Study on the multifrequency Langevin ultrasonic transducer

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Langevin ultrasonic transducers are widely used in high-power ultrasonics and underwater sound. In ultrasonic cleaning, a matching metal horn rather than a metal cylinder is used as the radiator in order to enhance the radiating surface and improve the acoustic matching between the transducer and the processed medium. To raise the effect of ultrasonic cleaning, the standing wave in the cleaning tank should be eliminated. One method to eliminate the standing wave in the tank is to use the multifrequency ultrasonic transducer. In this paper, the Langevin ultrasonic horn transducer, with two resonance frequencies, is studied. The transducer consists of two groups of piezoelectric ceramic elements: the back metal cylinder, the middle metal cylinder and the front matching metal horn. The vibrational modes of the transducer are analysed, and resonance frequency equations of the transducer in the half-wave and the all-wave vibrational modes are derived. According to the resonance frequency equations, transducers with two resonance frequencies are designed and made. The resonance frequencies, the effective electromechanical coupling coefficients and the equivalent electric impedances of the transducers are measured. It is shown that the measured resonance frequencies are in good agreement with the computed results, and the transducer can be excited to vibrate at two resonance frequencies, which correspond to the half-wave and the all-wave vibrational modes of the transducer.

Keywords: transducers; multifrequency Langevin ultrasonic transducers; resonance frequency; ultrasonic cleaning; vibrational modes

As new performance demands are placed on advanced electronic, optical and mechanical products, the need arises for different elements that have clean and non-destructive surfaces. This presents a new subject for ultrasonic cleaning. A solution to the problem is to utilize the technology of multifrequency ultrasonic cleaning.

In traditional ultrasonic cleaning[1–4], the transducer is operated on one resonance frequency, so that a standing wave is built in the cleaning tank. At the antinode of sound pressure, the effect of ultrasonic cleaning is perfect, while at the node of sound pressure, the effect of ultrasonic cleaning is not good; therefore, the elements to be cleaned should be mechanically moved to the antinode. This is somewhat cumbersome and so it is difficult to improve the cleaning quality of the elements. However, in the case of multifrequency ultrasonic cleaning, the situation is different. As there exist waves with different frequencies in the cleaning tank, the standing wave is destroyed and the difference between the nodes and antinodes is decreased. Therefore, mechanical movement is no longer needed and the cleaning quality can be improved. However, to produce ultrasound with different frequencies in the tank, a multifrequency ultrasonic transducer should be used.

In this paper, the Langevin ultrasonic horn transducers with two resonance frequencies are analysed, the vibrational modes are studied, and the resonance frequency equations are derived, which can be used to design ultrasonic transducers with different dimensions and resonance frequencies.

Theoretical analysis of the Langevin ultrasonic horn transducer with two resonance frequencies

Figure 1 shows the transducer with two resonance frequencies. In this figure, L_1, L_2 and L_3 are the lengths of the back metal cylinder, the middle metal cylinder and the front metal horn, respectively, each having a circular cross-section. Two groups of piezoelectric ceramic driving elements, whose length is L_0, are separated by the middle metal cylinder. To simplify the theoretical analysis, one-dimensional theory is used so that the lateral dimensions must be much smaller than the longitudinal dimensions. According to traditional experience, it is necessary that the lateral dimensions should be smaller than a quarter of the longitudinal wavelength. Based on the one-dimensional equivalent circuit of the longitudinal transducer, the equivalent circuit of the Langevin ultrasonic horn transducer with two resonance frequencies is shown in *Figure 2*, where Z_{BL} and Z_{FL} are the back and front load impedances of the transducer.

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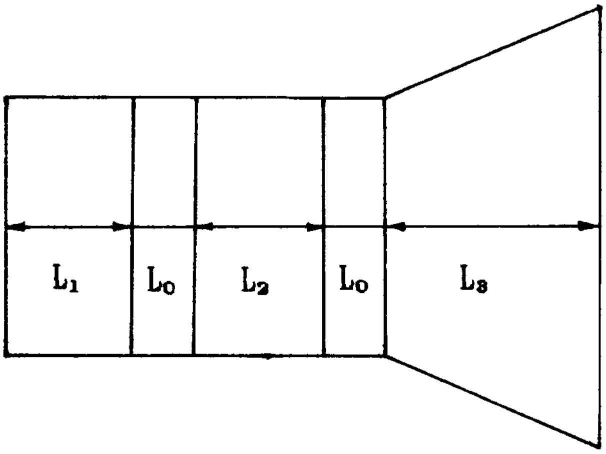

