

2.3.3. Macro-Cosmological Matter-Waves and Gravitation

In case of micro-universe of atoms and elementary particles, de Broglie matter waves are manageable using the following relations: $\tilde{E} = hf = E_k$, $\lambda = h/p$, $u = \lambda f = \omega/k = E_k/p$, $v = d\omega/dk = dE_k/dp$. Let us now try to construct or exercise what could be the macro-universe equivalent to de Broglie matter-waves concept. The idea here is to show that planets and similar macro objects are also respecting certain periodicity and “macro matter-waves packaging rules”, like de Broglie matter waves in a micro universe (see the introduction to such concept given by equations (2.11.5)-(2.11.9), (2.11.9-1)-(2.11.9-4) and (2.9.5-1)-(2.9.5-5)).

The best for exercising such brainstorming is to start from the Kepler’s third law (of planetary orbital motions), which is also applicable to all satellite motions around certain planet or big mass. Let us temporarily focus our attention only to pure circular rotations, where radius of rotation is r , in order to use simpler mathematical expressions. Kepler’s third law is showing that period T of a small-mass m , which presents planetary (or satellite) orbiting around a big mass $M \gg m$ (or its sun), is given by (2.11.10),

$$T^2 = \left(\frac{4\pi^2}{G(M+m)} \right) r^3 \cong \left(\frac{4\pi^2}{GM} \right) r^3 . \quad (2.11.10)$$

We will later need the expression for a maximal orbital or escape speed v_e (when planet, rocket or satellite would start escaping its stationary circular orbit), which can be found as (2.11.11),

$$v_e = \sqrt{\frac{2GM}{r}} . \quad (2.11.11)$$

Kepler laws are also showing the intrinsic tendency of (mutually approaching) motional masses to eventually stabilize in a form of rotational, orbital motions (see introductory elaborations in this chapter around equations (2.4-11) – (2.4-17)), and for stability of certain orbital motion, the principal request is that its total orbital momentum is conserved (meaning constant). If this were not the global tendency, our universe would collapse in a process of permanent masses agglomeration. We also know that the conservation of orbital and spin moments is valid on a micro-world scale. The major tendency valid for micro and elementary particles is also to create stable orbital motions, based on balancing between involved attractive forces with repulsive centrifugal forces (see new trends in modeling atoms and elementary particles in [16] to [22], Bergman, Lucas, Kanarev and others). Orbital, planetary motions are in the same time inertial (uniform) motions, which are coincidentally conserving their linear and orbital moments and potentially (also still hypothetically) hosting standing matter waves formations, as shown in (2.9.1). In addition, consequences of stable, standing waves formations (which are also directly related to periodical motions) are various energy, spin and orbital momentum quantizing.

Let us now attempt to show that (like in case of de Broglie matter waves applied on Bohr’s hydrogen, planetary atom model) a circular planetary (or satellite) orbit, or its perimeter, is susceptible to host some kind of gravitational (or inertial), orbital

standing matter-wave. Such macro matter wave should have orbital frequency f_o , wavelength $\lambda_o = \frac{2\pi r}{n}$, $n=1,2,3\dots$, group or orbital speed v , and associated phase speed $u = \lambda_o f_o$. Similar conceptualization is already initiated earlier in this chapter around equations (2.9.1), (2.9.2), (2.11.5), and shown applicable (in cases of vortex shedding flowmeters) in the chapter 4.1, around equations (4.3-0) and (4.3-0)-a,b.

Effectively, here we are attempting to present an orbiting planet as an equivalent matter-wave packet or wave group (which is the concept often and successfully applied in micro world physics). It is also appropriate to underline that certain planetary rotation around its sun has a period T and frequency of such (mechanical) rotation $f_m = 1/T$. f_m is not necessarily the frequency f_o of the associated, standing and macro matter-wave. For underlining possible difference between mechanical revolving frequency f_m and orbital macro matter-wave frequency f_o , we will first assume (and prove later) that $f_m \neq f_o$. Since the framework of this exercise is implicitly accepting that relevant planetary or satellite speeds are much lower compared to the light speed ($v \ll c$), we could safely say that certain planetary or group velocity (or its orbital velocity) should be two times higher than its phase velocity, $v = 2u = 2\lambda_o f_o$ (see better explanation why and when $v = 2u$ in the chapter 4.0, equations (4.0.78) – (4.0.81)). Now we can find mentioned orbital frequency, wavelength, group and phase speed of such (hypothetical) standing wave as (2.11.12),

$$\left\{ \begin{array}{l} 2\pi r = n\lambda_o, T = \frac{1}{f_m} = \frac{2\pi}{\omega_m}, v = \frac{2\pi r}{T} = 2\pi r f_m = \omega_m r \cong 2u, n = 1, 2, 3, \dots \\ u = \lambda_o f_o \cong \frac{1}{2} v = \frac{\pi r}{T} = \pi r f_m, T^2 = \left(\frac{4\pi^2}{GM} \right) r^3 = \frac{1}{f_m^2}, v_e = \sqrt{\frac{2GM}{r}}, \\ v = u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda}, u = \frac{v}{1 + \sqrt{1 - v^2/c^2}} \end{array} \right\} \Rightarrow \quad (2.11.12)$$

$$\lambda_o = \frac{2\pi r}{n}, f_o = \frac{u}{\lambda_o} = n \frac{u}{2\pi r} = n \frac{f_m}{2} = \frac{n}{2T} = \frac{n\sqrt{GM}}{4\pi r^{3/2}}, f_m = \frac{2f_o}{n} = \frac{1}{T} = \frac{\sqrt{GM}}{2\pi r^{3/2}},$$

$$u = \frac{v}{2} = \frac{1}{2} \sqrt{\frac{GM}{r}} = \frac{1}{2\sqrt{2}} v_e, v = 2u = \sqrt{\frac{GM}{r}} = \frac{1}{\sqrt{2}} v_e \ll c, m \ll M, \forall n = 1, 2, 3, \dots$$

Based on the group or planet's speed $v = \frac{2\pi r}{T}$, (2.11.12), the wave energy or kinetic energy of an orbiting planet, which has mass $m \ll M$, can be expressed as:

$$\begin{aligned} \tilde{E} = E_k &= \frac{1}{2} m v^2 = m v u = p u = 2m u^2 = \frac{1}{4} m v_e^2 = \frac{GmM}{2r} = \frac{1}{2} \cdot \left(\frac{GmM}{r^2} \right) \cdot r = \frac{1}{2} \cdot F_{m-M} \cdot r = \\ &= \frac{m}{2} \left(\frac{2\pi r}{T} \right)^2 = \frac{8m\pi^2 r^2}{n^2} f_o^2 = 2m(\pi r f_m)^2 = (2\pi m r^2 f_m) \cdot (\pi f_m) = L \pi f_m = \left(\frac{2\pi}{n} L \right) \cdot f_o = H f_o, \\ \left\{ \begin{array}{l} L = n \frac{H}{2\pi} = 2\pi m r^2 f_m, p = m v = \frac{\tilde{E}}{u} = \frac{H f_o}{u} = \frac{H}{\lambda_o} = n \frac{H}{2\pi r} = m \sqrt{\frac{GM}{r}}, G = 6.67 \cdot 10^{-11} \text{Nm}^2 \text{kg}^{-2}, \\ F_{m-M} = F_g = \frac{GmM}{r^2} = \frac{m v^2}{r}, \lambda_o = \frac{H}{p} = \frac{2\pi r}{n}, f_o = \frac{n f_m}{2} = \frac{n}{2T} = \frac{n\sqrt{GM}}{4\pi r^{3/2}}, H = \frac{2\pi}{n} L = \text{const..} \end{array} \right. \end{aligned} \quad (2.11.13)$$

In order to get a more tangible feeling what should be measurable effects of such “standing gravitational waves”, or gravitational field phenomena that has phase velocity $u = \lambda_o f_o \cong \frac{1}{2} v = \frac{\pi r}{T} = \pi r f_m$ we could analyze tidal waves on our planet Earth in relation to the Moon rotation around the Earth and establish predictable and measurable correlations (which should confirm (2.11.12) and (2.11.13)). Of course, we will need to take into account proper values for $u, \lambda_o, f_o, v, r, T, f_m$, valid for Moon’s rotation around Earth.

Obviously, the magnitude of the angular momentum L from (2.11.13), of an orbiting planet or satellite, its relevant orbital radius r , and associated characteristic speeds are quantized. Such orbital quantization is based on here associated resonant or standing waves, respecting simple geometrical fittings ($2\pi r = n\lambda_o$), and can be found as (see [43], M. Pitkänen, [38], [39], F. Florentin Smarandache and Vic Christianto, and [40], D. Da Rocha and Laurent Nottale):

$$L = pr = mvr = mr^2 \omega_m = mr^2 \frac{2\pi}{T} = 2\pi mr^2 f_m = m\sqrt{GM}r = n \frac{H}{2\pi} = n\hbar_{gr.} = \text{const}, m \cong \mu = \frac{mM}{m+M} \Rightarrow$$

$$\Rightarrow \left\{ \begin{array}{l} r = r_n = n^2 \frac{H^2}{4\pi^2 GMm^2} = n^2 \frac{GM}{v_o^2} = \frac{GM}{v_n^2} = n \frac{\lambda_o}{2\pi} = n \frac{\hbar_{gr.}}{mv}, \left[H = \text{const.} \Rightarrow \frac{r_1}{r_2} = \frac{n_1^2 m_2^2}{n_2^2 m_1^2} = \left(\frac{n_1 m_2}{n_2 m_1} \right)^2 \right] \\ v = v_n = \frac{v_o}{n} = \frac{2\pi}{nH} GMm = 2u = \frac{v_e}{\sqrt{2}} = \sqrt{\frac{GM}{r_n}}, \quad u = u_n = \frac{1}{2} \sqrt{\frac{GM}{r_n}} = \frac{v_o}{2n} \cong \frac{v}{2}, \\ v_o = nv_n = \frac{2\pi}{H} GMm = \frac{GMm}{\hbar_{gr.}} = 2un = \frac{nv_e}{\sqrt{2}} = n \sqrt{\frac{GM}{r_n}}, \\ f_o = n \frac{f_m}{2} = n \frac{1}{2T} = n \frac{\sqrt{GM}}{4\pi r^{3/2}}, \quad T_o = \frac{1}{f_o} = \frac{2T}{n}, \quad E_k = \tilde{E} = \tilde{E}_n = Hf_o = n \frac{Hf_m}{2}, \quad n, n_i = 1, 2, 3, \dots \end{array} \right. \quad (2.11.14)$$

There is an increasing evidence from astronomical measurements (spectral red-shifts; - [37] Tifft, 1978, [40] Nottale; [41] Rubčić, A., & J. Rubčić; [43] M. Pitkänen) that v_o (appearing in (2.11.14)) is a common velocity parameter valid for here considered planetary systems (like other universal constants known in Physics) having the value $v_o = 144.7 \pm 0.7$ km/s. This is establishing legitimacy of all other quantized parameters (from (2.11.14)) like orbital radius, phase and group velocity r_n, u_n, v_n , etc., implicating that some kind of gravitational or inertial standing-waves field structure (whatever that means) really exist around (and behind) planets in stationary orbital motions (see more of supporting remarks later, related to measured red-shifts, around equations (2.11.15) to (2.11.19)). Analog to vortex shedding phenomenology, known in fluid motions as “Karman vortex street”, we could say that certain kind of “Planetary Karman street” is on some way following planets and astronomic size objects, like an oscillatory tail. As shown in the chapter 4.1, around equations (4.3-0) and (4.3-0)-a,b, such vortex shedding is in formal agreement with matter waves quantization $\lambda = H/p, \tilde{E} = Hf$, or on a similar way equivalent to (2.11.12), (2.11.13) and (2.11.14).

What is relatively new and original in this paper is the introduction of mutually-related group and phase orbital velocities, which is implicitly introducing and defending the

concept of a wave-group or wave-packet in relation to orbital, planetary matter-waves (very much analogical to Quantum theory wave packets and wave functions). Later, we could enrich and extend the same modeling by applying Bohr-Sommerfeld's quantization conditions. In reality, creations of N. Bohr and his followers (in the early steps of old quantum mechanics) are much more natural and applicable to here elaborated matter-waves of planetary systems, than to the atom model, but it is also clear that certain analogy between such micro and macro world conceptualization really exist.

Using equations and relations from (2.11.14), we can predict and verify surprisingly exact quantization of celestial orbits in certain solar system (see such extremely well documented analyses in [38] and [39]). This is additionally confirming, or at least supporting, validity of planetary standing waves-group field structure (as elaborated in this paper). Since, planets' orbiting periods are very long, involved frequencies are almost meaningless, but this is only a matter of our perception, scaling and measurements reference systems. What matters here is that the same mathematics and similar concepts are applicable and working both on micro and macro world scale (of course, not taking it literally, and without intellectual flexibility). After establishing such grounds and analogical platforms, it will not be very difficult to apply the framework of Schrödinger's, wave quantum mechanics, backwards to orbiting planetary systems (see such mathematical modeling later; -equations (2.11.20 – (2.11.23)). The significant difference between Schrödinger's, wave quantum mechanics (as presently established; - Copenhagen interpretation) and its analogical application on planetary systems (as promoted here) is that we will find out that intrinsically probabilistic and ontologically stochastic wave function and associated concepts are not absolutely necessary, natural and best choice.

How good predictions in (2.11.14) are, is related to the facts that we are approximating the real situation in the following aspects:

- a) That all relevant planetary masses are very small compared to their common central mass or sun, and that the sun is in the state of rest (without rotation) in their common center of mass. The reality is that complete planetary system, including its sun, is rotating around its common center of mass (and the sun is not fixed to the common center of mass).
- b) That all of them (planets and sun) are in the same flat plane, what is not the general case (also not valid for our solar system).
- c) In addition, we are neglecting interplanetary interactions, considering that relevant forces and fields between every planet and its sun are dominant.
- d) Moreover, we are still not taking into account planets self-spinning.

♣ COMMENTS & FREE-THINKING CORNER:**2.3.3-1 Binary Systems, Kepler and Newton Laws and Matter Waves Hosting**

The essential fact in the background of (2.11.10) - (2.11.14) is that gravitation is the central force. Its direction is always along a radius, either towards or away from a point we are using as an origin, or force center. Magnitude of such central force depends solely upon the distance from its origin, r . We can present such forces as, $F_{m-M} = \frac{GmM}{r^2} = F(r)$, $\vec{F}(r) = F(r) \cdot \frac{\vec{r}}{r}$. Central forces are interesting because

we find them very often in physics. The gravitational and electrostatic forces are central forces (as well as forces between permanent magnets). Much of classical mechanics or physics can be placed in the framework of elaborated applications of Newton Laws. Let us start with the Second Newton Law, $\vec{F} = m\vec{a} = m \frac{d\vec{v}}{dt}$, and express the associated torque and angular momentum as, $\vec{\tau} = \vec{r} \times \vec{F} = \vec{r} \times m\vec{a}$,

$\vec{L} = \vec{r} \times \vec{p}$. Since a torque is the time derivative of angular momentum, let us find the torque for central forces (where $\vec{F}(r)$ is parallel with \vec{r}) as, $\left(\vec{\tau} = \frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r} \times \vec{p}) = \vec{r} \times m\vec{a} = \vec{r} \times \vec{F} = 0 \right) \Rightarrow \vec{L} = \text{const.}$

Consequently, an orbital planetary motion has constant angular momentum, because gravitational force is the central force. Little bit later we will see that this is the real origin of quantization in physics (equally applicable to Coulomb Law related analogical situations), as well as to micro-world of atoms and elementary particles where building blocks have constant angular moments or spin characteristics.

