Ljubisa Peric, D.Sc. Mech. Eng. Bul. Nemanjica 71/19, 18000 Nis, Serbia and Montenegro, April, 11th 2004.
e-mail: njperic@ptt.yu, ljubisa@rpknis.co.yu, phone: +38118 534171, +38118 534172, +38163 422329

NEW

it was derived:

Universal Formula for Electric Impedance of Piezoelectric Ceramic Z_p

Electric Impedance, Piezoelectric Ceramic and Materials, Circular-ring Plate
Mechanics of Coupled Fields, Electrodynamics, Oscillations, Coupled Tensors of State

Dear Sirs,

I have derived an **Universal Formula for Simulation of Electric Impedance Model** $Z_p = V/I\ [\Omega]$ of piezoelectric ceramic. Formula stands for all shapes of piezoelectric bodies: circular plate, circular-ring plate, rectangular plate, cylinder, circular-ring cylinder, circular-ring sectional cylinder, sphere, etc. Piezoceramic bodies may have n-th number of surfaces loaded by equal or different external loads: F_j^D or F_j^E (v_j^D or v_j^E - motion velocities of contour surfaces loaded by F_j^D or F_j^E) as interaction with outer medium, i.e., external mechanical impedances: Z_j^D or Z_j^E.

Formula enables precise determination of resonant frequencies of numerous radial, transversal, longitudinal, and lateral modes of oscillation even at design stage of piezoelectric transducers, and before manufacturing of piezoceramic elements and experimental measuring.

Application field of piezo-sensors and actuators is very wide, and of special interest for **military industry** and **space research**. It may be also used for significant improvement of existing methods of modeling FEM, BEM, etc.

The formula is based on two constitutive system of equations:

Isothermal func. of internal energy $U(S_{ij}, D_i)$:	Isothermal func. of electric potential $H(S_{ij}, E_i)$:
$T_{rr}^D = c_{11}^D S_{rr} + c_{12}^D S_{\theta\theta} + c_{13}^D S_{zz} - h_{31} D_z,$	$T_{rr}^E = c_{11}^E S_{rr} + c_{12}^E S_{\theta\theta} + c_{13}^E S_{zz} - e_{31} E_z,$
$T_{\theta\theta}^D = c_{12}^D S_{rr} + c_{11}^D S_{\theta\theta} + c_{13}^D S_{zz} - h_{31} D_z,$	$T_{\theta\theta}^E = c_{12}^E S_{rr} + c_{11}^E S_{\theta\theta} + c_{13}^E S_{zz} - e_{31} E_z,$
$T_{zz}^D = c_{13}^D (S_{rr} + S_{\theta\theta}) + c_{33}^D S_{zz} - h_{33} D_z,$	$T_{zz}^E = c_{13}^E (S_{rr} + S_{\theta\theta}) + c_{33}^E S_{zz} - e_{33} E_z,$
$E_z^D = -h_{31}(S_{rr} + S_{\theta\theta}) - h_{33} S_{zz} + D_z / \varepsilon_{33}^S.$	$D_z^E = e_{31}(S_{rr} + S_{\theta\theta}) + e_{33} S_{zz} + \varepsilon_{33}^S E_z.$

Formula of electric impedance Z_p, for piezoelectric body with n surfaces is a function of numerous parameters.

Sum of all forms of energy in a conservative system is constant (First Law of Thermodynamics, or Law of Conservation of Energy):

$$\sum_{j=1}^{n-1} F_j^D v_j^D + V^D I = \sum_{j=1}^{n-1} F_j^E v_j^E + I^E V, \qquad n = 2, 3, 4, \ldots, k.$$

Then follows:

$$Z_p = \frac{V}{I} = \sqrt{\frac{z_{nn}^D + \sum_{i=1}^{n-1} z_{in}^D \bar{v}_i^D + \sum_{j=1}^{n-1} z_{nj}^D \bar{v}_j^D + \sum_{i=1}^{n-1}\sum_{j=1}^{n-1} z_{ij}^D \bar{v}_i^D \bar{v}_j^D}{z_{nn}^E + \sum_{i=1}^{n-1} z_{in}^E \bar{v}_i^E + \sum_{j=1}^{n-1} z_{nj}^E \bar{v}_j^E + \sum_{i=1}^{n-1}\sum_{j=1}^{n-1} z_{ij}^E \bar{v}_j^E \bar{v}_i^E}}, \qquad n = 2, 3, 4, \ldots, k.$$

z_{ij}^D, z_{nj}^D, z_{nn}^D, z_{ij}^E, z_{nj}^E and z_{nn}^E - internal mechanical and electric impedances, transfer functions of system (black box).

