Laboratory #1 – Chap 3

Linear System Response: general case

Objectives

- Understand the difference and the relationship between a step and impulse response.
- Determine the limits of validity of an approximated impulse response.

Theory

The idealized impulse function has some of the following properties :

$$\delta(t) = \lim_{a \to 0} \frac{u(t) - u(t-a)}{a} \qquad \delta(t) = \frac{d}{dt} u(t)$$

R-C first order low-pass filter:

$$H(j\omega) = \frac{1}{1 + j\omega\tau} \quad ==> \quad H(s) = \frac{1}{1 + s\tau} = \frac{\frac{1}{\tau}}{s + \frac{1}{\tau}} \quad ==> \quad h(t) = \frac{1}{\tau} \cdot e^{-\frac{t}{\tau}} \cdot u(t)$$

Function	Laplace Transform	Fourier Transform
f(t)	**F(s)**	**F(jω)**
$d\,f(t)/dt$	$s\,F(s) - f(0^+)$	$j\omega\,F(j\omega)$
$\int_{-\infty}^{t} f(\tau)\,d\tau$	$F(s)/s$	$F(j\omega)/j\omega$
$\delta(t)$	1	1
1	1/s	-
e^{-at}	1/(s+a)	1/(jω+a)

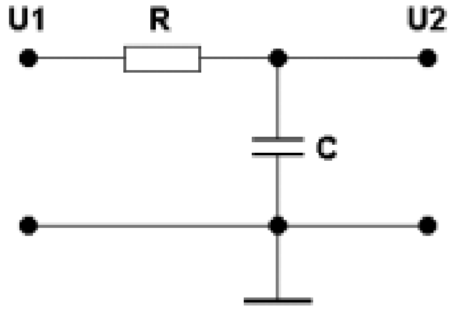
Step Response: $Y(s) = H(s)/s \;==>\; y(t) = \int h(\tau)\,d\tau$

Impulse Response: $h(t) = d\,y(t)/dt$

==> ***The linear system impulse response is the derivative of its step response***

Procedure

1) Build the following R-C circuit:



With: R = Ω , C = nF ==> $\tau = R \cdot C =$ s

2) u1(t) is a square-wave with $u1_{low} = 0V$ and $u1_{high} = 1V$, 50% duty cycle.
Choose an appropriate frequency in order to make sure that the period is at least equal to **$10 \cdot \tau$.** Verify that u2(t) is equal to the integral of this circuit impulse response! i.e:

$$u2(t) = 1 - e^{-t/\tau} \quad \text{(rising edge at } t = 0\text{)}$$
$$u2(t) = e^{-(t-to)/\tau} \quad \text{(falling edge at } t = to\text{)}$$

3) What happens if the square-wave frequency is not chosen correctly?

4) u1(t) is a rectangle pulse with $u1_{low} = 0V$ and $u1_{high} = 1V$.

ON-time: $t_{on} = \tau / 10$ OFF-time: $t_{off} = 10 \cdot \tau$

Verify that u2(t) is approximately equal to the impulse response of this circuit

Remember the scaling factor!!!

5) What happens if t_{on} and t_{off} are not chosen correctly ?
Consider several situations.

Laboratory #2 – Chap 3

Linear System Response: general case

Objectives

- Apply the Laplace Transform theory in a practical example.
- Understand the relationship between a filter bandwidth and its rise and/or fall time (transient mode)

Theory

From Laplace Transform theory: $u2(t) = L^{-1}[L[u1(t)]\cdot L[h(t)]] = L^{-1}[U1(s)\cdot H(s)]$

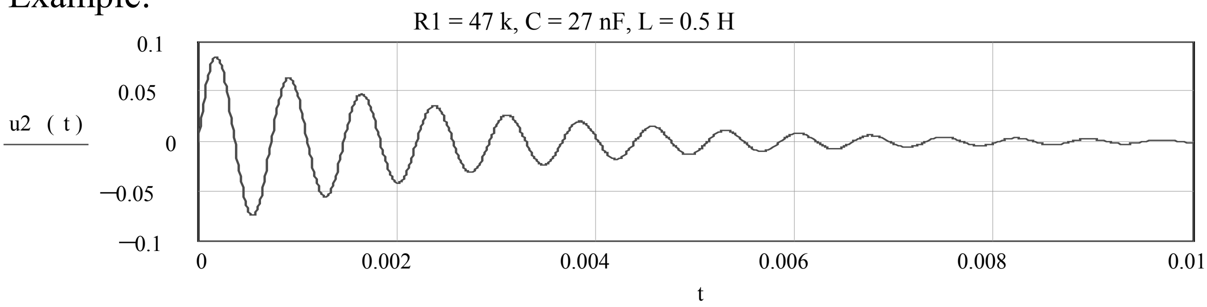
h(t): second order band-pass filter (Rp → ∞), u1(t): unit step: => U1(s) = 1/s

