

# The considerations and guides of the wattmeter method for measuring output acoustical power of Langevin-type transducers – I: theory

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## Abstract

The vibrational amplitude ratios of a number of positions in some typical vibration systems under different loads, especially of amplitudes at positions close to nodes and anti-nodes (related to no-load conditions) are given. Results according to the wattmeter method for measuring output acoustical power and electro-acoustic efficiency of high-power ultrasonic vibration systems (Langevin-type transducers and horns) have been simulated, and theoretical errors of this method calculated. This points out that there are some limits for the wattmeter method in high load conditions, but the method has practical values for measuring output acoustical power, especially for sonochemistry. Finally, based on the above analysis, some guidelines and suggestions for convenient on-line measurement are proposed. The vibration velocity of the radiation surface is discussed as an important quantity in analyzing acoustic input in high-intensity ultrasound applications. © 1997 Elsevier Science B.V.

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## 1. Introduction

With high-intensity ultrasound applications becoming more prevalent in sonochemistry, ultrasonic cleaning, degassing, levitation, and ultrasonic therapy, it is increasingly more important to measure acoustic parameters, e.g. sound pressure, sound field distribution and acoustical power. However, it is difficult to measure these parameters in high-intensity sound fields, including nonlinear acoustic effects, especially in low-frequency sound fields (below 100 kHz). Many experiments concerning power ultrasonic applications have used quantities of electric generators as equivalent acoustic parameters [1], but the efficiencies of ultrasonic vibration systems and electric circuits may cause output acoustical power to show irregular relations to electric input. The calorimeter method [2] is often used in measuring acoustic power. However, not only does acoustic power change to thermal energy, but it is also associated with sonoluminescence, particle levitation, chemical reaction, cavitation erosions, etc. [3] and some

heat is transferred from the vibration system itself. On the other hand, the wattmeter method [4–6] for measuring output acoustical power and electro-acoustic efficiency of power ultrasonic transducers, due to only considering the input electric power of ultrasonic vibration system without considering the above disturbance, has its special merits. At the Annual meeting in 1994 of the UIA (Ultrasonic Industry Association) [7], this method was considered as the standard of measuring acoustical power. In fact, it has been the industrial standard of Japan. According to the method:

$$P_a = P - P_e - P_0 + P_{e0}, \quad (1)$$

where  $P_a$  is output acoustical power,  $P$  and  $P_e$  are input electric power and dielectric losses under load conditions, respectively, and  $P_0$  and  $P_{e0}$  are input electric power and dielectric losses under no-load conditions, respectively. The values of  $P$  and  $P_0$  are measured at the same vibration velocity of the radiation surface of the transducer. The method is based on the assumption that the losses of the transducer system are composed of mechanical and dielectric losses, that mechanical losses are equal between load and no-load conditions provided the vibration amplitudes of the radiation sur-

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face of the transducer are kept equal, and the dielectric losses are determined by voltage on piezoelectric ceramics. The above quantities are based on the same frequency. However, when the amplitude of the radiation surface is maintained the same in two load conditions, are the other positions of the transducer system the same too? Are the displacement distributions of the transducer system the same? Otherwise, the mechanical losses between load and no-load conditions are obviously different even if the amplitudes of the radiation surface are unchanged. Then how do loads affect them? How about theoretical errors of measurement? And how about clamped transducer systems (two transducers having almost the same characteristics are clamped for measuring efficiency of transducer and dielectric losses)? The answers to these questions will promote the standardization and application of the wattmeter method. Until now, few works have been found which consider these questions carefully. This paper will attempt to answer these questions from theoretical results. First, the distribution of the transformer system (horn), the transducer-horn system and the clamped transducer system are analyzed, and then the theoretical errors of the wattmeter method are examined. It may shed some light on the measurement methods for acoustical power in high-power ultrasonic applications or in intensity sound fields in liquid media such as sonochemistry. At the same time, the results will produce some guidelines to the method.

**2. The vibration velocity amplitude of transformer systems(horn)**

From ultrasonic transformer design theory [8], the vibrational equations of horns could be simplified to an equivalent matrix so a transformer system could be expressed as some connected matrixes as in Fig. 1, where  $F$  and  $V$  are force and amplitude of vibration velocity, respectively, and  $Z_L$  is the load. Thus, the velocity amplitude distribution can be obtained. The vibration velocity amplitude ratios between front and back end face of three stainless steel horns were calculated (Figs. 2 and 3). The area ratios between the front and back end faces of the transformer systems are 1:1 (straight horn), 4:25 (tapered horn) and 25:9 (trumpet-type horn), respectively, and all the diameters of the back end faces of the transformer systems are 50 mm. The sound velocity and density of stainless steel are 5.2 km/s and

