

*Measuring Properties of  
Piezoelectric Ceramics*

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## 1. INTRODUCTION :

The familiar dielectric, elastic and piezoelectric constants for piezoelectric ceramics may readily be employing generally available laboratory equipment plus simple specimen holders, which may be constructed from standard components.

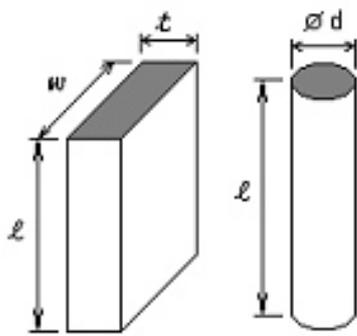
The purpose of this bulletin is to lay down specific procedures for making measurements in accordance with the preferred methods described in " IRE Standards on Piezoelectric Crystals, 1961", Proc. IRE, pp. 1162 - 1169; July, 1961.

This brochure will be limited to the following familiar constants:

- Coupling factors ....  $k_{33}, k_{31}, k_p$ .
- Free relative dielectric constant ....  $K_3^T$ .
- Dissipation factor (or  $\tan \delta$  ) ....  $D$ .
- Elastic compliances ....  $s_{33}^D, s_{11}^D, s_{11}^E, s_{33}^E$ .
- Piezoelectric d and g constants ....  $d_{33}, g_{33}, d_{31}, g_{31}$ .
- Mechanical Q ....  $Q_m$ .

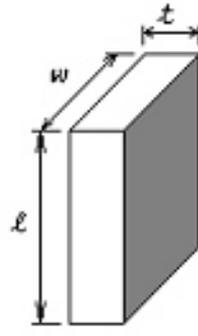
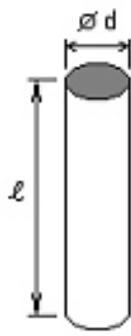
## 2. TEST SPECIMENS

To avoid large errors in the determination of some of the constants, it is necessary to employ specimens of certain shapes. Different shapes are required for the determination of different constants. In general, practical shapes used in transducers are not suitable for measurement of many of the constants. To determine all of the constants listed below, it is necessary to have specimens of three different configurations as shown below:



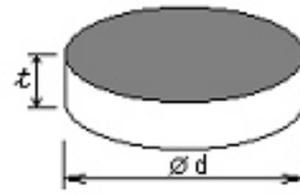
Both ends  
fully electroded

**Fig. 1 (a)**



Major faces  
fully electroded

**Fig. 1 (b)**



**Fig. 1 (c)**

<b>Dimensional Requirements</b>	$l \geq 2.5 w, t, d$	$l \geq 3.5 w, t$	$d \geq 10 t$
<b>Suggested Dimensions</b>	$l = 15 \text{ mm}$ $d = 5 \text{ mm}$ $t = 3 \text{ mm}$	$l = 40 \text{ mm}$ $w = 10 \text{ mm}$	$d = 40 \text{ mm}$ $t = 3 \text{ mm}$
<b>Suitable for the determination of</b>	$k_{33}$ $K_3^T$ $D$ $S_{33}^D$ $S_{33}^E$ $d_{33}$ $g_{33}$ $Q_m$ density	$k_{31}$ $K_3^T$ $D$ $S_{11}^D$ $S_{11}^E$ $d_{31}$ $g_{31}$ $Q_m$ density	$k_p$ $K_3^T$ $D$ $Q_m$

**Notations:**

$l$  = length.

$t$  = thickness.

$w$  = width.

$d$  = diameter.

### 3. EQUIPMENT

The measurements that must be performed on the test specimens are:

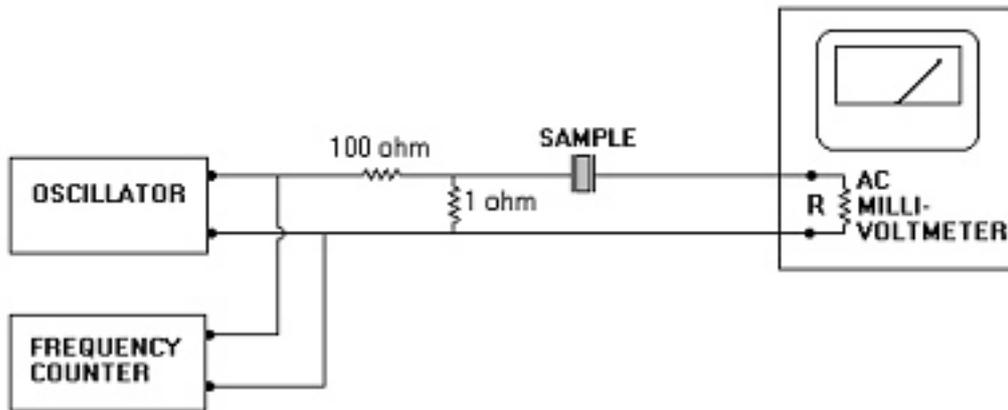
1. Weight or density.
2. Physical dimensions
3. Free capacitance and dissipation factor - (low field).
4. Frequency of minimum impedance and frequency of maximum impedance.
5. The magnitude of the minimum impedance.