Comparison of derived formula by computer simulation using software package MATLAB, with analogous characteristic obtained by experimental measuring on Automatic Network Analyzer HP4194A, shows results at least 50% better than all known results published until now in scientific literature available to me.

Formula is applicable on: 1D, 2D, and 3D oscillatin models of piezoelectric ceramic bodies, whose number of surfaces $S_n[m^2]$ may extend to infinity $(n \to \infty)$. Also, it can join different asumed oscillation models (any combination of: 1D, 2D, and 3D model) of the same piezoceramic specimen loaded under equal conditions in the same period of time.

1 – Case: $n = 2$ - **1D model** (transversal oscillation of the: circular plate, circular-ring plate, rectangular plate etc; or longitudinal oscillation of a: free beam, cantilever, cylinder, circular-ring cylinder, circular-ring sectional cylinder, etc.):

$$F_1^D v_1^D + V^D I = F_1^E v_1^E + I^E V,$$

$$Z_p = \frac{V}{I} = \sqrt{\frac{(z_{11}^D \overline{v}_1^D + z_{12}^D)\overline{v}_1^D + z_{21}^D \overline{v}_1^D + z_{22}^D}{(z_{11}^E \overline{v}_1^E + z_{12}^E)\overline{v}_1^E + z_{21}^E \overline{v}_1^E + z_{22}^E}}.$$

2 – Case: $n = 3$ - **2D model** (radial oscillation of the: circular plate, circular-ring plate, rectangular plate, free beam, cantilever, cylinder, circular-ring cylinder, circular-ring sectional cylinder, etc.):

$$F_1^D v_1^D + F_2^D v_2^D + V^D I = F_1^E v_1^E + F_2^E v_2^E + I^E V,$$

$$Z_p = \frac{V}{I} = \sqrt{\frac{(z_{11}^D \overline{v}_1^D + z_{12}^D \overline{v}_2^D + z_{13}^D)\overline{v}_1^D + (z_{21}^D \overline{v}_1^D + z_{22}^D \overline{v}_2^D + z_{23}^D)\overline{v}_2^D + z_{31}^D \overline{v}_1^D + z_{32}^D \overline{v}_2^D + z_{33}^D}{(z_{11}^E \overline{v}_1^E + z_{12}^E \overline{v}_2^E + z_{13}^E)\overline{v}_1^E + (z_{21}^E \overline{v}_1^E + z_{22}^E \overline{v}_2^E + z_{23}^E)\overline{v}_2^E + z_{31}^E \overline{v}_1^E + z_{32}^E \overline{v}_2^E + z_{33}^E}}.$$

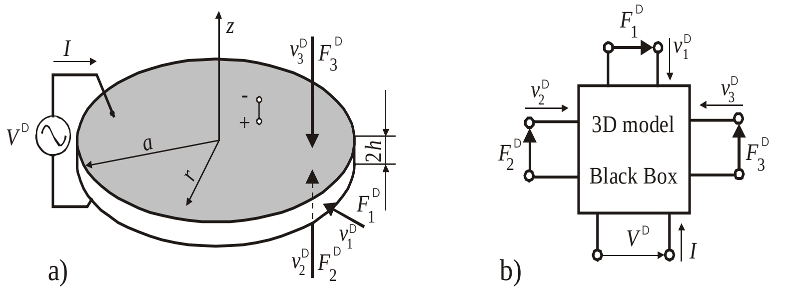
3 – Case: $n = 4$ - **3D model** (3D oscillation of the: circular plate, circular-ring plate, rectangular plate, free beam, cantilever, cylinder, circular-ring cylinder, circular-ring sectional cylinder, etc.):

$$F_1^D v_1^D + F_2^D v_2^D + F_3^D v_3^D + V^D I = F_1^E v_1^E + F_2^E v_2^E + F_3^E v_3^E + I^E V,$$