Figure 1 The geometrical diagram of the Langevin horn transducer with two resonance frequencies

As the external loads of the transducer are difficult to determine, it is the usual procedure to regard the transducer as unloaded. When the transducer is without external loads, $Z_{BL}=Z_{FL}=0$, the transducer vibrates freely. This is the case to be studied in the following analysis of the paper. R_0 and C_0 are the dielectric loss impedance and the clamped capacitance, n is the electromechanical transformation coefficient, Z_{11}, Z_{12}, Z_{13}, Z_{21}, Z_{22}, Z_{23}, Z_{31}, Z_{32}, Z_{33} and Z_{P1}, Z_{P2}, Z_{P3} are the series and parallel impedances of the T-networks of the back metal cylinder, the middle metal cylinder, the front metal horn (conical shape) and the piezoelectric ceramic pile. Their expressions are as follows

$$Z_{11}=Z_{12}=\mathrm{j}Z_1\tan(k_1L_1/2) \quad (1)$$

$$Z_{13}=Z_1/[\mathrm{j}\sin(k_1L_1)] \quad (2)$$

$$Z_{21}=Z_{22}=\mathrm{j}Z_2\tan(k_2L_2/2) \quad (3)$$

$$Z_{23}=Z_2/[\mathrm{j}\sin(k_2L_2)] \quad (4)$$

$$Z_{31}=-\mathrm{j}Z_3[(S_{32}/S_{31})^{1/2}-1]/(k_3L_3)-\mathrm{j}Z_3\cot(k_3L_3)+\mathrm{j}Z_3(S_{32}/S_{31})^{1/2}/[\sin(k_3L_3)] \quad (5)$$

$$Z_{32}=\mathrm{j}Z_3(S_{32}/S_{31})^{1/2}[(S_{32}/S_{31})^{1/2}-1]-\mathrm{j}Z_3\cot(k_3L_3)(S_{32}/S_{31})+\mathrm{j}Z_3(S_{32}/S_{31})^{1/2}/[\sin(k_3L_3)] \quad (6)$$

$$Z_{33}=-\mathrm{j}Z_3(S_{32}/S_{31})^{1/2}/[\sin(k_3L_3)] \quad (7)$$

$$Z_{P1}=Z_{P2}=\mathrm{j}Z_0\tan(k_0L_0/2) \quad (8)$$

$$Z_{P3}=Z_0/[\mathrm{j}\sin(k_0L_0)] \quad (9)$$

where $Z_1=\rho_1C_1S_1$, $Z_2=\rho_2C_2S_2$, $Z_0=\rho_0C_0S_0$, $Z_3=\rho_3C_3S_{31}$, $k_1=\omega/C_1$, $k_2=\omega/C_2$, $k_0=\omega/C_0$, $k_3=\omega/C_3$. S_1, S_2, S_0, S_{31} and S_{32} are the cross-sectional areas, ρ_1, ρ_2, ρ_0, ρ_3 and C_1, C_2, C_0, C_3 are the densities and sound speeds, k_1, k_2, k_0 and k_3 are the wave-numbers. For the conical front metal horn, when the external force acting on its front surface is zero, its input mechanical impedance can be derived as

$$Z_h=\mathrm{j}\frac{Z_3}{k_3L_3F}\,\frac{k_3L_3-[1+k_3^2L_3^2F(F+1)]\tan(k_3L_3)}{\tan(k_3L_3)-k_3L_3(F+1)} \quad (10)$$

where $F=R_{31}/(R_{32}-R_{31})$, R_{31} and R_{32} are the radii of the small and large ends of the horn. Equation (10) can be rewritten as

$$Z_h=\mathrm{j}Z_3\tan(k_3L_{3e}) \quad (11)$$

where

$$\tan(k_3L_{3e})=\frac{k_3L_3-[1+k_3^2L_3^2F(F+1)]\tan(k_3L_3)}{k_3L_3F[\tan(k_3L_3)-k_3L_3(F+1)]} \quad (12)$$

L_{3e} is called the equivalent cylinder length of the front conical horn and Z_h can be regarded as the load impedance of the piezoelectric ceramic pile. From *Figure 2* and the above equations, we can obtain the resonance frequency equations of the horn transducer with two frequencies.