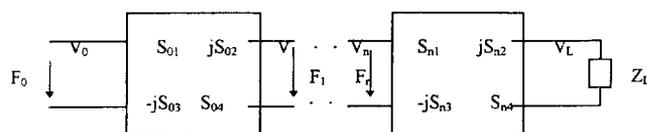
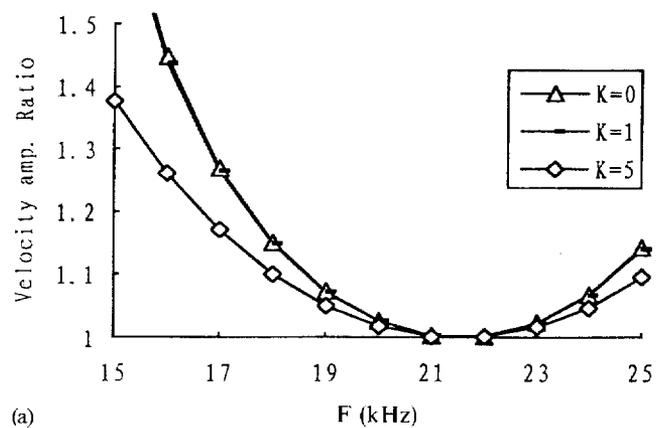
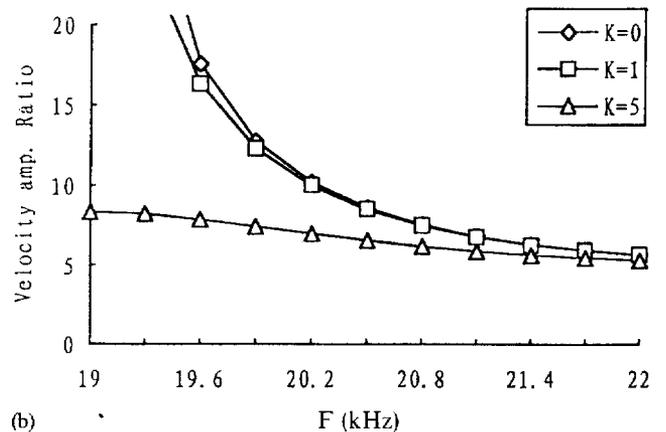


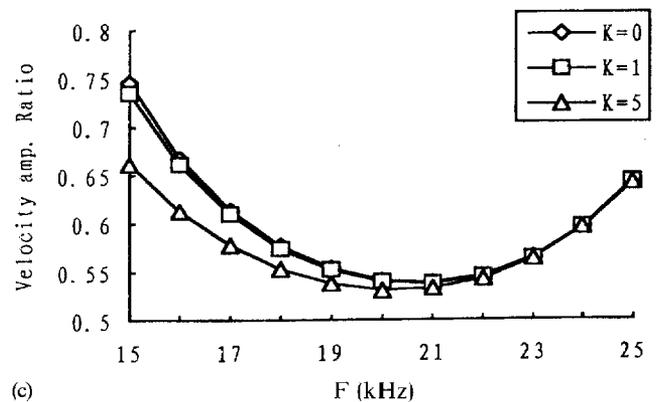
Fig. 1. The equivalent matrix of transformer systems.



(a)



(b)



(c)

Fig. 2. The velocity amplitude ratios of front and back end face of the transformer vs. frequencies and loads; (a) area ratio 1:1, (b) area ratio 4:25, (c) area ratio 25:9.

7800 kg/m<sup>3</sup> respectively. The load  $Z_L = K\rho VS$ , where  $K$  is the load coefficient,  $\rho$  is the density of water,  $V$  is the sound velocity in water and  $S$  is the area of radiation surface. In this paper, let  $\rho = 10^3$  kg/m<sup>3</sup> and  $V = 1500$  m/s. It is obvious that the vibration amplitude and ratios are affected by loads. The effects of loads on the velocity amplitude at positions which are close to the displacement node and on tapered transformers are much bigger. The heavier the load, the bigger the effects.