The data obtained from these measurements are used to calculate the various constants.

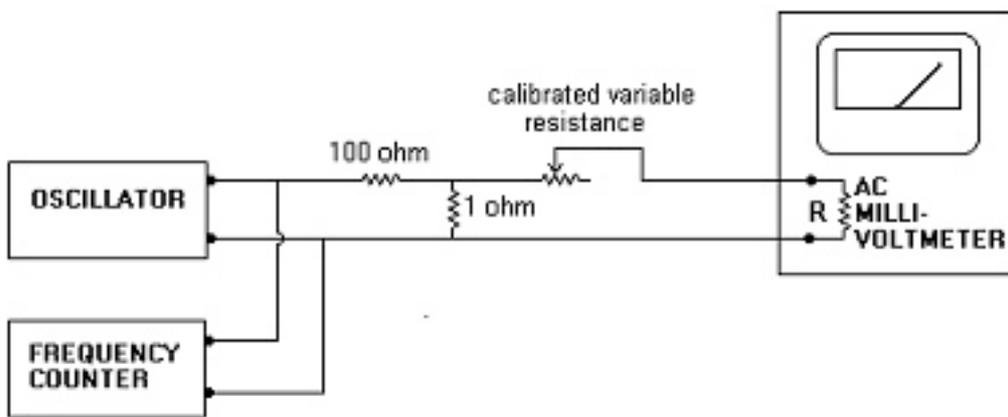
To perform these measurements, the following equipment is required:

1. **Laboratory Balance** capable of weighing to accuracy of about 10 milligram.
2. **Micrometer** with range to measure all dimensions of the test specimens.
3. **Capacitance (and dissipation) bridge** capable of covering the range of about 10 pF to 10,000 pF with accuracy of better than 1 %. For measurements of small end-electroded rods (Fig 1(a)), an external variable standard capacitor, should be added.
4. **Oscillator**. Very low distortion is a requirement. Frequency range to about 2 MHz. Output impedance not over 1000 ohms. Good short term stability is necessary.
5. **Frequency Counter** for accurately determining the frequency of the oscillator.
6. **Sensitive AC millivoltmeter**. Frequency range upto 2 MHz. Maximum sensitivity (0.001 volts full scale).
7. **Decade resistance boxes**. 0.01 ohms steps to 111.1 ohms and 1 ohm steps to 11,110 ohms.
8. **DPDT switch box and specimen holder**

The frequencies of minimum and maximum impedance are determined by using the circuit of Fig.2(a), as shown overleaf. The meter deflection peaks sharply at the frequency of minimum impedance, and a sharp null indicates the frequency of maximum impedance. The magnitudes of minimum and maximum impedance differ so greatly that it is necessary to change the value of resistance R for the two measurements.