$$Z_p = \frac{V}{I} = \sqrt{\frac{(z_{11}^D \overline{v}_1^D + z_{12}^D \overline{v}_2^D + z_{13}^D \overline{v}_3^D + z_{14}^D)\overline{v}_1^D + (z_{21}^D \overline{v}_1^D + z_{22}^D \overline{v}_2^D + z_{23}^D \overline{v}_3^D + z_{24}^D)\overline{v}_2^D + (z_{31}^D \overline{v}_1^D + z_{32}^D \overline{v}_2^D + z_{33}^D \overline{v}_3^D + z_{34}^D)\overline{v}_3^D + z_{41}^D \overline{v}_1^D + z_{42}^D \overline{v}_2^D + z_{43}^D \overline{v}_3^D + z_{44}^D}{(z_{11}^E \overline{v}_1^E + z_{12}^E \overline{v}_2^E + z_{13}^E \overline{v}_3^E + z_{14}^E)\overline{v}_1^E + (z_{21}^E \overline{v}_1^E + z_{22}^E \overline{v}_2^E + z_{23}^E \overline{v}_3^E + z_{24}^E)\overline{v}_2^E + (z_{31}^E \overline{v}_1^E + z_{32}^E \overline{v}_2^E + z_{33}^E \overline{v}_3^E + z_{34}^E)\overline{v}_3^E + z_{41}^E \overline{v}_1^E + z_{42}^E \overline{v}_2^E + z_{43}^E \overline{v}_3^E + z_{44}^E}}.$$

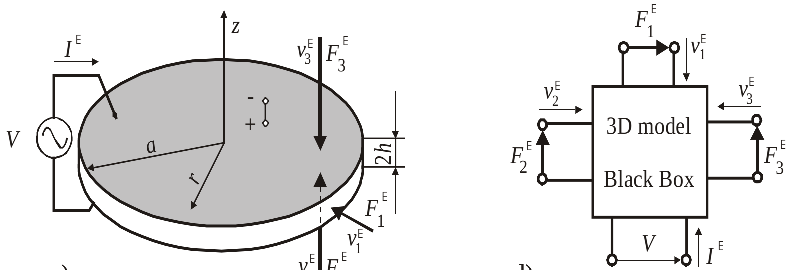
For PZT8 circular plate with dimensions: $2a_2 = 50\ [mm]$, $2h = 3[mm]$, $\rho = 7600[\frac{kg}{m^3}]$, whose coefficients are:

$c_{11}^E = 13{,}7 \cdot 10^{10}[N/m^2]$, $c_{12}^E = 6{,}97 \cdot 10^{10}$, $c_{13}^E = 7{,}16 \cdot 10^{10}$, $c_{33}^E = 12{,}4 \cdot 10^{10}$, $h_{31} = -7{,}8 \cdot 10^8[V/m]$, $h_{33} = 26{,}9 \cdot 10^8[V/m]$,
$c_{11}^D = 14 \cdot 10^{10}[N/m^2]$, $c_{12}^D = 7{,}28 \cdot 10^{10}$, $c_{13}^D = 6{,}08 \cdot 10^{10}$, $c_{33}^D = 16{,}1 \cdot 10^{10}$, $e_{31} = -4[C/m^2]$, $e_{33} = 13{,}8$, $\varepsilon_{33}^S / \varepsilon_0 = 582$,



a) b)