$$H(s)=\frac{\dfrac{S\cdot L\cdot\frac{1}{S\cdot C}}{S\cdot L+\frac{1}{S\cdot C}}}{R1+\dfrac{S\cdot L\cdot\frac{1}{S\cdot C}}{S\cdot L+\frac{1}{S\cdot C}}}=\frac{\frac{1}{C\cdot R}\cdot s}{s^2+\frac{s}{C\cdot R1}+\frac{1}{L\cdot C}} \quad => \quad Y(s)=\frac{1}{s}\cdot H(s)=\frac{\frac{1}{C\cdot R}\cdot s}{s^2+\frac{s}{C\cdot R1}+\frac{1}{L\cdot C}}$$

From Laplace Transform table: $k\cdot\frac{\omega o}{\left(s^2+a\right)^2+\omega o^2}$ => $k\cdot e^{-a\cdot t}\cdot\sin(\omega o\cdot t)=k\cdot e^{\frac{-t}{\tau}}\cdot\sin(\omega o\cdot t)$

After some algebra: $k=\frac{1}{\sqrt{\frac{C\cdot R1^2}{L}-\frac{1}{4}}}$ $a=\frac{1}{2\cdot C\cdot R1}$ $\tau=2\cdot C\cdot R1$ $\omega o=\sqrt{\frac{1}{L\cdot C}-\frac{1}{4\cdot C^2\cdot R1^2}}$

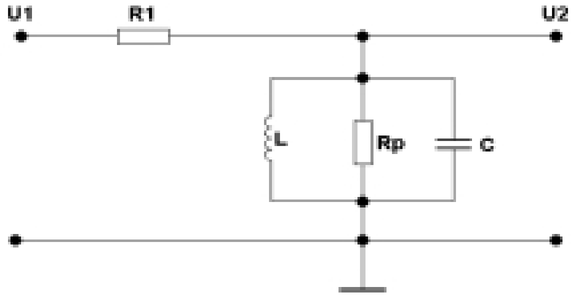
Example:



k = 0.092 τ = 2.53 ms fo = 1.37 kHz

Procedure

1) Build the following second order R-L-C band-pass filter:



R1 = kΩ L = H C = nF ==> fo = kHz, Bw ≈ 1/(2·π·Req·C) = Hz (with Q = fo/Bw > 5)

2) u1(t) is a square-wave with $u1_{low}$= 0V and $u1_{high}$= 1V, 50% duty cycle. Choose an appropriate frequency in order to make sure that the period is at least equal to **5/Bw**. Measure ***k***, $\boldsymbol{\tau}$ and ***fo*** and compare your results with the theoretical values.

3) What happens if the square-wave frequency is not chosen correctly?

4) With the same signal generator parameters, divide R1 by 3. What happens?

5) With the same signal generator parameters, divide R1 by 10. What happens?

6) Impulse response measurement: From theory, we know that:

$$h(t) = \frac{d}{dt} y_{step}(t) = k \cdot e^{\frac{-t}{\tau}} \cdot \left(\omega o \cdot \cos(\omega o \cdot t) - \frac{1}{\tau} \cdot \sin(\omega o \cdot t) \right)$$

if $\omega o > 10 \cdot \frac{1}{\tau}$ => $h_{approx}(t) = k \cdot \omega o \cdot e^{\frac{-t}{\tau}} \cdot \cos(\omega o \cdot t)$

u1(t) is a rectangle pulse with $u1_{low}$= 0V and $u1_{high}$= 1V.

ON-time: t_{on} = 1 / (20 · fo) OFF-time: t_{off} = 5 · Bw

Verify that u2(t) is approximately equal to the impulse response of this circuit

Remember the scaling factor !!!