The resonance frequency equations of the transducer in the half-wave vibrational mode

In this case, the transducer can be regarded as a longitudinal vibrator of half a wavelength[5–8], its two ends are the displacement antinodes, and there is a displacement node in the transducer. Let the displacement node be located at the middle cylinder of the transducer, and divide the middle metal cylinder into two cylinders of L_{21} and L_{22}, then we have

$$L_2=L_{21}+L_{22} \quad (13)$$

The part before the displacement node can be regarded as a longitudinal transducer of a quarter wavelength and it consists of the metal cylinder of length L_{22}, the piezoelectric ceramic pile and the front metal horn. From *Figure 2*, its input mechanical impedance Z_{mf} can be obtained as

$$Z_{mf}=\mathrm{j}\frac{[Z_0Z_2\cot(k_2L_{22})\cot(k_0L_0)-Z_0Z_3\tan(k_3L_{3e})\times\cot(k_0L_0)-Z_2Z_3\cot(k_2L_{22})\tan(k_3L_{3e})-Z_0^2]}{2Z_0\tan(k_0L_0/2)+Z_3\tan(k_3L_{3e})-Z_2\cot(k_2L_{22})} \quad (14)$$

When Z_{mf} is equal to zero, its resonance frequency equation can be obtained as

$$(Z_3/Z_2)\tan(k_3L_{3e})\tan(k_2L_{22})+(Z_3/Z_0)\tan(k_3L_{3e})\times\tan(k_0L_0)+(Z_0/Z_2)\tan(k_0L_0)\tan(k_2L_{22})=1 \quad (15)$$

The quarter wavelength transducer after the displacement node consists of the metal cylinder of length L_{21}, the piezoelectric ceramic pile and the back cylinder, its input mechanical impedance Z_{mb} can be obtained as follows

$$Z_{mb}=\mathrm{j}\frac{[Z_0Z_2\cot(k_2L_{21})\cot(k_0L_0)-Z_0Z_1\tan(k_1L_1)\times\cot(k_0L_0)-Z_2Z_1\cot(k_2L_{21})\tan(k_1L_1)-Z_0^2]}{2Z_0\tan(k_0L_0/2)+Z_1\tan(k_1L_1)-Z_2\cot(k_2L_{21})} \quad (16)$$

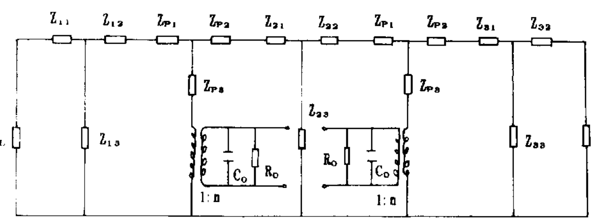

Figure 2 The equivalent circuit of the Langevin horn transducer with two resonance frequencies

Its resonance frequency equation can be derived according to the similar procedure

$$(Z_1/Z_2)\tan(k_1L_1)\tan(k_2L_{21})+(Z_1/Z_0)\tan(k_1L_1)\times\tan(k_0L_0)+(Z_0/Z_2)\tan(k_0L_0)\tan(k_2L_{21})=1 \quad (17)$$

When the resonance frequency of the transducer in half-wave vibrational mode is designated, the transducer can be designed and the dimensions can be calculated from the equations.

The resonance frequency equations of the transducer in the all-wave vibrational mode

In this case, apart from the two displacement antinodes at the ends of the transducer there is also a displacement antinode in the transducer. The transducer can be regarded as a combination of two transducers of half a wavelength. When the displacement antinode is located at the middle metal cylinder, it divides the middle cylinder into two cylinders of L_{a1} and L_{a2}, and we can get

$$L_2=L_{a1}+L_{a2} \quad (18)$$

The half wavelength transducer before the displacement antinode in the middle cylinder consists of the metal cylinder of length L_{a2}, the piezoelectric ceramic pile and the front metal horn. When the transducer is unloaded, the stress at the displacement antinode in the middle metal cylinder is equal to zero, therefore, the half wavelength transducer vibrates freely. From *Figure 2*, its input mechanical impedance Z_{af} can be obtained as

$$Z_{af}=j\frac{[Z_2Z_3\tan(k_3L_{3e})\tan(k_2L_{a2})-Z_0Z_3\tan(k_3L_{3e})\times\cot(k_0L_0)-Z_2Z_0\tan(k_2L_{a2})\cot(k_0L_0)-Z_0^2]}{2Z_0\tan(k_0L_0/2)+Z_3\tan(k_3L_{3e})+Z_2\tan(k_2L_{a2})} \quad (19)$$