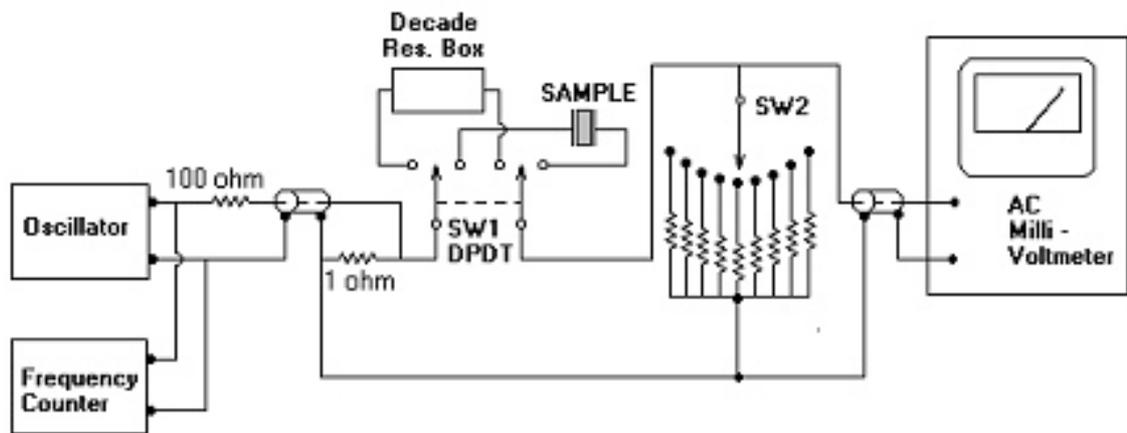
**FIG 2 (a)**



The magnitude of the minimum impedance is determined by adjusting the oscillator to the frequency of minimum impedance as in Fig 2(a); and then substituting a calibrated variable resistance such as shown in Fig 2(b) and adjusting it for the same meter indication.

**FIG 2 (b)**

The 100 : 1 resistance voltage divider formed by the 100 ohm resistor in series with the input and the 1 ohm resistor across the input cable has two functions. It serves to isolate the measuring circuit from any reactive component that may exist in the output impedance of the oscillator; and it provides a low resistance source for maximum sensitivity when adjusting the oscillator for the frequency of minimum impedance. The 1 ohm resistor should be a composition type to avoid inductance. The detailed circuit diagram is shown in Fig. 3, overleaf.



**FIG 3 : Detailed circuit for determining the frequencies of minimum and maximum impedance and magnitude of minimum impedance**

When using the circuit of Fig 2(a) to determine the frequency of maximum impedance, stray capacitance across the test specimen causes the maximum impedance to occur at a frequency which is too low. Due to the high dielectric constants of most piezoelectric ceramics, this error is negligible when the specimen is thin and of extended electrode area as in Figs. 1(b) and 1(c). However, when the specimen is an end electroded rod as shown in Fig. 1(a), a serious error may be introduced by stray capacitance. To avoid this error, the specimen is required to be shielded as shown in Fig 4 on page 8.

## 4. MEASUREMENT PROCEDURE

The steps necessary to determine the constants are as follows:

1. From Fig. 1, select specimen shape or shapes required for the constants to be determined.
2. From table I, select the equipment required to measure the desired constants.
3. Make the measurements as indicated in Table II. See the following paragraphs for detailed instructions.
4. Calculate the constants from the measured data as explained in detail later in this brochure.

Constants to be determined														EQUIPMENT REQD.
$k_{33}$	$k_{31}$	$k_p$	$K^T_3$	$D$	$s^D_{33}$	$s^E_{33}$	$s^D_{11}$	$s^E_{11}$	$d_{33}$	$d_{31}$	$g_{33}$	$g_{31}$	$Q_m$	
					X	X	X	X	X	X	X	X		Laboratory Balance
			X		X	X	X	X	X	X	X	X		Micrometer
			X	X					X	X	X	X	X	Capacitance Dissipation Bridge
X	X	X			X	X	X	X	X	X	X	X	X	Oscillator
X	X	X			X	X	X	X	X	X	X	X	X	Frequency Counter
X	X	X			X	X	X	X	X	X	X	X	X	AC Millivoltmeter
													X	Decade Resistance Box
X	X	X	X	X	X	X	X	X	X	X	X	X	X	Specimen Holders
X					X	X			X		X			Shielded Holder (Fig 4)

**TABLE I : Equipment required for determination of various constants**

Constants to be determined														MEASUREMENTS REQD.
$k_{33}$	$k_{31}$	$k_p$	$K^T_3$	$D$	$s^D_{33}$	$s^E_{33}$	$s^D_{11}$	$s^E_{11}$	$d_{33}$	$d_{31}$	$g_{33}$	$g_{31}$	$Q_m$	
					X	X	X	X	X	X	X	X		Weight or Density
			X		X	X	X	X	X	X	X	X		Dimensions
			X	X					X	X	X	X	X	Cap. & Dissipation Factor
X	X	X					X	X	X	X	X	X	X	Freq. of Minimum Impedance
X	X	X			X	X			X	X	X	X	X	Freq. of Maximum Impedance
													X	Mag. of Minimum Impedance

**TABLE II : Measurements required for determination of various constants**

**Weight:** Weigh the test specimen in air. This is required, together with dimensions, for determination of density. If preferred, the density may be determined by weighing the specimen in air and water, then it is only necessary to measure the frequency controlling dimension ( the largest dimension).