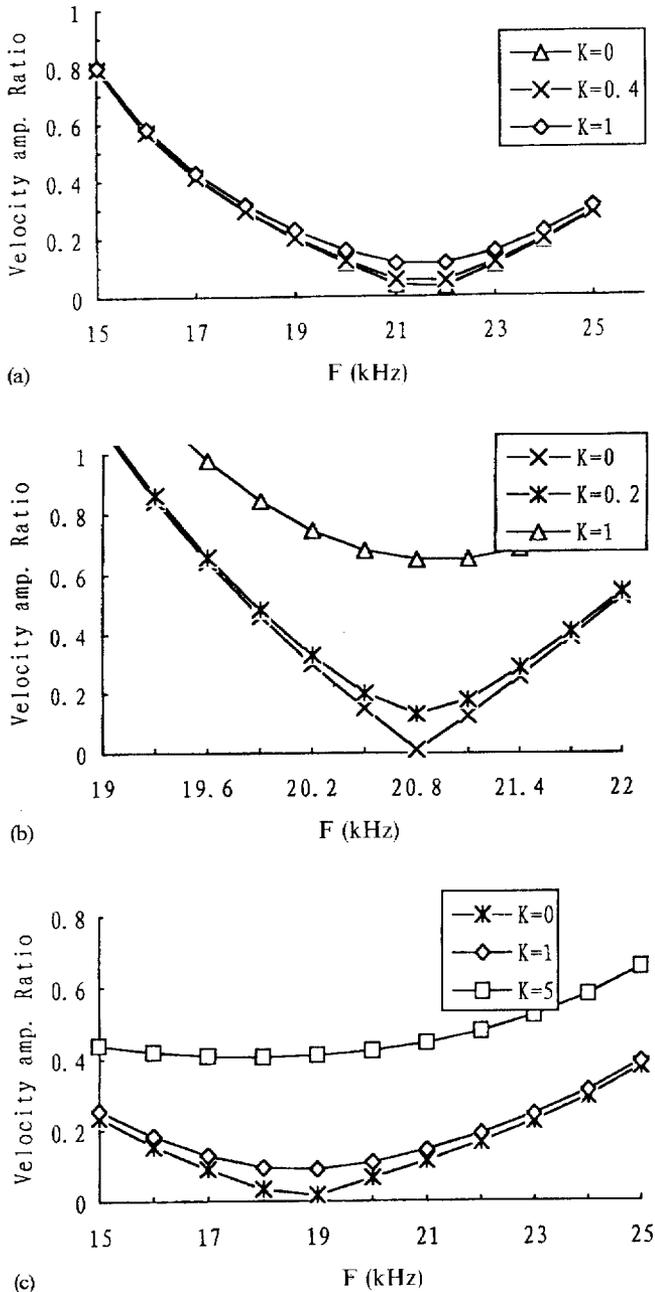


Fig. 3. Velocity amplitude ratios of a position (near node) and back end face of the transformer vs. frequencies and loads; (a) area ratio 1:1, (b) area ratio 4:25, (c) area ratio 25:9.

On the other hand, the amplitude ratios between front and back end face are almost equal at a certain scope of loads.

### 3. The vibration velocity amplitude of transducer-horn systems

The Langevin-type transducer systems can be simplified to an equivalent circuit [9,10] as in Fig. 4. The losses of the transducer itself are considered as a refer-

ence [11].  $C_0$  is the static capacity and  $R_0$  is the dielectric losses resistance. From these, the vibration velocity amplitude ratios, voltage transforming coefficients ( $V/U$ ) and electro-acoustic efficiency ( $V^2 R_w / P$ ) can be calculated in different load conditions.  $U$  is the input voltage of the transducer,  $R_w$  is equivalent resistance of the load and  $V$  is vibration velocity amplitude of radiation surface of the system. The transducer analyzed in this paper is shown in Fig. 5. The transducer-horn system defined in this paper consists of a transducer and a transformer, which includes two types of transformers, one being a straight horn with area an ratio of the front and back end faces of 1:1, and a radiation surface diameter of 50 mm; the other is a tapered horn with an area ratio of 4:25 and a radiation surface diameter of 20 mm. The results are shown in Figs. 6–9. Figs. 6 and 7 show that there is a great influence of loads on the voltage transforming coefficients and electro-acoustic efficiencies of the system, and with loads increasing, this influence tends to smooth out. From Fig. 8, the vibration velocity amplitude ratios between the front and back end surfaces of the transducer in the systems are changed with loads, and with loads increasing, the ratios decrease. The ratios of the tapered system (4:25) are much more affected by loads. From Fig. 9, the velocity amplitude of the position close to the displacement node obviously varies with loads; this suggests that the mechanical loss at that position also varies with loads.

### 4. Analysis of clamped transducer systems

In the wattmeter method [4–6], the dielectric losses are measured through a clamped transducer system as in Fig. 10. When one transducer is driven by an electric generator and another is connected to electrical impedance  $Z$ , it can be used to measure the electro-acoustic efficiency of the system, then the electro-acoustic efficiency of the transducer. The importance is that the values of  $Z$  are known explicitly, so it can be used to verify the analysis of this paper. When two transducers are driven with the same electric generator at the same time, the vibration velocities of radiation surfaces are very small. When the ceramics of group B are connected to electrical impedance  $Z$ , it can be simplified to Fig. 11. Similar to Section 3, results are obtained as in Figs. 12 and 13. The numbers in the frames of the figure are values ( $\Omega$ ) of  $Z$ , which is electric resistance. The voltage transforming coefficients are changed sharply with loads and frequencies near the resonance frequency and the amplitude ratios of the front and back end faces of the driven transducer in the clamped system decrease with the loads. The electro-acoustic efficiency of the clamped

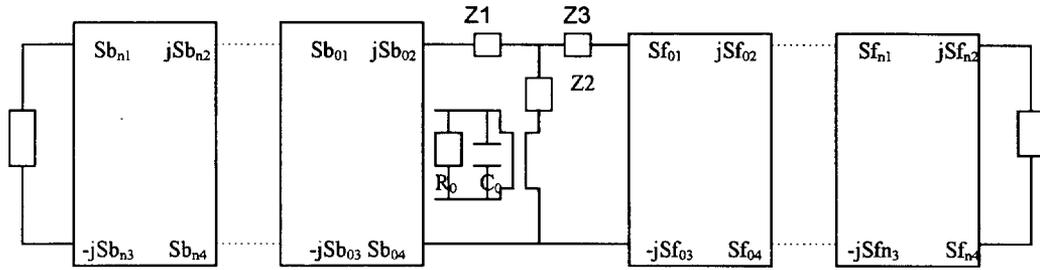


Fig. 4. The equivalent circuit of a transducer-horn system.