7) What happens if t_{on} and t_{off} are not chosen correctly ?
Consider several situations.

8) Apply a FSK signal (VCF input of your function generator driven by a square-wave of appropriate amplitude and frequency!) to your band-pass filter. Determine the fastest data rate allowing data recovery. Conclusion?

Choose: $freq_{high}$ ≈ 1.5 $freq_{low}$, $freq_{high}$ = fo

Laboratory #3 – Chap 3

Linear System Response: general case

Objectives

- Understand multiplication in the time domain and the corresponding convolution in the frequency domain.
- Verify that the product of two periodic signals generates sums and differences of frequencies contained in the periodic signals.

Theory

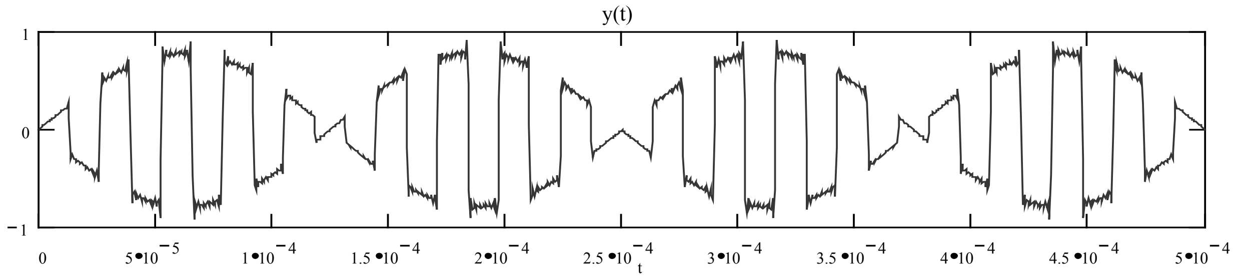
$y(t) = x1(t) \cdot x2(t)$

$x1(t) = A1 \cdot \sin(2 \cdot \pi \cdot freq_1 \cdot t)$,   x2(t): symmetrical square-wave of frequency $freq_2$

$$y(t) = A_1 \cdot \sin(2 \cdot \pi \cdot freq_1 \cdot t) \cdot A_2 \cdot \sum_{n=1}^{10} \frac{1}{2 \cdot n - 1} \cdot \sin\left[2 \cdot \pi \cdot freq_2 \cdot (2 \cdot n - 1) \cdot t\right]$$

$A_1 = 1$ $A_2 = 1$

$freq_1 = 4 \cdot 10^3$ $freq_2 = 3.8 \cdot 10^4$



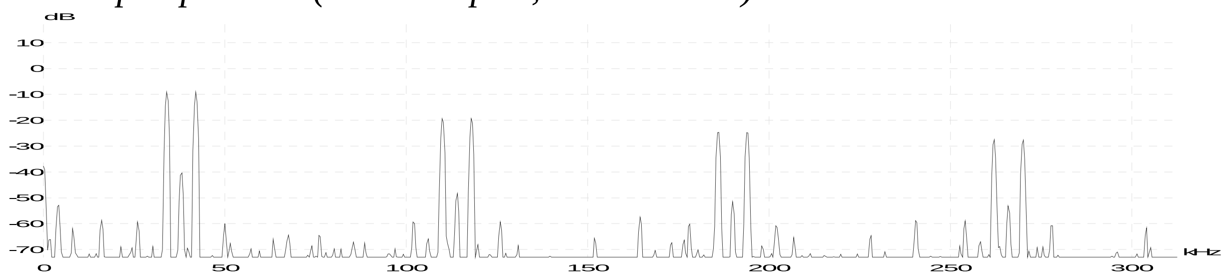
Time domain analysis:

Reminder: sin(a) sin (b) = 0.5 [-cos(a+b) + cos(a-b)]

Frequency components of y(t):