Let us now analyze the simplified case of gravitational attraction between two masses $m_1 = m$ and $m_2 = M$ from the point of view of Binary Systems relations in their center of mass coordinate system.

The total separation between the centers of two masses is $\vec{r} = \vec{r}_1 + \vec{r}_2$. We may define center of mass point placed between two objects through the equations,

$$m_1 r_1 = m_2 r_2, r = r_1 + r_2, r_1 = \frac{m_2}{m_1 + m_2} r, r_2 = \frac{m_1}{m_1 + m_2} r. \quad (2.11.14-1)$$

From the point of view of gravitational attraction between m_1 and m_2 nothing will change if we imagine that m_1 and m_2 may be in a uniform rotational motion around their common center of mass, since masses will in the same time experience mutually repulsive balancing centrifugal force (possible spinning is not taken into account). Let us imagine that m_1 and m_2 are rotating (around their common center of mass) with certain angular speed $\omega = \frac{2\pi}{T}$, what can be described with another set of equations,

$$\omega = \frac{v_1}{r_1} = \frac{v_2}{r_2} = \frac{v}{r}, \frac{v_1}{v_2} = \frac{r_1}{r_2}, p_1 = m_1 v_1 = m_2 v_2 = p_2 = p = p_r = m_r v_r, \vec{p}_1 + \vec{p}_2 = 0, \quad (2.11.14-2)$$

$$\vec{v}_1 = \frac{d\vec{r}_1}{dt}, \vec{v} = \frac{d\vec{r}}{dt} = \vec{v}_r = \vec{v}_1 + \vec{v}_2, v_1 v_2 = \omega^2 r_1 r_2, v_i = \omega r_i, \frac{v_1 v_2}{v^2} = \frac{r_1 r_2}{r^2}.$$

Here v_1 and v_2 are tangential velocities of m_1 and m_2 . In cases of such circular, rotational motions, every mass is experiencing certain centrifugal (mutually opposed) force with a tendency to separate them, as for example,

$$F_c = \frac{m_1 v_1^2}{r_1} = \frac{m_2 v_2^2}{r_2} = \frac{dp}{dt} = \frac{dp_r}{dt} = \frac{m_r v_r^2}{r} = \frac{p_r v_r}{r} \Leftrightarrow m_1 v_1 = m_2 v_2 (= p = m_r v_r = p_r) \Rightarrow$$

$$\left. \begin{aligned} &\Rightarrow \text{after integration} \Rightarrow \\ &\left\{ v_r = \frac{dr}{dt} = \frac{p_0}{m_r} \cdot \frac{r}{r_0}, p_r = m_r v_r = p_0 \cdot \frac{r}{r_0}, F_r = F_c = \frac{dp_r}{dt} = \frac{p_0}{r_0} v_r = p_0 \omega \cdot \frac{r}{r_0}, [p_0, r_0] = \text{constants} \right\} \end{aligned} \right\} \quad (2.11.14-3)$$

If the distance between two masses m_1 and m_2 is remaining unchanged (stable orbital motions), mutually opposed (or repulsive) centrifugal forces should be balanced with similar (central) attractive force between them, which is Newton force of gravitation F_g . Conceptualizing given case of a stable Binary System this way, we are developing and formulating Kepler's third law, as follows.

$$\left\{ \begin{aligned} F_g = G \frac{m_1 m_2}{r^2} = F_c = \frac{m_1 v_1^2}{r_1} = \frac{m_2 v_2^2}{r_2} = m_1 v_1 \omega = m_2 v_2 \omega = m_1 r_1 \omega^2 = m_2 r_2 \omega^2 = \frac{m_1 m_2}{m_1 + m_2} v_r \omega = \frac{m_1 m_2}{m_1 + m_2} r \omega^2 = m_r r \omega^2 = \frac{m_r v_r^2}{r} \\ m_r = \frac{m_1 m_2}{m_1 + m_2}, v_r = \omega r = \frac{dr}{dt}, v_1 = \frac{m_2}{m_1 + m_2} v_r = \omega r_1, v_2 = \frac{m_1}{m_1 + m_2} v_r = \omega r_2 \end{aligned} \right\} \Rightarrow$$

$$\Rightarrow \omega = \sqrt{G \frac{m_1 + m_2}{r^3}} = \frac{2\pi}{T}, \Leftrightarrow \left(\frac{T}{2\pi} \right)^2 = \frac{r^3}{G(m_1 + m_2)}. \quad (2.11.14-4)$$

Another conclusion radiating from here is that natural tendency of masses (regarding stable Binary Systems, or multi mass systems) is to create uniform or stationary rotational motions (around their common center of mass), this way balancing attractive Newton force with associated centrifugal force. If such rotation is not a visible case, at least mathematically and by respecting relevant conservation laws every Binary System could be equally presentable as a case of mutually coupled rotating bodies (including rotating disks, toroids...). The coupling force in question (for instance in cases of electromagnetically neutral bodies) is the gravitation.

We could also say that boundary or asymptotic tendency (or just mathematically equivalent state in the same center of mass coordinates) of Binary Systems is that initial masses m_1 and m_2 can be effectively replaced by one bigger central mass which is equal $m_c = m_1 + m_2$ and placed in their common center of mass position (being there in a state of rest). In addition to such central mass m_c there is another, (mathematically generated) reduced mass $m_r = \frac{m_1 m_2}{m_1 + m_2}$, which is rotating around central mass m_c . Such reduced mass m_r will have the total kinetic energy and orbital moment of masses m_1 and m_2 . The distance between m_r and m_c (or relevant circle radius) is again the same as before $r = r_1 + r_2, r_1 = \frac{m_2}{m_1 + m_2} r, r_2 = \frac{m_1}{m_1 + m_2} r, m_1 r_1 = m_2 r_2 \Rightarrow p = m_1 v_1 = m_2 v_2$. Angular (mechanical rotating) velocity ω of the new Binary System m_r and m_c will stay the same as found before for Binary System of masses m_1 and m_2 ($\omega = v_1 / r_1 = v_2 / r_2 = v_r / r = 2\pi f_m$). The attractive gravitational force between m_1 and m_2 will be the same as the attractive force between m_r and m_c , as for instance:

$$F_g = G \frac{m_1 m_2}{r^2} = G \frac{m_r m_c}{r^2} = \frac{m_r v_r^2}{r} = \frac{m_1 v_1^2}{r_1} = \frac{m_2 v_2^2}{r_2} = \frac{m_1 v_1^2 + m_2 v_2^2}{r_1 + r_2} = m_r r \omega^2 = m_1 r_1 \omega^2 = m_2 r_2 \omega^2. \quad (2.11.14-5)$$

We can also find involved orbital moments of rotating masses m_1 , m_2 and m_r , taking into account that the total orbital moment of a Binary System is conserved (constant).

$$\left\{ \begin{array}{l} \mathbf{L}_1 = p_1 r_1 = m_1 v_1 r_1 = m_1 r_1^2 \omega = \frac{m_1 v_1^2}{\omega} = \mathbf{J}_1 \omega, \\ \mathbf{L}_2 = p_2 r_2 = m_2 v_2 r_2 = m_2 r_2^2 \omega = \frac{m_2 v_2^2}{\omega} = \mathbf{J}_2 \omega, \end{array} \right\} \& \left\{ \begin{array}{l} \frac{\mathbf{L}_1}{\mathbf{L}_2} = \frac{\mathbf{J}_1}{\mathbf{J}_2} = \frac{r_1}{r_2} = \frac{v_1}{v_2}, p_1 = p_2 = p = p_r \\ \mathbf{L}_r = p_r r = m_r v_r r = m_r r^2 \omega = \frac{m_r v_r^2}{\omega} = \mathbf{J}_r \omega \end{array} \right\} \Rightarrow$$

$$\vec{\mathbf{L}}_i = \vec{r}_i \times \vec{p}_i \Rightarrow \mathbf{L}_i = m_i v_i r_i = m_i r_i^2 \omega = \frac{m_i v_i^2}{\omega} = \mathbf{J}_i \omega = \frac{2}{\omega} E_{ki}, \quad (2.11.14-6)$$

$$\begin{aligned} \mathbf{L} &= \mathbf{L}_1 + \mathbf{L}_2 = (\mathbf{J}_1 + \mathbf{J}_2) \omega = \frac{2}{\omega} \left(\frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2} \right) = \frac{2}{\omega} (E_{k1} + E_{k2}) = \frac{2}{\omega} E_{\text{orbital}} = \frac{m_r v_r^2}{\omega} = \frac{\mathbf{J}_r \omega^2}{\omega} = \\ &= \mathbf{J}_r \omega = \mathbf{L}_r = (\mathbf{J}_1 + \mathbf{J}_2) \sqrt{G \frac{m_1 + m_2}{r^3}} = \text{const.}, \mathbf{J}_r = \mathbf{J}_1 + \mathbf{J}_2, v_r = v_1 + v_2 = \omega r, \\ E_{ki} &= \frac{1}{2} m_i v_i^2 = \frac{1}{2} p_i v_i = \frac{1}{2} m_r v_r^2 \frac{r_i}{r} = \frac{1}{2} \mathbf{J}_i \omega^2 = \frac{1}{2} \mathbf{L}_i \omega, E_{kr} = E_{ki} \frac{v_r}{v_i} = E_{k1} + E_{k2}, \\ \frac{E_{k1}}{v_1} &= \frac{E_{k2}}{v_2} = \frac{E_{kr}}{v_r} = \frac{E_{ki}}{v_i} = \frac{m_i v_i}{2} = \frac{m_r v_r}{2} = \frac{1}{2} p, v_i = \omega r_i. \end{aligned}$$

Now we will be able to show that for (isolated) Binary Systems which are conserving total orbital moment, specific orbital (or kinetic, or motional) energy is in some way quantized, or given by similar expression like Planck's energy of a photon (except that new Planck-like H-constant will be much bigger compared to Planck constant of micro-world).

$$\begin{aligned} E_{\text{orbital}} &= E_{k1} + E_{k2} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} \mathbf{J}_1 \omega^2 + \frac{1}{2} \mathbf{J}_2 \omega^2 = \frac{1}{2} (\mathbf{J}_1 + \mathbf{J}_2) \omega^2 = \frac{1}{2} (\mathbf{L}_1 + \mathbf{L}_2) \omega = \\ &= \frac{1}{2} m_r v_r^2 \frac{r_1}{r} + \frac{1}{2} m_r v_r^2 \frac{r_2}{r} = \frac{1}{2} m_r v_r^2 \left(\frac{r_1}{r} + \frac{r_2}{r} \right) = \frac{1}{2} m_r v_r^2 = \frac{1}{2} \mathbf{J}_r \omega^2 = E_r = \frac{1}{2} [(\mathbf{J}_1 + \mathbf{J}_2) \omega] \cdot \omega = \\ &= \frac{1}{2} [\text{const}] \cdot \omega = \text{Const} \cdot \omega = H \cdot f = H \cdot f_0, \omega = 2\pi f_m = \frac{2\pi}{T} = \frac{H}{\text{Const}} \cdot f_0, f = f_0 \neq f_m. \end{aligned} \quad (2.11.14-7)$$

The next significant remark here (relevant for Binary Systems) is that ***in order to experience an attractive gravitational force, rotating bodies should rotate in the same direction (both having mutually collinear angular speed, and angular moments vectors)***. If rotation is not externally (or macroscopically) detectable, it should be in some ways internally (intrinsically) present in Binary Systems relations. Simply, gravitation without rotation cannot be explained. We can also find expressions for such intrinsically associated angular velocity and angular momentum of Binary Systems as,

$$\begin{aligned} F_g &= G \frac{m_1 m_2}{r^2} = G \frac{m_r m_c}{r^2} = F_c = \frac{m_r v_r^2}{r} = \frac{m_1 v_1^2}{r_1} = \frac{m_2 v_2^2}{r_2} = m_1 v_1 r_1 \frac{v_1}{r_1^2} = m_2 v_2 r_2 \frac{v_2}{r_2^2} = \mathbf{J}_1 \omega \frac{v_1}{r_1^2} = \mathbf{J}_2 \omega \frac{v_2}{r_2^2} = \mathbf{L}_1 \frac{v_1}{r_1^2} = \mathbf{L}_2 \frac{v_2}{r_2^2} = \mathbf{J}_r \omega \frac{v_r}{r^2} = \mathbf{L}_r \frac{v_r}{r^2}, \\ F_g r &= G \frac{m_1 m_2}{r^2} r = G \frac{m_1 m_2}{r} = G \frac{m_r m_c}{r} = F_c r = \frac{m_r v_r^2}{r} r = m_r v_r^2 = m_r r^2 \omega^2 = \mathbf{J}_r \omega^2 = \mathbf{L}_r \omega = 2E_r = 2E_{\text{orbital}} = 2Hf, \\ (m_1 r_1 &= m_2 r_2, m_1 v_1 = m_2 v_2 = m_r v_r = p) \Rightarrow F_i = \frac{dp}{dt} = F_r, \left(\vec{\tau} = \vec{r} \times \vec{F}_i = \frac{d\vec{L}}{dt}, \vec{F}_i = \frac{\vec{\tau}}{r} = \frac{\vec{r} \times \vec{F}_i}{r} = \frac{1}{r} \frac{d\vec{L}}{dt} \right), \end{aligned}$$

$$\begin{aligned} \mathbf{L}_r v_r &= G m_1 m_2 = \frac{\mathbf{L}_1 \mathbf{L}_2}{\mathbf{L}_1 + \mathbf{L}_2} v_r \Leftrightarrow G = \frac{\mathbf{L}_1 \mathbf{L}_2}{\mathbf{L}_1 + \mathbf{L}_2} \frac{v_r}{m_1 m_2} = \mathbf{L}_r \frac{\omega r}{m_1 m_2} = \frac{\mathbf{J}_1 \mathbf{J}_2}{\mathbf{J}_1 + \mathbf{J}_2} \frac{\omega^2 r}{m_1 m_2} = \mathbf{J}_r \frac{\omega^2 r}{m_1 m_2}, \\ \mathbf{J}_r &= \frac{\mathbf{J}_1 \mathbf{J}_2}{\mathbf{J}_1 + \mathbf{J}_2} = \mathbf{J}_1 + \mathbf{J}_2 = \sqrt{\mathbf{J}_1 \cdot \mathbf{J}_2} = \frac{m_1 m_2}{m_1 + m_2} r^2 = m_r r^2, \\ \mathbf{L}_r &= \mathbf{J}_r \omega = \mathbf{L} = \frac{\mathbf{L}_1 \mathbf{L}_2}{\mathbf{L}_1 + \mathbf{L}_2} = \mathbf{L}_1 + \mathbf{L}_2 = \sqrt{\mathbf{L}_1 \cdot \mathbf{L}_2} = \sqrt{G \frac{m_1 m_2 \mathbf{J}_r}{r}} \Rightarrow \omega = \sqrt{G \frac{m_1 m_2}{\mathbf{J}_r r}} = \sqrt{G \frac{m_r m_c}{\mathbf{J}_r r}}. \end{aligned} \quad (2.11.14-8)$$

Another conclusion to draw is that gravitational constant G is the measure of here elaborated intrinsic rotation or angular (mechanical revolving) speed of Binary Systems, leading to another alternative form of Kepler's third law as,

$$\frac{1}{\omega^2} = \frac{1}{(2\pi f_m)^2} = \left(\frac{T}{2\pi}\right)^2 = \frac{J_r r}{G m_1 m_2} = \frac{r^3}{G(m_1 + m_2)}, \quad (2.11.14-9)$$

f_m (=) Mechanical (planet or satellite) revolving or orbiting frequency.