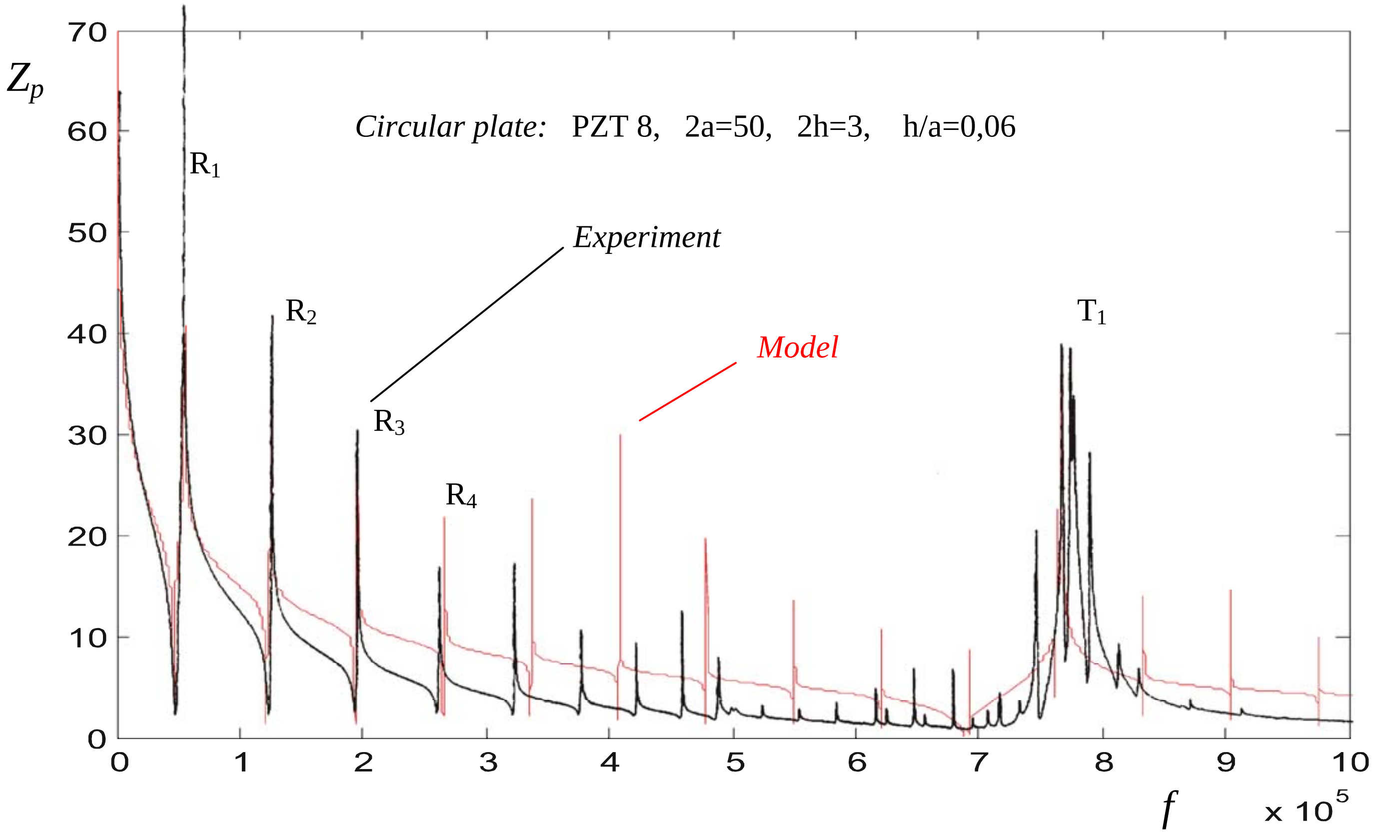
$$\begin{Bmatrix} F_1^D \\ F_2^D \\ F_3^D \\ V^D \end{Bmatrix} = \begin{bmatrix} z_{11}^D & z_{12}^D & z_{13}^D & z_{14}^D \\ z_{21}^D & z_{22}^D & z_{23}^D & z_{24}^D \\ z_{31}^D & z_{32}^D & z_{33}^D & z_{34}^D \\ z_{41}^D & z_{42}^D & z_{43}^D & z_{44}^D \end{bmatrix} \begin{Bmatrix} v_1^D \\ v_2^D \\ v_3^D \\ I \end{Bmatrix}.$$



c) d)

$$\begin{Bmatrix} F_1^E \\ F_2^E \\ F_3^E \\ I^E \end{Bmatrix} = \begin{bmatrix} z_{11}^E & z_{12}^E & z_{13}^E & z_{14}^E \\ z_{21}^E & z_{22}^E & z_{23}^E & z_{24}^E \\ z_{31}^E & z_{32}^E & z_{33}^E & z_{34}^E \\ z_{41}^E & z_{42}^E & z_{43}^E & z_{44}^E \end{bmatrix} \begin{Bmatrix} v_1^E \\ v_2^E \\ v_3^E \\ V \end{Bmatrix}.$$

Formula provides following simulated characteristic of electric impedance model, which is compared with analogous, obtained by measuring on Automatic Network Analyzer HP4194A:



Knowing values of resonant frequencies is an initial condition during design of piezoceramic elements. From the picture above one may see that by this formula one may determine several exact positions of radial resonant modes R_1, R_2, R_3 and R_4, and transversal mode T_1, which are the most frequently used in technical practice and application.

