}{\Theta} = \frac{b_{sj}}{B_s}, \frac{\theta_{pk}}{\Theta} = \frac{b_{sk}}{B_s}, \\ \Theta = \frac{\theta_{pi}}{b_{si}} B_s = \frac{\theta_{pj}}{b_{sj}} B_s = \frac{\theta_{pk}}{b_{sk}} B_s, B_s = \frac{b_{si}}{\theta_{pi}} \Theta = \frac{b_{sj}}{\theta_{pj}} \Theta = \frac{b_{sk}}{\theta_{pk}} \Theta = \sqrt{b_{si}^2 + b_{sj}^2 + b_{sk}^2} = \frac{1}{3}\left(\frac{b_{si}}{\theta_{pi}} + \frac{b_{sj}}{\theta_{pj}} + \frac{b_{sk}}{\theta_{pk}}\right) \cdot \Theta \end{bmatrix}$$

***Another challenging project** could be to show, or make it presentable, that mutually coupled electric and magnetic fields of the same event will behave like a couple of an original and its Hilbert transform function (eventually creating a corresponding, complex Analytic signal functions and Phasors; -see more in chapter 3. of this book). The starting steps in describing such an objective will be,*

$$E, H\ (=)\ \text{electric and magnetic fields},\ \mathcal{H}[\ \ ]\ (=)\ \text{Hilbert transform},$$

$$\bar{E}_4 = \bar{E}(E,\hat{E}) = (E,I\hat{E}) = (E,\alpha H),\ \alpha H = \alpha(v)\cdot H = \mathcal{H}[E] = \hat{E}$$
$$\bar{H}_4 = \bar{H}(H,\hat{H}) = (H,I\hat{H}) = (H,\beta E),\ \beta E = \beta(v)\cdot E = \mathcal{H}[H] = \hat{H}\ ,I^2 = -1.$$

Since products between 4-vectors are also 4-vectors, we could define (and later develop) corresponding scalar and vectors' products, such as,

$$\bar{E}\cdot\bar{H} = (E,I\hat{E})\cdot(H,I\hat{H}) = (E,I\alpha H)\cdot(H,I\beta E) = ... \quad \text{and / or} \quad \bar{E}\times\bar{H} = (\vec{E},I\hat{\vec{E}})\times(\vec{H},I\hat{\vec{H}}) = (\vec{E},I\alpha\vec{H})\times(\vec{H},I\beta\vec{E}) = ... ,$$

(where $\alpha(v) = \vec{\alpha}$ and $\beta(v) = \vec{\beta}$ could also be vectors). This should (later, when united with some creative imagination) produce Poynting vectors, and we will be closer to creation of relevant electromagnetic, energy-momentum 4-vectors and their complex (or hypercomplex) Phasor functions.

Innovative Modeling of Electromagnetic Waves

This approach could lead to groundbreaking modeling of associated electromagnetic waves. Refer to Figure 10.1.1, which illustrates two configurations: on the left, phase-shifted and mutually orthogonal electric and magnetic fields, representing an ideal electromagnetic wave or photon. This configuration aligns with the hypothesis that electric and magnetic field vectors behave as Hilbert transforms of the real and imaginary parts of an Analytic Signal Electromagnetic wave.

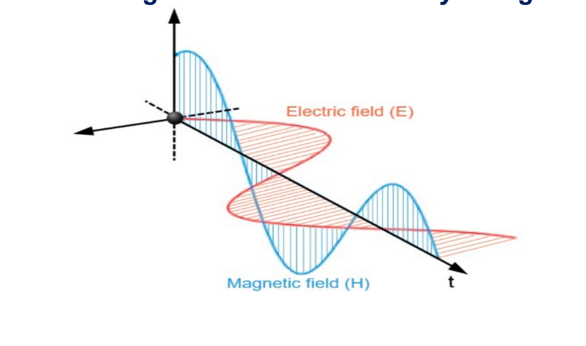
Currently, our description of electromagnetic waves and photons (shown on the right in Figure 10.1.1) inaccurately assumes that electric and magnetic field vectors are mutually orthogonal and in phase. This assumption implies that energy cannot fluctuate or oscillate between these fields, which is inconsistent with energy conservation principles. As such, Maxwell's equations may require updates to reflect this more accurate understanding (see Chapter 3 for further details).

Power in an electromagnetic wave must be continuously balanced and oscillate between electric and magnetic field energies, always adhering to conservation laws, not merely on average. It is crucial to consider the phase difference between the electric and magnetic field components, particularly in relation to complex impedance loads, which include resistive, capacitive, and inductive components. This phase difference influences the efficiency of energy transfer from the source to the load.

From this perspective, electromagnetic power and energy flow can be categorized into various types: Active, Reactive, Apparent, and represented as average, effective, or RMS power, among others. For a more comprehensive discussion, see Chapter 4.0, particularly section "4.0.11. Generalized Wave Functions and Unified Field Theory."

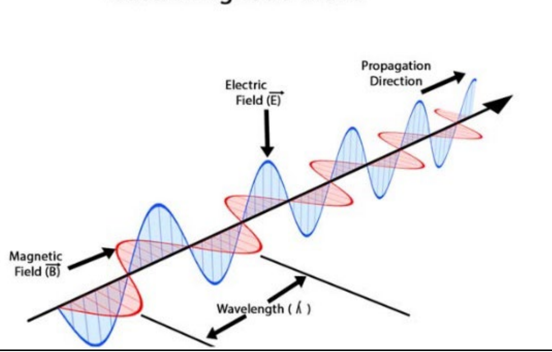
The essential insight here is that the spatial, temporal, source, and load-related properties of matter waves or wavefunctions are intrinsically interconnected. When comparing this characterization with contemporary Quantum Theory, as detailed in Chapter 4.0 and familiar equations (3.7-1) and (3.7-2) from Chapter 3, it becomes clear that the current wavefunction concept in orthodox Quantum Theory should be revised.

Electromagnetic Wave as an Analytic Signal



New (analytic signal related and still hypothetical) concept of electromagnetic waves

Electromagnetic Wave



Present (or old, still considered as valid) concept of electromagnetic vawes

Fig. 10.1.1 Propagation of Electromagnetic waves

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