

Available online at www.sciencedirect.com



Ultrasonics 41 (2003) 15-23

*Illtrasonics* 

www.elsevier.com/locate/ultras

# Open-circuit test of a PZT vibrator and its applications

Kuo-Tsi Chang<sup>a,b,\*</sup>, Minsun Ouyang<sup>a</sup>

<sup>a</sup> Department of Engineering and System Science, National Tsing-Hua University, 101, Section 2 Kuang Fu Road, Hsinchu 30043, Taiwan, ROC <sup>b</sup> Department of Electrical Engineering, National Lien-Ho College of Technology, 1 Lien-Kung Road, Kungjing Li, Miaoli 36012, Taiwan, ROC

Received 25 February 2002; received in revised form 14 June 2002; accepted 25 June 2002

## Abstract

This study investigates a terminal transient response of a Langevin-type PZT vibrator theoretically and experimentally to quantify an electrical shock and refine an equivalent circuit of the vibrator. The shock is induced immediately after an AC sinusoidal voltage of the vibrator is switched off. Then, the transient response involves a DC part and an AC part, which approach zero at the DC and AC times, respectively, and the vibrator is placed on a sponge in air. To do so, we should propose an open-circuit test to find the AC and DC times in addition to the maximum amplitude of the transient response. Thus the DC times exceeds the AC time, and the AC and DC times are used to estimate the resistances in the equivalent circuit presenting the real mechanical and dielectric losses, respectively. Therefore, the resistances in the equivalent circuit are sensitive to the vibration amplitude, but the inductance and capacitances are not. Furthermore, the maximum amplitude is required to cause the shock, and depends on the frequency of the source and the open-circuited time, and is about 65 times the amplitude of the source. © 2002 Elsevier Science B.V. All rights reserved.

*PACS:* 43.35.Z; 81.70.C *Keywords:* Open-circuit test; Equivalent circuit; PZT vibrator; Electric shock; Mechanical loss

## 1. Introduction

Electric shocks with high-voltage transient responses at the terminals of vibrators under high-power excitation are induced after the related drives are switched off. These shocks have not yet been investigated. An "a  $R_s$ - $L_s$ - $C_s$  branch parallels parameters of  $R_p$  and  $C_p$ " impedance model has not been built into an impedance analyzer (HP4194A) to measure resistance as the dielectric loss of vibrators. For a resistance equivalent to the real mechanical loss, the high-power constant differs from the low-power constant measured by the analyzer.

A differential circuit has been proposed to measure separately the mechanical loss and the dielectric loss by detecting the motional admittance under high-power excitation [1]. Moreover, terminal voltage and current have been used to determine dielectric loss just after switching on the high-power drive, and the force factor and the attenuation constant have been used to estimate the high-power equivalent resistance [2]. This factor resembles the ratio of the amplitude of the motional current to the amplitude of the vibration velocity and was used to estimate short-circuit transient response of a terminal current after the drive was switched off. These methods constitute very complicated ways to calculate the equivalent resistance and cannot analyze electric shocks for purely AC transient response.

Accordingly, an open-circuit test is proposed to elucidate complete terminal transient responses of vibrators under high-power excitation for quantifying electric shocks and estimating equivalent resistive constants. The test can reveal how to reduce the highest voltage of the shock and simply refine the high-power equivalent circuit.

# 2. Equivalent circuit of the vibrator

An equivalent circuit of a PZT vibrator with initial conditions, illustrated in Fig. 1(a), is employed to derive a terminal voltage with an open-circuit transient response. Another equivalent circuit connected to an AC voltage  $v_s(t) = V_s \sin \omega_s t$ , shown in Fig. 1(b), is applied

<sup>&</sup>lt;sup>\*</sup>Corresponding author. Address: Department of Electrical Engineering, National Lien-Ho College of Technology, 1 Lien-Kung Road, Kungjing Li, Miaoli 36012, Taiwan, ROC.



Fig. 1. Equivalent circuits of a PZT vibrator with: (a) initial conditions and (b) an AC voltage.