Shall we have a repulsive gravitational force in cases when masses in Binary Systems are not rotating in the same direction (when relevant angular moments' vectors are mutually opposed or maybe not collinear) is one of logical questions to ask here? Let us exercise what could be the answer on a similar question if mentioned masses are in addition self-spinning (having finite spin moments \vec{L}_{s1} and \vec{L}_{s2} , $\vec{L}_i \rightarrow (\vec{L}_i + \vec{L}_{si})$), and how such spinning moments would influence the attractive force/s between them?

$$F_g = F_c = G \frac{m_1 m_2}{r^2} = G \frac{m_r m_c}{r^2} = \left(\frac{\pi G}{c^4}\right) \frac{m_1 c^2 m_2 c^2}{\pi r^2} = \left(\frac{G}{c^4}\right) \frac{m_r c^2 m_c c^2}{r^2} = \left(\frac{\pi G}{c^4}\right) \frac{E_{r1} E_{r2}}{\pi r^2} = \left(\frac{\pi G}{c^4}\right) \frac{E_{r1} E_{r2}}{\pi r^2}, \quad (2.11.14-10)$$

$$\left\{ \begin{array}{l} (v_i \ll c) \Rightarrow E_{ri} = m_i c^2 = \gamma_i m_i c^2 \Rightarrow E_{ri} \cong m_i c^2 + \frac{1}{2} m_i v_i^2 = m_i c^2 + \frac{1}{2} J_i \omega_i^2 = m_i c^2 + \frac{1}{2} \vec{L}_i \cdot \vec{\omega}_i \\ \left(\vec{L}_i \rightarrow (\vec{L}_i + \vec{L}_{si}), \vec{L}_i = J_i \vec{\omega}_i, \vec{L}_{si} = J_i \vec{\omega}_{si} \right) \Rightarrow E_{ri} \cong m_i c^2 + \frac{1}{2} (\vec{L}_i + \vec{L}_{si}) \cdot \vec{\omega}_i \end{array} \right\} \Rightarrow$$

$$F_g \cong \left(\frac{G}{c^4}\right) \frac{\left[m_1 c^2 + \frac{1}{2} (\vec{L}_1 + \vec{L}_{s1}) \cdot \vec{\omega}_1 \right] \left[m_2 c^2 + \frac{1}{2} (\vec{L}_2 + \vec{L}_{s2}) \cdot \vec{\omega}_2 \right]}{r^2} = \left(\frac{G}{c^4}\right) \frac{\left[m_1 c^2 + \frac{1}{2} (\vec{L}_r + \vec{L}_{sr}) \cdot \vec{\omega}_r \right] \left[m_c c^2 + \frac{1}{2} (\vec{L}_c + \vec{L}_{sc}) \cdot \vec{\omega}_c \right]}{r^2}.$$

If there is a stable ground in here hypothesized exercise about gravitational force, presence of spin and orbital moments (of participants) can increase or decrease the total gravitational force between two bodies in a Binary System (depending on relative mutual positions of relevant orbital and spin moments). Most probably, such contributive spin-related members are too small compared to other involved energy-related members (in cases of planetary or solar systems), and it has been not easy to notice such possibility for addressing modifications of the old Newton Law. **Here we should enrich the same situation by paying more attention to matter-waves nature of such binary interactions by additional elaborations around equations (2.4-11) to (2.4-17) from the same chapter.**

Obviously, in absence of repulsive centrifugal forces, planets (or orbits) of certain Solar System would collapse and unite masses with their Sun if there are no orbital rotations. Since the repulsive (centrifugal) gravitational force (as formulated here) is something exclusively related to rotation, most probably that the hidden nature of Gravitation itself is on a similar effective way intrinsically and essentially also related to certain (equivalent) rotation inside of matter substance of gravitational masses.

Another step in exercising and hypothesizing the same situation (regarding decoding essence of Gravitation in Binary Systems relations) is to notice relations between different aspects of (involved) energy components and work of "matter vortices" characterized by orbital and spin moments which should have certain torque. There is a very small imaginative step from here to start thinking how to conceptualize rest masses as some kind of "frozen or self-stabilized matter vortices' states" (since dimensionally torque is measured by the same units as energy).

Until here we did not address any of relativistic aspects of motional masses, since by the nature of astronomic, gravitational Binary Systems (or planetary systems), we can consider that in majority of relevant cases relevant orbital velocities are much smaller compared to the speed of light, and in such cases it is clearly valid,

$$(v_{1,2} \ll c) \Rightarrow F_g = G \frac{m_1 m_2}{r^2} = G \frac{m_r m_c}{r^2} \Leftrightarrow m_1 m_2 = m_r m_c, m_r = \frac{m_1 m_2}{m_1 + m_2}, m_c = m_1 + m_2. \quad (2.11.14-11)$$

Let us now imagine that some of Binary Systems (not necessarily of exclusively gravitational nature) could be orbital speed sensitive, and let us analyze the consequences (again in reference to the relevant center of mass).

$$\left\{ m_i \rightarrow \gamma_i m_i = \frac{m_i}{\sqrt{1 - \frac{v_i^2}{c^2}}} = m_i^* \right\} \Rightarrow \left\{ \begin{array}{l} \left(m_r = \frac{m_1 m_2}{m_1 + m_2} \right) \rightarrow \frac{\gamma_1 m_1 \gamma_2 m_2}{\gamma_1 m_1 + \gamma_2 m_2} = \frac{m_1^* m_2^*}{m_1^* + m_2^*} = m_r^* = \gamma_r m_r \\ (m_c = m_1 + m_2) \rightarrow \gamma_1 m_1 + \gamma_2 m_2 = m_1^* + m_2^* = m_c^* \\ (m_1 m_2 = m_r m_c) \rightarrow \gamma_1 m_1 \gamma_2 m_2 = m_r^* m_c^* \end{array} \right\} \Rightarrow \quad (2.11.14-12)$$

$$\Rightarrow \left\{ \begin{array}{l} F_g = G \frac{m_1 m_2}{r^2} = G \frac{m_r m_c}{r^2} = G \frac{m_1^* m_2^*}{r^2} = G \frac{m_r^* m_c^*}{r^2} \\ \gamma_r = \frac{\gamma_1 \gamma_2}{\gamma_1 \frac{m_1}{m_1 + m_2} + \gamma_2 \frac{m_2}{m_1 + m_2}} \end{array} \right\}.$$

In order to make a simple validity test of here elaborated relativistic masses relations it would be very indicative and almost sufficient to present the case when one of masses is enormously bigger compared to other,

$$(m_1 = m \ll m_2 = M) \Rightarrow \left\{ \begin{array}{l} m_r = \frac{m_1 m_2}{m_1 + m_2} \cong m = m_1 \Rightarrow \gamma_r = \frac{\gamma_1 \gamma_2}{\gamma_1 \frac{m_1}{m_1 + m_2} + \gamma_2 \frac{m_2}{m_1 + m_2}} \cong \gamma_1 \\ E_{k1} + E_{k2} = E_{kr} = E_{ki} \frac{v_r}{v_i} = (\gamma_r - 1) m_r c^2 \cong (\gamma_1 - 1) m_1 c^2 = E_{k1} \end{array} \right\} \quad (2.11.14-13)$$

what already looks as correct result (see also equations (2.4-11) - (2.4-18)). There are number of imaginable consequences starting from here. For instance, any stable planetary system (with one big solar mass M_s) and number of orbiting planets with masses $\{m_i\}_{i=1}^n$ can be decomposed and analyzed as an ensemble of simple Binary Systems with masses m_j and $(M_c - m_j)$, as for example,

$$m_j \cdot (M_c - m_j) = m_{r,j} \cdot M_c, \quad M_c = M_s + \sum_{i=1}^n m_i, \quad m_{r,j} = \frac{m_j \cdot [M_c - m_j]}{M_c}, \quad \forall j \in (1, n), \quad (2.11.14-14)$$

where m_j and M_s are only an approximation of a Binary System masses when

$$M_c \cong M_s \gg \sum_{i=1}^n m_i.$$

Of course, similar strategy can be extended to n-body problems. In the familiar mainstream of thinking, we can implement laws of linear and orbital moments' conservation in order to establish a powerful analyzing framework that will take into account mutual interactions of many-body systems. This will give us a chance to explore non-Newtonian gravitation-related interactions between masses with spin and orbital moment's attributes.

2.3.3-2 Quantizing and Matter Waves Hosting

Circular orbits of stable Binary Systems (including most of stable solar or planetary systems), as conceptualized here, are presenting uniform, stationary, periodical and inertial motions. For inertial motions, we have seen in (2.9.1) and (2.9.2) that coincident validity and applicability of relevant linear and orbital momentum conservation is directly linked to standing matter waves formations. Consequently, stable Binary and Planetary Systems' Orbits as inertial motions, besides hosting orbiting masses could also host certain mutually synchronized standing matter waves formations, where synchronizing (or waves packing, or quantizing) criteria in relation to the relevant center of mass coordinates system should be,

$$\left[\begin{array}{l} 2\pi r_i = n_i \lambda_i, L_i = p_i r_i = m_i v_i r_i = m_i r_i^2 \omega = \frac{m_i v_i^2}{\omega} = J_i \omega = \frac{H}{\lambda_i} r_i, \lambda_i = \frac{2\pi r_i}{n_i} = \frac{H}{p_i}, H = \text{const.}, \\ \frac{L_1}{L_2} = \frac{J_1}{J_2} = \frac{r_1}{r_2} = \frac{v_1}{v_2} = \frac{n_1}{n_2} = \frac{E_{k1}}{E_{k2}}, \frac{n_1}{r_1} = \frac{n_2}{r_2} = \frac{n_r}{r} = \frac{n_i}{r_i} = \frac{2\pi}{\lambda_i}, n_i \in [1, 2, 3, \dots], \\ \omega = \frac{v_1}{r_1} = \frac{v_2}{r_2} = \frac{v_i}{r_i} = \frac{v_1 + v_2}{r_1 + r_2} = \frac{v}{r}, p_1 = m_1 v_1 = m_2 v_2 = p_2 = p = p_r, v_1 v_2 = \omega^2 r_1 r_2 \end{array} \right] \Rightarrow$$

$$\frac{H}{2\pi} = L_1 \frac{\lambda_1}{2\pi r_1} = L_2 \frac{\lambda_2}{2\pi r_2} = L_r \frac{\lambda}{2\pi r} = (L_1 + L_2) \frac{\lambda_1 + \lambda_2}{2\pi(r_1 + r_2)} =$$

$$= \frac{L_1}{n_1} = \frac{L_2}{n_2} = \frac{L_1 + L_2}{n_1 + n_2} = \frac{L_r}{n_r} = \frac{L_i}{n_i} = \frac{J_i \omega}{n_i} = \frac{2 J_i \omega^2}{n_i \omega} = \frac{2}{n_i \omega} E_{ki} = \hbar_{gr.}, \quad (2.11.14-15)$$

$$(v_i \ll c) \Rightarrow E_{ki} = \frac{n_i \omega}{2} \frac{H}{2\pi} = \frac{n_i v_i}{2r_i} \frac{H}{2\pi} \cong \frac{n_i 2u_i}{2r_i} \frac{H}{2\pi} = \frac{n_i u_i}{r_i} \frac{H}{2\pi} = H \frac{n_i \lambda_i f_i}{2\pi r_i} = H f_i,$$

$$\omega r_i = v_i \cong 2u_i = 2\lambda_i f_i, \omega = \frac{2\pi}{T} = 2\pi f_m \cong 2\lambda_i \frac{f_i}{r_i} = 2\lambda \frac{f}{r} = \frac{4\pi}{n} f, f_m \cong \frac{\lambda}{\pi r} f = \frac{2}{n} f = \frac{2}{n_i} f_i, f = f_0,$$

$$E_{orbital} = E_{k1} + E_{k2} = H(f_1 + f_2) = Hf = \frac{1}{2} F_g r = \frac{1}{2} G \frac{m_1 m_2}{r^2} r = \frac{1}{2} G \frac{m_1 m_2}{r} = \frac{1}{2} G \frac{m_r m_c}{r} = \frac{1}{2} F_c r = \frac{1}{2} \frac{m_r v_r^2}{r} =$$

$$= \frac{1}{2} m_r v_r^2 = \frac{1}{2} m_r r^2 \omega^2 = \frac{1}{2} J_r \omega^2 = \frac{1}{2} L_r \omega = E_r = \frac{H}{4\pi} (n_1 + n_2) \omega = \frac{H}{2} (n_1 + n_2) f_m = \frac{H}{2} n f_m, n = n_1 + n_2 = n_r.$$

We could again attempt to characterize and quantify unity of orbital moments of certain stable Solar System (L_s, n_s) with number of orbiting planets (L_i, n_i), considering the Sun as enormously bigger mass compared to any of related planets, $m_s \gg m_i$) on a similar way, as for instance,

$$\left[\frac{H}{2\pi} = \frac{L_1}{n_1} = \frac{L_2}{n_2} = \frac{L_1 + L_2}{n_1 + n_2} = \frac{L_r}{n_r} = \frac{L_i}{n_i} \right] \Rightarrow \frac{H}{2\pi} = \frac{L_s}{n_s} = \frac{L_i}{n_i} = \frac{\sum_{(i)} L_i}{\sum_{(i)} n_i} = \frac{L_s + \sum_{(i)} L_i}{n_s + \sum_{(i)} n_i} = \hbar_{gr.}, \quad (2.11.14-16)$$

where L_s and n_s are common parameters of the common Sun, and L_i and n_i are related to each planet. This way the same Solar System can be decomposed on number of elementary binary systems (where each planet and the sun are presenting one elementary Binary System). Of course, such strategy should be additionally elaborated and united with masses decomposition criteria from (2.11.14-14).