**Dimensions:** Measure the dimensions of the test specimen with a micrometer. It is advisable to measure at several points and use the averages of these measurements.

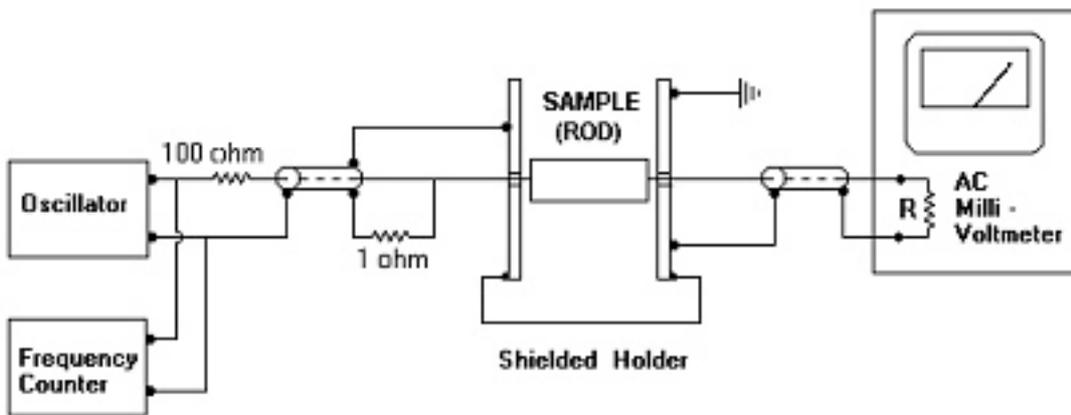
**Capacitance and dissipation factor:** Measure the capacitance  $C$  and dissipation factor  $D$  at 1000 Hz. Care should be exercised to avoid error due to stray capacitance, especially when the specimen is proportioned as in Fig 1(a). Keep the test voltage across the specimen below:  $4 \times$  (distance between the electrodes measured in mm) volts.

**Frequencies of Minimum and Maximum Impedance:** To determine the frequency of minimum impedance of any of the test specimens, and the frequency of maximum impedance of specimens 1(b) or 1(c), set up equipment as shown in Fig. 3. If determination of mechanical  $Q$  is not required, the decade resistance box may be omitted. The frequency of maximum impedance for end electroded rods ( Fig. 1(a).) requires a different set-up discussed later.

Turn switch SW1 to sample. Set switch SW2 to 1 ohm. Starting with the oscillator at a frequency somewhat below the expected resonance frequency of the specimen, slowly start increasing the frequency until a sharp maximum indication is obtained on the meter. It may be necessary to turn switch 2 to higher resistance position to obtain sufficient meter deflection. Always use the lowest value of resistance that permits mid-scale indication on the meter (.001 volt full scale range). Very carefully adjust the frequency for peak meter reading. The oscillator is now set to the frequency of minimum impedance  $f_m$ . Read the frequency at the counter and record it as  $f_m$ .

Next, turn switch 2 to 1000 ohms and increase the frequency until a sharp minimum indication is obtained on the meter. It may be necessary to turn switch SW2 to higher resistance position to maintain adequate sensitivity. Always use the lowest value of resistance that permits a sharp indication on the meter (.001 volts full scale range). Very carefully adjust the oscillator for minimum reading. the oscillator now is set to the frequency of maximum impedance  $f_n$ . Read the frequency at the counter and record it as  $f_n$ .

**Caution:** The small residual harmonic content of a low distortion oscillator is sufficient to cause false indications. For example, if  $f_m$  is 100 kHz, and the oscillator has some second harmonic content then when the oscillator is set at 50 kHz, it delivers in addition to 50 kHz, a small 100 kHz signal which will cause a sharp but small increase in meter indication. With a good oscillator, usually it is a simple matter to determine which indications are spurious, as the correct indications are always very much stronger.



For end-electroded rods (Fig 1(a)), shield the rods effectively against any stray capacitance when determining the frequency of maximum impedance. Set up as shown in Fig 4 and adjust for  $f_n$  as described above.

**FIG 4 : Circuit for determining frequency of maximum impedance of end electroded rods**

**Magnitude of Minimum Impedance:** The set-up for this measurement is the same as for the measurement of frequencies of minimum impedance (Fig 3). Adjust the oscillator to the frequency of minimum impedance as described above (switch SW2 set to low resistance, tune for peak). Note the meter reading on the AC Millivoltmeter. Then substitute the decade resistance box by turning switch SW1 to the Decade Resistance box position and adjust the resistance to obtain the same meter deflection. Record this value as  $Z_m$ .