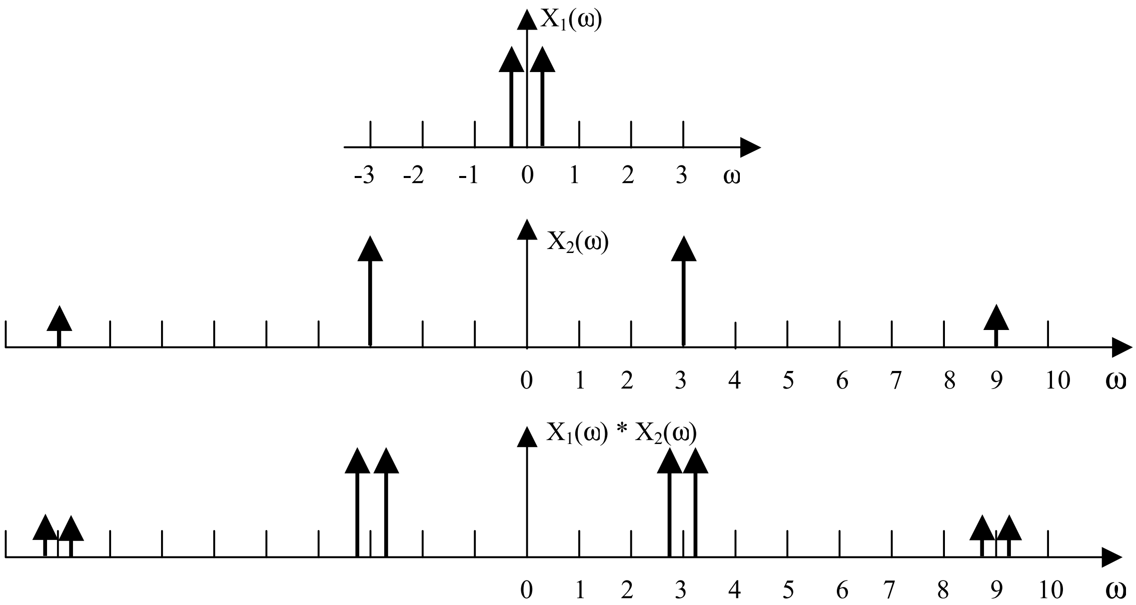
$1 \cdot 38$ kHz ± 4 kHz	relative amplitude:	1
$3 \cdot 38$ kHz ± 4 kHz		1/3
$5 \cdot 38$ kHz ± 4 kHz		1/5
$m \cdot 38$ kHz± 4 kHz		1/m

PicoScope Spectrum (4096 samples, linear scales)



Frequency domain analysis (convolution):

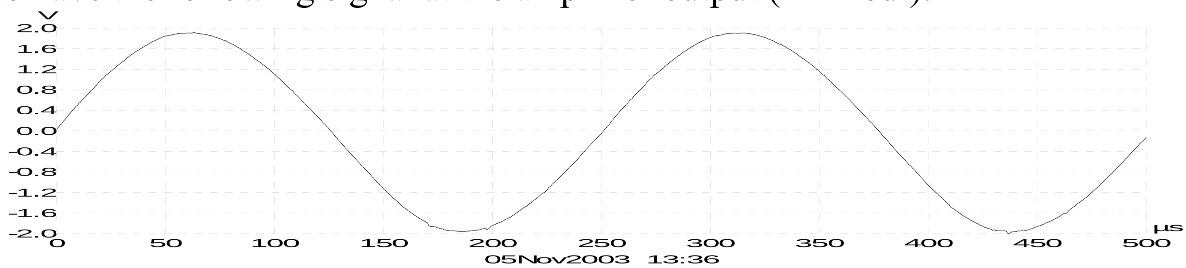
Concept:



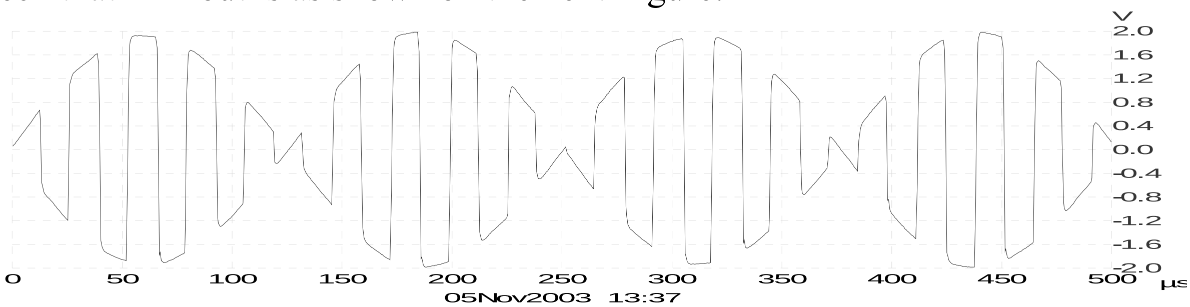
Procedure (with AMPMIX board)

Board extra-gain: 0 dB, Band-pass filter: OFF

Input a 4 kHz sine-wave to the AMPMIX board (Input). Adjust its amplitude so as to have the following signal at the amplifier output (AMPout):



Check that MIXout is as shown on the next Figure:



Use “PicoScope Spectrum Analyser” (No of spectrum bands: 4096, 1.5 MHz) to compare your display with the theory.