Table 1Description of the parameters in Fig. 1

Parameters	Descriptions
R <sub>d</sub>	Equivalent resistance indicating real dielectric loss
$R_{ m m}$	Equivalent resistance indicating real mechanical loss
$C_{\rm d}$	Damped capacitance of the piezoelectric ceramic
$L_{\rm m}$	Equivalent inductance indicating equivalent mass
$C_{\mathrm{m}}$	Equivalent capacitance indicating equivalent stiffness

to derive steady responses of  $v_{\rm cm}$  and  $i_{\rm m}$ . Table 1 describes the parameters in Fig. 1. The mechanical resonant frequencies of vibrators are given calculated by  $f_{\rm m} = 1/(2\pi (L_{\rm m}C_{\rm m})^{1/2})$  and  $f_{\rm ar} = 1/(2\pi (L_{\rm m}C_{\rm md})^{1/2})$  where  $C_{\rm md} = C_{\rm m}C_{\rm d}/(C_{\rm m} + C_{\rm d})$ . Here,  $f_{\rm m}$  and  $f_{\rm ar}$  are the resonant frequency and the anti-resonant frequency, respectively. Then the mechanical quality factor,  $Q_{\rm m}$ , is determined using,  $Q_{\rm m} = \omega_{\rm m}L_{\rm m}/R_{\rm m}$  where  $\omega_{\rm m} = 2\pi f_{\rm m}$ .

#### 3. Analysis of transient and steady responses

From Eq. (A.2) in Appendix A, the diagrams of the voltage  $v_{oc}$  are indicated in Fig. 2, and  $f_{oc} = \beta/2\pi \cong f_{ar}$ 

where  $f_{oc}$  represents the oscillated frequency of  $v_{oc}$ . The maximum amplitudes of DC and AC responses of  $v_{oc}$  are related to the initial conditions,  $V_{od}$ ,  $V_{om}$  and  $I_0$ . The DC response  $be^{-\alpha_{dc}t}$  almost vanishes after DC steady-state time  $t_{dc}$  (=  $5R_dC_d$ ) is reached, and the AC response  $fe^{-\alpha_{ac}t}\cos(\beta t - \varphi)$  nearly vanishes after AC steady-state time  $t_{ac}$  (=  $10L_m/R_m$ ) is reached. Consequently,  $R_d$  and  $R_m$  can be calculated since  $t_{dc}$ ,  $t_{ac}$ ,  $C_d$  and  $L_m$  are known.

Next, the initial conditions are calculated by the following steady-state responses: let  $v_s(t) = V_s \sin \omega_m t$ where *h* denotes the high-power excitation at the first resonant frequency, then  $v_{cm}(t) = Q_m V_s \sin(\omega_m t - \pi/2)$ ,  $i_m(t) = I_{mh} \sin \omega_m t$  and  $I_{mh} = V_s/R_m$ . Table 2 lists various initial conditions and their corresponding amplitudes of *b* and *f* in Eq. (A.2). The high-power highest voltage,  $V_{oc.max}^h$ , is determined by (Fig. 3)

$$V_{\text{oc.max}}^{\text{h}} \cong |b| + f \cong (2Q_{\text{m}}C_{\text{m}}/C_{\text{d}})V_{\text{s}}.$$
(1)

Moreover, let  $v_s(t) = V_s \sin \omega t$  where 1 represents the low-power excitation near and above the first resonant frequency, then  $v_{\rm cm}(t) = G_{\rm cm}V_s \sin(\omega t - \varphi_{\rm im} - \pi/2)$ ,  $i_{\rm m}(t) = I_{\rm ml} \sin(\omega t - \varphi_{\rm im})$ ,  $G_{\rm cm} = 1/(\omega C_{\rm m}|Z(\omega)|)$ ,



Fig. 2. Schematic waveforms of the transient response: (a) the DC and AC parts individually and (b) a complete transient response.