## 5. CALCULATIONS

**Density:** The density in kilograms/meter<sup>3</sup> is required in later calculations.

$$\text{Density} = \frac{\text{weight in kilograms}}{\text{volume in meter}^3}$$

$$\text{or } \rho = \frac{(\text{weight in air in kilograms})}{(\text{weight in air} - \text{weight in water}) \text{ in kg.}} \times 10^3$$

**Free Relative Dielectric Constant  $K_3^T$ :** This is calculated from the measured values of capacitance and physical dimensions of the specimen.

$$K_3^T = \frac{\text{distance between electrodes (meters)} \times C(\text{pF})}{\text{area of one electrode (meter}^2) \times 8.85}$$

The dissipation factor is determined while measuring capacitance is the required figure, no calculation being necessary.

**Coupling:** Coupling is calculated from the frequencies of minimum and maximum impedance. The applicable formula depends on the vibration mode as shown below. The frequencies of minimum and maximum impedance are not the exact frequencies required for these calculations, and a small correction theoretically should be made as described in the IRE Standards. However, when dealing with barium titanate and lead zirconate-lead titanate compositions, the error resulting from omitting the correction is not significant. Accordingly, the corrections generally are not made and will not be included here.

**Coupling  $k_{33}$ :** This is applicable only to length-poled rods as shown in Fig. 1(a).

$$k_{33} = \sqrt{\frac{\pi}{2} \frac{f_m}{f_n} \cdot \tan \left[ \frac{\pi}{2} \frac{(f_n - f_m)}{f_n} \right]}$$

Use the curve of Fig. 5 to determine  $k_{33}$ .

**Coupling  $k_{31}$ :** This is applicable only to long, slim, thickness-poled specimens as shown in Fig. 1 (b).

$$k_{31} = \frac{A}{\sqrt{1 + A}} \quad A = \frac{\pi}{2} \frac{f_n}{f_m} \cdot \tan \left[ \frac{\pi}{2} \frac{(f_n - f_m)}{f_m} \right]$$

or use the curve of Fig. 6 to determine  $k_{31}$ .

