

High power characteristics at antiresonance frequency of piezoelectric transducers

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Abstract

First in this paper, the loss in piezoelectric ceramics is described. Antiresonance is the vibration under constant D (electric displacement) driving, and therefore electro-mechanical loss becomes almost zero: resonance is the vibration under constant E (electric field) driving, and then there exists large electro-mechanical loss. The relations between antiresonance and the constant D driving are explained. Next, a method of measuring the high-power characteristics is described for antiresonance frequency. Experimental results for the quality factor and temperature rise and other equivalent constants are then shown as high-power characteristics obtained at the antiresonance frequency. Finally, some considerations for the stable-state driving of the high-power piezoelectric devices are described.

Keywords: High-power characteristics; Antiresonance; Constant current driving

1. Introduction

There exist two kinds of electrical resonance in piezoelectric transducers. One is the so-called resonance and the other is antiresonance, and both resonances are mechanical ones. The former represents the mechanical resonance vibrating under the electric short-circuit condition, while the latter represents the mechanical resonance vibrating under the electric open-circuit condition. In this paper, losses at the resonance and antiresonance are described, and the measuring method and experimental results of the high-power characteristics at antiresonance are given.

First in this paper, losses in the piezoelectric ceramics are described. Under constant D (electric displacement) driving, electro-mechanical loss becomes almost zero, while under constant E (electric field) driving, there exists large electro-mechanical loss. The former situation is related to antiresonance, and the latter situation is related to the resonance.

Next, methods of measuring the high-power characteristics – that is, the vibrational level dependence of the

equivalent circuit constants – are described for the antiresonance frequency. Some experimental results are then shown. Furthermore, some considerations on the stable-state driving of the high power piezoelectric devices are given.

2. Antiresonance as a vibration under constant electric displacement

In the case of piezoelectric transverse coupling, the fundamental equations of the piezoelectric transducer under constant electric displacement are given by

$$S = s_{11}^D T + g_{31} D, \quad (1)$$

$$E = -g_{31} T + \beta_{33}^T D, \quad (2)$$

where S , T , E and D are the strain, stress, electric field and electric displacement, respectively. s_{11}^D , g_{31} and β_{33}^T are the elastic compliance, piezoelectric constant and dielectric impermeability, respectively.

By using following boundary conditions,

$$u(x=0) = u_0, \quad u(x=\ell/2) = 0, \quad (3)$$

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longitudinal displacement u is given by

$$u = u_0(\cos kx - \cot(k\ell/2) \sin kx), \quad (4)$$

where x is the horizontal coordinate, ℓ is the length of the transducer and k is the wave-number. Stress T is represented, as follows, by using the strain S ($=\partial u/\partial x$),

$$T = S/s_{11}^D - g_{31}/s_{11}^D D \\ = -k/s_{11}^D u_0(\sin kx + \cot(k\ell/2) \cos kx) - g_{31}/s_{11}^D D. \quad (5)$$

At $x=0$, stress T equals to zero, then

$$u_0 = -(g_{31}/k)D \tan(k\ell/2). \quad (6)$$

By substituting Eqs. (5) and (6) into Eq. (2), electric field E is given by

$$E = [-g_{31}^2/s_{11}^D(\cos kx + \tan(k\ell/2) \sin kx) \\ + \beta_{33}^T/(1 - k_{31}^2)]D. \quad (7)$$

Here, we consider a certain value E_0 , which is constant over the whole length, and then the induced electric displacement D' is obtained as

$$D' = \epsilon_{33}^T(E - E_0). \quad (8)$$

This electric displacement refers to the movement of charge when the electrode is fabricated on the whole surface, and then the electric field E becomes E_0 (constant) over the whole length. When E_0 is put as

$$E_0 = (2 - k_{31}^2)/(1 - k_{31}^2)(D/\epsilon_{33}^T), \quad (9)$$

the integral of D' with respect to x , that is, $\int_0^\ell D' dx$, becomes zero. From this relation, the following equation is derived,

$$\tan(k\ell/2)/(k\ell/2) = -(1 - k_{31}^2)/k_{31}^2. \quad (10)$$

Eq. (10) gives the antiresonance frequency. Hence, it is concluded that the antiresonance is the vibration under constant D and, after fabricating the electrode over the whole transducer surface, because of the piezoelectric reaction, the induced charge is cancelled; the electric field then becomes constant over the whole length.

3. Loss in piezoelectric ceramics

The Gibbs free energy G is given by

$$G = -\frac{1}{2}PE - \frac{1}{2}ST, \quad (11)$$

where P is the polarization.