Table 2Maximum amplitudes and drive conditions

AC voltages at	At open-circuited	Drive conditions			Maximum amplitudes, $V_{\rm oc}^+ = b + f(b > 0)$ or	
frequencies times in Fig. 3		V <sub>od</sub>	Vom	$I_0$	$V_{\rm oc}^- = b + f(b < 0)$	
$f_{\rm s} = f_{\rm m}$	$t_1$	0	$Q_{\rm m}V_{ m s}$	0	$V_{\rm och}^+ = 2(C_{\rm m}Q_{\rm m}/C_{\rm d})V_{\rm s}$	
	$t_2$	$-V_{\rm s}$	0	$-I_{\rm mh}$	$V_{ m och}^0 = -V_{ m s} + I_{ m mh}/eta C_{ m d}$	
	$t_3$	0	$-Q_{\rm m}V_{\rm s}$	0	$V^{ m och} = (-1)  imes V^+_{ m och}$	
$f_{\rm s} > f_{\rm m}$	$t_4$	$-V_{\rm s}$	$G_{\rm cm}V_{\rm s}$	0	$V_{\rm ocl}^+ = 2(C_{\rm m}G_{\rm cm}/C_{\rm d}-1)V_{\rm s}$	
	$t_5$	0	0	$-I_{\rm ml}$	$V_{ m ocl}^0 = I_{ m ml}/eta C_{ m d}$	
	$t_6$	$V_{\rm s}$	$-G_{\rm cm}V_{\rm s}$	0	$V_{ m ocl}^- = (-1)  imes V_{ m och}^+$	



Fig. 3. Schematic waveforms of steady-state responses: (a) for resonance and (b) for above resonance.

 $I_{\rm ml} = V_{\rm s}/|Z(\omega)|$ ,  $\varphi_{\rm im} = \arctan(1/(\omega C_{\rm m}R_{\rm m}))$  and  $Z(\omega) = R_{\rm m} + j(\omega L_{\rm m} - 1/(\omega C_{\rm m}))$ . The low-power highest voltage,  $V_{\rm oc.max}^{\rm l}$ , is calculated by

$$V_{\rm oc,max}^{\rm l} \cong |b| + f \cong (2G_{\rm cm}C_{\rm m}/C_{\rm d} - 1)V_{\rm s}.$$
 (2)

## 4. Design of drive system

The drive system of the open-circuit test includes an inverter circuit, signal circuits and a switch system, as illustrated in Fig. 4. From Fig. 4(a) [3], the inverter circuit consists of a DC/DC buck converter, a full-bridge switching chopper, a filter and an equivalent circuit of the vibrator, and thus generates AC sinusoidal voltages of the vibrator. When  $S_1$  and  $S_4$  are ON and  $S_2$  and  $S_3$  are OFF, the amplitude of  $v_{sqr}$  is  $V_{in}$ . When  $S_1$  and  $S_4$  are OFF and  $S_2$  and  $S_3$  are ON, the amplitude of  $v_{sqr}$  is  $-V_{in}$ . Meanwhile, the frequency of  $v_{sqr}$  is identical to that of rectangular gate-to-source voltages,  $v_{gs1-4}$ , with an onduty cycle of 50%. The switches,  $S_{1-4}$ , are implemented by MOSFET devices (IRF840).

From Fig. 4(b), pulse-width modulation and voltagecontrolled oscillator circuits are used to control the voltage and frequency of AC sources, respectively, and implemented by UC3524 and LM566, respectively. Next, the switch system shown in Fig. 4(c) performs the switch SW displayed in Fig. 4(a), and comprises a function generator (Agilent, 33120A), a MOSFET device (IRF840) and a relay (DC 12 V, JZC-6F, 4098). Then, the MOSFET  $S_{sw}$  is controlled by a rectangular gate-to-source voltage,  $v_{sw}$  (1 Hz, 0 V/12 V) that is obtained from the function generator. In Fig. 4(d), a schematic waveform of  $v_{oc}$  is obtained after time  $t_{dsw}$  at the turned-on time of the MOSFET  $S_{sw}$ .