### THICKNESS COUPLING FACTOR

$k_{33}$  VS  $\frac{f_n - f_m}{f_m}$

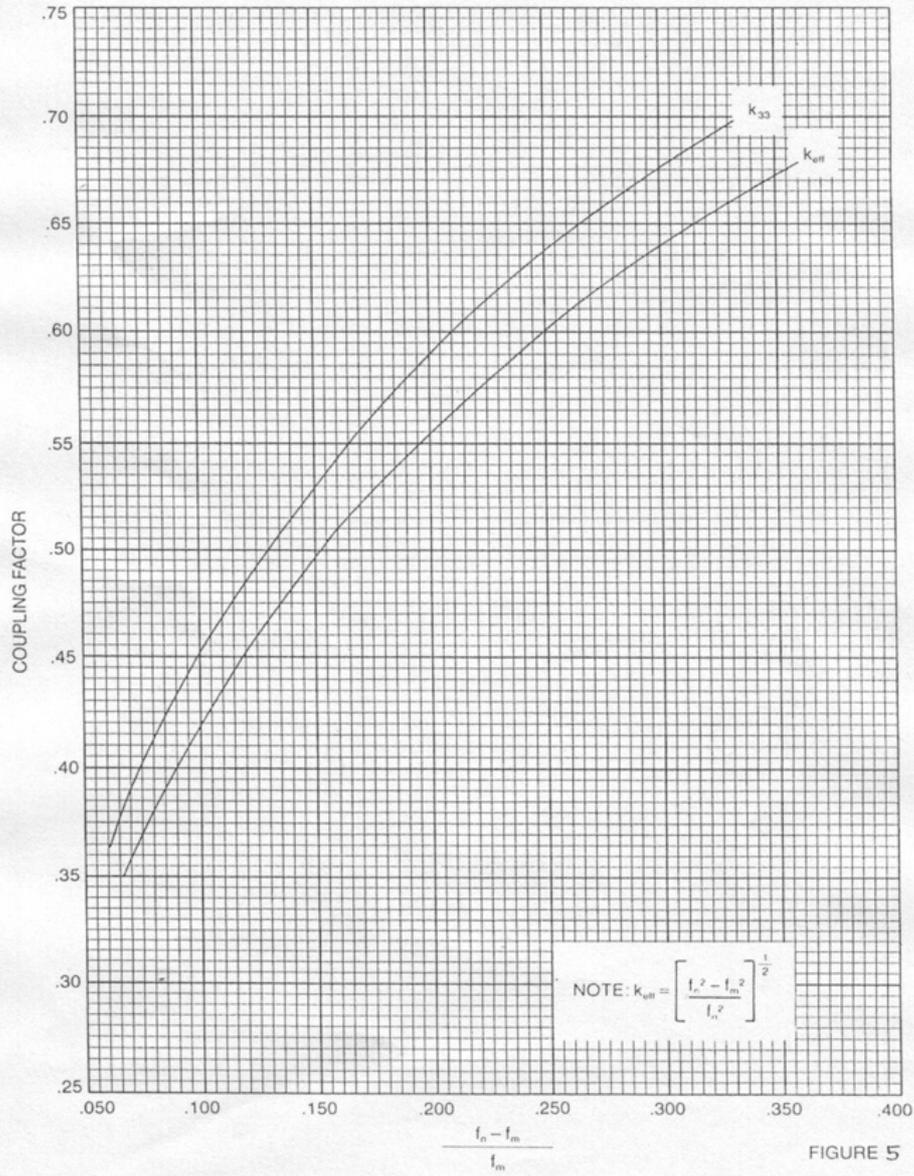


FIGURE 5