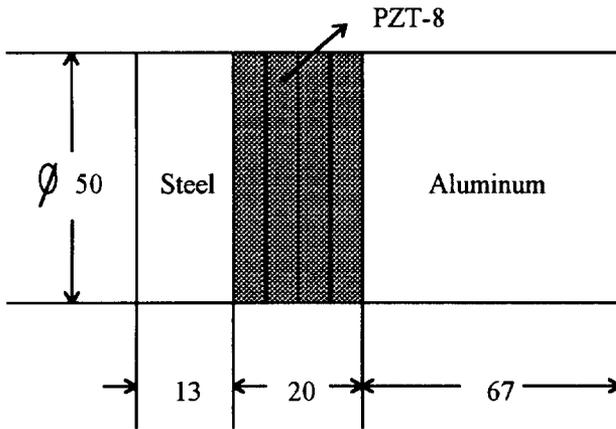


Fig. 5. The dimensions of the transducer.

system is:

$$\eta = \left[ \frac{(V_i - V_{i+1})NZ_w}{Z} \right]^2 R / \left( U \frac{U}{Z_i} \cos \theta \right) = \left[ \frac{(A_i - A_{i+1})NZ_w}{Z} \right]^2 \frac{R}{Z_i \cos \theta}, \quad (2)$$

where  $\eta$  is electro-acoustic efficiency,  $1/Z_w = 1/Z + 1/R_0 + j\omega C_0$ ,  $R$  is the real part of  $Z$ ,  $Z_i$  is input electric impedance of the system,  $\theta$  is the phase angle of  $Z_i$ , and  $A_i$  and  $A_{i+1}$  are the voltage transforming coefficients of back and front surface of ceramics group B, respectively. The electro-acoustic efficiency versus load and frequency is shown in Fig. 14 which shows that the electro-acoustic efficiency increases sharply with loads at smaller electric resistance load conditions, and varies obviously with frequency. When the load exceeds a certain value, the efficiency decreases slowly. With multi-group ceramics theory [11], when two transducers of the clamped transducer system are driven by the same electric generator at the same time, the voltage transforming coefficients of the transducer can be calculated and are shown in Fig. 15. It is obvious that vibration velocity of the transducer in a clamped system is very small compared with that of a free single transducer. The transducer is really ‘clamped’. When the value of connected electrical impedance  $Z$  is changed, the resonance frequency of the clamped system is changed, too.

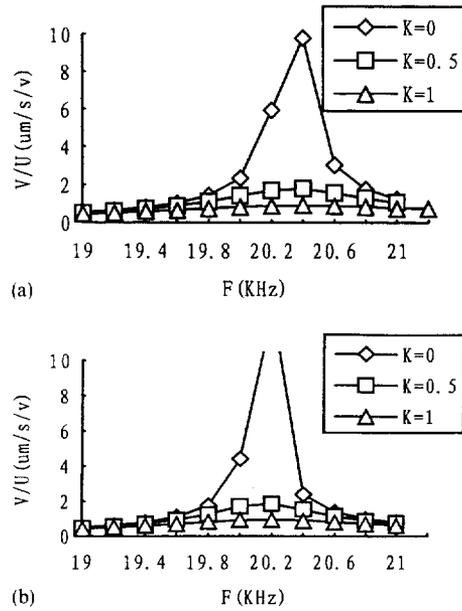


Fig. 6. Voltage transforming coefficients of a transducer-horn system vs. frequencies and loads; (a) area ratio of horn 1:1, (b) area ratio of horn 4:25.

Fig. 16 presents the resonance frequency of the clamped system versus inductance connected to ceramics of group B. At values of inductance close to  $1/\omega^2 C_0$  ( $\omega$  is resonance angle frequency at  $Z=0$  and  $C_0$  is the static capacitance of group B), the resonance frequency changes obviously, but the voltage transforming coefficients become relatively lower (Fig. 17). This indicates the difference between load and no-load conditions, which should be considered in the measuring procedure. Figs. 16 and 17 also indicate the possibility for designing a frequency-turning transducer, but the changing range of frequency is limited by  $C_0$ .

### 5. Error analysis of the wattmeter method for measuring output acoustical power

The above analysis has shown that the vibration velocity distribution of a Langevin-type transducer system is different under different loads and frequencies.

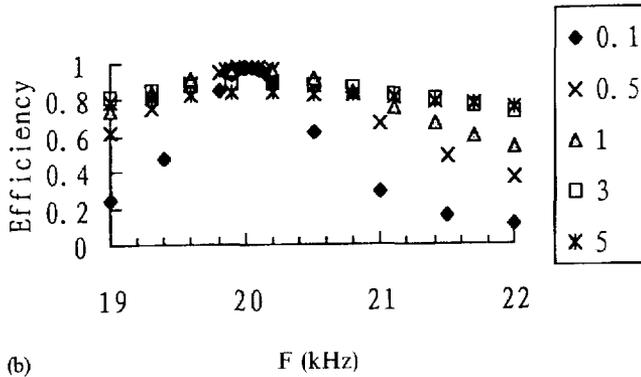
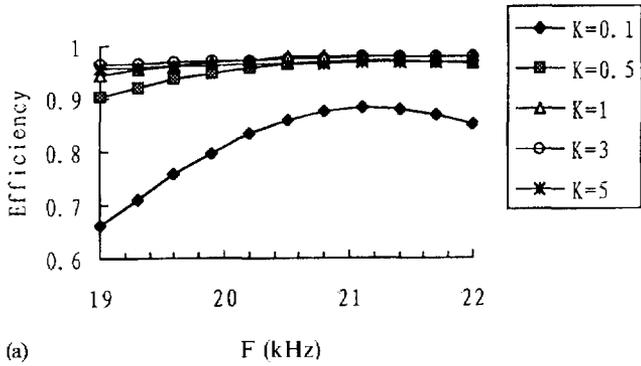


Fig. 7. Efficiencies of transducer-horn system vs. frequencies and loads; (a) area ratio of horn 1:1, (b) area ratio of horn 4:25.