3.1. At resonance frequency

Under E constant driving, from each term in Eq. (11), loss W in the piezoelectric ceramics is given by

$$W = \frac{1}{2}\epsilon E^2 \tan \delta + \frac{1}{2}sT^2 \tan \phi \\ + \frac{1}{2}(d^2/s)E^2 \tan \theta + \frac{1}{2}(d^2/\epsilon)T^2 \tan \theta, \quad (12)$$

where ϵ , s and d are the dielectric constant, elastic compliance and the piezoelectric constant. The first term in Eq. (12) represents the dielectric loss due to P - E hysteresis with the loss angle δ . The second term is the mechanical loss due to S - T hysteresis with the loss angle ϕ . The third term is the electro-mechanical loss due to S - E hysteresis with the loss angle θ . The final term also gives the electro-mechanical loss due to P - T hysteresis with the loss angle θ . Under such a driving condition, the domain moves easily, and therefore the hysteresis loop between S and E is fairly large. Resonance is the vibration under E constant driving; therefore, at resonance, all the losses in Eq. (12) must be considered. In particular, the electro-mechanical loss increases markedly with increasing vibrational displacement amplitude.

3.2. At antiresonance frequency

Under D (almost equal to P) constant driving, loss W in the piezoelectric ceramics is given by

$$W = \frac{1}{2}(P^2/\epsilon) \tan \delta + \frac{1}{2}sT^2 \tan \phi \\ + \frac{1}{2}(d^2/\epsilon)P^2 \tan(\theta - \delta) + \frac{1}{2}(d^2/\epsilon)T^2 \tan(\theta - \delta). \quad (13)$$

Each term in Eq. (13) coincides with each one in Eq. (12). However, in this case, electro-mechanical losses, represented as the third and fourth terms, become almost zero because θ almost equals δ , since $\epsilon \gg \epsilon_0$ in piezoelectric ceramics. Under D constant driving, the domain cannot move so easily, therefore the hysteresis loop between S and D (or P) is small. Antiresonance is the vibration under D constant driving, therefore the loss at the antiresonance frequency becomes lower than that at the resonance frequency.

4. Measurement method at the antiresonance frequency

Fig. 1 shows the improved, advanced equivalent circuit, which is called the impedance-type equivalent circuit

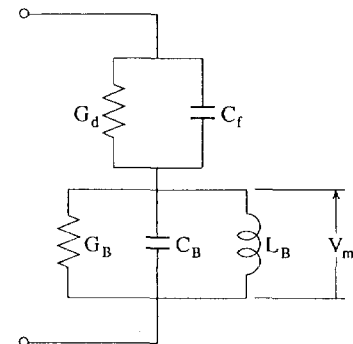


Fig. 1. Advanced impedance-type equivalent electric circuit.

[1]. Here, the conductance G_B means only mechanical loss. G_d is the conductance resulting from the electro-mechanical loss, and written as; $G_d = \omega C_d \tan \theta$. C_f is called the free capacitance and written by using the damped capacitance C_d and the coupling coefficient k as follows; $C_f = C_d / (1 - k^2)$.

4.1. Measuring procedure

(1) Quality factor Q_B .

Using the frequency perturbation method [2], the quality factor at antiresonance frequency Q_B can be obtained by

$$Q_B = \frac{2f_B}{f_2 - f_1} \frac{\sqrt{K_p(1 - K_p)}}{1 - 2K_p}, \quad (14)$$

where f_1 and f_2 are frequencies very close to the antiresonance frequency f_B , and have the relation $f_1 < f_B < f_2$. In addition, K_p is the perturbation ratio, which is given by the following equation under the constant velocity control,

$$K_p = (I - I_0) / I_0, \quad (15)$$

where I_0 is the terminal current at the antiresonance frequency f_B , and I is that at the frequency f_1 or f_2 . These values are measured by the digital voltmeters and transferred to the micro-computer through GP-IB.

(2) Electro-mechanical loss factor $\tan \theta$.

By using resonance frequency f_A and quality factor Q_A at resonance, and antiresonance frequency f_B and Q_B and the capacitance ratio C_B/C_f at antiresonance, the electro-mechanical loss factor $\tan \theta$ ($\equiv \tan \delta$) can be given as follows;

$$\tan \theta = C_B/C_f (1/Q_A - f_B/f_A 1/Q_B). \quad (16)$$

The measuring method for the other equivalent constants is shown in Ref. [3].