# TRANSVERSE COUPLING FACTOR

$$k_{21} \text{ VS } \frac{t_1 - t_2}{t_1}$$

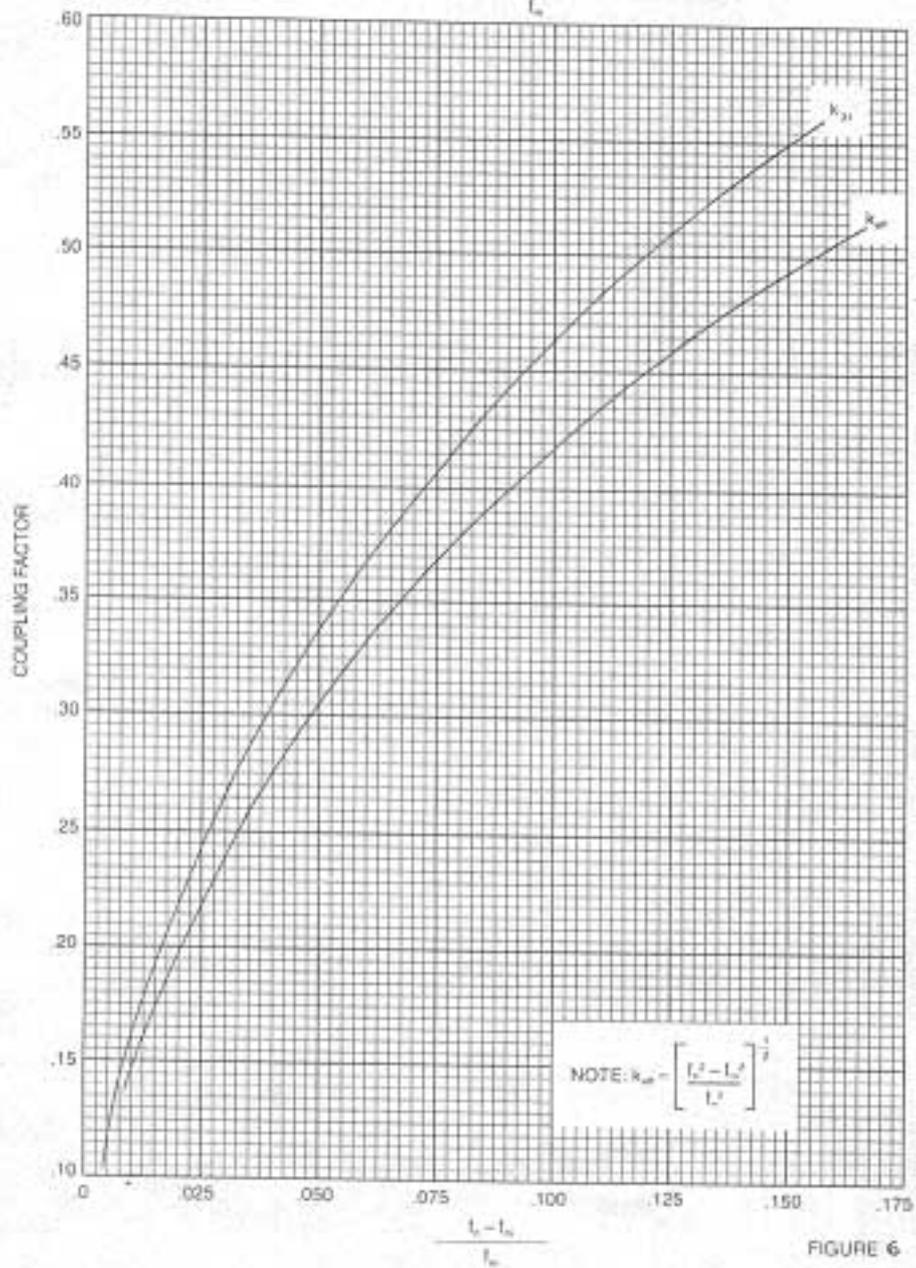
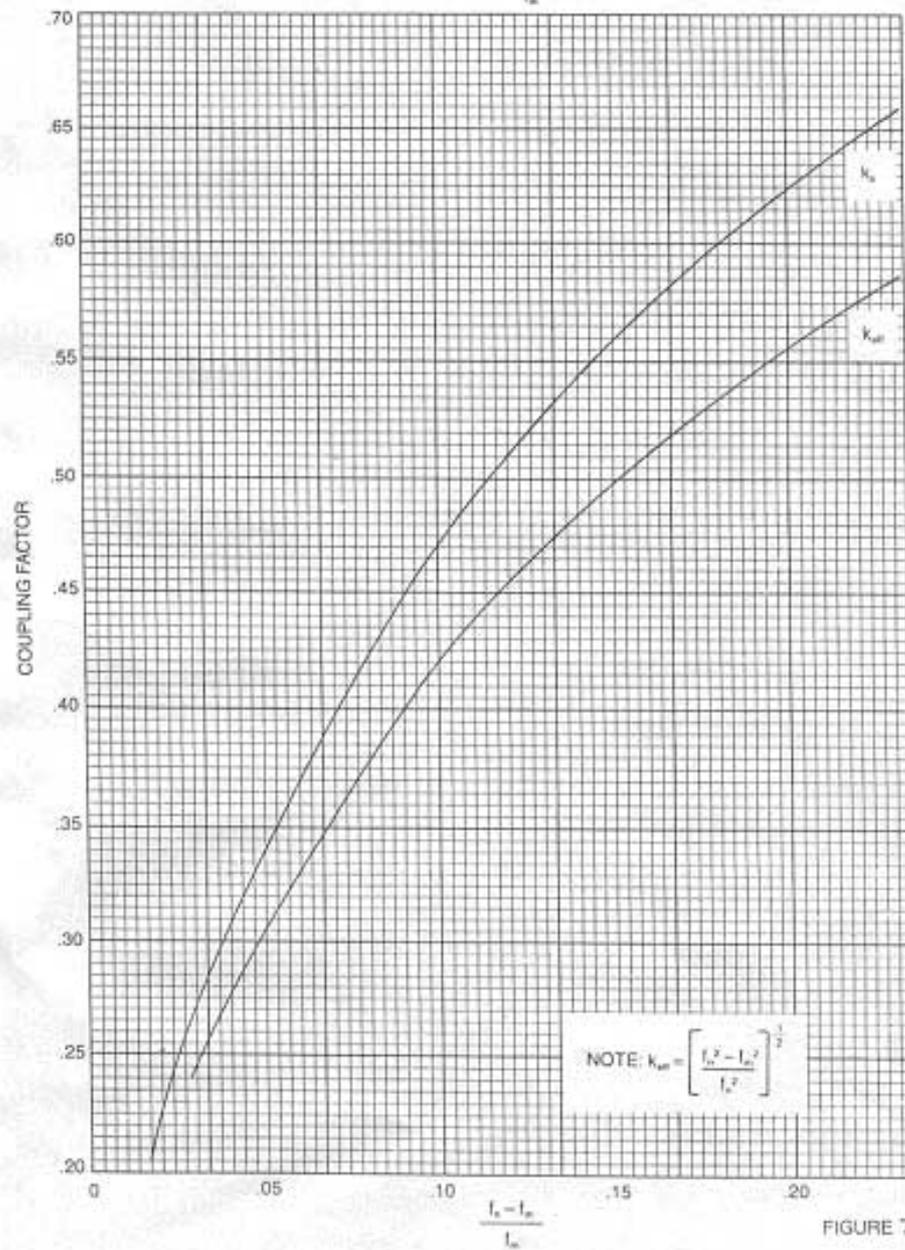