#### 5. Verification results

5.1. Measured results of the transient response and AC voltage

A Langevin-type PZT vibrator (BLT-45282H) (diameter, 45 mm; length, 79.5 mm; mass, 411 g)



Fig. 4. Block diagrams of drive system: (a) an inverter circuit, (b) function generators, (c) a switch system, and (d) a schematic waveform of the transient response obtained after time  $t_{dsw}$  at the turned-on time of the MOSFET  $S_{sw}$ .

manufactured by Increase-More Industrial Corp. in Taiwan is used in the test. The parameters of the vibrator are then measured using an impedance analyzer HP4194A:  $R_m^A = 174.6 \Omega$ ,  $L_m = 122.1 \text{ mH}$ ,  $C_m = 0.243 \text{ nF}$ ,  $C_d = 3.79 \text{ nF}$ ,  $f_m = 29.22 \text{ kHz}$  and  $f_{ar} = 30.13 \text{ kHz}$ . Moreover, the waveforms of  $v_{oc}$  in Figs. 5, 6, 7(a, b) and (c, d) are induced by sinusoidal voltages  $v_{s1}$  (10 V/29.22 kHz),  $v_{s2}$  (10 V/29.41 kHz),  $v_{s3}$  (30 V/29.22 kHz) and  $v_{s4}$ (30 V/29.41 kHz), respectively. Meanwhile, the voltages are obtained using the parameters,  $L_f$  ( $\cong$  143 µH) and  $C_f$ ( $\cong 0.1 \mu$ F) in Fig. 4(a), and the waveforms are measured using a probe with a ratio of 1/100 and an oscilloscope (YOKOGAWA, DL1520). From Fig. 5, the observed waveforms are obtained after about 2 ms at the turnedon time of the MOSFET  $S_{sw}$  in Fig. 4(c) due to the delay of the switch system. Additionally, the waveforms of the DC part of  $v_{oc}$  are measured by the smooth-and-filter function of the oscilloscope, and thus  $b \cong 280$  V obtained from Fig. 5(a) or (b) and  $b \cong 320$  V measured from Fig. 5(c) or (d).

#### 5.2. The measured times and the estimated results

According to Table 3, the DC time (about 180 ms) exceeds the AC time (around 46 or 48 ms), and thus  $R_m^1 \cong R_m^3 \cong 26.5 \ \Omega$  estimated using  $t_{ac}^1 \cong t_{ac}^3 \cong 46 \text{ ms}$ ;  $R_m^2 \cong R_m^4 \cong 25.4 \ \Omega$  obtained using  $t_{ac}^1 \cong t_{ac}^3 \cong 48 \text{ ms}$ , where the superscripts 1–4 represent the results



Fig. 5. Observed waveforms induced after the source  $v_{s1}$  is switched off at various times: (a)–(d) at time  $t_1$ , (e) and (f) at time  $t_2$ , and (g) and (h) at time  $t_3$ . The waveforms in (b) and (d) are measured using the smooth-and-filter function of the oscilloscope. The times, involving  $t_1$ ,  $t_2$  and  $t_3$ , are displayed in Fig. 3(a).



Fig. 6. Observed waveforms induced after the source  $v_{s2}$  is switched off at various times: (a) and (b) at time  $t_4$ , (c) and (d) at time  $t_6$ , and (e) near time  $t_4$ . The waveforms in (f) indicates the sources of  $v_{sqr}$  and  $v_{s2}$ . Both  $t_4$  and  $t_6$  are displayed in Fig. 3(b).

concerning the voltages  $v_{s1-4}$ , respectively. Then,  $R_d \cong 190 \text{ M}\Omega$  and  $R_{d1} \cong 9.5 \text{ M}\Omega$  estimated by  $R_d = R_{d1}R_{\text{scope}}/(R_{\text{scope}} - R_{d1})$  and  $R_{d1} \cong t_{dc}/5(C_d + C_{\text{scope}})$ , respectively, and  $t_{dc} \cong 180$  ms obtained from Table 3. Here,  $R_{\text{scope}}$  and  $C_{\text{scope}}$  represent the impedance of the probe and oscilloscope, as shown in Fig. 8(b);  $R_{\text{scope}} \cong 10 \text{ M}\Omega$  and  $C_{\text{scope}} \cong 3.73 \text{ pF}$  measured by the impedance analyzer. Therefore,  $R_{\text{scope}}(\ll R_d)$  influences the DC time and the

estimated result of  $R_d$  at the open state, but  $C_{\text{scope}} (\ll C_d)$  does not.