5. Experiments [4]

Quality factors Q_A and Q_B , temperature rises, capacitance ratio C_B/C_f and force factor A_B at the antiresonance frequency have been investigated on a PZT ceramic rectangular bar. The vibrational mode considered here is the fundamental longitudinal mode. The experimental results of Q_A , Q_B and the temperature rises are illustrated in Fig. 2 as functions of vibrational velocity v_0 , which was measured at the end of the ceramic bar using a 'Fotonic Sensor'. In addition, the temperature rise was measured at the center of the ceramic rectangular bar by using a thermocouple. In Fig. 2, the configuration and the dimensions of the test sample are illustrated.

From Fig. 2, it can be seen that Q_B is higher than Q_A

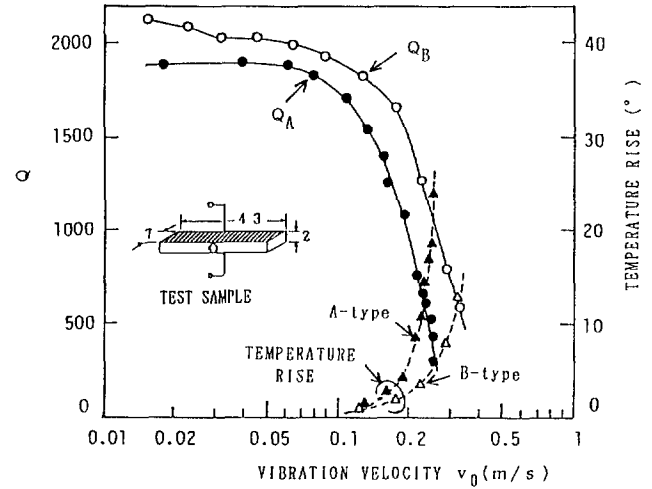


Fig. 2. Vibrational velocity dependence of the quality factor and temperature rise for both resonance and antiresonance of a PZT ceramic longitudinally vibrating transducer.

over the whole vibrational velocity range investigated here; from $v_0 \approx 0.02$ to about 0.3 m s^{-1} , and the difference between Q_A and Q_B becomes greater with increasing vibrational velocity. The temperature rise of antiresonance is less than that of resonance because $1/Q_B$ is smaller than $1/Q_A$. These facts are caused by the presence of electro-mechanical loss and its nonlinearity.

In Fig. 3, capacitance ratio C_B/C_f , force factor A_B and the electro-mechanical loss factor $\tan \theta$ are shown. $\tan \theta$ was obtained using the experimental results of Q_A , Q_B and C_B/C_f . For comparison, dielectric loss factor $\tan \delta_0$, which is directly measured at a sufficiently low frequency (1 kHz), is also shown in the figure. $\tan \delta_0$ includes an elastic loss brought by a quasi-static strain [5], therefore it is larger than $\tan \theta$ ($\equiv \tan \delta$) in a small vibrational velocity. C_B/C_f and A_B can be viewed as almost constant over the whole velocity range considered here. However,

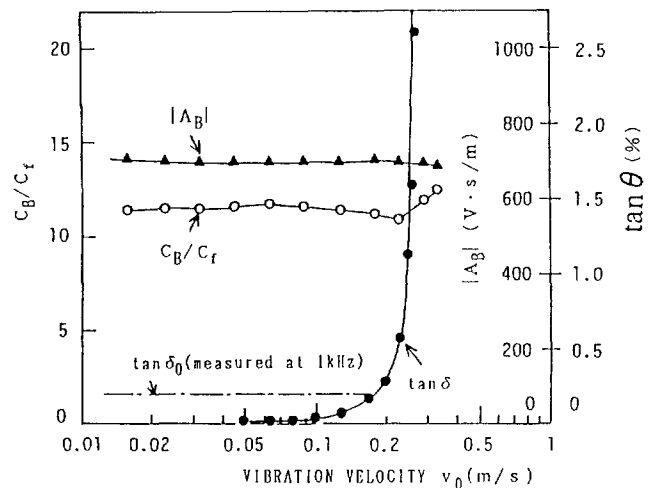


Fig. 3. Vibrational velocity dependence of the capacitance ratio and force factor of antiresonance and the electro-mechanical loss factor.

$\tan \theta$ has become markedly larger in the large vibrational velocity above about 0.2 m s^{-1} .

6. Driving frequency and efficiency

The equivalent circuit shown in Fig. 1 can be converted equivalently to the one shown in Fig. 4. This figure shows an equivalent electric circuit including the two resistances R_d and R_m . Here, resistance R_d indicating electro-mechanical loss is given by

$$R_d = \tan \theta / (\omega_A C_d), \quad (17)$$

where ω_A is the resonance angular frequency and C_d is the damped capacitance. R_m is the equivalent resistance indicating mechanical vibration loss, which is given as

$$R_m = (\omega_A L_A) / Q_B, \quad (18)$$

where L_A is the equivalent inductance in the series resonance circuit accompanying the equivalent capacitance C_A in Fig. 4.