FIGURE 6

### PLANAR COUPLING FACTOR

$k_p$  VS  $\frac{l_s - l_m}{l_m}$



**Coupling  $k_p$ :** This is applicable only to thin discs as shown in Fig. 1 (c).

$$\text{Calculate : } \frac{f_n - f_m}{f_m}$$

Use the curve of Fig.7 to determine  $k_p$ . Fig. 7 is valid only for piezoelectric ceramic having Poisson's ratio near 0.3. Presently available lead titanate-lead zirconate compositions meet this requirement.

**Elastic Constants:** Elastic constants are calculated from the frequency controlling dimension, density, and the frequencies of minimum or maximum impedance as shown below. Physical dimensions must be in meters.

$s_{33}^D$  . This is applicable only to length poled rods as shown in Fig. 1 (a).

$$s_{33}^D = \frac{1}{4 \cdot \rho \cdot f_n^2 \cdot l^2} \quad \frac{\text{meter/meter}}{\text{Newton/ meter}^2}$$

$s_{11}^E$  . This is applicable only to long, slim thickness-poled specimens as shown in Fig. 1 (b).

$$s_{11}^E = \frac{1}{4 \cdot \rho \cdot f_m^2 \cdot l^2} \quad \frac{\text{meter/meter}}{\text{Newton/ meter}^2}$$

$$s_{11}^D = (1 - k_{31}^2) \cdot s_{11}^E \quad \frac{\text{meter/meter}}{\text{Newton/ meter}^2}$$

$$s_{33}^E = \frac{s_{33}^D}{1 - k_{33}^2} \quad \frac{\text{meter/meter}}{\text{Newton/ meter}^2}$$

**Piezoelectric Constants:** The d and g constants are calculated from values of coupling, dielectric constant, and elastic constant previously calculated.

$$d_{33} = k_{33} \cdot \sqrt{8.85 \times 10^{-12} \text{ K}_3^T \cdot s_{33}^E} \quad \frac{\text{meter/meter}}{\text{volt/meter}} \quad \text{or} \quad \frac{\text{coulombs/ meter}^2}{\text{Newton/ meter}^2}$$

$$d_{31} = k_{31} \cdot \sqrt{8.85 \times 10^{-12} \text{ K}_3^T \cdot s_{11}^E} \quad \frac{\text{meter/meter}}{\text{volt/meter}} \quad \text{or} \quad \frac{\text{coulombs/ meter}^2}{\text{Newton/ meter}^2}$$

$$g_{33} = \frac{d_{33}}{8.85 \times 10^{-12} \cdot \text{K}_3^T} \quad \frac{\text{volts/meter}}{\text{Newtons/ meter}^2} \quad \text{or} \quad \frac{\text{meter/meter}}{\text{coulombs/ meter}^2}$$

$$g_{31} = \frac{d_{31}}{8.85 \times 10^{-12} \cdot \text{K}_3^T} \quad \frac{\text{volts/meter}}{\text{Newtons/ meter}^2} \quad \text{or} \quad \frac{\text{meter/meter}}{\text{coulombs/ meter}^2}$$

**Mechanical Q:** This is calculated from the frequencies of minimum and maximum impedance, the magnitude of minimum impedance and capacitance.

$$Q_m = \frac{f_n^2}{2\pi \cdot f_m \cdot Z_m \cdot C \cdot (f_n^2 - f_m^2)}$$

This is a dimensionless quantity.

## 6. TERMINOLOGY USED

S.No.	Term	Description
1	$k_{33}, k_{31}, k_p$	Piezoelectric Coupling Factors
2	$K_3^T$	Free Relative Dielectric Constant
3.	D or $\tan \delta$	Dissipation factor
4.	$S_{33}^D, S_{11}^D, S_{11}^E, S_{33}^E$	Elastic Constants
5.	$d_{33}, d_{31}$	Piezoelectric Charge Constants
6.	$g_{33}, g_{31}$	Piezoelectric Voltage Constants
7.	$Q_m$	Mechanical Quality Factor
8.	$f_m$	Frequency of minimum impedance or resonance frequency.
9.	$f_n$	Frequency of maximum impedance or anti-resonance frequency
10.	$Z_m$	Impedance at resonance frequency or frequency of minimum impedance.