#### 5.3. The maximum amplitudes and drive conditions

From Table 4, the magnitude of  $V_{\text{oc,max}}^1$  ( $\cong 650$  V) exceeds that of  $V_{\text{oc,max}}^2$  ( $\cong 140$  V), and the magnitude of  $A_{\text{oc,max}}^1$  ( $\cong 65$ ) exceeds that of  $A_{\text{oc,max}}^2$  ( $\cong 14$ ); the mag-



Fig. 7. Observed waveforms induced after the sources  $v_{s3}$  and  $v_{s4}$  are switched off at various times: (a) and (b) at time  $t_1$ , and (c) and (d) at time  $t_3$ .

Table 3 Magnitude of the measured times and estimated results

From figures	The mea- sured times		Estima	ated resul	ts	
	t <sub>ac</sub> (ms)	t <sub>dc</sub> (ms)	$\frac{R_{\rm m}}{(\Omega)}$	$R_{\rm dl}$ (M $\Omega$ )	$R_{\rm d}$ (M $\Omega$ )	$Q_{\rm m}$
5 and 7(a, b) 6 and 7(c, d)	46 48	180 180	26.5 25.4	9.5 9.5	190 190	844 881

nitude of  $V_{\text{oc,max}}^3$  ( $\cong$  1080 V) exceeds that of  $V_{\text{oc,max}}^4$ ( $\cong$  330 V), and the magnitude of  $A_{\text{oc,max}}^3$  ( $\cong$  36) exceeds that of  $A_{\text{oc,max}}^4$  ( $\cong$  11). Here,  $V_{\text{oc,max}}^i = \max \{V_{\text{oc,max}}^{i(+)}, |V_{\text{oc,max}}^{i(-)}|\}$  and  $A_{\text{oc,max}}^i = V_{\text{oc,max}}^i/V_{\text{si}}$  for i = 1-4, where  $V_{\text{oc,max}}^{(+)}(= b + f, b > 0, f > 0)$  and  $V_{\text{oc,max}}^{(-)}(= -|b| - f, b < 0, f > 0)$  represent the positive and negative maximum amplitude of  $v_{\text{oc,max}}$  are obtained at the opencircuited time  $t_1$  and  $t_3$  in Fig. 3(a), respectively, and  $V_{\text{oc,max}}^{3(+)}$  and  $V_{\text{oc,max}}^{3(-)}$  are also. For the above resonance operations,  $V_{\text{oc,max}}^{3(+)}$  and  $V_{\text{oc,max}}^{2(-)}$  are obtained at the opencircuited time  $t_4$  and  $t_6$  in Fig. 3(b), respectively, and  $V_{\text{oc,max}}^{4(-)}$  and  $V_{\text{oc,max}}^{2(-)}$  are also. Regarding the sign of b, b > 0, obtained at the opencircuited time in the interval between  $t_1$  and  $t_2$  in Fig. 3(a), as shown in Figs. 5(a)–(d) and 7(a), or between  $t_4$ and  $t_5$  in Fig. 3(b), as displayed in Figs. 6(a, b) and 7(c). Then, b < 0, obtained at the open-circuited time in the interval between  $t_2$  and  $t_3$ , as shown in Figs. 5(g, h) and 7(b), or between  $t_5$  and  $t_6$ , as displayed in Figs. 6(c, d) and 7(d). Moreover,  $b \approx 0$ , obtained at the open-circuited time  $t_2$  or  $t_5$ , as shown in Fig. 5(e,f).

#### 6. Discussion

The following observations prove that  $L_{\rm m}$ ,  $C_{\rm m}$  and  $C_{\rm d}$  are insensitive to the vibration amplitude of the vibrator. (1) The oscillated frequency (about 30.12 kHz in Fig. 6(e)) almost equals the anti-resonant frequency (around 30.12 kHz). (2) The maximum voltage of the transient response and the maximum current of the vibrator are obtained by the source at the resonant frequency (about 29.22 kHz). (3) The magnitude of  $R_{\rm m}^{\rm A}$  ( $\cong$  174.6  $\Omega$ ) exceeds that of  $R_{\rm m}^{\rm 1}$  ( $\cong R_{\rm m}^{\rm 3} \cong 26.5 \Omega$ ) or  $R_{\rm m}^{\rm 2}$  ( $\cong R_{\rm m}^{\rm 4} \cong 25.4 \Omega$ ), according to Table 3. (4) From



Fig. 8. Modification of equivalent circuit: (a) a diagram of the measuring system, (b) equivalent circuits of the vibrator and the measuring system, and (c) a modified equivalent circuit.