Laboratory #4 – Chap 3

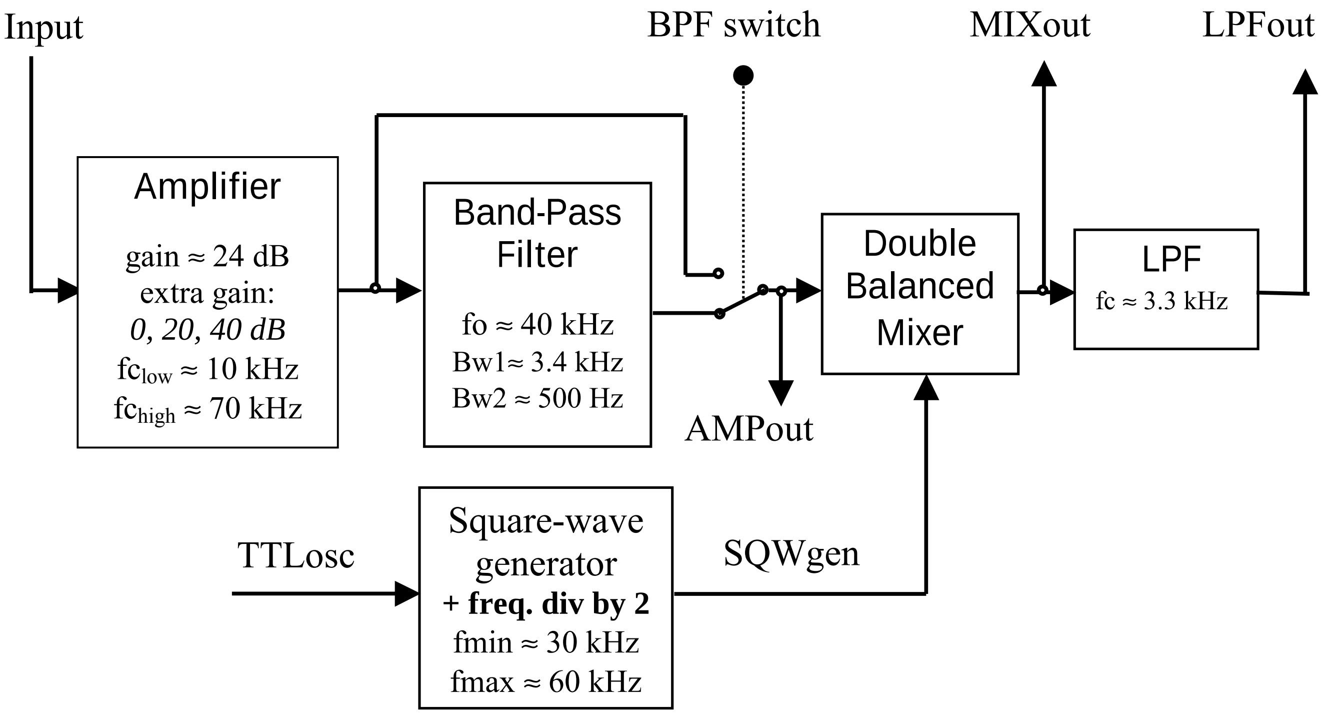
Linear System Response: general case

Objectives

- Understand *″Frequency down-conversion″*.
- Verify that *″undesired frequencies (e.g. mirror image)″* can also convert down to *base-band.*

Theory

″AMPMIX″ board bloc diagram:



Power supply: 5 (8 mA) to 18V (20 mA), Input impedance: 5 kΩ (AC coupling)

Amplifier: $fc_{low} \approx 10$ kHz, $fc_{high} \approx 70$ kHz
gain: ≈ 24 dB, additional gain: + 20 dB or + 40 dB