When Z_{af} is equal to zero, the resonance frequency equation of the half wavelength transducer before the displacement antinode is obtained as

$$(Z_0/Z_2)\cot(k_0L_0)\cot(k_2L_{a2})+(Z_0/Z_3)\cot(k_0L_0)\cot(k_3L_{3e})+[Z_0^2/(Z_2Z_3)]\cot(k_2L_{a2})\cot(k_3L_{36})=1 \quad (20)$$

The half wavelength transducer after the displacement antinode consists of the metal cylinder of length L_{e1}, the piezoelectric ceramic pile and the back metal cylinder. Its input mechanical impedance can be obtained as

$$Z_{ab}=j\frac{[Z_2Z_1\tan(k_1L_1)\tan(k_2L_{a1})-Z_0Z_1\tan(k_1L_1)\times\cot(k_0L_0)-Z_2Z_0\tan(k_2L_{a1})\cot(k_0L_0)-Z_0^2]}{2Z_0\tan(k_0L_0/2)+Z_1\tan(k_1L_1)+Z_2\tan(k_2L_{a1})} \quad (21)$$

The resonance frequency equation of the half wavelength transducer after the displacement antinode is

$$(Z_0/Z_2)\cot(k_0L_0)\cot(k_2L_{a1})+(Z_0/Z_1)\cot(k_0L_0)\cot(k_1L_1)+[Z_0^2/(Z_2Z_1)]\cot(k_2L_{a1})\cot(k_1L_1)=1 \quad (22)$$

Equations (20) and (22) are the resonance frequency equations of the Langevin horn transducer in the all-wave vibrational mode. When the material parameters and dimensions are designated, the resonance frequency of the transducer in the all-wave vibrational mode can be computed.

In the above analysis, the resonance frequency equations of the Langevin horn transducer with two resonance frequencies are obtained, and they can be used to design the transducer with two frequencies for ultrasonic cleaning. According to the above analysis, the procedures for designing an ultrasonic transducer with two frequencies are as follows. (a) When the material parameters of the components of the transducer and the dimensions of the piezoelectric ceramic pile are given, designate the length of the back cylinder, the front horn and the resonance frequency of the transducer in the half-wave vibrational mode, from the resonance frequency equations (Equations (15) and (17)), such that the length of the middle cylinder can be determined, that is, $L_2=L_{21}+L_{22}$. (b) After procedure (a), the transducer is entirely determined, and the next procedure is to determine the resonance frequency of the transducer in the all-wave vibrational mode.

In the resonance frequency equations (Equations (20) and (22)), there are three unknowns. They are L_{a1}, L_{a2} and the resonance frequency of the transducer in the all-wave vibrational mode. As $L_2=L_{a1}+L_2$ has been determined in procedure (a) there are, therefore, two separate unknowns and they can be entirely determined from Equations (20) and (22). As Equations (20) and (22) are transcendental ones, the analytic solutions are difficult to obtain; therefore, numerical methods must be used. The procedures to solving Equations (20) and (22) are as follows. (1) Give a value of the resonance frequency of the transducer in the all-wave vibrational mode, and from Equations (20) and (22) and $L_2=L_{a2}+L_{a2}$, two values of L_{a1} or L_{a2} can be found. (2) Change the values of the resonance frequency of the transducer in the all-wave vibrational mode until the two values of L_{a1} or L_{a2} are equal.

Experiments

To verify the design theory of the Langevin horn transducer with two resonance frequencies, three Langevin horn transducers were designed and made according to the design theory of this paper. The material of the middle cylinder and the front horn is hard aluminium, the material of the back cylinder is steel. The piezoelectric ceramic element is PZT-4, its thickness, outer and inner diameters are 4.4×10^{-3}(m), 15.5×10^{-3}(m) and 38.0×10^{-3}(m). In the transducer, there are two groups of piezoelectric ceramic elements, and every group consists of two elements. The resonance and anti-resonance frequencies of the transducer in the half-wave and the all-wave vibrational modes, and the equivalent electric impedances are measured using the transmission line method, while the effective electromechanical coupling coefficient K_{eff} is calculated according to

$$K_{eff}^2=1-(f_r/f_a)^2 \quad (23)$$

where f_r and f_a are the resonance and anti-resonance frequencies of the transducer. The measured results are shown in *Tables 1* and *2*, where f_{r1} and f_{r2} are the calculated resonance frequencies of the transducer in the half-wave and the all-wave vibrational modes, f_{rm1}, f_{am1} and f_{rm2}, f_{am2} are the measured frequencies of the transducers in the half-wave and the all-wave vibrational modes, R_1 and R_2 are the measured equivalent electric