Table 4 Magnitude of the maximum amplitudes

Items From figures		AC voltages		At open-circuited	Maximum amplitudes	
		Amplitude $V_{\rm s}$ (V)	Frequency $f_s$ (kHz)	time $t_{1-6}$ in Fig. 3	Magnitude V <sub>oc,max</sub> (V)	Sign (+ or -)
$V_{\rm oc.max}^{1(+)}$	5(c)	10	29.22	$t_1$	620	+
$V_{\rm oc,max}^{1(-)}$	5(g)			<i>t</i> <sub>3</sub>	650	-
$V_{\rm oc,max}^{2(+)}$	6(a)		29.41	$t_4$	140	+
$V_{ m oc,max}^{2(-)}$	6(c)			$t_6$	140	-
$V_{\rm oc,max}^{3(+)}$	7(a)	30	29.22	$t_1$	1080	+
$V_{\rm oc,max}^{3(-)}$	7(b)			$t_3$	1080	-
$V_{\rm oc,max}^{4(+)}$	7(c)		29.41	$t_4$	320	+
$V_{\rm oc,max}^{4(-)}$	7(d)			$t_6$	330	_

Table 5

Comparison of open-circuit test and impedance analyzer methods

Methods	Magnitude of $R_{\rm m}$ ( $\Omega$ )	AC times $t_{ac}$ (ms)	Maximum amplitudes		AC voltages	AC voltages	
			$V_{\rm oc}, \max$ (V)	$A_{\rm oc,max}$	Amplitude V <sub>s</sub> (V)	Frequency f <sub>s</sub> (kHz)	
Open-circuit test	26.5 (15.2%)	46	650 1080	65 36	10 30	29.22	
Impedance analyzer	174.6 (100%)	7.0	93 164	9.3 5.5	10 30	29.22	

Table 5, the magnitude of  $V_{\text{oc,max}}^1$  ( $\cong 650$  V) exceeds largely that of  $V_{\text{oc,max}}^{1A}$  ( $\cong 93$  V) and the magnitude of  $V_{\text{oc,max}}^3$  ( $\cong 1080$  V) exceeds greatly that of  $V_{\text{oc,max}}^{3A}$  ( $\cong 164$ V). Here,  $V_{\text{oc,max}}^{1A}$  and  $V_{\text{oc,max}}^{3A}$  are calculated using  $R_{\text{m}}^{\text{A}} =$ 

174.6  $\Omega$ . Then, the magnitude of  $R_m^1 \cong 26.5 \Omega$ ) is about 15.2 percentages of the magnitude of  $R_m^A \cong 174.6 \Omega$ ). These observations (1)–(4) prove that the magnitude of  $R_m$  is sensitive to the vibration amplitude. Furthermore,

the open-circuit method applied to estimate the values of  $R_{\rm m}$  and  $R_{\rm d}$  is more effective than the published methods [1,2], and the impedance of the measuring system influences the estimation of  $R_{\rm d}$ .

# 7. Conclusions

The waveforms of the transient response are obtained after about 2 ms at the open-circuited time due to the delay of the switch system, and the maximum amplitude of waveforms is obtained at the open-circuited time  $t_1$  or  $t_3$  in Fig. 3(a). Then, the maximum amplitude is about 65 times the voltage of the source at the resonant frequency, and used to quantify the electrical shock. Moreover, the DC time exceeds the AC time; the AC and DC times are used to estimate  $R_{\rm m}$  and  $R_{\rm d}$ , respectively, and the estimated result of  $R_{\rm m}^{\rm 1}$  is about 15.2 percentages of the magnitude of  $R_{\rm m}^{\rm A}$ . Thus the resistances in the equivalent circuit are sensitive to the amplitude of vibration, but the inductance and capacitances are not. Furthermore, the impedance of the measuring system influences the DC time and the estimation of  $R_d$ . The refined circuit will be applied to accurately estimate the real losses and the motional current.