Fig. 5 shows the experimental results of R_d and R_m in increasing the vibrational velocity v_0 measured at the end of the rectangular bar PZT ceramic transducer. In Fig. 5, R_A includes both mechanical and electro-mechanical loss, and it is obtained by the ratio of driving voltage to driving current.

As shown in Fig. 5, R_A almost equals $R_d + R_m$. At a smaller vibration level, R_d is much smaller than R_m . With increasing vibrational level, R_m increases in proportion to v_0^2 . However, the rate of increase of R_d is larger than that of R_m ; therefore, at a vibrational level larger than about $v_0 = 0.25 \text{ m s}^{-1}$, R_d becomes larger than R_m . As a result, a significant amount of input power will be consumed by R_d ; therefore, the vibrational velocity cannot be increased further.

Two of mechanical terminals, indicated by b and b' in Fig. 4, are positioned to obtain mechanical power. For simulating the efficiency characteristics, mechanical load resistance r_L is connected to the mechanical terminals b and b'. Here, we assume that the vibrational mode of the transducer does not vary before and/or after connection of r_L . With the equivalent circuit shown in Fig. 4, efficiency η (=mechanical output power/electric input power) can be calculated. Maximum efficiency can be obtained at the antiresonance frequency [3].

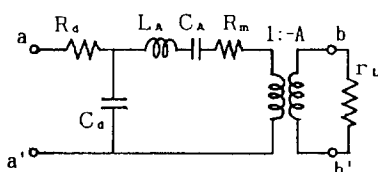


Fig. 4. Improved admittance type equivalent electric circuit with a mechanical load resistance.

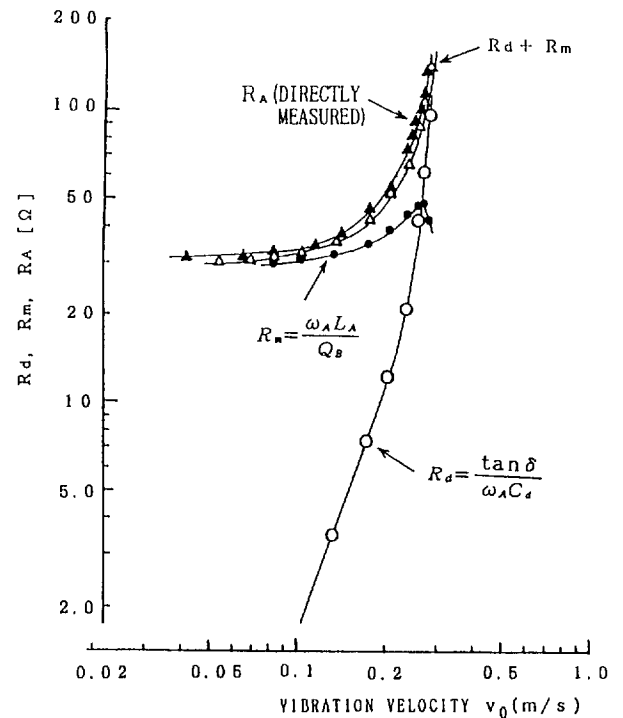


Fig. 5. Vibrational velocity dependence of resistances R_d , R_m and R_A .

7. Conclusions

In this paper, losses in the piezoelectric ceramics have been described. Under constant D driving, electro-mechanical loss becomes almost zero, while under constant E driving, there exists large electro-mechanical loss. The former situation is related to the antiresonance, and the latter situation is related to the resonance. Therefore, it is concluded that the loss at the antiresonance frequency is lower than that at the resonance frequency.

Experimental results of the vibrational level dependence on the equivalent electric circuit constants have been obtained for antiresonance frequency. It has been shown that the quality factor Q_A of resonance is always smaller than that of antiresonance Q_B and the difference between them becomes larger with increasing vibrational velocity, and the temperature rise of antiresonance is less than that of resonance. The electro-mechanical loss factor $\tan \theta$ has become markedly larger for the large vibrational velocity above about 0.2 m s^{-1} .

By using the experimental results of the equivalent constants, including the electro-mechanical loss, the efficiency of the transducers under high-power use can be calculated. Maximum efficiency can be obtained at the antiresonance frequency. In addition, at this frequency, the temperature rise is very small. It is then concluded that stable-state driving of high-power piezoelectric devices, such as an ultrasonic motor, can be achieved at the antiresonance frequency.

This study was conducted for the design of piezoelectric power devices. In addition, by comparing high-power characteristics on various piezoelectric materials, materials with high performance can be selected.

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