4 – Case: $n = 5$ - **3D model** (rectangular plate, free beam, etc.):

$$F_1^D v_1^D + F_2^D v_2^D + F_3^D v_3^D + F_4^D v_4^D + V^D I = F_1^E v_1^E + F_2^E v_2^E + F_3^E v_3^E + F_4^E v_4^E + I^E V,$$

$$Z_p = \frac{V}{I} = \sqrt{\frac{(z_{11}^D \bar{v}_1^D + z_{12}^D \bar{v}_2^D + z_{13}^D \bar{v}_3^D + z_{14}^D \bar{v}_4^D + z_{15}^D)\bar{v}_1^D + \ldots + (z_{41}^D \bar{v}_1^D + z_{42}^D \bar{v}_2^D + z_{43}^D \bar{v}_3^D + z_{44}^D \bar{v}_4^D + z_{45}^D)\bar{v}_4^D + z_{51}^D \bar{v}_1^D + z_{52}^D \bar{v}_2^D + z_{53}^D \bar{v}_3^D + z_{54}^D \bar{v}_4^D + z_{55}^D}{(z_{11}^E \bar{v}_1^E + z_{12}^E \bar{v}_2^E + z_{13}^E \bar{v}_3^E + z_{14}^E \bar{v}_4^E + z_{15}^E)\bar{v}_1^E + \ldots + (z_{41}^E \bar{v}_1^E + z_{42}^E \bar{v}_2^E + z_{43}^E \bar{v}_3^E + z_{44}^E \bar{v}_4^E + z_{45}^E)\bar{v}_4^E + z_{51}^E \bar{v}_1^E + z_{52}^E \bar{v}_2^E + z_{53}^E \bar{v}_3^E + z_{54}^E \bar{v}_4^E + z_{55}^E}}.$$

5 – Case: $n = 6$ - **3D model** (rectangular plate with a circular or rectangular hole, etc.):

$$F_1^D v_1^D + F_2^D v_2^D + F_3^D v_3^D + F_4^D v_4^D + F_5^D v_5^D + V^D I = F_1^E v_1^E + F_2^E v_2^E + F_3^E v_3^E + F_4^E v_4^E + F_5^E v_5^E + I^E V,$$

$$Z_p = \frac{V}{I} = \sqrt{\frac{(z_{11}^D \bar{v}_1^D + z_{12}^D \bar{v}_2^D + z_{13}^D \bar{v}_3^D + z_{14}^D \bar{v}_4^D + z_{15}^D \bar{v}_5^D + z_{16}^D)\bar{v}_1^D + \ldots + (z_{51}^D \bar{v}_1^D + z_{52}^D \bar{v}_2^D + z_{53}^D \bar{v}_3^D + z_{54}^D \bar{v}_4^D + z_{55}^D \bar{v}_5^D + z_{56}^D)\bar{v}_6^D + z_{61}^D \bar{v}_1^D + z_{62}^D \bar{v}_2^D + z_{63}^D \bar{v}_3^D + z_{64}^D \bar{v}_4^D + z_{65}^D \bar{v}_5^D + z_{66}^D}{(z_{11}^E \bar{v}_1^D + z_{12}^E \bar{v}_2^E + z_{13}^E \bar{v}_3^E + z_{14}^E \bar{v}_4^E + z_{15}^E \bar{v}_5^E + z_{16}^E)\bar{v}_1^E + \ldots + (z_{51}^E \bar{v}_1^E + z_{52}^E \bar{v}_2^E + z_{53}^E \bar{v}_3^E + z_{54}^E \bar{v}_4^E + z_{55}^E \bar{v}_5^E + z_{56}^E)\bar{v}_6^E + z_{61}^E \bar{v}_1^E + z_{62}^E \bar{v}_2^E + z_{63}^E \bar{v}_3^E + z_{64}^E \bar{v}_4^E + z_{65}^E \bar{v}_5^E + z_{66}^E}}.$$

…, etc.

If you are interested in this matter, I am willing to cooperate.

Sincerely Yours,

Ljubisa Peric, D.Sc. Mech. Eng.