This implies that mechanical losses are different between load and no-load conditions even keeping the vibration amplitude of the radiation surface of the transducer unchanged. So just from a theoretical point, there are errors in the wattmeter method. How much are these errors caused by loads? According to Eq. (1), the error can be expressed as:

$$E = \frac{P - P_0 - P_e - P_{e0} - P\eta}{P\eta} \quad (3)$$

According to the wattmeter method,  $V_0 = V$ . For a transducer-horn system, Eq. (3) can be expanded as follows:

$$E = \frac{\frac{A_0}{A} - 1 - \frac{A_0}{A} \frac{Z_i}{\cos \theta R_0} + \frac{Z_{i0}}{\cos \theta_0 R_0} - \frac{A_0}{A} \eta}{(A_0/A)\eta} \quad (4)$$

where  $A_0 = V_0^2/P_0$ ,  $A = V^2/P$ ,  $V_0$  and  $V$  are the vibration velocity amplitudes of the radiation surface of the transducer without and with load, respectively,  $Z_i$  is the input electric impedance of the system with load, and  $Z_{i0}$  is that without load,  $\theta$  and  $\theta_0$  are the phase angles of  $Z_i$  and  $Z_{i0}$  respectively, and  $\eta$  is the electro-acoustic efficiency of the system.

For a clamped transducer system, Eq. (3) can be

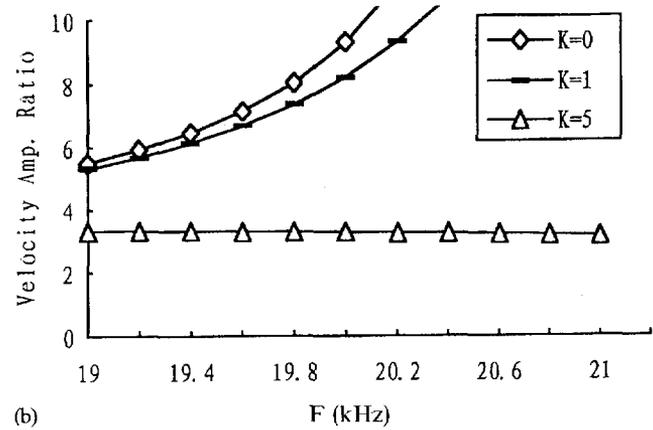
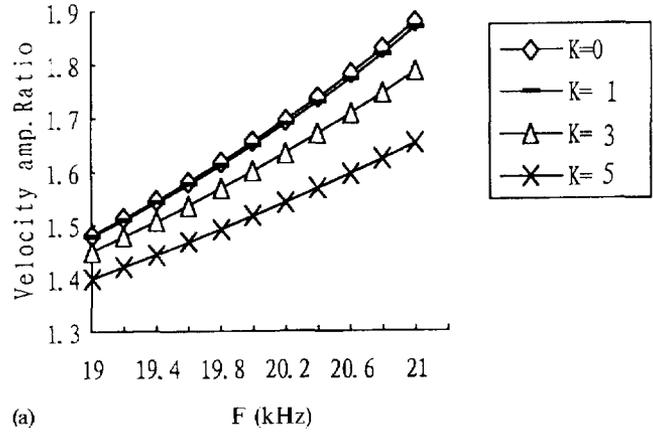


Fig. 8. Velocity amplitude ratios of front and back end face of the transducer in a transducer-horn system vs. frequencies and loads; (a) area ratio of horn 1:1, (b) area ratio of horn 4:25.

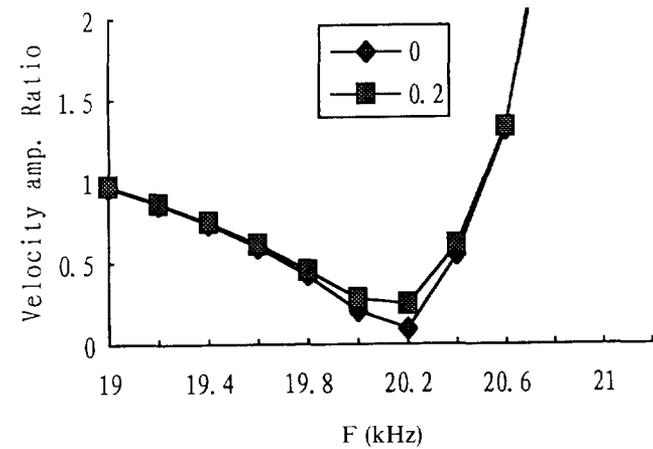


Fig. 9. Velocity amplitude ratios of a position (near node) and back end face of the transducer in a transducer-horn system (4:25) vs. frequencies and loads.

expanded as follows:

$$E = \frac{\frac{A_0}{A} - 1 - \frac{A_0}{A} \frac{Z_i}{\cos \theta R_0} + \frac{Z_{i0}}{\cos \theta_0 R_0} - \frac{A_0}{A} \eta - \frac{A}{A_0} \eta \frac{Z^2}{RR_0}}{(A_0/A)\eta} \quad (5)$$

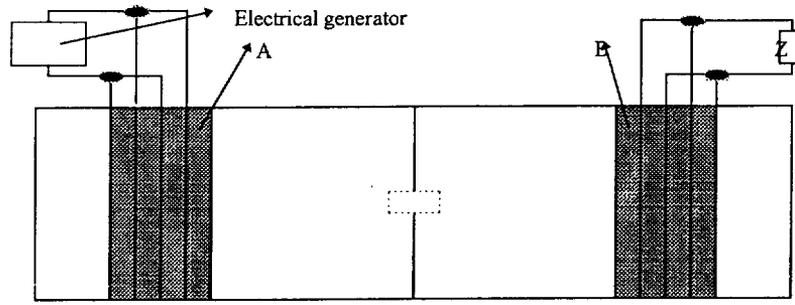


Fig. 10. Clamped transducer system.