There is still certain confusion and ambiguity in physics literature regarding relations between mechanical revolving (or orbital, rotating) frequency $f_m = \omega / 2\pi$ and associated, specific orbital, matter wave frequency $f = \omega_0 / 2\pi = f_0$, and resolutions of such discrepancies are being explained by *postulating correspondence principles* (what is obviously not a real and very scientific explanation). The background of mentioned discrepancies is closely related to the nature of wave motions, to particle-wave duality and to specific relations between a group and phase velocity of certain matter wave packet (which represents an energy-momentum wave model of a moving particle). For

instance, the relation between group and phase velocity (where group velocity is in the same time real, measurable particle velocity) can be found as (see chapters 4.0 and 4.1),

$$v = v_g = u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda}, \quad u = \lambda f, \quad u = \frac{v}{1 + \sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow \left[\begin{array}{l} (v \ll c) \Rightarrow \omega r_i = v_i \cong 2u_i = 2\lambda_i f_i \\ (v \approx c) \Rightarrow \omega r_i = v_i \cong u_i = \lambda_i f_i \end{array} \right] \Rightarrow$$

$$\Rightarrow \left[\begin{array}{l} \omega = \frac{2\pi}{T} = 2\pi f_m = \frac{v_i}{r_i} \cong \frac{2u_i}{r_i} \cong 2\lambda_i \frac{f_i}{r_i} = 2\lambda \frac{f}{r} = \frac{4\pi}{n} f, \quad f_m \cong \frac{\lambda}{\pi r} f = \frac{2}{n} f = 2 \frac{f_i}{n_i}, \quad f_i = \frac{n_i}{n} f, (v \ll c) \\ \omega = \frac{2\pi}{T} = 2\pi f_m = \omega_m = \frac{v_i}{r_i} \cong \frac{u_i}{r_i} \cong \lambda_i \frac{f_i}{r_i} = \lambda \frac{f}{r} = \frac{2\pi}{n} f, \quad f_m \cong \frac{\lambda}{2\pi r} f = \frac{1}{n} f = \frac{f_i}{n_i}, \quad f_i = \frac{n_i}{n} f, (v \approx c) \end{array} \right] \Rightarrow$$

$$\Rightarrow E_{ki} = \frac{n_i \omega}{2} \frac{H}{2\pi} = \frac{n_i v_i}{2r_i} \frac{H}{2\pi} = \frac{2n_i u_i}{2r_i} \frac{H}{2\pi} = H \frac{n_i \lambda_i f_i}{2\pi r_i} = H f_i \Rightarrow \quad (2.11.14-17)$$

$$\Rightarrow E_{\text{orbital}} = E_{k1} + E_{k2} = H(f_1 + f_2) = Hf, \quad f_1 + f_2 = f, \quad n_1 + n_2 = n, \quad n_i \lambda_i = 2\pi r_i,$$

where,

$$\left[\begin{array}{l} (v \ll c) \Rightarrow v \cong 2u = 2\lambda f = u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda} \Leftrightarrow u \cong -\lambda \frac{du}{d\lambda} \Leftrightarrow 2 \frac{d\lambda}{\lambda} = -\frac{df}{f} \Rightarrow \ln \left| \frac{\lambda}{\lambda_0} \right| \left| \frac{f}{f_0} \right| = 0 \Rightarrow \left| \frac{\lambda}{\lambda_0} \right| \left| \frac{f}{f_0} \right| = 1 \\ \Leftrightarrow \lambda^2 f = \lambda_0^2 f_0, \quad u\lambda = u_0 \lambda_0, \quad \lambda = \lambda_0 \sqrt{\frac{f_0}{f}} = \frac{H}{p}, \quad u = u_0 \frac{\lambda_0}{\lambda}, \quad (f_0, \lambda_0) = \text{const.}, \quad p = \frac{Hf}{c} \sqrt{\frac{f_0}{f}} = \frac{nHf_m}{2c} \sqrt{\frac{f_0}{f}} \end{array} \right], \quad (2.11.14-18)$$

$$\left[\begin{array}{l} (v \approx c) \Rightarrow v \cong u = \lambda f = u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda} \Leftrightarrow \frac{du}{d\lambda} = \frac{fd\lambda + \lambda df}{d\lambda} \cong 0 \Leftrightarrow \frac{d\lambda}{\lambda} = -\frac{df}{f} \Rightarrow \ln \left| \frac{\lambda}{\lambda_0} \right| \left| \frac{f}{f_0} \right| = 0 \Rightarrow \\ \left| \frac{\lambda}{\lambda_0} \right| \left| \frac{f}{f_0} \right| = 1 \Leftrightarrow u = \lambda f = \lambda_0 f_0 = u_0 = c = \text{const.}, \quad (f_0, \lambda_0) = \text{const.}, \quad \lambda = \lambda_0 \frac{f_0}{f} = \frac{c}{f} = \frac{H}{p}, \quad p = \frac{Hf}{c} = \frac{nHf_m}{c} \end{array} \right].$$

Until here we analyzed a Binary System composed of two rotating bodies (m_1 and m_2) around their common center of mass (where a total kinetic energy of both rotating participants is $E_{\text{orbital}} = E_{k1} + E_{k2} = H(f_1 + f_2) = Hf = E_r$). Alternatively we can present the same situation as another (artificial and equivalent) Binary System where one of involved masses $m_c = m_1 + m_2$ is much bigger than other ($m_1 = m \ll m_2 = M \cong m_c$) and staying at rest in their common center of mass (having zero orbital, kinetic energy), and second (much smaller) mass $m_r = \frac{m_1 m_2}{m_1 + m_2} \cong m \ll m_c \cong M$ is rotating around much bigger mass m_c , having again the same total orbital energy as before ($E_{\text{orbital}} = E_{k1} + E_{k2} = E_{k1} = Hf_1 = Hf = Hf_0$).

If we imagine that the last phase of a Binary System evolution is its collapse towards creation of a single spinning mass $m_c = m_1 + m_2$ (where mechanical spinning of m_c is characterized by ω_c), we will have (still in a center of mass coordinate system),

$$E_{\text{orbital}} = E_{k1} + E_{k2} = \frac{1}{2} m_r v_r^2 = H(f_1 + f_2) = Hf = E_r = \frac{1}{2} \mathbf{J}_r \omega^2 = \frac{1}{2} (\mathbf{J}_1 + \mathbf{J}_2) \omega^2 = \frac{1}{2} \mathbf{J}_c \omega_c^2 \Rightarrow$$

$$(\mathbf{J}_1 + \mathbf{J}_2) \omega^2 = \mathbf{J}_c \omega_c^2, \quad \omega_c = 2\pi f_c = \omega \sqrt{\frac{\mathbf{J}_1 + \mathbf{J}_2}{\mathbf{J}_c}} = \frac{2\pi}{T} \sqrt{\frac{\mathbf{J}_1 + \mathbf{J}_2}{\mathbf{J}_c}} = 2\pi f_m \sqrt{\frac{\mathbf{J}_1 + \mathbf{J}_2}{\mathbf{J}_c}} = \left\{ \begin{array}{l} \frac{4\pi f_0}{n} \sqrt{\frac{\mathbf{J}_1 + \mathbf{J}_2}{\mathbf{J}_c}}, \quad v_r \ll c \\ \frac{2\pi f_0}{n} \sqrt{\frac{\mathbf{J}_1 + \mathbf{J}_2}{\mathbf{J}_c}}, \quad v_r \approx c \end{array} \right\}. \quad (2.11.14-19)$$

Binary Systems (as conceptualized here) are planar motional systems, meaning that involved circular motions are in the same fixed plane, and this is the reason why quantizing or synchronizing, or standing-waves packing criteria is related only to one orbital quantum number. Here, we should not

forget that involved mechanical rotations and spinning have much different angular velocities ω_m, ω_c , compared to associated (surrounding) mater waves angular velocities $\omega_0 = 2\pi f_0 = 2\pi f$. Of course, all of that is an idealization or approximation, since more real are multi-body systems, like planetary or solar systems (including micro-world and subatomic systems), where orbital single-plane circular motions are becoming multi-planar elliptic-paths motions (having quantized inclinations for relevant planetary orbits). Consequently, new quantizing or waves synchronizing rules are getting additional angular quantum numbers, like in semi-classical quantization of angular momentum (see [40], D. Da Roacha and L. Nottale). In mentioned Multi-component Systems (including Binary Systems), very appropriate quantizing and generalizing approach will be to apply, *creatively and with intellectual flexibility*, Wilson-Bohr-Sommerfeld Action Integrals, combined with familiar theoretical concepts published by Anthony D. Osborne, & N. Vivian Pope (see [36]). Ironically, the early days Classical Quantum Physics related to N. Bohr Hydrogen Atom Model is much more a Quantum approach to macrocosmic, real planetary orbital motions, than anything what explains or conceptualize a real nature of hydrogen atom. Here (in relation with Binary Solar Systems) we are still not specifying what kind of matter waves we are talking about, but a very strong candidate (beside others related to inertial effects, rotation and gravitation) that cannot be excluded are electromagnetic fields and waves.

Quantization in Physics is simply a consequence of existence of stable Binary and Multi-component Systems and energy-momentum communications between them (but we should not forget that other, transient and non-stable systems have a place in our universe). This is also the area where modern-days Quantum Theory started being complex and fuzzy, since for managing such situations (in absence of real, clear, natural and obvious conceptualization), it was necessary to establish new, primarily mathematically operating theories and postulates, which have been deductively generating "second-hand", luckily useful results.

2.3.3-3 Standing-Waves Resonators and Gravitation

Another aspect of imaginable, stable standing-waves field structures in relation to gravitation is the fact that every two masses (of certain Binary System, including static masses that are mutually touching) can be presented as a kind of half-wave ($\lambda / 2$) resonator, or a gravitation-dipole, where the distance between two of such masses is equal to $r = \lambda / 2 = c_{gr} / 2f_{gr}$. Here c_{gr} is the radial (central) gravitational-waves velocity acting along the distance r connecting centers of masses in question, and f_{gr} is the relevant resonant frequency of the associated standing wave (see (2.11.14-15) and (2.11.14-16)). This can mathematically be described as,

$$\left\{ \begin{array}{l} r = \frac{\lambda}{2} = \frac{c_{gr}}{2f_{gr}} = r_1 + r_2, r_1 = \frac{m_2}{m_1 + m_2} r, r_2 = \frac{m_1}{m_1 + m_2} r, m_1 r_1 = m_2 r_2, \\ E_{orbital} = E_{k1} + E_{k2} = H(f_1 + f_2) = Hf = \frac{1}{2} F_g r = \frac{1}{2} G \frac{m_1 m_2}{r^2} r = \frac{1}{2} G \frac{m_1 m_2}{r} = \frac{1}{2} G \frac{m_r m_c}{r} = \\ = \frac{1}{2} F_c r = \frac{1}{2} \frac{m_r v_r^2}{r} r = \frac{1}{2} m_r v_r^2 = \frac{1}{2} m_r r^2 \omega^2 = \frac{1}{2} J_r \omega^2 = \frac{1}{2} L_r \omega = E_r = \frac{H}{4\pi} (n_1 + n_2) \omega = \\ = \frac{H}{2} (n_1 + n_2) f_m = \frac{H}{2} n f_m, n = n_1 + n_2 = n_r, n f_m = 2f, L_1 \rightarrow (L_1 + L_{s1}), L_2 \rightarrow (L_2 + L_{s2}), n \rightarrow (n + n_s) \\ \frac{H}{2\pi} = \frac{L_1}{n_1} = \frac{L_2}{n_2} = \frac{L_1 + L_2}{n_1 + n_2} = \frac{L_r}{n_r} = \frac{L_i}{n_i} = \frac{L_1 + L_2}{n} = \frac{(L_1 + L_{s1}) + (L_2 + L_{s2})}{n + n_s} \end{array} \right\} \Rightarrow$$

$$E_{orbital} = E_{k1} + E_{k2} = H(f_1 + f_2) = Hf = \frac{H}{2} (n_1 + n_2) f_m = \frac{H}{2} n f_m = \frac{1}{2} F_g r = \frac{G}{c_{gr}} m_1 m_2 f_{gr} = \frac{G}{c_{gr}} m_r m_c f_{gr} \Rightarrow$$

$$\Rightarrow H = H \cdot \frac{n f_m}{2f} = \frac{G}{c_{gr}} \frac{m_1 m_2 f_{gr}}{f} = \frac{G}{c_{gr}} \frac{m_r m_c f_{gr}}{f} = 2 \frac{G}{c_{gr}} \frac{m_1 m_2 f_{gr}}{n f_m} = 2 \frac{G}{c_{gr}} \frac{m_r m_c f_{gr}}{n f_m} =$$

$$= 2 \frac{F_g}{c_{gr}} \frac{f_{gr}}{n f_m} r^2 = \frac{F_g}{c_{gr}} \frac{f_{gr}}{f} r^2 = 2\pi \frac{L_1 + L_2}{n} = \text{constant}, \frac{n f_m}{2f} = \frac{G}{c_{gr} H} \frac{m_1 m_2 f_{gr}}{f} = \frac{G}{c_{gr} H} \frac{m_r m_c f_{gr}}{f} = 1 \Rightarrow$$

$$\Rightarrow F_g = \frac{\pi c_{gr} n f_m}{n f_{gr}} \frac{(L_1 + L_2)}{r^2} = \frac{2\pi c_{gr} f}{n f_{gr}} \frac{(L_1 + L_2)}{r^2} = \frac{4\pi f}{n} \frac{(L_1 + L_2)}{r} = 2\pi f_m \frac{(L_1 + L_2)}{r} =$$

$$= \omega_m \frac{(L_1 + L_2)}{r} = v \frac{(L_1 + L_2)}{r^2} = G \frac{m_1 m_2}{r^2} = G \frac{m_r m_c}{r^2},$$

$$\omega_m = \omega = \frac{2\pi}{T} = 2\pi f_m = \frac{v}{r} = \frac{v_1}{r_1} = \frac{v_2}{r_2} = \frac{v_r}{r}, \frac{v_1}{v_2} = \frac{r_1}{r_2}, v(L_1 + L_2) = G m_1 m_2,$$

$$p_1 = m_1 v_1 = m_2 v_2 = p_2 = p = p_r = m_r v_r, \vec{p}_1 + \vec{p}_2 = \vec{0}, \vec{r} = \vec{r}_1 + \vec{r}_2,$$

$$\vec{v}_i = \frac{d\vec{r}_i}{dt}, \vec{v} = \frac{d\vec{r}}{dt} = \vec{v}_r = \vec{v}_1 + \vec{v}_2, v_1 v_2 = \omega^2 r_1 r_2, v_i = \omega r_i, \frac{v_1 v_2}{v^2} = \frac{r_1 r_2}{r^2}.$$

(2.11.14-20)

There is number of challenging (still hypothetical) options regarding understanding the Gravitation starting from results found in (2.11.14-20). One of such options, offering replacement for Newton Law

($F_g = v \frac{(L_1 + L_2)}{r^2} = G \frac{m_1 m_2}{r^2}$), is that gravitational force is directly dependent on total resulting vector

of angular and spin moments of Binary System participants. Such angular moments ($\vec{L} = \vec{L}_1 + \vec{L}_2$) are externally visible (and measurable), and some of their components could be states related to spinning, or to other kind of hidden rotation of belonging subatomic entities (see also (2.2), (2.4-5), (2.5) and (2.11)). What is significant here is that all three vectors $\vec{r}, \vec{v}, \vec{L}$ are mutually orthogonal, meaning that relevant vectors' product will produce vector of gravitational force \vec{F}_g collinear with \vec{r} . Consequently, now we can be sure that origin of gravitation is in an interaction between angular, orbital and/or spin moments of mutually attracting masses.