Band-Pass filter: Second order, resonant frequency adjustable around 40 kHz.
Bandwidth: *Wide* (3.4 kHz) or *Narrow* (≈ 500 Hz)

Square-Wave generator: Tunable fmin ≈ 30 kHz, fmax ≈ 60 kHz
External frequency control (TTLosc), $SQWgen_{freq.} = TTLosc_{freq}/2$

Double balanced mixer: MIXout(t) = AMPout(t) · SQWgen(t), i.e. AMPout multiplied by **± 1**

LPF: Three cascaded R-C low-pass filters

Procedure (with AMPMIX board)

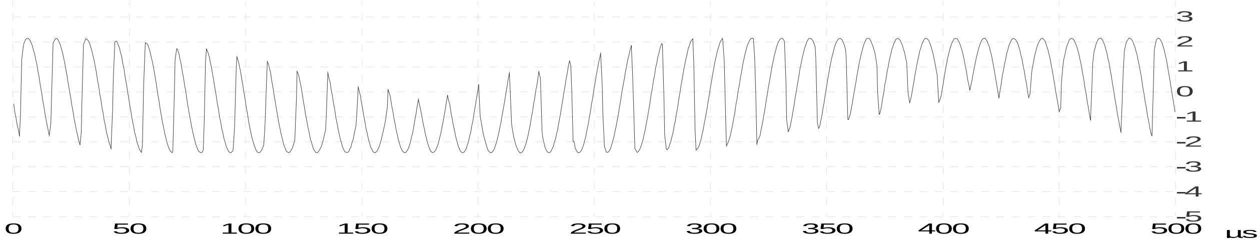
Part 1:

Input: sine-wave - 40 kHz - 100 mV peak-to-peak

Extra gain: *OFF* BPF: *OFF* SQWgen: *Local (≈ 38 kHz)*

Verify:

- AMPout: ″clean″ 40 kHz sine-wave (i.e. no saturation), ≈ 1.6V pp
- MIXout: must look as follows:



- LPFout: ″clean″ 2 kHz sine-wave, amplitude: ?

- Change the input sine-wave frequency in order to determine the output LPF cut-off frequency.
- Verify the ″mirror″ frequency concept (i.e. input signal frequency ≈ 36 kHz).
- Verify that if the input signal frequency is 2 kHz higher (or lower) than a local oscillator harmonic″, then down-conversion does occur and LPFout is also a 2 kHz sine-wave. What can you say of LPFout signal amplitude?

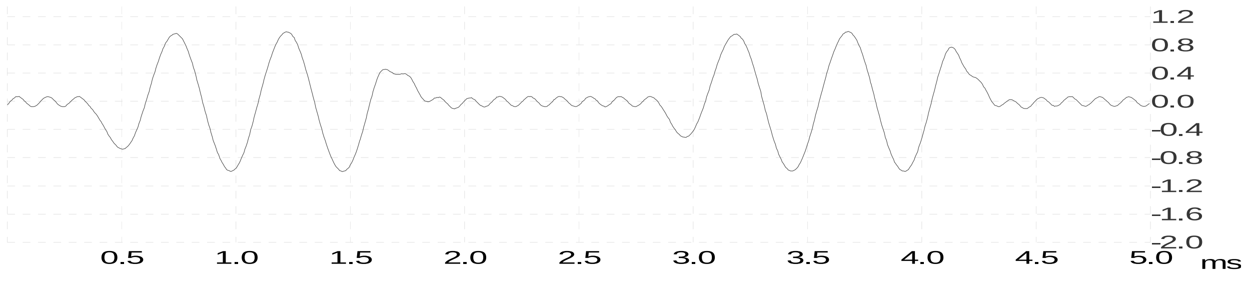
Part 2:

Input: FSK (freq$_L$: 30 kHz and freq$_H$: 40 kHz), 1000 bits/s - 100 mV pp

Extra gain: *OFF* BPF: *OFF* SQWgen: *Local (≈ 38 kHz)*

Verify:

- AMPout: ″clean″ 30 kHz and 40 kHz sine-wave (i.e. no saturation)
- LPFout looks as follows:



- Explain why!
- Slowly decrease freq$_H$. What happens?

Laboratory #5 – Chap 3

Linear System Response: general case

Objectives

- Understand..... Band-Pass Filter rise-time
- Prove....

Theory

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Procedure

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