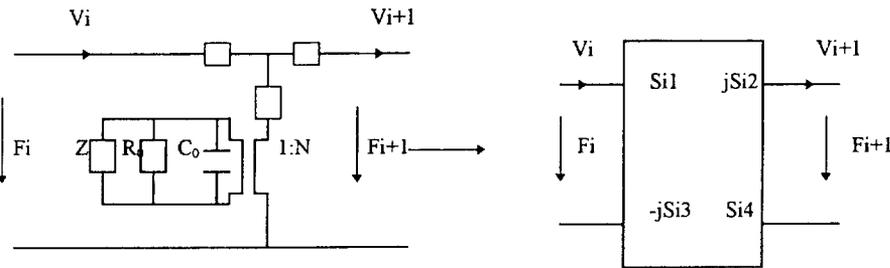


Fig. 11. Simplified equivalent circuit and matrix of ceramics groups connected with Z.

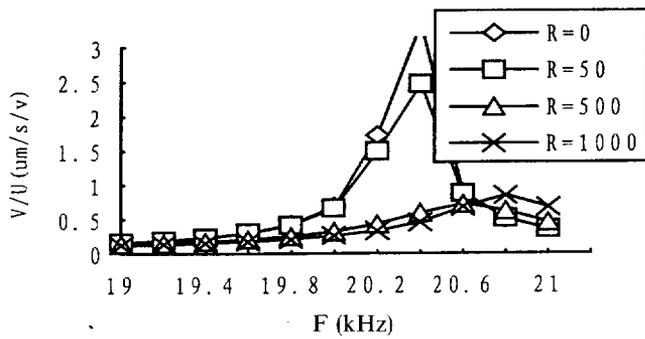


Fig. 12. Voltage transforming coefficients of the driven transducer in a clamped transducer system vs. loads and frequencies.

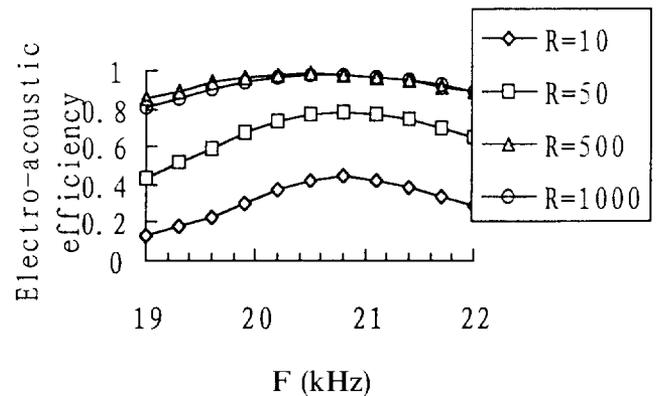


Fig. 14. Efficiencies of a clamped transducer system vs. frequencies and loads.

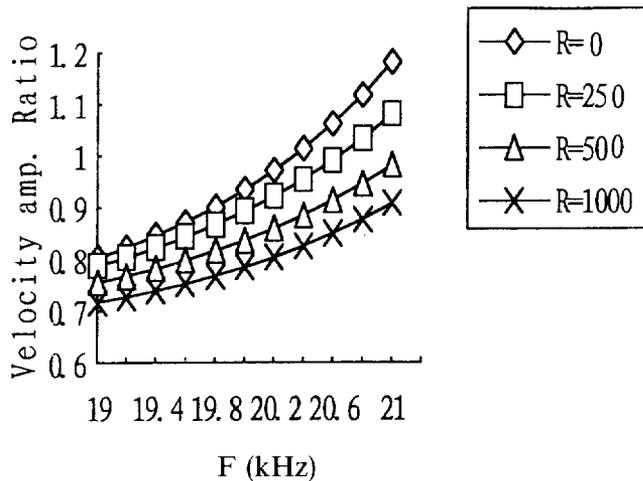


Fig. 13. Velocity amplitude ratios of front and back end face of the driven transducer in a clamped transducer system vs. loads and frequencies.

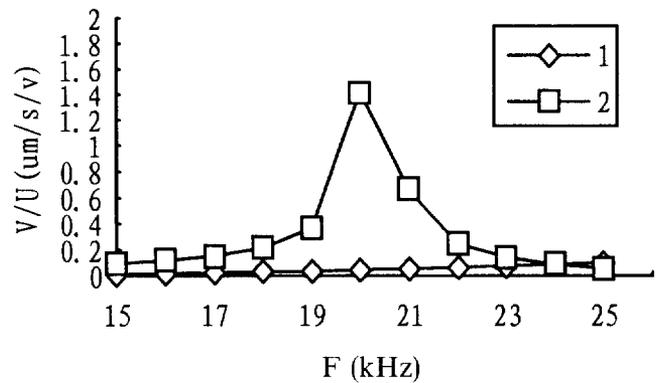


Fig. 15. Voltage transforming coefficients of the transducer vs. frequencies; (1) in a clamped transducer system driven with the same electrical generator at the same time, (2) single transducer.