Half-wave resonator, as an intuitive concept for explaining gravitational attraction between two pulsating or oscillating masses (elaborated in (2.11.14-20)) can also be approximated and modeled as the situation when two masses in question (Binary System) are mutually connected by certain springs. Such springs (obviously having non-linear spring coefficients k_1 and k_2) are effectively realizing Newton gravitational force, between masses in question and can be supported by the following (at least dimensionally correct, and still hypothetical) relations,

$$\left\{ \begin{array}{l} F_g = k_1 r_1 = k_2 r_2 = G \frac{m_1 m_2}{r^2}, m_1 r_1 = m_2 r_2 = m_r r, m_r = \frac{m_1 m_2}{m_1 + m_2}, \\ f_{gr} = \frac{1}{2\pi} \sqrt{\frac{k_1}{m_1}} = \frac{1}{2\pi} \sqrt{\frac{k_2}{m_2}} \quad (=) [\text{Hz}], \\ r_1 = \frac{m_2}{m_1 + m_2} r = \frac{\lambda_1}{4}, r_2 = \frac{m_1}{m_1 + m_2} r = \frac{\lambda_2}{4}, \lambda_i = \frac{c_{gr-i}}{f_{gr}} = \frac{H}{p_{gr-i}}, \\ r = r_1 + r_2 = \frac{\lambda_1}{4} + \frac{\lambda_2}{4} = \frac{\lambda}{2} = \frac{c_{gr1}}{4f_{gr}} + \frac{c_{gr2}}{4f_{gr}} = \frac{c_{gr1} + c_{gr2}}{4f_{gr}} = \frac{c_{gr}}{2f_{gr}}, \\ F_g r = G \frac{m_1 m_2}{r} = G \frac{m_r m_c}{r} = k_1 r_1^2 + k_2 r_2^2 = 2Hf = nHf_m = \frac{2Gm_1 m_2}{c_{gr}} f_{gr}. \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} \frac{k_1}{k_2} = \frac{m_1}{m_2} = \frac{r_2}{r_1} = \frac{c_{gr2}}{c_{gr1}} = \frac{\lambda_2}{\lambda_1} \\ c_{gr-i} = \lambda_i f_{gr} = 4r_i f_{gr} = \frac{2r_i}{\pi} \sqrt{\frac{k_i}{m_i}} \end{array} \right.$$

$$\Rightarrow \left\{ \begin{array}{l} \lambda_1 = \frac{4m_2}{m_1 + m_2} r = \frac{c_{gr-1}}{f_{gr}} = \frac{H}{p_{gr-1}}, \lambda_2 = \frac{4m_1}{m_1 + m_2} r = \frac{c_{gr-2}}{f_{gr}} = \frac{H}{p_{gr-1}}, \\ H = \lambda_1 p_{gr-1} = \lambda_2 p_{gr-2} = \frac{4m_2}{m_1 + m_2} p_{gr-1} r = \frac{4m_1}{m_1 + m_2} p_{gr-2} r = \frac{c_{gr-1}}{f_{gr}} p_{gr-1} = \frac{c_{gr-2}}{f_{gr}} p_{gr-2} = \\ = H \cdot \frac{nf_m}{2f} = \frac{G}{c_{gr}} \frac{m_1 m_2 f_{gr}}{f} = \frac{G}{c_{gr}} \frac{m_r m_c f_{gr}}{f} = 2 \frac{G}{c_{gr}} \frac{m_1 m_2 f_{gr}}{nf_m} = 2 \frac{G}{c_{gr}} \frac{m_r m_c f_{gr}}{nf_m} = \\ = 2 \frac{F_g}{c_{gr}} \frac{f_{gr}}{nf_m} r^2 = \frac{F_g}{c_{gr}} \frac{f_{gr}}{f} r^2 = 2\pi \frac{L_1 + L_2}{n} = \text{constant} \end{array} \right.$$

$$\Rightarrow \left\{ \begin{array}{l} p_{gr-1} = m_1 \cdot \frac{G(m_1 + m_2)}{4c_{gr} r} \frac{f_{gr}}{f} = m_1 \cdot \frac{G(m_1 + m_2)}{2c_{gr} r} \frac{f_{gr}}{nf_m} = m_1 \cdot v_1^*, \\ p_{gr-2} = m_2 \cdot \frac{G(m_1 + m_2)}{4c_{gr} r} \frac{f_{gr}}{f} = m_2 \cdot \frac{G(m_1 + m_2)}{2c_{gr} r} \frac{f_{gr}}{nf_m} = m_2 \cdot v_2^*, \\ v_1^* = v_2^* = \frac{G(m_1 + m_2)}{4c_{gr} r} \frac{f_{gr}}{f} = \frac{G(m_1 + m_2)}{2c_{gr} r} \frac{f_{gr}}{nf_m} = v^* \end{array} \right. \quad (2.11.14-21)$$

What is interesting in (2.11.14-21) is that Binary Systems relations are effectively showing that two masses, mutually exercising the Newton force of gravitation (as a Binary System), can be analyzed in a certain approximate way as two weakly coupled mass-spring oscillators (linked to their common center of mass), having the same resonant frequency on both sides. In order to achieve a global forces balance (like in cases of stable planetary systems, where attractive gravitational force is balanced by repulsive centrifugal force), attractive forces of such non-linear springs (between masses) should be compensated by equal repulsive forces of other two springs (connected in line with two masses in question in the mutually opposing directions). This way we can represent gravitational attraction between each of masses and a rest of the universe. This way (see Fig.2.5), we will be able to analyze (almost) independently, each of two mass-spring systems as an equivalent, macro $\lambda/4$ resonator, as already practiced in (2.11.14-21).

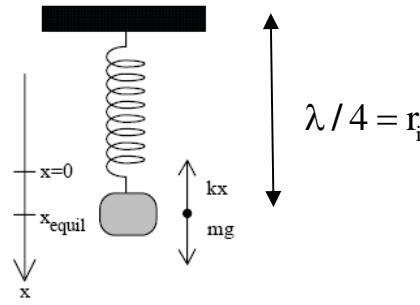


Fig.2.5. Simple Mass-Spring oscillator

The mass-spring oscillations (where mass m_i is oscillating with certain amplitude Δr_i , Fig.2.5) can be mathematically presented by simple harmonic function $x = (\Delta r_i) \cos(\omega t + \varphi)$. In reality, we could imagine that (valid for both masses) distance r_i between a mass m_i and common center of both masses is pulsating (or harmonically oscillating) between two values: $r_i + \Delta r_i$ and $r_i - \Delta r_i$. This will (after applying few of mathematical steps valid for mass-spring systems, and applicable to particle-wave duality situations) extend the relation of proportionality between relevant elements of a Binary System in question (found in (2.11.14-21)) to,

$$\frac{k_1}{k_2} = \frac{m_1}{m_2} = \frac{r_2}{r_1} = \frac{c_{gr2}}{c_{gr1}} = \frac{\lambda_2}{\lambda_1} = \frac{(\Delta r_2)^2}{(\Delta r_1)^2}, \quad (2.11.14-22)$$

$$\left(r = r_1 + r_2 = \frac{\lambda_1}{4} + \frac{\lambda_2}{4} = \frac{c_{gr1}}{4f_{gr}} + \frac{c_{gr2}}{4f_{gr}} = \frac{\langle c_{gr} \rangle}{2f_{gr}} = \frac{c_{gr}}{2f_{gr}} \right).$$

If such (standing waves and resonant) oscillations really exist between two astronomic objects, we should be able to detect them on some way. For instance if one of masses is our Sun and the other of masses is our planet Earth, the light beam coming from the Sun and detected on the Earth (by certain prism) should be wavelength-modulated producing that every specific color should have its bandwidth, directly proportional to the oscillatory speed amplitude $\omega \Delta r_i \lll c$ (like kind of Doppler effect). Such bandwidths can be measured (for number of specific colors) on Equator and somewhere far from Equator (as well as from some satellite observatory), and we should notice the differences between corresponding bandwidths. Since, here we are talking about modulated and standing waves motions (between two masses), we can apply generally valid relations between group and phase velocity, where: group velocity (of relevant gravitational wave) is $v = c_{gr} = v_{gr}$, phase velocity is $u = u_{gr}$, modulating planetary oscillating speed is $\Delta v = \omega \Delta r_i \lll c$, and mean group and phase velocities are $\bar{v} = \bar{c}_{gr} = \bar{v}_{gr}$, $\bar{u} = \bar{u}_{gr}$.

This would give us an idea how to establish relations between relevant frequency and wavelengths bandwidths, as follows,

$$\left\{ \begin{array}{l} v = u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda} = u + p \frac{du}{dp} = \frac{d\omega}{dk} = \frac{d\tilde{E}}{dp} = H \frac{df}{dp} = \frac{df}{df_s} = \frac{2u}{1 + \frac{uv}{c^2}}, \\ u = \lambda f = \frac{\omega}{k} = \frac{\tilde{E}}{p} = \frac{Hf}{p} = \frac{f}{f_s} = \frac{v}{1 + \sqrt{1 - \frac{v^2}{c^2}}} = \frac{E_k}{p}, f_s = k/2\pi, \\ \Rightarrow 0 \leq 2u \leq \sqrt{uv} \leq v \leq c \end{array} \right\} \Rightarrow$$

$$\Rightarrow \left\{ \begin{array}{l} d\tilde{E} = Hdf = mc^2 d\gamma, \quad \frac{df}{f} = \left(\frac{dv}{v}\right) \cdot \frac{1 + \sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{v^2}{c^2}} \\ (v \pm \Delta v) = (u \pm \Delta u) - (\lambda \mp \Delta \lambda) \frac{d(u \pm \Delta u)}{d(\lambda \mp \Delta \lambda)} \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} \frac{\Delta f}{\bar{f}} = \left(\frac{\Delta v}{\bar{v}}\right) \cdot \frac{1 + \sqrt{1 - \frac{\bar{v}^2}{c^2}}}{1 - \frac{\bar{v}^2}{c^2}} \cong \frac{2\Delta v}{\bar{v}} = \frac{\Delta v}{\bar{u}}, \\ \frac{du}{\Delta u} = \frac{d\lambda}{\Delta \lambda} = 2 \frac{du}{\Delta v} = \frac{dv}{\Delta v}, \\ \bar{v} \cong 2\bar{u} = 2\bar{\lambda} \cdot \bar{f}, \quad \Delta v \cong 2\Delta u, \\ 2 \frac{\Delta u}{\bar{v}} = \frac{\Delta v}{\bar{v}} = \frac{\Delta v}{2\bar{u}} = \frac{\Delta f}{2\bar{f}} = \frac{\Delta v}{2\bar{\lambda} \cdot \bar{f}}, \quad \bar{\lambda} \cdot \Delta f = \Delta v \end{array} \right\}. \quad (2.11.14-22)$$

If we continue developing similar ideas about standing waves communications between masses, we should be able to explain “redshifts and blueshifts” of the light spectra from deep and remote cosmic areas, captured by astronomic observatories on our planet.

No doubts that here we are faced with an oversimplified and accelerated mathematical and brainstorming conceptualization which is mostly good as the first step towards familiarization with gravitational standing waves as an explanation of the nature of gravitational attractive force. Taking and proving-valid such approach will have consequences on better understanding of origins of Gravitation.

2.3.3-4 Central Forces, Newton and Coulomb Laws

Next challenging question here is why central forces, like those that Newton and Coulomb laws are describing, are inversely dependent from the square of the relevant distance, $F(r) = \frac{C}{r^2}$, $C = \text{const.}$?

We can indirectly explain such situation ($F(r) = \frac{C}{r^2}$) by analyzing force components involved in orbital motions under central force. Since in cases of central forces relevant orbital momentum is constant $\frac{d\vec{L}}{dt} = 0 \Rightarrow \vec{L} = \vec{r} \times \vec{p} = \overline{\text{const.}}$, we can conclude that vector \vec{L} is perpendicular to the plane defined by the vector \vec{r} and the momentum \vec{p} . The fact that \vec{L} remains constant is saying that relevant plane (\vec{r} , \vec{p}) also remains constant (or stable), and that every orbital motion (on such plane) under central force is a stable, planar, and two-dimensional motion (which can naturally host standing waves structures without big need to give probabilistic or stochastic meaning to any of such waves). This is very much similar to astronomic observations documenting that many solar systems are planar, facilitating involved mathematical processing, as for example,

$$\begin{aligned} \vec{L} = L(r, \theta) &= m r^2 \frac{d\theta}{dt} = \text{const.}, \vec{F}(r, \theta) = \vec{F}(r) + \vec{F}(\theta) = \vec{F}(r) = \overline{\left(m \frac{d^2 \vec{r}}{dt^2} \right)} = m \vec{a}_r + m \vec{a}_\theta = \left[m \frac{d^2 r}{dt^2} - m r \left(\frac{d\theta}{dt} \right)^2 \right], \\ a_r &= \frac{d^2 r}{dt^2} - r \left(\frac{d\theta}{dt} \right)^2, a_\theta = r \frac{d^2 \theta}{dt^2} + 2 \frac{dr}{dt} \cdot \frac{d\theta}{dt}, m r \frac{d\theta}{dt} + 2 m \frac{dr}{dt} \frac{d\theta}{dt} = 0 \Rightarrow \frac{d\theta}{dt} = \frac{L}{m r^2} = \frac{L}{m} \rho^{-2}, \rho = \frac{1}{r}, \\ \frac{d^2 \rho}{dt^2} + \rho &= -\frac{m}{L^2} \cdot \frac{1}{\rho^2} F\left(\frac{1}{\rho}\right), F\left(\frac{1}{\rho}\right) = F(r) = \frac{C}{r^2} = C \cdot \rho^2 \Rightarrow \frac{d^2 \rho}{dt^2} + \rho = -\frac{mC}{L^2} \Rightarrow \rho = A \cos(\theta - \theta_0) - \frac{mC}{L^2}, \\ r &= \frac{1}{A \cos(\theta - \theta_0) - \frac{mC}{L^2}}. \end{aligned}$$

The last equation is describing conic curves $r = r(\theta)$, such as ellipse, parabola and hyperbola, depending on constants A , θ_0 , m , C , \vec{L} . If we chose the reference coordinates where $\theta_0 = 0$, we will get for a planetary and satellite orbits $r = \frac{1}{A \cos \theta - \frac{mC}{L^2}}$ that is a conic section, which can also be

transformed to $r = r_0 \frac{1+e}{1+e \cdot \cos \theta}$, where e presents eccentricity of the orbit. For $e < 1$, the orbit is an ellipse, for $e = 1$, the orbit is parabola, for $e > 1$ the orbit is hyperbola, and for $e = 0$ the orbit is a circle (covering all cases of planetary, asteroids and satellite orbits). ----- ♣]

It is becoming obvious that expression for planetary macro-wave (motional or kinetic) energy, $E_k = Hf_o$ from (2.11.13), is directly analog to Planck's wave-quantum energy (of a photon, $\tilde{E} = hf$, for instance), as well as macro equivalent for a wavelength $\lambda = \frac{H}{p}$ is also analog to a micro-world de Broglie matter wavelength $\lambda = \frac{h}{p}$ where new "macro-world Planck-like constant" H is,

$$\begin{aligned} H = H(m, r, n) &= \frac{2\pi}{n} \mathbf{L} = \frac{2\pi}{n} \frac{GMm}{v} = \frac{2\pi m \sqrt{GM r_n}}{n} = \frac{2\pi GMm}{v_0} = \\ &= \frac{2\pi r_n v m}{n} = \frac{\tilde{E}}{f_o} = \tilde{E}T = \text{const.} \gg h, \quad (n \in [1, 2, 3 \dots], v \ll c, m \ll M) \end{aligned} \quad (2.11.15)$$

$$\Rightarrow \left(\hbar = \frac{h}{2\pi} \right) \text{ analog to } \left(\hbar_{gr.} = \frac{H}{2\pi} = \frac{\mathbf{L}}{n} = \frac{m \sqrt{GM r_n}}{n} = \frac{GMm}{v_0} \right).$$

The difference between Planck's constant h and analog constant of planetary macro waves H is that h is already known as universally valid constant, and H can be different for every planetary and/or satellite system.... Of course, for certain planetary system, and for a sufficiently high integer $n = N$ in (2.11.15), we should be able to find when H will be equal to h , as for instance,

$$\begin{aligned} H = h &= \frac{2\pi m \sqrt{GM r}}{N} = \frac{2\pi GMm}{v_0} = 6.626 \ 0693(11) \cdot 10^{-34} \text{ J} \cdot \text{s}, \\ N &= v_0 \sqrt{\frac{r}{GM}} = 0.948252278 \cdot 10^{34} \cdot m \sqrt{GM r} = 0.77443828 \cdot 10^{29} \text{ m} \sqrt{Mr}, \\ (v_0 &= 0.77443828 \cdot 10^{29} \sqrt{G \cdot m \cdot M}), \end{aligned}$$

but such high quantum numbers are obviously unrealistic.