#### Appendix A

From Fig. 1(a), the terminal voltage,  $V_{oc}(S)$ , is derived using the Laplace transformation and the Kirchoff current law, and then expressed by,

$$V_{\rm oc}(S) = L\{v_{\rm oc}(t)\} = \frac{b}{S + \alpha_{\rm dc}} + \frac{c(S + \alpha_{\rm ac})}{(S + \alpha_{\rm ac})^2 + \beta^2} + \frac{d(\beta)}{(S + \alpha_{\rm ac})^2 + \beta^2}$$
(A.1)

where

$$\begin{split} \alpha_{\rm dc} &= \frac{1}{R_{\rm d}C_{\rm d}}, \quad \alpha_{\rm ac} = \frac{R_{\rm m}}{2L_{\rm m}} \gg \alpha_{\rm dc}, \\ \beta &= \sqrt{\frac{1}{L_{\rm m}} \left(\frac{1}{C_{\rm m}} + \frac{1}{C_{\rm d}}\right) - \frac{R_{\rm m}^2}{4L_{\rm m}^2}}, \\ b &= V_{\rm od} + \frac{(\alpha_{\rm dc}I_0/C_{\rm d}) + (V_{\rm om}/L_{\rm m}C_{\rm d})}{\alpha_{\rm dc}^2 - 2\alpha_{\rm dc}\alpha_{\rm ac} + (1/L_{\rm m}C_{\rm m})}, \quad c = V_{\rm od} - b \end{split}$$

and

$$d = \frac{\left(\alpha_{\rm dc} - \alpha_{\rm ac}\right)\frac{V_{\rm om}}{L_{\rm m}C_{\rm d}} + \frac{I_0}{C_{\rm d}}\left(\alpha_{\rm dc}\alpha_{\rm ac} - \frac{1}{L_{\rm m}C_{\rm m}}\right)}{\beta\left(\alpha_{\rm dc}^2 - 2\alpha_{\rm dc}\alpha_{\rm ac} + \frac{1}{L_{\rm m}C_{\rm m}}\right)}.$$

Generally,

$$b \approx V_{\rm od} + \frac{C_{\rm m}}{C_{\rm d}} (\alpha_{\rm dc} L_{\rm m} I_0 + V_{\rm om}),$$
  
$$d \approx \frac{1}{\beta C_{\rm d}} (C_{\rm m} (\alpha_{\rm dc} - \alpha_{\rm ac}) V_{\rm om} - I_0),$$
  
$$\beta \approx \sqrt{\frac{1}{L_{\rm m}} \left(\frac{1}{C_{\rm m}} + \frac{1}{C_{\rm d}}\right)}.$$

Furthermore, the voltage,  $v_{oc}(t)$ , is further calculated by the inverse Laplace transformation, and expressed by,

$$v_{\rm oc}(t) = L^{-1}\{V_{\rm oc}(S)\} \cong b \mathrm{e}^{-\alpha_{\rm dc}t} + f \mathrm{e}^{-\alpha_{\rm ac}t} \cos(\beta t - \varphi)$$
(A.2)

where  $f = \sqrt{c^2 + d^2}$  and  $\varphi = \arctan\left(\frac{d}{c}\right)$ .

#### References

- S. Hirose, New method for measuring mechanical vibration loss and dielectric loss of piezoelectric transducer under high-power excitation, Jpn. J. Appl. Phys. 33 (1994) 2945–2948.
- [2] M. Umeda, K. Nakamura, S. Ueha, The measurement of highpower characteristics for a piezoelectric transducer based on the electrical transient response, Jpn. J. Appl. Phys. 37 (1998) 5322– 5325.
- [3] G.C. Hsieh, C.H. Lin, J.M. Li, Y.C. Hsu, A study of seriesresonant DC/AC inverter, IEEE Trans. Power Electron. 11 (1996) 641–652.