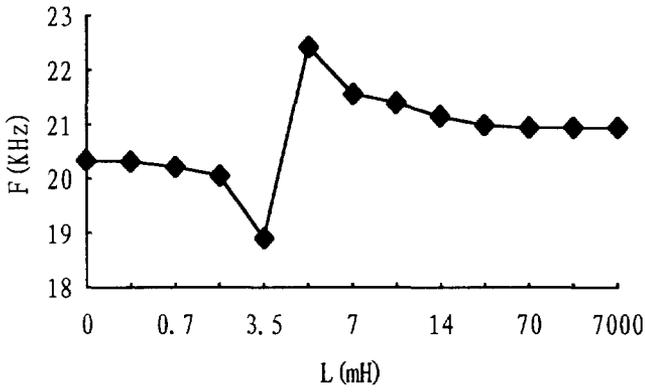


Fig. 16. Variation of the resonance frequency of a clamped transducer system with inductance  $L$ . One transducer was driven, the other was connected with inductance  $L$ .

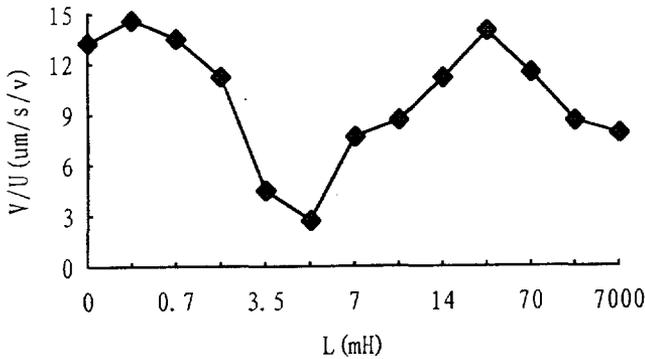


Fig. 17. Variation of voltage transforming coefficients of the driven transducer in a clamped transducer system at resonance frequency with inductance  $L$ . One transducer was driven, the other was connected with inductance  $L$ .

The final item considers the dielectric losses of the ceramics group B. If dielectric losses of the systems are neglected, the errors should be expressed as:

$$E = \frac{P - P_0 - P\eta}{P\eta} \quad (6)$$

The results of the above equations are shown in Figs. 18 and 19. Fig. 18 concerns the errors without considering the dielectric losses. The errors increase with loads, and are bigger for a tapered transducer–horn system [parts (c) of Figs. 18 and 19]. Parts (a) of Figs. 18 and 19 imply the losses of a clamped system become small with loads increasing below a certain value of  $R$ . Parts (b) and (c) of Figs. 18 and 19 imply the losses of a transducer–horn system become large with load increasing. Fig. 19(a) shows that when considering dielectric losses, the errors become big for a clamped system, while Fig. 19(b and c) show that errors become small obviously for transducer–horn systems.

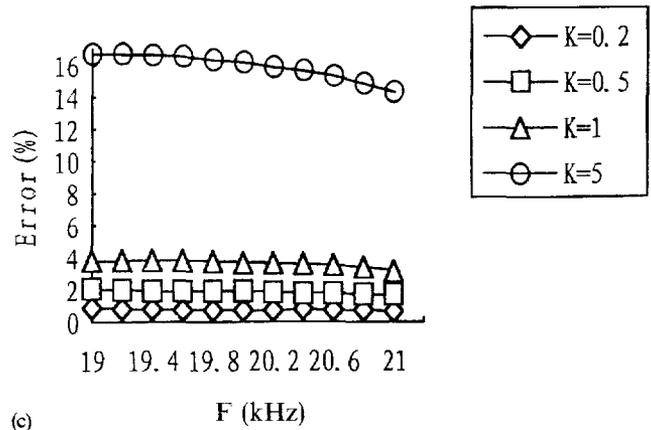
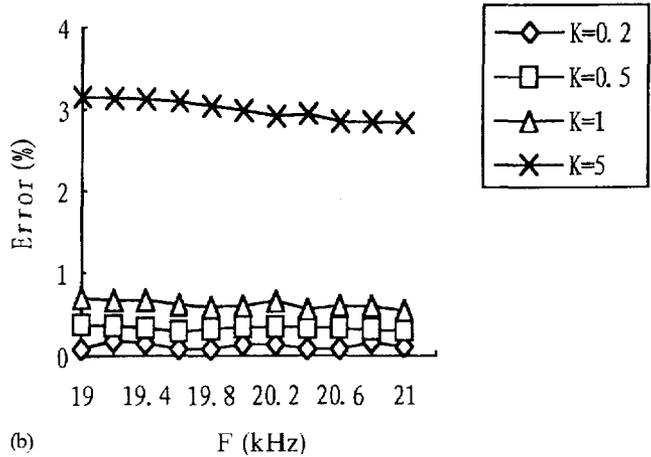
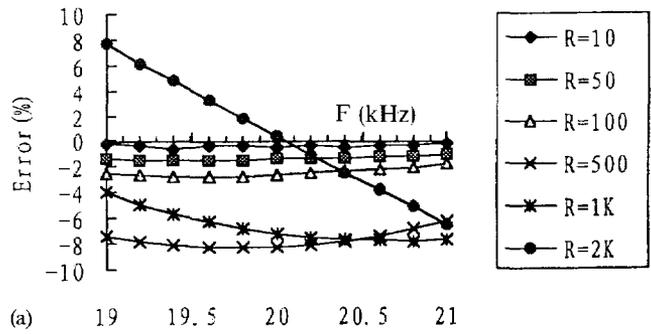


Fig. 18. Theoretical errors of the wattmeter method for measuring output acoustical power under different loads and frequencies without considering the dielectric losses; (a) clamped transducer system, (b) transducer–horn system (1:1), (c) transducer–horn system (4:25).