In fact, for certain planetary system (where each of planets in relation to a common Sun could be approximated as a Binary System), we should be able to find H that will be the same constant for each planet. For instance, if the ratio between any two of H constants, (2.11.16), applied for planets (with circular orbits) from the same solar system, will be equal to one, than we can say that H is at least locally valid constant, as for instance,

$$\begin{aligned} \frac{H_1}{H_2} &= \frac{H(m_1, r_1, n_1)}{H(m_2, r_2, n_2)} = \frac{n_2}{n_1} \frac{m_1}{m_2} \sqrt{\frac{r_1}{r_2}} = \frac{n_2}{n_1} \frac{m_1}{m_2} \frac{v_2}{v_1} = 1, \quad (n_1, n_2) \in [1, 2, 3 \dots], \\ H = H_1 = H_2 &\Rightarrow n_2 \cdot m_1 \cdot \sqrt{r_1} = n_1 \cdot m_2 \cdot \sqrt{r_2} \Leftrightarrow n_2 \cdot m_1 \cdot v_2 = n_1 \cdot m_2 \cdot v_1, \end{aligned} \quad (2.11.16)$$

$$\frac{H}{2\pi} = \frac{\mathbf{L}_1}{n_1} = \frac{\mathbf{L}_2}{n_2} = \frac{\mathbf{L}_1 + \mathbf{L}_2}{n_1 + n_2} = \frac{\mathbf{L}_i}{n_i} = \frac{GMm}{v_0}, \quad \frac{\mathbf{L}_2}{\mathbf{L}_1} = \frac{m_2}{m_1} \frac{v_1}{v_2} = \frac{n_2}{n_1}, \quad \frac{\mathbf{L}_2}{n_2} \frac{n_1}{\mathbf{L}_1} = \frac{n_1}{n_2} \frac{m_2}{m_1} \frac{v_1}{v_2} = 1.$$

The limitations of H constant expressions related to (2.11.15) and (2.11.16) are, that we are still approximating all orbits with circles (which are in the same plane) and we are not taking into account any planetary "self-spinning" momentum S . Another

limitation involved here is that orbital and spin moments conservation is fully valid only if we take in consideration a total resulting moment/s of all planets, moons and satellites of a solar system in question (see [36], Anthony D. Osborne, & N. Vivian Pope). Consequently, after implementing more elaborated analyses, we should be able to find more general and more precise expressions for \mathbf{H} . Present comments regarding planetary-world \mathbf{H} constant are still indicative brainstorming directions serving to establish the grounds for defending utility of such constant/s. Another interesting situation here is how to explain that micro-world \mathbf{h} -constant (or Planck constant) is unique and universally valid for all atomic and subatomic entities. Do we have (in our Universe) a succession of \mathbf{H} -constants, starting from certain big \mathbf{H} -numbers (for galactic formations) which are gradually descending towards smaller numbers with unique and constant \mathbf{h} -value at the opposite subatomic end, could be a question to answer?

It is obvious that here we are combining dynamics of orbital motions with certain kind of stable space packing expressed by necessity of standing waves formation, which is in a very close relation to relevant angular (and spin) moments conservation, applicable to inclinations of planetary orbits. In addition, since certain wave-like space-periodicity and stable packing in periodical planetary motions exists, it could also be presentable as integer multiple “ n_α ” of angular segments $\alpha = \frac{2\pi}{n_\alpha}$, capturing

the angle of a full circle that is equal $n_\alpha \alpha = 2\pi = n\lambda_o / r$, $n_\alpha = 1, 2, 3, \dots$ (of course, valid for an idealized and oversimplified situation),

$$\mathbf{L} = n \frac{\mathbf{H}}{2\pi} = \frac{n}{n_\alpha} \cdot \frac{\mathbf{H}}{\alpha} = \frac{\mathbf{H}}{\lambda_o} r = \frac{2\pi\sqrt{GM}}{n\lambda_o} \cdot mr^{3/2} = m\sqrt{GM}r, \quad (2.11.17)$$

$$\alpha = \frac{2\pi}{n_\alpha} = \frac{n}{n_\alpha} \cdot \frac{\mathbf{H}}{\mathbf{L}} = \frac{n}{n_\alpha} \cdot \frac{\lambda_o}{r} = \frac{2\pi\sqrt{GM}}{n_\alpha \mathbf{L}} \cdot mr^{1/2}, \quad (n, n_\alpha) \in [1, 2, 3, \dots].$$

For instance, spherical coordinate system (which should naturally be most applicable here) has one radial coordinate and two angular, and we should use minimum three different quantum numbers for describing such spatial standing waves packaging. The more general approach regarding inclinations of planetary orbits, instead (2.11.17) should be an angular or spatial quantizing of relevant orbital mater waves, like in A. Sommerfeld quantization and semi-classical quantization of angular momentum (see [40], D. Da Roacha and L. Nottale). See later more of supporting background under “Wavelength analogies in different frameworks”, T.4.2, as well as extended matter-waves conceptualization with equations 4.3-1, 4.3-2, 4.3-3 and Fig.4.1.2, Fig.4.1.3 and Fig.4.1.4, all from the chapter 4.1.

The ideas, modeling and documented astronomic observations about solar systems quantization of orbital radius and relevant velocities (like results in (2.11.14)) are already known from the publications of: William Tiff, Rubčić, A., & J. Rubčić, V. Christianto, Nottale, L. and their followers (see literature under [37], [38], [39], [40], [41], and [42]). The possibility, suggested by the observation of velocity quantization (72 km/s, Tiff, 1978, [37]) in the redshifts of galaxies, that wave-particle duality with a much larger value of Planck's constant may apply at galactic distances is also examined. For instance in (2.11.14), we have the phase velocity found as

$u = u_n = \frac{1}{2} \sqrt{\frac{GM}{r}} = \frac{v_0}{2n} \cong \frac{v}{2}$. The galactic (phase) velocity redshifts measured by Tiffit are very often found to be around 72 km/s (see [37]), and the best-known estimate for specific velocity is $v_0 = 144.7 \pm 0.7 \left[\frac{\text{km}}{\text{s}} \right] = 2 \times 72.35 \pm 0.35 \left[\frac{\text{km}}{\text{s}} \right]$ (see (2.11.14)). Of course, for higher values of quantum numbers $n = 2, 3, \dots$ we should be able to detect other galactic (phase-velocity related) red-shifts such as $u = \frac{v_0}{2n} \cong \frac{144}{2n} = \frac{72}{n} \left[\frac{\text{km}}{\text{s}} \right] \Rightarrow u \in \left(\frac{72}{2} = 36, \frac{72}{3} = 24, \frac{72}{4} = 18 \dots \right) \left[\text{km/s} \right]$. It is also found that orbital velocities (see

(2.11.14)) of planets and satellites belonging to our Solar System, $v_n = \sqrt{\frac{GM}{r_n}}$, multiplied by n ($n = 1, 2, 3 \dots$) are equal to the multiple of a fundamental velocity which is close to $24 \left[\text{km/s} \right]$. In addition, increments of the intrinsic galactic redshifts are found to be $\cong 24 \left[\text{km/s} \right]$ (see [40] Nottale; [41] Rubčić, A., & J. Rubčić; [43] M. Pitkänen), very much similar to the predictable situation regarding quantized orbital, planetary and satellite velocities from (2.11.14). Surprisingly, in here mentioned literature regarding redshifts, nobody related such situation to phase velocity on the way as conceptualized here (related to orbital, macro-cosmological matter waves). Since measured spectral redshifts are really affected by orbital phase velocity $u = u_n$ it is almost obvious that here hypothesized standing-waves field structure should exist. All of that (about measured redshifts) is in addition, implicitly suggesting that electromagnetic dipoles polarizations between rotating (and spinning) astronomic objects could also be involved here (as speculated earlier in this chapter; -see equations from (2.4-6) to (2.4-10)). W. Tiffit type of redshift can analogically be related to (still hypothetical) **“Planetary Vortex Shedding”** phenomenology, known in fluid motions as “Karman Vortex Street”. If certain kind of “Planetary Karman Street” is on some way following planets and astronomic size objects (like an oscillatory, or helix tail), the light coming from distant sources, and passing such spatial zones of “Planetary Karman Streets” will be velocity-modulated, causing measured redshifts, and probably in some cases **“blueshifts”**. See more about vortex shedding in the chapter 4.1, around equations (4.3-0) and (4.3-0)-a,b. If measured spectral frequency shifts of electromagnetic waves (also applicable to any other cosmic rays, x-rays, neutrinos, ...) coming from distant astronomical sources and passing solar or planetary systems, are showing certain quantized and predictable repetition nature (based on Doppler Effect), and if such quantization can be numerically related to quantizing of planetary group and phase velocity, we will have almost direct confirmation of innovative concepts regarding gravitation, as promoted in this paper. Of course, here we should not neglect other types of periodical signal modulations, like detecting amplitude and phase modulations caused by complex structure of fields associated to planetary orbital motions.

Anyway, our macro-universe is known to behave as a big and very precise astronomic clock, where periodical motions are its intrinsic property. It will be just a matter of finding or fitting proper integers $(n, n_\alpha) \in [1, 2, 3, \dots]$ into above given (or similar) macro matter-waves relations, in order to support here presented concept. Of course, the

situation analyzed here is presently addressing only purely circular planetary orbits (for having mathematical simplicity and faster introduction), and in later analyses we would need to take into account elliptic and other self-closed planetary orbits (this way, most probably generating additional quantum numbers, or integers like n, n_α). Quantizing of planetary orbital motions presented here is realized using extremely simple, geometrical concepts analog to N. Bohr atom modeling $n\lambda = 2\pi r_n, \lambda = h/p = h/mv_n$. The natural development of such quantized model of planetary systems will be in some ways similar to the evolution of Bohr's planetary atom model towards Sommerfeld's atom model (related to the period before the wave and probabilistic quantum mechanics and Schrödinger equations started to be dominant theoretical approach). For micro-world, we are simply implementing or associating universal quantization by default, since this is well known and tested in many cases (Planck, L. de Broglie, Einstein, Sommerfeld...). We should not forget that analogical planetary, orbital quantizing is not possible without global, macro-universe conservation of relevant orbital moments.

Also, as a significant theoretical background and support to the innovative concept of Macro-Cosmological stability and gravitation (presented here) the following reference should be taken into account: [36], Anthony D. Osborne, & N. Vivian Pope, "An Angular Momentum Synthesis of 'Gravitational' and 'Electrostatic' Forces".

♣ COMMENTS & FREE-THINKING CORNER:

For stable and planar solar systems we already know quantization rules (2.11.14) applicable for an orbit radius and relevant planetary (or tangential) velocity. Most of stable solar systems are not necessarily planar and we should consider existence of similar quantization (or spatial orbits packaging) regarding angular orbits positions (or orbit inclination towards certain reference orbital plane).

In order to generalize the same concept (already elaborated with equations from (2.11.13) to (2.11.17)) for any closed planetary orbit, we can apply Wilson-Bohr-Sommerfeld action integrals (used in supporting N. Bohr's Planetary Atom Model). Wilson-Bohr-Sommerfeld action integrals (see [9]), related to any periodical motion on a self-closed stationary orbit C_n , applied over one period of the motion, present the kind of general quantifying rule (for all self-closed standing waves, which are energy carrying structures, having constant angular momentum). Sommerfeld (see chapter 5; equations (5.4.1)) extended Bohr atom model to cover elliptic (and circular) electron (or planetary) orbits, where semi-major axis is "a" and semi-minor axis is "b". We can (just to initiate brainstorming in that direction) analogically (also still hypothetically, and highly speculatively) apply the same strategy on a planet which has mass m and which is rotating around its sun, which has mass $M \gg m$, on the following way,

$$\left. \begin{aligned} \left\{ \oint_{C_n} L d\alpha = n_\alpha H, L = \text{Constant}, 0 \leq \alpha \leq 2\pi \right\} &\Rightarrow L = n_\alpha \frac{H}{2\pi}, n_\alpha = 1, 2, 3, \dots, n \\ \left\{ \oint_{C_n} p_r dr = n_r H \right\} &\Rightarrow L \left(\frac{a}{b} - 1 \right) = n_r \frac{H}{2\pi}, n_r = 0, 1, 2, 3, \dots \end{aligned} \right\} \Rightarrow \quad (2.11.18)$$

$$\Rightarrow a = \frac{m+M}{(mM)^2} \frac{\left(\frac{H}{2\pi}\right)^2}{G} n^2 \cong \frac{\left(\frac{H}{2\pi}\right)^2}{GMm^2} n^2 = a_0 n^2, b = a \frac{n_\alpha}{n} = a_0 n_\alpha n, n \equiv n_\alpha + n_r = 1, 2, 3, 4, \dots$$

In addition to (2.11.18), for a certain stable (planar) planetary system with number of planets (or even for our universe) it should also be valid that its total angular momentum is constant (including spinning moments of planets, moons and asteroids),

$$\left. \begin{aligned} \left[\begin{aligned} \bar{\omega}_c &= \frac{\sum_{(i)} J_i \bar{\omega}_i}{\sum_{(i)} J_i} = \frac{\sum_{(i)} \bar{L}_i}{\sum_{(i)} J_i} \\ \oint_{C_n} L d\alpha &= n_\alpha H, \quad \oint_{C_n} p dr = n_r H \\ [n_\alpha, n_r &\in [1, 2, 3, \dots]] \end{aligned} \right] \Rightarrow \left[\begin{aligned} \bar{L} &= \sum_{(i)} \bar{L}_i = \sum_{(i)} J_i \bar{\omega}_i = \bar{\omega}_c \sum_{(i)} J_i = \text{const.} (= n \frac{H}{2\pi}) \\ \sum_{(i)} \oint_{C_n} L_i d\alpha &= \sum_{(i)} n_i H_i = nH = \text{const.} \end{aligned} \right] \\ &\& \\ \left[\left(\bar{p}, \frac{E}{c} \right)^2 = \text{inv.} \Rightarrow p^2 - \left(\frac{E}{c} \right)^2 &= - \left(\frac{E_0}{c} \right)^2 \Leftrightarrow p^2 c^2 - E^2 = -E_0^2 \Leftrightarrow p^2 c^2 - (\gamma m c^2)^2 = -(m c^2)^2 \right] \\ \Rightarrow \left[\begin{aligned} \left(\frac{\bar{r}}{r} \times \left(\bar{p}, \frac{E}{c} \right) \right)^2 &= \left(\frac{1}{r} \left(\bar{L}, \frac{E}{c} \bar{r} \right) \right)^2 = \text{inv.}, \Rightarrow \frac{1}{r_1^2} \left[\bar{L}_1^2 - \left(\frac{E}{c} \right)^2 r_1^2 \right] = \frac{1}{r_2^2} \left[\bar{L}_2^2 - \left(\frac{E_0}{c} \right)^2 r_2^2 \right] \\ \gamma &= \gamma_1, \gamma_2 = 1, \quad \bar{L} = \text{const.} \end{aligned} \right] \\ \Rightarrow L &= L_1 = L_2 = \frac{r_1 r_2}{c} \sqrt{\frac{E^2 - E_0^2}{r_2^2 - r_1^2}} = m c r_1 r_2 \sqrt{\frac{\gamma_1^2 - 1}{r_2^2 - r_1^2}} = \text{const.}, \Rightarrow r_1 r_2 \sqrt{\frac{\gamma_1^2 - 1}{r_2^2 - r_1^2}} = \text{const.} \end{aligned} \right\} \Rightarrow \quad (2.11.19)$$

Of course, if we have combination/s of orbital (**L**) and spin moments (**S**), we would need to replace **L** with **L+S**. As we can see in [36], Anthony D. Osborne, & N. Vivian Pope effectively and analogically (could be unintentionally, too) made a very significant extension of Sommerfeld concept to the macro-world of planets, stars and galaxies, and it is obvious that the new chapter of Cosmology and Astronomy is being initiated. ♣]

Elements of certain stable space-time structure with periodical motions (planetary systems, for instance) are mutually coupled by fields and forces integrating them into a stable macro system, and important associated condition or consequence regarding such stability is the creation of standing waves of involved fields. Positions and paths of planets (inside such systems) are defined by stable or stationary energy-momentum conditions of the system in question, which are related to system minimal energy dissipation, or maximal mechanical quality factor conditions (for instance, found by solving relevant Euler-Lagrange-Hamilton equations). In order to give an idea how we could evolve this quantum-like conceptualization of Gravitation it would be very useful to see the Appendix (at the end of this book) that is innovatively treating “**Bohr’s model of hydrogen atom and particle-wave dualism**”. Of course, some other time, ideas paved with (2.11.10) - (2.11.21) should be better elaborated, extended and verified, *but significant and innovative brainstorming breakthrough is already made*. What we should, conceptually and imaginatively, visualize and upgrade here is that we are no more dealing only with time-stable and spatially isolated linear (and circular) planetary orbits and discrete planetary masses. Masses of planets in orbital motions are embedded in certain energy-momentum, time and spatial distributions that are presenting material extensions, links and bridges between all elements of such planetary systems. What we see as planetary masses and orbits (described by Kepler and Newton laws) are only space-time localized effective centers (and/or channels) of such energy-momentum agglomerations that are (in a wider space-time frame) structured as standing waves.

Based on the planetary macro waves conceptualization which is presented from (2.11.12) until (2.11.19) we can also create kind of Schrödinger equation valid for such standing-waves situations of quantized mass (or energy-momentum) distributions. Here we should consider relevant (equivalent) mass in its extended meaning as relativistic, velocity-dependent, spatially distributed and coupled with surrounding energy-momentum states and fields (familiar to conceptualization given in “**2.2. Generalized Coulomb-Newton Force Laws**”, equations (2.3) - (2.4-3)). This time, relevant wave function Ψ is directly related to a planet or satellite self-closed path or orbit, or to a relevant radius of orbiting (like having standing waves on a self-closed and oscillating circular string). Geometrically and analogically, this is a modeling based on formulating closed space structures where all relevant and mutually coupled motional elements with certain periodicity are becoming stationary and stable. The first association related to any standing waves formation is that this should also be a kind of resonance. In addition, such structures or states have integer number of certain elementary wavelengths or integer number of other relevant elementary domains, and such states can be qualified as quantized states. Really, there is nothing more significant to implement or profit from Quantum Theory here.

For getting better idea about such “standing waves *packing*” it is useful to see: *Chapter 4.1, T.4.2., Wavelength analogies in different frameworks*, and *Chapter 5, T.5.3., Analogical Parallelism between Different Aspects of Mater Waves*. If we insist

to create some simplified, preliminary and conceptual visualization of planetary orbital motions with associated gravitational matter-waves structure (as spatially distributed energy-momentum states, enclosed in **toroidal** forms), this could be intuitively linked to the illustration on Fig. 2.6. and strongly related to future creative modeling and innovative solutions resulting from equations (2.11.20) to (2.11.23).

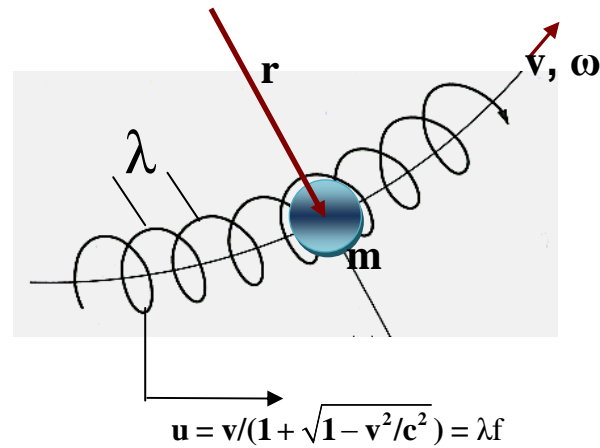


Fig.2.6. Gravitational matter waves and orbital planetary motion

Natural way of modeling such geometrical forms (standing waves) is directly related to Schrödinger equation, which has its conceptual, historical and analogical origins in generalization of d'Alembert equation, which is related to standing waves oscillations of an ordinary string. In addition, the same, classical wave equation (in certain analogical form of d'Alembert equation) has been known in different fields of Classical Mechanics, Fluid Mechanics, Acoustics and Maxwell Electromagnetic Theory long before being "renamed, modified and analogically applied" to waves phenomenology in micro Physics by Schrödinger and other Quantum Theory founders. The establishment of Schrödinger equation is well known and in this paper additionally elaborated and generalized later, in the chapter 4.3. Here, we will analogically apply such generalized Schrödinger-like equations to planetary orbital motions, since planets are also respecting certain periodicity and "*macro matter-waves packaging rules*", like standing, de Broglie matter waves in a micro universe (see explanations around equations (2.11.5)-(2.11.9), (2.11.9-1)-(2.11.9-4) , (2.9.5-1)-(2.9.5-5) and (2.11.12)- (2.11.14)). Another striking analogy (showing that a mass in motion has some kind of associated helix-spinning field, or tail) is related to fluid flow vortices, as speculated in the chapter 4.1; -see equations (4.3-0), (4.3-0)-a, (4.3-0)-b.

Let us briefly specify set of the final forms of such equation (by applying full analogy with (4.10) from the Chapter 4.3), which will address deterministic (non stochastic) planetary and satellites orbital motions in a field of gravitation, as for instance,

$$\left[\begin{array}{l}
\frac{\hbar^2}{\tilde{m}} \left(\frac{u}{v} \right) \Delta \bar{\Psi} + (\tilde{E} - U_p) \bar{\Psi} = \left(\frac{\tilde{E} - U_p}{\hbar} \right)^2 \cdot \bar{\Psi} + \frac{\partial^2 \bar{\Psi}}{\partial t^2} = \frac{\partial \bar{\Psi}}{\partial t} + \mathbf{u} \nabla \bar{\Psi} = \mathbf{0}, \\
\nabla = \mathbf{i} \frac{\partial}{\partial x} + \mathbf{j} \frac{\partial}{\partial y} + \mathbf{k} \frac{\partial}{\partial z} = \mathbf{grad}, \Delta = \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}, \bar{\Psi} = \Psi + \mathbf{j} \hat{\Psi}, \mathbf{j}^2 = -1
\end{array} \right] \quad (4.10), \text{chapter 4.3}$$

&

$$\left[\begin{array}{l}
\mathbf{v} = \omega_m \mathbf{r} \cong 2\mathbf{u} = 2\lambda_0 \mathbf{f}_0 = \frac{2\pi \mathbf{r}}{T} = 2\pi r \mathbf{f}_m = \sqrt{\frac{GM}{r}} \ll c, \left(\mathbf{r} = r_0 \frac{1+e}{1+e \cdot \cos \theta} \right), \\
\omega_m = 2\pi f_m = \frac{4\pi f_0}{n} = \frac{2\pi}{T} = \frac{\sqrt{GM}}{r^{3/2}} = \frac{v}{r}, \mathbf{p} = m\mathbf{v} = \frac{\mathbf{H}}{\lambda_0} = \frac{n\mathbf{H}}{2\pi r}, n = 1, 2, 3, \dots \quad (2.11.13) \\
\mathbf{L} = n \frac{\mathbf{H}}{2\pi} = \mathbf{p}r = m\mathbf{v}r = m r^2 \omega_m = m r^2 \frac{2\pi}{T} = 2\pi m r^2 f_m = \sqrt{GM} r \cdot m, \quad (2.11.14), \quad (2.11.15) \\
\tilde{E} = E_k = \frac{1}{2} m v^2 = H f_0 = \frac{GMm}{2r}, U_p = -\frac{GMm}{r}, E_k - U_p = \frac{3}{2} \frac{GMm}{r}
\end{array} \right] \Rightarrow$$

&

$$\left[\begin{array}{l}
\mathbf{h} \leftrightarrow \mathbf{H} = \frac{2\pi \sqrt{GM} r}{n} \cdot m, \hbar = \frac{h}{2\pi} \leftrightarrow \hbar_{gr.} = \frac{H}{2\pi} = \frac{\sqrt{GM} r}{n} \cdot m = \frac{GMm}{v_0}, \\
\mathbf{m} \leftrightarrow \frac{mM}{m+M} \cong m \ll M.
\end{array} \right]$$

$$\Rightarrow \left[\begin{array}{l}
\frac{\left(\frac{H}{2\pi} \right)^2}{2m} \Delta \bar{\Psi} + (E_k - U_p) \bar{\Psi} = \left(2\pi \frac{E_k - U_p}{H} \right)^2 \cdot \bar{\Psi} + \frac{\partial^2 \bar{\Psi}}{\partial t^2} = \frac{\partial \bar{\Psi}}{\partial t} + \frac{v}{2} \nabla \bar{\Psi} = \mathbf{0} \Rightarrow \\
\frac{H^2}{8\pi^2 m} \Delta \bar{\Psi} + \left(\frac{3GMm}{2r} \right) \bar{\Psi} = \left(\frac{3\pi GMm}{Hr} \right)^2 \cdot \bar{\Psi} + \frac{\partial^2 \bar{\Psi}}{\partial t^2} = \frac{\partial \bar{\Psi}}{\partial t} + \frac{v}{2} \nabla \bar{\Psi} = \mathbf{0} \Rightarrow \\
\Rightarrow \frac{GMmr}{2n^2} \Delta \bar{\Psi} + \frac{3GMm}{2r} \bar{\Psi} = \frac{9GMn^2}{4r^3} \cdot \bar{\Psi} + \frac{\partial^2 \bar{\Psi}}{\partial t^2} = \frac{\partial \bar{\Psi}}{\partial t} + \frac{1}{2} \sqrt{\frac{GM}{r}} \nabla \bar{\Psi} = \mathbf{0}, \\
\frac{r}{2n^2} \Delta \bar{\Psi} + \frac{3}{2r} \bar{\Psi} = \frac{9n^2}{4mr^3} \cdot \bar{\Psi} + \frac{1}{GMm} \frac{\partial^2 \bar{\Psi}}{\partial t^2} = \frac{1}{GMm} \frac{\partial \bar{\Psi}}{\partial t} + \frac{1}{2\sqrt{GM} r m} \nabla \bar{\Psi} = \mathbf{0}, \\
\Rightarrow \frac{r^2}{3n^2} \Delta \bar{\Psi} + \bar{\Psi} = \mathbf{0}, \frac{9GMn^2}{4r^3} \cdot \bar{\Psi} + \frac{\partial^2 \bar{\Psi}}{\partial t^2} = \mathbf{0}, \frac{2r^{1/2}}{\sqrt{GM}} \frac{\partial \bar{\Psi}}{\partial t} + \nabla \bar{\Psi} = \mathbf{0}.
\end{array} \right] \quad (2.11.20)$$

Since we know that planets and satellites are often spinning (beside rotating around big masses), in all equations from (2.11.13) until (2.11.20) we would need to increase orbital moment \mathbf{L} for the amount of associated relevant spin moment \mathbf{S} , or $\mathbf{L} \rightarrow \mathbf{L} + \mathbf{S}$. The consequences of solving (2.11.20) will show that planetary and satellite orbits are presenting closed circular or elliptic lines only approximately (in average). In reality, certain harmonic, standing-waves orbit-line, or relevant radius amplitude modulation with toroidal field envelope should be measurable when planets and satellites' orbits are being very precisely monitored (because of rotation and mutual interactions among participants).

It is obvious that a future development of here introduced macro-cosmological matter-waves concept will significantly enrich our understanding of Gravitation. For instance, some form of extended and more precise Bode's law (including other angular and space quantization and time-dependent waving properties) could result from the solutions of (2.11.20),

$$\begin{aligned} \frac{rH^2}{12\pi^2GMm^2} \Delta \bar{\Psi} + \frac{3}{r} \bar{\Psi} &= \frac{GM}{v_0^2} \Delta \bar{\Psi} + \frac{3}{r} \bar{\Psi} = \frac{\hbar_{gr.}}{mv_0} \Delta \bar{\Psi} + \frac{3}{r} \bar{\Psi} = 0, \\ \left(\frac{3\pi GMm}{Hr}\right)^2 \cdot \bar{\Psi} + \frac{\partial^2 \bar{\Psi}}{\partial t^2} &= \frac{9v_0^2}{4r^2} \cdot \bar{\Psi} + \frac{\partial^2 \bar{\Psi}}{\partial t^2} = 0, \\ \frac{\partial \bar{\Psi}}{\partial t} + \frac{v}{2} \nabla \bar{\Psi} &= 0. \end{aligned} \quad (2.11.21)$$

In order to get generally valid and fully natural solutions $\bar{\Psi} = \bar{\Psi}(\mathbf{r}, \theta, \varphi, t)$ of (2.11.20), involved operators ($\nabla, \nabla^2 = \Delta$) should be applied in spherical, polar coordinates $(\mathbf{r}, \theta, \varphi)$. In addition, elliptic planetary orbits should be taken into account (of course, after upgrading all equations from (2.11.12) until (2.11.18), which are valid for ideal circular orbits, into new and equivalent expressions applicable for elliptic planetary orbits, and consequently, all of that will slightly modify differential equations found in (2.11.20)).

In fact, full and correct explanation of the ideas found in (2.4)-(2.11.20) could be much more complex than here presented, but for the purpose of introducing new concepts about Quantum Gravitation, Particle-Wave Duality, force-field charges and unification between linear and rotational elements of every motion, given conceptual platform is already sufficiently clear. In order to understand wider meaning of wave functions it is useful to see the chapter "4.3 Wave Function and Generalized Schrödinger Equation"; -equations: (4.33-1), (4.41-1) to (4.45), T.4.2 and T.4.3, as well as mater-waves conceptualization around equations (4.3) and (4.3-1) in the chapter 4.1.