### 6. Conclusions and discussion

(1) For Langevin-type ultrasonic transducer systems, the distributions of displacement, stress and vibrational velocity amplitude vary with loads. When loads are heavy ( $> \rho VS$ ), the difference of amplitude ratios between load and no-load conditions become great, and the errors cannot be neglected. However, when transducers are used in liquid processing, such as

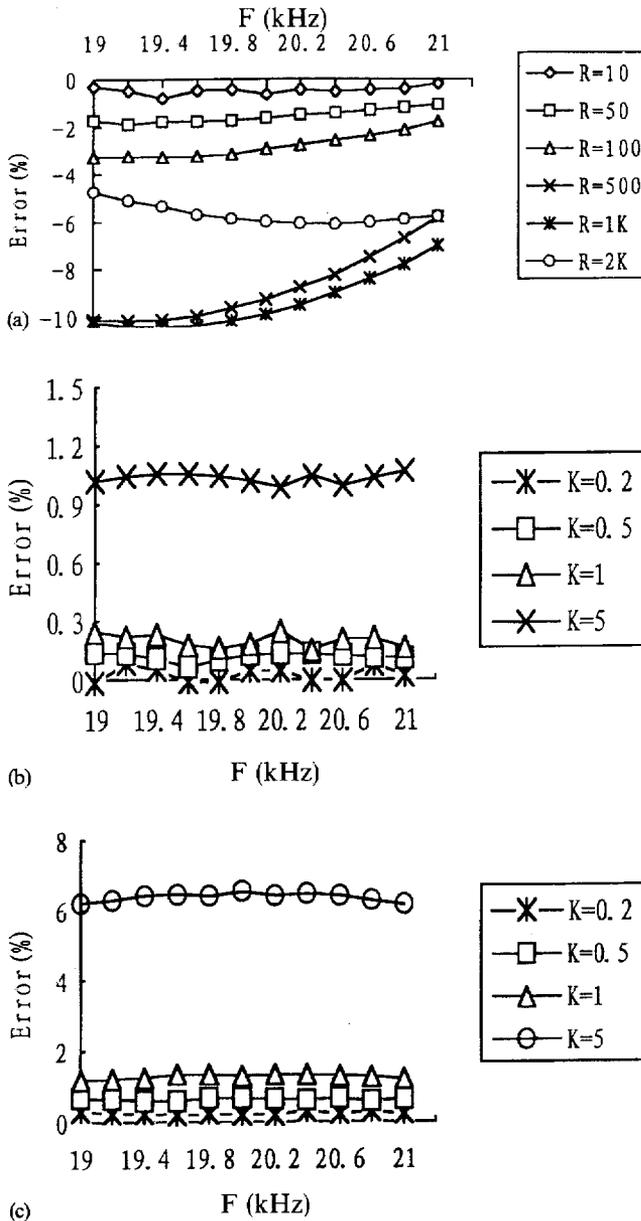


Fig. 19. Theoretical errors of the wattmeter method for measuring output acoustical power under different loads and frequencies; (a) clamped transducer system, (b) transducer-horn system (1:1), (c) transducer-horn system (4:25).

sonochemistry, the loads are usually smaller than  $\rho VS$  [12]. Therefore, in applications of power ultrasound applied in liquid media, the wattmeter method is proper and accurate.

(2) The velocity amplitude of the position close to the displacement node (no-load conditions) varies obviously even under light load ( $<\rho VS$ ) conditions. Therefore, the amplitude chosen to measure should be at a position close to the anti-node. The amplitude ratios between two anti-nodes hardly vary when loads are smaller than  $\rho VS$ . Thus, in the measuring process,

the amplitude of the back end surface of the transducer or another convenient position close to one of the anti-nodes can substitute that of the radiation surface of the transducer.

(3) At zero load conditions, the voltage transforming coefficients of the systems change clearly with frequency so when parameters are measured at zero load conditions, a frequency must be maintained close to that of the system at load conditions.

(4) For a transducer-horn system, the bigger the voltage transforming coefficient, the greater the measuring errors with the wattmeter method so errors in measuring the acoustic power of a transducer used in an ultrasonic cell disruptor are greater than those of an ultrasonic cleaner.

(5) In low-accuracy situations and light-load conditions, the dielectric losses of ceramics can be neglected.

(6) The effect of load on a clamped transducer system is different from that on a transducer-horn system. For the former, the losses of the transducer itself are bigger at no-load conditions than at load conditions, whereas the opposite holds for the latter. However, a clamped transducer system can be used to perform some meaningful experiments to verify the wattmeter method because of the explicit loads.

(7) For different radiation surfaces of transducer-horn systems, the output acoustic power is not able to reflect the effects of the intensity of ultrasound. In fact, the intensity of ultrasound in liquid media, and even the sound pressure, is connected directly with the vibration velocity of the radiation surface. Therefore, the vibration velocity amplitude may be an important quantity for intensity ultrasound applications in liquid media. Thus, it is convenient and practical for the repetition of research experiments.

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