In addition, based on Parseval's identity that is universally valid and connecting time and frequency domains of any wave function (see chapter 4.0, equations (4.0.4)), the wave function $\bar{\Psi} = \bar{\Psi}(\mathbf{r}, \theta, \varphi, t)$, instead of being an oscillating amplitude, or displacement, or orbital radius (like in (2.11.20) and (2.11.21)), could also, analogically get an extended meaning of spatially-distributed power (see (2.11.22)), where number of closely related conditions and relations should be satisfied, such as,

$$\Psi^2 = \frac{dE}{dt} = \frac{dE_k}{dt} = \frac{d\tilde{E}}{dt} = v \frac{dp}{dt} = \omega_m \frac{dL}{dt} = vF = \omega_m \tau = \text{Power} = P,$$

$$\left. \begin{aligned} E_k = \tilde{E} &= \frac{1}{2} mv^2 = mvu = pu = 2mu^2 = \frac{1}{4} mv_c^2 = \frac{GmM}{2r} = \frac{1}{2} \cdot \left(\frac{GmM}{r^2} \right) \cdot r = \frac{1}{2} \cdot F_{m-M} \cdot r = \\ &= \frac{m}{2} \left(\frac{2\pi r}{T} \right)^2 = \frac{8m\pi^2 r^2}{n^2} f_o^2 = 2m(\pi r f_m)^2 = (2\pi m r^2 f_m) \cdot (\pi f_m) = L\pi f_m = \left(\frac{2L\pi}{n} \right) \cdot f_o = Hf_o = \\ &= \int_{-\infty}^{+\infty} \Psi^2(t) dt = \int_{-\infty}^{+\infty} \hat{\Psi}^2(t) dt = \frac{1}{2} \int_{-\infty}^{+\infty} |\bar{\Psi}(t)|^2 dt = \frac{1}{2} \int_{-\infty}^{+\infty} a^2(t) dt = \int_{-\infty}^{+\infty} \left[\frac{a(t)}{\sqrt{2}} \right]^2 dt = \\ &= \frac{1}{2\pi} \int_{-\infty}^{+\infty} |\bar{U}(\omega)|^2 d\omega = \int_{-\infty}^{+\infty} \left| \frac{\bar{U}(\omega)}{\sqrt{2\pi}} \right|^2 d\omega = \frac{1}{\pi} \int_0^{+\infty} [A(\omega)]^2 d\omega = \int_0^{+\infty} \left[\frac{A(\omega)}{\sqrt{\pi}} \right]^2 d\omega = \\ &= \int_{-\infty}^{+\infty} P(t) dt (=) [J] \end{aligned} \right\}, \quad (2.11.22)$$

$$\left. \begin{aligned} v = u - \lambda \frac{du}{d\lambda} &= -\lambda^2 \frac{df}{d\lambda} = u + p \frac{du}{dp} = \frac{d\omega}{dk} = \frac{d\tilde{E}}{dp} = H \frac{df}{dp} = \frac{df}{df_s} = \frac{2u}{1 + \frac{uv}{c^2}}, \\ u = \lambda f &= \frac{\omega}{k} = \frac{\tilde{E}}{p} = \frac{Hf}{p} = \frac{f}{f_s} = \frac{v}{1 + \sqrt{1 - \frac{v^2}{c^2}}} = \frac{E_k}{p}, f_s = k/2\pi, \lambda = \frac{H}{p}, \\ \Rightarrow 0 \leq 2u &\leq \sqrt{uv} \leq v \leq c \end{aligned} \right\},$$

because it is clear that here (regarding orbital planetary motions) we are dealing with harmonic and periodical wave functions which are creating stable, self-closed standing waves and resonant-like field states with very high quality factors. It is also important to mention that certain unbounded, creative and intellectually flexible approach should be implemented in fruitful merging between the wave function environment from (2.11.22) and wave equations (2.11.21) in order to formulate new wave and quantum gravity theory. What we have here so far, are just early brainstorming and intuitive steps.

♣ COMMENTS & FREE-THINKING CORNER:

An innovative extension of N. Bohr's atom model (presented at the end of this book: Appendix, "8. BOHR'S MODEL OF HYDROGEN ATOM AND PARTICLE-WAVE DUALISM") is elaborating that such structures (like hydrogen atom, or planetary systems; - see equations (8.64) until (8.74), and "8.3. Structure of the Field of Subatomic Forces") should be surrounded with complex force field $\mathbf{F}(r, \theta, \phi, t)$, which has at least two force components $\mathbf{F}_1(r, \theta, \phi, t) + \mathbf{F}_2(r, \theta, \phi, t)$ (one potential and the other solenoidal vector field), as for instance:

$$\mathbf{F}(r, \theta, \phi, t) = \mathbf{F}_1(r, \theta, \phi, t) + \mathbf{F}_2(r, \theta, \phi, t), \nabla \times \mathbf{F} \neq 0, \nabla \mathbf{F} \neq 0 \Rightarrow$$

$$\Rightarrow \begin{cases} \nabla \times \mathbf{F}_1 = 0, \nabla \mathbf{F}_1 \neq 0 \\ \nabla \times \mathbf{F}_2 \neq 0, \nabla \mathbf{F}_2 = 0 \end{cases} \quad (2.11.23)$$

We could also try modeling field of gravitation on a similar way by exploring the possibility that gravitational force is a composition (or superposition) of one solenoidal and one potential vector field, just to mention possibilities how and where to make the next step. Gravitational attraction is implicitly suggesting that there is certain energy fluctuation and gravitational potential (which has certain associated speed) between mutually attracting masses (see supporting elaborations around equations (2.4-11) – (2.4-17)). Presence of such energy fluctuations (also related to standing waves phenomenology) would alternate the meaning of particle velocity (in a field of gravitation that has solenoidal and potential vector components). If we decode, evolve and apply ideas of an extraordinary and unusual freethinker, as Evangelos Karamihas is, (see [48]), we will be able to develop (with certain level of creativity and intellectual flexibility) another form of relativistic mass, as follows,

$$\begin{aligned}
\mathbf{F}_{1-2} = -\mathbf{F}_{2-1}, |\mathbf{F}_{1-2}| = |\mathbf{F}_{2-1}| &= G \frac{m_1 m_2}{r^2} = m_1 \mathbf{a}_{1g} = m_2 \mathbf{a}_{2g} = m_1 \frac{d\mathbf{v}_{1g}}{dt} = m_2 \frac{d\mathbf{v}_{2g}}{dt} \Rightarrow \\
\Rightarrow \left\{ \begin{array}{l} \mathbf{a}_{1g} = \frac{d\mathbf{v}_{1g}}{dt} = \frac{d^2 \mathbf{r}_1}{dt^2} = G \frac{m_2}{r^2}, \mathbf{a}_{2g} = \frac{d\mathbf{v}_{2g}}{dt} = \frac{d^2 \mathbf{r}_2}{dt^2} = G \frac{m_1}{r^2} \\ \mathbf{v}_{1g} = \frac{d\mathbf{r}_1}{dt}, \mathbf{v}_{2g} = \frac{d\mathbf{r}_2}{dt} \end{array} \right\} &\Rightarrow \left\{ \begin{array}{l} \sqrt{r} d\mathbf{r}_1 = \sqrt{Gm_2} \cdot dt \\ \sqrt{r} d\mathbf{r}_2 = \sqrt{Gm_1} \cdot dt \\ \mathbf{r} = \mathbf{r}_1 + \mathbf{r}_2 \end{array} \right\} \Rightarrow \\
\Rightarrow \left\{ \begin{array}{l} v_{1g}^2 = \frac{Gm_2}{r}, v_{2g}^2 = \frac{Gm_1}{r} \\ \mathbf{a}_{1g} = \frac{v_{1g}^2}{r}, \mathbf{a}_{2g} = \frac{v_{2g}^2}{r} \\ \mathbf{m}(\mathbf{v}) = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \end{array} \right\} &\Rightarrow \left\{ \begin{array}{l} m_1 = \frac{m_{10}}{\sqrt{1 - \frac{v_{1g}^2}{c^2}}} \\ m_2 = \frac{m_{20}}{\sqrt{1 - \frac{v_{2g}^2}{c^2}}} \\ \mathbf{m}(\mathbf{v}_g) = \frac{m_0}{\sqrt{1 - \frac{v_g^2}{c^2}}} \\ v^2 = v_g^2 + (\mathbf{v}^*)^2, (\mathbf{v}^*)^2 = v^2 - v_g^2 \\ \mathbf{m}^* = \frac{m_0}{\sqrt{1 - \frac{(\mathbf{v}^*)^2}{c^2}}} = \frac{m_0}{\sqrt{1 - \frac{v^2 - v_g^2}{c^2}}} \end{array} \right\} \Rightarrow \\
\Rightarrow \left\{ \begin{array}{l} \mathbf{m}^*_{(v \rightarrow 0)} = \frac{m_0}{\sqrt{1 + \frac{v_g^2}{c^2}}} \cong m_0 \\ \mathbf{m}^*_{(v \rightarrow c)} = m_0 \frac{c}{v_g} < \infty \end{array} \right\}. & \quad (2.11.24)
\end{aligned}$$

As we can see from (2.11.24), mass \mathbf{m}^* in (relativistic) relation to its velocity \mathbf{v}^* (in the field of gravitation) would never reach infinity, even in a case of moving by maximal speed of light, c . ♣]

2.4. How to unite Gravitation, Rotation and Electromagnetism

It should already be obvious that all possible forces and fields in our Universe are anyway united, regardless whether we know how to formulate the Unified Field Theory. In given (philosophical) frontiers we can just mention the best starting points for creating a new (analogous and hypothetical) field structure of rectilinear and rotational motions, which would (conveniently) follow Faraday-Maxwell electromagnetic field definition. For instance, Lorentz and Laplace forces are the explicit connection between rectilinear motion of electrical charge (current) and magnetic field, and can be transformed to analogue forms of certain interaction between participants being in rectilinear and rotational motion (using concepts of analogies already elaborated here). The Ampere-Maxwell's, Biot-Savart's, and Faraday's induction laws can serve to complete mathematically the previous situation, for more precise description of "rectilinear rotational" field/s (just by transforming mentioned laws into corresponding analogue expressions).

A number of attempts are already known in modern science regarding formulation of Maxwell-like theory of gravitation and explaining the origin/s of inertia. Traditional formulation of gravitation field takes mass as a (primary) source of gravity. However, in this paper it is demonstrated that more essential (and dominant) source/s of gravity-related phenomenology should be found in different interactions between moving objects, such as, between their linear and/or angular (or spinning) moments, which are coupled with certain electric and magnetic moments and/or dipoles, and between their motional and state of rest energies. (See also chapter 4.1 of this paper for more supporting elements regarding associated de Broglie, matter waves). Revitalizing and updating Wilhelm Weber's force law (to cover electromagnetic and gravity related interactions, with combined linear and rotational motion elements) would be a very healthy platform towards establishing new Maxwell-like theory of gravitation (see literature [28] and [29]).

Most probably that many force/fields manifestations and components of constant, or accelerated movements, (such as Coriolis, centrifugal, centripetal, gyroscope effect, pendulum oscillations, inertial and similar forces, Gravitomagnetic induction from General Relativity Theory etc.) could conveniently be incorporated, interpreted and mutually united with here proposed concepts about gravitation. It is conceptually already clear what the author of this paper is proposing related to links between rotation, linear motion and electromagnetism. See also equations (4.18), (4.22)-(4.29), (5.15) and (5.16).

After establishing new platform for understanding the complementary nature of "linear-rotational" field/s and motions (see chapters 4.0 to 4.3 of this paper), we shall have an open way for creating full set of "Gravity-Rotation" field equations, making them initially analogue with Maxwell equations of electromagnetic field. Later on, we could modify and/or upgrade such equations up to the most meaningful and useful forms that will correspond to reality of different natural fields and forces (see development of equations (4.22)-(4.29)). Later (in chapter 3), it will be shown that Maxwell Theory should also be slightly upgraded in order to become compatible for unification with upgraded theory of Gravitation (see also literature [23] – [26]).

Anyway, in order to present a significant and new insight regarding gravitation we would need to introduce very new and original concepts that do not present only redundant and

analogical variations of already known field theories. Let us initiate one of such concepts, as follows.

2.5. New Platforms for Understanding Gravitation

Gravitation could also be conceptualized by making analogies with mechanical or acoustical resonators. *Let us imagine that our macro-universe or cosmos effectively presents a kind of fluid-like substance with different particle or mass agglomerations submersed (or hanging) in such substance. Mentioned particle agglomerations (in the frames of this conceptualization) would be different cosmic objects, planets, stars, galaxies, dust, atoms etc. Let us now imagine that such composite cosmic fluid is being mechanically vibrated by certain constant frequency (from an external, presently unknown source of mechanical vibrations). In case of performing a real experiment (just to visualize the concept and make relevant analogies), in a vessel filled with liquid that is mixed with solid particles, by vibrating such vessel we will notice creation of three-dimensional standing wave/s structure, where submersed (and suspended) particles would make higher mass density agglomerations in nodal areas of standing waves. Nodal areas in this case are zones where oscillating velocities are minimal (or zero) and oscillating forces are maximal. If we intentionally introduce a small test particle somewhere in a vicinity of any of such nodal areas with elevated mass density (while vessel filled with liquid and other particles is resonating), we will notice that the test particle will be attracted by the closest nodal zone (or closest particle). Of course, here we are temporarily excluding cases of involvements of possible electromagnetic forces in order to make the situation very simple in its first brainstorming steps. Similar attractive force (in a vicinity of a nodal zone) could be observed in the case of resonant, standing wave oscillations of half-wavelength solid resonators or multiple half-wavelength resonators, known in ultrasonic technology). If external vibrations that are driving mentioned resonators are suddenly switched off, the attractive force/s towards nodal areas will disappear. Now we could conceptualize our universe as an equivalent (or analogical) mechanical fluid-like system that is permanently in a state of **very low frequency** resonant and standing wave oscillations. Such standing-waves oscillations are forcing all astronomical objects to take only certain stable nodal positions of the easiest agglomerating areas, which are kind of its space-matrix texture (apart from other linear and rotational motions involved). Placed around such astronomical objects (planets, stars, galaxies...), every test mass would experience only an attractive force (similar like in cases of gravitation). Later, the same initial concept can be upgraded by considering linear and rotational motions (of submersed particles, or astronomical objects) that are again forced to comply with agglomeration rules around global standing-waves nodal areas, complying with the framework of Euler-Lagrange-Hamilton mechanics. **Understanding of mass, as conceptualized here, is indirectly considering that any mass is a storage or modus of energy packing and/or agglomerating (and in the same time kind of “frozen, rotating energy states”).** The problem here could be the fact that we know that between astronomical objects in our universe there is significant “empty space of vacuum states”, and our imaginative fluid-substance (which should mechanically resonate) would have problems regarding performing mechanical vibrations. Again, we would need to understand the specific nature of such fluid-like substance that is a carrier of externally introduced mechanical vibrations on some new innovative way.*

Contemporary physics presented many elaborations to show that ether-type fluids are still not something what could be experimentally confirmed (at least applicable to imaginative, static and stable, universal reference-states based on certain spatial, ether-matrix structure). In mechanics and acoustics, we already know that vacuum cannot be a carrier of mechanical vibrations, and for supporting here introduced concept of gravitation, we really need to have kind of mechanical resonant and standing waves states of our universe. Most probably that some kind of electromagnetic, magnetostrictive and/or electrostrictive coupling nature should also be involved here (in a framework of coupled oscillators) in order to realize penetration of mechanical vibrations through vacuum and empty space states (and certainly vacuum in our universe is not at all an empty space). Anyway, the situation regarding explaining gravitation, as initiated here, could be richer and different compared to old Newton, Kepler and Einstein framework, since none of them is explaining why gravitation is only manifesting as an attractive force. In the same time, we know that standing-waves mechanical resonators are easily showing existence of such (only) attractive forces in their nodal areas, and by analogy, we could make predictions regarding what behind force of gravitation should be. The remaining question to answer here would be where and what the source of mentioned vibrations is? **Since all constituents of our universe are mutually connected and interacting, as well as in permanent relative motions, and we know that matter-waves are associated to mass motions, this should be an important element of the answer regarding origin of mentioned intrinsic vibrations and their standing waves (in the context of understanding the nature of gravitation).** Einstein's General Relativity Theory is already explaining gravitation from the point of view of specific space and fields' geometry-related modifications and deformations, taking this as a fact, not speculating (as here) that specific space-matrix texture could be a consequence of complex resonating, standing wave/s formations. Since everything what exists in our Universe is anyway mutually cross-linked, coupled and united (in some cases most probably without our best and full knowledge about it), any new theory about Gravitation should take into account electromagnetic and other forces coupled with gravitation. Of course, the ordinary Newton (static) gravitation force is for many orders of magnitude weaker than all other forces (electromagnetic, nuclear...), compared on the same scale, making that we usually neglect interactions between gravitation and other fields. If the concept proposed here has enough realistic grounds, this would be a breakthrough in novel and better understanding of gravitation (maybe also applicable to other forces like electromagnetic ones). Another contemporary field unification theory (which is going much deeper and wider in conceptualizing a multi-dimensional space with its elementary and vibrating building blocks that are taking forms of strings and membranes) that could be in some ways familiar to here introduced concepts is the Superstrings or M-theory (which is still evolving and searching for its best foundations). We should not forget that any new concept of gravitation should be simple, elegant and well integrated into remaining chapters of physics that are already working well, and some attempts in creating such modeling will be made later.